535514: Reinforcement Learning

Lecture 14 — DPG and DDPG

Ping-Chun Hsieh

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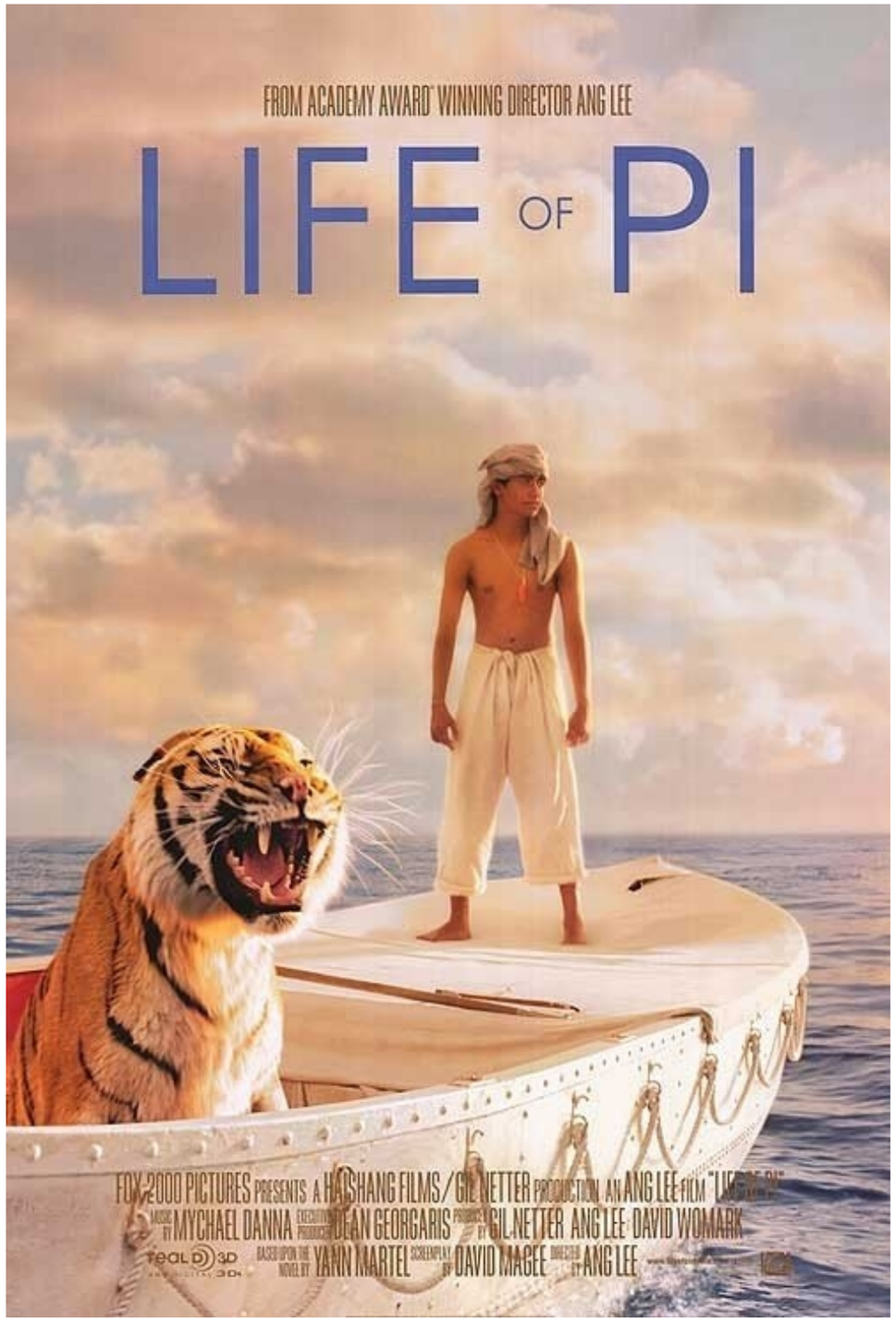


DDPG: Two Interpretations

- Version 1: Maximize $V^{\pi_\theta}(\mu)$ by policy gradient

- Version 2: Search & Mimic

- Which version of the story do you believe?

Recall: Deterministic Policy Gradient (DPG)

- Consider continuous actions and deterministic policy: $a = \pi_\theta(s)$
- Assumptions: $\nabla_a Q(s,a), \nabla_a P(s'|s,a), \nabla_\theta \pi_\theta(s), \nabla_a R(s,a)$ exist

- **Deterministic Policy Gradient Theorem**:

$$\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma} \mathbb{E}_{s \sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)} \right]$$

Recall: On-Policy Deterministic Actor-Critic

Deterministic PG: $$\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_\mu^{\pi_\theta}}\left[\nabla_\theta \pi_\theta(s)\,\nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)}\right]$$

- Deterministic Actor-Critic (DAC):

Step 1: Initialize θ_0, w_0 and step sizes α_θ, α_w

Step 2: Sample a trajectory $\tau = (s_0, a_0, r_1, \cdots) \sim P_\mu^{\pi_\theta}$

For each step of the current trajectory $t = 0,1,2,\cdots$

$$\Delta w_k \leftarrow \Delta w_k + \alpha_w\left(r_t + \gamma Q_{w_k}(s_{t+1}, a_{t+1}) - Q_{w_k}(s_t, a_t)\right)\nabla_w Q_w(s_t, a_t)|_{w=w_k}$$

$$\Delta\theta_k \leftarrow \Delta\theta_k + \alpha_\theta\gamma^t\left(\nabla_\theta \pi_\theta(s_t)\,\nabla_a Q_w(s_t, a)|_{a=\pi_\theta(s_t)}\right)$$

$$= \nabla_\theta Q_w(s_t, \pi_\theta(s_t))|_{\theta=\theta_k}$$

$$\theta_{k+1} \leftarrow \theta_k + \Delta\theta_k, w_{k+1} \leftarrow w_k + \Delta w_k$$

A Quick Remark on DPG Expression

- In Deterministic Actor-Critic:

$$\Delta\theta_k \leftarrow \Delta\theta_k + \alpha_\theta \gamma^t \Big(\nabla_\theta \pi_\theta(s_t) \nabla_a Q_w(s_t, a)|_{a=\pi_\theta(s_t)} \Big)$$

$$= \nabla_\theta Q_w(s_t, \pi_\theta(s_t))|_{\theta=\theta_k}$$

- In the original DPG expression:

$$\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma} \mathbb{E}_{s\sim d_\mu^{\pi_\theta}} \Big[\nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s, a)|_{a=\pi_\theta(s)} \Big]$$

$$\frac{\partial Q^{\pi_\theta}(s, \pi_\theta(s))}{\partial \theta_i} := \lim_{\Delta\theta_i \to 0} \frac{Q^{\pi_{\theta+\Delta\theta_i}}(s, \pi_{\theta+\Delta\theta_i}(s)) - Q^{\pi_\theta}(s, \pi_\theta(s))}{\Delta\theta_i}$$

$$= \nabla_\theta Q^{\pi_\theta}(s, \pi_\theta(s))|_{\theta=\theta_k} \; ??$$

- Question: Any issue with deterministic policies?

Insufficient exploration

- Question: Is it possible to learn π but act under another policy β?

"*Off-policy learning!*"

Off-Policy Learning with Deterministic Policy Gradients

On-Policy vs Off-Policy

- On-policy:

  Learned policy = Policy used to interact with the environment

- Off-policy:

  Learned policy ≠ Policy used to interact with the environment

  Called “behavior policy”

Kazami Hayato
(learning agent)

Asurada
(policy for interaction)

Off-Policy Learning

Off-policy learning

1. Learn a target policy $\pi_\theta(a \mid s)$ and compute $V^{\pi_\theta}(s)$ or $Q^{\pi_\theta}(s, a)$
2. In the meantime, follow a behavior policy $\beta(a \mid s)$

$$\{s_0, a_0, r_1, s_1, a_1, r_2, \cdots s_{T-1}, a_{T-1}, r_T\} \sim \beta$$

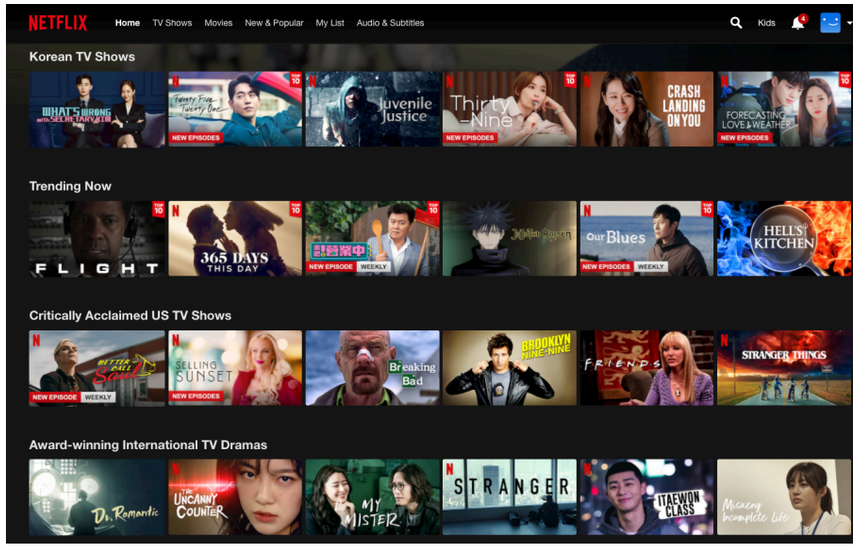
- Why is off-policy learning useful?
  1. Learn from observing humans or other agents
  2. Reuse experience generated from old policies $\pi_1, \pi_2 \cdots, \pi_{k-1}$
  3. Learn about optimal policy while following an exploratory policy
  4. Learn about multiple policies while following one policy

(Slide Credit: David Silver)

Off-Policy Learning is Essential in Many “Real-World” Problems

- Recommender Systems

➡ Deploy a safe policy for collecting user data without losing user’s interest



➡ Learn a better policy from these data

- Robot Control

➡ Deploy a safe policy for collecting robot data without hurting the machine

➡ Learn a good policy from these data

What are the behavior policies in the above applications?

Deterministic PG in Off-Policy Learning

(On-policy) DPG: $$\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma} \mathbb{E}_{s\sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)} \right]$$

- Question: Does the deterministic PG remain the same in off-policy learning?

Nope! (state visitation distribution shall change)

- Consider a new objective for off-policy learning: (Why reasonable?)

$$J_\beta(\pi_\theta) := \sum_s d_\mu^\beta(s) V^{\pi_\theta}(s)$$

Maximizing $J_\beta(\pi_\theta)$ is "equivalent" to maximizing $V^{\pi_\theta}(\mu)$ (as long as $d_\mu^\beta(s) > 0$ for all states s)

- "Off-policy" deterministic PG: Let's use $\nabla_\theta J_\beta(\pi_\theta)$!

$$V^{\pi_\theta}(\mu) := \sum_s d_\mu^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

Off-Policy Deterministic PG

- **Off-Policy Deterministic Policy Gradient**:

$$\nabla_\theta J_\beta(\pi_\theta) \approx \mathbb{E}_{s\sim d_\mu^\beta}\left[\nabla_\theta \pi_\theta(s)\, \nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)}\right]$$

- Question: Is the above easy to operate with?

- Derivation:

$$\nabla_\theta J_\beta(\pi_\theta) = \nabla_\theta \left(\sum_s d_\mu^\beta(s) V^{\pi_\theta}(s) \right)$$

Bellman expectation equation

$$\nabla_\theta Q^{\pi_\theta}(s, \pi_\theta(s))$$

$$= \nabla_\theta \left(\sum_s d_\mu^\beta(s) Q^{\pi_\theta}(s, \pi_\theta(s)) \right)$$

The DPG paper (Silver, ICML 2014) dropped a term $\nabla_\theta Q^{\pi_\theta}(s,a)$

$$\approx \sum_s d_\mu^\beta(s) \left(\nabla_\theta \pi_\theta(s)\, \nabla_a Q^{\pi_\theta}(s,a)|_{a=\mu_\theta(s)} \right)$$

Why is $\nabla_\theta Q^{\pi_\theta}(s,a)$ Difficult to Evaluate?

- Recall from the expression of deterministic PG:

$$\nabla_\theta V^{\pi_\theta}(s) = \frac{1}{1-\gamma_\infty}\mathbb{E}_{s'\sim d_\mu^{\pi_\theta}}\Big[\nabla_\theta \pi_\theta(s')\,\nabla_a Q^{\pi_\theta}(s',a)|_{a=\pi_\theta(s')}\Big]$$

$$= \sum_{s'}\sum_{t=0}^{\infty}\gamma^t P(s\to s',t,\pi_\theta)\,\nabla_\theta\pi_\theta(s')\,\nabla_a Q^{\pi_\theta}(s',a)\Big|_{a=\pi_\theta(s')}$$

Accordingly, we have

$$\nabla_\theta Q^{\pi_\theta}(s,a) = \nabla_\theta\Big(R(s,a)+\gamma\sum_{s'}P(s'|s,a)V^{\pi_\theta}(s')\Big)$$

$$= \gamma\sum_{s'}P(s'|s,a)\,\nabla_\theta V^{\pi_\theta}(s')$$

$$= \gamma\sum_{s'}P(s'|s,a)\sum_{s''}\sum_{t=0}^{\infty}\gamma^t P(s'\to s'',t,\pi_\theta)\,\nabla_\theta\pi_\theta(s'')\,\nabla_a Q^{\pi_\theta}(s'',a)\Big|_{a=\pi_\theta(s'')}$$

hard to evaluate in off-policy learning (why?)

Off-Policy Deterministic Actor-Critic (OPDAC)

- Off-Policy Deterministic Actor-Critic (OPDAC):
  - Critic: estimate $Q_w \approx Q^{\pi_\theta}$ by TD bootstrapping
  - Actor: updates policy parameters θ by off-policy deterministic PG

Step 1: Initialize θ_0, w_0 and step sizes α_θ, α_w

Step 2: Sample a trajectory $\tau = (s_0, a_0, r_1, \cdots) \sim P_\mu^\beta$

For each step of the current trajectory $t = 0,1,2,\cdots$

$$\Delta w_k \leftarrow \Delta w_k + \alpha_w \left(r_t + \gamma Q_{w_k}(s_{t+1}, \pi_\theta(s_{t+1})) - Q_{w_k}(s_t, a_t) \right) \nabla_w Q_w(s_t, a_t)|_{w=w_k}$$

$$\Delta \theta_k \leftarrow \Delta \theta_k + \alpha_\theta \gamma^t \left(\nabla_\theta \pi_\theta(s_t) \nabla_a Q_{w_k}(s_t, a)|_{a=\pi_\theta(s_t)} \right)$$

$$= \nabla_\theta Q_{w_k}(s_t, \pi_\theta(s_t))|_{\theta=\theta_k}$$

$$\theta_{k+1} \leftarrow \theta_k + \Delta \theta_k, w_{k+1} \leftarrow w_k + \Delta w_k$$

- Question: Can you identify differences between OPDAC and DAC?

Let's Prove the DPG Together

$$
\begin{aligned}
\nabla_\theta V^{\pi_\theta}(s) &= \nabla_\theta Q^{\pi_\theta}(s, \pi_\theta(s)) \\
&= \nabla_\theta \left(R(s, \pi_\theta(s)) + \int_{\mathcal{S}} \gamma P(s' \mid s, \pi_\theta(s)) V^{\pi_\theta}(s') \, \mathrm{d}s' \right) \\
&= \nabla_\theta \pi_\theta(s) \nabla_a R(s,a)|_{a=\pi_\theta(s)} + \nabla_\theta \int_{\mathcal{S}} \gamma P(s' \mid s, \pi_\theta(s)) V^{\pi_\theta}(s') \, \mathrm{d}s' \\
&= \nabla_\theta \pi_\theta(s) \nabla_a R(s,a)|_{a=\pi_\theta(s)} \\
&\quad + \int_{\mathcal{S}} \gamma \left(P(s' \mid s, \pi_\theta(s)) \nabla_\theta V^{\pi_\theta}(s') + \nabla_\theta \pi_\theta(s) \nabla_a P(s' \mid s, a)|_{a=\pi_\theta(s)} V^{\pi_\theta}(s') \right) \mathrm{d}s' \\
&= \nabla_\theta \pi_\theta(s) \nabla_a \left(R(s,a) + \int_{\mathcal{S}} \gamma P(s' \mid s, a) V^{\pi_\theta}(s') \, \mathrm{d}s' \right) \Big|_{a=\pi_\theta(s)} \\
&\quad + \int_{\mathcal{S}} \gamma P(s' \mid s, \pi_\theta(s)) \nabla_\theta V^{\pi_\theta}(s') \, \mathrm{d}s' \\
&= \nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)} + \int_{\mathcal{S}} \gamma P(s \to s', 1, \pi_\theta) \nabla_\theta V^{\pi_\theta}(s') \, \mathrm{d}s'
\end{aligned}
$$

product rule

By Bellman equation, this is $Q^{\pi_\theta}(s,a)$

This is again a recursion! (Similar to the proof of (P2) of stochastic PG)

Let's Prove the DPG Together (Cont.)

The remaining proof can be done by "peeling off":

$$\nabla_\theta V^{\pi_\theta}(s) = \nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)}$$
$$+ \int_{\mathcal{S}} \gamma P(s \to s', 1, \pi_\theta) \nabla_\theta \pi_\theta(s') \nabla_a Q^{\pi_\theta}(s',a)\Big|_{a=\pi_\theta(s')} \mathrm{d}s'$$
$$+ \int_{\mathcal{S}} \gamma^2 P(s \to s', 2, \pi_\theta) \nabla_\theta V^{\pi_\theta}(s') \mathrm{d}s'$$
$$\vdots$$
$$= \int_{\mathcal{S}} \sum_{t=0}^{\infty} \gamma^t P(s \to s', t, \pi_\theta) \nabla_\theta \pi_\theta(s') \nabla_a Q^{\pi_\theta}(s',a)\Big|_{a=\pi_\theta(s')} \mathrm{d}s'$$

$\Downarrow$

$\downarrow_S^{\pi_\theta}(s')$

Deep Deterministic Policy Gradient (DDPG)

(= OPDAC with Deep Neural Nets)

Lillicrap et al., “Continuous control with deep reinforcement learning”, ICLR 2016

What is DDPG?

- DDPG: Combine OPDAC with NN nonlinear VFA
  - Off-policy: Exploration
  - Nonlinear VFA: Convergence issue

- To tackle the above issues, DDPG applies several techniques:

(T1) Experience replay (for data-efficient off-policy learning)

(T2) Ornstein-Uhlenbeck process for exploration (optional)

(T3) Target networks

(T1) Experience Replay

- Main idea:

  1. Store the previous experiences (s, a, s', r) into a buffer

  2. Sample a mini-batch from the buffer at each step

(similar to mini-batch SGD in supervised learning)

- Purposes:

1. Better estimate of DPG: Break correlations between successive steps in a trajectory (“more stable learning”, as stated in many papers)

2. Better data efficiency: Fewer interactions with environment needed for convergence

(T2) Ornstein-Uhlenbeck Process for Exploration

- Issue with Gaussian noise exploration $a_t = \pi_\theta(s_t) + N(0,\sigma^2)$?

- Ornstein-Uhlenbeck (OU) process: Similar to Gaussian policies, but with temporal correlation

Brownian motion

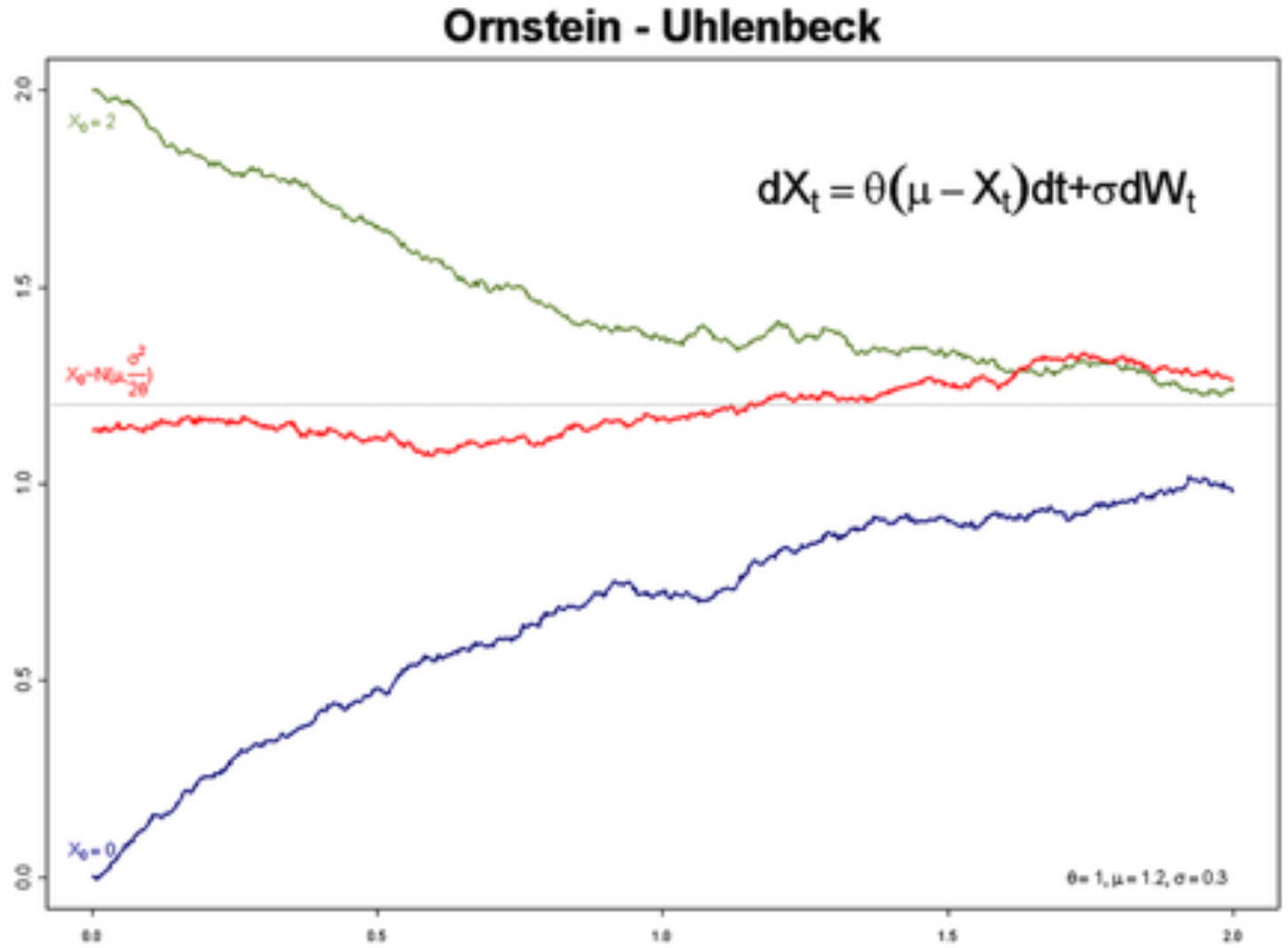
$$dx_t = \theta(\mu - x_t)dt + \sigma \cdot dW_t$$

- Discrete-time approximation of OU:

$$X_{t+1} - X_t = \theta(\mu - X_t)\Delta t + \sigma \cdot \Delta W_t$$

i.i.d. normal random variables $\sim \mathcal{N}(0,\Delta_t)$

Example of OU Process



(Same OU process with 3 different initial conditions)

How about a sequence of i.i.d. Gaussian random variables?

(Figure Source: Wikipedia)

(T3) Target Networks

- Idea: Use separate *target networks* ($\bar{\pi}_\theta$ for actor, $\bar{Q}_w$ for critic) that are updated only periodically
- For DDPG, the critic update with target networks

$$\Delta w_k \leftarrow \Delta w_k + \alpha_w \Big(r_t + \gamma \underbrace{\bar{Q}_{w_k}(s_{t+1}, \bar{\pi}_\theta(s_{t+1}))}_{\text{target}} - \underbrace{Q_{w_k}(s_t, a_t)}_{\text{update}} \Big) \nabla_w Q_w(s_t, a_t)|_{w=w_k}$$

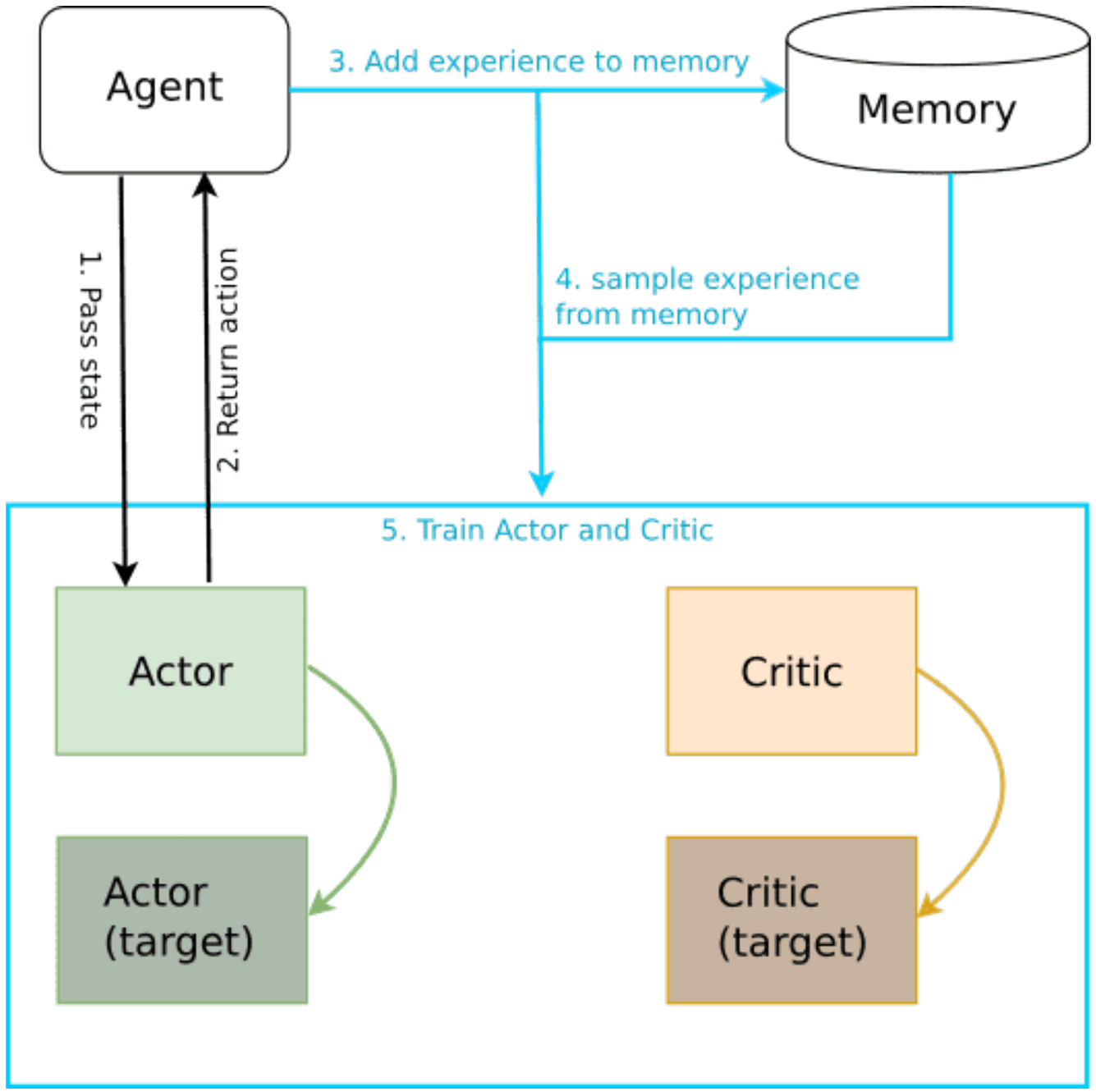
- Similar to value iteration:

$$\underbrace{V(s)}_{\text{update}} \leftarrow \max_a R(s,a) + \gamma \sum_{s'} P(s' \mid s, a) \underbrace{\bar{V}(s)}_{\text{target}}$$

- Purpose: Mitigate divergence

(Slide Credit: Pascal Poupart)

DDPG Architecture



 (Figure Credit: https://www.katnoria.com/ddpg-reacher/)

Pseudo Code of DDPG Algorithm

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$

2 evaluation networks and 2 target networks

Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**

action drawn from a deterministic policy with exploration

    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R

experience replay

    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

Update actor and critic

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

→ This can be viewed as the gradient of Q w.r.t. θ

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

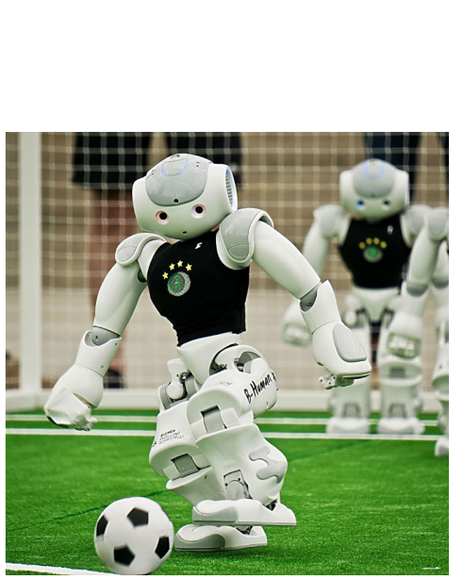
Update target networks (small τ for stability)

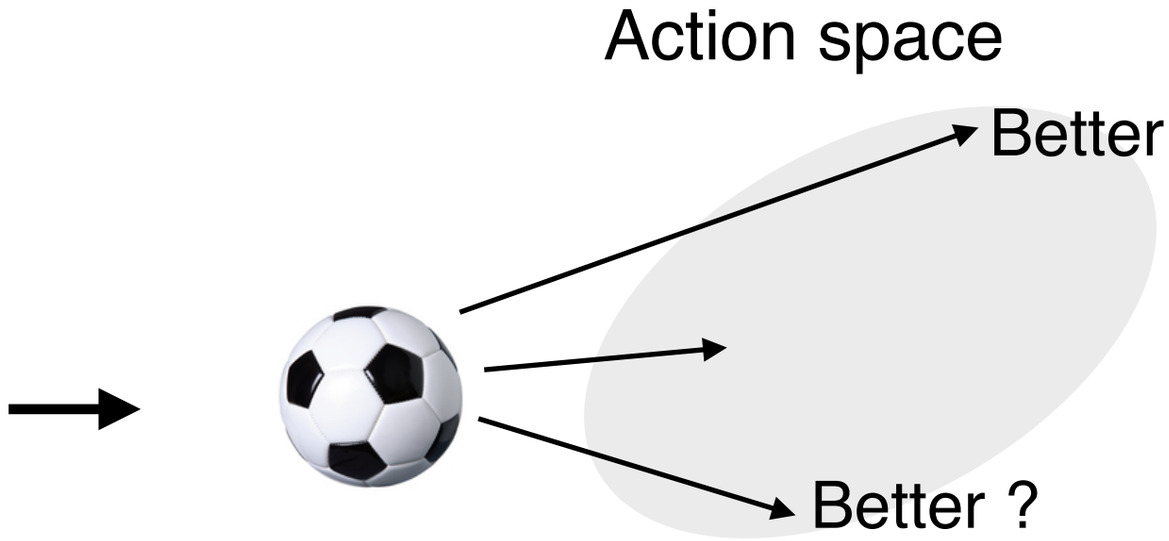
  end for
end for

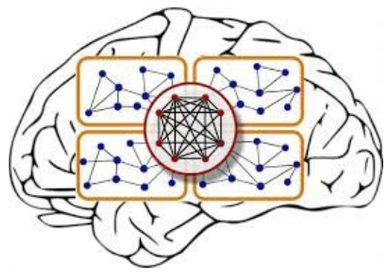
An Alternative Interpretation of DDPG

A Motivating Example of “Search & Mimic”

(A Robocup example)







Current state

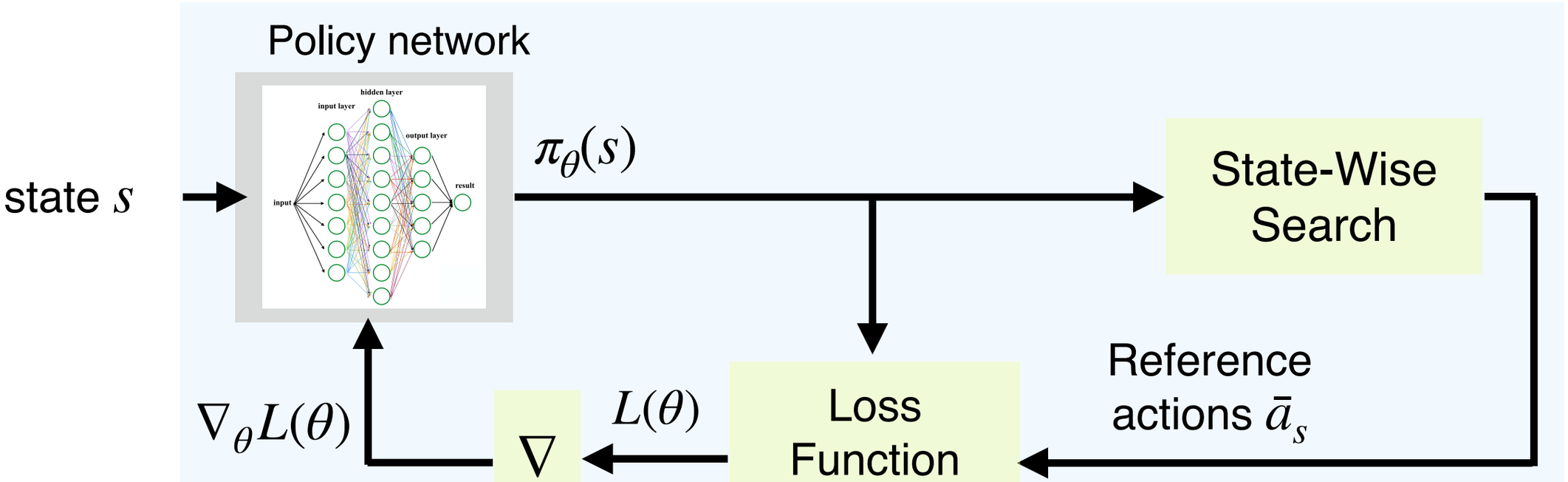
Search for a better reference action (higher Q value)

Mimic (update the policy towards the reference action)

RL = Repeatedly “search & mimic” in different scenarios

An Alternative Interpretation of DDPG

Let's formally write down "Search & Mimic" approach:



$$\bar{a}_s = \pi_{\bar{\theta}}(s) + \eta \nabla_a Q_w(s,a)$$

$$L(\theta) = \frac{1}{|B|} \sum_{s \in B} \left(\pi_\theta(s) - \bar{a}_s\right)^2$$

$$\nabla_\theta L(\theta) = \frac{1}{|B|} \sum_{s \in B} \nabla_\theta \left(\pi_\theta(s) - \bar{a}_s\right)^2 = \frac{1}{|B|} \sum_{s \in B} \nabla_\theta \left(\pi_\theta(s) - \left(\pi_{\bar{\theta}}(s) + \eta \nabla_a Q_w(s,a)\right)\right)^2$$

One Surprising Fact: “Search & Mimic” and DDPG Are Equivalent!

Theorem: $\Delta\theta_{DDPG}$ & $\Delta\theta_{S\&M}$ are parallel

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$$\Delta\theta_{DDPG} \propto \nabla_\theta J_\mu(\pi_\theta) = \frac{1}{|B|}\sum_{s\in B} \nabla_a Q_w(s,a)\,\nabla_\theta \pi_\theta(s) \qquad \text{(By DPG theorem)}$$

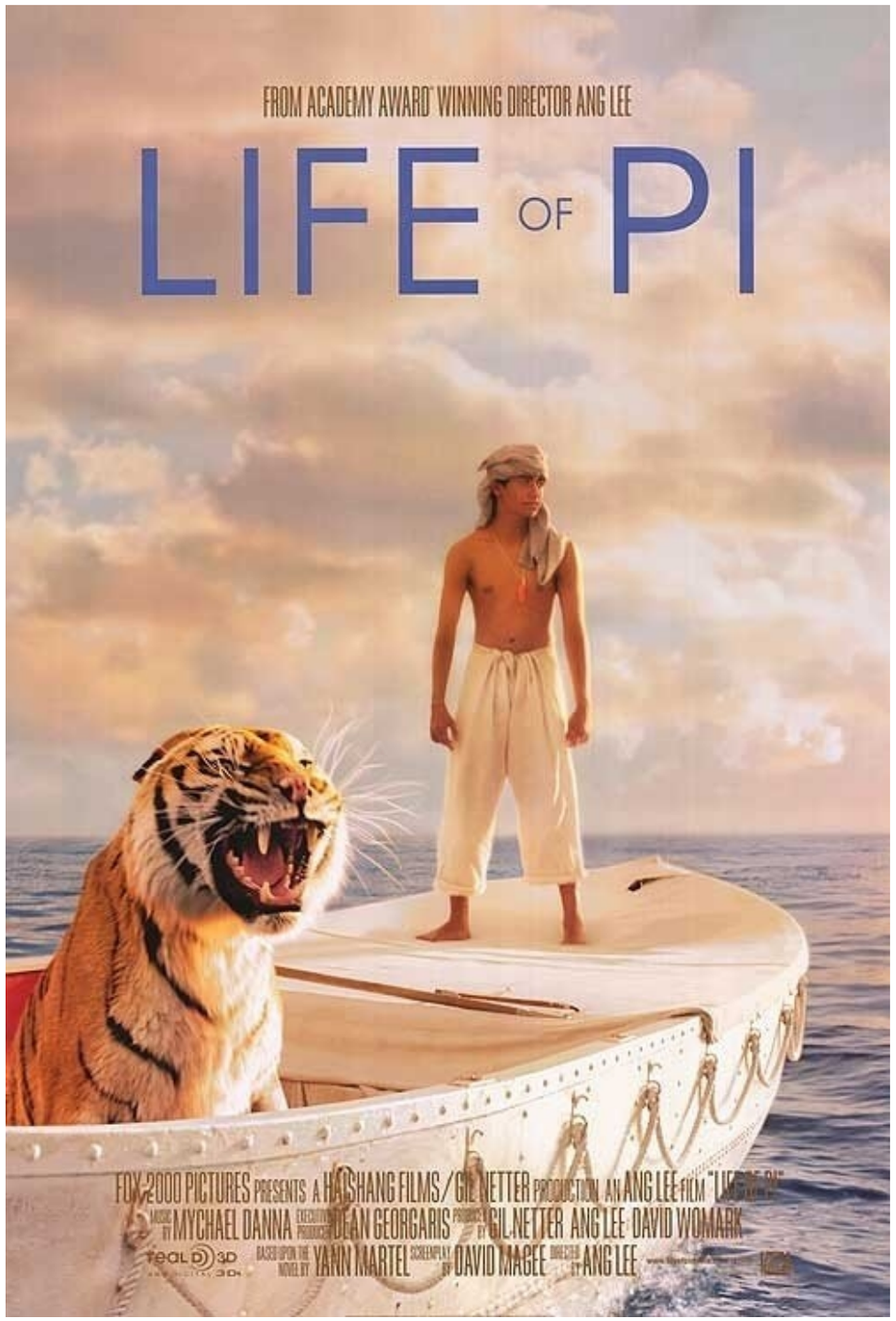
$$\Delta\theta_{S\&M} \propto \nabla_\theta L(\theta) = \frac{1}{|B|}\sum_{s\in B} \nabla_\theta \Big(\pi_\theta(s) - \bar{a}_s\Big)^2 \qquad \text{(By Search \& Mimic)}$$

$$= \frac{1}{|B|}\sum_{s\in B} \nabla_\theta \Big(\pi_\theta(s) - \big(\pi_{\bar{\theta}}(s) + \eta\,\nabla_a Q_w(s,a)\big)\Big)^2$$

$$= \frac{1}{|B|}\sum_{s\in B} \nabla_a Q_w(s,a)\,\nabla_\theta \pi_\theta(s)$$

(For more details, please refer to our UAI 2021 paper, available at https://arxiv.org/pdf/2102.11055.pdf)

Rethinking DDPG: Two Interpretations



What does DDPG actually do?

- Version 1: Maximize $V^{\pi_\theta}(\mu)$ by policy gradient

- Version 2: State-wise search in action space & mimicking

- Which version of the story better describes our learning process?