Homework 4

FE621 Computational Methods in Finance
due 23:55ET, Friday Dec 7, 2018

Specifications. For all the problems in this assignment you need to design and use a computer program, output and present the results in nicely formatted tables and graphs. The computer program may be written in any programming language you want. Please submit an archive containing a written report (pdf), where you detail your results and copy your code into an Appendix. The archive should also contain the code with comments. Any part of the problems that asks for implementation should contain a reference to the relevant code submitted.

1 Comparing different Monte Carlo schemes. (30 points)

Consider the Black-Scholes setup (geometric Brownian motion) with $r = 6\%$, $\delta = 0.03, \sigma = 20\%$, $S_0 = 100$, and assume we want to price an European option with strike $K = 100$ and maturity $T = 1$.

(a) Implement a simple Monte Carlo scheme using m simulation trials for European Call and Put options. This should be a function of n (number of time steps) and m. In all practical applications you should use at least 300 time steps and at least 1 million simulated paths. Furthermore, implement a calculation of the standard error of the estimate of the option price and a way to time the simulation routine.

(b) Implement a Monte Carlo scheme for European call and put options using the antithetic variates method (see section 4.3 of the textbook), the delta–based control variate (section 4.5 of the textbook) with $\beta_1 = -1$, and the combined antithetic variates with delta-based control variate method. Report the values obtained in four columns: Monte Carlo (MC), MC with Antithetic Variates, MC with Delta-based Control Variate, and MC with both Antithetic Variates and Delta-based Control Variate. Report the estimated option values, the corresponding standard deviations, as well as the time it takes to obtain each result. Write a paragraph comparing the results you obtained. Discuss the methods implemented.

2 Simulating the Heston model. (30 points)

Consider the Heston stochastic volatility model with parameters: $S_0 = 100$, $V_0 = 0.010201$, $\kappa = 6.21$, $\theta = 0.019$, $\sigma = 0.61$, $\rho = -0.7$, $r = 3.19\%$. Apply the Euler discretization schemes as presented in Table 1 of the paper [1]. Please implement all the five schemes listed there and use Monte Carlo simulations to price a call option with strike $K = 100$ and maturity $T = 1$. The exact call option price (benchmark) is in this case $C_0 = 6.8061$. Provide a table listing the estimated call option price, the bias, the root mean square error (RMSE), and the computation time in seconds. Report the results for each of the five schemes in one table.

3 Multiple Monte Carlo Processes (20 points)

Assume a portfolio of 10 million dollars is invested as follows:

- 40% in IBM stock
- 30% in a 10 year Treasury Bill
- 30% in Chinese Yuan (which depends on the Yuan Dollar exchange rate).

We assume that the processes X, Y, Z described next are modeling the evolution of the IBM share price, unit of TBill, and the number of Yuan obtained for \$1 respectively.

$$
\begin{aligned}
dX_t &= 0.01X_t dt + 0.3X_t dW_t^1, \quad X_0 = 80 \\
dY_t &= 100(90000 + 1000t - Y_t)dt + \sqrt{Y_t}dW_t^2, \quad Y_0 = 90000 \\
dZ_t &= 5(6 - Z_t) + 0.01\sqrt{Z_t}dW_t^3, \quad Z_0 = 6.1
\end{aligned}
$$

1. Calculate the number of shares and the amount in Yuan that the portfolio contains when it is started.
2. Assume the Brownian motions are independent and perform Monte Carlo simulations for all assets for 10 days ($t = 10/252$). Use 3 million simulations, and use $\Delta t = 0.001$. Calculate VAR for the portfolio ($\alpha = 0.01$, $N = 10$ days).
3. Calculate the CVAR (conditional value at risk).

(BONUS 1) SABR parameter estimation (40points)

From Fabrice Douglas Rouah's paper [2], the author shows how to estimate the parameters in a SABR model. In this question, you are required to use swaption data from the file "2017_2_15_mid.xlsx" to estimate SABR parameters. Here is the detailed information of the data structure in that file: Each column represents the maturity of a swaption contract (exercise time). Specifically, 1Yr; 2Yr;...; 30Yr represent the maturity T in equation (3) in the Rouah paper. In this data set, all swaptions are European Call options with the underlying a swap. Each row, which is marked with 1Mo, 2Mo, ... , 30Yr represent the maturity of the underlying swap. You have the option to enter into this swap at the exercising time of the swaption (T). "Vol" is at-the-money volatility for each swaption contract and "strike" is the strike price K for each of the contracts. NOTE all the contracts are AT-THE-MONEY contracts (i.e. forward rate equals strike). Pick one maturity (1Yr, 2Yr, ..., 30Yr) you are interested in (one column) and answer the following questions.

1. Assume $\beta = 0.5$, implement equation (5) to estimate parameters α, ρ and ν. To solve this equation, you can use any standard optimization method such as Newton-Raphson method. For this question, you could use any available package to solve it.

2. Set $\beta = 0.7$ and 0.4, and repeat part 1. That is, estimate the corresponding α, ρ and ν values.

3. Compare the parameters you obtained based on different β choice values. What do you observe? Write a paragraph to explain your observations.

4. Using the maximum (minimum) of the function you optimized, tell us which model would give you best estimation.

5. Using the best parameter values do the following: Select another contract you have not used in the previous question (a different column) and treat it as a benchmark. That is, calculate at-the-money volatility using the parameters you obtained and compare the numbers you see in the benchmark column. What do you observe? Please describe.

(BONUS 2) Sim.DiffProc question (30points)

For this problem please research and understand how the approximate Likelihood method works. Describe the Euler approach then the Ozaki method using your own words.

Generate your own path (or using the R package) from the process:

$$dS_t = (\theta_1 + \theta_2 S_t)dt + \theta_3 S_t^{\theta_4} dW_t$$

Please use $S_0 = 100$, $\theta_1 = 1000$, $\theta_2 = -10$, $\theta_3 = 0.8$, $\theta_4 = 0.5$. Generate daily data $\Delta t = 1/365$ for 4 years.

Then use each of the methods in the Sim.DiffProc to estimate parameters. Finally, create a table with the real values of the parameters and each of the estimations. Report which method gives best estimates. Also report the AIC values for each method. Discuss.

(BONUS 3) Any other problem assigned in class and mentioned as Bonus (10points)

References

[1] Lord, Roger, Remmert Koekkoek, and Dick Van Dijk. *A comparison of biased simulation schemes for stochastic volatility models.* Quantitative Finance 10.2 (2010): 177-194.

[2] Rouah, Fabrice Douglas. "The SABR Model." ht tp://www. volopta. com.