Introduction to Analysis Homework 3

September, 19, 2024

1. Let $S \subset \mathbb{R}^m$. Show that S is closed if and only if every accumulation point of S is in S.

2. Let $S \subset \mathbb{R}^m$. Show that the closure of S is the union of S and the set of all accumulation points of S.

3. Let $S \subset \mathbb{R}^m$. Show that x is an accumulation point of S if and only if for all $\epsilon > 0$, $B_\epsilon(x) \cap S$ has infinitely many points.

4. Let A_n be subsets of a metric space M, $A_{n+1} \subseteq A_n$, and $A_n \neq \emptyset$ for all $n \in \mathbb{N}$, but assume that $\cap_{n=1}^{\infty} A_n = \emptyset$. Suppose $x \in \cap_{n=1}^{\infty} \overline{A_n}$. Show that x is an accumulation point of A_1.

5. Let A be a nonempty set of real numbers, which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. Prove that

$$\inf A = -\sup(-A).$$

(Note that $\mathbb{R}$ has the greatest-lower-bound property. As a result, $\inf A$ exists. In this homework, you do not need to explain the existence of $\inf A$ anymore.)