Introduction to Analysis Homework 1

September, 5, 2024

1. Show that the Euclidean space $\mathbb{R}^k$ is a metric space with each of the following metric:
$$d_p(\mathbf{x}, \mathbf{y}) = \left(\sum_{i=1}^{k} |x_i - y_i|^p\right)^{\frac{1}{p}},$$
for all $p \in \mathbb{N}$.

2. Let $S \subset \mathbb{R}^k$. Show that S is open $\iff$ for any $\mathbf{x} \in S$, there exists $r > 0$ such that $B_r(\mathbf{x}) \subset S$.

3. Show that the union of any collection of open sets is open.

4. Show that the union of finitely many closed sets is closed.

5. Show that if S and T are two subsets of $\mathbb{R}^k$ such that $S \cap T = \emptyset$ and T is open, then $\overline{S} \cap T = \emptyset$.