Homework 2

FE621 Computational Finance

due 23:55, Sunday September 30, 2018

For all the problems in this assignment you need to design and use a computer program, output results and present the results in nicely formatted tables and figures. The computer program may be written in any programming language you want. Please write comments to all the parts of your code. They are a requirement and they will be graded.

You need to submit a PDF containing the report. Please use a word processor such as Microsoft Word, LaTeX,or whatever Apple uses to create your report. You will be judged by the quality of the writing and interpreting the results.

Problem 1: Binomial/Trinomial Tree Basics (60 points)

1. Construct *an additive binomial tree* to calculate the values of the European Call option and one for the European Put option. *Hint.* See the book [1].

2. Download Option prices (you can use the Bloomberg Terminal, Yahoo! Finance, etc.) for an equity, for 3 different maturities (1 month, 2 months, and 3 months) and 20 strike prices. For each strike price in the data, calculate the corresponding implied volatility. You may use the data and calculations you have done for Homework 1. Also use the current short-term interest rate.

   Calculate the option prices (European Calls and Puts) using the binomial tree, and compare the results with the same prices obtained applying the Black–Scholes formula. Use at least 200 steps in your construction.

3. Compute and plot the absolute error ε_N as a function of $N \in \mathbb{N}^*$:

$$\varepsilon_N = \left| C^{BSM}(S_0, K, T, r; \sigma) - C_N^{BT}(S_0, K, T, r; \sigma) \right|,$$

   where $C^{BSM}(S_0, K, T, r; \sigma)$ and $C_N^{BT}(S_0, K, T, r; \sigma)$ are the Black–Scholes–Merton price and the price calculated using a binomial tree with N steps, respectively. Consider $N \in \{50, 100, 150, 200, 250, 300, 350, 400\}$. What do you observe?

4. Implement *an additive binomial tree* to calculate the American option, both Call and Put. Repeat the steps in part 2) and calculate the respective option prices as if they are American.

5. Create a table which contains the following columns: Bid and Ask values, Black Scholes price, European and American prices calculated using the binomial tree. Then write comments about the observed differences between the various option valuations and how they compare with the actual bid/ask values.

6. Using the binomial tree for American Calls and Puts, calculate the implied volatility corresponding to the data you have downloaded in part (2). You will need to use the bisection or Newton/secant method of finding roots with the respective binomial trees. Compare these values of the implied volatility with the volatilities from Homework 1. Write detailed observations.

7. Implement a trinomial tree to price an American Put option. *Hint.* See Chapter 3 in [1]. Use the trinomial tree with the data in this problem and compare with the results obtained using the binomial tree.

Problem 2. Adapting the Binomial tree to time varying (deterministic) coefficients (30 points). Please refer to Section 2.9 in [1]. Consider the following model for $X_t = \log S_t$ the logarithm of the stock value:

$$dX_t = \nu_t dt + \sigma_t dW_t,$$

where $\nu_t = r_t - \frac{\sigma_t^2}{2}$. This model is under the equivalent martingale measure. Assume that the interest rate and volatility functions evolve according to:

$$r_t = 0.05(1 + 0.01t)$$
$$\sigma_t = 0.3(1 + 0.005t)$$

Please answer he following questions.

1. For a conveniently chosen $\Delta x \in (0.02, 0.08)$, calculate the time steps Δt_i as well as the time varying probabilities p_i to construct an approximating binomial tree for the continuous time model. Present a table with the first 12 entries, that is each row should contain $i, r_i, \sigma_i, \nu_i, p_i$ for all $i \in \{1, 2, \ldots, 11\}$.

2. Apply one of the numerical methods learned thus far to determine the optimal value of Δx such that $\sum_{i=0}^{1} 1\Delta t_i = 0.5$, that is the tree spans a half year period.

3. Using the tree you just constructed please calculate the price of an European option with $X_0 = \log S_0 = \log 10$, strike $K = 10$, maturity $T = 0.5$.

Problem 3. Dealing with discrete cash dividends (10 points). A stock price follows the typical geometric Brownian motion process with $r = 0.001$ and $\sigma = 0.4$. Suppose we price everything at time $t = 0$ when stock price is $S_0 = 100$. We want to price a call option and a put option both with strike $K = 90$ and maturity 2 months (i.e., $T = 2/12$). However, we also know that the stock will pay a \$0.6 cash dividend in 2 weeks at time $\tau = 1/24$. Please construct a tree that is capable of estimating the price of both derivatives. Please report the result. We will provide points for any alternative method to the tree construction you may provide.

Bonus Problems

Generally, the bonus problems are here to help the students who work hard. You will not need any bonus problem to get an A given that all the assignments deserve an A. However, they are here to make up missed work or generally because they are interesting problems. Because of this please avoid turning in nonsense in the hope that it will earn you partial bonus points. I will give partial credit for significant work and well thought out arguments to solve the problem.

Bonus Problem 1 (40 points). A two-dimensional tree for the Heston model. Beliaeva and Nawalkha (2010) have developed in [5] a path-independent two-dimensional tree for the Heston model. In their approach, separate trees for the stock price and for the variance are constructed independently of one another, and then recombined, please see Chapter 8 in [3] for a detailed presentation.

1. Price an American Put Option using the Beliaeva and Nawalkha method. Please note that the code provided in the book is incomplete (marked with "...”). Consider the same numerical values for the parameters of interest as in [4].

2. Price an European Call Option using the Beliaeva and Nawalkha method. Compare this result with the prices obtained in Homework 1, Problem 3 via the analytical formula. Consider the same numerical values as in [4]. What can you observe?

Bonus Problem 2 (50 points). A multinomial recombining tree for general Stochastic Volatility models. We consider here an interesting method of option pricing under general assumptions, involving a multinomial recombining tree and particle filtering techniques. Please read the paper [6], and pay special attention to sections 3 and 4.1, 4.2.

1. Using synthetic parameters, i.e., chosen by you, estimate the probability distribution for the volatility process Y_t at discrete time points $t_1, t_2, \ldots, t_n$. To this end, implement the particle filter described in Section 3 of [6]. You should store from this step the particles $\{\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_n\}$ together with their corresponding probabilities $\{p_1, p_2, \ldots, p_n\}$.

2. Construct the successors in the multinomial tree with synthetic parameters, see Section 4.1. Please note that two separate cases were analyzed.

3. You built in part (2) the one period model. Implement now the multiperiod model described in Section 4.2. Price for illustrative purposes the European Call option and try to replicate the results in Section 6 of the paper.

4. Build a multinomial tree as in [6] to price an American Put Option. Compare with the results in bonus problem 1(a)

5. The most important limitation here is that the two Brownian Motions W_t and Z_t in equation (1.1) are assumed independent, i.e.

$$\mathbb{E}[dW_t dZ_t] = 0.$$

To circumvent this, let $\sigma(y) = \sqrt{y}$, $\psi(y) = \gamma\sqrt{y}$, and $\mathbb{E}[dW_t dZ_t] = \rho dt$ in (1.1), i.e., the Heston Model, and consider the transformation

$$X_t = \ln S_t - \frac{\rho}{\gamma} Y_t - H_t, \text{ where } H_t = \left(\mu - \frac{\rho\alpha\nu}{\gamma}\right) t$$

Apply Ito's Lemma on X_t and show that dX_t can be written as $dX_t = \mu_X(t)dt + \sigma_X(t)dB_t$, where B_t is uncorrelated with W_t, and μ_X, σ_X are real–valued functions. Build a multinomial tree as in [6] to price an American Put Option. Compare your results with the results in bonus problem 1 (1) and part (4) of this problem.

References

[1] Clewlow, Les and Strickland, Chris. *Implementing Derivative Models (Wiley Series in Financial Engineering)*, John Wiley and Sons 1996.

[2] Niklas Westermark. *Barrier Option Pricing*, Degree Project in Mathematics, First Level. KTH Royal Institute of Technology, Stockholm, Sweden.

[3] Rouah, F. D. *The Heston Model and Its Extensions in Matlab and C*, 2013, John Wiley and Sons.

[4] Mikhailov, Sergei and Nögel, Ulrich. *Heston's stochastic volatility model: Implementation, calibration and some extensions* 2004, John Wiley and Sons.

[5] Beliaeva, Natalia A and Sanjay K, Nawalkha, *A simple approach to pricing American options under the Heston stochastic volatility model.* The Journal of Derivatives, Vol. 17, No. 4, pp. 25–43, 2010.

[6] Florescu, Ionuţ, Viens, Frederi G, *Stochastic volatility: option pricing using a multinomial recombining tree*, Applied Mathematical Finance, Vol. 15, No. 2, pp. 151–181, 2008, Taylor & Francis.