Linked Lists, Queues, and Priority Queues

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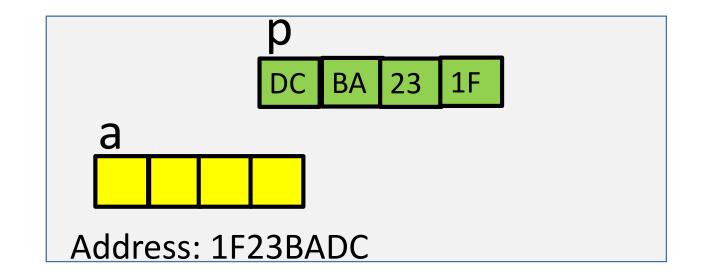
Linked List

Why do we need a linked list?

A linked list is efficient for storing and managing a varying number of elements.

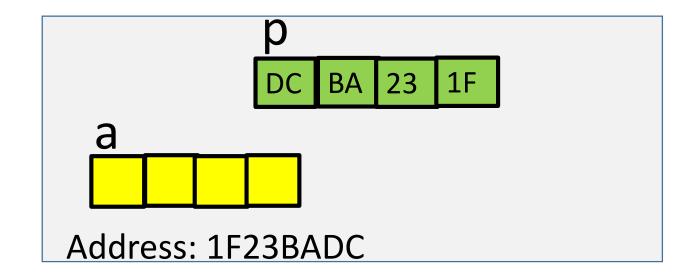
```
int a;  // an integer variable
int *p;  // a pointer to an integer

p = &a;  // assign the address of a to p
```



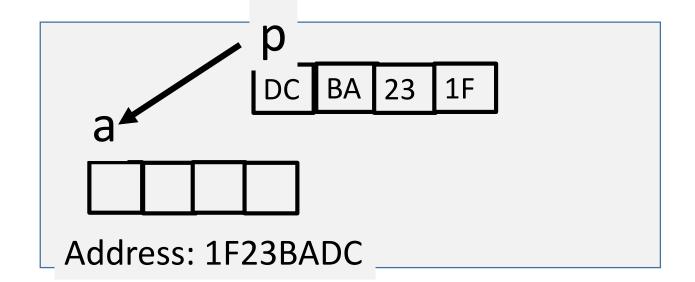
```
int a;  // an integer variable
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p = &a;  // assign the address of a to p
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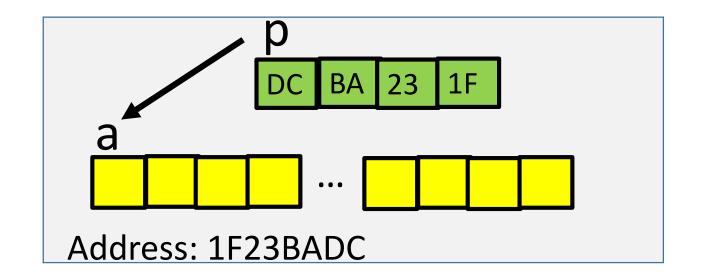


```
int a;  // an integer variable
int *p;  // a pointer to an integer

p = &a;  // assign the address of a to p
```



```
X a;  // an object of X
X *p;  // a pointer to an object of X
p = &a;  // assign the address of a to p
```



```
X a, b; // objects of X
X *p; // a pointer to an object of X
p = &a; // assign the address of a to p
```

```
DC BA 23 1F
a
...
Address: 1F23BADC
```

```
class X {
      void foo();
      double d;
p = &a;
p-> foo();
p->d;
```

- A linked list consists of nodes.
- Each node contains an element, and a pointer which points to its next neighbor.

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- Each node contains an element, and a pointer which points to its next neighbor.

```
template
<typename T = int>
class Node {
   T element;
   Node *next;
};
```

- A linked list consists of nodes.
- Each node contains an element, and a pointer which points to its next neighbor.

```
template
<typename T = int>
class Node {
   T element;
   Node *next;
};
```

```
Node *head, *tail;
Node node1, node2, ...,node98, node99;
head = &node1; node1.next = &node2; ... node98.next = &node99; ...; node99.next = 0;
tail = &node99.
```

- A linked list consists of nodes.
- Each node contains an element, and a pointer which points to its next neighbor.

```
template
<typename T = int>
class Node {
   T element;
   Node *next;
};
```

head and tail point to the first and the last nodes, respectively.

```
Node *head, *tail;
Node node1, node2, ...,node98, node99;
head = &node1; node1.next = &node2; ... node98.next = &node99; ...; node99.next = 0;
tail = &node99.
```

- A linked list consists of nodes.
- Each node contains an element, and a pointer which points to its next neighbor.

```
template
<typename T = int>
class Node {
    T element;
    Node *next;
};
tail

tail

element
next

element
next

next
```

```
Node *head, *tail;
Node node1, node2, ...,node98, node99;
head = &node1; node1.next = &node2; ... node98.next = &node99; ...; node99.next = 0;
tail = &node99.
```

#define NULL 0

```
nullptr
```

//C11

```
template
<typename T = int>
class Node {
   T element;
   Node *next;
};
```

```
head element next element next tail
```

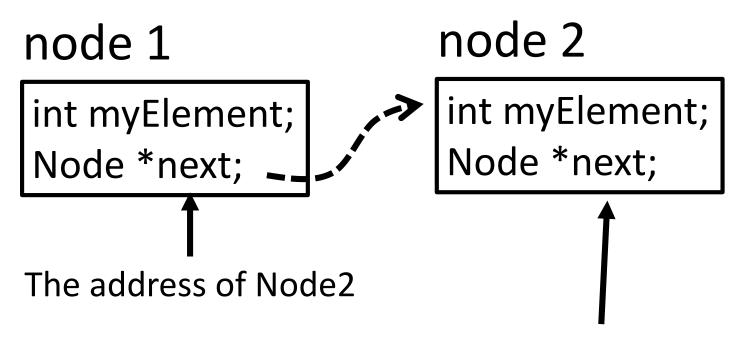
```
Node *head, *tail;

Node node1, node2, ...,node98, node99;

head = &node1; node1.next = &node2; ... node98.next = &node99; ...; node99.next = 0;

tail = &node99.
```

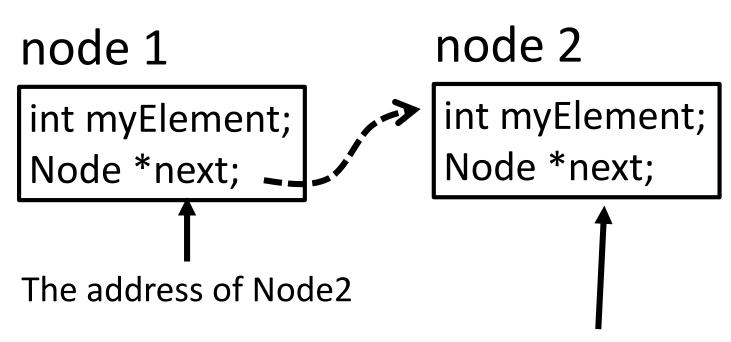
Each node contains an element, and a pointer which points to its next neighbor.



The address of next element.

If next == 0, there is no more nodes.

Each node contains an element, and a pointer which points to its next neighbor.

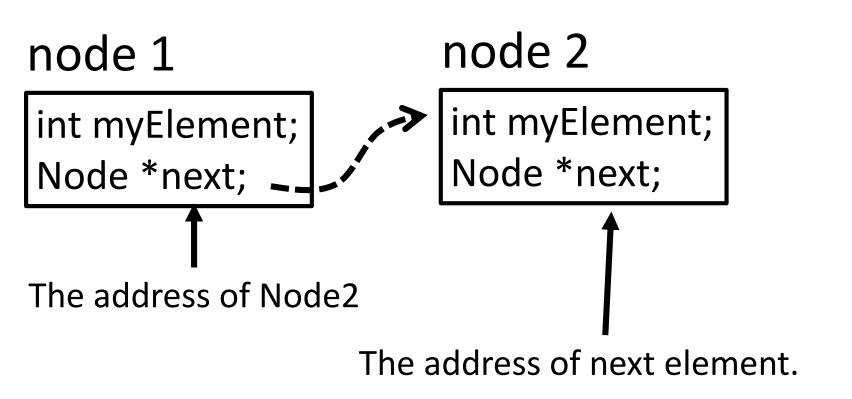


Node node1, node2; node1.next = &node2; node2.next = 0;

The address of next element.

If next == 0, there is no more nodes.

Each node contains an element, and a pointer which points to its next neighbor.



Node node1, node2; node1.next = &node2; node2.next = 0;

```
Node *node1, *node2;

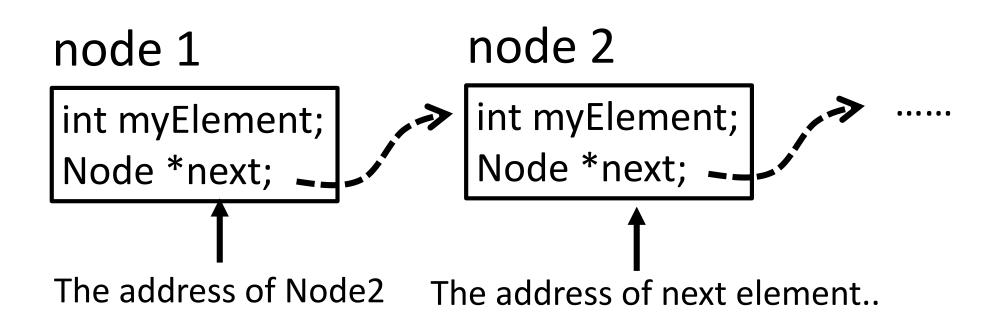
node1 = new Node;

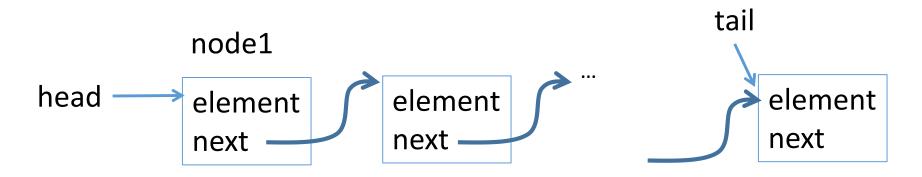
node2 = new Node;

node1->next = node2;

node2.next = 0;
```

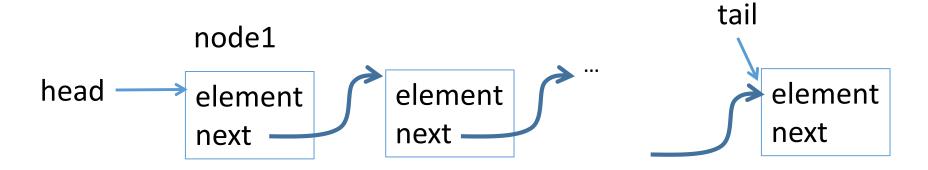
If next == 0, there is no more nodes.



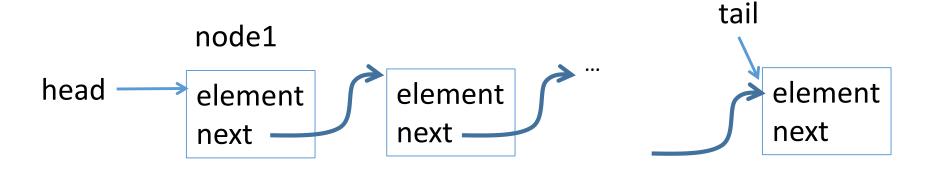


How to tell a node is the last one? Maintain a tail pointer. Node *tail;

bool flg_last_node = tail == &node;

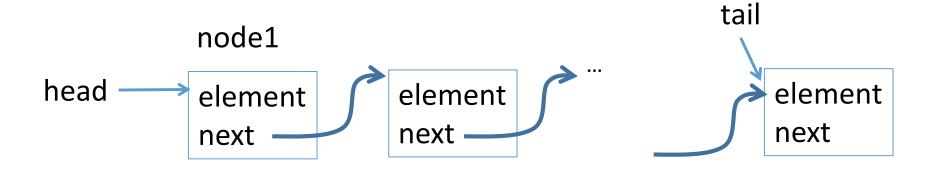


How to tell a node is the last one? Maintain a tail pointer. Node *tail;



How to tell a node is the last one? Maintain a tail pointer. Node *tail;

bool flg_last_node = !node.next;



The Node Class

```
template < typename T >
class Node {
public:
  T element; // The element contained in the node
  Node* next; // A pointer which points to the next node
  Node() { // No-arg constructor
    next = NULL;  // next = 0; or next = nullptr in C11
  Node (const T &element) { // Constructor
    this->element = element;
    next = NULL;
                              21
```

The head and tail pointers

• If the list is A1 both A2 and A3 should be A4. In this case, both pointers do not point to any node.

The head and tail pointers

• If the list is empty, both head and tail should be NULL. In this case, both pointers do not point to any node.

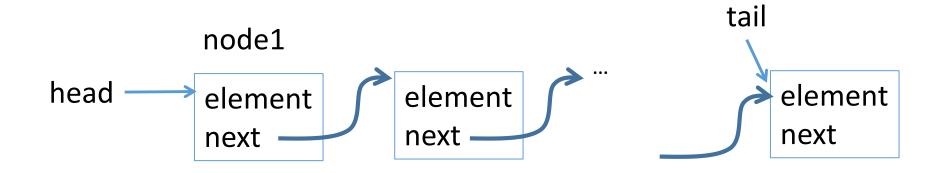
NULL is defined in <iostream> and <cstddef>.

Implementation of a linked list

- Add the first node
- Add the other node(s)

What should be done?

- 1. head points to the first element.
- 2. tail points to the last element.
- 3. Set tail->next = 0



Implementation of a linked list

- Add the first node
- Add the other node(s)

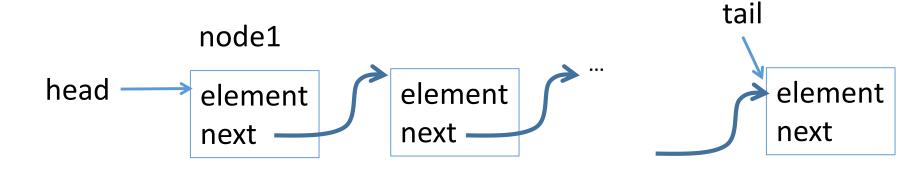
What should be done?

Maintain the head

Maintain the tail

Maintain the next pointer(s)

Set tail->next = 0



Node Class

```
template < typename T >
class Node {
public:
  T element; // The element contained in the node
  Node* next; // A pointer which points to the next node
  Node() { // No-arg constructor
   next = NULL;
  Node(T element) { // Constructor
    this->element = element;
    next = NULL;
```

The approach

- 1. Initialize the linked list
- 2. Add the first node (add as the last element)
- 3. Add the second node (add as the last element)
- 4. Add the third node (add as the last element)

When a node is added, the head and tail pointers must be updated properly.

- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
- 4. Add the third node

```
template < typename T = int >
class Node {
public:
  T element; Node* next;
  Node() { next = NULL; }
  Node(T element) {
    this->element = element;
    next = NULL;
Set next = 0;
```

- 1. Initialize the linked list
- 2. Add the first node
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- 4. Add the third node

```
Initialize();
```

```
Node *head, *tail;
void initialize( ) {
  head = tail = 0; // NULL
```

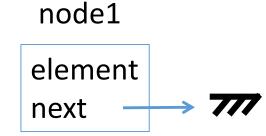
```
head \longrightarrow 7777 tail \longrightarrow 7777
```

- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
- 4. Add the third node

```
Initialize();
Node *node1 = new Node; ...
addNode(node1);
```

```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  tail = node;
  tail->next = 0;
```

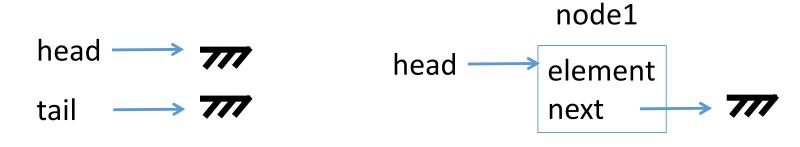
```
head \longrightarrow 777 tail \longrightarrow 777
```



- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
- 4. Add the third node

```
Initialize();
Node *node1 = new Node; ...
addNode(node1);
```

```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  tail = node;
  tail->next = 0;
```

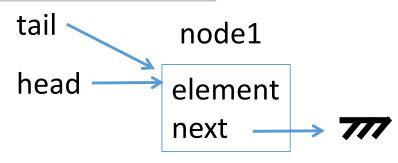


- 1. Initialize the linked list
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Initialize();
Node *node1 = new Node; ...
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```

```
void addNode( Node *node ) {
  if (head == 0) {
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```



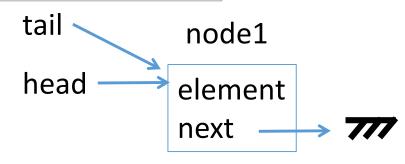


- 1. Initialize the linked list
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- 4. Add the third node

```
Initialize();
Node *node1 = new Node; ...
addNode(node1);
```

```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  tail = node;
  tail->next = 0; // required! Why?
```

```
head \longrightarrow 7777 tail \longrightarrow 7777
```



- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
- 4. Add the third node

```
...
Node *node2 = new Node; ...
addNode(node2);
```

```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
Node *node2 = new Node; ...
addNode(node2);
```

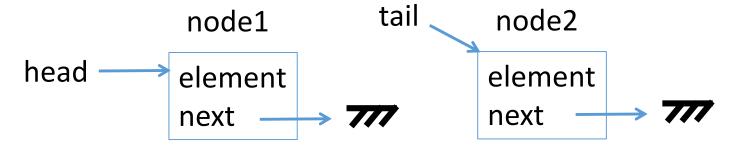
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
...
Node *node2 = new Node; ...
addNode(node2);
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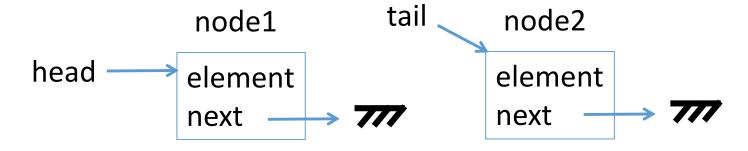
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
...
Node *node2 = new Node; ...
addNode(node2);
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void addNode( Node *node ) {
  if (head == 0) {
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- 1. Initialize the linked list
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...
Node *node2 = new Node; ...
addNode(node2);
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void addNode( Node *node ) {
  if (head == 0) {
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  tail->next = 0;
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- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
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```
...
Node *node2 = new Node; ...
addNode(node2);
```

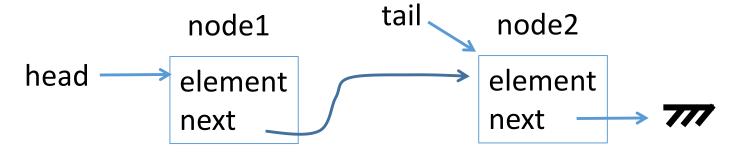
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
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```
...
Node *node2 = new Node; ...
addNode(node2);
```

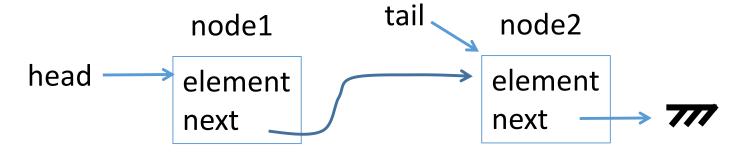
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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- 3. Add the second node
- 4. Add the third node

```
...
Node *node2 = new Node; ...
addNode(node2);
```

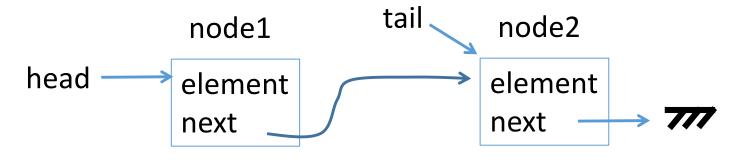
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
- 2. Add the first node
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```
...
Node *node2 = new Node; ...
addNode(node2);
```

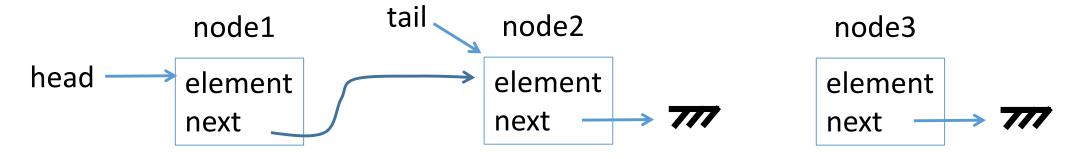
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
...
Node *node3 = new Node; ...
addNode(node3);
```

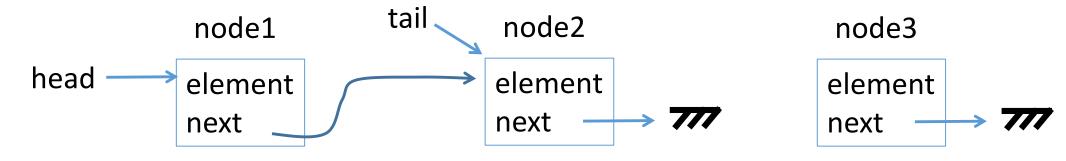
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
...
Node *node3 = new Node; ...
addNode(node3);
```

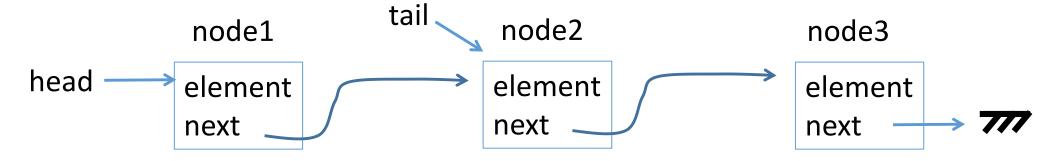
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
...
Node *node3 = new Node; ...
addNode(node3);
```

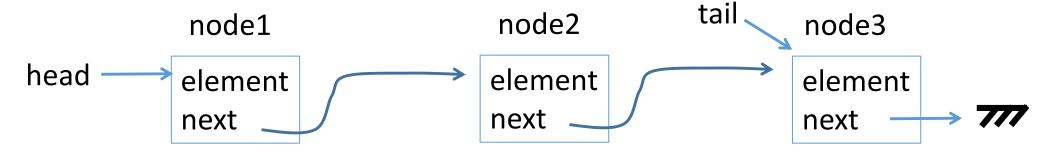
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
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```
...
Node *node3 = new Node; ...
addNode(node3);
```

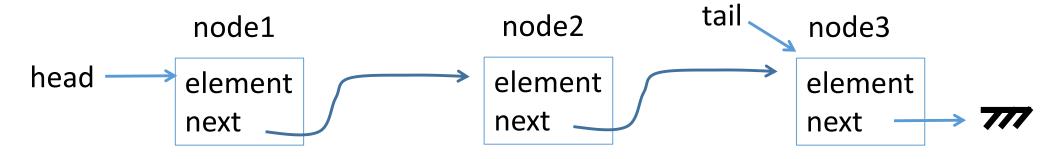
```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



- 1. Initialize the linked list
- 2. Add the first node
- 3. Add the second node
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```
...
Node *node3 = new Node; ...
addNode(node3);
```

```
void addNode( Node *node ) {
  if (head == 0) {
     head = node;
  if (tail) tail->next = node;
  tail = node;
  tail->next = 0;
```



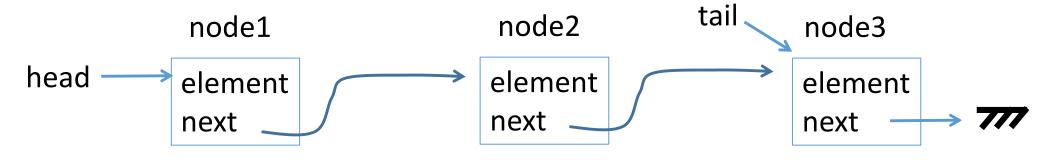
- 1. Initialize the linked list
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```
...
Node *node3 = new Node; ...
addNode(node3);
```

```
void addNode( Node *node ) {
  if (head == 0) {
    head = node;
}

if (tail) tail->next = node;
tail = node;
tail->next = 0;
}

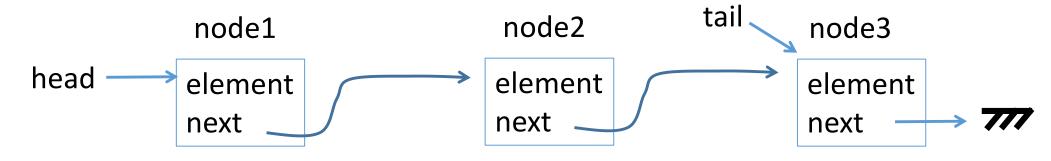
How to eliminate the if-structure?
```



- 1. Initialize the linked list
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- 3. Add the second node
- 4. Add the third node

```
...
Node *node3 = new Node; ...
addNode(node3);
```

```
void addNode( Node *node ) {
  if (head == 0) {
     head = tail = node;
     tail->next = 0;
     return;
  tail->next = node;
  tail = node;
  tail->next = 0;
```

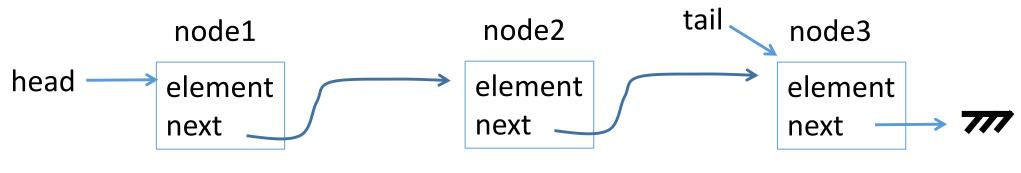


```
Node* cur = head; // current
while (cur != NULL)
  cout << cur->element << endl;</pre>
  cur = cur->next;
                                        tail
                             node2
           node1
                                              node3
  head
                            element
                                              element
           element
                            next
           next
                                              next
```

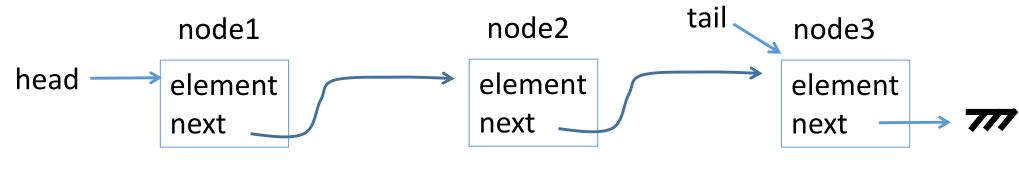
50

```
Node* cur = head; // current
while (current)
  cout << cur->element << endl;</pre>
  cur = cur - > next;
                                         tail
                              node2
            node1
                                               node3
  head
                             element
                                               element
           element
                             next
           next
                                               next
```

```
for (Node* cur = head; cur != NULL; cur = cur->next)
{
  cout << cur->element << endl;
}</pre>
```



```
for (Node* cur = head; cur; cur = cur->next)
{
  cout << cur->element << endl;
}</pre>
```



LinkedList

```
template<typename T>
class LinkedList {
public:
  LinkedList();
  void addNode( Node<T> *node );
  Node<T> *getLast() const;
  Node<T> *getFirst() const;
  bool removeFirst() const;
  bool removeLast() const;
  bool empty() const;
```

```
template<typename T>
void LinkedList<T>::addFirst(T element)
  Node<T>* newNode = new Node<T>(element);
  newNode->next = head;
  head = newNode;
                                                 tail
  size++;
                                  head
  if (!tail) tail = head;
                                         next
                                                        next
```

```
template<typename T>
void LinkedList<T>::addFirst(T element)
  Node<T>* newNode = new Node<T>(element);
  newNode->next = head;
  head = newNode;
                                                 tail
  size++;
                                  head
  if (!tail) tail = head;
                          next
                                         next
                                                        next
                          newNode
```

```
template<typename T>
void LinkedList<T>::addFirst(T element)
  Node<T>* newNode = new Node<T>(element);
  newNode->next = head;
  head = newNode;
                                                  tail
  size++;
                    head
  if (!tail)
    tail = head;
                                         next
                          next
                                                         next
                          newNode
```

```
template<typename T>
void LinkedList<T>::addFirst(T element)
  Node<T>* newNode = new Node<T>(element);
  newNode->next = head;
  head = newNode;
                                                  tail
  size++;
                    head
  if (!tail)
    tail = head;
                                         next
                          next
                                                         next
                          newNode
```

```
template<typename T>
void LinkedList<T>::addFirst(T element)
  Node<T>* newNode = new Node<T>(element);
  newNode->next = head;
  head = newNode;
                   head
                             tail
  size++;
  if (!tail)
    tail = head;
                          next
// head and tail
                         newNode
   are NULL
```

Implementing addLast(T element). Add a new object behind the last object.

```
template<typename T>
void LinkedList<T>::addLast(T element)
  if (tail == NULL)
    head = tail = new Node<T>(element);
  else {
    tail->next = new Node<T>(element);
    tail = tail->next;
  size++;
```

The new node's next must be set to NULL in the constructor.

Visualizing the memory content addLast(T *node)

```
%node := FA234Ah

//Assume the list is non-empty.

tail->next = node;

tail->next = 0;

%node := FA234Ah

FA234Ah

fa234Ah

onext = FA234Ah

read = FA234Ah

fa234Ah

fa234Ah

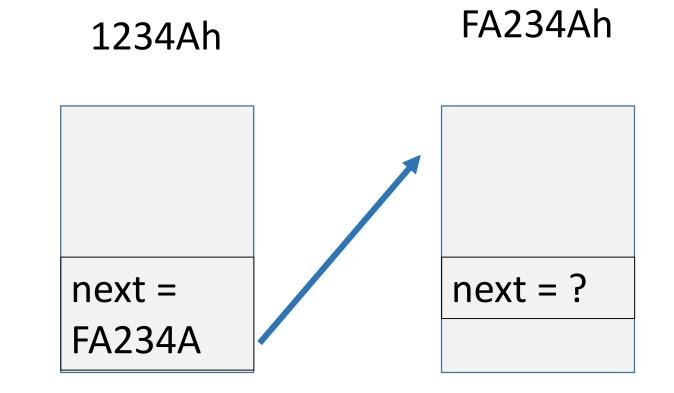
next = 0;
```

tail = 1234Ah

addLast(T *node)

node = FA234Ah

```
tail->next = node;
tail = node;
tail->next = 0;
```



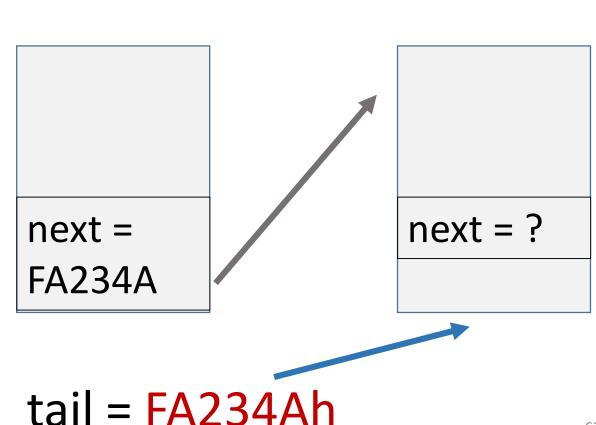
tail = 1234Ah

addLast(T *node)

node = FA234Ah

FA234Ah

```
tail->next = node;
tail = node;
tail->next = 0;
```



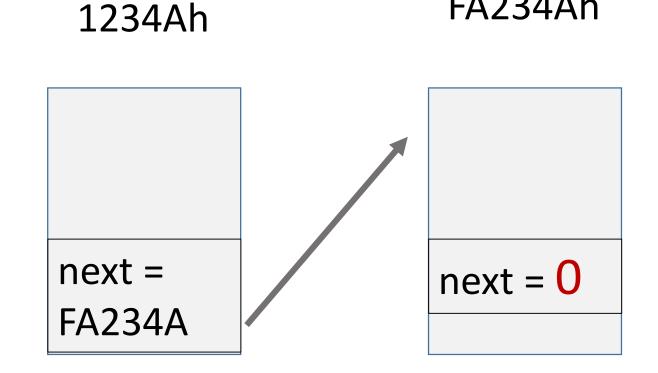
1234Ah

addLast(T *node)

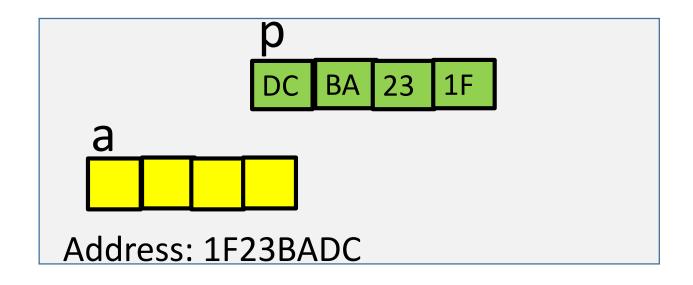
node = FA234Ah

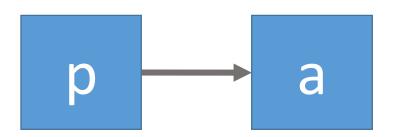
FA234Ah

```
tail->next = node;
tail = node;
tail->next = 0;
```



tail = FA234Ah

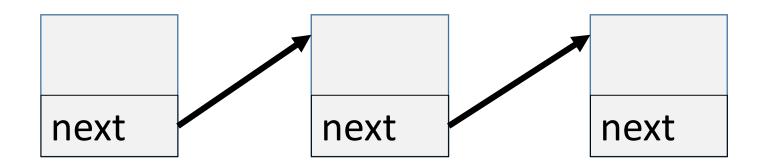




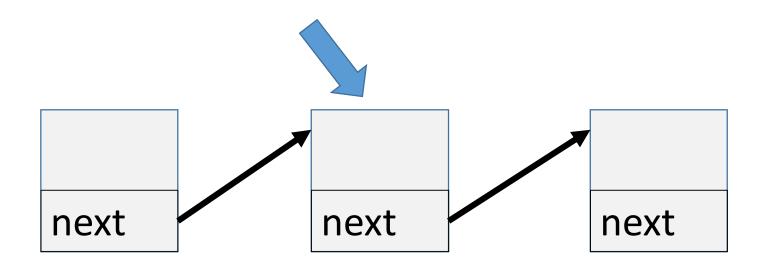
Implementing removeFirst(). How do you remove the first object? Do this exercise on your own.

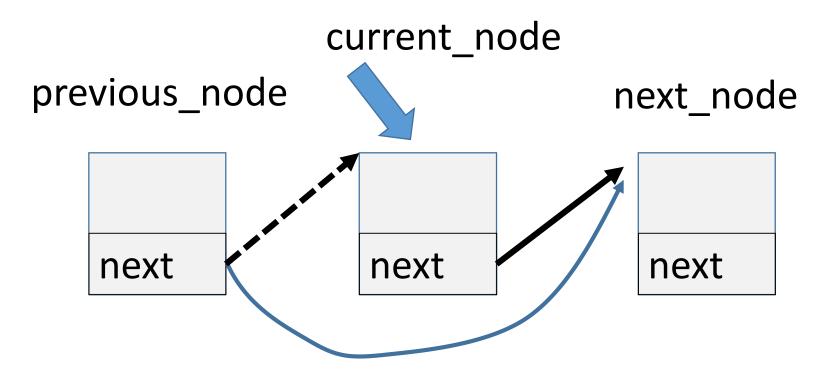
```
template<typename T>
T LinkedList<T>::removeFirst() {
  if (size == 0) return NULL;
  else {
    Node < T > * temp = head;
    head = head->next;
    size--;
    if (!head) tail = NULL;
    T element = temp->element;
    delete temp; // do we need to delete it or not?
    return element;
```

Node removal

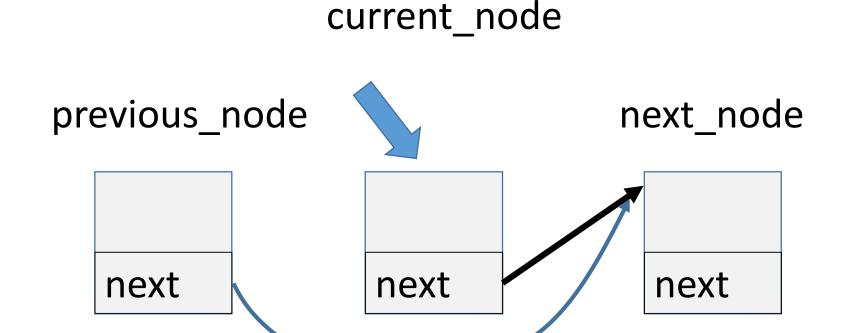


Node removal



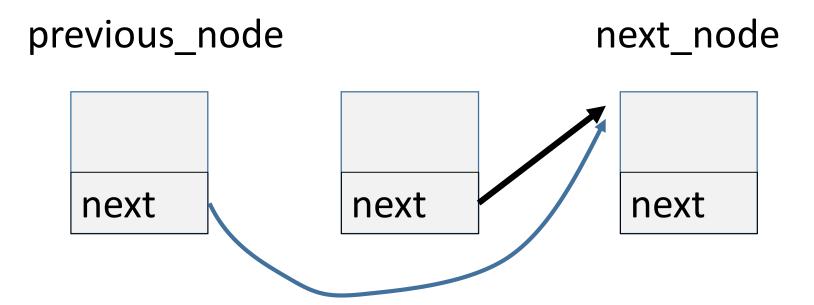


previous node->next = current node->next



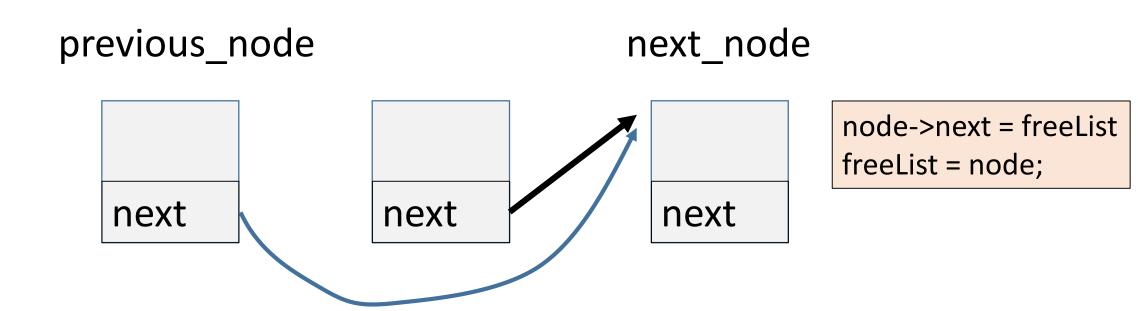
previous_node->next = current_node->next

current_node



recycle the node: delete it or put it to a free list.

current_node



recycle the node: delete it or put it to a free list.

Node removal

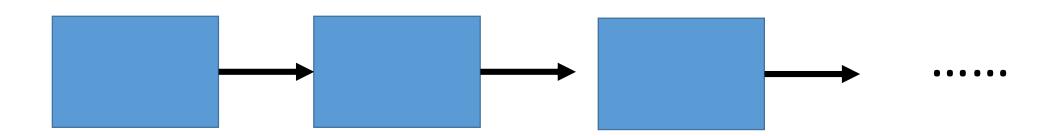
```
void removeNode( Node<T> *node) {
  if (node==head) {
     removeFirstNode();
     return;
  if (node==tail) {
     removeLastNode();
     return;
  Node<T> *prev = findPreviousNode(node);
  prev->next = node->next;
                                                                     node->next
                                   prev
                                   next
                                                      next
                                                                         next
```

Node removal

```
void removeNode( Node<T> *node) {
  if (node==head) {
    removeFirstNode();
                                                       Implement
    return;
                                                   findPreviousNode
  if (node==tail) {
    removeLastNode();
    return;
  Node<T> *prev = findPreviousNode(node);
  prev->next = node->next;
                                                                   node->next
                                   prev
                                  next
                                                     next
                                                                        next
```

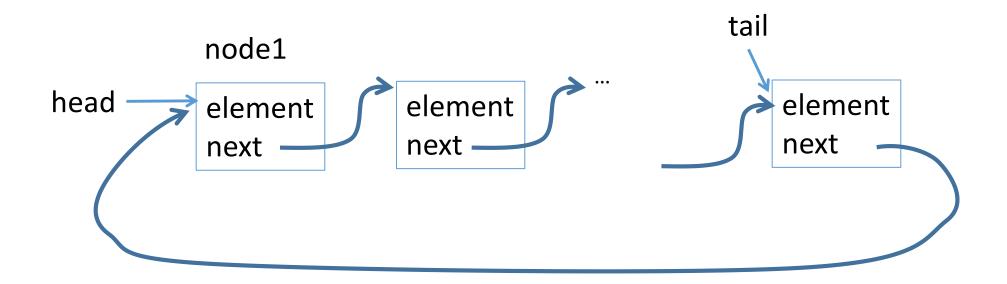
Variations of Linked Lists

A singly linked list has one pointer, i.e., next, which points to the next node.



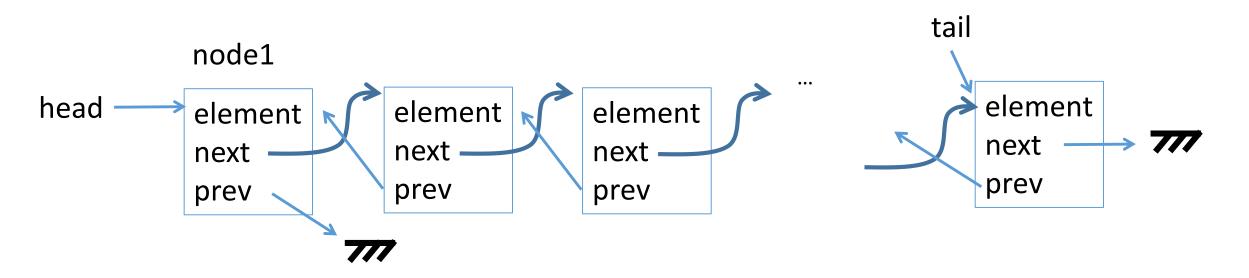
Circular Linked Lists

- A circular, singly linked list
- The last node points back to the first node.



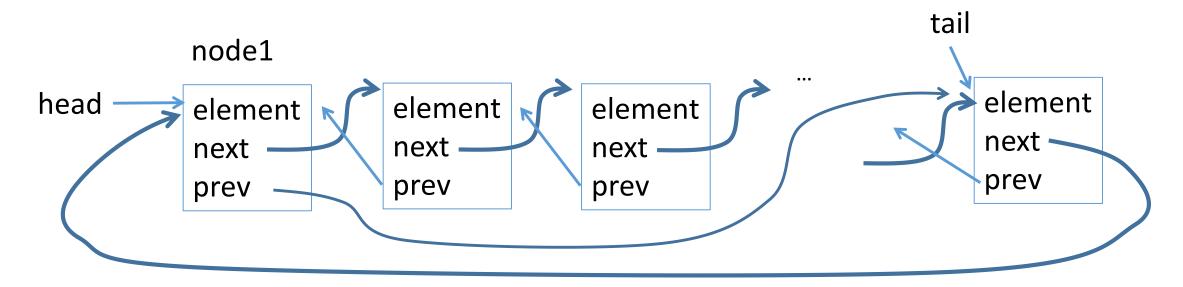
Doubly Linked Lists

- A doubly linked list contains the nodes with two pointers.
- The forward pointer points to the next node.
- The backward pointer points to the previous node.



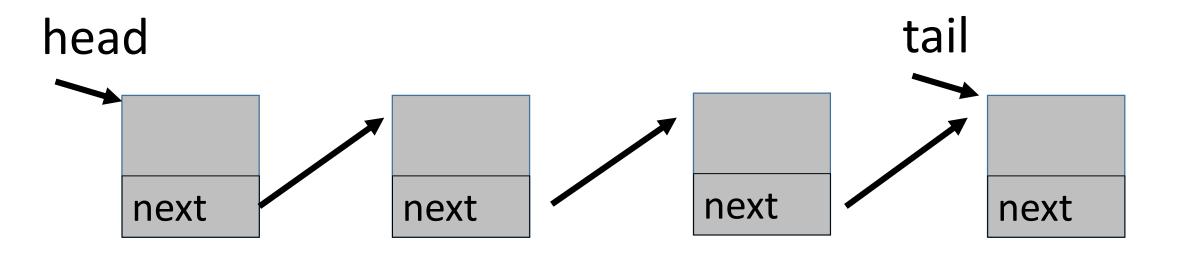
Circular, Doubly Linked Lists

- A linked list contains the nodes with two pointers.
- The forward pointer points to the next node.
- The backward pointer points to the previous node.
- The last node (forward pointer) points to the first node.
- The first node (backward pointer) points to the last node.



Stacks

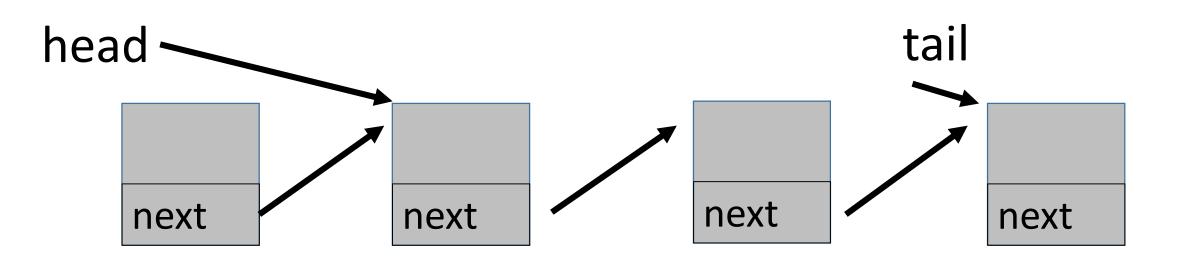
Stack can be implemented using an array or a linked list.



head = head->next

Stacks

Stack can be implemented using an array or a linked list.



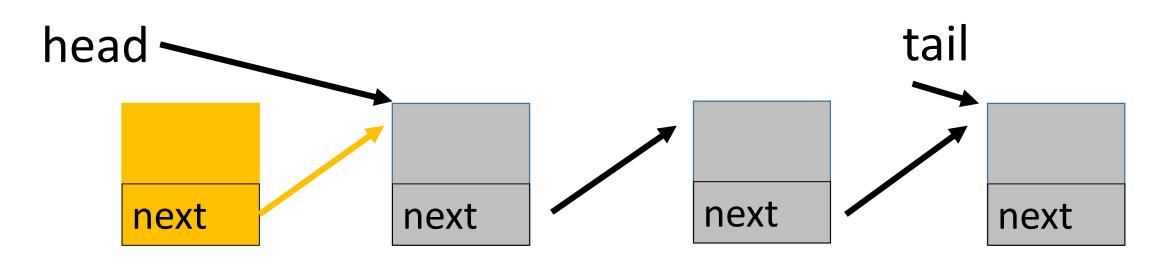
head = head->next

Stacks (Last-In-First-Out LIFO)

Stack can be implemented using an array

or

a linked list.



head = head->next

A queue: 1) remove the first node; 2) add a node as the last node.

We can adopt two ways to implement a queue.

 Using inheritance: You can declare the queue class by extending the linked list class.

Using composition: You can declare a linked list as a data field in the queue class.

We can adopt two ways to implement a queue.

Using inheritance: We can derive the queue class from the linked list class.

Using composition: We can declare a linked list as a data field in the queue class.

```
template<typename E>
class Queue: public LinkedList<E> {
    ...
};
```

We can adopt two ways to implement a queue.

Using inheritance: We can derive the queue class from the linked list class.

Using composition: We can declare a linked list as a data field in the queue class.

```
template<typename E>
class Queue: public LinkedList<E> {
    ...
};
```

```
template<typename E>
class Queue {
...
LinkedList L;
Node *addLast( const E &e );
Node *removeFirst( );
};
```

We can adopt two ways to implement a queue.

Using inheritance: We can derive the queue class from the linked list class.

Using composition: We can declare a linked list as a data field in the queue class.

```
template<typename E>
class Queue: public LinkedList<E> {
    ...
};
```

```
template<typename E>
class Queue {
    ...
    LinkedList L;
    Node *enqueue( const E &e ); //last
    Node *dequeue(); // remove first
    int getSize() const;
};
```

Supplemental Materials

Iterators

We can use iterators to perform a uniform way for traversing elements in various types of containers.

```
vector<int>::iterator p1;
cout << "Traverse the vector: ";
for (p1 = intVector.begin(); p1 != intVector.end(); p1++)
 cout << * D 1 << "";
```



```
vector<int>::iterator p1;
cout << "Traverse the vector: ";</pre>
for (p1 = intVector.begin(); p1 != intVector.end(); p1++)
 cout <<
```

```
Datatype::iterator p1;
cout << "Traverse the vector: ";
for (p1 = intVector.begin(); p1 != intVector.end(); p1++)
{
   cout << *p1 << " ";
}</pre>
```

```
Datatype::iterator p1;
cout << "Traverse the vector: ";
for (p1 = intVector.begin(); p1 != intVector.end(); p1++)
{
   cout << *p1 << " ";
}</pre>
```

```
vector<int> a(10, 1);
for ( int i = 0; i < a.size( ); ++i ) {</pre>
    cout << "i:" << "\t" << a[i] << endl;
vector<int>::iterator it;
for ( it = a.begin( ); it != a.end( ); ++it) {
    cout << "i:" << "\t" << (*it) << endl;
```

The Iterator Class

- Iterators can be viewed as encapsulated pointers. In a linked list, you can use pointers to traverse the list.
- But iterators have more functions than pointers. Iterators are objects.
- Iterators contain functions for accessing and manipulating elements.

The Iterator Class for LinkedList

Operation	Description
operator++(): Iterator	Get the iterator for the next pointer
operator*(): T &	Return the element from the node pointed by the iterator
operator==(itr:interator <t>&: bool</t>	
operator!=(itr:interator <t>&: bool</t>	

Advantages of Iterators

An iterator function likes a pointer.

 We may use pointers, array indexes, or other data structures to implement iterator functions.

Priority Queues

Elements are assigned with priorities.

 The element with the highest priority is accessed or removed first.

• A priority queue has a largest-in, first-out behavior.

Use vector to implement priority queue

```
class PQ {
protected:
vector<int> q;
public:
    int getElement() {
         int key = -1;
         return key;
```

Use vector to implement priority queue

```
vector<int> q;
public:
      // assume that a valid key is non-negative
      // return -1: no more element
      int getElement() {
            int key = -1;
            int key index;
            for ( int i = 0; i < q.size( ); ++i ) {
                  if ( key < 0 | | q[i] > key ) {
                        key = q[i];
                        key index = i;
            return key;
```

Use a heap structure to implement a priority queue structure

Exercise

Use a structure with a sorting ability to implement a priority queue structure

Exercise