Distributional Temporal Difference Learning for Finance: Dealing with Leptokurtic Rewards

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Brain, Mind and Markets Laboratory

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An overview on reinforcement learning (RL)

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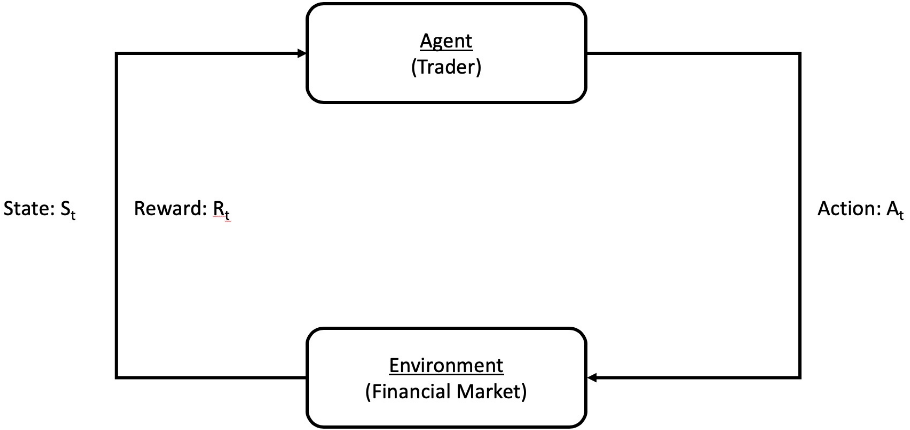

An overview on reinforcement learning (RL)

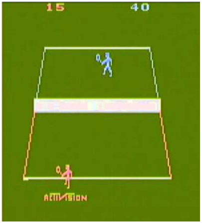

Figure: Atari games (2013) [6]

Figure: Board games (Nature, 2016) [8]

Figure: Poker (Science, 2019) [2]

Figure: RTS games (Nature, 2019) [9]

An overview on reinforcement learning (RL)

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- We model the interaction as a stationary Markov Decision Process (MDP): (S, A, R, P)
- S: set of states, A: set of actions
- $R: S \times A \rightarrow F$ is a reward function that maps each state-action pair to a reward that lives in outcome space F
- $P(s^{'}|s,a)$: state transition probability distribution
- Policy $\pi(a|s)$: the probability of action a in state s

Bellman principle

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In economics and finance, we intend to find an optimal value of a state $V(s)$ where

$$V(s) = \max_a \left\{ \mathbb{E}[R|s,a] + \gamma \mathbb{E}[V(s^{'})|s,a] \right\}$$

- We do not know the functional form of V, nor do we know the expectations

Reinforcement Learning provides a solution: we start from state-action "Q" values

$$Q(s,a) = \left\{ \hat{\mathbb{E}}[R|s,a] + \gamma \hat{\mathbb{E}}[Q(s^{'},a^{'})|s,a] \right\}$$

- The question: how do we update Q values dynamically, and ensure that the optimal Q values converge to the Bellman values?

The SARSA approach

- **SARSA** stands for "State, Action, Reward, Next State, Next Action"
- When the agent is in state s, she chooses actions a that is the optimal one (according to learned Q values).
- This generates a reward R, next state $s^{'}$ and she uses this to update Q for state s and action a recursively:

$$Q(s,a) \leftarrow Q(s,a) + \alpha(R + \gamma Q(s^{'}, a^{'}) - Q(s,a)),$$

  where α: learning rate (step size) and γ: discount rate
- With "sufficient" exploration (enough visits in all state-action pairs), the value function estimation converges to true value function [10].

Distributional reinforcement learning (2017-)

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Rather than using recursive estimation, we keep track of the **entire distribution** (history) of realized Q values for a state s and action a and take the weighted average to get an estimate of $Q(s, a)$ (weights = discounting the past)

DRL: How to estimate the distribution of Qs?

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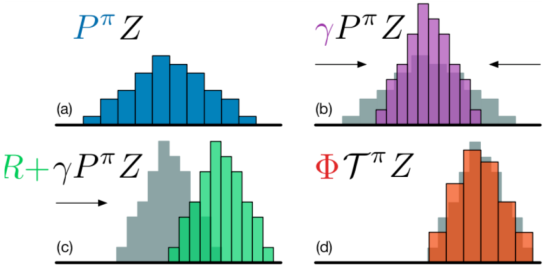
- Categorical: parametric densities fit to probability histograms [1]



- Quantile: parametric fitting of quantiles in cumulative density function [4, 5]
- Expectile: parametric fitting of expectiles in cumulative density function [7]

The Q value is then obtained as a simple integration over the estimated distribution (e.g. in categorical approach, $Q(s,a) = \sum_i z_i p_i(s,a)$ where $p_i(s,a)$ is the estimated probability that Q takes the value z_i)

Efficient estimation and DRL

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- If we keep track of the entire distribution anyway, why not use the BEST estimation of the expectation?!
- This may be beneficial, particularly in the context of finance, where rewards are heavy tailed (leptokurtosis), and hence the simple average is not an "efficient" estimator.

(Efficient estimator: reaches the Cramér-Rao lower bound, or Chapman-Robbins lower bound [3].)

Leptokurtic reward

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- Leptokurtosis: daily returns on the S&P500 index generate a distribution with a kurtosis of 10 or higher ("heavy tails")
- Neither SARSA nor distributional RL are "well-behaved" under leptokurtic rewards (see evidence later)
  - Neither method obtains optimal state-action values $Q(s,a)$ due to frequent reward outliers
- *There is more*: Q value updating rule uses the sum of two random variables with (asymptotically) very different distributions:
  1. Rewards are (always) leptokurtic
  2. Realized Q values eventually will only depend on state transitions and actions, so their distributions are NOT leptokurtic (provided state transitions are not)

Our approach: efficient estimation

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We utilize efficient estimation from mathematical statistics:

- Instead of processing the reward R directly, we estimate the mean reward, and perform standard SARSA with estimated mean reward.

$$Q(s,a) \leftarrow Q(s,a) + \alpha(\hat{\mathbb{E}}[R|s,a] + \gamma Q(s^{'},a^{'}) - Q(s,a))$$

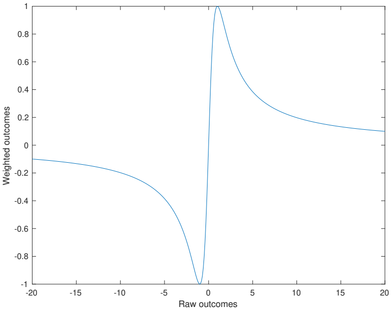
- We use $\hat{\mathbb{E}}[R|s,a]$ instead of (one) realized R; We compare two versions of $\hat{E}$:
  1. **d-RL**: sample average of past n rewards:

  $$\hat{\mathbb{E}}(R(s,a)) = \frac{1}{n}\sum_{i=0}^{n} R_i$$

  2. **d-RL-MLE**: We use MLE, which reaches the Cramér-Rao lower bound (under certain conditions; if MLE is not efficient, we use something better)

MLE

- We need a good assumption about the nature of the reward/return distribution; We use student-t with low degrees of freedom.
- MLE estimator of the mean (reward) is complex; we use the Expectation-Maximization (EM) algorithm
  - MLE does not merely "truncate" samples!!
  - Instead, MLE projects outliers towards the middle.



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Environment: financial decision making task

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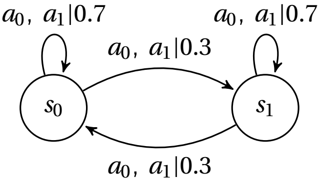
- Two-states, two-actions MDP
- State transition is exogenous: $P(s^{'}|s,a) = P(s^{'}|s)$



$R_t(s,a) \sim$	Gaussian		Leptokurtic		S&P500	
	s_0	s_1	s_0	s_1	s_0	s_1
a_0	$\boldsymbol{N_{0,1}+2}$	$N_{0,1}+1$	$\boldsymbol{T_{1.1}+1.5}$	$T_{1.1}+1$	$\boldsymbol{\mu_d+0.5}$	μ_d
a_1	$N_{0,1}+1$	$\boldsymbol{N_{0,1}+2}$	$T_{1.1}+1$	$\boldsymbol{T_{1.1}+1.5}$	μ_d	$\boldsymbol{\mu_d+0.5}$

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"Episodes" in Machine Learning

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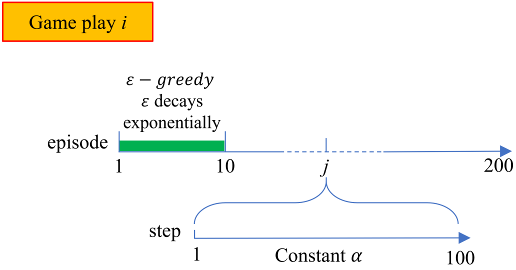
- Step/trial = each frame
- Episode: all steps until pole falls
- Game: the entire play (after pulling up pole again repeatedly).

Pole Balancing

(https://www.youtube.com/watch?v=46wjA6dqxOM)

Results

- 100 game plays (i). $\alpha = 0.1$, constant over all episodes; Agent explores (chooses random action) with probability ε in the first 10 episodes
- We check at the end of each episode *after* exploration whether the agent learns the correct policy (s_0, a_0) and (s_1, a_1); If the agent passes all 190 "checks" in a game play i, we deem it one "optimal policy convergence"
- We count how many games the agents obtain such convergence



Results

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% of optimal policy convergence	Gaussian $N(0,1)$	Student-t $(\upsilon = 1.1)$	Empirical S&P500
SARSA	82%	2%	47%
dis-RL-Cat	80%	4%	-
d-RL	**100%**	33%	95%
d-RL-MLE	**100%**	**95%**	**97%**

Table: Percentage of game plays where the agent computed the optimal policy at the end of *all* 190 episodes, averaged across two states.

Contamination of estimated distribution by outliers except in the case of d-RL-MLE

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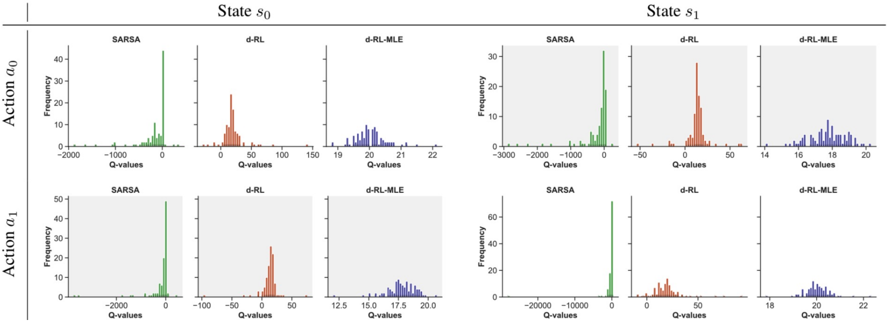
- In an environment with leptokurtic rewards, it is likely that a huge outlier reward (+ve/-ve) can dominate $Q(s,a)$ updates, hence causing the robots to switch between optimal and sub-optimal actions overtime.



Conclusions

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- Distributional RL can be exploited to obtain the most efficient way to estimate mean action-values across states, and hence, enhance control.
- In a leptokurtic environment, availability of the distribution allows one to estimate the mean in a more efficient way, ensuring that outliers do not cause policy non-convergence or policy instability.
- From a broader perspective, our results show the importance of bringing prior domain-specific knowledge to machine learning algorithms using the tools of mathematical statistics.
  - A non-leptokurtic example: If the reward distribution is shifted-exponential, then the best estimation of the expected reward given an action and a state is to using the running Minimum/Maximum, not the sample average!

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