

数学基础

指数

- $X^a X^b = X^{a+b}$
- $\frac{X^a}{X^b} = X^{a-b}$
- $(X^a)^b = X^{ab}$
- $X^a + X^a = 2X^a$
- $2^a + 2^a = 2^{a+1}$

对数

一般, 以 2 为底数

定义: $X^a = B$, 则当且仅当 $\log_x B = a$

- $\log_A B = \frac{\log_c B}{\log_c A}$

证明:

$$\begin{aligned} C^x &= B, C^y = A, A^z = B \\ \Rightarrow C^x &= B = A^z = (C^y)^z \\ \Rightarrow C^x &= (C^y)^z \\ \Rightarrow \log_A B &= z = x/y = \frac{\log_C B}{\log_C A} \end{aligned}$$

- $\log AB = \log A + \log B$
- $\log A/B = \log A - \log B$
- $\log(A^B) = B \log A$
- $\log X < X$ (对所有的 $X > 0$ 成立)
- $\log 1 = 0, \log 2 = 1, \log 1024 = 10, \log 1048576 = 20$

级数

- $\sum_{i=0}^N 2^i = 2^{N+1} - 1$
- $\sum_{i=0}^N A^i = \frac{A^{N+1}-1}{A-1}$, 若 $0 < A < 1$, 则 $\sum_{i=0}^N A^i \leq \frac{1}{1-A}$

证明:

$$S = \sum_{i=1}^{+\infty} A^i, (0 < A < 1)$$

$$S = 1 + A + A^2 + A^3 + \dots$$

$$AS = A + A^2 + A^3 + A^4 + \dots$$

将两式相减, $S - AS = 1 + (A^{+\infty} - > 0)$

$$\text{当 } S \rightarrow +\infty, S = \frac{1}{1-A},$$

$$\text{因 } N \leq +\infty, \text{ 故 } \sum_{i=0}^N A^i = S_N \leq S = \frac{1}{1-A},$$

$$\bullet \sum_{i=1}^{+\infty} i/2^i = 2$$

证明:

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \dots$$

$$2 * S, \text{ 得 } 2S = 1 + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} + \dots$$

$$\text{两式相减, } S = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots$$

等差数列, $S = 2$

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