

Recursive Functions

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Recursive Functions

- A recursive function calls itself in a calling sequence.
- For example

```
void f( ... ) {  
    ...  
    f( ... )  
    ...  
}
```

```
void g( ... ) {  
    ...  
    f( ... )  
    ...  
}  
void f( ... ) {  
    ...  
    g( ... )  
    ...  
}
```

Recursive Functions

- A recursive function solves a problem by solving the associated sub-problems and combining the solutions of the sub-problems.

```
void f( ... ) {  
    ...  
    f( ... )  
    ...  
}
```

```
void g( ... ) {  
    ...  
    f( ... )  
    ...  
}  
void f( ... ) {  
    ...  
    g( ... )  
    ...  
}
```

Recursive Functions

What is the purpose of the code fragment?

```
int s = 1;  
if ( n == 0 ) return s;  
s = n*f(n-1);
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    int s = 1;  
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    return s;  
}
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If n is zero, return s.
Otherwise, set s as $n * f(n-1)$.
Then return s.

Recursive Functions

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Recursive Functions

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Need to give the high level idea
about the function f.

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Need to give the high level idea
about the function f.

f() performs the same action
every time.
But n is reduced by 1 every time.

Recursive Functions

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int f( int n ) {  
    int s = 1;  
    if ( n == 0 ) return s;  
    s = n*f(n-1);  
    return s;  
}
```

n = 7

```
f(7) {  
    ...  
    s = 7*f(6);  
    ...  
}
```

Dry-run: tracing the function call(s)

If n is zero, return s.
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n = 6

```
f(6) {  
    ...  
    s = 6*f(5);  
    ...  
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```
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    ...  
}
```

n = 5

```
f(5) {  
    ...  
    s = 5*f(4);  
    ...  
}
```

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    int s = 1;  
    if ( n == 0 ) return s;  
    s = n*f(n-1);  
    return s;  
}
```

When should
be
terminated?

n = 7

```
f(7) {  
    ...  
    s = 7*f(6);  
    ...  
}
```

n = 6

```
f(6) {  
    ...  
    s = 6*f(5);  
    ...  
}
```

n = 5

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    ...  
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When should
be

Need boundary case(s)

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```

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n = 5

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    ...  
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If n is zero, return s.
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f(6) {  
    ...  
    s = 6*f(5);  
    ...  
}
```

n = 5

```
f(5) {  
    ...  
    s = 5*f(4);  
    ...  
}
```

.....

```
f(n=?) {  
    ...  
    ...  
}
```

If n is zero, return s.
Otherwise, set s as $n*f(n-1)$.
Then return s.

Need to give the high level idea
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Dry-run: tracing the function call(s)

For the case(s), f() does not call itself. (directly or indirectly).

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    ...  
    s = 5*f(4);  
    ...  
}
```

.....

```
f(0) {  
    ...  
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```

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Need to give the high level idea
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For the case(s), f() does not call itself. (directly or indirectly)⁷.

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    ...  
    s = 5*f(4);  
    ...  
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```

.....

```
f(0) {  
    ...  
    return 1;  
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Combining the results of sub-problems

If n is zero, return s.
Otherwise, set s as $n*f(n-1)$.
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When should
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Combining the results of sub-problems
 $s = 7 * f(6)$

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.....

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    ...  
}
```

$s = 7 * 6 * f(5)$

If n is zero, return s.
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.....

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f(0) {  
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$s = 7 * 6 * 5 * f(4)$

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    ...  
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```

.....

```
f(0) {  
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    ...  
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```

s=7*6*5*f(4).....

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```

.....

```
f(0) {  
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    return 1;  
    ...  
}
```

$s = 7 * 6 * 5 * f(4) * \dots * 1$

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To understand a recursive function
(It does one job.)

- ① Dry-run: tracing the function call(s)
- ② Combining the results of sub-problems
- ③ Generalize the results

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$s=7*6*5*4*3*2*.....*1$

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$s = 7*6*5*4*3*2*1*.....*1$

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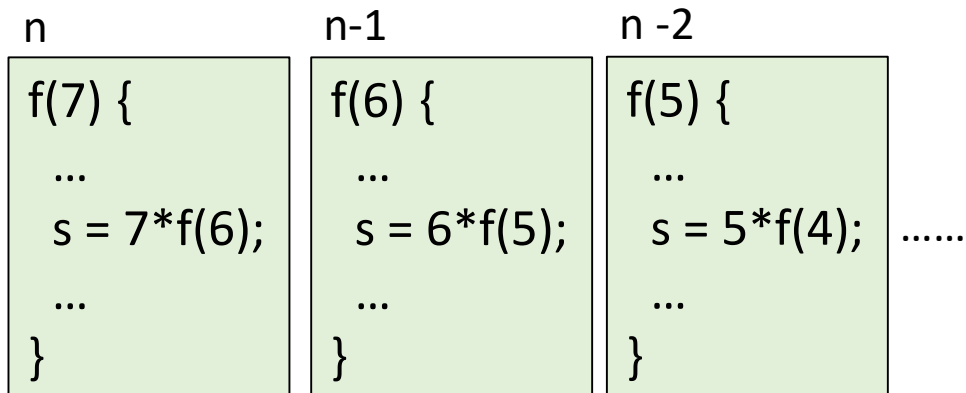
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$s=7*6*5*4*3*2*1 = 7!$

For the case(s), f() does not call itself. (directly or indirectly).

Recursive Functions

```
int f( int n ) {  
    int s = 1;  
    if ( n == 0 ) return s;  
    s = n*f(n-1);  
    return s;  
}
```



When should
be

Need boundary case(s)

0
<pre>f(0) { ... return 1; ... }</pre>

To understand a recursive function
(It does one job.)

- ① Dry-run: tracing the function call(s)
- ② Combining the results of sub-problems
- ③ Generalize the results

If n is zero, return s.
Otherwise, set s as $n*f(n-1)$.
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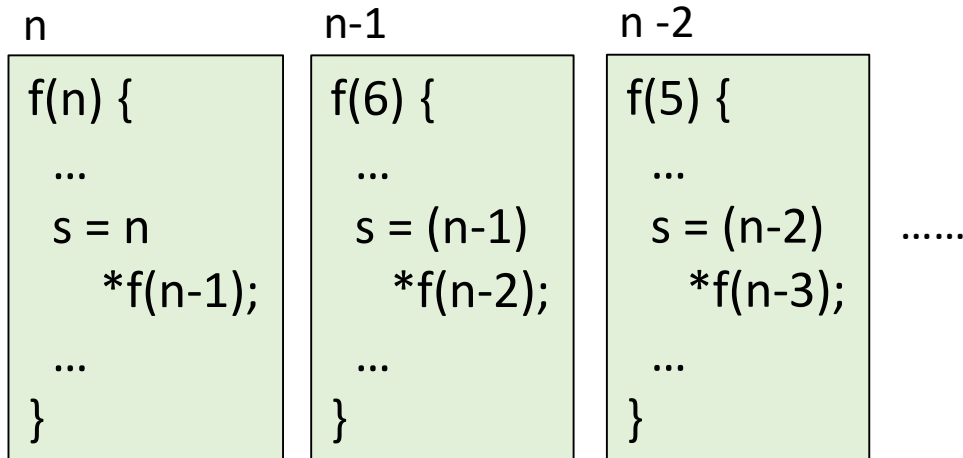
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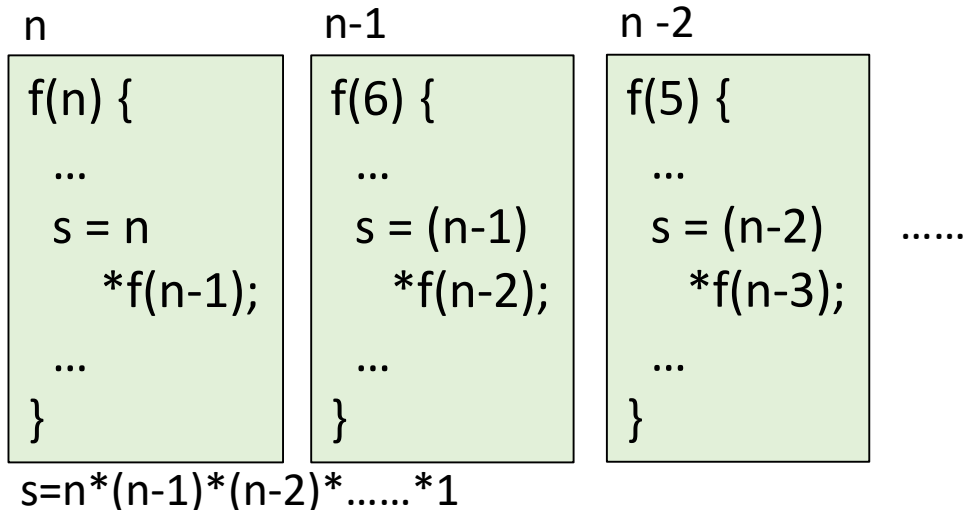
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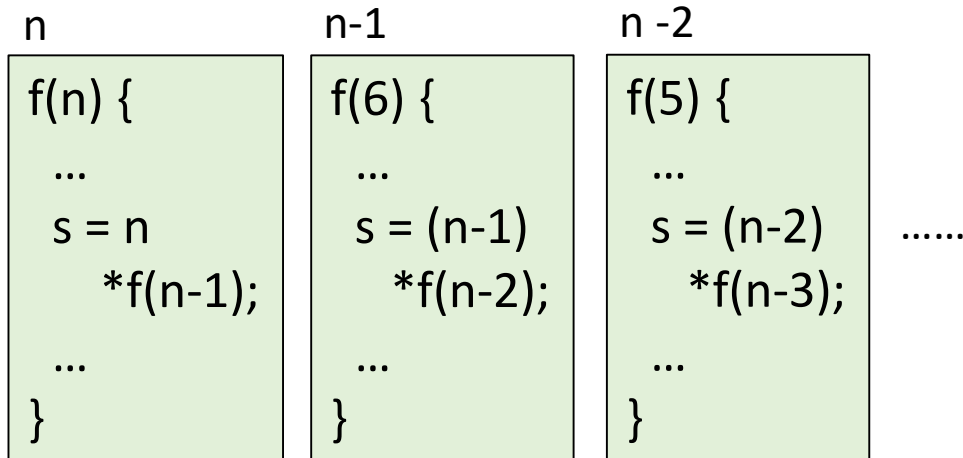
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}
```



$s = n * (n-1) * (n-2) * \dots * 1 * 1 = n!$

To understand a recursive function
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- ② Combining the results of sub-problems
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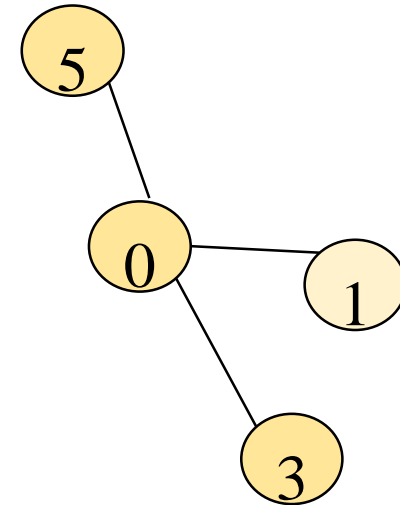
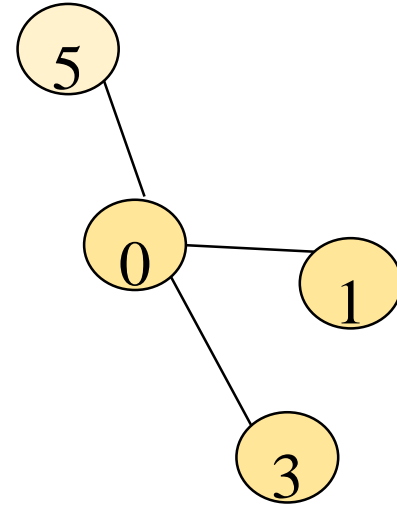
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Depth-First Search

Traverse a graph or a tree in a depth-first manner. That's that, a node is explored if it has not been visited.

```
dfs(vertex v)
  visit v;
  for each neighbor w of v
    if (w is not visited)
      dfs(w);
```

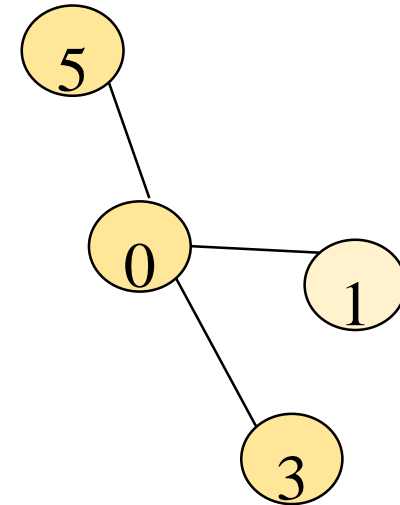
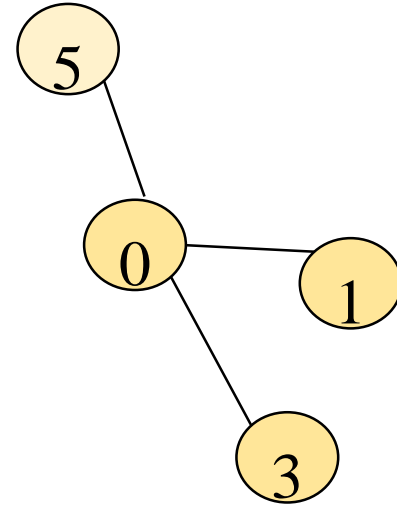


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Invoke `dfs(Node 0)`. Possible result(s):



Depth-First Search

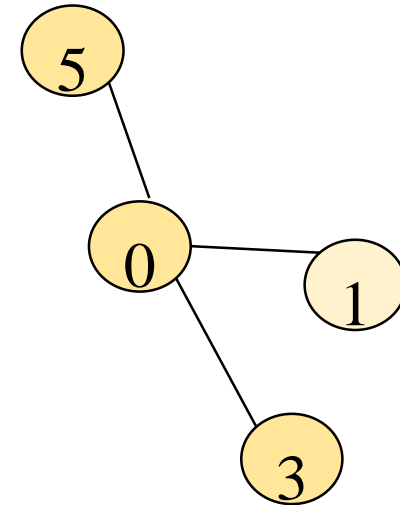
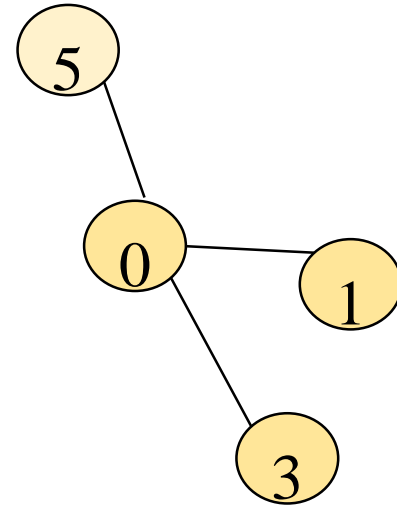
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Invoke `dfs(Node 0)`. Possible result(s):

3, 0, 1, 5

3, 0, 5, 1



Depth-First Search

Traverse a graph or a tree in a depth-first manner. That's that, a node is explored if it has not been visited.

```
dfs(vertex v)
    visit v;
    for each neighbor w of v
        if (w is not visited)
            foo(w);
```

Depth-First Search

Given a node $v = v_0$

```
visit v;  
for each neighbor w of v  
    if (w is not visited)  
        foo(w);
```

Depth-First Search

Given a node $v = v_0$

What do you do?

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Depth-First Search

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1. What do you do?
2. visit v: display v and mark v as “visited”
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Depth-First Search

Given a node $v = v_1$

```
visit v;  
for each neighbor w of v  
    if (w is not visited)  
        foo(w);
```

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Depth-First Search

Given a node $v = v_2$

```
visit v;  
for each neighbor w of v  
    if (w is not visited)  
        foo(w);
```

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Depth-First Search

Given a node $v = v_j$

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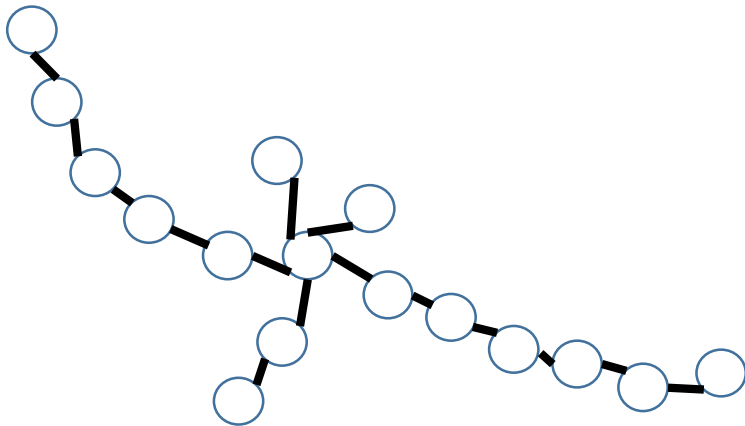
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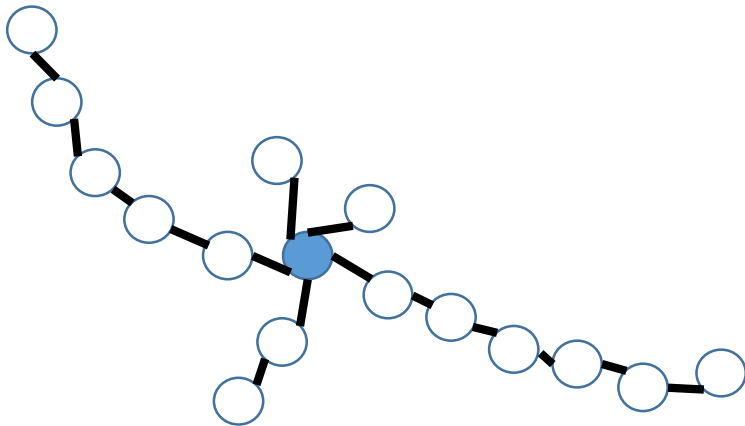
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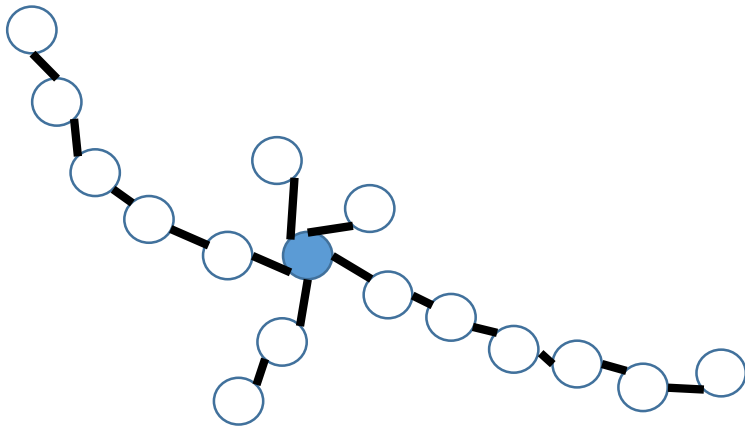
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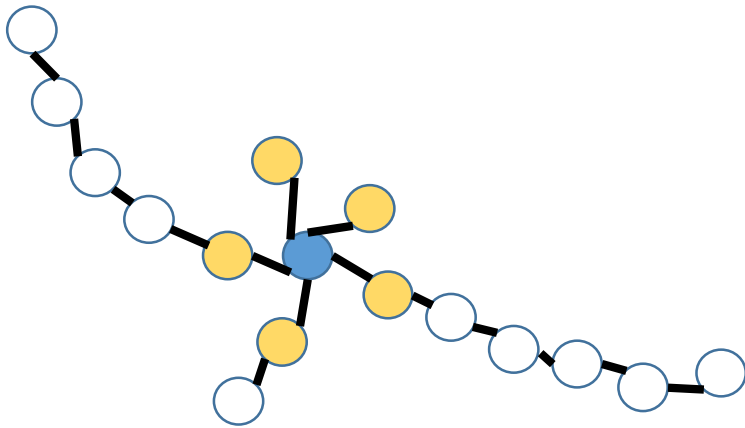
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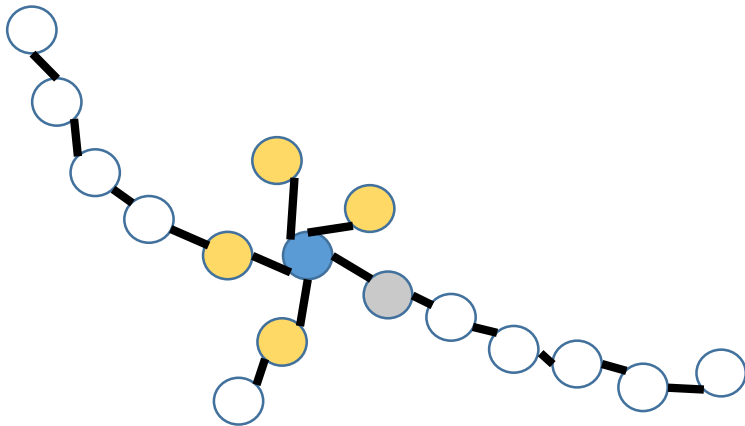
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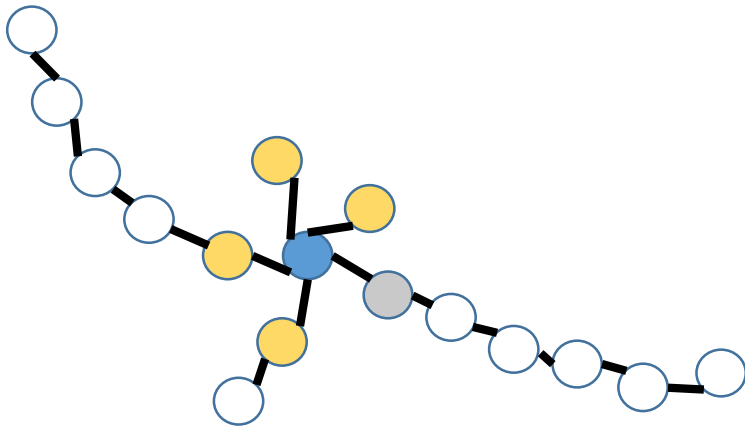
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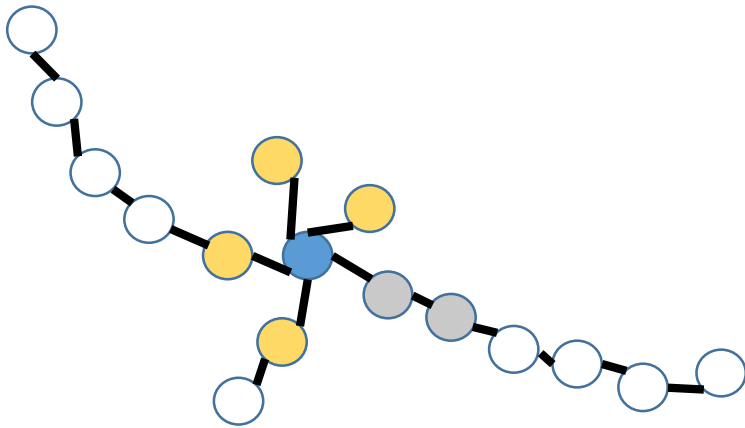
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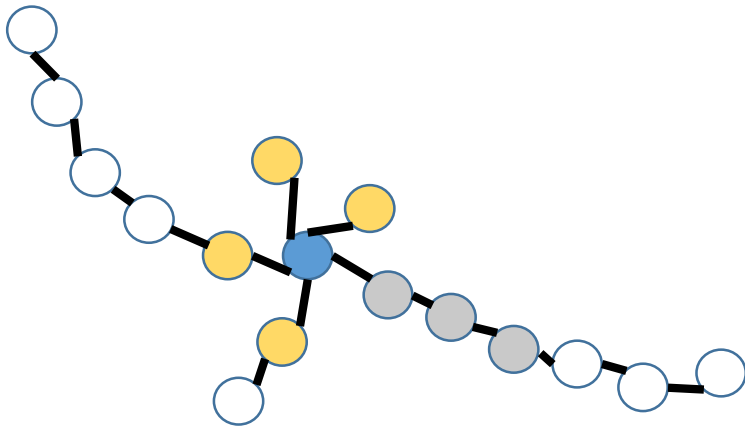
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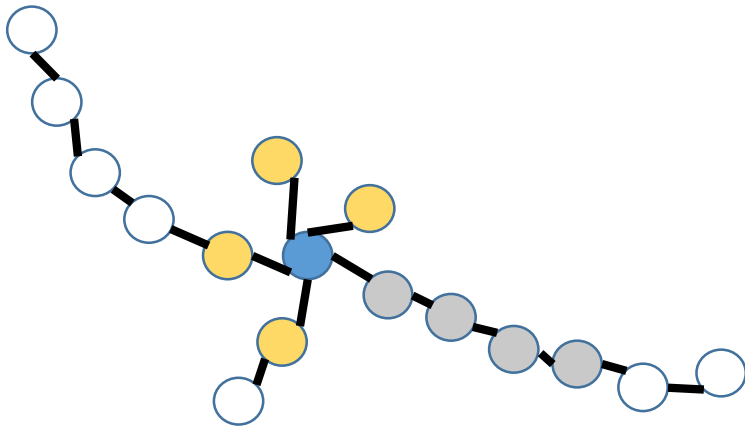
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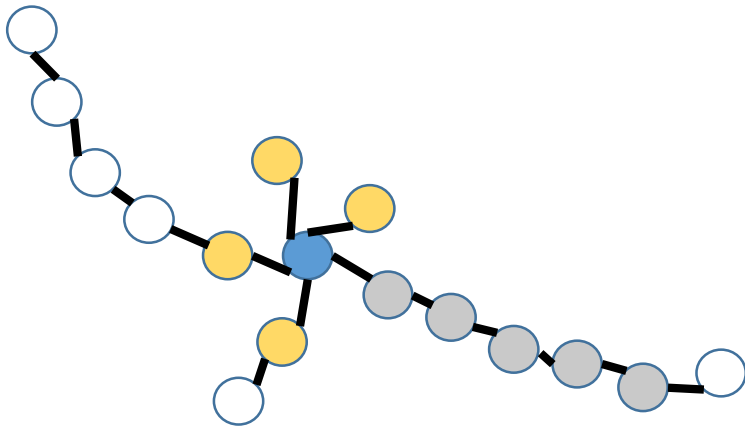
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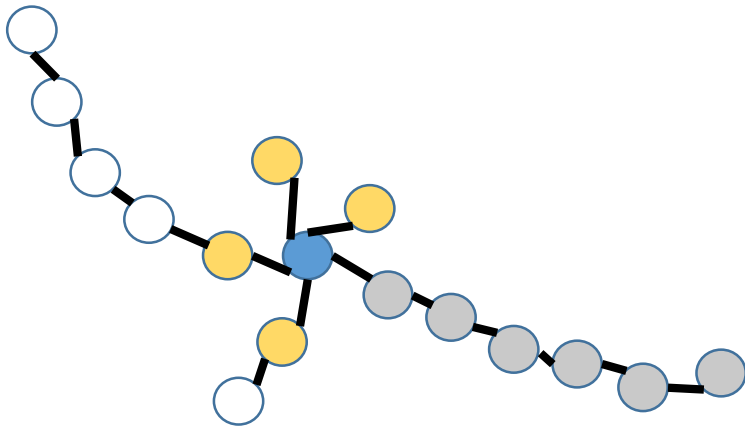
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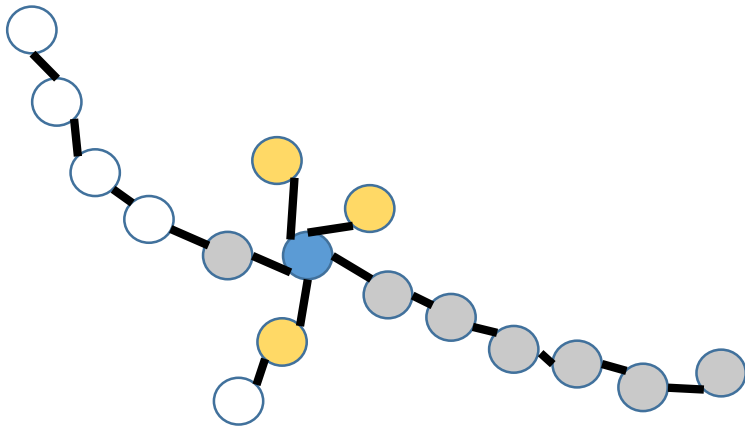
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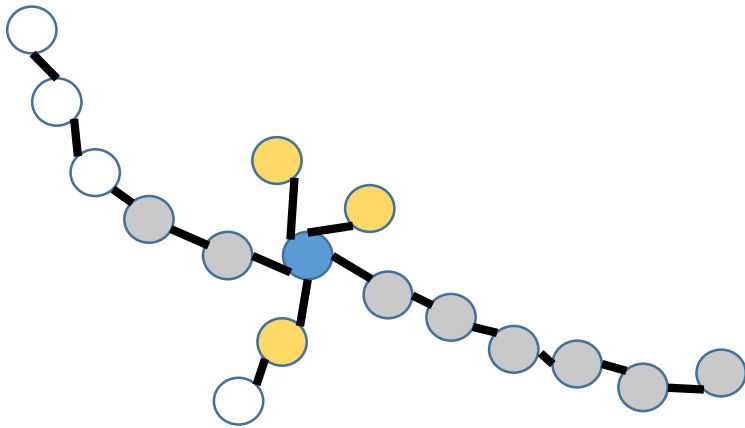
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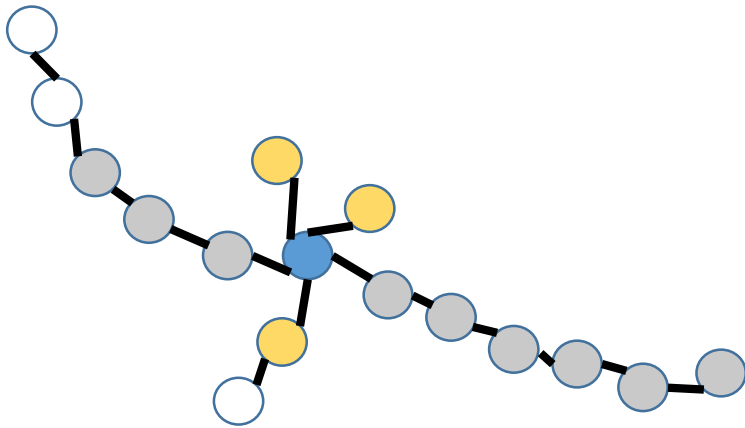
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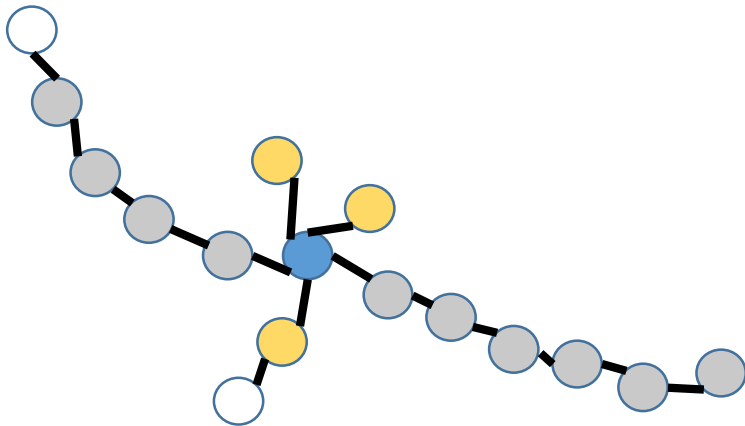
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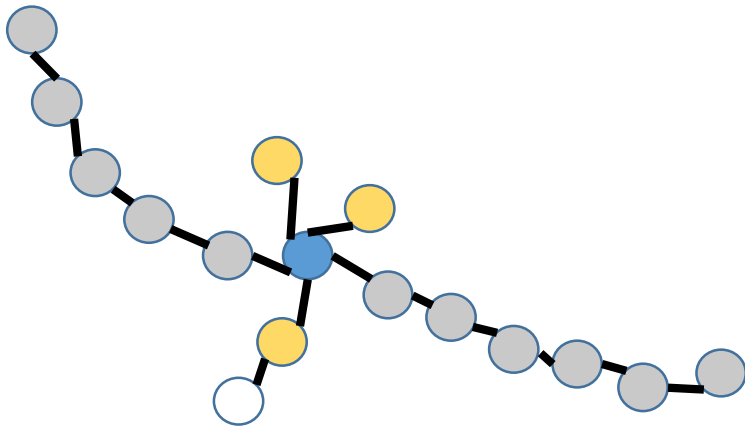
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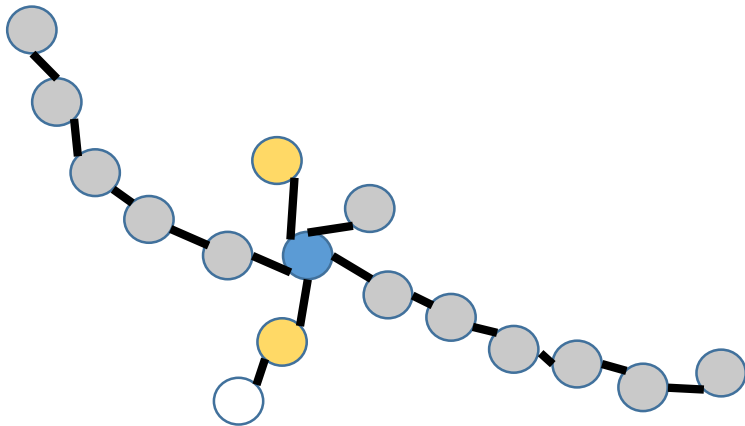
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Now, you should generalize your findings.

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4. call $\text{foo}(w)$ if w is not visited

But, wait...

What is the purpose of function dfs ?

Need to generalize the idea.

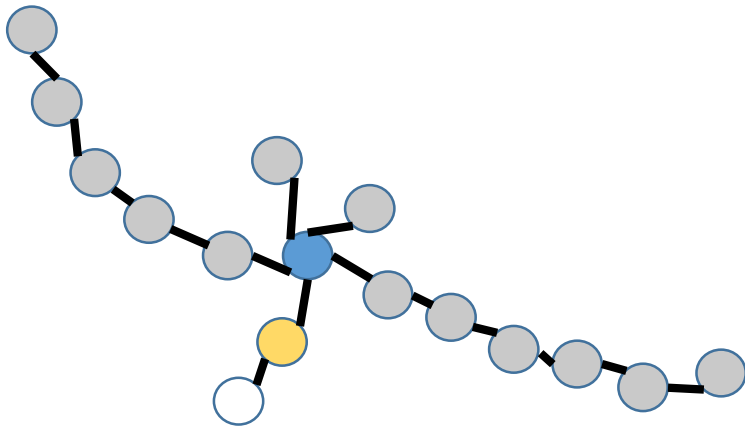
Work on some GOOD examples to get insights

Now, you should generalize your findings.

Depth-First Search

Given a node $v = v_j$

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  for each neighbor w of v
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      dfs(w);
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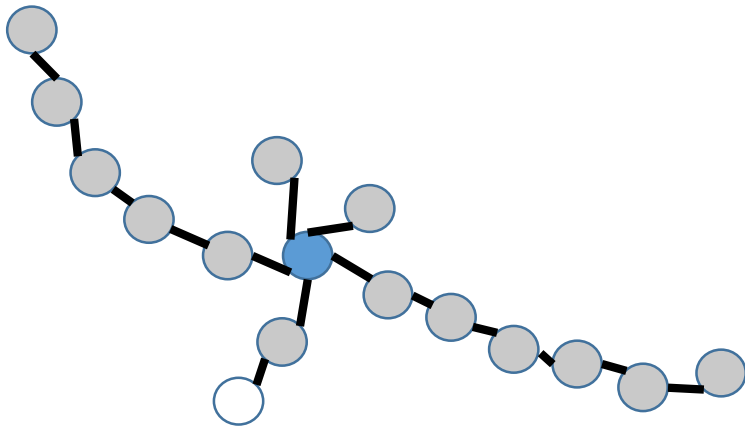
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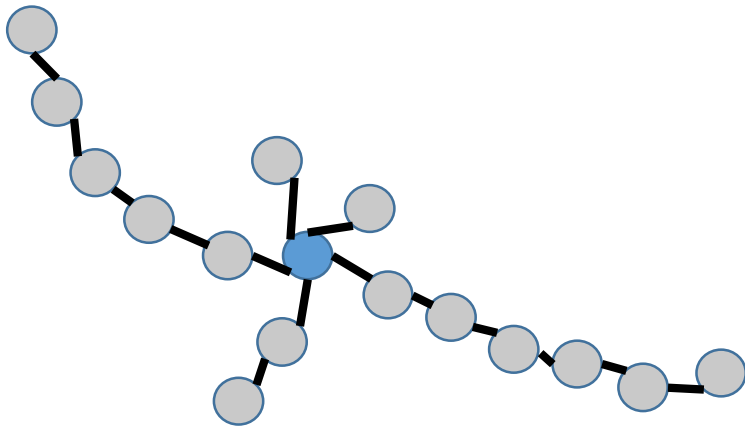
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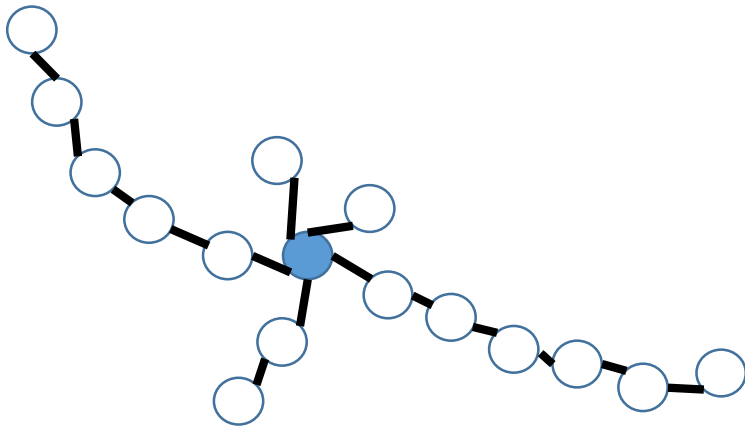
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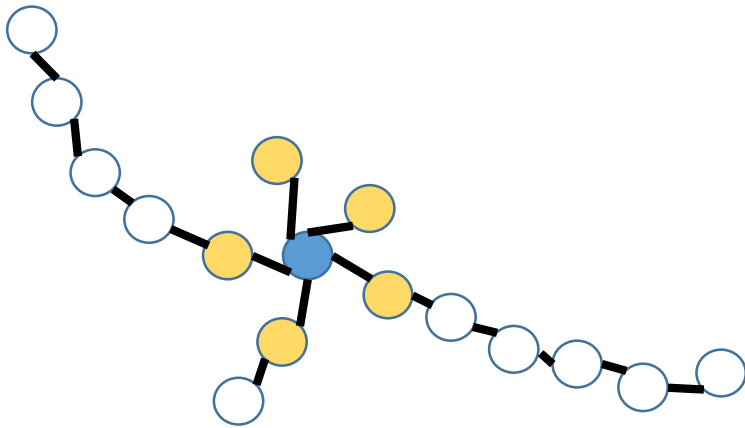
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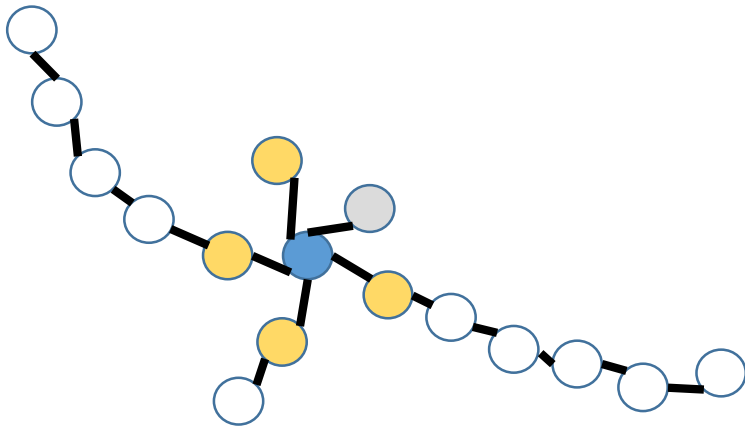
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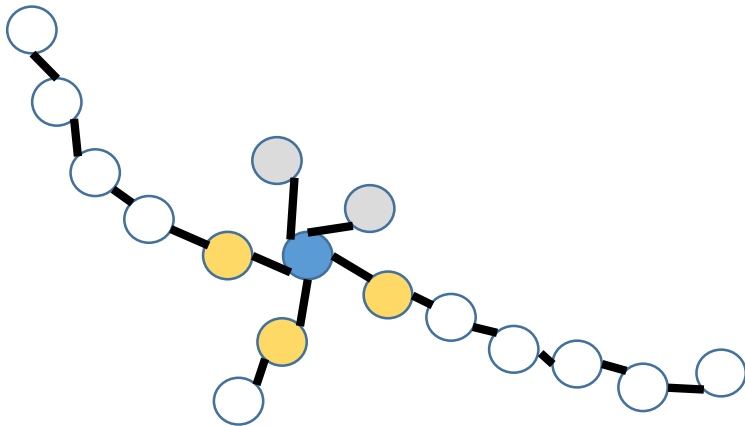
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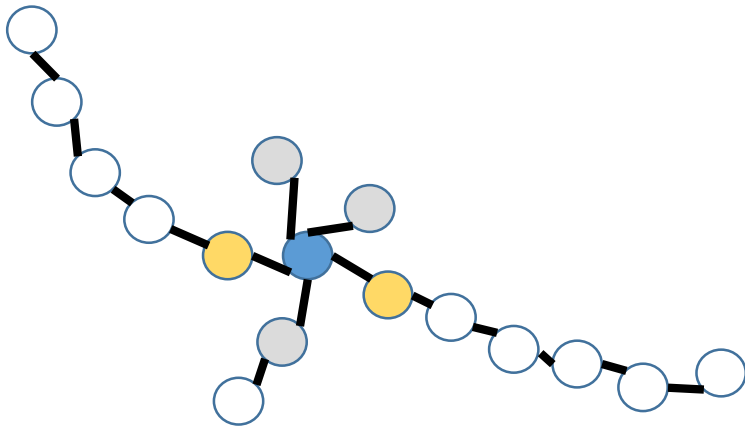
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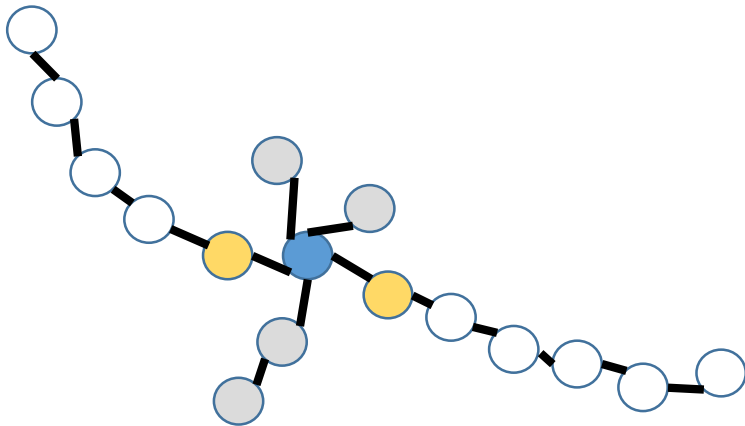
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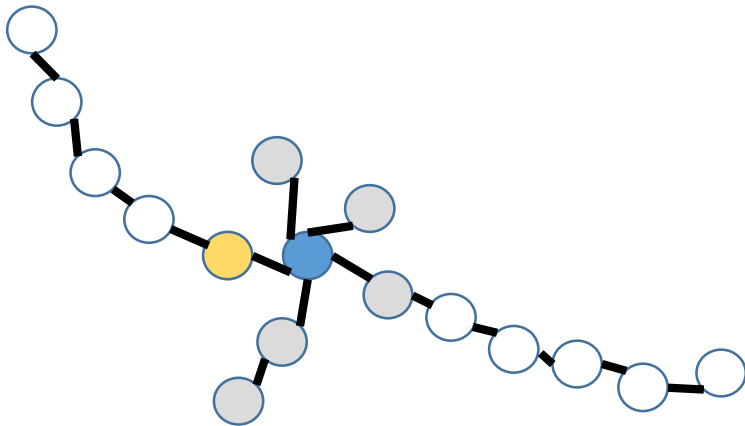
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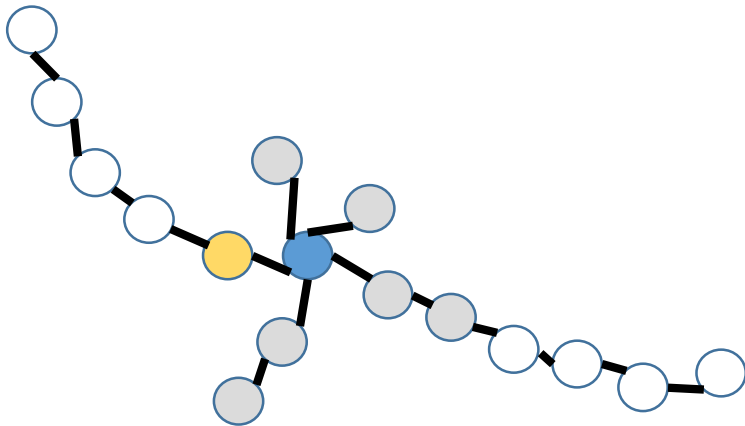
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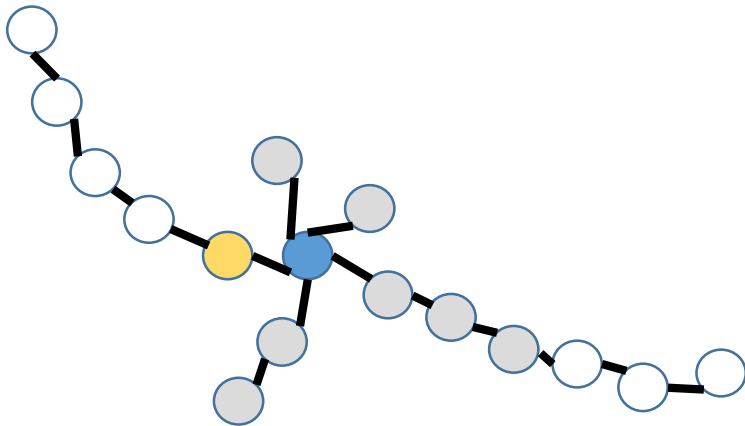
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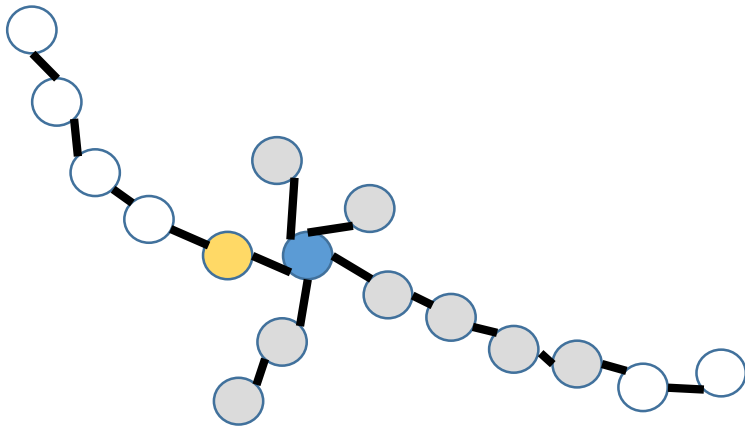
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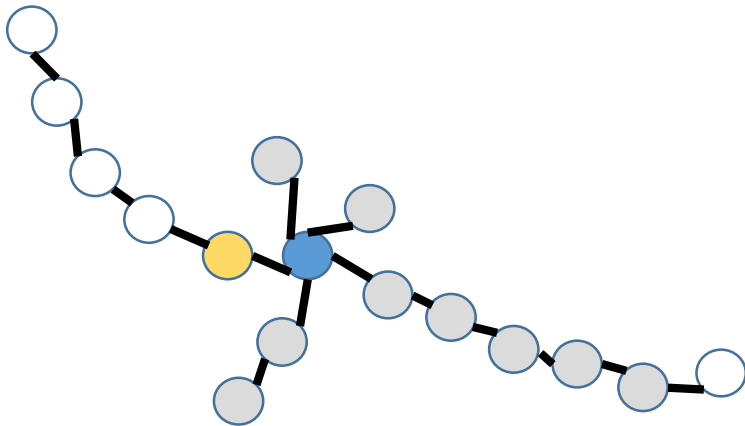
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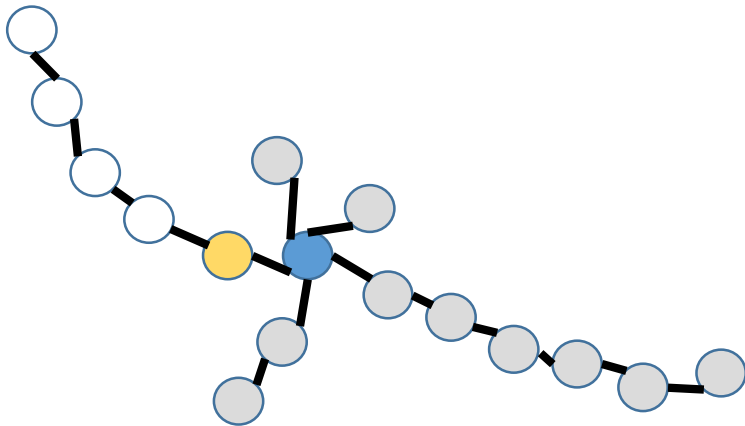
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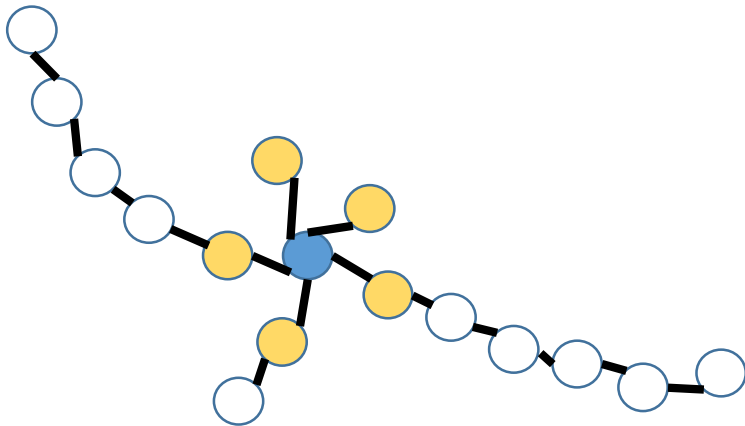
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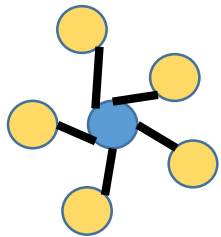
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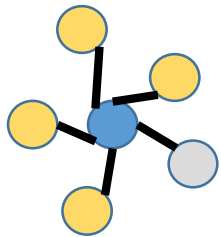
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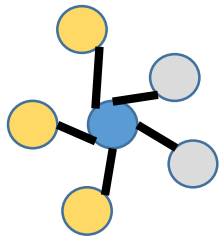
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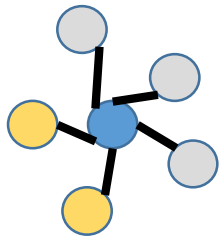
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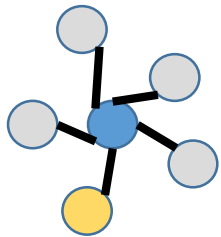
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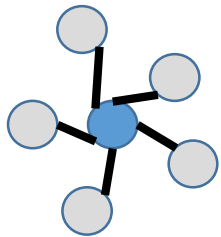
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A bad example does not give you a good insight.

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