Computational Methods in Finance

- Please note: for the problems using data we are not interested in all the minor steps performed. You should explain your broad reasoning and hand in a final report.
- Do not include any failed attempts at modeling just to give volume to the final report.
- You need to argument with graphs or selected numerical results all the conclusions you draw. Put important plots and tables within the report and relegate any non essential ones to an appendix at the end of the document.
- Please add supporting arguments (e.g., output) in the appendix. The whole report should be constructed as a regular journal article.
- The final write-up (excluding the appendix) should not be more than 10-15 pages.
- Communication with other students either physical or virtual is strictly forbidden.

For instructor's use only

Problem	Points	Score
A	100	
B	100	
Bonus	40	
Total	200	

Problem A. Pricing basket options.

(a) Let the correlation matrix A be defined as:

$$A = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 0.5 & 1.0 & -0.4 \\ 0.2 & -0.4 & 1.0 \end{bmatrix}.$$

Consider three assets, starting with $S(0) = [100, 101, 98]$. The assets are assumed to follow a standard geometric Brownian motion of the form

$$dS_i(t) = \mu_i S_i(t)dt + \sigma_i S_i(t)dW_i(t).$$

We assume $\mu = [0.03, 0.06, 0.02]$, the volatility $\sigma = [0.05, 0.2, 0.15]$, and the BM's have the correlation matrix A (e.g., $d < W_1, W_2 >= a_{12}dt = 0.5dt$).

(b) We take maturity $T = 100$ days and the number of simulated paths is $m = 1000$, Consider one day sampling frequency, i.e. $\Delta t = 1/365$. Plot these 1000 sample paths.

(c) Basket options are options on a basket of assets. A commonly traded basket option is a vanilla call/put option on a linear combination of assets. To clarify, suppose $S_i(t), i = 1, \ldots, N$ are the prices of N stocks at time t and let $a_i, i = 1, \ldots, N$ are real constants. Set

$$U(t) = \sum_{i=1}^{N} a_i S_i(t)$$

A vanilla basket option is simply a vanilla option on $U(T)$. Specifically, on the exercise date T, the payoff of the option is $\max\{\alpha(U(T) - K), 0\}$, where K is the exercise price and $\alpha = 1$ for a call and $\alpha = -1$ for a put. Price an European call option and an European put option with $K = 100$ on the 3 asset basket given in part (b), using Monte Carlo simulations. Consider a simple average basket $a_1 = a_2 = a_3 = 1/3$. Make sure that the option price is close to the true price. Report a 95% confidence interval of the option price.

(d) Next price the following exotic options on the basket in part (b). Here we use $B = 104$, $K = 100$, and $T = 100$ days.

(i) If the asset 2 hits the barrier (i.e., $B < S_2(t)$) for some t then the payoff of the option is equal to an European Call option written on the basket $(U(T) - K)_+$ similar to part (c); If the barrier is not hit then the option is worthless

(ii) If $\max_{t\in[0,T]} S_2(t) > \max_{t\in[0,T]} S_3(t)$, then the payoff of the option is $(S_2^2(T) - K)_+$; otherwise, the option is a vanilla call option on the basket, similar to part (c) of this problem.

(iii) Take $A_i(0,T) := \sum_{t=1}^{T} S_i(t)$, the average of the daily values for stock i. If $A_2(0,T) > A_3(0,T)$, then the payoff is $(A_2(0,T) - K)_+$; otherwise the payoff is 0

Problem B. Principal Component Analysis.

1. Download daily prices for the components of DJIA for the last 5 years. Construct the corresponding matrix of standardized returns.

$$Y_{it} = \frac{R_{it} - \overline{R}_i}{\overline{\sigma}_i}$$

$$i = 1, ..., N \text{ and } t = 1, ..., T$$

with

$$\overline{R}_i = \frac{\sum_{t=1}^{T} R_{it}}{T} \text{ and } \overline{\sigma}_i = \sqrt{\frac{\sum_{t=1}^{T} (R_{it} - \overline{R}_i)^2}{T}}$$

2. Calculate the sample correlation matrix

$$C_{ij} = \frac{1}{T} \sum_{t=1}^{T} Y_{it} Y_{jt}$$

3. Calculate the eigenvalues and eigenvectors of the matrix C_{ij} and graph the eigenvalues $\lambda_1 > \lambda_2 > ....$ What percent of the trace is explained by summing the first 5 eigenvalues?

4. Consider the first eigenvector and denote it by $(V_1, V_2, ..., V_N)$. Define the factor

$$F_t = \frac{1}{\sqrt{\lambda}} \sum_{i=1}^{N} \frac{V_i}{\overline{\sigma}_i} R_{it} \text{ with } t = 1, 2, ..., T$$

Note: F_t represents the returns of a portfolio which invests $\frac{1}{\sqrt{\lambda_1}} \frac{V_i}{\overline{\sigma}_i}$ dollars in the i^{th} stock. Calculate the sample mean and sample standard deviation of the factor F.

5. Consider a series of daily returns of the DIA ETF for the same time period. Perform a linear regression of the returns of F with the standardized returns of DIA. Calculate the R-squared of the regression and discuss whether F and the capitalization-weighted market portfolio are good proxies for each other. Discuss the result and argue why F and the particular market index might be related.

6. Consider the 5 eigenportfolios (factors) corresponding to the top 5 eigenvalues in the PCA conducted previously. That is, each of the 5 factors are constructed in a similar way with the construction in part4. Let F_{kt} denote the daily return of the eigenportfolio k on date t. For each of the 30 equity in the DJIA do the following:

   - Estimate the mean μ_s and standard deviation σ_s for return of each equity R^s. Standardize the returns $r_k^s = (R_k^s - \mu_s)/\sigma_s$, where k denote the time.
   - Run a regression with the 5 factors and obtain the parameters β_{sk} in:
   $$r_k^s = \sum_{i=1}^{5} \beta_{sk} F_k + \varepsilon_k^s.$$
   Please note that since the returns are standardized the regression intercept should be zero.

   Next we will be using the following factor model for stock returns:
   $$R^s = \mu_s + \sigma_s \Big(\sum_{k=1}^{5} \beta_{sk} F_k \Big) + \sigma_s \sqrt{ \Big(1 - \sum_{k=1}^{5} \beta_{sk}^2 \Big) } G_s$$
   where F_k with $k = 1, 2, ..., 5$ are uncorrelated Student-t variables with 3.5 degrees of freedom. We assume that G_s is also a Student-t variable with the same degrees of freedom, which is uncorrelated with F_k and with $G_{s'}$, if s' is a different stock. We will assume that each day return follows this structure. Generate realizations for R^s for the next 10 days for all the index components. Calculate the return of a sample portfolio equally weighted in its components. Repeat the experiment 100,000 times. Calculate 99% one-week VAR.

Bonus. Research paper For this problem refer to the paper at: `https://www.sciencedirect.com/science/article/pii/S0378437108010479`. The paper may be accesed from the Stevens network and a preprint may be found at: `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2066888`. In this paper the authors apply a Re-scaled Range (R/S) method and a Detrended Fluctuation (DFA) method to estimate the degree of long memory effects in high frequency financial data. The study uses High frequency data during a particular day. Repeat the two studied with any day data. Use a Bloomberg terminal to download high frequency data.