Sunday, 17. May 2020 14:56

Sets $G=(N,A)$, where $N\in\{F\cup H\cup S\}$ (set of nodes)

$V=\{\text{Truck, Transporter}\}$

Decision Variables

"Should a hub be built at a location $i\in H$?"

$$y_i = \begin{cases} 1 & | \text{ hub is built at } i\in H \\ 0 & | \text{ hub is not built at } i\in H \end{cases}$$

→ possible: define $y_i=0 \quad \forall i\notin H$

Parameters

node-based parameters

d_i : demand at node $i\in N$, $i\notin S \Rightarrow d_i=0$

s_i : supply at node $i\in N$, $i\notin F \Rightarrow s_i=0$

c_i^f : fixed cost of operating $i\in N$, $i\notin H \Rightarrow c_i^f=0$

c_i^h : handling cost per log at $i\in N$, $i\notin H \Rightarrow c_i^h=0$

p : price per log delivered (no subscript here, has to be added in the objective function!)

vehicle- and arc-based parameters: I don't think this becomes important in any of the functions

cap_v : capacity of vehicle $v\in V$, where $cap_{Truck} > cap_{Trans}$

$c_{ij,v}^t$: transport cost for vehicle $v\in V$ per log on arc $(i,j)\in A$

Variables

x_{ij} : number of logs transported on arc $(i,j)\in A$

$u_{ij,v}$: decision: $v\in V$ used on arc $(i,j)\in A$, $u_{ij,v}\in\{1,0\}$, $\forall v\in V$

Objective: maximise:

$\sum_{j\in N^S} p\cdot x_{ij}$ − (total revenue)

(we can add y_i, but it already follows from constraints)

$\sum_{i\in N^H}\left[\sum_{j\in N}(c_i x_{ji})\right]$ − (handling cost)

$-\sum_{i\in N^H} y_i c_i^f$ − (fixed cost for hubs)

$-\sum_{(i,j)\in A}\left[x_{ij}\sum_{v\in V}\left[c_{ij,v}^t u_{ij,v}\right]\right]$ (transportation cost)

ass 1: no back-transfer (from sawmills, to forests)

ass 2: hubs cannot store logs; handling costs are incurred for every log passing a hub

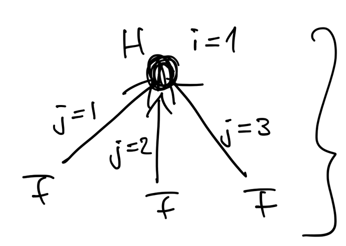
fixed costs : $\sum_{i\in N^H} y_i c_i^f$

transport costs

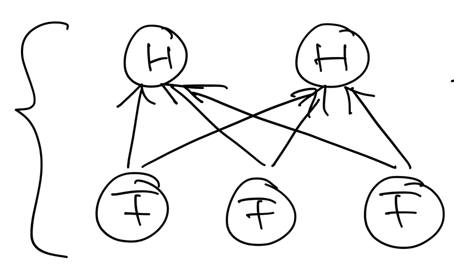
alt: $p\,x_{ij}\,I_{\{j\in N^S\}}$ → $I_{condition} = \begin{cases} 1 & | \text{ condition} \\ 0 & | \text{ } \overline{\text{condition}} \end{cases}$

$j\in N^S \Rightarrow I=1$ else $I=0$

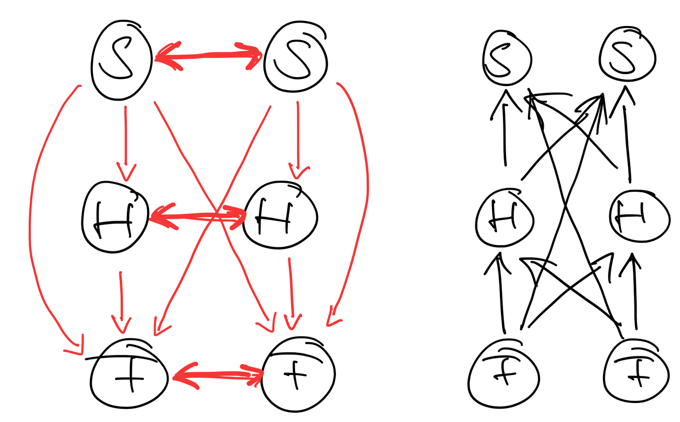
→ easy case: hubs can not store logs



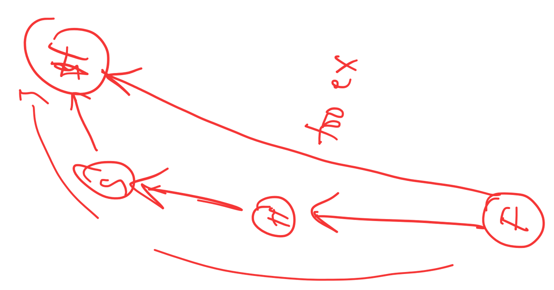
$\sum_{j\in N} c_i x_{ji}$ X_i



$\sum_{i\in N^H} X_i$

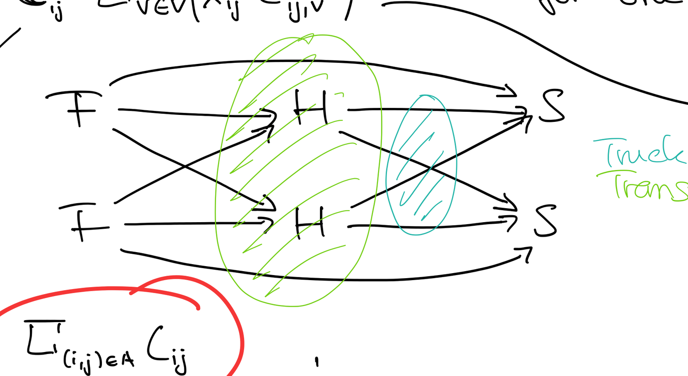


are these arcs allowed?



$i \longrightarrow N_j$

$C_{ij} = \sum_{v\in V}(x_{ij}\, c_{ij,v}^t)$ ← for one specific arc $(i,j)\in A$



$\sum_{(i,j)\in A} C_{ij}$

→ $\sum_{v\in V}\left[x_{ij}\, c_{ij,v}^t\, u_{ij,v}\right] = x_{ij}\sum_{v\in V}\left[c_{ij,v}^t\, u_{ij,v}\right]$

→ $(x_{ij}\, c_{ij,Truck}^t) + x_{ij}\, c_{ij,Trans}^t$

either

$y_i=0 \Rightarrow \sum_{j\in N} x_{ij} = \sum_{j\in N} x_{ji} = 0$

Constraints

FLOW CONSERVATION CONSTRAINTS

all params ≥ 0

$x_{ij} \leq \sum_{v\in V}\left[u_{ij,v}\, cap_v\right]$, $(i,j)\in A$

$\sum_{i\in N^H}(1-y_i)x_{ij}=0$ FROM HUB

| if hub is built $\Rightarrow y_i=1 \Rightarrow$ always fulfilled
| if hub not built $\Rightarrow (1-y_i)=1$
$\Rightarrow$ only fulfilled if $x_{ij}=0$

$\sum_{i\in N^H}(1-y_i)x_{ji}=0$ TO HUB

$\Rightarrow \sum_{i\in N^H}\left[(1-y_i)(x_{ji}+x_{ij})\right]=0$

hub is built $\Rightarrow$ no constraint on x_{ij}

hub not built $\Rightarrow x_{ij}$

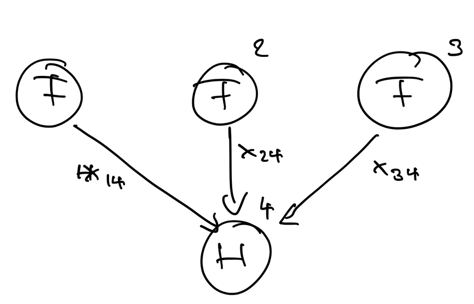
Trucks cannot reach forest-nodes: $\sum_{i\in N^F}\left[u_{ij,Truck}+u_{ji,Truck}\right]=0$

Logs cannot be transported between two hubs of the same type:

$x_{ij}=0$ if i and j are in the same set

EXTENSIONS

- additional parameter: p_i : number of forest areas a hub location $i\in N^H$ can take logs from
- constraint: $\sum_{j\in N^F} I_{\{x_{ji}>0\}} \leq p_i$, $\forall i\in N^H$,



$p_4=2$ $\quad I_{\{x_{i4}>0\}}$

x_{14}	x_{24}	x_{34}	Σ		?	
10	0	0	10	✓	1	✓
0	0	1	1	✓	1	✓
~~7~~	~~1~~	~~1~~	9	✗	3	✗
~~100~~	~~4~~	~~100~~	204	✗	3	✗
0	5	100	105	✓	2	✓

→ $? \leq 2$

$?_i \leq p_i$

$I_{\{x_{14}>0\}}$	$I_{\{x_{24}>0\}}$	$I_{\{x_{34}>0\}}$	$\Sigma = ?$
1	0	0	1
0	0	1	1
1	1	1	3
1	1	1	3
0	1	1	2

→ $\sum_{j\in N^F} I_{\{x_{j4}>0\}} \quad \forall i\in N^H$

→ $\sum_{j\in N^F} I_{\{x_{ji}>0\}} \leq p_i$, $\forall i\in N^H$