数学基础

指数

- $X^aX^b = X^{a+b}$
- $\frac{X^a}{X^b} = X^{a-b}$
- $(X^a)^b = X^{ab}$
- $X^a + X^a = 2X^a$
- $2^a + 2^a = 2^{a+1}$

对数

一般，以 2 为底数

定义：$X^a = B$, 则当且仅当$log_xB = a$

- $log_AB = \frac{log_cB}{log_cA}$

  证明：

$$\begin{aligned} & C^x = B, C^y = A, A^z = B \\ => \ & C^x = B = A^z = (C^y)^z \\ => \ & C^x = (C^y)^z \\ => \ & log_AB = z = x/y = \frac{log_CB}{log_CA} \end{aligned}$$

- $logAB = logA + logB$
- $logA/B = logA - logB$
- $log(A^B) = B\,logA$
- $logX < X$(对所有的$X > 0$成立)
- $log1 = 0, log2 = 1, log1024 = 10, log1048576 = 20$

级数

- $\sum_{i=0}^{N} 2^i = 2^{N+1} - 1$
- $\sum_{i=0}^{N} A^i = \frac{A^{N+1}-1}{A-1}$, 若$0 < A < 1$, 则 $\sum_{i=0}^{N} A^i \leq \frac{1}{1-A}$

  证明：

$$S=\sum_{i=1}^{+\infty}A^i,(0<A<1)$$

$$S=1+A+A^2+A^3+...$$

$$AS=A+A^2+A^3+A^4+...$$

$$将两式相减,S-AS=1+(A^{+\infty}->0)$$

$$当S->+\infty,S=\frac{1}{1-A},$$

$$因N\leq+\infty,故\sum_{i=0}^{N}A^i=S_N\leq S=\frac{1}{1-A},$$

- $\sum_{i=1}^{+\infty}i/2^i$ = 2
  证明：

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...$$

$$2*S,得2S=1+\frac{2}{2^1}+\frac{3}{2^2}+\frac{4}{2^3}+...$$

$$两式相减,S=1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+...$$

$$等差数列,S=2$$

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