Homework 10

1. Show that the following Taylor series for elementary functions

(i) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ on $(-1, 1)$.

(ii) $ln(1 + x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}$ on $(-1, 1]$.

2.Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} e^{\frac{-1}{x}} & , \text{ if } x > 0, \\ 0 & , \text{ if } x \leq 0. \end{cases}$$

Show that f is analytic on $\mathbb{R} \setminus \{0\}$, and is not analytic at the origin.

3.(i) Find the absolute maximum and minimum of $f(x, y) = x^2 + y^2 + y$ on the disc $x^2 + y^2 \leq 1$. (Use Lagrange's method)

(ii) The two planes $x + z = 4$ and $3x - y = 6$ intersect in a line L. Use Lagrange's method to find the point on L that is closest to the origin. (Hint: Minimize the square of the distance)

4. Let $f : (a, b) \to \mathbb{R}$ be a C^r function on (a,b), where $r \in \mathbb{N}$, and let $c \in (a, b)$. If $f^{(n)}(c) = 0$ for all $1 \leq n < r$ and $f^{(r)}(c) \neq 0$.

Show that if r is even and $f^{(r)}(c) > 0$, then f has a local minimum at c.