Homework 6

1. Let $A := \{1/n : n \in \mathbb{N}\} \cup \{0\} \subset \mathbb{R}$. Prove that A is compact directly from the definition (without using the Heine-Borel theorem).

Let $\{S_n\}_{n=1}^{\infty}$ be a sequence of compact and nonempty subsets in a metric space E. Then the followings hold.

2. If $S_n \supset S_{n+1}$ for all $n \geq 1$, then $\bigcap_{n=1}^{\infty} S_n \neq \phi$ and is compact.

3. If, in addition,$f : E \to E$ is a function, then $f(\bigcap_{n=1}^{\infty} Sn) \subset \bigcap_{n=1}^{\infty} f(S_n)$.

4. If, in addition, f is continuous, then $f(\bigcap_{n=1}^{\infty} Sn) = \bigcap_{n=1}^{\infty} f(S_n)$.