Homework 2

1. Show that continuity implies separate continuity, but the converse does not hold (given example).

2. Let $U, V \subset \mathbb{R}^m$ be open and $f : U \to V$ be a one-to-one and onto function such that f and f^{-1} are both continuous. Let S, U satisfy $\partial S \subset U$. Show that the following holds:

(i) $f(\partial S) \subset \partial(f(S))$

(ii) $\partial(f(S)) \subset V \iff \partial(f(S)) \subset f(\partial(S))$

(iii) It might happen that $f(\partial S) \neq \partial(f(S))$ (given example)

(iv) If $U = V = \mathbb{R}^m$, then $f(\partial S) = \partial(f(S))$

3. Let I be an interval and let $p \in I$. Let f, g, and h be real-valued functions defined on I, except possibly at p.

(i) Show that if $g(x) \leq f(x) \leq h(x)$ for all $x \in I \setminus \{p\}$ and

$$\lim_{x \to p} g(x) = \lim_{x \to p} h(x) = A$$

for some $A \in \mathbb{R}$, then $lim_{x \to p} f(x) = A$.

(ii) Show that if $lim_{x \to p} f(x) = 0$ and g is bounded on $I \backslash \{p\}$, then $lim_{x \to p} f(x) g(x) = 0$.