# Overview of Linear Algebra (I)

翁志文

Department of Applied Mathematics, National Yang Ming Chiao Tung University

2024 Fall

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

# Outline



國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

# Syllabus

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

# Course descriptions and objectives

Linear Algebra learns the technique and theory to deal with large data by using matrices. The first semester focuses on computation and the second on theory. The following four topics will be covered in the first semester

1. Matrix multiplication;
2. Similar matrices;
3. Determinants;
4. Orthogonal matrices.

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

# Description of Course Details

- **Textbook:** Textbook: Introduction to Linear Algebra (6th Edition, 2023) by Gilbert Strang, Wellesley-Cambridge Press (滄海書局代理, 04-27088787)
- **Grading Policy:** Appearance, homework and presentations: 40%; Four exams with each 15% .
- **Pedagogy and other supplementary information :** 1. Use E3 system. 2. Every student has duty to give presentation and ask questions by using the lecture note provided by the instructor. This takes 40% of the grading.
- **Office hours:** Monday 15:30-17:20 or anytime you find me in my office.
- **Contact:** SA 350, weng@math.nctu.edu.tw, 03-5731610

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

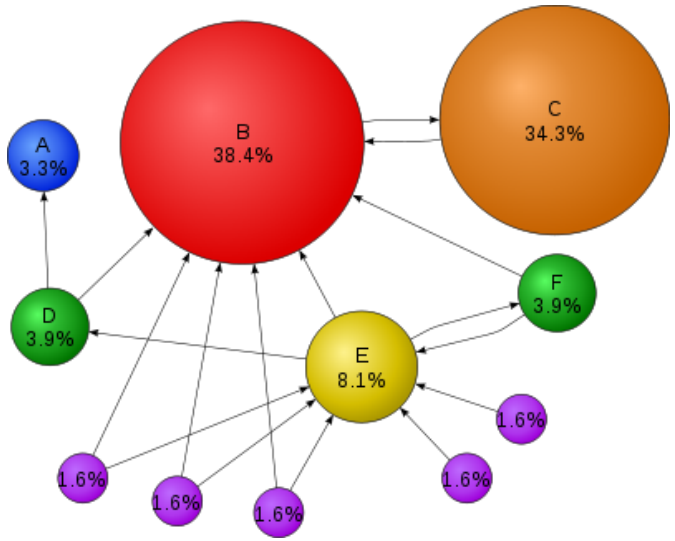
# Google page-rank

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

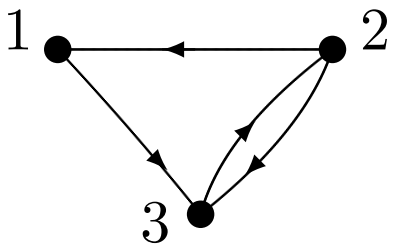
# 谷歌網頁評價

http://en.wikipedia.org/wiki/PageRank

PageRank was developed at Stanford University by Larry Page (hence the name Page-Rank) and Sergey Brin as part of a research project about a new kind of search engine.



# Example




$\alpha \in (0,1)$ 是守規矩的機率。

$$
\begin{aligned}
1 &= p_1 + p_2 + p_3, \\
p_1 &= \frac{1-\alpha}{3} + \alpha\left(\frac{p_2}{2}\right), \\
p_2 &= \frac{1-\alpha}{3} + \alpha\left(\frac{p_3}{1}\right), \\
p_3 &= \frac{1-\alpha}{3} + \alpha\left(\frac{p_1}{1} + \frac{p_2}{2}\right).
\end{aligned}
$$

$$
1 \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} \frac{1-\alpha}{3} & \frac{1-\alpha}{3} + \frac{\alpha}{2} & \frac{1-\alpha}{3} \\ \frac{1-\alpha}{3} & \frac{1-\alpha}{3} & \frac{1-\alpha}{3} + \alpha \\ \frac{1-\alpha}{3} + \alpha & \frac{1-\alpha}{3} + \frac{\alpha}{2} & \frac{1-\alpha}{3} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}.
$$

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

# 流量密碼

$$\begin{array}{ccccccc} \rho(M) & \cdot & P & = & M & \times & P \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{網譜半徑} & & \text{流量密碼} & & \text{網路} & & \\ \text{(spectral radius)} & & \text{(eigenvector)} & & \text{(graph)} & & \end{array}$$

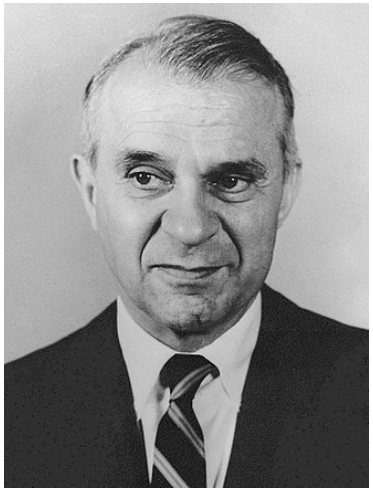
**研究體裁：**網譜 $\leftrightarrow$ 網路 $\leftrightarrow$ 流量密碼。

# Leontief's input-output analysis

國立陽明交通大學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

# Wassily Wassilyevich Leontief

Leontief was a Russian-American economist known for his research on **input-output analysis** and how changes in one economic sector may affect other sectors.



Leontief won the Nobel Committee's Nobel Memorial Prize in Economic Sciences in 1973, and four of his doctoral students have also been awarded the prize (Paul Samuelson 1970, Robert Solow 1987, Vernon L. Smith 2002, Thomas Schelling 2005).

# Input-output matrix

**假設：**

- 每一個體需要所有商品以製造其唯一商品。
- 商品 $j$ 的總價以 $p_j$ 表示。$p_j$ 也是個體 $j$ 的收入。
- 以 $a_{ij}$ 表商品 $j$ 分配給個體 $i$ 的比例。所以 $a_{1j}+a_{2j}+a_{3j}=1$ 且 $0 \le a_{ij} \le 1$。
- 個體支出不能超過收入。也就是 $a_{i1}p_1+a_{i2}p_2+a_{i3}p_3 \le p_i$。

| 個體\商品 | 食物 | 衣服 | 房子 | 花費 | 收入 |
|---|---|---|---|---|---|
| 農夫 | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{11}p_1+a_{12}p_2+a_{13}p_3$ | $p_1$ |
| 裁縫師 | $a_{21}$ | $a_{22}$ | $a_{23}$ | $a_{21}p_1+a_{22}p_2+a_{23}p_3$ | $p_2$ |
| 木匠 | $a_{31}$ | $a_{32}$ | $a_{33}$ | $a_{31}p_1+a_{32}p_2+a_{33}p_3$ | $p_3$ |
| 總價 | $p_1$ | $p_2$ | $p_3$ | | |

**以矩陣表示問題：**已知 $AP \le P$，求 $P$。