Homework 1

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Part 1

1. Gather necessary Data from Yahoo

```
library(TTR)
library(quantmod)
```

```
## Warning: package 'quantmod' was built under R version 3.4.4
```

```
## Loading required package: xts
```

```
## Warning: package 'xts' was built under R version 3.4.4
```

```
## Loading required package: zoo
```

```
## Warning: package 'zoo' was built under R version 3.4.4
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Version 0.4-0 included new data defaults. See ?getSymbols.
```

```
library(curl)
```

```
## Warning: package 'curl' was built under R version 3.4.4
```

```
library(jsonlite)
```

```
## Warning: package 'jsonlite' was built under R version 3.4.4
```

```
Bothdata=function(symbol){
  price <<- getSymbols(symbol,src = "yahoo", from='2018-09-10',to='2018-09-11',auto.assign = FALSE)
  option <<-getOptionChain(symbol,Exp ='2018-11-16') # pick any expiration date to make code valid
  }
Bothdata('AAPL') # AAPL is example
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```

```
##
## WARNING: There have been significant changes to Yahoo Finance data.
## Please see the Warning section of '?getSymbols.yahoo' for details.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.yahoo.warning"=FALSE).
```

```r
print(price)
```

```
##            AAPL.Open AAPL.High AAPL.Low AAPL.Close AAPL.Volume
## 2018-09-10    220.95    221.85   216.47     218.33    39516500
##            AAPL.Adjusted
## 2018-09-10        218.33
```

```r
head(option)
```

```
## $calls
##                    Strike   Last         Chg    Bid    Ask  Vol    OI
## AAPL181116C00095000     95 130.85  0.00000000 128.85 129.45   10    17
## AAPL181116C00100000    100 120.28  0.00000000 126.20 130.20    2     7
## AAPL181116C00110000    110 113.60  0.00000000 111.25 112.70   10    13
## AAPL181116C00115000    115  92.50  0.00000000  93.05  93.80    2    15
## AAPL181116C00120000    120 104.75  0.00000000 107.75 110.35    3   149
## AAPL181116C00125000    125 103.43  0.00000000 101.40 105.40    1    21
## AAPL181116C00130000    130  79.70  0.00000000  78.20  78.95    2   174
## AAPL181116C00135000    135  81.50 16.75000000  81.55  82.35    1    23
## AAPL181116C00140000    140  86.43  0.00000000  84.10  84.65    2   505
## AAPL181116C00145000    145  73.38 10.42999650  73.35  74.20    1    72
## AAPL181116C00150000    150  74.64  3.29000100  74.20  74.80    2  1418
## AAPL181116C00155000    155  74.06  0.00000000  69.25  69.85    2   327
## AAPL181116C00160000    160  61.60  0.00000000  64.30  64.85    2   925
## AAPL181116C00165000    165  59.90 -3.65999980  59.40  59.95   30  8676
## AAPL181116C00170000    170  54.66 -3.29000100  54.50  55.00   13  1432
## AAPL181116C00175000    175  50.59 -0.68999860  49.60  50.15    7  3114
## AAPL181116C00180000    180  44.62 -3.00000000  44.75  45.25   30  3189
## AAPL181116C00185000    185  40.12 -3.08000180  39.95  40.40    9  2607
## AAPL181116C00190000    190  35.41 -1.59000020  35.20  35.65    6  5773
## AAPL181116C00195000    195  30.74 -1.75000190  30.60  31.00   33  5110
## AAPL181116C00200000    200  26.33 -2.52000050  26.20  26.50  373 21196
## AAPL181116C00205000    205  22.25 -1.87000080  22.00  22.20   65  2242
## AAPL181116C00210000    210  18.15 -2.30000110  18.15  18.25  268  9992
## AAPL181116C00215000    215  14.49 -2.25000000  14.55  14.70  549  6470
## AAPL181116C00220000    220  11.45 -1.94000050  11.40  11.55 1645 12303
## AAPL181116C00225000    225   8.70 -1.75000000   8.70   8.80 3700  9183
## AAPL181116C00230000    230   6.45 -1.50000000   6.45   6.55 2382 12458
## AAPL181116C00235000    235   4.73 -1.17000010   4.65   4.75 1003  9003
## AAPL181116C00240000    240   3.35 -0.95000030   3.25   3.35 1084  7912
## AAPL181116C00245000    245   2.32 -0.73000000   2.29   2.34  260  4014
## AAPL181116C00250000    250   1.62 -0.53000010   1.59   1.63  643  5666
## AAPL181116C00255000    255   1.12 -0.40999997   1.09   1.13  176  1368
## AAPL181116C00260000    260   0.78 -0.31000006   0.77   0.80  276  1825
## AAPL181116C00265000    265   0.55 -0.19999999   0.54   0.57   91  1244
## AAPL181116C00270000    270   0.41 -0.13000003   0.39   0.41   56  1041
## AAPL181116C00275000    275   0.31 -0.13000000   0.28   0.30   81  1020
## AAPL181116C00280000    280   0.23 -0.07000001   0.21   0.23   61   653
## AAPL181116C00285000    285   0.17 -0.08000000   0.15   0.17    6   513
## AAPL181116C00290000    290   0.15 -0.03999999   0.11   0.13   45   569
## AAPL181116C00295000    295   0.13  0.00000000   0.09   0.10    1   348
## AAPL181116C00300000    300   0.07 -0.02000000   0.06   0.08    2  4518
## AAPL181116C00305000    305   0.06  0.00000000   0.04   0.05  293  3510
## AAPL181116C00310000    310   0.04 -0.01000000   0.03   0.04   20  5255
## AAPL181116C00315000    315   0.03  0.00000000   0.01   0.03  249  1003
## AAPL181116C00320000    320   0.02  0.00000000   0.00   0.03    1  1253
## AAPL181116C00325000    325   0.03  0.00000000   0.00   0.03   10   117
## AAPL181116C00330000    330   0.02  0.00000000   0.00   0.03   55   338
## AAPL181116C00340000    340   0.06  0.00000000   0.00   0.06   25    61
## AAPL181116C00345000    345   0.01  0.00000000   0.00   0.04  250   219
## AAPL181116C00350000    350   0.02  0.00000000   0.00   0.04    5   128
## 
## $puts
##                     Strike  Last         Chg   Bid   Ask  Vol    OI
## AAPL181116P00095000     95  0.01 -0.02000000  0.00  0.02   10   291
## AAPL181116P00105000    105  0.01  0.00000000  0.01  0.02   20   345
## AAPL181116P00110000    110  0.03  0.00000000  0.00  0.04   91   727
## AAPL181116P00115000    115  0.03  0.00000000  0.00  0.03   91  6880
## AAPL181116P00120000    120  0.04  0.00000000  0.00  0.03 1300  2214
## AAPL181116P00125000    125  0.04 -0.01000000  0.01  0.04  250  5188
## AAPL181116P00130000    130  0.06 -0.03000000  0.03  0.07   10  1765
## AAPL181116P00135000    135  0.13  0.00000000  0.07  0.09   10  2076
## AAPL181116P00140000    140  0.12  0.00000000  0.10  0.12    1  4017
## AAPL181116P00145000    145  0.16  0.00000000  0.12  0.14  576  2406
## AAPL181116P00150000    150  0.17 -0.02000000  0.14  0.17    2  4147
## AAPL181116P00155000    155  0.21  0.00000000  0.20  0.21  213  1976
## AAPL181116P00160000    160  0.26  0.00000000  0.25  0.27   38  4474
## AAPL181116P00165000    165  0.33  0.00000000  0.31  0.33   17 11002
## AAPL181116P00170000    170  0.39 -0.02000001  0.38  0.41  284 13585
## AAPL181116P00175000    175  0.49  0.00000000  0.49  0.51   65  4456
## AAPL181116P00180000    180  0.62  0.00000000  0.62  0.64   24  7963
## AAPL181116P00185000    185  0.82  0.02999997  0.81  0.83   60  6231
## AAPL181116P00190000    190  1.08  0.04000008  1.07  1.10   30 11290
## AAPL181116P00195000    195  1.48  0.08000004  1.44  1.48   71  2623
## AAPL181116P00200000    200  2.10  0.25999987  1.99  2.05  160 17224
## AAPL181116P00205000    205  2.80  0.26000000  2.78  2.84  182  8490
## AAPL181116P00210000    210  3.90  0.40000010  3.85  3.95  368 15654
## AAPL181116P00215000    215  5.40  0.59000015  5.30  5.40  641  5650
## AAPL181116P00220000    220  7.30  0.80000020  7.20  7.30  353  6473
## AAPL181116P00225000    225  9.57  0.97999954  9.50  9.65  401  5152
## AAPL181116P00230000    230 12.30  0.80000020 12.30 12.40   73  3688
## AAPL181116P00235000    235 15.55  1.10000040 15.55 15.65   40  1306
## AAPL181116P00240000    240 19.05  0.96999930 19.15 19.30   10  1346
## AAPL181116P00245000    245 23.15  1.14999960 23.15 23.40   47   721
## AAPL181116P00250000    250 27.45  1.45000080 27.45 27.60   25   393
## AAPL181116P00255000    255 32.35  2.64999770 31.85 32.30   16    47
## AAPL181116P00260000    260 32.15  0.00000000 38.50 39.80    8    31
## AAPL181116P00265000    265 49.35  0.00000000 48.70 49.10    6     0
## AAPL181116P00270000    270 42.60  0.00000000 42.55 44.25    4     7
## AAPL181116P00275000    275 48.34  0.00000000 52.90 54.30    5    53
## AAPL181116P00280000    280 61.40  0.00000000 56.00 56.55    2     2
## AAPL181116P00300000    300 76.75  0.00000000 75.95 76.55    2     2
```

2. Download both Option and Equity data for AMZN,SPY and VIX.

```r
#Get first Option Maturity Data for Amazon,SPY,VIX(Expire in one month)
Aoption=getOptionChain("AMZN",Exp = '2018-10-19',src = "yahoo")
SPoption=getOptionChain("SPY",src = "yahoo", Exp = '2018-10-19')
Voption=getOptionChain("^VIX",src = "yahoo",Exp ="2018-10-17" )
```

```r
#Get second and third Option for Amazon,SPY,VIX(Expire in two and three month)
#Amazon
Amdata=function(symbol){
  Aprice1 <<- getSymbols(symbol,src = "yahoo", from='2018-09-13',to='2018-09-14',auto.assign = FALSE)
  Aoption1 <<-getOptionChain(symbol,Exp ='2018-11-02') #expire in two month
  }
Amdata('AMZN')



#SPY
SPdata=function(symbol){
  SPprice1<<-getSymbols(symbol,src = "yahoo", from='2018-09-13',to='2018-09-14',auto.assign = FALSE)
  SPoption1<<-getOptionChain(symbol,Exp ='2018-11-02') #expire in two month
}
SPdata('SPY')


#VIX
Vdata=function(symbol){
  Vprice1<<-getSymbols(symbol,src = "yahoo", from='2018-09-13',to='2018-09-14',auto.assign = FALSE)
  Voption1<<-getOptionChain(symbol,Exp ='2018-11-21') #expire in two month
}
Vdata('^VIX')

#DATA2
#Amazon
Amdata2=function(symbol){
  Aprice2 <<- getSymbols(symbol,src = "yahoo", from='2018-09-14',to='2018-09-15',auto.assign = FALSE)
  Aoption2 <<-getOptionChain(symbol,Exp ='2019-02-15') #expire in five month  ##DUe to some wrong moves
}
Amdata2('AMZN')


#SPY
SPdata2=function(symbol){
  SPprice2<<-getSymbols(symbol,src = "yahoo", from='2018-09-14',to='2018-09-15',auto.assign = FALSE)
  SPoption2<<-getOptionChain(symbol,Exp ='2018-12-31')
}
SPdata2('SPY')

#VIX
Vdata2=function(symbol){
  Vprice2<<-getSymbols(symbol,src = "yahoo", from='2018-09-14',to='2018-09-15',auto.assign = FALSE)
  Voption2<<-getOptionChain(symbol,Exp ='2018-12-19')
}
Vdata2('^VIX')
```

Bonus

```r
multiprice=getSymbols(c('AMZN','SPY'),src = "yahoo", from="2018-09-01", to="2018-09-10")
Adata=data.matrix(AMZN)
Sdata=data.matrix(SPY)
write.csv(Sdata,file = "Sdata.csv")
write.csv(Adata,file = "Adata.csv")
```

3.Three symbols:“AMZN”,“SPY”,“VIX”

“AMZN” is the stock symbol that traded on the Nasdaq which represents Amzon.com lnc. The expiration date is on the third Friday of each month.

“SPY” is the symbol for SPDR S&P 500 ETF Trust. It is the largest exchange-traded fund that traded on NYSE. It aims to track S&P500 stock market index. The expiration date is the

“VIX” is the ticker symbol of CBOE Volatility Index which measures the stock markets’ expectation of volatility, commonly implied by S&P500. Usually, it is published by Chicago Board Option Exchange. The expiration date is on every Wednesday.

4. The undeying equity price at the moment downloaded Day1

Amazon: 1989.87. Time to maturity respectly:36 days,50 days,154 days SPY:290.83. Time to maturity: 36 days,50 days,109 days vix:12.37. Time to maturity: 34 days,69 days,97 days

The undeying equity price at the moment downloaded Day2 Amazon: 1970.19. Time to maturity respectly:35 days,49 days,153 days SPY:290.88. Time to maturity: 35 days,49 days,108 days vix:12.07. Time to maturity: 34 days,68 days,96 days

Part 2 Analysis of the data

5.Black-Scholes formula

```
blackscholes = function(S0, Sigma, t, K, r, optionType = 'call'){
  d1 = (log(S0/K) + (r + (Sigma^2)/2) * t)/(Sigma * sqrt(t))
  d2 = d1 - Sigma * sqrt(t)
  if(optionType == 'call'){
    call = S0 * pnorm(d1) - K * exp(-r * t) * pnorm(d2)
    return(call)
  }else if(optionType == 'put'){
    put = K * exp(-r * t) * pnorm(-d2) - S0 * pnorm(-d1)
    return(put)
  }
}
```

6. Implement Bisection method

```
vrange=c(0,1)
t1=0
bisection=function(f, vrange, tol = 1e-6){
  while((vrange[2]-vrange[1]) >= tol){
    x=0.5*(vrange[1]+vrange[2])
if(f(x)*f(vrange[2])>0)
  {
  vrange[2]=x
}else if(f(x)*f(vrange[2])<0){
  vrange[1]=x
}
  t1=1+t1
}
return(x)
}
```

Bonus: Implement the Secent method (Bonus)

```
t2=0
secant=function(g, vrange,tol = 1e-6){
difference=vrange[2]-vrange[1]
repeat{
x=vrange[2]-(vrange[2]-vrange[1])/(g(vrange[2])-g(vrange[1]))*g(vrange[2])
vrange[1]=vrange[2]
vrange[2]=x
difference=vrange[2]-vrange[1]
t2=1+t2
if(abs(difference)>=tol) break
}
return(x)
}
```

calculate implied vol corresponding to each strike price for Amazon Options by using Bisection method

```
#calculate implied vol for first Amazon Option, mature at 10-19-2018, for each strike price
# Call
Aimpcall=Aimpcall1=Aimpcall2=Aimput=Aimput1=Aimput2=c()
Aoptioncall=Aoption$calls$Strike

for(i in 1:length(Aoptioncall)){
  Amvolc = function(sigma){
    blackscholes(1989.87, sigma, 36/252,Aoptioncall[i], 0.0216, 'call') - 0.5 * (Aoption$calls$Bid[i] +
  }
  Aimpcall[i] = bisection(Amvolc, vrange, 1e-6)
}

#Put
Aoptionput=Aoption$puts$Strike
for(i in 1:length(Aoptionput)){
  Amvolp = function(sigma){
    blackscholes(1989.87, sigma, 36/252,Aoptionput[i], 0.0216, 'put') - 0.5 * (Aoption$puts$Bid[i] + Ao
  }
  Aimput[i] = bisection(Amvolp, vrange, 1e-6)
}


#calculate implied vol for second Amazon Option, mature at 11-02-2018, for each strike price
# Call

Aoption1call=Aoption1$calls$Strike

for(i in 1:length(Aoption1call)){
  Amvolc1 = function(sigma){
    blackscholes(1989.87, sigma, 50/252,Aoption1call[i], 0.0216, 'call') - 0.5 * (Aoption1$calls$Bid[i]
  }
  Aimpcall1[i] = bisection(Amvolc1, vrange, 1e-6)
}

#Put
Aoption1put=Aoption1$puts$Strike
for(i in 1:length(Aoption1put)){
  Amvolp1 = function(sigma){
    blackscholes(1989.87, sigma, 50/252,Aoption1put[i], 0.0216, 'put') - 0.5 * (Aoption1$puts$Bid[i] + A
  }
  Aimput1[i] = bisection(Amvolp1, vrange, 1e-6)
}


#calculate implied vol for third Amazon Option, mature at 02-15-2018, for each strike price
# Call
Aoption2call=Aoption2$calls$Strike

for(i in 1:length(Aoption2call)){
  Amvolc2 = function(sigma){
    blackscholes(1989.87, sigma, 154/252,Aoption2call[i], 0.0216, 'call') - 0.5 * (Aoption2$calls$Bid[i]
  }
  Aimpcall2[i] = bisection(Amvolc2, vrange, 1e-6)
}

#Put
Aoption2put=Aoption2$puts$Strike
for(i in 1:length(Aoption2put)){
  Amvolp2 = function(sigma){
    blackscholes(1989.87, sigma, 154/252,Aoption2put[i], 0.0216, 'put') - 0.5 * (Aoption2$puts$Bid[i] +
  }
  Aimput2[i] = bisection(Amvolp2, vrange, 1e-6)
}
```

calculate implied vol corresponding to each strike price for SPY Options

```
#first SPY Option, mature at 10-19-2018
# Call
Simpcall=Simpcall1=Simpcall2=Simput=Simput1=Simput2=c()
SPoptioncall=SPoption$calls$Strike

for(i in 1:length(SPoptioncall)){
  Simvolc = function(sigma){
    blackscholes(29.83, sigma, 36/252,SPoptioncall[i], 0.0216, 'call') - 0.5 * (SPoption$calls$Bid[i] +
  }
  Simpcall[i] = bisection(Simvolc, vrange, 1e-6)
}

#Put
SPoptionput=SPoption$puts$Strike
for(i in 1:length(SPoptionput)){
  Smvolp = function(sigma){
    blackscholes(290.83, sigma, 36/252,SPoptionput[i], 0.0216, 'put') - 0.5 * (SPoption$puts$Bid[i] + SP
  }
  Simput[i] = bisection(Smvolp, vrange, 1e-6)
}


#calculate implied vol for second SPY Option, mature at 11-02-2018, for each strike price
# Call

SPoption1call=SPoption1$calls$Strike
```

```
for(i in 1:length(SPoption1call)){
  Simvolc1 = function(sigma){
    blackscholes(290.83, sigma, 50/252,SPoption1call[i], 0.0216, 'call') - 0.5 * (SPoption1$calls$Bid[i
  }
  Simpcall1[i] = bisection(Simvolc1, vrange, 1e-6)
}

#Put
SPoption1put=SPoption1$puts$Strike
for(i in 1:length(SPoption1put)){
  Simvolp1 = function(sigma){
    blackscholes(290.83, sigma, 50/252,SPoption1put[i], 0.0216, 'put') - 0.5 * (SPoption1$puts$Bid[i] +
  }
  Simput1[i] = bisection(Simvolp1, vrange, 1e-6)
}


#calculate implied vol for third SPY Option, mature at 12-31-2018, for each strike price
# Call
SPoption2call=SPoption2$calls$Strike

for(i in 1:length(SPoption2call)){
  Simvolc2 = function(sigma){
    blackscholes(290.83, sigma, 109/252,SPoption2call[i], 0.0216, 'call') - 0.5 * (SPoption2$calls$Bid[
  }
  Simpcall2[i] = bisection(Simvolc2, vrange, 1e-6)
}

#Put
SPoption2put=SPoption2$puts$Strike
for(i in 1:length(SPoption2put)){
  Simvolp2 = function(sigma){
    blackscholes(290.83, sigma, 109/252,SPoption2put[i], 0.0216, 'put') - 0.5 * (SPoption2$puts$Bid[i]
  }
  Simput2[i] = bisection(Simvolp2, vrange, 1e-6)

}
```

calculate implied vol corresponding to each strike price for Amazon Options by using Scant method

```
#calculate implied vol for first Amazon Option, mature at 10-19-2018, for each strike price
# Call
SAimpcall=SAimpcall1=SAimpcall2=SAimput=SAimput1=SAimput2=c()
Aoptioncall=Aoption$calls$Strike

for(i in 1:length(Aoptioncall)){
  Amvolc = function(sigma){
    blackscholes(1989.87, sigma, 36/252,Aoptioncall[i], 0.0216, 'call') - 0.5 * (Aoption$calls$Bid[i] +
  }
  SAimpcall[i] = secant(Amvolc, vrange, 1e-6)
}

#Put
Aoptionput=Aoption$puts$Strike
```

```
for(i in 1:length(Aoptionput)){
  Amvolp = function(sigma){
    blackscholes(1989.87, sigma, 36/252,Aoptionput[i], 0.0216, 'put') - 0.5 * (Aoption$puts$Bid[i] + Aop
  }
  SAimput[i] = secant(Amvolp, vrange, 1e-6)
}


#calculate implied vol for second Amazon Option, mature at 11-02-2018, for each strike price
# Call

Aoption1call=Aoption1$calls$Strike

for(i in 1:length(Aoption1call)){
  Amvolc1 = function(sigma){
    blackscholes(1989.87, sigma, 50/252,Aoption1call[i], 0.0216, 'call') - 0.5 * (Aoption1$calls$Bid[i]
  }
  SAimpcall1[i] = secant(Amvolc1, vrange, 1e-6)
}

#Put
Aoption1put=Aoption1$puts$Strike
for(i in 1:length(Aoption1put)){
  Amvolp1 = function(sigma){
    blackscholes(1989.87, sigma, 50/252,Aoption1put[i], 0.0216, 'put') - 0.5 * (Aoption1$puts$Bid[i] +
  }
  SAimput1[i] = secant(Amvolp1, vrange, 1e-6)
}


#calculate implied vol for third Amazon Option, mature at 02-15-2018, for each strike price
# Call
Aoption2call=Aoption2$calls$Strike

for(i in 1:length(Aoption2call)){
  Amvolc2 = function(sigma){
    blackscholes(1989.87, sigma, 154/252,Aoption2call[i], 0.0216, 'call') - 0.5 * (Aoption2$calls$Bid[i
  }
  SAimpcall2[i] = secant(Amvolc2, vrange, 1e-6)
}

#Put
Aoption2put=Aoption2$puts$Strike
for(i in 1:length(Aoption2put)){
  Amvolp2 = function(sigma){
    blackscholes(1989.87, sigma, 154/252,Aoption2put[i], 0.0216, 'put') - 0.5 * (Aoption2$puts$Bid[i] +
  }
  SAimput2[i] = secant(Amvolp2, vrange, 1e-6)
}
```

Average all the implied Vol for Options

```
# Amazon Call options
Atable=data.frame(Aoptioncall,Aimpcall)# First option
```

```
Atable1= data.frame(Aoption1call,Aimpcall1)# Second Option
Atable2=data.frame(Aoption2call,Aimpcall2)# Third
A.mean=mean(Atable$Aimpcall)
A.mean1=mean(Atable1$Aimpcall1)
A.mean2=mean(Atable2$Aimpcall2)
A.vol=c(A.mean,A.mean1,A.mean2)
head(A.vol)
```

```
## [1] 0.1606432 0.2543520 0.1897861
```

```
# Amazon Put Option
Atable4=data.frame(Aoptionput,Aimput) # First put
Atable5= data.frame(Aoption1put,Aimput1)# Second Put
Atable6=data.frame(Aoption2put,Aimput2)# Third Put
A.mean3=mean(Atable4$Aimput)
A.mean4=mean(Atable5$Aimput1)
A.mean5=mean(Atable6$Aimput2)
A.volp=c(A.mean,A.mean1,A.mean2)
head(A.volp)
```

```
## [1] 0.1606432 0.2543520 0.1897861
```

```
# SPY call Option
Stable=data.frame(SPoptioncall,Simpcall)# First option
Stable1= data.frame(SPoption1call,Simpcall1)# Second Option
Stable2=data.frame(SPoption2call,Simpcall2)# Third
S.mean=mean(Stable$Simpcall)
S.mean1=mean(Stable1$Simpcall1)
S.mean2=mean(Stable2$Simpcall2)
S.vol=c(S.mean,S.mean1,S.mean2)
head(S.vol)
```

```
## [1] 9.536743e-07 5.614221e-02 2.423605e-02
```

```
# SPY Put Option
Stable4=data.frame(SPoptionput,Simput) # First put
Stable5= data.frame(SPoption1put,Simput1)# Second Put
Stable6=data.frame(SPoption2put,Simput2)# Third Put
S.mean3=mean(Stable4$Simput)
S.mean4=mean(Stable5$Simput1)
S.mean5=mean(Stable6$Simput2)
S.volp=c(S.mean,S.mean1,S.mean2)
head(S.volp)
```

```
## [1] 9.536743e-07 5.614221e-02 2.423605e-02
```

```
avergeimp=cbind(A.vol,A.volp,S.vol,S.volp)
colnames(avergeimp)=c("Amzn Call","Amzn Put","SPY Call","SPY Put")
rownames(avergeimp)=c("First mean","Second Mean","Third Mean")
avergeimp # create a table for implied volatility mean
```

```
##             Amzn Call  Amzn Put     SPY Call      SPY Put
## First mean  0.1606432 0.1606432 9.536743e-07 9.536743e-07
## Second Mean 0.2543520 0.2543520 5.614221e-02 5.614221e-02
## Third Mean  0.1897861 0.1897861 2.423605e-02 2.423605e-02
```

7. Present a table report implied Vol

```
library(plyr)
```

```
## Warning: package 'plyr' was built under R version 3.4.4
```

```
list1=list()
list1[[1]]=data.frame(t(Aimpcall))
list1[[2]]=data.frame(t(Aimpcall1))
list1[[3]]=data.frame(t(Aimpcall2))
list1[[4]]=data.frame(t(Aimput))
list1[[5]]=data.frame(t(Aimput1))
list1[[6]]=data.frame(t(Aimput2))
list1[[7]]=data.frame(t(Simpcall))
list1[[8]]=data.frame(t(Simpcall1))
list1[[9]]=data.frame(t(Simpcall2))
list1[[10]]=data.frame(t(Simput))
list1[[11]]=data.frame(t(Simput1))
list1[[12]]=data.frame(t(Simput2))
imptable=rbind.fill(list1)
Imptable=t(imptable)
colnames(Imptable)=c("Aimpcall","Aimpcall1","Aimpcall2","Aimput","Aimput1","Aimput2","Simpcall","Simpcal
row.names(Imptable)=c()
Imptable
```

```
##           Aimpcall Aimpcall1    Aimpcall2       Aimput   Aimput1
##   [1,] 9.536743e-07 0.2455416 9.536743e-07 8.266439e-01 0.3999510
##   [2,] 9.536743e-07 0.2482939 9.536743e-07 7.875624e-01 0.3736944
##   [3,] 9.536743e-07 0.2513723 9.536743e-07 8.684225e-01 0.3685732
##   [4,] 9.536743e-07 0.2523298 7.658682e-01 7.048826e-01 0.3650599
##   [5,] 9.536743e-07 0.2519197 9.536743e-07 6.969690e-01 0.3636503
##   [6,] 9.536743e-07 0.2522860 9.536743e-07 6.476736e-01 0.3614130
##   [7,] 9.536743e-07 0.2509222 9.536743e-07 6.364527e-01 0.3611155
##   [8,] 9.536743e-07 0.2524405 9.536743e-07 6.204786e-01 0.3465147
##   [9,] 9.536743e-07 0.2531080 9.536743e-07 5.933905e-01 0.3387098
##  [10,] 9.536743e-07 0.2534552 9.536743e-07 6.025553e-01 0.3362875
##  [11,] 9.536743e-07 0.2520418 9.536743e-07 5.340185e-01 0.3344870
##  [12,] 9.536743e-07 0.2534361 9.536743e-07 5.348177e-01 0.3325872
##  [13,] 9.536743e-07 0.2535810 9.536743e-07 5.237856e-01 0.3308306
##  [14,] 9.536743e-07 0.2540579 9.536743e-07 4.921846e-01 0.3309641
##  [15,] 9.536743e-07 0.2538939 9.536743e-07 5.158262e-01 0.3285532
##  [16,] 9.536743e-07 0.2539110 9.536743e-07 4.759645e-01 0.3285818
##  [17,] 9.536743e-07 0.2541342 9.536743e-07 4.664907e-01 0.3264246
##  [18,] 9.536743e-07 0.2527132 9.536743e-07 4.720125e-01 0.3231535
##  [19,] 9.536743e-07 0.2531729 9.536743e-07 4.622221e-01 0.3221521
##  [20,] 9.536743e-07 0.2544928 9.536743e-07 4.342852e-01 0.3212919
##  [21,] 9.536743e-07 0.2553072 9.536743e-07 4.414854e-01 0.3200960
##  [22,] 9.536743e-07 0.2545118 4.918013e-01 4.509287e-01 0.3195448
##  [23,] 9.536743e-07 0.2548914 9.536743e-07 4.495974e-01 0.3174982
##  [24,] 9.332952e-01 0.2529421 4.686689e-01 4.325972e-01 0.3163977
##  [25,] 9.085436e-01 0.2547235 9.536743e-07 4.298162e-01 0.3163309
##  [26,] 9.536743e-07 0.2550344 9.536743e-07 4.276323e-01 0.3159075
##  [27,] 8.351450e-01 0.2549810 9.536743e-07 4.382429e-01 0.3159857
##  [28,] 9.536743e-07 0.2549715 9.536743e-07 3.980532e-01 0.3161783
##  [29,] 9.536743e-07 0.2549620 4.340391e-01 4.340620e-01 0.3163366
##  [30,] 9.536743e-07 0.2550230 9.536743e-07 4.092131e-01 0.3160906
##  [31,] 9.536743e-07 0.2541914 9.536743e-07 4.180059e-01 0.3154688
##  [32,] 9.536743e-07 0.2541552 9.536743e-07 4.198008e-01 0.3164988
##  [33,] 9.536743e-07 0.2539454 9.536743e-07 4.186544e-01 0.3160467
##  [34,] 9.536743e-07 0.2539930 9.536743e-07 4.371462e-01 0.3155775
##  [35,] 7.602949e-01 0.2542505 9.536743e-07 4.583826e-01 0.3158464
##  [36,] 7.529783e-01 0.2537165 9.536743e-07 4.334536e-01 0.3159437
##  [37,] 9.536743e-07 0.2541742 4.023046e-01 3.983488e-01 0.3163748
##  [38,] 9.536743e-07 0.2545958 9.536743e-07 4.287615e-01 0.3159895
##  [39,] 7.246523e-01 0.2553835 9.536743e-07 4.323225e-01 0.3158674
##  [40,] 7.191057e-01 0.2549028 3.917570e-01 4.075079e-01 0.3157187
##  [41,] 9.536743e-07 0.2550554 9.536743e-07 4.054270e-01 0.3159018
##  [42,] 9.536743e-07 0.2559366 9.536743e-07 4.198160e-01 0.3145609
##  [43,] 6.966314e-01 0.2558451 3.858576e-01 3.948488e-01 0.3140192
##  [44,] 9.536743e-07 0.2565374 9.536743e-07 3.989038e-01 0.3140326
##  [45,] 9.536743e-07 0.2696657 9.536743e-07 3.971109e-01 0.3140192
##  [46,] 9.536743e-07 0.2693930 9.536743e-07 4.103651e-01 0.3139963
##  [47,] 9.536743e-07        NA 9.536743e-07 4.078150e-01 0.3257990
##  [48,] 9.536743e-07        NA 9.536743e-07 4.057360e-01 0.3321466
##  [49,] 9.536743e-07        NA 3.724623e-01 4.031172e-01 0.3459482
##  [50,] 9.536743e-07        NA 1.810198e-01 4.074259e-01 0.3519545
##  [51,] 6.363974e-01        NA 9.536743e-07 3.721151e-01        NA
##  [52,] 9.536743e-07        NA 9.536743e-07 3.962469e-01        NA
##  [53,] 9.536743e-07        NA 9.536743e-07 3.933916e-01        NA
##  [54,] 6.148977e-01        NA 9.536743e-07 3.905172e-01        NA
##  [55,] 9.536743e-07        NA 2.076960e-01 3.893366e-01        NA
##  [56,] 9.536743e-07        NA 9.536743e-07 3.656054e-01        NA
##  [57,] 9.536743e-07        NA 9.536743e-07 3.630381e-01        NA
##  [58,] 9.536743e-07        NA 9.536743e-07 3.592119e-01        NA
##  [59,] 9.536743e-07        NA 9.536743e-07 3.597136e-01        NA
##  [60,] 9.536743e-07        NA 3.506536e-01 3.570261e-01        NA
##  [61,] 9.536743e-07        NA 9.536743e-07 3.666258e-01        NA
##  [62,] 9.536743e-07        NA 9.536743e-07 3.561182e-01        NA
##  [63,] 5.371866e-01        NA 9.536743e-07 3.544092e-01        NA
##  [64,] 9.536743e-07        NA 9.536743e-07 3.677149e-01        NA
##  [65,] 9.536743e-07        NA 2.293787e-01 3.499994e-01        NA
##  [66,] 9.536743e-07        NA 1.585264e-01 3.503637e-01        NA
##  [67,] 9.536743e-07        NA 1.697512e-01 3.463316e-01        NA
##  [68,] 9.536743e-07        NA 2.337351e-01 3.593893e-01        NA
##  [69,] 9.536743e-07        NA 2.346888e-01 3.564920e-01        NA
##  [70,] 4.925089e-01        NA 2.350321e-01 3.528051e-01        NA
##  [71,] 9.536743e-07        NA 2.362947e-01 3.456507e-01        NA
##  [72,] 9.536743e-07        NA 3.245039e-01 3.324308e-01        NA
##  [73,] 9.536743e-07        NA 2.376776e-01 3.313589e-01        NA
##  [74,] 9.536743e-07        NA 1.976881e-01 3.492785e-01        NA
##  [75,] 9.536743e-07        NA 2.390089e-01 3.437872e-01        NA
##  [76,] 9.536743e-07        NA 2.404966e-01 3.474379e-01        NA
##  [77,] 9.536743e-07        NA 2.407923e-01 3.348627e-01        NA
##  [78,] 9.536743e-07        NA 2.086821e-01 3.378077e-01        NA
##  [79,] 9.536743e-07        NA 2.107267e-01 3.311319e-01        NA
##  [80,] 9.536743e-07        NA 3.079176e-01 3.331060e-01        NA
##  [81,] 9.536743e-07        NA 2.147112e-01 3.148527e-01        NA
##  [82,] 9.536743e-07        NA 2.435560e-01 3.296328e-01        NA
##  [83,] 9.536743e-07        NA 2.181749e-01 3.273592e-01        NA
##  [84,] 9.536743e-07        NA 2.194796e-01 3.263845e-01        NA
##  [85,] 9.536743e-07        NA 2.441950e-01 3.188810e-01        NA
##  [86,] 9.536743e-07        NA 2.217436e-01 3.065863e-01        NA
##  [87,] 9.536743e-07        NA 2.450991e-01 3.101511e-01        NA
##  [88,] 9.536743e-07        NA 2.454214e-01 3.080378e-01        NA
##  [89,] 9.536743e-07        NA 2.455797e-01 3.128519e-01        NA
##  [90,] 9.536743e-07        NA 2.456331e-01 3.000746e-01        NA
##  [91,] 3.865252e-01        NA 2.452040e-01 2.995901e-01        NA
##  [92,] 9.536743e-07        NA 2.459040e-01 3.095675e-01        NA
##  [93,] 9.536743e-07        NA 2.467108e-01 3.062677e-01        NA
##  [94,] 9.536743e-07        NA 2.269545e-01 2.942686e-01        NA
##  [95,] 9.536743e-07        NA 2.468042e-01 2.915049e-01        NA
##  [96,] 9.536743e-07        NA 2.467031e-01 2.978792e-01        NA
##  [97,] 9.536743e-07        NA 2.451982e-01 2.884092e-01        NA
##  [98,] 9.536743e-07        NA 2.466574e-01 2.986917e-01        NA
##  [99,] 9.536743e-07        NA 2.468367e-01 2.885008e-01        NA
## [100,] 9.536743e-07        NA 2.466631e-01 2.853689e-01        NA
## [101,] 9.536743e-07        NA 2.451372e-01 2.933111e-01        NA
## [102,] 9.536743e-07        NA 2.451143e-01 2.899199e-01        NA
## [103,] 1.262465e-01        NA 2.449236e-01 2.836847e-01        NA
## [104,] 1.159983e-01        NA 2.458544e-01 2.896433e-01        NA
## [105,] 1.404123e-01        NA 2.471933e-01 2.783632e-01        NA
## [106,] 1.325121e-01        NA 2.455835e-01 2.771902e-01        NA
## [107,] 1.496706e-01        NA 2.450552e-01 2.794142e-01        NA
## [108,] 1.533728e-01        NA 2.451029e-01 2.754049e-01        NA
## [109,] 1.495523e-01        NA 2.469549e-01 2.815390e-01        NA
## [110,] 1.597319e-01        NA 2.459307e-01 2.799826e-01        NA
## [111,] 1.532755e-01        NA 2.459307e-01 2.791643e-01        NA
## [112,] 1.643095e-01        NA 2.449942e-01 2.773657e-01        NA
## [113,] 1.598368e-01        NA 2.472715e-01 2.765493e-01        NA
## [114,] 1.579504e-01        NA 2.464952e-01 2.754507e-01        NA
## [115,] 1.613760e-01        NA 2.467699e-01 2.729139e-01        NA
## [116,] 1.681681e-01        NA 2.465162e-01 2.731009e-01        NA
## [117,] 1.706781e-01        NA 2.468290e-01 2.701006e-01        NA
## [118,] 1.711416e-01        NA 2.347708e-01 2.704630e-01        NA
## [119,] 1.710978e-01        NA 2.466936e-01 2.679873e-01        NA
## [120,] 1.709871e-01        NA 2.462072e-01 2.691793e-01        NA
## [121,] 1.742773e-01        NA 2.466879e-01 2.673807e-01        NA
## [122,] 1.738596e-01        NA 2.339506e-01 2.670431e-01        NA
## [123,] 1.748991e-01        NA 2.455187e-01 2.650938e-01        NA
## [124,] 1.745214e-01        NA 2.466898e-01 2.657442e-01        NA
## [125,] 1.767206e-01        NA 2.467337e-01 2.646933e-01        NA
## [126,] 1.781397e-01        NA 2.465372e-01 2.607622e-01        NA
## [127,] 1.783800e-01        NA 2.464647e-01 2.622538e-01        NA
## [128,] 1.775770e-01        NA 2.461519e-01 2.603579e-01        NA
## [129,] 1.772280e-01        NA 2.467318e-01 2.600393e-01        NA
## [130,] 1.819334e-01        NA 2.464647e-01 2.600355e-01        NA
## [131,] 1.802893e-01        NA 2.464418e-01 2.603331e-01        NA
## [132,] 1.798563e-01        NA 2.358980e-01 2.599592e-01        NA
## [133,] 1.801863e-01        NA 2.350435e-01 2.595091e-01        NA
## [134,] 1.790152e-01        NA 2.461939e-01 2.596312e-01        NA
## [135,] 1.811056e-01        NA 2.466383e-01 2.594786e-01        NA
## [136,] 1.813650e-01        NA 2.459784e-01 2.588739e-01        NA
## [137,] 1.802988e-01        NA 2.468863e-01 2.590036e-01        NA
## [138,] 1.794901e-01        NA 2.466059e-01 2.589502e-01        NA
## [139,] 1.825380e-01        NA 2.465792e-01 2.582769e-01        NA
## [140,] 1.829348e-01        NA 2.464724e-01 2.590132e-01        NA
## [141,] 1.835203e-01        NA 2.464972e-01 2.583437e-01        NA
## [142,] 1.818361e-01        NA 2.464437e-01 2.554388e-01        NA
## [143,] 1.835833e-01        NA 2.363405e-01 2.566786e-01        NA
## [144,] 1.861658e-01        NA 2.458525e-01 2.577276e-01        NA
## [145,] 1.871691e-01        NA 2.458162e-01 2.578268e-01        NA
## [146,] 1.882334e-01        NA 2.455759e-01 2.553053e-01        NA
## [147,] 1.852522e-01        NA 2.458944e-01 2.572508e-01        NA
## [148,] 1.875887e-01        NA 2.362223e-01 2.556581e-01        NA
## [149,] 1.868258e-01        NA 2.457476e-01 2.566614e-01        NA
## [150,] 1.869364e-01        NA 2.449751e-01 2.582369e-01        NA
## [151,] 1.882524e-01        NA 2.363653e-01 2.563047e-01        NA
## [152,] 1.886206e-01        NA 2.456980e-01 2.569551e-01        NA
## [153,] 1.879988e-01        NA 2.365904e-01 2.616873e-01        NA
## [154,] 1.865263e-01        NA 2.456827e-01 2.581663e-01        NA
## [155,] 1.883707e-01        NA 2.374830e-01 2.606516e-01        NA
## [156,] 1.909056e-01        NA 2.365618e-01 2.609720e-01        NA
## [157,] 1.891775e-01        NA 2.545710e-01 2.621374e-01        NA
## [158,] 1.902151e-01        NA 2.456007e-01 2.631483e-01        NA
## [159,] 1.908674e-01        NA 2.464190e-01 2.630739e-01        NA
## [160,] 1.909971e-01        NA 2.463980e-01 2.654696e-01        NA
## [161,] 1.876059e-01        NA 2.462530e-01 2.656832e-01        NA
## [162,] 1.913652e-01        NA 2.461653e-01 2.657404e-01        NA
## [163,] 1.878672e-01        NA 2.450991e-01 2.675390e-01        NA
## [164,] 1.938524e-01        NA 2.462606e-01 2.692919e-01        NA
## [165,] 1.894350e-01        NA 2.456636e-01 2.707396e-01        NA
## [166,] 1.949854e-01        NA 2.442217e-01 2.702074e-01        NA
## [167,] 1.948175e-01        NA 2.372866e-01 2.734137e-01        NA
## [168,] 1.942968e-01        NA 2.458372e-01 3.293161e-01        NA
## [169,] 1.896296e-01        NA 2.454309e-01 2.750330e-01        NA
## [170,] 1.945162e-01        NA 2.462988e-01 2.755518e-01        NA
## [171,] 1.939421e-01        NA 2.377367e-01 2.809591e-01        NA
## [172,] 1.887484e-01        NA 2.465982e-01 2.772856e-01        NA
## [173,] 1.899405e-01        NA 2.378626e-01 2.832670e-01        NA
## [174,] 1.904707e-01        NA 2.466249e-01 2.811460e-01        NA
## [175,] 1.906538e-01        NA 2.465391e-01 2.887869e-01        NA
## [176,] 1.988363e-01        NA 2.462683e-01 2.849798e-01        NA
## [177,] 1.914835e-01        NA 2.459116e-01 2.869406e-01        NA
## [178,] 1.948977e-01        NA 2.448931e-01 2.892008e-01        NA
## [179,] 1.999445e-01        NA 2.497244e-01 2.960825e-01        NA
## [180,] 1.938467e-01        NA 2.454634e-01 9.852123e-02        NA
## [181,] 1.936636e-01        NA 2.469854e-01 2.958651e-01        NA
## [182,] 1.947546e-01        NA 2.489729e-01 2.983961e-01        NA
## [183,] 1.948805e-01        NA 2.492971e-01 3.703394e-01        NA
## [184,] 2.009783e-01        NA 2.387476e-01 3.036528e-01        NA
## [185,] 1.960955e-01        NA 2.490053e-01 3.794165e-01        NA
## [186,] 1.973333e-01        NA 2.391577e-01 3.112535e-01        NA
## [187,] 1.976023e-01        NA 2.452955e-01 3.184767e-01        NA
## [188,] 1.979647e-01        NA 2.466974e-01 3.271914e-01        NA
## [189,] 1.994677e-01        NA 2.474108e-01 3.994474e-01        NA
## [190,] 2.000532e-01        NA 2.453547e-01 3.340693e-01        NA
## [191,] 2.051382e-01        NA 2.455492e-01 3.361197e-01        NA
## [192,] 2.044401e-01        NA 2.464209e-01 4.099493e-01        NA
## [193,] 2.054358e-01        NA 2.453909e-01 4.130335e-01        NA
## [194,] 2.020922e-01        NA 2.436552e-01 4.166861e-01        NA
## [195,] 2.070093e-01        NA 2.480707e-01 9.536743e-07        NA
## [196,] 2.130556e-01        NA 2.455702e-01 4.315634e-01        NA
## [197,] 2.080984e-01        NA 2.452955e-01 3.768568e-01        NA
## [198,] 2.072001e-01        NA 2.479525e-01 3.872728e-01        NA
## [199,] 2.062120e-01        NA 2.454176e-01 3.908587e-01        NA
## [200,] 2.071218e-01        NA 2.456961e-01 3.944216e-01        NA
## [201,] 2.092581e-01        NA 2.450399e-01 4.803991e-01        NA
## [202,] 2.104654e-01        NA 2.457933e-01 5.049810e-01        NA
## [203,] 2.135839e-01        NA 2.480536e-01 9.536743e-07        NA
## [204,] 2.124872e-01        NA 2.495165e-01 5.233603e-01        NA
## [205,] 2.130537e-01        NA 2.452631e-01 4.582834e-01        NA
## [206,] 2.184057e-01        NA 2.466383e-01 4.649534e-01        NA
## [207,] 2.334299e-01        NA 2.506456e-01 7.147169e-01        NA
## [208,] 2.246218e-01        NA 2.541876e-01 7.855597e-01        NA
## [209,] 2.256517e-01        NA 2.490396e-01           NA        NA
## [210,] 2.263937e-01        NA           NA           NA        NA
## [211,] 2.229815e-01        NA           NA           NA        NA
## [212,] 2.229910e-01        NA           NA           NA        NA
## [213,] 2.293997e-01        NA           NA           NA        NA
## [214,] 2.302408e-01        NA           NA           NA        NA
## [215,] 2.312841e-01        NA           NA           NA        NA
## [216,] 2.356138e-01        NA           NA           NA        NA
## [217,] 2.358446e-01        NA           NA           NA        NA
## [218,] 2.368059e-01        NA           NA           NA        NA
## [219,] 2.415953e-01        NA           NA           NA        NA
## [220,] 2.695303e-01        NA           NA           NA        NA
## [221,] 2.404184e-01        NA           NA           NA        NA
## [222,] 2.421064e-01        NA           NA           NA        NA
## [223,] 2.469797e-01        NA           NA           NA        NA
## [224,] 2.464247e-01        NA           NA           NA        NA
## [225,] 2.521372e-01        NA           NA           NA        NA
## [226,] 2.534819e-01        NA           NA           NA        NA
## [227,] 2.549562e-01        NA           NA           NA        NA
## [228,] 2.575102e-01        NA           NA           NA        NA
## [229,] 2.599478e-01        NA           NA           NA        NA
## [230,] 2.569113e-01        NA           NA           NA        NA
## [231,] 2.619009e-01        NA           NA           NA        NA
## [232,] 2.474356e-01        NA           NA           NA        NA
## [233,] 2.472334e-01        NA           NA           NA        NA
## [234,] 2.822580e-01        NA           NA           NA        NA
## [235,] 2.616911e-01        NA           NA           NA        NA
## [236,] 2.640848e-01        NA           NA           NA        NA
## [237,] 2.587900e-01        NA           NA           NA        NA
## [238,] 2.642031e-01        NA           NA           NA        NA
## [239,] 2.970991e-01        NA           NA           NA        NA
## [240,] 2.937403e-01        NA           NA           NA        NA
## [241,] 2.984743e-01        NA           NA           NA        NA
## [242,] 3.045034e-01        NA           NA           NA        NA
## [243,] 3.210230e-01        NA           NA           NA        NA
## [244,] 3.484354e-01        NA           NA           NA        NA
## [245,] 3.297300e-01        NA           NA           NA        NA
## [246,] 3.279734e-01        NA           NA           NA        NA
## [247,] 3.292704e-01        NA           NA           NA        NA
## [248,] 3.393412e-01        NA           NA           NA        NA
## [249,] 3.446531e-01        NA           NA           NA        NA
## [250,] 3.205709e-01        NA           NA           NA        NA
## [251,] 3.365736e-01        NA           NA           NA        NA
##          Aimput2     Simpcall    Simpcall1    Simpcall2     Simput
##   [1,] 0.4968939 9.536743e-07 9.536743e-07 9.536743e-07 0.83812809
##   [2,] 0.4843054 9.536743e-07 5.535221e-02 9.536743e-07 0.84353161
##   [3,] 0.4764986 9.536743e-07 5.623531e-02 9.536743e-07 0.76589870
##   [4,] 0.4750090 9.536743e-07 5.773640e-02 9.536743e-07 0.73222828
##   [5,] 0.4477530 9.536743e-07 5.865192e-02 9.536743e-07 0.69999409
##   [6,] 0.4184484 9.536743e-07 5.868435e-02 9.536743e-07 0.72832394
##   [7,] 0.4168787 9.536743e-07 5.861759e-02 9.536743e-07 0.69617748
##   [8,] 0.3818541 9.536743e-07 5.839443e-02 9.536743e-07 0.61076450
##   [9,] 0.3943186 9.536743e-07 5.827618e-02 9.536743e-07 0.58319187
##  [10,] 0.4067507 9.536743e-07 5.774593e-02 9.536743e-07 0.60660267
##  [11,] 0.3947744 9.536743e-07 5.708599e-02 9.536743e-07 0.60521221
##  [12,] 0.3906889 9.536743e-07 5.643749e-02 9.536743e-07 0.55179501
##  [13,] 0.3842402 9.536743e-07 5.584049e-02 9.536743e-07 0.52566242
##  [14,] 0.3801985 9.536743e-07 5.548763e-02 9.536743e-07 0.50030231
##  [15,] 0.4153376 9.536743e-07 5.525494e-02 9.536743e-07 0.49785519
##  [16,] 0.3700018 9.536743e-07 5.476284e-02 9.536743e-07 0.47289753
##  [17,] 0.3477926 9.536743e-07 5.427074e-02 9.536743e-07 0.46333408
##  [18,] 0.3463030 9.536743e-07 5.403614e-02 9.536743e-07 0.43896389
##  [19,] 0.3437796 9.536743e-07 5.419254e-02 9.536743e-07 0.42595577
##  [20,] 0.3434629 9.536743e-07 5.430698e-02 9.536743e-07 0.42116261
##  [21,] 0.3503408 9.536743e-07 5.442905e-02 9.536743e-07 0.41639233
##  [22,] 0.3500090 9.536743e-07 5.491734e-02 9.536743e-07 0.41164494
##  [23,] 0.3428679 9.536743e-07 5.557919e-02 9.536743e-07 0.41569042
##  [24,] 0.3462553 9.536743e-07 5.856609e-02 9.536743e-07 0.41089725
##  [25,] 0.3444796 9.536743e-07 6.289577e-02 9.536743e-07 0.40612698
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##  [28,] 0.3341017 9.536743e-07           NA 9.536743e-07 0.39194012
##  [29,] 0.3352346 9.536743e-07           NA 9.536743e-07 0.38725185
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##  [33,] 0.3335733 9.536743e-07           NA 9.536743e-07 0.37563229
##  [34,] 0.3307352 9.536743e-07           NA 9.536743e-07 0.37097073
##  [35,] 0.3263407 9.536743e-07           NA 9.536743e-07 0.39629650
##  [36,] 0.3269434 9.536743e-07           NA 9.536743e-07 0.41223812
##  [37,] 0.3234224 9.536743e-07           NA 9.536743e-07 0.36303997
##  [38,] 0.2860556 9.536743e-07           NA 9.536743e-07 0.35838795
##  [39,] 0.3216105 9.536743e-07           NA 9.536743e-07 0.35375309
##  [40,] 0.3207426 9.536743e-07           NA 9.536743e-07 0.38245678
##  [41,] 0.3231363 9.536743e-07           NA 9.536743e-07 0.37749767
##  [42,] 0.3259153 9.536743e-07           NA 9.536743e-07 0.37255764
##  [43,] 0.3209867 9.536743e-07           NA 9.536743e-07 0.36091900
##  [44,] 0.3146276 9.536743e-07           NA 9.536743e-07 0.33585262
##  [45,] 0.2789392 9.536743e-07           NA 9.536743e-07 0.36386395
##  [46,] 0.3176680 9.536743e-07           NA 9.536743e-07 0.33119678
##  [47,] 0.2757578 9.536743e-07           NA 9.536743e-07 0.34811878
##  [48,] 0.2754183 9.536743e-07           NA 9.536743e-07 0.35185146
##  [49,] 0.3159876 9.536743e-07           NA 9.536743e-07 0.32146740
##  [50,] 0.3149309 9.536743e-07           NA 9.536743e-07 0.33935070
##  [51,] 0.3169413 9.536743e-07           NA 9.536743e-07 0.31227207
##  [52,] 0.3145838 9.536743e-07           NA 9.536743e-07 0.33963108
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##  [54,] 0.3141146 9.536743e-07           NA 9.536743e-07 0.30215549
##  [55,] 0.3130140 9.536743e-07           NA 9.536743e-07 0.30089664
##  [56,] 0.3130846 9.536743e-07           NA 9.536743e-07 0.29629612
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##  [62,] 0.3052969 9.536743e-07           NA 9.536743e-07 0.27457905
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##  [65,] 0.3030176 9.536743e-07           NA 9.536743e-07 0.26577473
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##  [68,] 0.3017397 9.536743e-07           NA 9.536743e-07 0.25417995
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##  [70,] 0.3010454 9.536743e-07           NA 9.536743e-07 0.24703884
##  [71,] 0.3059912 9.536743e-07           NA 9.536743e-07 0.24438953
##  [72,] 0.3008547 9.536743e-07           NA 9.536743e-07 0.23974705
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##  [75,] 0.3046675 9.536743e-07           NA 8.365917e-02 0.22941685
##  [76,] 0.2973318 9.536743e-07           NA 8.806324e-02 0.22642231
##  [77,] 0.2985983 9.536743e-07           NA 9.536743e-07 0.22333813
##  [78,] 0.2967234 9.536743e-07           NA 9.536743e-07 0.21864033
##  [79,] 0.2965593 9.536743e-07           NA 6.172657e-02 0.21545315
##  [80,] 0.2987146 9.536743e-07           NA 7.165432e-02 0.21074390
##  [81,] 0.2970285 9.536743e-07           NA 9.536743e-07 0.20745945
##  [82,] 0.2967825 9.536743e-07           NA 7.800770e-02 0.20409870
##  [83,] 0.2950563 9.536743e-07           NA 9.536743e-07 0.20001507
##  [84,] 0.2557707 9.536743e-07           NA 8.104420e-02 0.19590473
##  [85,] 0.2968645 9.536743e-07           NA 8.193302e-02 0.19238377
##  [86,] 0.2963438 9.536743e-07           NA 8.261967e-02 0.18879604
##  [87,] 0.3002329 9.536743e-07           NA 8.298779e-02 0.18514347
##  [88,] 0.2953482 9.536743e-07           NA 8.308697e-02 0.18142605
##  [89,] 0.2945223 9.536743e-07           NA 8.318806e-02 0.17817211
##  [90,] 0.2948580 9.536743e-07           NA 8.304882e-02 0.17431545
##  [91,] 0.2933836 9.536743e-07           NA 8.279705e-02 0.17040157
##  [92,] 0.2943449 9.536743e-07           NA 8.223629e-02 0.16689396
##  [93,] 0.2935762 9.536743e-07           NA 8.158207e-02 0.16329479
##  [94,] 0.2925234 9.536743e-07           NA 5.585766e-02 0.16002750
##  [95,] 0.2919931 9.536743e-07           NA 8.040142e-02 0.15623379
##  [96,] 0.2904482 9.536743e-07           NA 7.950306e-02 0.15274143
##  [97,] 0.2913656 9.536743e-07           NA 7.872295e-02 0.14913082
##  [98,] 0.2909288 9.536743e-07           NA 7.780361e-02 0.14576054
##  [99,] 0.2893076 9.536743e-07           NA 7.681370e-02 0.14224720
## [100,] 0.2904234 9.536743e-07           NA 7.624149e-02 0.13859463
## [101,] 0.2898474 9.536743e-07           NA 7.513523e-02 0.13511181
## [102,] 0.2894068 9.536743e-07           NA 7.419491e-02 0.13204098
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## [104,] 0.2879133 9.536743e-07           NA 6.505489e-02 0.12551975
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## [107,] 0.2867861 9.536743e-07           NA 6.817150e-02 0.11578846
## [108,] 0.2867899 9.536743e-07           NA 7.203197e-02 0.11284161
## [109,] 0.2869425 9.536743e-07           NA 7.849979e-02 0.11013126
## [110,] 0.2861357 9.536743e-07           NA 8.343410e-02 0.10686016
## [111,] 0.2855349 9.536743e-07           NA 9.489536e-02 0.10408115
## [112,] 0.2861719 9.536743e-07           NA 9.654140e-02 0.10134411
## [113,] 0.2860384 9.536743e-07           NA 1.005564e-01 0.09858608
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## [115,] 0.2859926 9.536743e-07           NA           NA 0.09344196
## [116,] 0.2854891 9.536743e-07           NA           NA 0.09114170
## [117,] 0.2853937 9.536743e-07           NA           NA 0.08889484
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## [119,] 0.2852545 9.536743e-07           NA           NA 0.08574581
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## [121,] 0.2853804 9.536743e-07           NA           NA 0.08509541
## [122,] 0.2858419 9.536743e-07           NA           NA 0.08595753
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## [126,] 0.2858706 9.536743e-07           NA           NA 0.09925175
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## [129,] 0.2854414 9.536743e-07           NA           NA 0.11666203
## [130,] 0.2846022 9.536743e-07           NA           NA 0.12311268
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## [145,] 0.2872458 9.536743e-07           NA           NA 0.41903782
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## [147,] 0.3008757 9.536743e-07           NA           NA 0.45784283
## [148,] 0.3007593 9.536743e-07           NA           NA 0.38604259
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## [150,] 0.2878275 9.536743e-07           NA           NA 0.58336353
## [151,] 0.2238474 9.536743e-07           NA           NA 0.37756824
## [152,] 0.3032484           NA           NA           NA 0.63598728
## [153,] 0.2231588           NA           NA           NA 0.61302853
## [154,] 0.2899561           NA           NA           NA 0.55960369
## [155,] 0.2896280           NA           NA           NA 0.47656918
## [156,] 0.3047647           NA           NA           NA 0.62343311
## [157,] 0.3037500           NA           NA           NA         NA
## [158,] 0.2220850           NA           NA           NA         NA
## [159,] 0.3064909           NA           NA           NA         NA
## [160,] 0.2914572           NA           NA           NA         NA
## [161,] 0.2920465           NA           NA           NA         NA
## [162,] 0.2911005           NA           NA           NA         NA
## [163,] 0.2204943           NA           NA           NA         NA
## [164,] 0.2205439           NA           NA           NA         NA
## [165,] 0.2935534           NA           NA           NA         NA
## [166,] 0.2937574           NA           NA           NA         NA
## [167,] 0.2941465           NA           NA           NA         NA
## [168,] 0.3114119           NA           NA           NA         NA
## [169,] 0.2949781           NA           NA           NA         NA
## [170,] 0.2953672           NA           NA           NA         NA
## [171,] 0.2971144           NA           NA           NA         NA
## [172,] 0.2177305           NA           NA           NA         NA
## [173,] 0.3168344           NA           NA           NA         NA
## [174,] 0.3171701           NA           NA           NA         NA
## [175,] 0.2986650           NA           NA           NA         NA
## [176,] 0.3195543           NA           NA           NA         NA
## [177,] 0.2162619           NA           NA           NA         NA
## [178,] 0.2997942           NA           NA           NA         NA
## [179,] 0.2160292           NA           NA           NA         NA
## [180,] 0.3232012           NA           NA           NA         NA
## [181,] 0.3025141           NA           NA           NA         NA
## [182,] 0.2165518           NA           NA           NA         NA
## [183,] 0.2158957           NA           NA           NA         NA
## [184,] 0.3319101           NA           NA           NA         NA
## [185,] 0.3355036           NA           NA           NA         NA
## [186,] 0.2151670           NA           NA           NA         NA
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## [188,] 0.3382311           NA           NA           NA         NA
## [189,] 0.3144464           NA           NA           NA         NA
## [190,] 0.2151766           NA           NA           NA         NA
## [191,] 0.2151651           NA           NA           NA         NA
## [192,] 0.3527021           NA           NA           NA         NA
## [193,] 0.3537302           NA           NA           NA         NA
## [194,] 0.2162523           NA           NA           NA         NA
## [195,] 0.4077005           NA           NA           NA         NA
## [196,]        NA           NA           NA           NA         NA
## [197,]        NA           NA           NA           NA         NA
## [198,]        NA           NA           NA           NA         NA
## [199,]        NA           NA           NA           NA         NA
## [200,]        NA           NA           NA           NA         NA
## [201,]        NA           NA           NA           NA         NA
## [202,]        NA           NA           NA           NA         NA
## [203,]        NA           NA           NA           NA         NA
## [204,]        NA           NA           NA           NA         NA
## [205,]        NA           NA           NA           NA         NA
## [206,]        NA           NA           NA           NA         NA
## [207,]        NA           NA           NA           NA         NA
## [208,]        NA           NA           NA           NA         NA
## [209,]        NA           NA           NA           NA         NA
## [210,]        NA           NA           NA           NA         NA
## [211,]        NA           NA           NA           NA         NA
## [212,]        NA           NA           NA           NA         NA
## [213,]        NA           NA           NA           NA         NA
## [214,]        NA           NA           NA           NA         NA
## [215,]        NA           NA           NA           NA         NA
## [216,]        NA           NA           NA           NA         NA
## [217,]        NA           NA           NA           NA         NA
## [218,]        NA           NA           NA           NA         NA
## [219,]        NA           NA           NA           NA         NA
## [220,]        NA           NA           NA           NA         NA
## [221,]        NA           NA           NA           NA         NA
## [222,]        NA           NA           NA           NA         NA
## [223,]        NA           NA           NA           NA         NA
## [224,]        NA           NA           NA           NA         NA
## [225,]        NA           NA           NA           NA         NA
## [226,]        NA           NA           NA           NA         NA
## [227,]        NA           NA           NA           NA         NA
## [228,]        NA           NA           NA           NA         NA
## [229,]        NA           NA           NA           NA         NA
## [230,]        NA           NA           NA           NA         NA
## [231,]        NA           NA           NA           NA         NA
## [232,]        NA           NA           NA           NA         NA
## [233,]        NA           NA           NA           NA         NA
## [234,]        NA           NA           NA           NA         NA
## [235,]        NA           NA           NA           NA         NA
## [236,]        NA           NA           NA           NA         NA
## [237,]        NA           NA           NA           NA         NA
## [238,]        NA           NA           NA           NA         NA
## [239,]        NA           NA           NA           NA         NA
## [240,]        NA           NA           NA           NA         NA
## [241,]        NA           NA           NA           NA         NA
## [242,]        NA           NA           NA           NA         NA
## [243,]        NA           NA           NA           NA         NA
## [244,]        NA           NA           NA           NA         NA
## [245,]        NA           NA           NA           NA         NA
## [246,]        NA           NA           NA           NA         NA
## [247,]        NA           NA           NA           NA         NA
## [248,]        NA           NA           NA           NA         NA
## [249,]        NA           NA           NA           NA         NA
## [250,]        NA           NA           NA           NA         NA
## [251,]        NA           NA           NA           NA         NA
##           Simput1   Simput2
##   [1,] 0.19592571 0.2990561
##   [2,] 0.18044758 0.3090181
##   [3,] 0.16552639 0.3195467
##   [4,] 0.15789700 0.3131666
##   [5,] 0.15036488 0.2906466
##   [6,] 0.14305592 0.3135958
##   [7,] 0.13599873 0.3265123
##   [8,] 0.12914753 0.3100958
##   [9,] 0.12240696 0.2827959
##  [10,] 0.11596012 0.3064089
##  [11,] 0.10953045 0.3042078
##  [12,] 0.10486126 0.2759657
##  [13,] 0.10361385 0.3163710
##  [14,] 0.10252857 0.2729273
##  [15,] 0.10018826 0.2967558
##  [16,] 0.09803486 0.2851763
##  [17,] 0.09589100 0.2665167
##  [18,] 0.09371662 0.2813959
##  [19,] 0.09276295 0.2759104
##  [20,] 0.09188366 0.2907267
##  [21,] 0.09021664 0.2581091
##  [22,] 0.08895969 0.3010321
##  [23,] 0.08795643 0.2846785
##  [24,] 0.08746243 0.2745943
##  [25,] 0.08744717 0.2729616
##  [26,] 0.08745670 0.2621832
##  [27,] 0.08936977 0.2598314
##  [28,] 0.09221363 0.2858419
##  [29,] 0.10681248 0.2423773
##  [30,]         NA 0.2543440
##  [31,]         NA 0.2801199
##  [32,]         NA 0.2551451
##  [33,]         NA 0.2665129
##  [34,]         NA 0.2325163
##  [35,]         NA 0.2537031
##  [36,]         NA 0.2786512
##  [37,]         NA 0.2410898
##  [38,]         NA 0.2545099
##  [39,]         NA 0.2226820
##  [40,]         NA 0.2539473
##  [41,]         NA 0.2522097
##  [42,]         NA 0.2165995
##  [43,]         NA 0.2386770
##  [44,]         NA 0.2127237
##  [45,]         NA 0.2353563
##  [46,]         NA 0.2086935
##  [47,]         NA 0.2220259
##  [48,]         NA 0.2511454
##  [49,]         NA 0.2028322
##  [50,]         NA 0.2260885
##  [51,]         NA 0.1992826
##  [52,]         NA 0.1971598
##  [53,]         NA 0.1955271
##  [54,]         NA 0.1935778
##  [55,]         NA 0.1915846
##  [56,]         NA 0.1897860
##  [57,]         NA 0.1879320
##  [58,]         NA 0.2017031
##  [59,]         NA 0.1845064
##  [60,]         NA 0.1980543
##  [61,]         NA 0.1808348
##  [62,]         NA 0.1791124
##  [63,]         NA 0.2111177
##  [64,]         NA 0.1756601
##  [65,]         NA 0.2078829
##  [66,]         NA 0.1721182
##  [67,]         NA 0.1702471
##  [68,]         NA 0.1835966
##  [69,]         NA 0.1667986
##  [70,]         NA 0.1916285
##  [71,]         NA 0.1635103
##  [72,]         NA 0.1617441
##  [73,]         NA 0.1598997
##  [74,]         NA 0.1583948
##  [75,]         NA 0.1565237
##  [76,]         NA 0.1817503
##  [77,]         NA 0.1674032
##  [78,]         NA 0.1513166
##  [79,]         NA 0.1497202
##  [80,]         NA 0.1479063
##  [81,]         NA 0.1462240
##  [82,]         NA 0.1445475
##  [83,]         NA 0.1706152
##  [84,]         NA 0.1411924
##  [85,]         NA 0.1394014
##  [86,]         NA 0.1377935
##  [87,]         NA 0.1360655
##  [88,]         NA 0.1344023
##  [89,]         NA 0.1327944
##  [90,]         NA 0.1311407
##  [91,]         NA 0.1294394
##  [92,]         NA 0.1277704
##  [93,]         NA 0.1262026
##  [94,]         NA 0.1245642
##  [95,]         NA 0.1232386
##  [96,]         NA 0.1216650
##  [97,]         NA 0.1201525
##  [98,]         NA 0.1186113
##  [99,]         NA 0.1171789
## [100,]         NA 0.1159124
## [101,]         NA 0.1144457
## [102,]         NA 0.1131926
## [103,]         NA 0.1120672
## [104,]         NA 0.1109247
## [105,]         NA 0.1072454
## [106,]         NA 0.1081972
## [107,]         NA 0.1161070
## [108,]         NA 0.1307116
## [109,]         NA 0.1971350
## [110,]         NA 0.2130136
## [111,]         NA 0.3160982
## [112,]         NA 0.3802900
## [113,]         NA 0.3714647
## [114,]         NA 0.2469149
## [115,]         NA        NA
## [116,]         NA        NA
## [117,]         NA        NA
## [118,]         NA        NA
## [119,]         NA        NA
## [120,]         NA        NA
## [121,]         NA        NA
## [122,]         NA        NA
## [123,]         NA        NA
## [124,]         NA        NA
## [125,]         NA        NA
## [126,]         NA        NA
## [127,]         NA        NA
## [128,]         NA        NA
## [129,]         NA        NA
## [130,]         NA        NA
## [131,]         NA        NA
## [132,]         NA        NA
## [133,]         NA        NA
## [134,]         NA        NA
## [135,]         NA        NA
## [136,]         NA        NA
## [137,]         NA        NA
## [138,]         NA        NA
## [139,]         NA        NA
## [140,]         NA        NA
## [141,]         NA        NA
## [142,]         NA        NA
## [143,]         NA        NA
## [144,]         NA        NA
## [145,]         NA        NA
## [146,]         NA        NA
## [147,]         NA        NA
## [148,]         NA        NA
## [149,]         NA        NA
## [150,]         NA        NA
## [151,]         NA        NA
## [152,]         NA        NA
## [153,]         NA        NA
## [154,]         NA        NA
## [155,]         NA        NA
## [156,]         NA        NA
## [157,]         NA        NA
## [158,]         NA        NA
## [159,]         NA        NA
## [160,]         NA        NA
## [161,]         NA        NA
## [162,]         NA        NA
## [163,]         NA        NA
## [164,]         NA        NA
## [165,]         NA        NA
## [166,]         NA        NA
## [167,]         NA        NA
## [168,]         NA        NA
## [169,]         NA        NA
## [170,]         NA        NA
## [171,]         NA        NA
## [172,]         NA        NA
## [173,]         NA        NA
## [174,]         NA        NA
## [175,]         NA         NA
## [176,]         NA         NA
## [177,]         NA         NA
## [178,]         NA         NA
## [179,]         NA         NA
## [180,]         NA         NA
## [181,]         NA         NA
## [182,]         NA         NA
## [183,]         NA         NA
## [184,]         NA         NA
## [185,]         NA         NA
## [186,]         NA         NA
## [187,]         NA         NA
## [188,]         NA         NA
## [189,]         NA         NA
## [190,]         NA         NA
## [191,]         NA         NA
## [192,]         NA         NA
## [193,]         NA         NA
## [194,]         NA         NA
## [195,]         NA         NA
## [196,]         NA         NA
## [197,]         NA         NA
## [198,]         NA         NA
## [199,]         NA         NA
## [200,]         NA         NA
## [201,]         NA         NA
## [202,]         NA         NA
## [203,]         NA         NA
## [204,]         NA         NA
## [205,]         NA         NA
## [206,]         NA         NA
## [207,]         NA         NA
## [208,]         NA         NA
## [209,]         NA         NA
## [210,]         NA         NA
## [211,]         NA         NA
## [212,]         NA         NA
## [213,]         NA         NA
## [214,]         NA         NA
## [215,]         NA         NA
## [216,]         NA         NA
## [217,]         NA         NA
## [218,]         NA         NA
## [219,]         NA         NA
## [220,]         NA         NA
## [221,]         NA         NA
## [222,]         NA         NA
## [223,]         NA         NA
## [224,]         NA         NA
## [225,]         NA         NA
## [226,]         NA         NA
## [227,]         NA         NA
## [228,]         NA         NA
## [229,]         NA         NA
## [230,]         NA         NA
## [231,]         NA         NA
## [232,]         NA         NA
## [233,]         NA         NA
## [234,]         NA         NA
## [235,]         NA         NA
## [236,]         NA         NA
## [237,]         NA         NA
## [238,]         NA         NA
## [239,]         NA         NA
## [240,]         NA         NA
## [241,]         NA         NA
## [242,]         NA         NA
## [243,]         NA         NA
## [244,]         NA         NA
## [245,]         NA         NA
## [246,]         NA         NA
## [247,]         NA         NA
## [248,]         NA         NA
## [249,]         NA         NA
## [250,]         NA         NA
## [251,]         NA         NA
```

```r
fix(Imptable)
```

8.Put-Call parity

```r
# Amazon first option
putcall1=putcall2=c() # create dataframe to store the data

for (i in 1:length(Aimpcall)){ # Check put-call parity for all the implied volatilities
  putcall1[i] = blackscholes(1987.89, Aimpcall[i], 36/252,Aoptioncall[i], 0.0216, 'call') - blackscholes
  putcall2[i] = 1987.89 -Aoptioncall[i] * exp(-0.0216*36/252)
  Aparity = abs(putcall1 - putcall2) # if the result is close to zero, then the put-call parity holds
}
  Aparity
```

```
##   [1] 8.278910e-02 8.750845e-02 4.562973e-01 1.144104e-01 2.486540e-01
##   [6] 1.506630e-01 1.685910e-01 3.115942e-01 2.644425e-01 4.082957e-01
##  [11] 2.195169e-01 3.017237e-01 3.306584e-01 2.417202e-01 5.090807e-01
##  [16] 3.230491e-01 3.622608e-01 5.420480e-01 6.020814e-01 4.692731e-01
##  [21] 7.247786e-01 9.851803e-01 1.029935e+00 2.421951e+01 2.429600e+01
##  [26] 8.576827e-01 2.419654e+01 5.528035e-01 1.180896e+00 8.062814e-01
##  [31] 1.020759e+00 1.129307e+00 1.184820e+00 1.716551e+00 2.308929e+01
##  [36] 2.360232e+01 1.162237e+00 2.059777e+00 2.337914e+01 2.405317e+01
##  [41] 1.722607e+00 2.301097e+00 2.417178e+01 1.880954e+00 1.948951e+00
##  [46] 2.550735e+00 2.609977e+00 2.689320e+00 2.749029e+00 3.106599e+00
##  [51] 2.426241e+01 2.979431e+00 3.035239e+00 2.322068e+01 3.227444e+00
##  [56] 2.378022e+00 2.433731e+00 2.439315e+00 2.626403e+00 2.682973e+00
##  [61] 3.324597e+00 3.009143e+00 2.404168e+01 4.054666e+00 3.309838e+00
##  [66] 3.546203e+00 3.549669e+00 4.573558e+00 4.660150e+00 2.325471e+01
##  [71] 4.504015e+00 3.932928e+00 4.122502e+00 5.660359e+00 5.571992e+00
##  [76] 6.197975e+00 5.548535e+00 6.125949e+00 5.936396e+00 6.464811e+00
##  [81] 5.323626e+00 6.934956e+00 7.132718e+00 7.460063e+00 7.170891e+00
##  [86] 6.440956e+00 7.184950e+00 7.412622e+00 8.365043e+00 7.507978e+00
##  [91] 2.516984e+01 9.524680e+00 9.681452e+00 8.827397e+00 9.037017e+00
##  [96] 1.036507e+01 9.772764e+00 1.170024e+01 1.098252e+01 1.119995e+01
## [101] 1.297702e+01 1.319858e+01 1.150512e+01 1.356098e+01 1.046378e+01
## [106] 1.145498e+01 1.027412e+01 9.394144e+00 1.124877e+01 9.644772e+00
## [111] 1.076942e+01 8.666155e+00 9.464999e+00 9.732177e+00 8.628854e+00
## [116] 7.236301e+00 6.015137e+00 5.896868e+00 5.265985e+00 5.409435e+00
## [121] 4.007729e+00 3.825156e+00 2.859809e+00 2.838177e+00 1.683167e+00
## [126] 3.266058e-03 8.591150e-02 7.284571e-01 1.135953e+00 2.967961e+00
## [131] 2.890976e+00 3.364303e+00 4.108457e+00 4.274469e+00 5.511543e+00
## [136] 6.345857e+00 6.602463e+00 7.006703e+00 8.753202e+00 9.328260e+00
## [141] 1.037018e+01 1.140036e+01 1.226804e+01 1.343215e+01 1.441576e+01
## [146] 1.617622e+01 1.552735e+01 1.738606e+01 1.764837e+01 1.799110e+01
## [151] 1.966421e+01 2.034298e+01 1.960538e+01 2.103143e+01 2.156046e+01
## [156] 2.285559e+01 2.290066e+01 2.363669e+01 2.458069e+01 2.472236e+01
## [161] 2.473353e+01 2.627301e+01 2.586874e+01 2.733981e+01 2.688452e+01
## [166] 2.881064e+01 2.868795e+01 1.464006e+01 2.890191e+01 3.030933e+01
## [171] 2.960328e+01 3.048756e+01 2.994813e+01 3.128286e+01 3.022029e+01
## [176] 3.292115e+01 3.223606e+01 3.283474e+01 3.255564e+01 6.127575e+01
## [181] 3.329519e+01 3.354132e+01 1.761753e+01 3.436344e+01 1.345451e+01
## [186] 2.966181e+01 2.882963e+01 1.947383e+01 4.567865e+00 1.949356e+01
## [191] 2.007481e+01 4.957503e+00 5.125219e+00 4.883068e+00 3.416310e+01
## [196] 4.251892e+00 9.542531e+00 1.957411e+01 1.974323e+01 1.980461e+01
## [201] 4.850635e+01 7.386017e+01 3.638160e+01 8.892030e+01 8.506451e+01
## [206] 8.991800e+01 3.584220e+02 4.741400e+02           NA           NA
## [211]           NA           NA           NA           NA           NA
## [216]           NA           NA           NA           NA           NA
## [221]           NA           NA           NA           NA           NA
## [226]           NA           NA           NA           NA           NA
## [231]           NA           NA           NA           NA           NA
## [236]           NA           NA           NA           NA           NA
## [241]           NA           NA           NA           NA           NA
## [246]           NA           NA           NA           NA           NA
## [251]           NA
```

```
# Amazon second option

for (i in 1:length(Aimpcall1)){
  putcall1[i] = blackscholes(1987.89, Aimpcall1[i], 50/252,Aoption1call[i], 0.0216, 'call') - blackschol
  putcall2[i] = 1987.89 -Aoption1call[i] * exp(-0.0216*50/252)
  Aparity1 = abs(putcall1 - putcall2) # if the result is close to zero, then the put-call parity holds
}
  Aparity1
```

```
##   [1] 3.464859e+01 3.782843e+01 5.179572e+01 5.695481e+01 5.721939e+01
##   [6] 5.586426e+01 5.772125e+01 5.912180e+01 5.632177e+01 5.650756e+01
##  [11] 5.711415e+01 5.747273e+01 5.730571e+01 5.803341e+01 5.776566e+01
##  [16] 5.694419e+01 5.789777e+01 5.228638e+01 5.138777e+01 5.198662e+01
##  [21] 5.495202e+01 5.739520e+01 5.086569e+01 4.600974e+01 4.570193e+01
##  [26] 4.695889e+01 5.420529e+01 5.523646e+01 6.622618e+01 7.097607e+01
##  [31] 8.552156e+01 9.890930e+01 1.054876e+02 1.110813e+02 1.177564e+02
##  [36] 1.243822e+02 1.312549e+02 1.552094e+02 1.627026e+02 1.699982e+02
##  [41] 1.773919e+02 1.855330e+02 1.919931e+02 2.086804e+02 3.585076e+02
##  [46] 3.628688e+02 2.609977e+00 2.689320e+00 2.749029e+00 3.106599e+00
##  [51] 2.426241e+01 2.979431e+00 3.035239e+00 2.322068e+01 3.227444e+00
##  [56] 2.378022e+00 2.433731e+00 2.439315e+00 2.626403e+00 2.682973e+00
##  [61] 3.324597e+00 3.009143e+00 2.404168e+01 4.054666e+00 3.309838e+00
##  [66] 3.546203e+00 3.549669e+00 4.573558e+00 4.660150e+00 2.325471e+01
##  [71] 4.504015e+00 3.932928e+00 4.122502e+00 5.660359e+00 5.571992e+00
##  [76] 6.197975e+00 5.548535e+00 6.125949e+00 5.936396e+00 6.464811e+00
##  [81] 5.323626e+00 6.934956e+00 7.132718e+00 7.460063e+00 7.170891e+00
##  [86] 6.440956e+00 7.184950e+00 7.412622e+00 8.365043e+00 7.507978e+00
##  [91] 2.516984e+01 9.524680e+00 9.681452e+00 8.827397e+00 9.037017e+00
##  [96] 1.036507e+01 9.772764e+00 1.170024e+01 1.098252e+01 1.119995e+01
## [101] 1.297702e+01 1.319858e+01 1.150512e+01 1.356098e+01 1.046378e+01
## [106] 1.145498e+01 1.027412e+01 9.394144e+00 1.124877e+01 9.644772e+00
## [111] 1.076942e+01 8.666155e+00 9.464999e+00 9.732177e+00 8.628854e+00
## [116] 7.236301e+00 6.015137e+00 5.896868e+00 5.265985e+00 5.409435e+00
## [121] 4.007729e+00 3.825156e+00 2.859809e+00 2.838177e+00 1.683167e+00
## [126] 3.266058e-03 8.591150e-02 7.284571e-01 1.135953e+00 2.967961e+00
## [131] 2.890976e+00 3.364303e+00 4.108457e+00 4.274469e+00 5.511543e+00
## [136] 6.345857e+00 6.602463e+00 7.006703e+00 8.753202e+00 9.328260e+00
## [141] 1.037018e+01 1.140036e+01 1.226804e+01 1.343215e+01 1.441576e+01
## [146] 1.617622e+01 1.552735e+01 1.738606e+01 1.764837e+01 1.799110e+01
## [151] 1.966421e+01 2.034298e+01 1.960538e+01 2.103143e+01 2.156046e+01
## [156] 2.285559e+01 2.290066e+01 2.363669e+01 2.458069e+01 2.472236e+01
## [161] 2.473353e+01 2.627301e+01 2.586874e+01 2.733981e+01 2.688452e+01
## [166] 2.881064e+01 2.868795e+01 1.464006e+01 2.890191e+01 3.030933e+01
## [171] 2.960328e+01 3.048756e+01 2.994813e+01 3.128286e+01 3.022029e+01
## [176] 3.292115e+01 3.223606e+01 3.283474e+01 3.255564e+01 6.127575e+01
## [181] 3.329519e+01 3.354132e+01 1.761753e+01 3.436344e+01 1.345451e+01
## [186] 2.966181e+01 2.882963e+01 1.947383e+01 4.567865e+00 1.949356e+01
## [191] 2.007481e+01 4.957503e+00 5.125219e+00 4.883068e+00 3.416310e+01
## [196] 4.251892e+00 9.542531e+00 1.957411e+01 1.974323e+01 1.980461e+01
## [201] 4.850635e+01 7.386017e+01 3.638160e+01 8.892030e+01 8.506451e+01
## [206] 8.991800e+01 3.584220e+02 4.741400e+02           NA           NA
## [211]           NA           NA           NA           NA           NA
## [216]           NA           NA           NA           NA           NA
## [221]           NA           NA           NA           NA           NA
## [226]           NA           NA           NA           NA           NA
## [231]           NA           NA           NA           NA           NA
## [236]           NA           NA           NA           NA           NA
## [241]           NA           NA           NA           NA           NA
## [246]           NA           NA           NA           NA           NA
## [251]           NA
```

```
# Amazon third option
for (i in 1:length(Aimpcall2)){
  putcall1[i] = blackscholes(1987.89, Aimpcall2[i], 154/252,Aoption2call[i], 0.0216, 'call') - blacksche
  putcall2[i] = 1987.89 -Aoption2call[i] * exp(-0.0216*153/252)
  Aparity2 = abs(putcall1 - putcall2) # if the result is close to zero, then the put-call parity holds
}
  Aparity2
```

```
##   [1]  0.5068181  0.5018635  0.7168162 25.4520548  0.6348756
##   [6]  0.4016490  0.6621555  0.5963342  1.0365258  1.6754131
##  [11]  1.6246368  1.6592099  1.7897354  2.0192070  4.8153210
##  [16]  2.6927341  2.0965849  2.7304978  2.8558748  3.1173111
##  [21]  4.2349213 24.6268649  4.4103936 24.7107110  5.3643628
##  [26]  2.2920547  5.6912724  5.7647959 25.6335932  8.0033943
##  [31]   7.6513966   8.0287537   8.4564883   8.7331832   8.8088324
##  [36]  10.2942897  24.7672762   5.6648054  10.6210705  25.3215065
##  [41]  11.6523502  12.5325704  24.5462015  12.3333810   7.2772110
##  [46]  14.2451270   7.7818072   8.0588770  24.0120702  14.0638664
##  [51]  17.1887960  17.2398084  18.1945410  18.2705470  14.9557342
##  [56]  21.0888225  12.0915070  23.0757251  23.5805805  20.9452395
##  [61]  24.5614673  26.8267010  28.1849388  29.3913325  14.4517139
##  [66]  28.5999979  26.3552961  11.6407557  13.1349424  11.3667145
##  [71]  11.9940590  22.3165769   9.7875601  21.6093196   9.3606547
##  [76]   5.3209582   4.6435237  14.6943265  13.1356056  28.5026719
##  [81]   9.9436051   3.4103768   5.4975876  10.8145286   8.4027870
##  [86]   1.4302032   9.8049371  13.8673657  16.2747904  18.2304556
##  [91]  19.4815649  20.3070768  21.9823357  13.9732771  27.5484903
##  [96]  29.2735240  29.0719787  31.2224988  33.2487898  33.7468350
## [101]  34.2954691  33.3750183  32.7302221  32.1587874  31.9894014
## [106]  29.7914816  28.1700798  26.6979044  26.1761437  24.3798791
## [111]  23.0327507  20.3356397  19.9882684  17.8158459  18.5134490
## [116]  19.3610611  22.7165947  18.5298136  26.3086337  26.1447825
## [121]  27.0560761  19.3015663  21.4542356  27.9886610  29.6245880
## [126]  28.7598275  29.5965626  29.5076022  30.4178618  28.5200746
## [131]  19.4255789  16.8864430  13.5653795  21.4115337  21.3224233
## [136]  21.1846001  14.7259087  22.3298449  59.0214990  22.6271570
## [141]  22.8869820  60.4454776  17.4741422  23.0964816  23.6060967
## [146]  15.7036275  15.9643635  10.7905835  12.9035203  21.4021769
## [151]  55.6502885   9.8547558  53.3795228  18.6365846  14.5620277
## [156]   5.2169071  16.2675445  60.1543575  10.7314807  16.7410928
## [161]  16.6272634  17.4624520  61.8756166  62.6330570  20.5144433
## [166]  19.9784918  16.5047030  11.1189883  20.9327805  14.3526226
## [171]   5.7271143  50.0329911   4.5130164   3.6780931   7.1041414
## [176]   4.5528921  50.6090049   2.8080374  48.5347508   9.3599285
## [181]   0.6424762  43.7817016  19.1217244  42.5291727  51.1777923
## [186]   1.6536839  53.4206480  52.9365084  39.9936330   3.7793215
## [191]   3.7135582  99.4197893 100.1794336  42.6381589 368.4447885
## [196]          NA          NA          NA          NA          NA
## [201]          NA          NA          NA          NA          NA
## [206]          NA          NA          NA          NA          NA
## [211]          NA          NA          NA          NA          NA
## [216]          NA          NA          NA          NA          NA
## [221]          NA          NA          NA          NA          NA
## [226]          NA          NA          NA          NA          NA
## [231]          NA          NA          NA          NA          NA
## [236]          NA          NA          NA          NA          NA
## [241]          NA          NA          NA          NA          NA
## [246]          NA          NA          NA          NA          NA
## [251]          NA
```

```
# SPY First Option
for (i in 1:length(Simpcall)){
  putcall1[i] = blackscholes(290.83, Simpcall[i], 36/252, SPoptioncall[i], 0.0216, 'call') - blackschole
  putcall2[i] = 290.83 - SPoptioncall[i] * exp(-0.0216*36/252)
  Sparity = abs(putcall1 - putcall2)
}
 Sparity
```

```
##   [1] 4.959317e-03 9.922098e-03 4.955671e-03 4.953798e-03 4.951798e-03
##   [6] 1.486970e-02 1.486397e-02 4.945370e-03 4.942890e-03 1.484556e-02
##  [11] 2.475100e-02 1.483123e-02 1.482347e-02 1.481535e-02 2.470116e-02
##  [16] 2.468738e-02 3.456495e-02 3.454264e-02 4.440645e-02 4.440020e-02
##  [21] 4.439439e-02 4.438933e-02 5.427114e-02 5.426273e-02 5.425582e-02
##  [26] 5.424843e-02 5.423847e-02 5.423109e-02 5.422179e-02 1.135704e-01
##  [31] 6.408988e-02 1.135389e-01 6.406830e-02 6.405921e-02 1.233886e-01
##  [36] 1.827785e-01 7.390545e-02 7.389254e-02 7.387739e-02 1.529748e-01
##  [41] 1.529521e-01 1.529309e-01 1.331180e-01 8.368412e-02 1.726430e-01
##  [46] 9.352572e-02 1.527990e-01 1.824457e-01 1.033408e-01 1.724934e-01
##  [51] 1.032953e-01 2.119811e-01 1.131221e-01 1.130949e-01 1.229391e-01
##  [56] 1.229112e-01 2.117799e-01 1.327174e-01 1.326883e-01 2.709509e-01
##  [61] 1.424790e-01 1.424424e-01 1.424075e-01 1.522254e-01 1.620436e-01
##  [66] 1.619989e-01 1.668813e-01 1.717604e-01 1.815627e-01 1.815127e-01
##  [71] 1.913094e-01 1.912428e-01 2.010297e-01 2.108187e-01 2.107384e-01
##  [76] 2.205135e-01 2.302811e-01 2.301984e-01 2.399495e-01 2.398584e-01
##  [81] 2.496007e-01 2.593307e-01 2.641358e-01 2.689289e-01 2.786372e-01
##  [86] 2.883376e-01 2.980276e-01 3.076896e-01 3.222684e-01 3.319137e-01
##  [91] 3.415542e-01 3.560760e-01 3.705950e-01 3.899836e-01 4.044694e-01
##  [96] 4.238256e-01 4.431351e-01 4.673469e-01 4.915232e-01 5.156418e-01
## [101] 5.446425e-01 5.834079e-01 6.220986e-01 6.656595e-01 7.140435e-01
## [106] 7.672985e-01 8.253419e-01 8.980138e-01 9.853257e-01 1.067624e+00
## [111] 1.174340e+00 1.295651e+00 1.431603e+00 1.321586e+00 5.045857e-01
## [116] 2.781200e-01 1.031360e+00 1.740233e+00 2.384754e+00 2.969726e+00
## [121] 3.474933e+00 3.905069e+00 4.254748e+00 4.523700e+00 4.671517e+00
## [126] 4.854088e+00 4.945587e+00 4.971092e+00 5.046732e+00 5.066536e+00
## [131] 5.060940e+00 5.100574e+00 5.104687e+00 5.093511e+00 5.092325e+00
## [136] 5.080971e+00 1.973243e+00 1.936762e+00 1.925300e+00 5.004463e+00
## [141] 2.107931e+00 3.018927e+00 1.225597e-01 3.025387e+00 4.454020e+00
## [146] 2.017676e+00 8.437875e+00 4.828339e+00 1.960836e+00 1.375398e+01
## [151] 1.969117e+00 9.854756e+00 5.337952e+01 1.863658e+01 1.456203e+01
## [156] 5.216907e+00 1.626754e+01 6.015436e+01 1.073148e+01 1.674109e+01
## [161] 1.662726e+01 1.746245e+01 6.187562e+01 6.263306e+01 2.051444e+01
## [166] 1.997849e+01 1.650470e+01 1.111899e+01 2.093278e+01 1.435262e+01
## [171] 5.727114e+00 5.003299e+01 4.513016e+00 3.678093e+00 7.104141e+00
## [176] 4.552892e+00 5.060900e+01 2.808037e+00 4.853475e+01 9.359928e+00
## [181] 6.424762e-01 4.378170e+01 1.912172e+01 4.252917e+01 5.117779e+01
## [186] 1.653684e+00 5.342065e+01 5.293651e+01 3.999363e+01 3.779321e+00
## [191] 3.713558e+00 9.941979e+01 1.001794e+02 4.263816e+01 3.684448e+02
## [196]           NA           NA           NA           NA           NA
## [201]           NA           NA           NA           NA           NA
## [206]           NA           NA           NA           NA           NA
## [211]           NA           NA           NA           NA           NA
## [216]           NA           NA           NA           NA           NA
## [221]           NA           NA           NA           NA           NA
## [226]           NA           NA           NA           NA           NA
## [231]           NA           NA           NA           NA           NA
## [236]           NA           NA           NA           NA           NA
## [241]           NA           NA           NA           NA           NA
## [246]           NA           NA           NA           NA           NA
## [251]           NA
```

```r
# SPY Second Option
for (i in 1:length(Simpcall1)){
  putcall1[i] = blackscholes(290.83, Simpcall1[i], 50/252, SPoption1call[i], 0.0216, 'call') - blacksche
  putcall2[i] = 290.83 - SPoption1call[i] * exp(-0.0216*50/252)
  Sparity1 = abs(putcall1 - putcall2)
}
 Sparity1
```

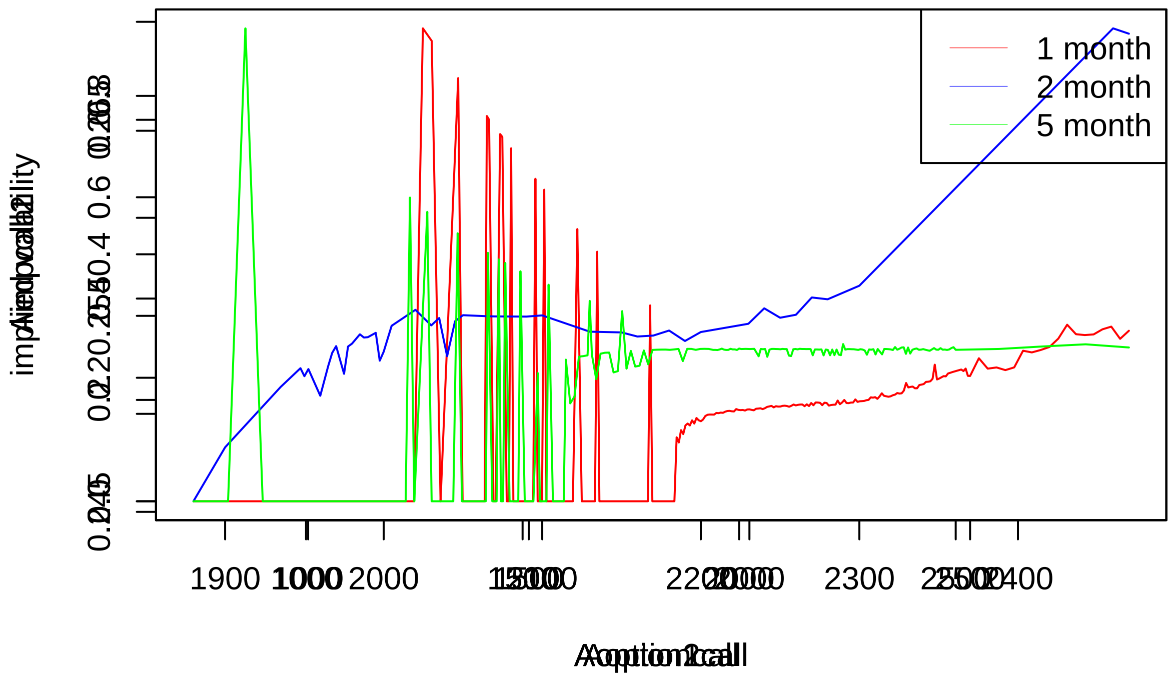
```
##   [1]   0.34099268   0.11201653   0.12615841   0.01364513   0.11433676
##   [6]   0.23777496   0.37691976   0.38407123   0.37722180   0.48474082
##  [11]   0.58402491   0.71828584   1.09127930   1.25166950   1.34982140
##  [16]   1.72139925   2.12852599   2.58109321   3.19548918   3.44717422
##  [21]   3.54818213   4.03256192   4.50740390   6.34619219   8.48526606
##  [26]  10.65438124  21.76934905   0.05423109   0.05422179   0.11357041
##  [31]   0.06408988   0.11353891   0.06406830   0.06405921   0.12338862
##  [36]   0.18277852   0.07390545   0.07389254   0.07387739   0.15297476
##  [41]   0.15295215   0.15293095   0.13311797   0.08368412   0.17264296
##  [46]   0.09352572   0.15279901   0.18244572   0.10334077   0.17249343
##  [51]   0.10329535   0.21198110   0.11312213   0.11309487   0.12293908
##  [56]   0.12291117   0.21177988   0.13271737   0.13268834   0.27095090
##  [61]   0.14247905   0.14244239   0.14240750   0.15222542   0.16204363
##  [66]   0.16199891   0.16688130   0.17176042   0.18156267   0.18151266
##  [71]   0.19130944   0.19124282   0.20102969   0.21081871   0.21073840
##  [76]   0.22051346   0.23028114   0.23019845   0.23994948   0.23985843
##  [81]   0.24960068   0.25933066   0.26413578   0.26892894   0.27863716
##  [86]   0.28833762   0.29802764   0.30768963   0.32226837   0.33191366
##  [91]   0.34155417   0.35607596   0.37059502   0.38998357   0.40446944
##  [96]   0.42382561   0.44313509   0.46734689   0.49152317   0.51564177
## [101]   0.54464249   0.58340789   0.62209856   0.66565953   0.71404353
## [106]   0.76729849   0.82534192   0.89801383   0.98532566   1.06762437
## [111]   1.17434030   1.29565130   1.43160335   1.32158583   0.50458568
## [116]   0.27812004   1.03136024   1.74023258   2.38475440   2.96972555
## [121]   3.47493305   3.90506901   4.25474758   4.52370003   4.67151749
## [126]   4.85408828   4.94558682   4.97109212   5.04673237   5.06653593
## [131]   5.06094049   5.10057406   5.10468667   5.09351065   5.09232456
## [136]   5.08097072   1.97324340   1.93676240   1.92530001   5.00446281
## [141]   2.10793112   3.01892702   0.12255967   3.02538689   4.45401984
## [146]   2.01767591   8.43787521   4.82833900   1.96083617  13.75398163
## [151]   1.96911654   9.85475583  53.37952275  18.63658455  14.56202765
## [156]   5.21690705  16.26754447  60.15435752  10.73148074  16.74109285
## [161]  16.62726335  17.46245196  61.87561664  62.63305702  20.51444334
## [166]  19.97849181  16.50470305  11.11898831  20.93278054  14.35262263
## [171]   5.72711428  50.03299105   4.51301636   3.67809312   7.10414138
## [176]   4.55289208  50.60900490   2.80803739  48.53475078   9.35992845
## [181]   0.64247623  43.78170162  19.12172438  42.52917267  51.17779233
## [186]   1.65368393  53.42064799  52.93650840  39.99363298   3.77932147
## [191]   3.71355820  99.41978934 100.17943358  42.63815885 368.44478852
## [196]           NA           NA           NA           NA           NA
## [201]           NA           NA           NA           NA           NA
## [206]           NA           NA           NA           NA           NA
## [211]           NA           NA           NA           NA           NA
## [216]           NA           NA           NA           NA           NA
## [221]           NA           NA           NA           NA           NA
## [226]           NA           NA           NA           NA           NA
## [231]           NA           NA           NA           NA           NA
## [236]           NA           NA           NA           NA           NA
## [241]           NA           NA           NA           NA           NA
## [246]           NA           NA           NA           NA           NA
## [251]           NA
```

```
# SPY Third Option
for (i in 1:length(Simpcall2)){
  putcall1[i] = blackscholes(290.83, Simpcall2[i], 109/252, SPoption2call[i], 0.0216, 'call') - blacksc
  putcall2[i] = 290.83- SPoption2call[i] * exp(-0.0216*109/252)
  Sparity2 = abs(putcall1 - putcall2)
}
 Sparity2
```

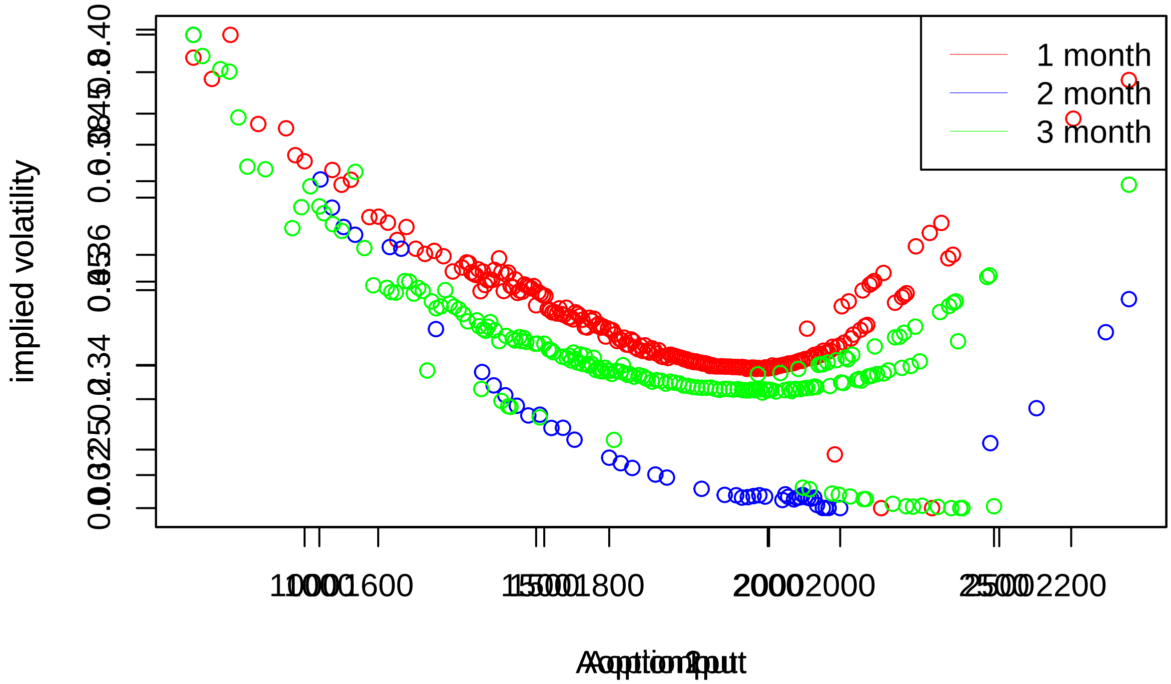
```
##   [1]   0.1649881   0.2284627   0.3116107   0.2919409   0.1843063
##   [6]   0.3407638   0.4730512   0.3650695   0.2084802   0.3893746
##  [11]   0.3990646   0.2229166   0.5802699   0.2422897   0.4524893
##  [16]   0.3789322   0.2615705   0.4031695   0.3834803   0.5645012
##  [21]   0.2904624   0.7800257   0.6130226   0.5344773   0.5538973
##  [26]   0.4705504   0.4801687   0.8670829   0.3675622   0.5285475
##  [31]   0.9448465   0.6210896   0.8363918   0.4252970   0.7427872
##  [36]   1.2478682   0.6542014   0.9232081   0.4926286   1.0402364
##  [41]   1.0742128   0.5358282   0.9460968   0.5694606   1.0139200
##  [46]   0.6030308   0.8859294   1.6210519   0.6606269   1.1832890
##  [51]   0.7086914   0.7278186   0.7566806   0.7806475   0.8046199
##  [56]   0.8334120   0.8621840   1.2622706   0.9294550   1.3490552
##  [61]   0.9966228   1.0350452   2.0823276   1.1166885   2.2327588
##  [66]   1.2030526   1.2461902   1.7734375   1.3469904   2.3353809
##  [71]   1.4622470   1.5198282   1.5773418   1.6495086   1.5824828
##  [76]   2.6921549   2.4436601   1.9325487   1.9819714   1.9677952
##  [81]   2.1917885   1.9785986   3.7746529   1.9837423   1.9786124
##  [86]   1.9780611   1.9724534   1.9714822   1.9653008   1.9587893
##  [91]   1.9470661   1.9450959   1.9476814   3.1850340   1.9367733
##  [96]   1.9333625   1.9197286   1.9056672   1.8962073   1.8613018
## [101]   1.8410386   1.8153687   0.1515985   2.3224716   3.9084086
## [106]   5.3302656   6.1823955   6.4504398   3.2883281   3.4843789
## [111]   2.2782222   9.1479427   7.3177987   1.3215858   0.5045857
## [116]   0.2781200   1.0313602   1.7402326   2.3847544   2.9697255
## [121]   3.4749330   3.9050690   4.2547476   4.5237000   4.6715175
## [126]   4.8540883   4.9455868   4.9710921   5.0467324   5.0665359
## [131]   5.0609405   5.1005741   5.1046867   5.0935107   5.0923246
## [136]   5.0809707   1.9732434   1.9367624   1.9253000   5.0044628
## [141]   2.1079311   3.0189270   0.1225597   3.0253869   4.4540198
## [146]   2.0176759   8.4378752   4.8283390   1.9608362  13.7539816
## [151]   1.9691165   9.8547558  53.3795228  18.6365846  14.5620277
## [156]   5.2169071  16.2675445  60.1543575  10.7314807  16.7410928
## [161]  16.6272634  17.4624520  61.8756166  62.6330570  20.5144433
## [166]  19.9784918  16.5047030  11.1189883  20.9327805  14.3526226
## [171]   5.7271143  50.0329911   4.5130164   3.6780931   7.1041414
## [176]   4.5528921  50.6090049   2.8080374  48.5347508   9.3599285
## [181]   0.6424762  43.7817016  19.1217244  42.5291727  51.1777923
## [186]   1.6536839  53.4206480  52.9365084  39.9936330   3.7793215
## [191]   3.7135582  99.4197893 100.1794336  42.6381589 368.4447885
## [196]          NA          NA          NA          NA          NA
## [201]          NA          NA          NA          NA          NA
## [206]          NA          NA          NA          NA          NA
## [211]          NA          NA          NA          NA          NA
## [216]          NA          NA          NA          NA          NA
## [221]          NA          NA          NA          NA          NA
## [226]          NA          NA          NA          NA          NA
## [231]          NA          NA          NA          NA          NA
## [236]          NA          NA          NA          NA          NA
## [241]          NA          NA          NA          NA          NA
## [246]          NA          NA          NA          NA          NA
## [251]          NA
```

9.Create a 2 dimensional plot of implied volatilities versus strike K

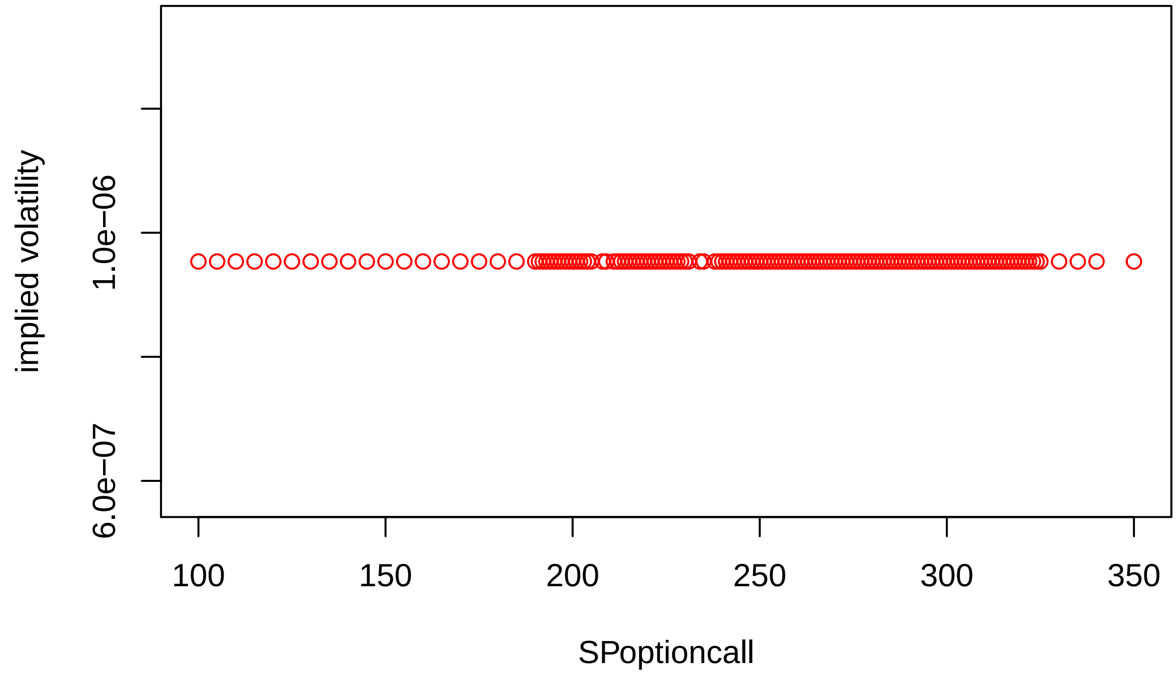
```
# Plot the implied volatilities
# Amazon Call Option
plot(Aimpcall~Aoptioncall, col = 'red', ylab = "implied volatility",type='l')
par(new = TRUE)
plot(Aimpcall1~Aoption1call, col = 'blue',type='l')
par(new = TRUE)
plot(Aimpcall2~Aoption2call, col = 'green',type='l')
par(new=TRUE)
legend('topright', c('1 month','2 month','5 month'), col = c('red','blue','green'), lwd = 0.3)
```



```
# Amazon Put Option
plot(Aimput~Aoptionput, col = 'red', ylab = "implied volatility")
par(new = TRUE)
plot(Aimput1~Aoption1put, col = 'blue', ylab = "implied volatility")
par(new = TRUE)
plot(Aimput2~Aoption2put, col = 'green', ylab = "implied volatility")
legend('topright', c('1 month','2 month','3 month'), col = c('red','blue','green'), lwd = 0.3)
```



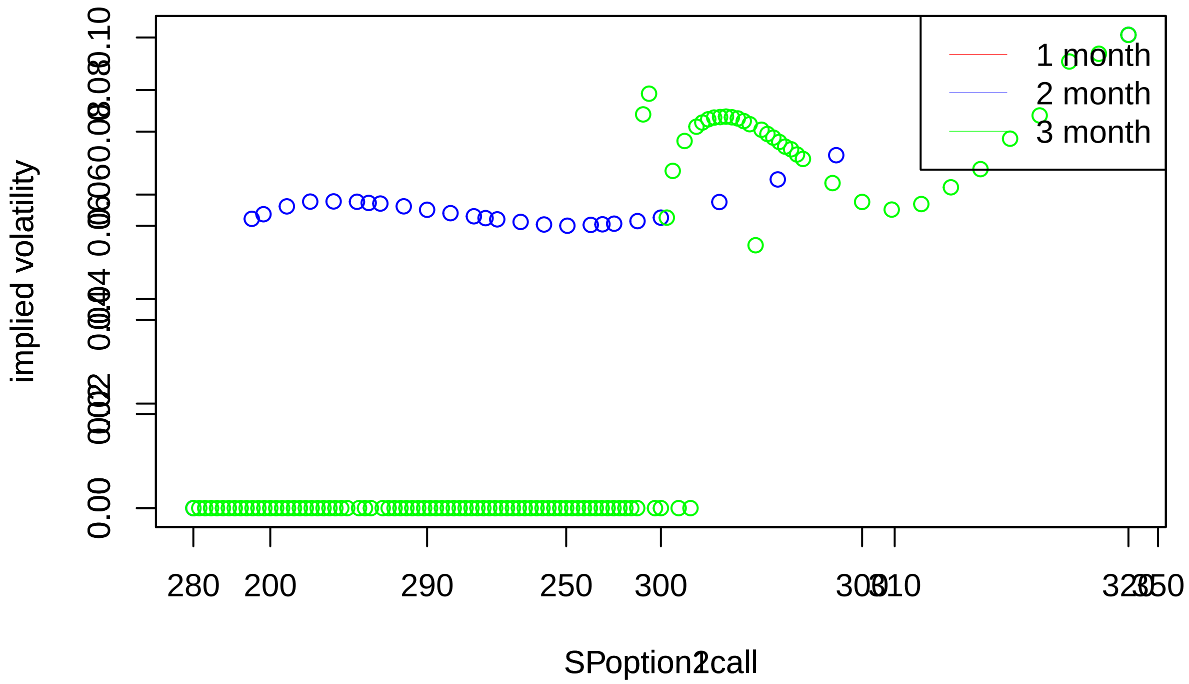
```
# SPY call
plot(Simpcall~SPoptioncall, col = 'red', ylab = "implied volatility")
```



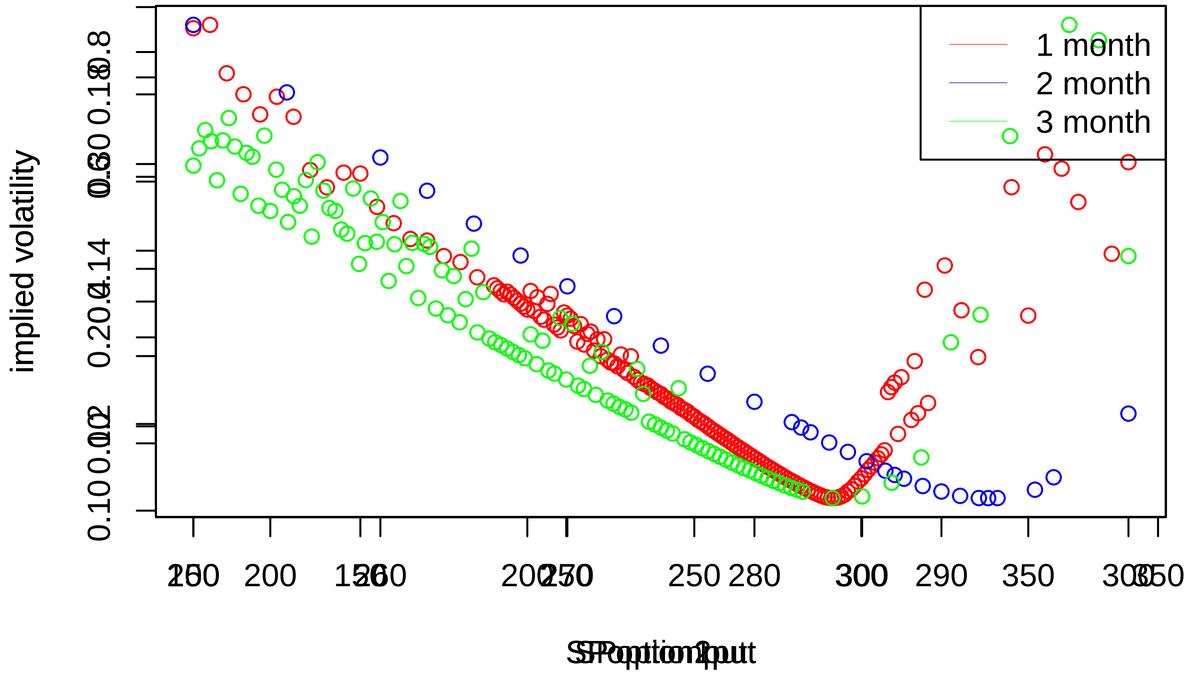
```
par(new)
```

```
## NULL
```

```
plot(Simpcall1~SPoption1call, col = 'blue', ylab = "implied volatility")
par(new = TRUE)
plot(Simpcall2~SPoption2call, col = 'green', ylab = "implied volatility")
legend('topright', c('1 month','2 month','3 month'), col = c('red','blue','green'), lwd = 0.3)
```



```r
# SPY put
plot(Simput~SPoptionput, col = 'red', ylab = "implied volatility")
par(new = TRUE)
plot(Simput1~SPoption1put, col = 'blue', ylab = "implied volatility")
par(new = TRUE)
plot(Simput2~SPoption2put, col = 'green', ylab = "implied volatility")
legend('topright', c('1 month','2 month','3 month'), col = c('red','blue','green'), lwd = 0.3)
```



I have no idea why the Amazon Plots look horrible, but fortunatly the SPY plots look good enough. I noticed that the implied volatility is very low near the spot price,afterwards it becomes larger away from the spot The path is more like NIke brand.

10. Calculate Delta,Vega, Gamma

```
# Call
# Mathematical Method

# Amazon Call Option
Adelta = Adelta1 = Adelta2= c()
Avega = Avega1 = Avega2 = c()
Agamma = Agamma1 = Agamma2 = c()

# First option
for ( i in 1:length(Aoptioncall)){
  S0 = 1989.87
  q = 0
  r = 0.0216
  t = 36/252
  d1 = (log(S0/Aoptioncall[i]) + (r + Aimpcall[i]^2/2) * t)/( Aimpcall[i] * sqrt(t))
  Adelta[i] = exp(-q * t) * pnorm(d1)
  Avega[i] = S0*((1/sqrt(2*pi))*exp(-(d1^2)/2))*sqrt(t)
  Agamma[i] = (exp(-r * t)/(S0 * Aimpcall[i] * sqrt(t))) * exp(-d1^2/2)/sqrt(2 * pi)
}
```

```r
# Second month
for ( i in 1:length(Aoption1call)){
  S0 = 1989.87
  q = 0
  r = 0.0216
  t = 50/252
  d1 = (log(S0/Aoption1call[i]) + (r + Aimpcall1[i]^2/2) * t)/( Aimpcall1[i] * sqrt(t))
  Adelta1[i] = exp(-q * t) * pnorm(d1)
  Avega1[i] = S0*((1/sqrt(2*pi))*exp(-(d1^2)/2))*sqrt(t)
  Agamma1[i] = (exp(-r * t)/(S0 * Aimpcall1[i] * sqrt(t))) * exp(-d1^2/2)/sqrt(2 * pi)
}



# Third month
for ( i in 1:length(Aoption2call)){
  S0 = 1989.87
  q=0
  r = 0.0216
  t = 154/252
  d1 = (log(S0/Aoption2call[i]) + (r + Aimpcall2[i]^2/2) * t)/( Aimpcall2[i] * sqrt(t))
  Adelta2[i] = exp(-q * t) * pnorm(d1)
  Avega2[i] = S0*((1/sqrt(2*pi))*exp(-(d1^2)/2))*sqrt(t)
  Agamma2[i] = (exp(-r * t)/(S0 * Aimpcall2[i] * sqrt(t))) * exp(-d1^2/2)/sqrt(2 * pi)
}




# SPY call option
Sdelta = Sdelta1 = Sdelta2= c()
Svega = Svega1 = Svega2 = c()
Sgamma = Sgamma1 = Sgamma2 = c()

#  First Option
for ( i in 1:length(SPoptioncall)){
  S0 = 290.83
  q = 0
  r = 0.0216
  t = 36/252
  d1 = (log(S0/SPoptioncall[i]) + (r + Simpcall[i]^2/2) * t)*( 1/Simpcall[i] * sqrt(t))
  Sdelta[i] = exp(-q * t) * pnorm(d1)
  Svega[i] = S0*((1/sqrt(2*pi))*exp(-(d1^2)/2))*sqrt(t)
  Sgamma[i] = (exp(-r * t)/(S0 * Simpcall[i] * sqrt(t))) * exp(-d1^2/2)/sqrt(2 * pi)
}



# Second Option
for ( i in 1:length(SPoption1call)){
  S0 = 290.83
  q = 0
  r = 0.0216
  t = 50/252
  d1 = (log(S0/SPoption1call[i]) + (r + Simpcall1[i]^2/2) * t)/( Simpcall1[i] * sqrt(t))
  Sdelta1[i] = exp(-q * t) * pnorm(d1)
  Svega1[i] = S0*((1/sqrt(2*pi))*exp(-(d1^2)/2))*sqrt(t)
  Sgamma1[i] = (exp(-r * t)/(S0 * Simpcall1[i] * sqrt(t))) * exp(-d1^2/2)/sqrt(2 * pi)
}



# Third Option
for ( i in 1:length(SPoption2call)){
  S0 = 290.83
  q = 0
  r = 0.0216
  t = 109/252
  d1 = (log(S0/SPoption2call[i]) + (r + Simpcall2[i]^2/2) * t)/( Simpcall2[i] * sqrt(t))
  Sdelta2[i] = exp(-q * t) * pnorm(d1)
  Svega2[i] = S0*((1/sqrt(2*pi))*exp(-(d1^2)/2))*sqrt(t)
  Sgamma2[i] = (exp(-r * t)/(S0 * Simpcall2[i] * sqrt(t))) * exp(-d1^2/2)/sqrt(2 * pi)
}

# Numerical Method
# Amazon options
r=0.0216
At=36/252
At2=50/252
At3=154/252
st3=109/252
error=1e-5
A2delta = A2delta1 = A2delta2= c()
A2vega = A2vega1 = A2vega2 = c()
A2gamma = A2gamma1 = A2gamma2 = c()

# First option
for ( i in 1:length(Aoptioncall)){
  S0=1989.87
  deltaf = function(S0, sigma, At, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At, K, r, "call") - blackscholes(S0 - error, sigma, At, K, r, "cal
  }
  A2delta[i] = deltaf(S0, Aimpcall[i], At, Aoptioncall[i], r, error = 1e-5 )
  vegaf = function(S0, sigma, At, K, r, error = 1e-5) {
    (blackscholes(S0, sigma + error, At, K, r) - blackscholes(S0, sigma - error, At, K, r))/(2 * error)
  }
 A2vega[i] = vegaf(S0, Aimpcall[i], At, Aoptioncall[i], r, error = 1e-5)
  gammaf = function(S0, sigma, At, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At, K, r, "call") - 2 * blackscholes(S0, sigma, At, K, r, "call") +
  }
  A2gamma[i] = gammaf(S0, Aimpcall[i], At, Aoptioncall[i], r, error = 1e-5)
}
```

```
# Second Option
for ( i in 1:length(Aoption1call)){
  S0=1989.87
  deltaf = function(S0, sigma, At2, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At2, K, r, "call") - blackscholes(S0 - error, sigma, At2, K, r, "ca
  }
  A2delta1[i] = deltaf(S0, Aimpcall1[i], At2, Aoption1call[i], r, error = 1e-5 )
  vegaf = function(S0, sigma, At2, K, r, error = 1e-5) {
    (blackscholes(S0, sigma + error, At2, K, r) - blackscholes(S0, sigma - error, At2, K, r))/(2 * erro
  }
 A2vega1[i] = vegaf(S0, Aimpcall1[i], At2, Aoption1call[i], r, error = 1e-5)
  gammaf = function(S0, sigma, At2, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At2, K, r, "call") - 2 * blackscholes(S0, sigma, At2, K, r, "call"
  }
  A2gamma1[i] = gammaf(S0, Aimpcall1[i], At2, Aoption1call[i], r, error = 1e-5)
}



# Third Option
for ( i in 1:length(Aoption2call)){
  S0=1989.87
  deltaf = function(S0, sigma, At3, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At3, K, r, "call") - blackscholes(S0 - error, sigma, At3, K, r, "ca
  }
  A2delta2[i] = deltaf(S0, Aimpcall2[i], At3, Aoption2call[i], r, error = 1e-5 )
  vegaf = function(S0, sigma, At3, K, r, error = 1e-5) {
    (blackscholes(S0, sigma + error, At3, K, r) - blackscholes(S0, sigma - error, At3, K, r))/(2 * erro
  }
 A2vega2[i] = vegaf(S0, Aimpcall2[i], At3, Aoption2call[i], r, error = 1e-5)
  gammaf = function(S0, sigma, At3, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At3, K, r, "call") - 2 * blackscholes(S0, sigma, At3, K, r, "call"
  }
  A2gamma2[i] = gammaf(S0, Aimpcall2[i], At3, Aoption2call[i], r, error = 1e-5)
}




# SPY Call Option
# First option
S2delta = S2delta1 = S2delta2= c()
S2vega = S2vega1 = S2vega2 = c()
S2gamma = S2gamma1 = S2gamma2 = c()

for ( i in 1:length(SPoptioncall)){
   S0=290.83
  deltaf = function(S0, sigma, At, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At, K, r, "call") - blackscholes(S0 - error, sigma, At, K, r, "call
  }
  S2delta[i] = deltaf(S0, Simpcall[i], At, SPoptioncall[i], r, error = 1e-5 )
  vegaf = function(S0, sigma, At, K, r, error = 1e-5) {
    (blackscholes(S0, sigma + error, At, K, r) - blackscholes(S0, sigma - error, At, K, r))/(2 * error)
  }
 S2vega[i] = vegaf(S0, Simpcall[i], At, SPoptioncall[i], r, error = 1e-5)
  gammaf = function(S0, sigma, At, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At, K, r, "call") - 2 * blackscholes(S0, sigma, At, K, r, "call") +
  }
  S2gamma[i] = gammaf(S0, Simpcall[i], At, SPoptioncall[i], r, error = 1e-5)
}



# Second Option
for ( i in 1:length(SPoption1call)){
  S0=290.83
  deltaf = function(S0, sigma, At2, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At2, K, r, "call") - blackscholes(S0 - error, sigma, At2, K, r, "ca
  }
  S2delta1[i] = deltaf(S0, Simpcall1[i], At2, SPoption1call[i], r, error = 1e-5 )
  vegaf = function(S0, sigma, At2, K, r, error = 1e-5) {
    (blackscholes(S0, sigma + error, At2, K, r) - blackscholes(S0, sigma - error, At2, K, r))/(2 * erro
  }
 S2vega1[i] = vegaf(S0, Simpcall1[i], At2, SPoption1call[i], r, error = 1e-5)
  gammaf = function(S0, sigma, At2, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At2, K, r, "call") - 2 * blackscholes(S0, sigma, At2, K, r, "call")
  }
  S2gamma1[i] = gammaf(S0, Simpcall1[i], At2, SPoption1call[i], r, error = 1e-5)
}



# Third Option
for ( i in 1:length(SPoption2call)){
  S0=290.83
  deltaf = function(S0, sigma, st3, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, st3, K, r, "call") - blackscholes(S0 - error, sigma, st3, K, r, "ca
  }
  S2delta2[i] = deltaf(S0, Simpcall2[i], st3, SPoption2call[i], r, error = 1e-5 )
  vegaf = function(S0, sigma, st3, K, r, error = 1e-5) {
    (blackscholes(S0, sigma + error, st3, K, r) - blackscholes(S0, sigma - error, st3, K, r))/(2 * erro
  }
 S2vega2[i] = vegaf(S0, Simpcall2[i], st3, SPoption2call[i], r, error = 1e-5)
  gammaf = function(S0, sigma, st3, K, r, error = 1e-5) {
    (blackscholes(S0 + error, sigma, At3, K, r, "call") - 2 * blackscholes(S0, sigma, st3, K, r, "call")
  }
  S2gamma2[i] = gammaf(S0, Aimpcall2[i], At3, Aoption2call[i], r, error = 1e-5)
}




#Create a table for the result
 library(plyr)
Ac1=cbind(Adelta,A2delta)
```

```
Ac2=cbind(Avega,A2vega)
Ac3=cbind(Agamma,A2gamma)
Ac4=cbind(Adelta1,A2delta1)
Ac5=cbind(Adelta2,A2delta2)
Ac6=cbind(Avega1,A2vega1)
Ac7=cbind(Avega2,A2vega2)
Ac8=cbind(Agamma1,A2gamma1)
Ac9=cbind(Agamma2,A2gamma2)
Sc1=cbind(Sdelta,S2delta)
Sc2=cbind(Svega,S2vega)
Sc3=cbind(Sgamma,S2gamma)
Sc4=cbind(Sdelta1,S2delta1)
Sc5=cbind(Sdelta2,S2delta2)
Sc6=cbind(Svega1,S2vega1)
Sc7=cbind(Svega2,S2vega2)
Sc8=cbind(Sgamma1,S2gamma1)
Sc9=cbind(Sgamma2,S2gamma2)
list2=list()
list2[[1]]=data.frame(t(Ac1))
list2[[2]]=data.frame(t(Ac2))
list2[[3]]=data.frame(t(Ac3))
list2[[4]]=data.frame(t(Ac4))
list2[[5]]=data.frame(t(Ac5))
list2[[6]]=data.frame(t(Ac6))
list2[[7]]=data.frame(t(Ac7))
list2[[8]]=data.frame(t(Ac8))
list2[[9]]=data.frame(t(Ac9))
list2[[10]]=data.frame(t(Sc1))
list2[[11]]=data.frame(t(Sc2))
list2[[12]]=data.frame(t(Sc3))
list2[[13]]=data.frame(t(Sc4))
list2[[14]]=data.frame(t(Sc5))
list2[[15]]=data.frame(t(Sc6))
list2[[16]]=data.frame(t(Sc7))
list2[[17]]=data.frame(t(Sc8))
list2[[18]]=data.frame(t(Sc9))

indextable=rbind.fill(list2)
row.names(indextable)=c()
Indextable=t(indextable)
head(Indextable)
```

```
##    [,1] [,2] [,3]     [,4] [,5]         [,6]      [,7]      [,8]      [,9]
## X1    1    1    0 62607495    0  0.000000000 0.7301189 0.7301189 1.0000000
## X2    1    1    0 60613657    0  0.004547474 0.6956446 0.6956446 1.0000000
## X3    1    1    0 59616738    0 -0.004547474 0.6345722 0.6345723 1.0000000
## X4    1    1    0 58619819    0  0.004547474 0.6124914 0.6124914 0.9575173
## X5    1    1    0 56625981    0  0.000000000 0.6082145 0.6082145 1.0000000
## X6    1    1    0 54632143    0 -0.004547474 0.6037369 0.6037369 1.0000000
##        [,10]    [,11]    [,12]    [,13]        [,14]       [,15]
## X1 1.0000000 293.0059 293.0059   0.0000 6.297869e+07 0.001512416
## X2 1.0000000 310.1806 310.1806   0.0000 6.001803e+07 0.001583319
## X3 1.0000000 333.2921 333.2921   0.0000 5.903114e+07 0.001680457
## X4 0.9575173 339.4535 339.4535 140.7543 1.407543e+02 0.001705028
## X5 1.0000000 340.5158 340.5158   0.0000 5.508360e+07 0.001713148
## X6 1.0000000 341.5828 341.5828   0.0000 5.409671e+07 0.001716022
##           [,16]        [,17]        [,18] [,19] [,20] [,21]   [,22] [,23]
## X1 -0.002273737 0.000000e+00 -0.004547474     1     1     0 9556905     0
## X2  0.000000000 0.000000e+00 -0.004547474     1     1     0 9307675     0
## X3  0.000000000 0.000000e+00 -0.004547474     1     1     0 9058445     0
## X4 -0.006821210 7.495583e-05  0.004547474     1     1     0 8809216     0
## X5  0.004547474 0.000000e+00  0.000000000     1     1     0 8559986     0
## X6  0.004547474 0.000000e+00  0.000000000     1     1     0 8310756     0
##    [,24]     [,25]     [,26] [,27] [,28]    [,29]        [,30] [,31]
## X1     0 1.0000000 1.0000000     1     1  0.00000 601371.61205     0
## X2     0 0.9138404 0.9138404     1     1 20.36407     20.36407     0
## X3     0 0.8985180 0.8985180     1     1 22.98033     22.98033     0
## X4     0 0.8651072 0.8651072     1     1 28.11150     28.11150     0
## X5     0 0.8295010 0.8295010     1     1 32.84368     32.84368     0
## X6     0 0.7932326 0.7932326     1     1 36.99526     36.99526     0
##      [,32]      [,33]      [,34] [,35]       [,36]
## X1 5278449 0.00000000 0.00000000     0 0.000000000
## X2 5228914 0.02182833 0.02216893     0 0.000000000
## X3 5179379 0.02424588 0.02330580     0 0.000000000
## X4 5129844 0.02888852 0.02899014     0 0.001101341
## X5 5080309 0.03322465 0.03325340     0 0.000000000
## X6 5030774 0.03740371 0.03808509     0 0.000000000
```

```
# The result of two method is nearly the same.
```

11.Calculate Option Pirce for DATA2

```
# Amazon Options Values
# Assign some value and dataframe

r = 0.0216
At=34/252
At2=48/252
At3=153/252
st3=107/252

Acall=Acall1=Acall2=Aput=Aput1=Aput2=c()
# first option
for (i in 1:length(Aoptioncall)){
  Acall = blackscholes(1970.19, Aimpcall,At , Aoptioncall[i], 0.0216, 'call')
}

for (i in 1:length(Aoptionput)){
  Aput = blackscholes(1970.19, Aimput, At, Aoptionput[i], 0.0216, 'put')
}


# Second Option
for (i in 1:length(Aoption1call)){
  Acall1 = blackscholes(1970.19, Aimpcall1, At2, Aoption1call[i], 0.0216, 'call')
}


for (i in 1:length(Aoption1put)) {
  Aput1 = blackscholes(1970.19, Aimput1, At2, Aoption1put[i], 0.0216, 'put')
}


#Third Option
for (i in 1:length(Aoption2call)){
  Acall2 = blackscholes(1970.19, Aimpcall2, At3, Aoption2call[i], 0.0216, 'call')
}

for (i in 1:length(Aoption2put)){
  Aput2 = blackscholes(1970.19, Aimput2, At3, Aoption2put[i], 0.0216, 'put')
}



# SPY Option Values
Scall=Scall1=Scall2=Sput=Sput1=Sput2=c()


# SPY First Option
for (i in 1:length(SPoptioncall)){
  Scall = blackscholes(290.88, Simpcall,At , SPoptioncall[i], 0.0216, 'call')
}

for (i in 1:length(SPoptionput)){
  Sput = blackscholes(290.88, Simput, At, SPoptionput[i], 0.0216, 'put')
}



# Second Option

for (i in 1:length(SPoption1call)){
  Scall1 = blackscholes(290.88, Simpcall1,At , SPoption1call[i], 0.0216, 'call')
}


for (i in 1:length(SPoption1put)){
  Sput1 = blackscholes(290.88, Simput1, At, SPoption1put[i], 0.0216, 'put')
}


# Third Option
for (i in 1:length(SPoption2call)){
  Scall2 = blackscholes(290.88, Simpcall2,st3 , SPoption2call[i], 0.0216, 'call')
}

for (i in 1:length(SPoption2put)){
  Sput2 = blackscholes(290.88, Simput2, st3, SPoption2put[i], 0.0216, 'put')
}
```

Part3. Numerical Integration of real-valued functions

1.trapezoidal Rule and Simpson Rule

```
TrapezoidRule = function(a, b, N, f) #trapezoid
{
h = (b-a)/(N-1)
theSum = 0.5 * h * (f(a)+f(b))
for (i in 1:(N-2)) {
ai = a + i*h
theSum = theSum + h*f(ai)
}
theSum
}
SimpsonRule = function(a, b, N, f) #simpson
{
N = N-1
h = (b-a)/N
x = seq(from=a, to=b, by=h/2)
y = f(x)
ix1 = seq(from=3, by=2, to=2*N+1)
ix2 = seq(from=2, by=2, to=2*N )
h/6 * (y[1] + 2*sum(y[ix1]) + 4*sum(y[ix2]) + y[2*N+1])
}
a = -10^6; b=10^6; N=10^6; n=6;
f = function(x) {ifelse(x==0,yes = 1,no = sin(x)/x)}
table1 = matrix(0, nrow = n, ncol = 4, dimnames = list(
c(1:n), c('N', 'a', 'trapezoid_definite_integral', 'simpson_definite_integral')))
for(i in 1:n){
table1[i,1] = N*i
table1[i,2] = abs(a)
table1[i,3] = TrapezoidRule(a, b, table1[i, 1], f)
table1[i,4] = SimpsonRule(a, b, table1[i, 1], f)
}
print(table1)
```

```
##       N     a trapezoid_definite_integral simpson_definite_integral
## 1 1e+06 1e+06                    3.141591                  3.141591
## 2 2e+06 1e+06                    3.141591                        NA
## 3 3e+06 1e+06                    3.141591                  3.141591
## 4 4e+06 1e+06                    3.141591                  3.141591
## 5 5e+06 1e+06                    3.141591                  3.141591
## 6 6e+06 1e+06                    3.141591                  3.141591
```

2.

```
table2 = matrix(0, nrow = n, ncol = 7, dimnames = list(
c(1:n), c('N', 'a', 'trapezoid_definite_integral', 'simpson_definite_integral',
'trapezoid_truncation_error', 'simpson_truncation_error', 'compare')))
for(i in 1:n) {
table2[i,1] = N*i
table2[i,2] = b*i
table2[i,3] = TrapezoidRule(-table2[i,2], table2[i,2], table2[i, 1], f)
table2[i,4] = SimpsonRule(-table2[i,2], table2[i,2], table1[i, 1], f)
table2[i,5] = table2[i,3] - pi
table2[i,6] = table2[i,4] - pi
table2[i,7] = (abs(table2[i,5]) - abs(table2[i,6]))
}
print(table2)
```

```
##       N     a trapezoid_definite_integral simpson_definite_integral
## 1 1e+06 1e+06                    3.141591                  3.141591
## 2 2e+06 2e+06                    3.141592                        NA
## 3 3e+06 3e+06                    3.141592                  3.141592
## 4 4e+06 4e+06                    3.141593                  3.141592
## 5 5e+06 5e+06                    3.141593                  3.141593
## 6 6e+06 6e+06                    3.141593                  3.141593
##   trapezoid_truncation_error simpson_truncation_error       compare
## 1              -1.202971e-06            -2.118625e-06 -9.156540e-07
## 2              -4.847779e-07                       NA            NA
## 3              -2.047636e-07            -5.157048e-07 -3.109412e-07
## 4              -4.463545e-08            -2.352273e-07 -1.905918e-07
## 5               5.530645e-08            -4.356161e-08  1.174484e-08
## 6               1.161388e-07             8.873893e-08  2.739992e-08
```

3.

```
a=-10^6; b=10^6; N=10; n=10; e=10^(-4);
table3 = matrix(0, nrow = n, ncol = 6, dimnames = list(
c(1:n), c('N', 'a', 'trapezoid_definite_integral', 'simpson_definite_integral',
'|I(k)-I(k-1)|_for_trapezoid', '|I(k)-I(k-1)|_for_simpson')))
for(k in 1:n) {
table3[k,1] = ifelse(k==1, yes=N, no=10*table3[k-1,1])
table3[k,2] = a
table3[k,3] = TrapezoidRule(a, b, table3[k, 1], f)
table3[k,5] = ifelse(k==1,yes=NA, no=abs(table3[k,3] - table3[k-1,3]))
if(k>1 && table3[k,5]<e) break
}
for(k in 1:n) {
table3[k,1] = ifelse(k==1, yes=N, no=10*table3[k-1,1])
table3[k,2] = a
table3[k,4] = SimpsonRule(a, b, table3[k, 1], f)
table3[k,6] = ifelse(k==1,yes=NA, no=abs(table3[k,4] - table3[k-1,4]))
if(k>1 && table3[k,6]<e) break
}
print(table3)
```

```
##        N      a trapezoid_definite_integral simpson_definite_integral
## 1  1e+01 -1e+06                   -2.905369              1.481463e+05
## 2  1e+02 -1e+06                   -3.160205              1.346802e+04
## 3  1e+03 -1e+06                    3.142395              1.335176e+03
## 4  1e+04 -1e+06                   -3.141266              1.309000e+02
## 5  1e+05 -1e+06                   -3.141622              1.361358e+01
## 6  1e+06 -1e+06                    3.141591              3.141591e+00
## 7  1e+07 -1e+06                    3.141591              3.141591e+00
## 8  0e+00  0e+00                    0.000000              0.000000e+00
## 9  0e+00  0e+00                    0.000000              0.000000e+00
## 10 0e+00  0e+00                    0.000000              0.000000e+00
##    |I(k)-I(k-1)|_for_trapezoid |I(k)-I(k-1)|_for_simpson
## 1                           NA                        NA
## 2                 2.548358e-01              1.346782e+05
## 3                 6.302600e+00              1.213284e+04
## 4                 6.283661e+00              1.204276e+03
## 5                 3.554346e-04              1.172865e+02
## 6                 6.283213e+00              1.047199e+01
## 7                 6.642856e-07              2.217864e-07
## 8                 0.000000e+00              0.000000e+00
## 9                 0.000000e+00              0.000000e+00
## 10                0.000000e+00              0.000000e+00
```

4.

```r
g = function(x) {1+exp(-x)*sin(8*x^(2/3))}
a=0; b=2; N=100; n=10; e=10^(-4);
table4 = matrix(0, nrow = n, ncol = 5, dimnames = list(
c(1:n), c('N', 'trapezoid_definite_integral', 'simpson_definite_integral',
'|I(k)-I(k-1)|_for_trapezoid', '|I(k)-I(k-1)|_for_simpson')))
for(k in 1:n) {
table4[k,1] = N*k
table4[k,2] = TrapezoidRule(a, b, table4[k, 1], g)
table4[k,4] = ifelse(k==1,yes=NA, no=abs(table4[k,2] - table4[k-1,2]))
if(k>1 && table4[k,4]<e) break
}
for(k in 1:n) {
table4[k,1] = N*k
table4[k,3] = SimpsonRule(a, b, table4[k, 1], g)
table4[k,5] = ifelse(k==1,yes=NA, no=abs(table4[k,3] - table4[k-1,3]))
if(k>1 && table4[k,5]<e) break
}
print(table4)
```

```
##      N trapezoid_definite_integral simpson_definite_integral
## 1  100                    2.014442                  2.022973
## 2  200                    2.015704                  2.019639
## 3  300                    2.015987                  2.018524
## 4  400                    2.016099                  2.017965
## 5  500                    2.016155                  2.017629
## 6  600                    0.000000                  2.017405
## 7  700                    0.000000                  2.017244
## 8  800                    0.000000                  2.017124
## 9  900                    0.000000                  2.017030
## 10   0                    0.000000                  0.000000
##    |I(k)-I(k-1)|_for_trapezoid |I(k)-I(k-1)|_for_simpson
## 1                           NA                        NA
## 2                 1.262165e-03              3.333319e-03
## 3                 2.833463e-04              1.115662e-03
## 4                 1.115527e-04              5.590632e-04
## 5                 5.626335e-05              3.359216e-04
## 6                 0.000000e+00              2.241798e-04
## 7                 0.000000e+00              1.602551e-04
## 8                 0.000000e+00              1.202669e-04
## 9                 0.000000e+00              9.358907e-05
## 10                0.000000e+00              0.000000e+00
```