2 次元極座標のデルタ

<div align="right">hsjoihs</div>

1.

$$\vec{x} = (x, y), \vec{x_0} = \left(x_0, y_0\right)$$

$$\delta\left(\vec{x} - \vec{x_0}\right) \equiv \delta\left(x - x_0\right)\delta\left(y - y_0\right)$$

$$x = r\cos\theta, y = r\sin\theta$$

$$x_0 = r_0\cos\theta_0, y_0 = r_0\sin\theta_0$$

$$\delta\left(\vec{x} - \vec{x_0}\right) = \lim_{n\to\infty} \frac{n}{\pi} e^{-n\left(x-x_0\right)^2 - n\left(y-y_0\right)^2}$$

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 = x^2 + y^2 - 2xx_0 - 2yy_0 + x_0^2 + y_0^2$$

$$= r^2 + r_0^2 - 2(x,y) \cdot \left(x_0, y_0\right) = r^2 + r_0^2 - 2rr_0\cos\left(\theta - \theta_0\right)$$

$$\delta\left(\vec{x} - \vec{x_0}\right) = \lim_{n\to\infty} \frac{n}{\pi} e^{-n\left(x-x_0\right)^2 - n\left(y-y_0\right)^2} = \lim_{n\to\infty} \frac{n}{\pi} e^{-n\left(r^2 + r_0^2 - 2rr_0\cos\left(\theta - \theta_0\right)\right)}$$

どうせ高次の項など効いてこないので（そういうとこやぞ）

$$= \lim_{n\to\infty} \frac{n}{\pi} e^{-n\left(r^2 + r_0^2 - 2rr_0\left(1 - \frac{1}{2}\left(\theta - \theta_0\right)^2\right)\right)}$$

$$= \lim_{n\to\infty} \frac{n}{\pi} e^{-n\left(r^2 + r_0^2 - 2rr_0 + rr_0\left(\theta - \theta_0\right)^2\right)}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n}{\pi}} e^{-n\left(r-r_0\right)^2} \cdot \sqrt{\frac{n}{\pi}} e^{-n\left(rr_0\left(\theta - \theta_0\right)^2\right)}$$

$$= \delta(r - r_0)\delta\!\left(\sqrt{rr_0}\,(\theta - \theta_0)\right)$$

$$\int\int f(r,\theta)\delta(\vec{x} - \vec{x_0})drd\theta = \int\int f(r,\theta)\delta(r - r_0)\delta\!\left(\sqrt{rr_0}\,(\theta - \theta_0)\right)drd\theta$$

$$= \int f(r_0,\theta)\delta\!\left(\sqrt{r_0 r_0}\,(\theta - \theta_0)\right)d\theta$$

$$= \int f(r_0,\theta)\frac{1}{r_0}\delta(\theta - \theta_0)d\theta$$

$$= \int\int f(r,\theta)\frac{1}{r}\delta(r - r_0)\delta(\theta - \theta_0)drd\theta$$

$$\delta(\vec{x} - \vec{x_0}) = \frac{1}{r}\delta(r - r_0)\delta(\theta - \theta_0)$$