Homework 3

FE621 Computational Methods in Finance
due 23:55ET, Oct 21, 2018

Specifications. For all the problems in this assignment you need to design and use a computer program, output and present the results in nicely formatted tables and figures. The computer program may be written in any programming language you want. Please submit an archive containing a written report (pdf), where you detail your results and copy your code into an Appendix. The archive should also contain the code with comments. Any part of the problems that asks for implementation should contain a reference to the relevant code submitted.

Problem 1 (40 points). Finite difference methods.

(a) Implement the Explicit Finite Difference method to price both European Call and Put options. *Hint.* See Chapter 3 in [1].

(b) Implement the Implicit Finite Difference method to price European Call and Put options.

(c) Implement the Crank-Nicolson Finite Difference method to price both European Call and Put options.

(d) Consider $S_0 = 100, K = 100$, $T = 1$ year, $\sigma = 25\%, r = 6\%, \delta = 0.03$. Calculate and report the price for European Call and Put using all the FD methods. Put the results of the 3 methods (EFD, IFD, CNFD) side by side in a table and write your observations.

(e) Using the parameters from part (d), plot on the same graph the implicit finite difference updating coefficients A, B, C as a function of σ, $\sigma \in \{0.05, 0.1, 0.15, ..., 0.6\}$. Write detailed comments on your observations.

(f) For both the Explicit and Implicit Finite Difference schemes estimate the numbers Δt, Δx as well as the total number N_j of points on the space grid x to obtain a desired error of $\varepsilon = 0.001$. *Hint.* You need to do this part in a theoretical way. Please use the convergence order as the actual error of the estimate.

(g) Repeat part (f) of this problem but this time get the empirical number of iterations for all three methods (including CNFD). Specifically, obtain the

Black Scholes price for the data in (d), then do an iterative procedure to figure out the Δx, Δt, n, and N to obtain an accuracy of $\varepsilon = 0.001$.

(h) Calculate the hedge sensitivities for the European call option using the Explicit Finite Difference method. You need to calculate Delta, Gamma, Theta, and Vega.

Problem 2 (30 points). Finite difference methods applied to market data. We will use here the algorithms implemented in Problem 1 to price European Call and Put options using market data, and compare these methods.

(a) Download Option prices (you can use the Bloomberg Terminal, Yahoo! Finance, etc.) for an equity, for 3 different maturities (1 month, 2 months, and 3 months) and 10 strike prices. Use the same method from Homework 1 to calculate the implied volatility. Set the current short-term interest rate equal to the one from the day you downloaded the data.

(b) Use the Explicit, Implicit, and Crank-Nicolson Finite Difference schemes implemented in Problem 1 to price European Call and Put options. Use the calculated implied volatility to obtain a space parameter Δx that insures stability and convergence with an error magnitude of no greater than 0.001.

(c) For the European style options above (puts and calls) calculate the corresponding Delta, Gamma, Theta, and Vega using the Explicit finite difference method.

(d) Create a table with the following columns: time to maturity T, strike price K, type of the option (Call or Put), ask price A, bid price B, market price $C_M = (A+B)/2$, implied volatility from the BSM model σ_{imp}, option price calculated with EFD, IFD, and CNFD. Plot on the same graph A, B, C_M, and the 3 option prices obtained with finite difference schemes, as a function of K and T. What can you observe?

Problem 3 (30 points). Design an EFD scheme We know that an option price under a certain stochastic model satisfies the following PDE:

$$\frac{\partial V}{\partial t} + 2\sin(S)\frac{\partial V}{\partial S} + 0.2S^{\frac{3}{2}}\frac{\partial^2 V}{\partial S^2} - rV = 0.$$

Assume you have an equidistant grid with points of the form $(i, j) = (i\Delta t, j\Delta x)$, where $i \in \{1, 2, \ldots, N\}$ and $j \in \{-N_S, N_S\}$. Let $V_{i,j} = (i\Delta t, j\Delta x)$.

1. Discretize the derivatives and give the finite difference equation for an Explicit scheme. Use the notation introduced above.

2. What do you observe about the process of updating coefficients?

3. Implement the scheme and calculate the value of an European Put option in this model. Use the parameter values: $S_0 = 100$, $K = 100$, $T = 1$ year.

Problem 4 (Bonus 10 points) Design and implement an Explicit Finite Difference scheme for the Heston model as described in [2] Chapter 10. Use this scheme to price a put option when $S_0 = 100, K = 100$, $T = 1$ year, $v_0 = 0.0625$, $r = 6\%$. The parameters in the model:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^1$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t} dW_t^2$$

are: $\kappa = 2$, $\theta = 0.09$, $\sigma = 0.5$, $\rho = 0.6$

References

[1] Clewlow, Les and Strickland, Chris. *Implementing Derivative Models (Wiley Series in Financial Engineering)*, John Wiley and Sons 1996.

[2] Rouah, F. D. *The Heston Model and Its Extensions in Matlab and C*, 2013, John Wiley and Sons.

[3] Mikhailov, Sergei and Nögel, Ulrich. *Heston's stochastic volatility model: Implementation, calibration and some extensions* 2004, John Wiley and Sons.

[4] Fusai, Gianluca and Roncoroni, Andrea. *Implementing models in quantitative finance: methods and cases.* 2007 Springer Science & Business Media.

[5] Carr, Peter and Madan, Dilip. *Option valuation using the fast Fourier transform.* Journal of computational finance 2.4 (1999): 61-73.

[6] Mikhailov, Sergei and Nögel, Ulrich. *Heston's stochastic volatility model: Implementation, calibration and some extensions* 2004, John Wiley and Sons.