# Homework 8

1. Let $S \subset \mathbb{R}^m$ be bounded and $f : S \to \mathbb{R}^k$ be **uniformly continuous** on $S$. Show that $f(S)$ is bounded in $\mathbb{R}^k$

2. Let $S \subset \mathbb{R}^m$ be bounded and $f : \mathbb{R}^m \to \mathbb{R}^k$ be **continuous** on $\mathbb{R}^m$. Show that

   (i) $f(S)$ is bounded in $\mathbb{R}^k$

   (ii) give example why uniform continuity in the problem 1 can not be replaced by continuity .

3. Suppose $g$ is a real function on $\mathbb{R}$ with bounded derivative (say $|g'| \leq M$). Fix $\epsilon > 0$, and define $f(x) = x + \epsilon g(x)$. Prove that f is one-to-one if $\epsilon$ is small enough.

4. Let $f : (a, b) \to \mathbb{R}$ be a differentiable function.

   (i)If $f'(x) = 0$ for all $x \in (a, b)$, then $f$ is constant on $(a, b)$.

   (ii)If $f'(x) > 0$ for all $x \in (a, b)$ except finitely many points at which $f'(x) = 0$ then f is strictly increasing on (a,b).

5. Let $f : [a, b] \to \mathbb{R}$ be continuous such that $f(a) = f(b)$. Then for any number $n \in \mathbb{N}$, there exists $c \in [a, b]$ such that $f(c) = f(c + \frac{b-a}{n})$.