

演算子法（またの名を：「ヘヴィサイドそういうとこだぞ」）

hsjoihs

1. 計算

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = x^2$$

$$D^2y + 4Dy + 3y = x^2$$

$$(D^2 + 4D + 3)y = x^2$$

$$(D + 1)(D + 3)y = x^2$$

$$y = \frac{1}{(D + 1)(D + 3)}x^2$$

$$y = \frac{1}{2}\left(\frac{1}{D + 1} - \frac{1}{D + 3}\right)x^2$$

$$\frac{1}{D + 1}x^2 = (1 - D + D^2 - D^3 + \dots)x^2 = x^2 - 2x + 2$$

$$\frac{1}{1 + \frac{D}{3}}x^2 = \left(1 - \frac{D}{3} + \frac{D^2}{9} - \frac{D^3}{27} + \dots\right)x^2 = x^2 - \frac{2x}{3} + \frac{2}{9}$$

$$y = \frac{1}{2} \frac{1}{D + 1}x^2 - \frac{1}{6} \frac{1}{1 + \frac{D}{3}}x^2 = \left(\frac{1}{2}x^2 - x + 1\right) - \left(\frac{1}{6}x^2 - \frac{x}{9} + \frac{1}{27}\right)$$

$$= \frac{1}{3}x^2 - \frac{8x}{9} + \frac{26}{27}$$

2. 検算

$$y'' = \frac{2}{3}$$

$$4y' = \frac{8}{3}x - \frac{32}{9}$$

$$3y = x^2 - \frac{8x}{3} + \frac{26}{9}$$