535514: Reinforcement Learning
Lecture 14 — DPG and DDPG

Ping-Chun Hsieh

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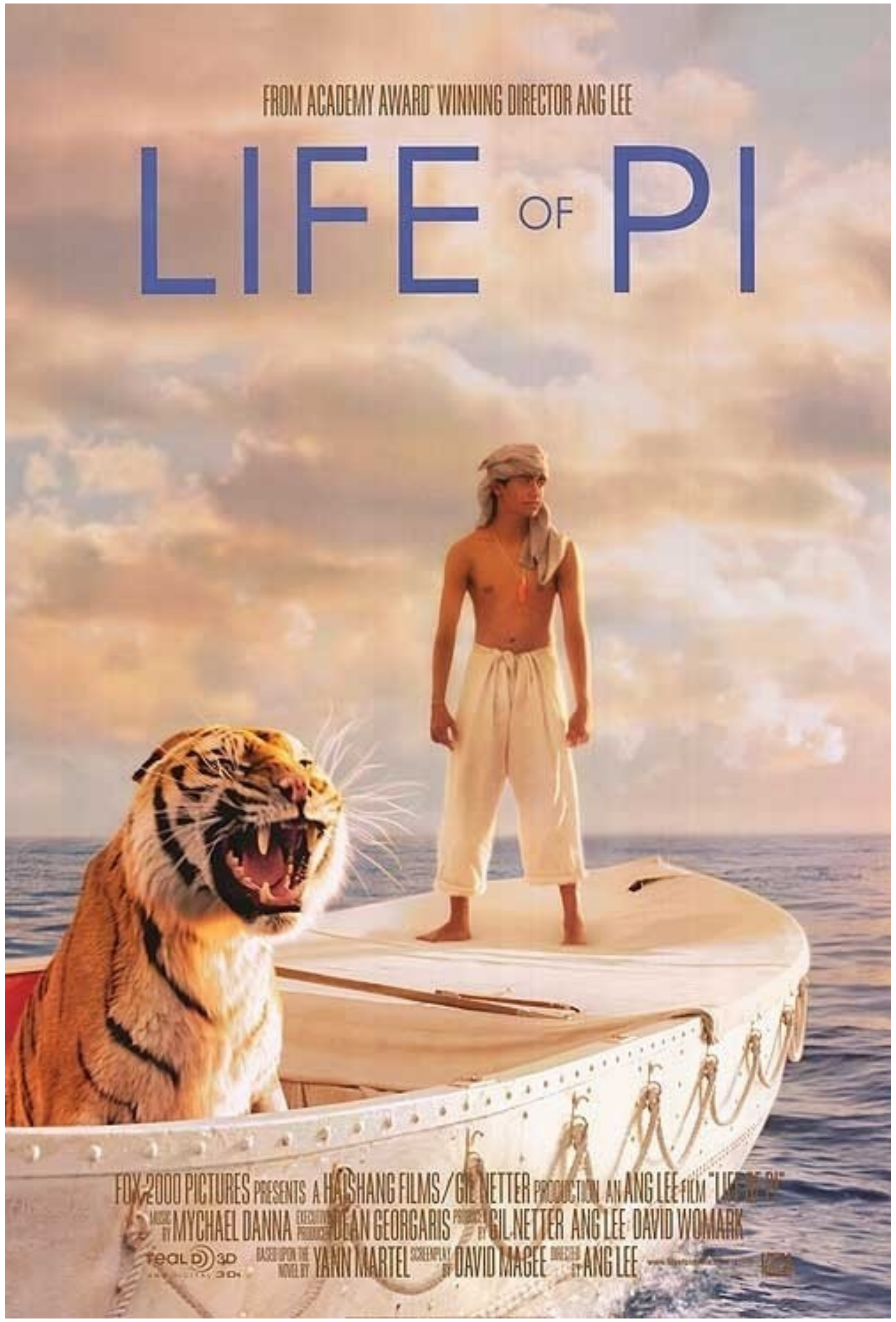


DDPG: Two Interpretations

- Version 1: Maximize $V^{\pi_\theta}(\mu)$ by policy gradient
- Version 2: Search & Mimic
- Which version of the story do you believe?

Recall: Deterministic Policy Gradient (DPG)

- Consider continuous actions and deterministic policy: $a = \pi_\theta(s)$

- Assumptions: $\nabla_a Q(s, a), \nabla_a P(s' \mid s, a), \nabla_\theta \pi_\theta(s), \nabla_a R(s, a)$ exist

- **Deterministic Policy Gradient Theorem**:

$$\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma} \mathbb{E}_{s \sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s, a)|_{a=\pi_\theta(s)} \right]$$

Recall: On-Policy Deterministic Actor-Critic

Deterministic PG: $\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_\mu^{\pi_\theta}}\left[\nabla_\theta \pi_\theta(s)\nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)}\right]$

- Deterministic Actor-Critic (DAC):

Step 1: Initialize θ_0, w_0 and step sizes α_θ, α_w

Step 2: Sample a trajectory $\tau = (s_0, a_0, r_1, \cdots) \sim P_\mu^{\pi_\theta}$

For each step of the current trajectory $t = 0,1,2,\cdots$

$$\Delta w_k \leftarrow \Delta w_k + \alpha_w\left(r_t + \gamma Q_{w_k}(s_{t+1}, a_{t+1}) - Q_{w_k}(s_t, a_t)\right)\nabla_w Q_w(s_t, a_t)|_{w=w_k}$$

$$\Delta \theta_k \leftarrow \Delta \theta_k + \alpha_\theta \gamma^t \underbrace{\left(\nabla_\theta \pi_\theta(s_t)\nabla_a Q_w(s_t, a)|_{a=\pi_\theta(s_t)}\right)}_{= \nabla_\theta Q_w(s_t, \pi_\theta(s_t))|_{\theta=\theta_k}}$$

$$\theta_{k+1} \leftarrow \theta_k + \Delta\theta_k,\ w_{k+1} \leftarrow w_k + \Delta w_k$$

A Quick Remark on DPG Expression

- In Deterministic Actor-Critic:

$$\Delta\theta_k \leftarrow \Delta\theta_k + \alpha_\theta \gamma^t \Big(\underbrace{\nabla_\theta \pi_\theta(s_t)\, \nabla_a Q_w(s_t, a)\big|_{a=\pi_\theta(s_t)}}_{= \nabla_\theta Q_w(s_t, \pi_\theta(s_t))|_{\theta=\theta_k}} \Big)$$

- In the original DPG expression:

$$\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_\mu^{\pi_\theta}}\Big[\underbrace{\nabla_\theta \pi_\theta(s)\, \nabla_a Q^{\pi_\theta}(s, a)\big|_{a=\pi_\theta(s)}}_{= \nabla_\theta Q^{\pi_\theta}(s, \pi_\theta(s))|_{\theta=\theta_k} \;??} \Big]$$

- Question: Any issue with deterministic policies?

Insufficient exploration

- Question: Is it possible to learn π but act under another policy β?

Off-policy learning!

Off-Policy Learning with Deterministic Policy Gradients

On-Policy vs Off-Policy

- On-policy:

  Learned policy $=$ Policy used to interact with the environment

- Off-policy:

  Learned policy $\neq$ $\underbrace{\text{Policy used to interact with the environment}}_{\text{Called “behavior policy”}}$

Kazami Hayato
(learning agent)

Asurada
(policy for interaction)

Off-Policy Learning

Off-policy learning

1. Learn a target policy $\pi_\theta(a \mid s)$ and compute $V^{\pi_\theta}(s)$ or $Q^{\pi_\theta}(s, a)$
2. In the meantime, follow a behavior policy $\beta(a \mid s)$

$$\{s_0, a_0, r_1, s_1, a_1, r_2, \cdots s_{T-1}, a_{T-1}, r_T\} \sim \beta$$

- Why is off-policy learning useful?
  1. Learn from observing humans or other agents
  2. Reuse experience generated from old policies $\pi_1, \pi_2 \cdots, \pi_{k-1}$
  3. Learn about optimal policy while following an exploratory policy
  4. Learn about multiple policies while following one policy

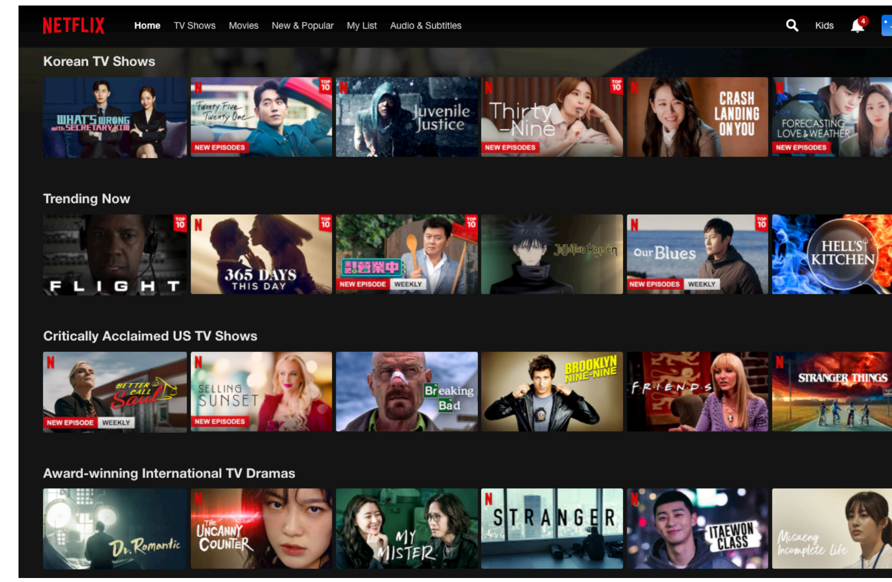
(Slide Credit: David Silver)

Off-Policy Learning is Essential in Many “Real-World” Problems

- Recommender Systems

➡ Deploy a safe policy for collecting user data without losing user’s interest

➡ Learn a better policy from these data



- Robot Control

➡ Deploy a safe policy for collecting robot data without hurting the machine

➡ Learn a good policy from these data

What are the behavior policies in the above applications?

Deterministic PG in Off-Policy Learning

(On-policy) DPG: $\nabla_\theta V^{\pi_\theta}(\mu) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_\mu^{\pi_\theta}}\left[\nabla_\theta \pi_\theta(s)\nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)}\right]$

- Question: Does the deterministic PG remain the same in off-policy learning?

  Nope! (state visitation distribution shall change)

- Consider a new objective for off-policy learning: (Why reasonable?)

$$J_\beta(\pi_\theta) := \sum_s d_\mu^\beta(s) V^{\pi_\theta}(s)$$

- "Off-policy" deterministic PG: Let's use $\nabla_\theta J_\beta(\pi_\theta)$!

Off-Policy Deterministic PG

- **Off-Policy Deterministic Policy Gradient**:

$$\nabla_\theta J_\beta(\pi_\theta) \approx \mathbb{E}_{s\sim d_\mu^\beta}\Big[\nabla_\theta \pi_\theta(s)\,\nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)}\Big]$$

- Question: Is the above easy to operate with?
- Derivation:

$$\nabla_\theta J_\beta(\pi_\theta) = \nabla_\theta\Big(\sum_s d_\mu^\beta(s) V^{\pi_\theta}(s)\Big)$$

$$= \nabla_\theta\Big(\sum_s d_\mu^\beta(s) Q^{\pi_\theta}(s,\pi_\theta(s))\Big)$$

The DPG paper (Silver, ICML 2014) dropped a term $\nabla_\theta Q^{\pi_\theta}(s,a)$

$$\approx \sum_s d_\mu^\beta(s)\Big(\nabla_\theta \pi_\theta(s)\,\nabla_a Q^{\pi_\theta}(s,a)|_{a=\mu_\theta(s)}\Big)$$

Why is $\nabla_\theta Q^{\pi_\theta}(s, a)$ Difficult to Evaluate?

- Recall from the expression of deterministic PG:

$$\nabla_\theta V^{\pi_\theta}(s) = \frac{1}{1-\gamma}\mathbb{E}_{s' \sim d_\mu^{\pi_\theta}}\left[\nabla_\theta \pi_\theta(s')\nabla_a Q^{\pi_\theta}(s', a)\big|_{a=\pi_\theta(s')}\right]$$

$$= \sum_{s'}\sum_{t=0}^{\infty}\gamma^t P(s \to s', t, \pi_\theta)\nabla_\theta \pi_\theta(s')\nabla_a Q^{\pi_\theta}(s', a)\Big|_{a=\pi_\theta(s')}$$

Accordingly, we have

$$\nabla_\theta Q^{\pi_\theta}(s, a) = \nabla_\theta\Big(R(s, a) + \gamma\sum_{s'} P(s'\,|\,s, a)V^{\pi_\theta}(s')\Big)$$

$$= \gamma \sum_{s'} P(s'\,|\,s, a)\nabla_\theta V^{\pi_\theta}(s')$$

$$= \gamma \sum_{s'} P(s'\,|\,s, a)\underbrace{\sum_{s''}\sum_{t=0}^{\infty}\gamma^t P(s' \to s'', t, \pi_\theta)}_{\text{hard to evaluate in off-policy learning (why?)}}\nabla_\theta \pi_\theta(s'')\nabla_a Q^{\pi_\theta}(s'', a)\Big|_{a=\pi_\theta(s'')}$$

hard to evaluate in off-policy learning (why?)

Off-Policy Deterministic Actor-Critic (OPDAC)

- Off-Policy Deterministic Actor-Critic (OPDAC):
  - Critic: estimate $Q_w \approx Q^{\pi_\theta}$ by TD bootstrapping
  - Actor: updates policy parameters θ by off-policy deterministic PG

Step 1: Initialize θ_0, w_0 and step sizes α_θ, α_w

Step 2: Sample a trajectory $\tau = (s_0, a_0, r_1, \cdots) \sim P_\mu^\beta$

For each step of the current trajectory $t = 0,1,2,\cdots$

$$\Delta w_k \leftarrow \Delta w_k + \alpha_w \left(r_t + \gamma Q_{w_k}(s_{t+1}, \pi_\theta(s_{t+1})) - Q_{w_k}(s_t, a_t) \right) \nabla_w Q_w(s_t, a_t)|_{w=w_k}$$

$$\Delta \theta_k \leftarrow \Delta \theta_k + \alpha_\theta \gamma^t \left(\nabla_\theta \pi_\theta(s_t) \nabla_a Q_{w_k}(s_t, a)|_{a=\pi_\theta(s_t)} \right)$$

$$\theta_{k+1} \leftarrow \theta_k + \Delta\theta_k, w_{k+1} \leftarrow w_k + \Delta w_k \qquad = \nabla_\theta Q_{w_k}(s_t, \pi_\theta(s_t))|_{\theta=\theta_k}$$

- Question: Can you identify differences between OPDAC and DAC?

Let's Prove the DPG Together

$$
\begin{aligned}
\underline{\nabla_\theta V^{\pi_\theta}(s)} =& \nabla_\theta Q^{\pi_\theta}\left(s, \pi_\theta(s)\right) \\
=& \nabla_\theta\left(R\left(s, \pi_\theta(s)\right)+\int_{\mathcal{S}} \gamma P\left(s^{\prime} \mid s, \pi_\theta(s)\right) V^{\pi_\theta}\left(s^{\prime}\right) \mathrm{d} s^{\prime}\right) \\
=& \nabla_\theta \pi_\theta(s) \nabla_a R(s, a)\big|_{a=\pi_\theta(s)}+\nabla_\theta \int_{\mathcal{S}} \gamma P\left(s^{\prime} \mid s, \pi_\theta(s)\right) V^{\pi_\theta}\left(s^{\prime}\right) \mathrm{d} s^{\prime} \\
=& \nabla_\theta \pi_\theta(s) \nabla_a R(s, a)\big|_{a=\pi_\theta(s)} \\
& +\int_{\mathcal{S}} \gamma\left(P\left(s^{\prime} \mid s, \pi_\theta(s)\right) \nabla_\theta V^{\pi_\theta}\left(s^{\prime}\right)+\nabla_\theta \pi_\theta(s) \nabla_a P\left(s^{\prime} \mid s, a\right)\big|_{a=\pi_\theta(s)} V^{\pi_\theta}\left(s^{\prime}\right)\right) \mathrm{d} s^{\prime} \\
=& \nabla_\theta \pi_\theta(s) \nabla_a\left(R(s, a)+\int_{\mathcal{S}} \gamma P\left(s^{\prime} \mid s, a\right) V^{\pi_\theta}\left(s^{\prime}\right) \mathrm{d} s^{\prime}\right)\bigg|_{a=\pi_\theta(s)} \\
& +\int_{\mathcal{S}} \gamma P\left(s^{\prime} \mid s, \pi_\theta(s)\right) \nabla_\theta V^{\pi_\theta}\left(s^{\prime}\right) \mathrm{d} s^{\prime} \\
=& \nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s, a)\big|_{a=\pi_\theta(s)}+\int_{\mathcal{S}} \gamma P\left(s \rightarrow s^{\prime}, 1, \pi_\theta\right) \underline{\nabla_\theta V^{\pi_\theta}\left(s^{\prime}\right)} \mathrm{d} s^{\prime}
\end{aligned}
$$

This is again a recursion! (Similar to the proof of (P2) of stochastic PG)

Let's Prove the DPG Together (Cont.)

The remaining proof can be done by "peeling off":

$$
\begin{aligned}
\nabla_\theta V^{\pi_\theta}(s) = {} & \nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s,a)|_{a=\pi_\theta(s)} \\
& + \int_{\mathcal{S}} \gamma P\left(s \to s', 1, \pi_\theta\right) \nabla_\theta \pi_\theta\left(s'\right) \nabla_a Q^{\pi_\theta}\left(s', a\right)\Big|_{a=\pi_\theta(s')} \mathrm{d}s' \\
& + \int_{\mathcal{S}} \gamma^2 P\left(s \to s', 2, \pi_\theta\right) \nabla_\theta V^{\pi_\theta}\left(s'\right) \mathrm{d}s' \\
& \vdots \\
= {} & \int_{\mathcal{S}} \sum_{t=0}^{\infty} \gamma^t P\left(s \to s', t, \pi_\theta\right) \nabla_\theta \pi_\theta\left(s'\right) \nabla_a Q^{\pi_\theta}\left(s', a\right)\Big|_{a=\pi_\theta(s')} \mathrm{d}s'
\end{aligned}
$$

Deep Deterministic Policy Gradient (DDPG)

(= OPDAC with Deep Neural Nets)

Lillicrap et al., "Continuous control with deep reinforcement learning", ICLR 2016

What is DDPG?

- DDPG: Combine OPDAC with NN nonlinear VFA
  - Off-policy: Exploration
  - Nonlinear VFA: Convergence issue

- To tackle the above issues, DDPG applies several techniques:

(T1) Experience replay (for data-efficient off-policy learning)

(T2) Ornstein-Uhlenbeck process for exploration (optional)

(T3) Target networks

(T1) Experience Replay

- Main idea:

  1. Store the previous experiences (s, a, s', r) into a buffer
  2. Sample a mini-batch from the buffer at each step

  (similar to mini-batch SGD in supervised learning)

- Purposes:

1. Better estimate of DPG: Break correlations between successive steps in a trajectory ("more stable learning", as stated in many papers)

2. Better data efficiency: Fewer interactions with environment needed for convergence

(T2) Ornstein-Uhlenbeck Process for Exploration

- Issue with Gaussian noise exploration $a_t = \pi_\theta(s_t) + N(0, \sigma^2)$?

- Ornstein-Uhlenbeck (OU) process: Similar to Gaussian policies, but with temporal correlation

Brownian motion

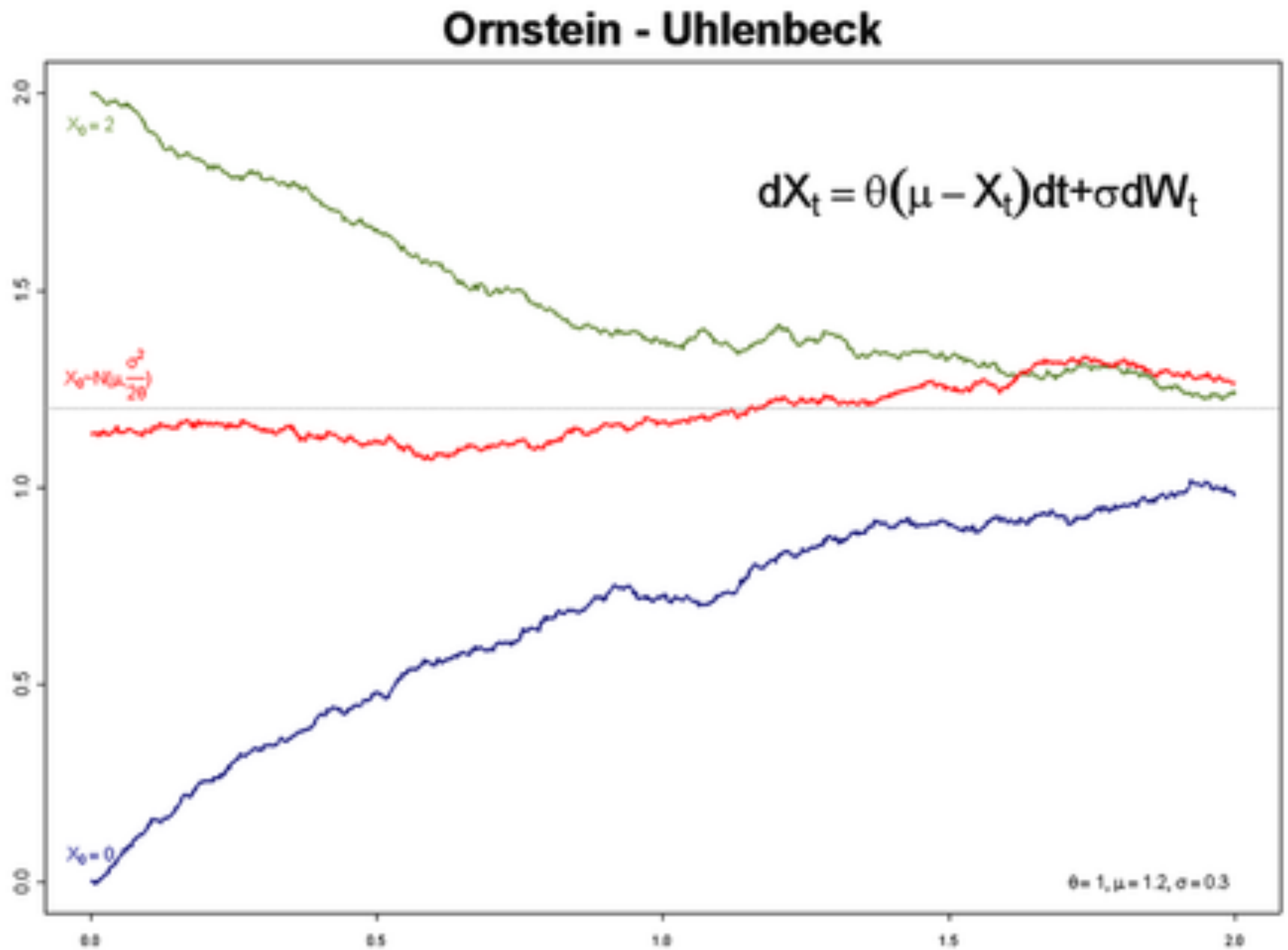
$$dx_t = \theta(\mu - x_t)dt + \sigma \cdot dW_t$$

- Discrete-time approximation of OU:

$$X_{t+1} - X_t = \theta(\mu - X_t)\Delta t + \sigma \cdot \Delta W_t$$

i.i.d. normal random variables $\sim \mathcal{N}(0, \Delta_t)$

Example of OU Process



(Same OU process with 3 different initial conditions)

How about a sequence of i.i.d. Gaussian random variables?

(Figure Source: Wikipedia)

(T3) Target Networks

- Idea: Use separate *target networks* ($\bar{\pi}_\theta$ for actor, $\bar{Q}_w$ for critic) that are updated only periodically
- For DDPG, the critic update with target networks

$$\Delta w_k \leftarrow \Delta w_k + \alpha_w \Big(\underbrace{r_t + \gamma \bar{Q}_{w_k}(s_{t+1}, \bar{\pi}_\theta(s_{t+1}))}_{\text{target}} - \underbrace{Q_{w_k}(s_t, a_t)}_{\text{update}} \Big) \nabla_w Q_w(s_t, a_t)|_{w=w_k}$$

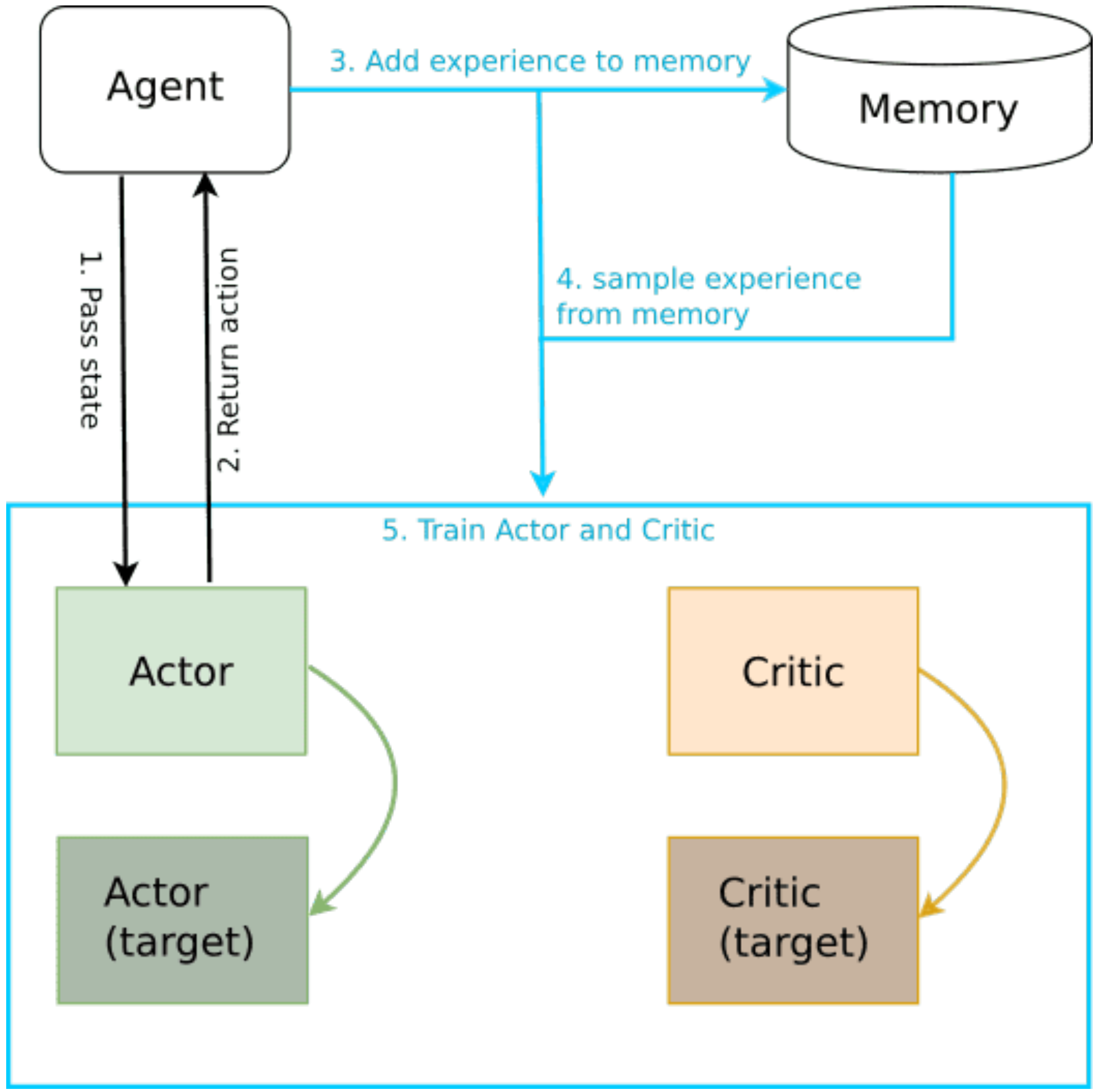
- Similar to value iteration:

$$\underbrace{V(s)}_{\text{update}} \leftarrow \max_a R(s,a) + \gamma \sum_{s'} P(s'|s,a) \underbrace{\bar{V}(s)}_{\text{target}}$$

- Purpose: Mitigate divergence

(Slide Credit: Pascal Poupart)

DDPG Architecture



 (Figure Credit: https://www.katnoria.com/ddpg-reacher/)

Pseudo Code of DDPG Algorithm

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$

2 evaluation networks and 2 target networks

Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise

action drawn from a deterministic policy with exploration

    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R

experience replay

    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

Update actor and critic

This can be viewed as the gradient of Q w.r.t. θ

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$

$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

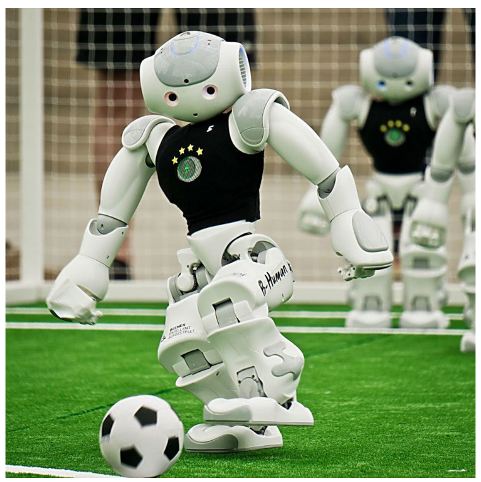
Update target networks (small τ for stability)

  end for
end for

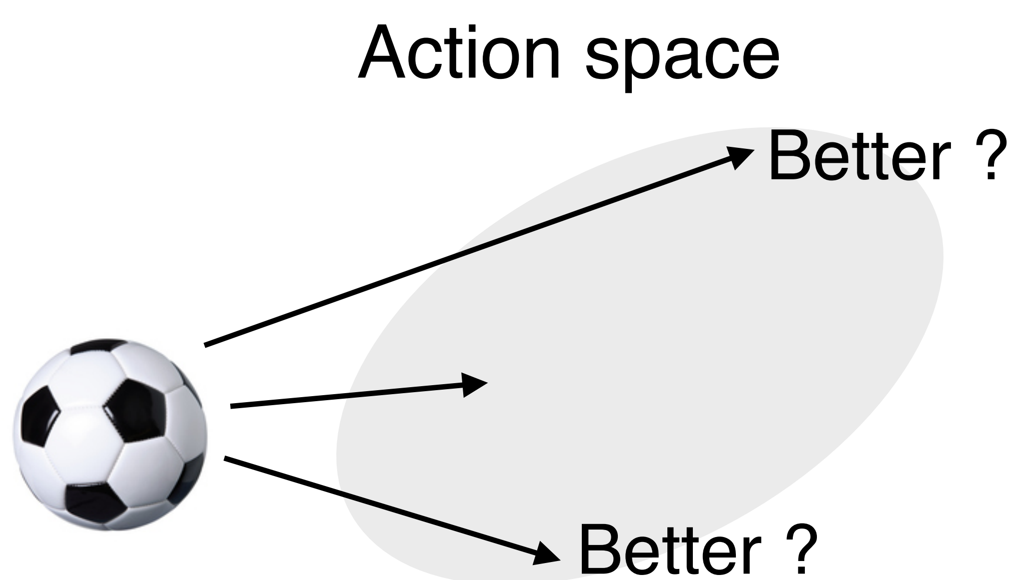
An Alternative Interpretation of DDPG

A Motivating Example of “Search & Mimic”

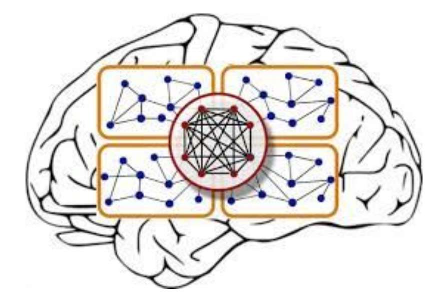
(A Robocup example)



Current state



Search for a better reference action (higher Q value)

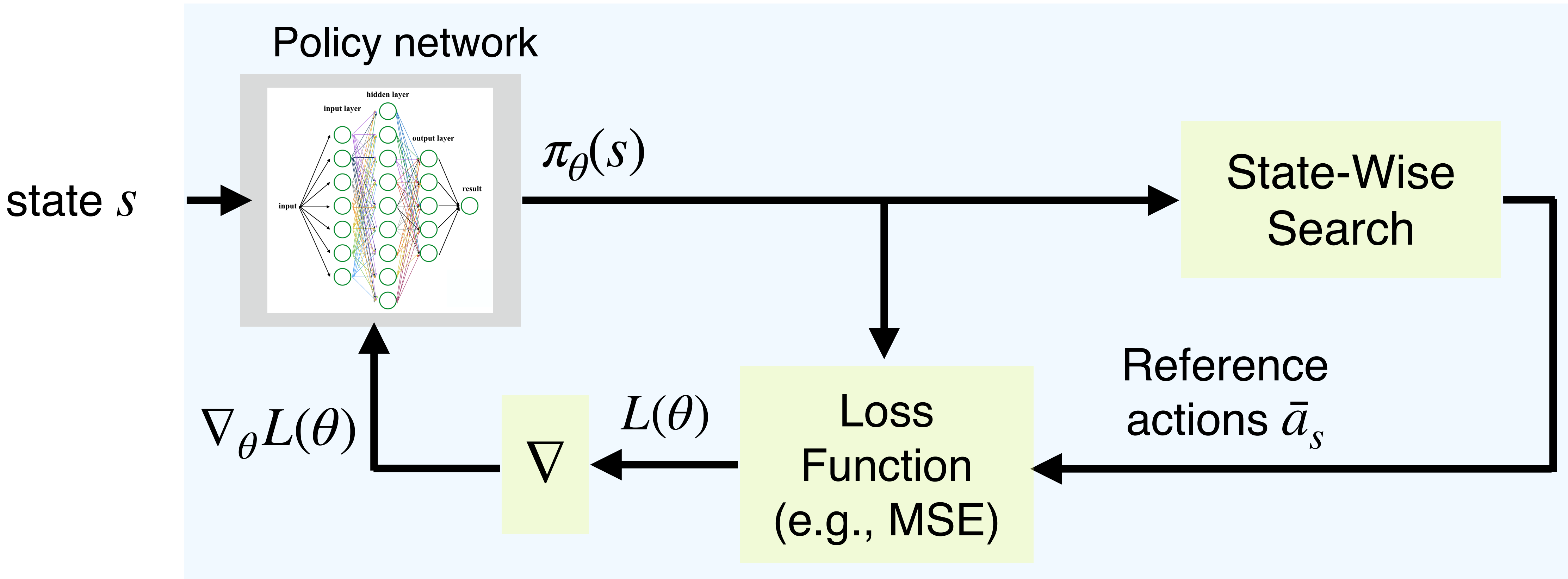


Mimic (update the policy towards the reference action)

RL = Repeatedly “search & mimic” in different scenarios

An Alternative Interpretation of DDPG

Let's formally write down "Search & Mimic" approach:



$$\bar{a}_s = \pi_{\bar{\theta}}(s) + \eta \nabla_a Q_w(s, a)$$

$$L(\theta) = \frac{1}{|B|} \sum_{s \in B} \left(\pi_\theta(s) - \bar{a}_s\right)^2$$

$$\nabla_\theta L(\theta) = \frac{1}{|B|} \sum_{s \in B} \nabla_\theta \left(\pi_\theta(s) - \bar{a}_s\right)^2 = \frac{1}{|B|} \sum_{s \in B} \nabla_\theta \Big(\pi_\theta(s) - \left(\pi_{\bar{\theta}}(s) + \eta \nabla_a Q_w(s, a)\right)\Big)^2$$

One Surprising Fact: “Search & Mimic” and DDPG Are Equivalent!

Theorem: $\Delta\theta_{DDPG}$ & $\Delta\theta_{S\&M}$ are parallel

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$$\Delta\theta_{DDPG} \propto \nabla_\theta J_\mu(\pi_\theta) = \frac{1}{|B|}\sum_{s\in B} \nabla_a Q_w(s,a)\,\nabla_\theta \pi_\theta(s)$$ (By DPG theorem)

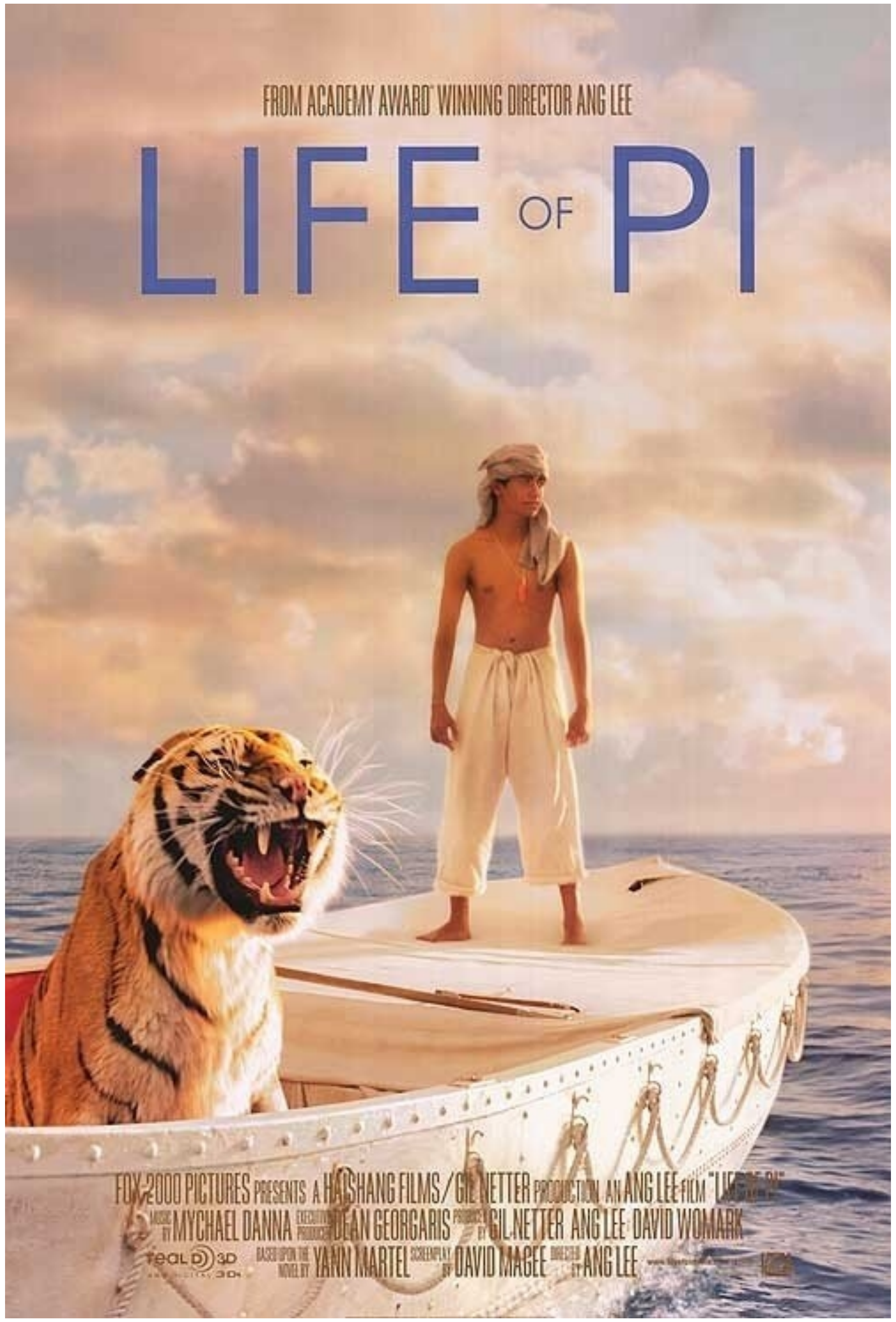
$$\Delta\theta_{S\&M} \propto \nabla_\theta L(\theta) = \frac{1}{|B|}\sum_{s\in B} \nabla_\theta\left(\pi_\theta(s) - \bar{a}_s\right)^2$$ (By Search & Mimic)

$$= \frac{1}{|B|}\sum_{s\in B} \nabla_\theta\left(\pi_\theta(s) - \left(\pi_{\bar{\theta}}(s) + \eta\,\nabla_a Q_w(s,a)\right)\right)^2$$

$$= \frac{1}{|B|}\sum_{s\in B} \nabla_a Q_w(s,a)\,\nabla_\theta \pi_\theta(s)$$

(For more details, please refer to our UAI 2021 paper, available at https://arxiv.org/pdf/2102.11055.pdf)

Rethinking DDPG: Two Interpretations



<u>*What does DDPG actually do?*</u>

- Version 1: Maximize $V^{\pi_\theta}(\mu)$ by policy gradient
- Version 2: State-wise search in action space & mimicking
- Which version of the story better describes our learning process?