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the submission should be in pdf -1P

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Solution submitted by:

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Task description

The company **LGLog** is active in forest and wood management, transporting logs from the forest to sawmills. The company operates in multiple forests and owns several sawmills. The forest can only be reached by very expensive and specialized forest transporters with high fuel consumption. Therefore, the company wants to build transshipment hubs near the harvested forest areas to move the logs to cheaper and more fuel efficient trucks that will transport the logs to the sawmills. The company has identified several potential transshipment hubs. Your job is to decide which preselected locations should be chosen.

Each transshipment hub has a fixed cost for being open. Vehicles have travel costs depending on the amount of logs they are carrying, which are arc dependent. Each type of vehicle has a capacity on the number of logs it can carry. Assume that the truck can handle more logs than a forest transporter. Additionally there is a uniform handling cost per log at the hubs. You can assume there are enough vehicles of both types available to carry all of the demand, but multiple vehicles of the same type may not travel on the same arc at the same time.

Assume you are aware of the demand at each sawmill and the supply in terms of logs at each forest. Note that these two quantities need not be balanced. LGLog is paid a fixed price per log delivered to any sawmill. Every hub can be reached from any forest and every sawmill can be reached from any hub with different distances in between them. It is also possible to travel directly from the forst to a sawmill. Transporting logs between hubs is prohibited. Furthermore, no logs are transported between sawmills nor between forest areas.

Model

Model components

Set

$$\text{Directed Graph: } G = (N, A)$$
$$\text{Nodes: } N = N^S \cup N^F \cup N^T$$
$$N^S\text{: sawmills}$$
$$N^F\text{: forests}$$
$$N^H\text{: potential transshipme}$$
$$\text{Vehicles: } V = \{\text{Truck, Transporter}\}$$

Definition of A is missing, since for example an arc from a hub to a forest is not included **-1P**

Decision Variable

$$x_{ij} : \text{number of logs transported on arc } (i, j) \in A$$

Parameters

Node-based parameters:

$$\begin{aligned} d_i &: \ \text{demand at node } i \in N^S \\ s_i &: \ \text{supply at node } i \in N^F \\ c_i^o &: \ \text{fixed operating cost for a hub i } i \in N^H \end{aligned}$$

When any of the above parameter is not not defined for a subset of N, it takes the value 0.

Vehicle- and arc-based parameters:

$$\begin{aligned} \text{cap}_v &: \text{capacity (max number of logs) of vehicle } v \in V \\ & (\text{cap}_{\text{Truck}} > \text{cap}_{\text{Transporter}}) \\ c_{ij,v}^t &: \text{travel cost per log of vehicle } v \in V \text{ on arc } (i, j) \in A \end{aligned}$$

It is assumed that, while trucks can not reach forests, transporters can in theory be used to transport logs between hubs and sawmills (or directly between forests and sawmills). See also the variable $u_{ij,v}$ and the third constraint. Only one of the vehicle types can travel an arc at the same time (a truck and a transporter can not travel the same arc).

> *Note: We thought about modelling a scenario where a truck and a transporter can travel on the same arc (e.g. between hubs and sawmills). We decided for the scenario described above.*

Other variables:

$$\begin{aligned} p &: \text{price per log delivered} \\ u_{ij,v} &: \text{vehicle of type } v \text{ used on arc } (i, j) \in A \\ & u_{ij,v} \in \{0, 1\}\ , \ \forall v \in V \\ c^h &: \ \text{handling cost per log} \end{aligned}$$

for each arc there is only one type given, therefore, the variables u_ijv is not neccessary.

p and c^h are parameters, not variables

Handling costs are assumed to occur whenever a log arrives at a hub.

Objective function:

$$\max \Big(p \cdot \sum_{i \in N^S} \sum_{j \in N} x_{ji}$$

each log delivered to a sawmill.

$$-c^h \cdot \sum_{i \in N^H} \sum_{j \in N} x_{ji}$$

occur when logs enter a hub.

$$-\boxed{\sum_{i \in N^H} \left(I_{\{x_{ji}>0\}} c_i^o\right)}$$

| Fixed costs of operating hubs occur only at hubs that h

$$-\boxed{\sum_{(i,j) \in A} \Big[x_{ij} \sum_{v \in V} (c^t_{ij,v} \cdot u_{ij,v})\Big]}\Big)$$

| Transport costs occur for every log through arc (i,j), d

this is not a linear formulation, instead you need a binary variable to indicate whether the hub is operating or not
-1P for the missing variable, and
-0.5P for this non-linear formulation

non-linear formulation again
-0.5P

-2P for the constraint for the activation of hub

these two constraints can be avoided by a better definition of arcs

aints:

$$u_{ij,v} \in \{0,1\} \quad | \quad \text{binarity of variable}$$

$$0 \le x_{ij} \le (u_{ij,v} \cdot cap_v) \quad | \quad \forall v \in V \; - \; \text{Non-negativi} \ldots \text{pac}$$

$$\sum_{i \in N^F} (u_{ij,v} + u_{ji,v} = 0) \quad | \quad \forall v \in \text{Trucks} \; - \; \text{Trucks} \ldots \text{s.}$$

the capacity constraint is ok, u_ijv can be avoided

$$\{i,j\} \underset{\text{exactly one}}{\in} \{N^S, N^F, N^H\} \Rightarrow x_{ij} = 0 \quad | \quad \text{Transporting logs between two nodes of the san}$$

?

$x_{ij} = 0 \quad \forall i,j \in \{N^S, N^F, N^H\}, i \neq j$

Flow conservation constraints:

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(i,j) \in A} x_{ji} \le s_i - d_i \quad | \quad \forall i \in \{N^F \cup N^S\}$$

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(i,j) \in A} x_{ji} = 0 \quad | \quad \forall i \in N^T$$

the flow conservation is ok

Since parameters take on the value 0 if they are not defined for a node, this statement is equivalent to the formulation with three separate lines. Since the total supply may differ from the total demand, the equality sign was replaced by inequality; the inequality represents scenarios where not all supply may be transported away from forests, or not all demand from sawmills may be met.

Extension

Modify the model to support the following problem extension:

Due to limited storage capacity, transshipment hubs may only transship logs from a limited number of forest areas. That is, each transshipment hub i may only take logs from at most pi forest areas.

Additional Parameter:

$$p_i \;: \text{max number of forest from which a hu}$$

Additional Constraint:

$$\sum_{j \in N^F} \boxed{I_{\{x_{ij}>0\}}} \le p_i$$

what is I? Instead you need a binary variable to define the whether one arc is used or not **-1P** for the missing variable definition, for example, z_ij, **-1P** for this constraint, **-2P** for an additional constraint for the relationship of x_ij and z_ij (using big-M)

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10 points for bonus

Task description

The following is based on problem 12.23 from "Model Building in Mathematical Programming, Fourth Edition" by H. Paul Williams (John Wiley & Sons, 2002). A small milk processing company is committed to **collecting milk from 20 farms and taking it back to the depot for processing**. The company has **one tanker truck with a capacity for carrying 80 000 liters** of milk. **Some of the farms are small and need a collection only every other day**. The **other farms need a collection every day**. The **route of the truck starts and ends at a depot**, and **it is possible to travel between all nodes**. The goal is to find the optimal route for the tanker truck on each day, bearing in mind that it has to:

1. Visit all the 'every day' farms,
2. Visit some of the 'every other day' farms, and
3. Work within its capacity.

On alternate days it must again visit the 'every day' farms, but also visit the 'every other day' farms not visited on the previous day.

Model

Sets

$$\text{Directed Graph: } G = (V, A),\ \ V = F \cup 0$$
$$\text{Farms: } F = \{1, \dots, 20\}$$
$$\text{Depot: } \{0\}$$
$$\text{Arcs: } A = V \times V \quad (\textit{it is possible to travel between all nodes})$$

Parameters

$$C : \text{capacity of the tanker truck } (C = 80,000)$$
$$d = \{1, 2\}$$
$$s_i : \text{Milk supply of farm } i \in F$$
$$c_{ij} : \text{Cost of traveling arc } (i, j) \in A$$
$$f_i = \begin{cases} 2 & | \quad \text{if the farm } i \text{ has to be visited every day} \\ 1 & | \quad \text{if the farm } i \text{ has to be visited every second day} \end{cases}$$

Decision Variables

$$x_{ijd} = \begin{cases} 1 & | \quad \text{if the truck travels directly from node i to j on day d} \\ 0 & | \quad \text{otherwise} \end{cases}$$

Objective

$$\min \sum_{i \in V} \sum_{j \in V} [c_{ij} \sum_{d=\{1,2\}} x_{ijd}]$$

Minimise the transport costs over both days (that is, minimise the sum of the transport costs on each alternating day).

Constraints

Each farm is visited accordingly:

Bigger farms have to be visited once every day, smaller farms only every second day.

$$\sum_{d=\{1,2\}} \sum_{j \in V} x_{jid} = f_i \quad | \, \forall i \in F$$

Vehicle flow

Whenever a farms is visited by the truck on a specific day, the truck also has to leave that farm:

$$\sum_{j \in V} x_{ijd} = \sum_{j \in V} x_{jid} \quad | \, \forall i \in F$$

Vehicle capacity

The sum of the supplies of all nodes visited on a certain day is not more than the capacity of the truck:

$$\sum_{i \in V} [s_i \sum_{j \in V} x_{ijd}] \leq C \quad | \, \forall d$$

Make sure the depot is visited

On each day, the the truck travels on exactly one arc to and exactly one arc away from the depot:

$$\sum_{j \in F} x_{0jd} = \sum_{j \in F} x_{j0d} = 1 \quad | \, \forall d$$

this constraint means that each farm is visited directly from depot each day once. This is wrong -2P

Make sure there are no cycles without the depot (SEC)

$$\sum_{i,j \in R} x_{ijd} < |R| - 1 \quad | \, \forall d, \; \forall \varnothing \subset R \subseteq F \backslash \{0\}$$

We assumed that the tanker truck can make only one tour per day, and that it can not re-visit nodes.

constraint of definition of variables is missing -1P

Extension

Assume there is a matrix of travel times between all farms and the depot and a maximum time for the route of the truck. Extend your mathematical model to support these situation. (Ignore the time it takes to load the milk into the truck at each node)

Additional parameter:

$$t_{ij} : \text{time it takes to travel arc } (i,j) \in A$$
$$T : \text{maximum time for the route of the truck}$$

Additional constraint:

Sum of the times for all arcs travelled on a route must be smaller then the overall time allowed for the route:

$$\sum_{i \in V} \sum_{j \in V} t_{ij} x_{ijd} \leq T \quad | \quad \forall d, \forall i \in V$$

Bonus extension

Assume now that there are **multiple vehicles** available to serve the customers. **Each vehicle has a capacity that may not be exceeded**, but the **travel times for the vehicles are the same**. Adjust the above model to support this case.

Adjusted Parameters

$$K : \text{number of vehicles}$$
$$\kappa_k : \text{capacity of vehicle for } k = 1, \ldots, K \quad \text{(replaces C)}$$

Adjusted Decision Variable

$$x_{ijdk} = \begin{cases} 1 & | \quad \text{if truck k travels directly from node i to j on day d} \\ 0 & | \quad \text{otherwise} \end{cases}$$

Adjusted Objective

Minimise the cost of travelling, which is the sum of the costs of each arc for every truck and day that arc is travelled.

$$\min \sum_{i \in V} \sum_{j \in V} \cancel{[} c_{ij} \sum_{k=\{1,\ldots,K\}} \sum_{d=\{1,2\}} x_{ijdk} \cancel{]}$$

Adjusted Constraints

Each farm is visited accordingly:

Bigger farms have to be visited once every day, smaller farms only every second day; a farm can only be visited by one vehicle each day.

$$\sum_{k=\{1,...,K\}} \sum_{d=\{1,2\}} \sum_{j \in V} x_{jidk} = f_i \quad | \, \forall i \in F$$

Vehicle flow

Whenever a farms is visited by the truck on a specific day, the truck also has to leave that farm:

$$\sum_{j \in V} x_{ijdk} = \sum_{j \in V} x_{jidk} \quad | \, \forall i \in F, \; \forall k \in 1, \ldots K$$

Vehicle capacity

The sum of the supplies of all nodes visited on a certain day is not more than the capacity of the truck:

$$\sum_{i \in V} [s_i \sum_{j \in V} x_{ijdk}] \leq \kappa_k \quad | \, \forall d, \; \forall k \in 1, \ldots K$$

Make sure the depot is visited

On each day, the the truck travels on exactly one arc to and exactly one arc away from the depot:

$$\boxed{\sum_{j \in F} x_{0jdk} = \sum_{j \in F} x_{j0dk} = 1 \quad | \quad \forall d, \; \forall k \in 1, \ldots K}$$

Make sure there are no cycles without the depot (SEC)

$$\sum_{i,j \in R} x_{ijdk} < |R| - 1 \quad | \, \forall d, \; \forall k, \; \forall \varnothing \subset R \subseteq F \backslash \{0\}$$

Constraint about using all vehicles is missing -2P