Must For All Maths Talent Search Exams & Olympiads

BMA'S

TALENT & OLYMPIAD

EXAMS RESOURCE BOOK

MATHEMATICS

CLASS IX

Strong Foundation for Better Results

BEST SELLER

SOLD Over 3.6 Million copies of this series since the 1st Edition

Revised Edition

Brain Mapping Academy

BMA's

TALENT & OLYMPIAD

EXAMS RESOURCE BOOK

CLASS IX

Mathematics

BRAIN MAPPING ACADEMY
Mapping Your Future

www.bmatalent.com

Published by:

Brain Mapping Academy

#16-11-16/1/B, First Floor, Farhath Hospital Road,
Saleem Nagar, Malakpet, Hyderabad-500 036.
✆ 040-66135169,65165169.
E-mail: info@bmatalent.com
Website: www.bmatalent.com

ISBN : **978-93-82058-53-3**

Disclaimer

Every care has been taken by the compilers and publishers to give correct, complete and updated information. In case there is any omission, printing mistake or any other error which might have crept in inadvertently, neither the compiler / publisher nor any of the distributors take any legal responsibility.

In case of any dispute, all matters are subjected to the exclusive jurisdiction of the courts in Hyderabad only.

First Edition : 2003

Second Edition : 2008

Revised Edition : 2015

Publisher's Note

Sometimes the understanding of fundamental concepts alone does not help the students to crack the competitive exams as most of them are objective in structure. Students need rigorous training to familiarize themselves to the style of the exams they are attempting. The board exams which are of qualifying, but not competitive, nature do not completely address the needs of students in testing them in objective type format.

To bridge this gap and to enable the students to face the reality of competitive exams, Brain Mapping Academy, brought out an all-objective questions reference book.

A crisp summary of the topics and useful equations were provided at the beginning of each chapter so that the students can memorize the important points.

Care has been taken to design thought-provoking questions. These should help students to attain a deeper understanding of principles. The questions have been reviewed to fill the gaps in problem coverage and to build the confidence in the students. They have also been expanded to impart reasoning/logical/analytical skills.

This book will cater all the requirements of the students who are approaching national/state level talent search examinations and all Olympiad exams. This book also complements the additional preparation needs of the students for the regular board exams.

We took utmost care to make this the best resource book available for the talent / olympiad exams aspirants. We welcome criticism from the students, teacher community and educators, especially concerning any errors and deficiencies which may have remained in this edition and the suggestions for improvement for the next edition.

This page is intentionally left blank.

For Your Information

nstse
Test • Assess • Achieve

NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Aim of this examination

The focus on fundamentals is so important that without a firm understanding of them, a child cannot be expected to face the reality of the competitive world once he/she finishes the formal education. Even while opting for higher studies the student has to go through a complete scan of what he/she knows. Exams like IIT-JEE, AIEEE, AIIMS, AFMC, CAT, SAT, GRE, GMAT, etc. are so designed to test the fundamental strength of a student. Hence the need of the hour is building the fundamental base as strong as possible.

A successful life emerges out from healthy and sound competition. Competition is the only way for the students to shake lethargy. It's the only way to get introduced for manly worthiness. Firm standards in education and competition are the tonic for a promising and talented future.

This exactly is the philosophy behind the Unified Council's NSTSE.

Organisation

National Science Talent Search Examination is conducted by Unified Council. Unified Council is India's first ISO 9001 certified organisation in the educational testing and assessment. Since its inception, Unified Council has put together the best brains in an endeavour to make the younger generation fundamentally stronger and nourish their brains for a bright and enterprising future.

Eligibility : Students of classes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 & 12 are eligible to participate in this examination.

Medium & Syllabus: This exam is conducted in only English medium and is suitable for all the students following CBSE/ICSE/State Board Syllabi.

Examination Pattern

There will be a separate question paper for each class. All questions are objective-type multiple choice with no negative marking for wrong answers.

Duration: 90 minutes

Date : Conducted every year on the last Sunday of January.

Test Centres : Spread across the country.

DIVISION OF MARKS

FOR CLASS I		
Mathematics	:	25 marks
General Science	:	15 Marks
FOR CLASS II		
Mathematics	:	25 marks
General Science	:	25 Marks
FOR CLASS III		
Mathematics	:	40 marks
General Science	:	35 Marks
FOR CLASSES IV & V		
Mathematics	:	45 marks
General Science	:	45 Marks
General Questions	:	10 marks

FOR CLASSES VI TO X		
Mathematics	:	25 marks
Physics	:	25 marks
Chemistry	:	20 marks
Biology	:	20 marks
General Questions	:	10 marks
FOR CLASS XI & XII(PCM)		
Mathematics	:	40 marks
Physics	:	25 marks
Chemistry	:	25 marks
General Questions	:	10 marks
FOR CLASS XI & XII(PCB)		
Biology	:	40 marks
Physics	:	25 marks
Chemistry	:	25 marks
General Questions	:	10 marks

Infrastructure

The Council makes use of ultra-modern equipment such as ***Optical Mark Recognition (OMR)*** *equipment to evaluate the answer papers to proficiently assess students' performance. The examination procedure is* ***completely computerised.***

Unique Service from Unified Council:

Unique analysis reports like Student's Performance Report for students, General School Report & Individual School Report for schools provided. These reports are very much helpful for students & schools to analyse their strengths and weaknesses.

General School Report (GSR) analyses the performance of students participating in the exam (subject-wise and class-wise). The report, in graphical format will have Ogive and Histogram Graphs, which are useful to schools that wish to improve their students' performance by benchmarking the areas of weaknesses and building upon them.

Individual School Report (ISR) analyses the performance of a particular school when compared to the rest of the students participating in this examination (subject-wise, class-wise and question-wise). This report acts as a tool for the schools to improve their students' performance in the future by benchmarking the areas of weaknesses and building upon them.

Awards & Scholarships:

Top 100 members in each class will be awarded with Awards & Medals etc.

UNIFIED COUNCIL

#16-11-16/1/B, Farhath Hospital Road, Saleem Nagar, Malakpet, Hyderabad-500 036
Phones : 040-24557708, 24545862, 66139917
E-mail: exam@unifiedcouncil.com, Website: www.unifiedcouncil.com

CONTENTS

Mathematics

This page is intentionally left blank.

CHAPTER 1

Number Systems

Synopsis

- **Rational numbers (Q):** The numbers of the form $\frac{p}{q}$ where 'p' and 'q' are integers and $q \neq 0$ are called rational numbers.

> ***Note:*** **A number of the form $\frac{a}{b}$ is a fraction. So all fractions are rational numbers. In the fraction $\frac{a}{b}$, 'a' is called the numerator and 'b' is called the denominator.**
>
> **e.g., $\frac{1}{2}, -\frac{2}{3}, \frac{7}{6}, \frac{6}{11}, -\frac{2}{9}, \ldots\ldots$**

(i) Zero is a rational number.

> ***Note:*** **0 by 0 is undefined.**

(ii) Every integer is a rational number.

(iii) A rational number, may or may not be an integer.

(iv) To write 'n' distinct rational numbers between any two rational numbers 'a' and 'b', we write $a = \frac{p_1}{q}$ and $b = \frac{p_2}{q}$ such that $(p_2 - p_1)$ is a positive integer greater than 'n'. (Here $a < b$); P_1, P_2 and q are integers $(q \neq 0)$.

A set of 'n' rational numbers can be written as $\frac{p_1+1}{q}, \frac{p_1+2}{q}, \ldots\ldots, \frac{p_1+n}{q}$

i.e., $a = \frac{p_1}{q} < \frac{p_1+1}{q} < \frac{p_1+2}{q} \ldots\ldots < \frac{p_1+n}{q} < \frac{p_2}{q} = b$.

(v) Between two given rational numbers *a* and *b*, there are infinitely many rational numbers.

- **Properties of rational numbers:**

(i) If $\frac{p}{q}$ is a rational number and 'm' is a non-zero integer, then $\frac{p}{q} = \frac{p \times m}{q \times m}$.

(ii) If $\frac{p}{q}$ is a rational number and 'm' is a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$

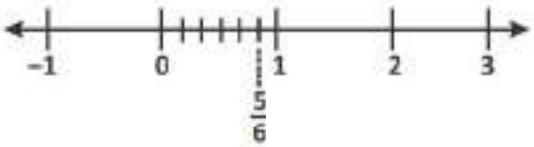
(iii) Two rational numbers are equivalent only when the product of the numerator of the first rational number and the denominator of the second is equal to the product of the denominator of the first and the numerator of the second.

Thus, $\frac{p}{q} = \frac{r}{s}$ only if $p \times s = q \times r$.

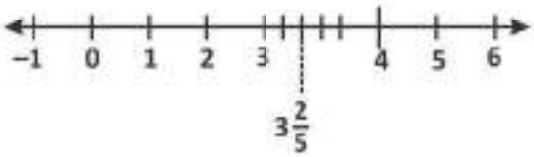
Note: $\frac{-p}{q} = \frac{p}{-q} = -\frac{p}{q}$

◆ **Representation of rational numbers on a number line:**

(i) Rational numbers of the form $\frac{m}{n}$ where $m < n$ are represented on the number line as shown below.



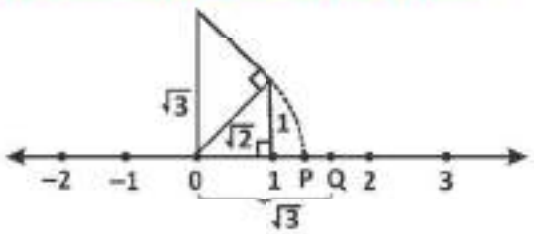
(ii) Rational numbers of the form $\frac{m}{n}$ where $m > n$ are represented on the number line as shown below.



◆ **Irrational numbers (Q'):** Any number which cannot be expressed in the form of $\frac{p}{q}$ (which is neither terminating nor repeating decimal) where p and q are integers and $q \neq 0$ is said to be an irrational number.

Note: π **is an irrational number.**

◆ **Representation of an irrational number on a number line:**



In the above figure, Q represents $\sqrt{3}$ on the number line.

◆ **Important properties of rational & irrational numbers:**

(i) The sum of two rational numbers is a rational number.

(ii) The sum of two irrational numbers may or may not be irrational.

(iii) The product of two rational numbers is a rational number.

(iv) The product of two irrational numbers may or may not be irrational.

(v) The difference of two rational numbers is a rational number.

(vi) The sum of a rational number and an irrational number is an irrational number.

(vii) The difference of a rational number and an irrational number is an irrational number.

(viii) The product of a rational number and an irrational number is an irrational number.

(ix) The quotient of two rational numbers is a rational number.

Exception : $\frac{\text{Any rational number}}{0} = \infty$

(x) The quotient of two irrational numbers may or may not be irrational.

(xi) The quotient of a rational number and an irrational number is an irrational number.

Exception : $\frac{0}{\text{Any irrational number}} = 0$

◆ **Real numbers (R) :** The union of all rational and irrational numbers is called the set of real numbers.

Definition : A number whose square is non-negative is called a real number.

Note : **(i) Every point on the number line, represents either a rational number or an irrational number i.e., a real number.**

(ii) Corresponding to every point on the number line, there is a unique real number. And corresponding to every real number, there is a unique point on the number line.

◆ **Decimal expansion of real numbers:**

(i) Every rational number $\frac{p}{q}$ $(p, q \in Z \text{ and } q \neq 0)$ can be expressed in the form of terminating or non-terminating recurring decimal.

(ii) Every irrational number can be expressed as a non-terminating non-recurring decimal.

◆ **Square root of a given positive real number :** Let 'a' be any positive real number. We can express $\sqrt{a} = b$ if and only if $b > 0$ and $b^2 = a$. The value of 'b' is called the positive square root of the positive real number 'a'.

◆ **Some results on square roots:**

(i) $(\sqrt{x})^2 = x$ where x is a positive real number.

(ii) $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$ where x and y are positive real numbers.

(iii) $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$ where x and y are positive real numbers.

(iv) $(\sqrt{x} + \sqrt{y}) \times (\sqrt{x} - \sqrt{y}) = x - y$ where x and y are positive real numbers.

(v) $(\sqrt{x}+\sqrt{y})^2 = x+y+2\sqrt{xy}$ and $(\sqrt{x}-\sqrt{y})^2 = x+y-2\sqrt{xy}$ where x and y are positive real numbers.

(vi) $(\sqrt{a}+\sqrt{b})\times(\sqrt{c}+\sqrt{d}) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$ where a, b, c and d are positive real numbers.

(vii) $(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$ where 'a' is any real number and 'b' is a positive real number.

◆ **Rationalizing factor :** In expressions like $\frac{a}{\sqrt{x}}, \frac{1}{a+b\sqrt{x}}, \frac{1}{\sqrt{x}+\sqrt{y}}$, and $\frac{1}{a\sqrt{x}+b\sqrt{y}}$ etc. to make the denominators free from square roots such as $\sqrt{x}$ and $\sqrt{y}$, we multiply the numerator and denominator by a suitable fator. This factor is called the rationalizing factor.

◆ **Rationalizing the denominator:** The process of converting the denominator of an expression containing a term with a square root to an equivalent expression where denominator is a rational number is called **rationalizing the denominator.**

◆ **Rationalizing factors:**

(i) The rationalizing factor of $\frac{a}{\sqrt{x}}$ is $\sqrt{x}$.

(ii) The rationalizing factor of $\frac{1}{a+b\sqrt{x}}$ is $a-b\sqrt{x}$.

(iii) The rationalizing factor of $\frac{1}{a\sqrt{x}+b\sqrt{y}}$ is $a\sqrt{x}-b\sqrt{y}$.

(iv) The rationalizing factor of $\frac{1}{a-b\sqrt{x}}$ is $\left(a+b\sqrt{x}\right)$.

(v) The rationalizing factor of $\frac{1}{a\sqrt{x}-b\sqrt{y}}$ is $a\sqrt{x}+b\sqrt{y}$.

◆ **Laws of exponents for real numbers :** If 'a' is any real number and 'n' is a natural number (positive integer), then $a^n = a\times a\times \ldots\ldots \times a$ ('n' factors).

Here, (a^n) is called the n^{th} power of a. The real number 'a' is called the base and positive integer 'n' is called the exponent.

For any rational base a; and any integers 'm' and 'n', we have

(i) $a^m \times a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\frac{a^m}{a^n} = a^{m-n}; m>n$

(iv) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}} (m<n)$ (v) $a^n \times b^n = (a\times b)^n$ (vi) $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n; b\neq 0$

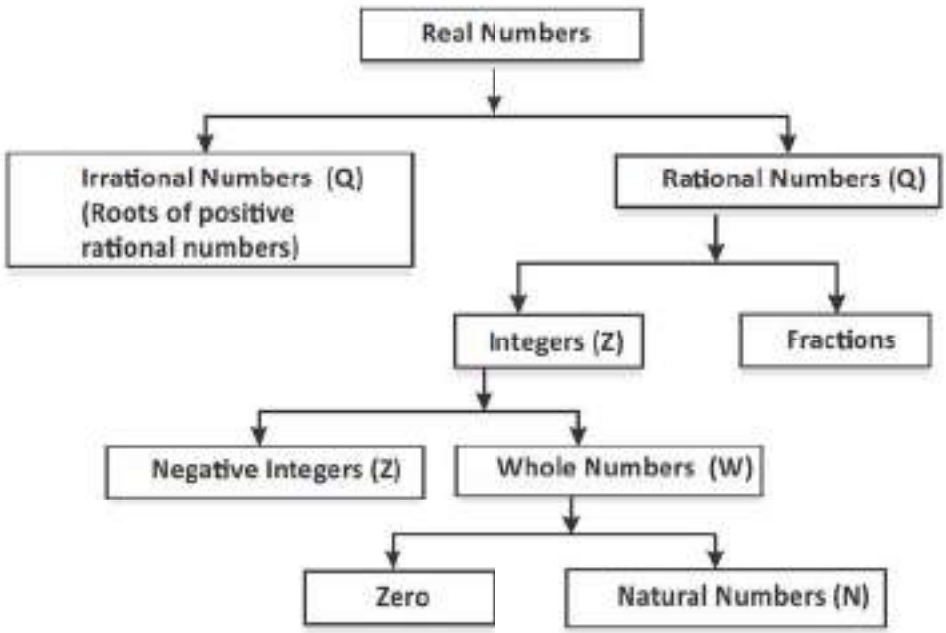
(vii) $a^0 = 1$ (viii) $\frac{1}{a^n} = a^{-n}$ (ix) $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$

- **n^{th} root of a positive real number :** For any natural number $n > 2$, we define $\sqrt[n]{a} = b$ as the n^{th} root of positive real number 'a' if $b^n = a$, and $b > 0$.
- **Laws of rational exponents of positive real numbers :** For a positive real number 'a', and any positive integers 'm' and 'n' ($\neq 0$), such that 'm' and 'n' have no common factor other than 1, we have

$a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (a^m)^{\frac{1}{n}}$ i.e., $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Note: **For any positive real number $a(\neq 1)$ if $a^p = a^q$, then $p = q$.**

- **Overall view of the number system:**



Multiple Choice Questions

A B C D

1. Which of the following is not a rational number?

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{9}$ (D) $\sqrt{16}$

2. Of which of the following is the set of natural numbers a subset?

(A) The set of even numbers.
(B) The set of odd numbers.
(C) The set of composite numbers.
(D) The set of real numbers.

3. Which is an irrational number between $\sqrt{2}$ and $\sqrt{3}$?

(A) $2^{\frac{1}{2}}$ (B) $3^{\frac{1}{4}}$ (C) $6^{\frac{1}{4}}$ (D) $6^{\frac{1}{8}}$

4. What type of a number is $(6+\sqrt{2})(6-\sqrt{2})$?

(A) Rational number
(B) Irrational number
(C) Prime number
(D) Negative Integer

5. What type of a number is $\left(\sqrt{2}+\sqrt{3}\right)^2$?

(A) A rational number
(B) An irrational number
(C) A fraction
(D) A decimal number

6. Given **X** : Every fraction is a rational number.

and **Y**: Every rational number is a fraction.

Which of the following is correct?

(A) X is False and Y is True.
(B) X is True and Y is False.
(C) Both X and Y are True.
(D) Both X and Y are False.

7. What is the rationalizing factor of $\left(2\sqrt[3]{5}\right)$?

(A) $\sqrt[3]{5}$ (B) $\sqrt[3]{5^2}$ (C) 5^2 (D) 5^3

8. What is the rationalizing factor of $\sqrt[5]{a^2b^3c^4}$?

(A) $\sqrt[5]{a^3b^2c}$ (B) $\sqrt[4]{a^3b^2c}$
(C) $\sqrt[3]{a^3b^2c}$ (D) $\sqrt{a^3b^2c}$

9. What is the rational denominator of $\frac{2\sqrt[3]{5}}{\sqrt[3]{9}}$?

(A) 1 (B) 2
(C) 3 (D) 4

10. Find the value of $\left(\sqrt[3]{27}-\sqrt{6\frac{3}{4}}\right)^2$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{2}$ (C) $\frac{\sqrt{3}}{4}$ (D) $\frac{3}{4}$

11. What is the product of $\sqrt[3]{4}$ and $\sqrt[3]{22}$?

(A) $2\sqrt[3]{11}$ (B) $\left(3\sqrt{11}\right)$
(C) $4\sqrt{11}$ (D) $8\sqrt[3]{11}$

12. What is the quotient when $\sqrt[6]{12}$ is divided by $\sqrt{3}\sqrt[3]{2}$?

(A) $\frac{1}{\sqrt[2]{3}}$ (B) $\frac{1}{\sqrt[3]{3}}$ (C) $\frac{1}{\sqrt[4]{3}}$ (D) $\frac{1}{\sqrt[5]{3}}$

13. If $\frac{-4}{7}=\frac{-32}{x}$, what is the value of x?

(A) -56 (B) 56 (C) -65 (D) 65

14. Find the value of $x-y^{x-3}$ when $x=2$ and $y=-2$.

(A) 18 (B) -18 (C) 14 (D) -14

15. If $9^{x+2}=240+9^x$, find x.

(A) 0.5 (B) 0.2 (C) 0.4 (D) 0.1

16. Which expression is equal to $(x^{-1}+y^{-1})^{-1}$?

(A) xy (B) $x+y$
(C) $\frac{xy}{x+y}$ (D) $\frac{x+y}{xy}$

17 If $10^x = 64$, what is the value of $10^{\frac{x}{2}+1}$?

(A) 18 (B) 42
(C) 80 (D) 81

18 If $\frac{x}{x^{1.5}} = 8x^{-1}$ and $x > 0$, find x.

(A) $\frac{\sqrt{2}}{4}$ (B) $2\sqrt{2}$
(C) 4 (D) 64

19 If $4^x - 4^{x-1} = 24$, what is the value of $(2x)^x$?

(A) $5\sqrt{5}$ (B) $\sqrt{5}$
(C) $25\sqrt{5}$ (D) 125

20 If $x = \sqrt{5} + 2$, find the value of $x - \frac{1}{x}$.

(A) $2\sqrt{5}$ (B) 4
(C) 2 (D) $\sqrt{5}$

21 If $x = \frac{2}{3-\sqrt{7}}$, find the value of $(x-3)^2$.

(A) 1 (B) 3 (C) 6 (D) 7

22 If $x = 7 + 4\sqrt{3}$ and $xy = 1$, what is the value of $\frac{1}{x^2} + \frac{1}{y^2}$?

(A) 64 (B) 134
(C) 194 (D) 1/49

23 If $\sqrt{5} = 2.236$ and $\sqrt{3} = 1.732$, find the value of $\frac{1}{\sqrt{5}-\sqrt{3}}$.

(A) 3.968 (B) $\frac{1}{3.968}$
(C) 1.984 (D) $\sqrt{0.504}$

24 Identify a rational number between −5 and 5.

(A) 0
(B) −7.3
(C) −5.7
(D) 1.101100110001...

25 The sum of the digits of a number is subtracted from the number, the resulting number is always divisible by which of the following numbers?

(A) 2 (B) 5
(C) 8 (D) 9

26 Find the difference between $1.\bar{4}$ and $0.\bar{2}$.

(A) $0.\bar{2}$ (B) $1.\bar{3}$
(C) $1\frac{1}{9}$ (D) $1\frac{2}{9}$

27 If $\sqrt{a}$ is an irrational number, what is a?

(A) Rational (B) Irrational
(C) 0 (D) Real

28 Which of the following is an irrational number?

(A) $\sqrt{9}$ (B) π
(C) $\frac{22}{7}$ (D) 2.5151515....

29 Which of the following numbers is an irrational number?

(A) $\sqrt{16} - 4$
(B) $(3-\sqrt{3})(3+\sqrt{3})$
(C) $\sqrt{5} + 3$
(D) $-\sqrt{25}$

30 If $x = 2 + 2^{1/3} + 2^{2/3}$, find the value of $(x^3 - 6x^2 + 6x)$.

(A) 2 (B) 4 (C) 8 (D) 6

31 If $x = \frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 10$.

(A) 14 (B) 8
(C) 4 (D) 16

32 If $a = 3$ and $b = 5$, find the value of $a^b + b^a$.

(A) 512 (B) 251
(C) 513 (D) 152

33 If $p = 3$ and $q = 5$, find the value of $\left(\frac{1}{p}+\frac{1}{q}\right)^{p}$.

(A) $\frac{3375}{512}$ (B) $\frac{512}{3375}$
(C) $\frac{512}{225}$ (D) $\frac{64}{225}$

34 Find the value of $\frac{1}{\sqrt{5}-\sqrt{4}}$.

(A) $\sqrt{5}-\sqrt{4}$ (B) $\sqrt{5}+\sqrt{4}$
(C) $\frac{\sqrt{5}+\sqrt{4}}{9}$ (D) $\sqrt{4}-\sqrt{5}$

35 Find the rational number for $0.\overline{5}$.

(A) $\frac{9}{5}$ (B) $\frac{5}{10}$
(C) $\frac{5}{9}$ (D) $\frac{10}{5}$

36 Identify the simplest form of $\frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}-3\sqrt{5}}$.

(A) $6+\sqrt{15}$ (B) $4+\sqrt{15}$
(C) $4+\sqrt{3}$ (D) $6+\sqrt{5}$

37 Find the value of $\frac{(0.6)^{0}-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^{3}+\left(\frac{-1}{3}\right)^{-1}}$.

(A) $\frac{-2}{3}$ (B) $\frac{3}{2}$
(C) $-\frac{3}{2}$ (D) $\frac{2}{3}$

38 m and n are integers and $\sqrt{mn}=10$. Which of the following cannot be a value of $m+n$?

(A) 25 (B) 52
(C) 101 (D) 50

39 If $2.5252525\ldots = \frac{p}{q}$ (in the lowest form) what is the value of $\frac{q}{p}$?

(A) 0.4 (B) 0.42525
(C) 0.0396 (D) 0.396

40 If $x^{x^{\sqrt{x}}} = \left(x\sqrt{x}\right)^{x}$, find the value of x.

(A) $\frac{3}{2}$ (B) $\frac{2}{9}$
(C) $\frac{9}{4}$ (D) $\frac{4}{9}$

Previous Contest Questions

1 Simplify $\frac{a^{\frac{1}{2}}+a^{-\frac{1}{2}}}{1-a}+\frac{1-a^{-\frac{1}{2}}}{1+\sqrt{a}}$

(A) 1 (B) 0
(C) $\frac{2}{1-a}$ (D) $1+a$

2 Which symbol is used to denote a collection of all positive integers?

(A) N (B) W (C) Z (D) Q

3 Find the value of 0.9999 ..... in the form of $\frac{p}{q}(p, q \in Z \text{ and } q \neq 0)$.

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$
(C) $\frac{9}{10}$ (D) 1

4 If $x=\sqrt[3]{2+\sqrt{3}}$, find the value of $x^{3}+\frac{1}{x^{3}}$.

(A) 2 (B) 4
(C) 8 (D) 9

5 Find the simplest rationalising factor of $5^{1/3}+5^{-1/3}$.

(A) $5^{2/3}+1+5^{-2/3}$ (B) $5^{1/3}-5^{-1/3}$
(C) $5^{2/3}-1+5^{-2/3}$ (D) $5^{2/3}+1-5^{-2/3}$

CHAPTER 2

Polynomials

Synopsis

- An expression of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$, where $a_0, a_1, a_2, ..., a_{n-1}, a_n$ are real numbers, 'n' is a non-negative integer and $a_n \neq 0$, is called a polynomial of degree 'n'.

  Each of $a_n x^n, a_{n-1} x^{n-1}, ... a_2 x^2, a_1 x$ and a_0 with $a_n \neq 0$ is called a term of the polynomial $p(x)$.

> ***Note:*** **The power of variable in a polynomial must be a whole number.**

- An expression of the form $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$, is called a rational expression.

> ***Note:*** **Every polynomial is a rational expression, but every rational expression need not be a polynomial.**

- A polynomial $d(x)$ is called a divisor of a polynomial $p(x)$ if $p(x) = d(x).q(x)$ for some polynomial $q(x)$.
- Polynomials of one term, two terms and three terms are called monomial, binomial and trinomial respectively.
- A polynomial of degree one is called a linear polynomial.
- A polynomial of degree two is called a quadratic polynomial.
- A polynomial of degree three is called a cubic polynomial.
- A polynomial of degree four is called a biquadratic polynomial.
- A real number 'a' is a zero of a polynomial $p(x)$ if $p(a) = 0$. 'a' is also called the root of the equation $p(x) = 0$.
- Every linear polynomial in one variable has a unique zero.
- A non-zero constant polynomial has no zero.
- Every real number is a zero of the zero polynomial.
- The degree of a non-zero constant polynomial is zero.
- The degree of a zero polynomial is not defined.
- If $p(x)$ and $g(x)$ are two polynomials such that degree of $p(x) \geq$ degree of $g(x)$ and $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x)\, q(x) + r(x)$.
- **Factor theorem :**

  Let $f(x)$ be a polynomial of degree $n \geq 1$ and 'a' be any real number. Then

(i) $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$.

(ii) $f(a) = 0$ if $(x - a)$ is a factor of $f(x)$.

If $x - 1$ is a factor of a polynomial of degree 'n' then the sum of its coefficients is zero.

- **Remainder theorem:**

If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the linear polynomial $x - a$ (where 'a' is any real number) then the remainder is $p(a)$.

We can express $p(x)$ as $p(x) = (x - a)\,q(x) + r(x)$ where $q(x)$ is the quotient and $r(x)$ is the remainder.

- The process of writing an algebraic expression as the product of two or more algebraic expressions is called factorization.

- **Some important identities:**

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + b^3 + 3a^2b + 3ab^2$
6. $(a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - b^3 - 3a^2b + 3ab^2$
7. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
8. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
9. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.
11. $(x + a)(x + b) = x^2 + (a + b)x + ab$

Multiple Choice Questions A B C D

1 Write the degree of the polynomial 0.

(A) 1 (B) 0
(C) N (D) Not defined

2 If the quotient is $3x^2 - 2x + 1$, remainder is $2x - 5$ and divisor is $x + 2$, what is the dividend?

(A) $3x^3 - 4x^2 + x - 3$
(B) $3x^3 - 4x^2 - x + 3$
(C) $3x^3 + 4x^2 - x + 3$
(D) $3x^3 + 4x^2 - x - 3$

3 If $(x - 2)$ is one factor of $x^2 + ax - 6 = 0$ and $x^2 - 9x + b = 0$, find $a + b$.

(A) 15 (B) 13
(C) 11 (D) 10

4 Given P = Product of x^2y and $\frac{x}{y}$ and Q = Quotient obtained when x^2 is divided by D. If P = Q, what is the value of D?

(A) 0 (B) 1
(C) x (D) $\frac{1}{x}$

5 Which of the following is one of the factors of $a^3 + 8b^3 - 64c^3 + 24abc$?

(A) $a + 2b - 4c$ (B) $a - 2b + 4c$
(C) $a + 2b + 4c$ (D) $a - 2b - 4c$

6 Which of the following is a trinomial in p?

(A) $x^2 + 5$ (B) $p^3 + p^2 + \sqrt{2}$
(C) $\sqrt{p}\left(1+\sqrt{2p}\right)$ (D) $y + \frac{1}{y} + \frac{1}{2}$

7 Identify one of the factors of $x^3 + \frac{1}{x^3} + 2 - 2x - \frac{2}{x}$ from the following.

(A) $x - \frac{1}{x}$ (B) $x + \frac{1}{x} - 1$
(C) $x + \frac{1}{x}$ (D) $x^2 + \frac{1}{x^2}$

8 If the factors of $a^2 + b^2 + 2(ab + bc + ca)$ are $(a + b + m)$ and $(a + b + nc)$, find the value of $m + n$.

(A) 0 (B) 2
(C) 4 (D) 6

9 If $(x^2 + 3x + 5)(x^2 - 3x + 5) = m^2 - n^2$, find m.

(A) $x^2 - 3x$ (B) $3x + 5$
(C) $x^2 + 5$ (D) $x^2 - 5$

10 If $\frac{x}{y} + \frac{y}{x} = -1 (x, y \neq 0)$, what is the value of $x^3 - y^3$?

(A) 1 (B) −1
(C) $\frac{1}{2}$ (D) 0

11 What are the factors of $x^2 + (a + b + c)x + ab + bc$?

(A) $(x + a)(x + b + c)$
(B) $(x + a)(x + a + c)$
(C) $(x + b)(x + a + c)$
(D) $(x + b)(x + b + c)$

12 Factorise: $a^4 + 4$

(A) $(a^2 + 2a - 2)(a^2 + 2a + 2)$
(B) $(a^2 - 2a - 2)(a^2 + 2a - 2)$
(C) $(a^2 + 2a - 2)(a^2 - 2a + 2)$
(D) $(a^2 + 2a + 2)(a^2 - 2a + 2)$

13 Identify one of the factors of $x^{12} - y^{12}$.

(A) $(x^2 - xy + y^2)$ (B) $(x^4 + x^2y^2 + y^4)$
(C) $(x^2 + xy - y^2)$ (D) $(x^4 - xy + y^4)$

14 If $x + y + z = 0$, what is the value of $x^3 + y^3 + z^3$?

(A) xyz (B) 2xyz
(C) 3xyz (D) 0

15 If $a + b + c = 0$, evaluate $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$.

(A) 1 (B) 2
(C) 3 (D) 4

16 Resolve into factors:
$6x^3 - 24xy^2 - 3x^2y + 12y^3$

(A) $3(2x - y)(x - 2y)(x + y)$
(B) $3(2x - y)(x + y)(x + 2y)$
(C) $3(2x - y)(x + 2y)(x - 2y)$
(D) $3(2x + y)(x - y)(x + 2y)$

17 Identify the degree of the polynomial $4 - x^2 - x^3 + \frac{5x^7}{3}$.

(A) 2 (B) 7
(C) 0 (A) 3

18 If $x + \frac{1}{x} = a + b$ and $x - \frac{1}{x} = a - b$, which of the following is true?

(A) $ab = 1$ (B) $a = b$
(C) $ab = 2$ (D) $a + b = 0$

19 Find the value of $2^{\frac{1}{4}}.4^{\frac{1}{8}}.16^{\frac{1}{16}}.256^{\frac{1}{32}}$.

(A) 1 (B) 2
(C) 4 (D) 8

20 What is the value of
$\frac{0.96 \times 0.96 \times 0.96 + 0.04 \times 0.04 \times 0.04}{0.96 \times 0.96 - 0.96 \times 0.04 + 0.04 \times 0.04}$?

(A) 0 (B) 2
(C) 1 (D) Not defined

21 Express $x - 8xy^3$ as product of factors.

(A) $x(1 - 2y)(1 + 2y + 4y^2)$
(B) $x(1 + 2y)(1 + 2y + 4y^2)$
(C) $x(1 - 2y)(1 - 2y + 4y^2)$
(D) $x(1 + 2y)(1 - 2y + 4y^2)$

22 Factorise $a^{2x} - b^{2x}$.

(A) $(a^x + b^x)(a^x - b^x)$ (B) $(a^x - b^x)^2$
(C) $(a^x + b^x)(a^2 - b^2)$ (D) $(a^x - b^x)(a^2 + b^2)$

23 If $p = (2 - a)$, what is $a^3 + 6ap + p^3 - 8$?

(A) 1 (B) 0 (C) 2 (D) 3

24 Find the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ when $a + b + c = 3x$.

(A) 3 (B) 2
(C) 1 (D) 0

25 Find the zero of the polynomial $f(x) = qx$, $q \neq 0$.

(A) 1 (B) b
(C) 0 (D) –1

26 If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$, which of the following is true?

(A) $x^3 + y^3 + z^3 = 0$
(B) $x + y + z = 27xyz$
(C) $(x + y + z)^3 = 27xyz$
(D) $x^3 + y^3 + z^3 = 27xyz$

27 Find the value of 'a' if the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x - a$ leave the same remainder when divided by $x - 1$.

(A) $a = 1$ (B) $a = -1$
(C) $a = 2$ (D) $a = -2$

28 Factorise: $ab(a + b)^2 - 3ab(a + b)$

(A) $(a + b)(a - b)ab$
(B) $(a + b - 5)(2a + b)(a - b)$
(C) $ab(a + b)(a + b - 3)$
(D) $ab(2a - b)(2a + b - 6)$

29 If $x - 1$ is a factor of $f(x) = x^3 - 6x^2 + 11x - 6$, which of the following is true?

(A) $f(-x) = 0$ (B) $f(-1) = 0$
(C) $f(x) = 0$ (D) $f(1) = 0$

30 If $y - 2$ and $y - \frac{1}{2}$ are the factors of $py^2 + 5y + r$, which of the following holds good?

(A) $p > r$ (B) $p = r$
(C) $p < r$ (D) Both (A) and (C)

31 Which of the following polynomials has –5 as a zero?

(A) $(p - 5)$ (B) $x^2 - 25$
(C) $p^2 - 5p$ (D) $x^2 + 5$

32 Factorise: $x^2 - z^2 + y^2 - p^2 + 2pz - 2xy$

(A) $(x - y - p - z)(x - y - p + z)$
(B) $(x - y + p - z)(x - y - p + z)$
(C) $(x + y + p - z)(x + y + p + z)$
(D) $(x + y - p + z)(x - y - p + z)$

33 Find the zeroes of the polynomial $p(z) = (4z + \pi)(z - 4\pi)$.

(A) $4\pi, -\frac{\pi}{4}$ (B) $-4\pi, \frac{\pi}{4}$

(C) $4\pi, \frac{\pi}{4}$ (D) $-4\pi, -\frac{\pi}{4}$

34 If $(x + 1)$ is a factor of $x^n + 1$, which of the following statements is true?

(A) n is an odd integer
(B) n is an even integer
(C) n is a negative integer
(D) n is a positive integer

35 When $x^3 - 2x^2 + ax - b$ is divided by $x^2 - 2x - 3$, the remainder is $x - 6$. Find the values of a and b.

(A) –2 and –6 (B) 2 and –6

(C) –2 and 6 (D) 2 and 6

36 Find the coefficient of x^2 in the product of $(x - 1)(1 - 2x)$.

(A) –3 (B) 3

(C) –2 (D) 1

37 If $x^2 + x + 1$ is a factor of the polynomial $3x^3 + 8x^2 + 8x + 3 + 5k$, what is the value of k?

(A) 0 (B) 2/5

(C) 5/2 (D) –1

38 If $(x - 1)$ is a factor of polynomial f(x) but not of g(x), it must be a factor of which of the following polynomials?

(A) $f(x)\,g(x)$ (B) $-f(x) + g(x)$

(C) $f(x) - g(x)$ (D) $[f(x) + g(x)]g(x)$

39 If $(x - a)(x - b)$ are factors of polynomial g(x), which of the following statements is correct?

(A) $g(a) = 0, g(b) \neq 0$
(B) $g(a) = 0, g(b) = 0$
(C) $g(a) \neq 0, g(b) \neq 0$
(D) $g(a) \neq 0, g(b) = 0$

40 If $x^{2014} + 2014$ is divided by $(x + 1)$, what is the remainder?

(A) 2014 (B) 1

(C) 2013 (D) 2015

41 If $x^2 + kx + 6 = (x + 2)(x + 3)$ for all x, what is the value of k?

(A) 1 (B) 6

(C) 5 (D) 3

42 If $x + \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$?

(A) 9 (B) 11

(C) 7 (D) 8

43 Find one of the factors of $(x - 1) - (x^2 - 1)$.

(A) $x^2 - 1$ (B) $x + 1$

(C) $x - 1$ (D) $x + 4$

44 What is the product of $\left(y - \frac{1}{y}\right)\left(y + \frac{1}{y}\right)\left(y^2 + \frac{1}{y^2}\right)$?

(A) $y^4 + \frac{1}{y^4}$ (B) $y^2 + \frac{1}{y^2} + 2$

(C) $y^4 - \frac{1}{y^4}$ (D) $y^3 + \frac{1}{y^3} - 2$

45 If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$, find k.

(A) 1 (B) 2

(C) 4 (D) 8

46 Factorise: $\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$.

(A) $\left(\frac{a}{b} + \frac{b}{a}\right)^2$ (B) $\left(1 + \frac{a}{b}\right)^2$

(C) $\left(1 - \frac{b}{a}\right)^2$ (D) $\left(\frac{b}{a} - \frac{a}{b}\right)^2$

47 Factorise: $4a^2 + 9b^2 + c^2 + 12ab + 4ac + 6bc$.

(A) $(2a + 3b + c)^2$ (B) $(a + 3b + 2c)^2$

(C) $(a - 3b + 2c)^2$ (D) $(2a + 3b - c)^2$

48 What is the remainder obtained when the polynomial p(x) is divided by $(b - ax)$?

(A) $p\left(\frac{-b}{a}\right)$ (B) $p\left(\frac{a}{b}\right)$

(C) $p\left(\frac{b}{a}\right)$ (D) $p\left(\frac{-a}{b}\right)$

49 If $a + b + c = 10$, $a^2 + b^2 + c^2 = 38$ and $a^3 + b^3 + c^3 = 160$, find the value of abc.

(A) 45 (B) 15
(C) 10 (D) 30

50 What are the factors of $x^3 + x^2 - \frac{1}{x^2} + \frac{1}{x^3}$?

(A) $\left(x^2+1\right)\left(x+\frac{1}{x}-1+\frac{1}{x^2}\right)$

(B) $(x+1)\left(x^2+\frac{1}{x^2}-1+\frac{1}{x}-x\right)$

(C) $\left(x+\frac{1}{x}\right)\left(x^2+x-1-\frac{1}{x}+\frac{1}{x^2}\right)$

(D) $\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}-1\right)$

51 Evaluate $\frac{(3.78)^2-(2.22)^2}{1.56}$

(A) 6 (B) 3
(C) 9 (D) 15

52 Let R_1 and R_2 be the remainders when the polynomials $f(x) = 4x^3 + 3x^2 - 12ax - 5$ and $g(x) = 2x^3 + ax^2 - 6x + 2$ are divided by $(x - 1)$ and $(x - 2)$ respectively. If $3R_1 + R_2 + 28 = 0$, find the value of 'a'.

(A) 0 (B) −1
(C) 1 (D) 32

53 What is the value of

$$\frac{(2.3)^3-0.027}{(2.3)^2+0.69+0.09}\ ?$$

(A) 2 (B) 3
(C) 2.327 (D) 2.273

54 If $(x-1)^7 = a_7x^7 + a_6x^6 + a_5x^5 + \ldots + a_1x + a_0$, what is the value of $a_7 + a_6 + a_5 + \ldots + a_1 + a_0$?

(A) 0 (B) 1
(C) 128 (D) 64

55 Resolve into factors: $1 + a + b + c + ab + bc + ca + abc$

(A) $(1 + a)(1 + b)(1 + c)$
(B) $(a + b + c + 1)(a - b - c)$
(C) $(a + b)(b + c)(c + a) + 1$
(D) $(a^2 + b^2 + c^2)(1 - a - b - c)$

Previous Contest Questions

1 Which of the following are the factors of $4x^2 - y^2 + 2x - 2y - 3xy$?

(A) $(x + y)$ and $(4x + y - 2)$
(B) $(x - y)$ and $(4x - y + 2)$
(C) $(x + y)$ and $(4x - y - 2)$
(D) $(x - y)$ and $(4x + y + 2)$

2 Find the value of the polynomial $x^2 - x - 1$ at $x = -1$.

(A) −3 (B) 1
(C) −1 (D) 0

3 Given that $(1 - x)(1 + x + x^2 + x^3 + x^4)$ is $\frac{31}{32}$ and x is a rational number, what is $1 + x + x^2 + x^3 + x^4 + x^5$?

(A) $\frac{31}{64}$ (B) $\frac{63}{32}$
(C) $\frac{63}{64}$ (D) $\frac{31}{32}$

4 If $p(x) = 4x^3 - 3x^2 + 2x + 1$, $q(x) = x^3 - x^2 + x + 1$ and $r(x) = x^2 - 2x + 1$ find the value of $3p(x) + 7q(x) + r(x)$.

(A) $19x^3 - 15x^2 + 11x + 11$
(B) $-19x^3 - 15x^2 + 11x - 11$
(C) $19x^3 - 15x^2 - 11x + 11$
(D) $19x^3 - 15x^2 - 11x - 11$

5 If $x^3 + y^3 + z^3 - 3xyz = k(x + y + z)\{(x - y)^2 + (y - z)^2 + (z - x)^2\}$, find k.

(A) 1 (B) $\frac{1}{4}$
(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

This page is intentionally left blank.

CHAPTER 3

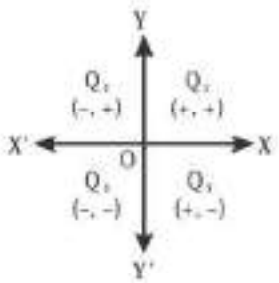
Co-ordinate Geometry

Synopsis

- **Co-ordinate Geometry:** The branch of mathematics in which geometric problems are solved through algebra by using the coordinate system is known as coordinate geometry.
- In coordinate geometry, every point is represented by an ordered pair, called coordinates of that point.
- A pair of numbers 'a' and 'b' listed in a specific order with 'a' at the first place and 'b' at the second place is called an ordered pair (a, b).

> ***Note:*** **(i) $(a, b) \neq (b, a)$**
> **(ii) If $(a, b) = (c, d)$ then $a = c$ and $b = d$.**

- The position of a point in a plane is determined with reference to two fixed mutually perpendicular lines called the coordinate axes.
- The horizontal line is called X-axis and the vertical line is called Y-axis.
- The point of intersection of the coordinate axes is called origin O(0, 0).
- In a point P(a, b), 'a' is called x-coordinate or first coordinate or abscissa and 'b' is called y-coordinate or second coordinate or ordinate.
- The axes divide the plane into four quadrants.
  - (i) Q_1 is the I quadrant. Here both x and y are positive i.e., $x > 0$ and $y > 0$. The ordered pair (a, b) belongs to this quadrant.
  - (ii) Q_2 is the II quadrant. Here x is negative and y is positive i.e., $x < 0$ and $y > 0$. The ordered pair (–a, b) belongs to this quadrant.
  - (iii) Q_3 is the III quadrant. Here both x and y are negative. i.e., $x < 0$ and $y < 0$. The ordered pair (–a, –b) belongs to the quadrant.
  - (iv) Q_4 is the IV quadrant. Here x is positive and y is negative i.e., $x > 0$ and $y < 0$. The ordered pair (a, –b) belongs to this quadrant.



- The coordinates of any point on X-axis is of the form (a, 0) [y-coordinate zero].
- The coordinates of any point on Y-axis is of the form (0, b) [x-coordinate zero].

Multiple Choice Questions

A B C D

1. What is the intersection of X and Y axes called?

(A) Origin (B) Null point
(C) Common point (D) Ordinate

2. What is the distance of the point (2, 3) from the X-axis?

(A) 2 units (B) 5 units
(C) 3 units (D) 6 units

3. What is the distance of the point (3, 2) from the Y-axis?

(A) 2 units (B) 3 units
(C) 5 units (D) 6 units

4. Where does the point (−2, 0) lie?

(A) On positive X-axis.
(B) On positive Y-axis.
(C) On negative X-axis.
(D) On negative Y-axis.

5. Where does the point (0, −2) lie?

(A) On positive X-axis.
(B) On positive Y-axis.
(C) On negative X-axis.
(D) On negative Y-axis.

6. Where does the point (3, 0) lie?

(A) On positve X-axis.
(B) On positive Y-axis.
(C) On negative X-axis.
(D) On negative Y-axis.

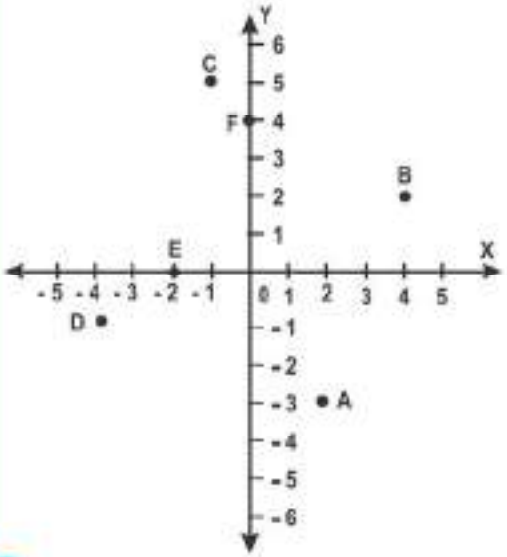
7. The points P(3, p) and Q(q, 5) represent the same point R. What are the respective values of p and q?

(A) 5 and 3 (B) −5 and 3
(C) 3 and 5 (D) 3 and −5

8. What is the distance of a point (0, −3) from the origin?

(A) 0 units
(B) −3 units
(C) cannot be determined
(D) 3 units

(9-12): Observe the given coordinate plane and answer the following questions.



9. What are the coordinates of point C?

(A) (0, 0) (B) (−1, 5)
(C) (2, −3) (D) (4, 2)

10. What are the coordinates of point E?

(A) (0, −2) (B) (1, −2)
(C) (−2, 1) (D) (−2, 0)

11. What are the coordinates of point D?

(A) (−4, −1) (B) (4, 2)
(C) (0, 4) (D) (−1, 5)

12. What is the y-coordinate of point F?

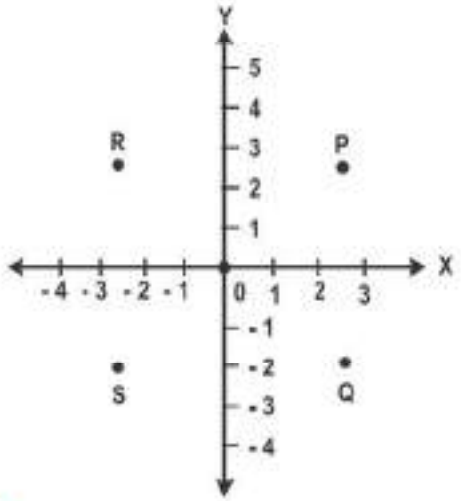
(A) −3 (B) 0
(C) 4 (D) −2

13. The points A(−a, −a), B(a, −a) C(−a, a), D(−a, a) form a polygon.

Where does the origin lie?

(A) On the vertex of the polygon.
(B) On the side of the polygon.
(C) Outside the polygon.
(D) At the point where the diagonals of the polygon meet.

(14-16): Observe the given coordinate plane and answer the following questions.



14 In which quadrant is the point (3, 3) ?

(A) Q_1 (B) Q_2
(C) Q_3 (D) Q_4

15 Which point is represented by the ordered pair (–3, –1) ?

(A) P (B) Q
(C) R (D) S

16 What are the coordinates of the point Q?

(A) (–2, 2) (B) (2, –2)
(C) (3, 3) (D) (–3, –1)

17 What are the coordinates of origin ?

(A) (0, 1) (B) (1, 0)
(C) (0, 0) (D) (1, 1)

18 Which of the following quadrants has/ have points with a positive abscissa?

(A) I and II quadrants
(B) I and IV quadrants
(C) I quadrant only
(D) IV quadrant only

19 Identify the co-ordinates of the point which lies on y-axis at a distance of 4 units in negative direction of y-axis.

(A) (0, 4) (B) (4, 0)
(C) (0, –4) (D) (–4, 0)

20 In which of the following quadrants ordinate of a point negative?

(A) III and IV quadrants
(B) III quadrant only
(C) II and III quadrants
(D) IV quadrant only

21 The points (–5, 2) and (2, –5) lie in which of the following ?

(A) The same quadrants.
(B) II and III quadrants respectively.
(C) II and IV quadrants respectively.
(D) IV and and III quadrants respectively.

22 Which of the following points lie on the y-axis ?

(A) (0, 4) (B) (–2, 0)
(C) (3, 0) (D) (–1, –1)

23 Identify the ordered pairs that result in a quadrilateral.

(A) (1, –1), (2, –2), (4, –4), (6, –6)
(B) (1, 0), (–3, 0), (6, 0), (–8, 0)
(C) (3, 2), (2, 3), (–4, 5), (5, –3)
(D) (0, –5), (0, 0), (0, 3), (0, 5)

24 The perpendicular distance of a point from the *x*-axis is 2 units and its perpendicular distance from the *y*-axis is 3 units. Find the co-ordinates of the point if it lies in the III Quadrant.

(A) (–3, –2) (B) (–2, –3)
(C) (3, –2) (D) (–3, 2)

25 The perpendicular distance of a point from the *x*-axis is 4 units and its perpendicular distance from the y-axis is 5 units. What are the co-ordinates of such a point if it lies in the II Quadrant?

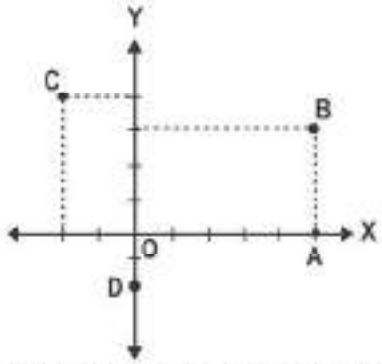
(A) (–4, 5) (B) (–5, 4)
(C) (–4, –5) (D) (5, –4)

26 Which of the following points lie on x-axis?

A(0, 2), B(0, 6), C(–3, 0), D(0, –3), E(0, 4), F(6, 0), G(3, 0)

(A) C, E and F (B) D, E and B
(C) C, H and F (D) C, H and B

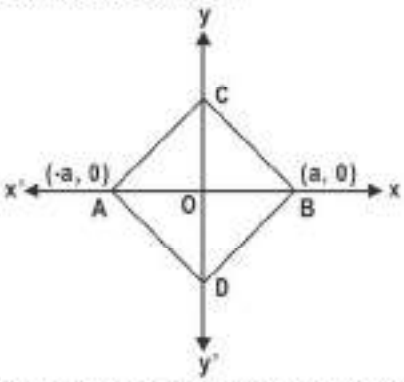
27 Study the given graph.



What are the co-ordinates of A, B, C and D?

(A) A (5, 3), B (0, 5), C (-2, 4), D (0, 2)
(B) A (0, 5), B (3, 5), C (4, -2), D (0, -2)
(C) A (5, 0), B (5, 3), C (-2, 4) D (-2, 0)
(D) A (5, 0), B (5, 3), C (-2, 4), D (0, -2)

28 In the given graph, Δ ABC and Δ ABD are equilateral triangles.



Find the respective coordinates of points C and D.

(A) $(0, a\sqrt{3})(0, a\sqrt{3})$
(B) $(0, -a\sqrt{3})(0, a\sqrt{3})$
(C) $(0, a\sqrt{3})(0, -a\sqrt{3})$
(D) $(a\sqrt{3}, 0)(-a\sqrt{3}, 0)$

29 Plot the points P(1, 0), Q(4, 0) and S(1, 3). Find the coordinates of the point R such that PQRS is a square.

(A) (3, 4) (B) (4, 3)
(C) (3, -4) (D) (-3, 4)

30 What is the number of quadrants of a cartesian plane?

(A) 1 (B) 2
(C) 3 (D) 4

31 Points A(–a, –a), B(a, –a), C(a, a) and D(x, y) form a square. In which of the following quadrants does the point D lie?

(A) II (B) III
(C) I (D) IV

32 What is the ordinate of a point on the y-axis ?

(A) A positive number
(B) A negative number
(C) 0
(D) All the above

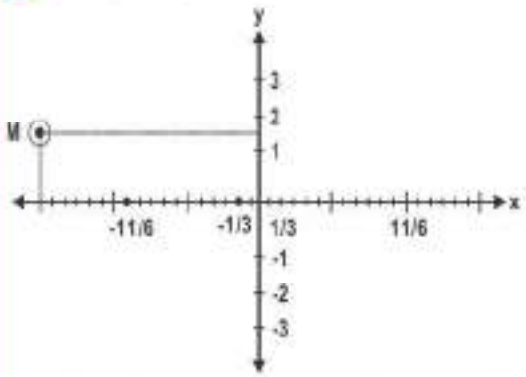
33 Which of the following are the signs of the coordinates of a point in the II quadrant?

(A) (+, +) (B) (–, +)
(C) (–, –) (D) (+, –)

34 What are the signs of the coordinates of a point in the III quadrant ?

(A) (+, +) (B) (–, +)
(C) (+, –) (D) (–, –)

35 Study the given graph.



Identify the coordinates of the point M.

(A) $\left(-\frac{13}{6}, \frac{1}{2}\right)$ (B) $\left(-3, \frac{3}{2}\right)$
(C) $\left(-\frac{13}{3}, \frac{1}{2}\right)$ (D) $\left(-\frac{3}{2}, 3\right)$

36 Which of the following statements is true?

(A) Ordinate is positive to the right of the origin.
(B) Ordinate is negative to the left of the origin.
(C) Ordinate is negative below x-axis.
(D) Ordinate is negative above x-axis.

37 Identify the correct statement.

(A) Ordinate is negative to the right of the origin.
(B) Abscissa is negative to the left of the origin.
(C) Ordinate is positive below x-axis.
(D) Abscissa is negative above x-axis.

38 Which of the following is true about the ordinate?

(A) It is positive above x-axis.
(B) It is positive above y-axis.
(C) It is positive to the right of the origin.
(D) It is negative to the left of the origin.

39 Which figure is formed by joining points O(0,0), B(16,0) and C(16,12) on a graph paper?

(A) A triangle
(B) A square
(C) A right angled triangle
(D) A line

40 Which quadrilateral is formed by joining the points (1, 1), (2, 4), (8, 4) and (10, 1)?

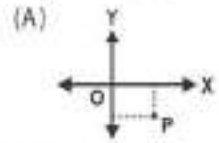
(A) A triangle (B) A square
(C) A rectangle (D) A trapezium

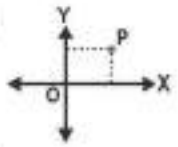
41 Which of the following represents the point (–2, 5)?

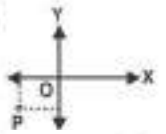
(A) Moving 2 units towards right and 5 units upwards.
(B) Moving 2 units towards left and 5 units upwards.
(C) Moving 2 units towards right and 5 units downwards.
(D) Moving 2 units towards left and 5 units downwards.

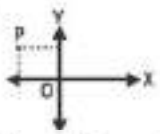
Previous Contest Questions

1 Which of the following graph represents the point P(–a, b) where $a < 0, b < 0$?

(A) 

(B) 

(C) 

(D) 

2 Reshma moves 5 units right and then 3 units downwards. She then moves 4 units to the left, finally stops at a point represented by (–2, –2) on the cartesian plane. What was her starting point on the plane?

(A) (–3, 1) (B) (0, 0)
(C) (2, –1) (D) (–5, –3)

3 What is the abscissa of any point on the X-axis?

(A) 0 (B) 1
(C) –1 (D) An integer

4 What is the ordinate of any point on the X-axis?

(A) 0 (B) 1
(C) 2 (D) –1

5 The perpendicular distance of a point from the x-axis is 4 units and its perpendicular distance from the y-axis is 5 units. What are the co-ordinates of such a point if it lies in the II Quadrant?

(A) (–4, 5) (B) (–5, 4)
(C) (–4, -5) (D) (5, –4)

CHAPTER 4

Linear Equations in Two Variables

Synopsis

- **Equation:** A statement of equality of two algebraic expressions involving a variable is called an equation.
- **Simple linear equation:** An equation which contains only one variable of degree 1 is called a simple linear equation.
- **Solution of an equation:** The value of the variable, which when substituted in the given equation, makes the two sides L.H.S (Left Hand Side) and R.H.S (Right Hand Side) of the equation equal is called the solution of that equation.
- **Transposition:** Any term of an equation may be taken to the other side with a change in its sign. This process is called transposition.
- **Cross multiplication:** If $\frac{ax+b}{cx+d}=\frac{p}{q}$, then $q(ax + b) = p(cx + d)$. This process is called cross multiplication.
- **Linear equation in one variable:** An equation of the form $ax + b = 0$ or $ax = c$, is a linear equation in one variable x, where $a \neq 0$ and a, b and c are real numbers.
- **Solution of a linear equation in one variable:** The value of the variable (x) for which both the sides of the equation become equal is said to be the solution of the equation.

> ***Note:*** **A solution is also called the 'root' of the equation.**

- **Linear equation in two variables:**
  - (i) An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that 'a' and 'b' are not both zero, is called a linear equation in two variables.
  - (ii) A linear equation in two variables has infinitely many solutions.
  - (iii) The graph of every linear equation in two variables is a straight line.
  - (iv) $x = 0$ is the equation of Y-axis.
  - (v) $y = 0$ is the equation of X-axis.
  - (vi) The graph of $x = a$ is a straight line parallel to the Y-axis.
  - (vii) The graph of $y = a$ is a straight line parallel to the X-axis.
  - (viii) An equation of the type $y = mx$ represents a line passing through the origin.
  - (ix) Every point on the graph of a linear equation in two variables is a solution of the linear equation. Conversely, every solution of the linear equation is a point on the graph of the linear equation.

Multiple Choice Questions

A B C D

1. How many line(s) pass through the point (0, 0) ?

(A) Only one (B) Two
(C) Infinitely many (D) Three

2. Which of the following is true about $y = 4x - 3$?

(A) It has a unique solution.
(B) It has only two solutions.
(C) It has infinitely many solutions.
(D) It is a linear equation in one variable.

3. If $x = -2$ and $y = 3$ is the solution of the equation $3x - 5y = k$, find 'k'.

(A) −21 (B) −9
(C) −18 (D) 19

4. What does an equation of the form $ax + by + c = 0$, where a and b are non-zero numbers represent?

(A) A straight line (B) A circle
(C) A triangle (D) A quadrilateral

5. Which of the following is the equation of a line parallel to y-axis?

(A) $y = -2$ (B) $y = 0$
(C) $y = 5$ (D) $x = -4$

6. What is the equation of X-axis?

(A) $x = 0$ (B) $x = -2$
(C) $y = 0$ (D) $y = -2$

7. For which condition does the equation $px + qy + r = 0$ represent a linear equation in two variables?

(A) $p \neq 0, q = 0$ (B) $q \neq 0, q = 0$
(C) $p = 0, q = 0$ (D) $p \neq 0, q \neq 0$

8. Which of the following is correct with respect to the line $x + 1 = 0$?

(A) It is parallel to y-axis.
(B) It passes through (0, −1).
(C) It is parallel to x-axis.
(D) It passes through (0, 0).

9. Which is the equation of a line passing through the origin?

(A) $y = 2$ (B) $x = 4$
(C) $y = 5x$ (D) $x = -7$

10. Identify the correct statement from the following with respect to the line $y - 2 = 0$.

(A) It is parallel to X-axis.
(B) It is parallel to Y-axis.
(C) It passes through the origin.
(D) It passes through $x = 2$.

11. If (3, 4) is a solution of the equation $5x - 2y = k$, find the value of k.

(A) 7 (B) 6
(C) 5 (D) 4

12. If the point (2, −3) lies on the graph of the equation $ay = 7x - 26$, what is the value of 'a'?

(A) 37 (B) 16
(C) −5 (D) 4

13. The cost of a shirt of a particular brand is ₹ 800. What is the linear equation when the cost of x number of shirts is ₹ y ?

(A) $x = 800y$ (B) $x = 800 + y$
(C) $y = 800x$ (D) $y = 800 + x$

14. Where does the point of the form (p, p) $\forall$ $p \neq 0$ always lie?

(A) x − axis
(B) origin
(C) on the line $y = x$
(D) on the line $x + y = 0$

15. What is the solution set of $\frac{5}{3} - \frac{2}{x} = \frac{8}{x}$ for $x \neq 0$?

(A) {2} (B) $\left(\frac{18}{5}\right)$
(C) $\left(\frac{26}{5}\right)$ (D) {6}

16 What is the solution set of $\sqrt{k+64} - 8 = -2$?

(A) {−28} (B) {−124}
(C) {4} (D) { }

17 What are all the roots for the equation $3|w - 14| - 6 = 21$?

(A) 19 (B) 23
(C) 5 and 23 (D) 9 and 19

18 What is the solution of the equation given in the box?

$$\frac{3y+4}{2} + \frac{2y-5}{3} = \frac{31}{2}$$

(A) $y = 1$ (B) $y = 6$
(C) $y = 7$ (D) $y = 13$

19 Veena heard that it is 82° Fahrenheit in Ooty. She knows that $F = \frac{9}{5}C + 32$, where F represents the temperature in degrees Fahrenheit and C represents the temperature in degrees Celsius. Which is closest to the temperature in Ooty, in degrees Celsius?

(A) 28 (B) 63
(C) 90 (D) 180

20 Identify the point at which the graph of the equation $7x - 9y - 21 = 0$ cuts x – axis.

(A) (21, 0) (B) (0, 9)
(C) (3, 0) (D) (3, 1)

21 Which of the following equations represents a straight line passing through the points (1, 6), (0, 4) and (−2, 0)?

(A) $2x - y = -4$ (B) $x - 2y = -4$
(C) $2x + y = 4$ (D) $x + 2y = -4$

22 Identify the equation of line parallel to y – axis and 5-units away from the origin.

(A) $y = 5$ (B) $x = -5$
(C) $x = 5$ (D) $y = -5$

23 Find the point at which the straight lines represented by linear equations $x - y = 2$ and $3x - 2y = 7$ intersect.

(A) (1, 1) (B) (1, 3)
(C) (2, 2) (D) (3, 1)

24 Which of the following is true?

(A) The equation of the x-axis is $y = 0$.
(B) The graph of every linear equation in two variable is a curve.
(C) $y = 3x + 5$ has a unique solution.
(D) The graph of the equation $2y - 5 = 0$ is parallel to y – axis.

25 If the point $(2p, p - 3)$ lies on the graph of the equation $3x + 2y + 12 = 0$, find the value of p.

(A) $-\frac{3}{4}$ (B) $\frac{7}{15}$
(C) $-\frac{1}{6}$ (D) $\frac{6}{7}$

26 If the expressions $y - 10$ and $24 - y$ are equal, what is the value of y?

(A) 34 (B) 14
(C) 17 (D) 12

27 How many kilograms of tea at ₹ 50 per kg should be mixed with 35 kg of tea costing ₹ 60 per kg so as to sell the mixture at ₹ 57 per kg without gaining or losing anything in the transaction?

(A) 5 kg (B) 7 kg
(C) 25 kg (D) 15 kg

28 Vihan spent ₹ 132 to buy movie tickets for 20 children and 4 adults. Adult tickets cost ₹ 3 more than child tickets. If A is the price of an adult ticket and S is the price of a child ticket, which system of equations could be used to find the price of each adult and child ticket?

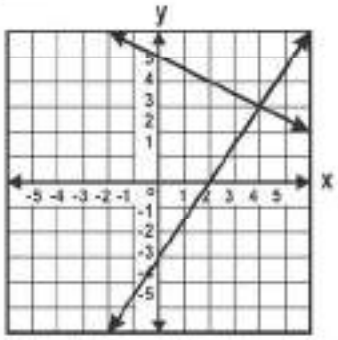
(A) $\begin{cases} S = A + 3 \\ 4A + 20S = 132 \end{cases}$

(B) $\begin{cases} A = S + 3 \\ 4A + 20S = 132 \end{cases}$

(C) $\begin{cases} A = S + 3 \\ 20A + 4S = 132 \end{cases}$

(D) $\begin{cases} A = S + 3 \\ A + S = 132 \end{cases}$

29 The graph of a system of linear equations is given.



Based upon the graph, which is the apparent solution to the system of equations?

(A) (2, 5) (B) (3, 4)
(C) (4, 3) (D) (5, 2)

30 Find 'x' if $\frac{x-2}{x+3}=\frac{x+2}{x-3}, x \neq 3, x \neq -3$.

(A) 3 (B) 0
(C) 1 (D) − 2

31 Three consecutive numbers such that twice the first, 3 times the second and 4 times the third together make 191. Find the least of the consecutive numbers.

(A) 18 (B) 21
(C) 19 (D) 20

32 Rishab rode his bike to a store 5 km from his house. The table given shows the distance from the store paired with the number of minutes after leaving his house.

Minutes (x)	*Kilometres from store (y)*
2	4.3
3	4
5	3.4
8	2.5

Which of following equations of line best fits for the given data?

(A) $y=-0.2x+4.3$ (B) $y=-0.2x+6.1$
(C) $y=-0.3x+4.9$ (D) $y=-0.3x+6.1$

33 A man is five times as old as his son. After 2 years the man will be four times as old as his son. What is the present age of the man?

(A) 35 years (B) 30 years
(C) 6 years (D) 31 years

34 Mega city High School earned ₹ 5100 on tickets sales for a play. The cost per ticket was ₹ 12. If t represents the number of tickets sold to the play, which of the following equations could be used to determine the number of tickets sold for the play?

(A) $12 = 5100t$ (B) $12t = 5100$
(C) $t = 5100 - 12$ (D) $t = 5100.12$

35 The function $f(x) = 35 + 15x$ represents the amount of money, in Rupees, Mr. Ramesh earns for working x hours. How much money does Mr. Ramesh earn for working 25 hours?

(A) ₹ 75 (B) ₹ 375
(C) ₹ 410 (D) ₹ 1250

36 Two planes start from a city and fly in opposite directions, one averaging a speed of 40 km/hour greater than the second. If they are 3400 km apart from 5 hours, find the sum of their average speeds.

(A) 680 km/h (B) 360 km/h
(C) 320 km/h (D) 640 km/h

37 Which equation represents the relationship between time, t, and distance, d?

Time (hours)	*Distance (km)*
2	90
3	135
4	180
5	225

(A) $d = t + 45$ (B) $d = 45t$
(C) $t = 45d$ (D) $t = \frac{45}{d}$

38 The angles of a pentagon are in the ratio 2 : 3 : 3 : 3 : 4. Find the least angle of the pentagon.

(A) 108° (B) 72° (C) 27° (D) 90°

39 Straight lines represented by linear equations $x + y = 2$ and $5x - 3y = 2$ intersect at which of the given points?

(A) (1, 2) (B) (1, 1) (C) (2, 1) (D) (3, 2)

40 A woman sells to the first customer half her stock and half an apple, to the second customer she sells half her remaining stock and half an apple, and so on to the third, and to a fourth customer. She finds that she has now 15 apples left. How many apples did she have before she started selling?

(A) 63 (B) 127 (C) 240 (D) 289

41 A student was asked to divide a number by 17/8. Instead, he actually multiplied it by 17/8 and hence got 225 more than the expected answer. What was the expected answer?

(A) 126 (B) 136 (C) 64 (D) 84

42 If a and b are real numbers, when does the equation $3x - 5 + a = bx + 1$ has a unique solution x?

(A) for all a and b (B) no root
(C) If $a \neq 6$ (D) If $b \neq 3$

43 In a class, $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent, what part of the total number of students are present?

(A) $\frac{23}{30}$ (B) $\frac{23}{36}$ (C) $\frac{18}{49}$ (D) $\frac{17}{25}$

44 How many solutions does a linear equation in two variable have?

(A) 1 (B) Infinite
(C) 2 (D) 0

45 The course of an enemy submarine as plotted on a set of rectangular axes is $2x + 3y = 5$. On the same axes the course of the destroyer is indicated by $x - y = 10$. What is the point (x, y) at which the submarine can be destroyed?

(A) (−7, 3) (B) (−3, 7)
(C) (3, −7) (D) (7, −3)

46 When a ball bounces, it rises to $\frac{3}{4}$ of the height from which it fell. If the ball is dropped from a height of 32 m, how high will it rise at the third bounce?

(A) $14\frac{1}{2}$ m (B) $13\frac{1}{2}$ m
(C) 13 m (D) 14

Previous Contest Questions

1 The breadth of a rectangular room is 2 m less than its length(l). If the perimeter of the room is 14 m, find the length(l) and breadth(b) of the room.

(A) $l = 2.5$ m, $b = 4.5$ m
(B) $l = 3.5$ m, $b = 3.5$ m
(C) $l = 4.5$ m, $b = 2.5$ m
(D) $l = 2.5$ m, $b = 5.5$ m

2 The total value of a collection of coins of denominations ₹ 1.00, 50 paise, 25 paise, 10 paise and 5 paise, is ₹ 380. If the number of coins of each denomination is the same, find the number of one-rupee coins.

(A) 160 (B) 180
(C) 200 (D) 220

3 $x = 3$ and $y = -1$ is a solution of which of the linear equations given?

(A) $x + y = 3$ (B) $2x + y = 3$
(C) $x + 2y = 1$ (D) $2x - y = 1$

4 Find the equation of the line that passes through the points (5, 15) and (10, 20).

(A) $y = x + 10$ (B) $y = x - 30$
(C) $y = x + 30$ (D) $y = x + 15$

5 Which of the following statements best models every linear equation in two variables x and y?

(A) A straight line parallel to x – axis
(B) A straight line parallel by y – axis
(C) A straight line
(D) A straight line that passes through the origin

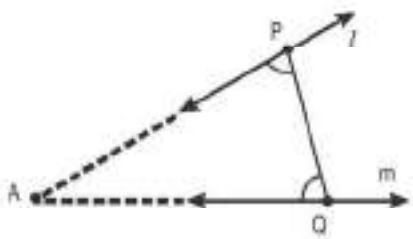
This page is intentionally left blank.

CHAPTER 5

Introduction to Euclid's Geometry

Synopsis

- **Axioms:** Axioms or postulates are the assumptions which are obvious universal truths and are not to be proved.
- Some of the axioms given by Euclid:
  - (i) Things which are equal to the same thing are equal to one another.
    i.e., if $a = c$ and $b = c$, then $a = b$.
  - (ii) If equals are added to equals, the wholes are equal.
    i.e., if $a = b$ and $c = d$, then $a + c = b + d$.
    Also $a = b \Rightarrow a + c = b + c$.
    Here, a, b, c and d are same kind of things.
  - (iii) If equals are subtracted from equals, the remainders are equal.
  - (iv) The things which coincide with one another are equal to one another.
  - (v) The whole is greater than the part.
    i.e., if $a > b$, then there exists 'c' such that $a = b + c$.
    Here, 'b' is a part of 'a' and therefore, 'a' is greater than 'b'.
  - (iv) Things which are double the same things are equal to one another.
  - (vii) Things which are halves of the same things are equal to one another.
- **Euclid's five postulates:**
  - (i) **Postulate 1:** A straight line may be drawn from any one point to any other point.
  - (ii) **Postulate 2:** A terminated line (i.e., a line segment) can be produced indefinitely on either side to form a line.
  - (iii) **Postulate 3:** A circle can be drawn with any centre and any radius.
  - (iv) **Postulate 4:** All right angles are equal to one another.
  - (v) **Postulate 5:** If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

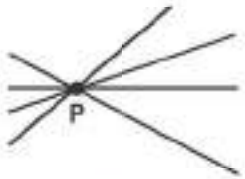


- Theorems or propositions are the properties which are to be proved, using definitions, axioms/postulates, previously proved statements and deductive reasoning.

- Any two distinct straight lines are either intersecting or parallel. If the two lines are intersecting, then they can be oblique to each other or perpendicular to each other.
- **Intersecting lines:** Two distinct straight lines which meet each other at a point are called intersecting straight lines.
- **Perpendicular lines:** Two intersecting lines are perpendicular to each other if one meets the other at a point and the angles made by the first with either side of the second at the point are each equal to one right angle.
- **Parallel lines:** Two distinct lines which are not intersecting are called parallel lines.
- **Axioms of points and lines:**

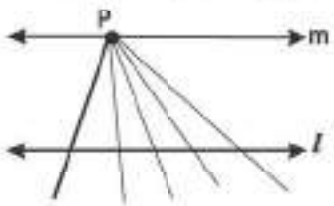
(i) **Axiom 1:** A line contains infinitely many points.

(ii) **Axiom 2:** Through a given point, infinitely many lines can be made to pass.



(iii) **Axiom 3:** Given two distinct points, there exists one and only one line passing through them.

- **Collinear points:** Three or more points are collinear if one and only one line can be made to pass through them.
- **Concurrent lines:** Three or more lines are said to be concurrent if they all pass through a unique point. The point is called the point of concurrence of the lines.
- Two distinct lines cannot have more than one point in common.
- **Playfair's axiom:** (Axiom for parallel lines): For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.



- Another version for the above axiom is stated as:
  Two distinct intersecting lines cannot be parallel to the same line.
- If a point C lies between two points A and B, then AC + BC = AB.
- If C is the midpoint of a line segment AB, then $AC = BC = \frac{1}{2}AB$.
- There is one and only one midpoint of a line segment.

Multiple Choice Questions

A B C D

1. If C is the midpoint of the segment AB, P and Q are midpoints of the segments AC and BC respectively, find BQ.

(A) $\frac{1}{2}$ AB (B) $\frac{1}{3}$ AB

(C) $\frac{1}{4}$ AB (D) $\frac{1}{5}$ AB

2. If l, m and n are three distinct lines such that $l \parallel m$ and $l \parallel n$, which of the following holds good?

(A) $m \perp n$ (B) $m \parallel n$

(C) $m = n$ (D) $l \perp n$

3. If a point C lies in between A and B, what is AC + BC equal to?

(A) AB (B) $\frac{1}{2}$ AB

(C) 2 AB (D) 4 AB

4. If C is the midpoint of the line segment AB, D and E are midpoints of the segments AC and BC respectively, what is the length of AB?

(A) 2 AD (B) 4 BC

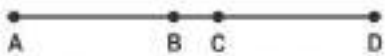
(C) 4 BE (D) 2 CD

5. If AC = PQ and CP = BQ, find the midpoint of the line segment AB.

(A) P (B) Q

(C) C (D) B

6. In the figure given, if AC = BD, what is the measure of AB?



(A) AD (B) BD

(C) CD (D) AC

7. Which of the following describe a surface?

(A) Length and Breadth
(B) Length only
(C) Breadth only
(D) Length and Height

8. How many points does a line contain?

(A) Two (B) Three

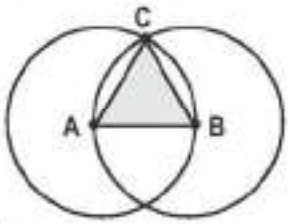
(C) Four (D) Infinitely many

9. If $l \parallel m$ and $l \perp n$, identify the correct option.

(A) $l \parallel n$ (B) $m \perp n$

(D) $m \parallel n$ (D) $m \perp l$

10. In the figure given, if A and B are the centres of the two intersecting circles, what type of a triangle is Δ ABC?



(A) A scalene triangle
(B) A right triangle
(C) An isosceles triangle
(D) An equilateral triangle

11. What are the boundaries of surfaces?

(A) Surfaces (B) Angles

(C) Lines (D) Points

12. In ancient India, what are the shapes of altars used for house hold rituals?

(A) Squares and circles
(B) Triangles and rectangles
(C) Trapeziums and pyramids
(D) Rectangles and squares

13. Which of the following is stated in the form 'Lines are parallel if they do not intersect'?

(A) An axiom (B) A definition

(C) A postulate (D) A proof

14. In Indus Valley Civilisation (about 300 B.C.), what were the dimensions of the bricks used for construction work?

(A) 1 : 3 : 4 (B) 4 : 2 : 1

(C) 4 : 4 : 1 (D) 4 : 3 : 2

15 How many interwoven isosceles triangles are there in Sriyantra (in the Atharvaveda)?

(A) Seven (B) Eight
(C) Nine (D) Eleven

16 Identify the incorrect statement.

(A) Only one line can pass through a single point.
(B) Only one line can pass through two distinct points.
(C) A terminated line can be produced indfinitely on both the sides.
(D) If two circles are equal, their radii are equal.

17 Which of the following is the shape of the base of a solid pyramid?

(A) A triangle (B) A square
(C) A rectangle (D) Any polygon

18 How many lines can pass through a given point?

(A) Two (B) None
(C) Only one (D) Infinitely many

19 What is the shape of the side faces of a pyramid?

(A) Triangles (B) Squares
(C) Polygons (D) Trapeziums

20 What is the number of line segments determined by three collinear points?

(A) Two (B) Three
(C) Only one (D) Four

21 To which of the following countries did Euclid belong?

(A) Babylonia (B) Egypt
(C) Greece (D) India

22 What is the number of line segments determined by three given non-collinear points?

(A) Two (B) Three
(C) Infinitely many (D) Four

23 Given four points such that no three of them are collinear, what is the number of lines that can be drawn through them?

(A) 2 lines (B) 4 lines
(C) 6 lines (D) 8 lines

24 What is formed when two planes intersect each other?

(A) Plane (B) Point
(C) Straight line (D) Angle

25 Which of the following are the three steps from solids to points?

(A) Solids - Surfaces - Lines - Points
(B) Solids - Lines - Surfaces - Points
(C) Lines - Points - Surfaces - Solids
(D) Lines - Surfaces - Points - Solids

26 Which of the following needs a proof ?

(A) Axiom (B) Theorem
(C) Postulate (D) Definition

27 How many dimensions does a solid has?

(A) 1 (B) 2
(C) 3 (D) 0

28 In which of the following forms did Euclid state that all right angles are equal to each other?

(A) An axiom (B) A defination
(C) A postulate (D) A proof

29 In which of the following forms did Euclid state that if equals are subtracted from equals, the remainders are equals?

(A) An axiom (B) A postulate
(C) A definition (D) A proof

30 Euclid divided his famous treatise "The Elements" into how many chapters?

(A) 13 chapters (B) 12 chapters
(C) 11 chapters (D) 9 chapters

31 If the point F lies in between M and N and C is the midpoint of MF which of the following is true?

(A) MC + FN = MN (B) MF + CF = MN
(C) MC + FN = MN (D) CF + CN = MN

32 It is known that if $x + y = 10, x + y + z = 10 + z$. Which Euclid's axiom illustrates this statement?

(A) First Axiom (B) Second Axiom
(C) Third Axiom (D) Fourth Axiom

33 What is the total number of propositions in the elements?

(A) 465 (B) 460
(C) 13 (D) 55

34 If a point P lies in between A and B, which of the following is true?

(A) $AP = \frac{1}{2}AB$ (B) $BP = \frac{1}{2}AB$
(C) $AP + PB = AB$ (D) $AP = BP$

35 Identify the incorrect statement.

(A) If a point P lies in between A and B, then $AP < AB$ and $BP < AB$.
(B) A line segment has fixed length.
(C) One point always determines a unique line.
(D) A line segment can be produced indefinitely on either side.

36 If X, Y, Z are the three points on a line and Y lies between X and Z, which of the following is true?

(A) $XY + YZ = XZ$.
(B) $XY + XZ = YZ$
(C) $XZ + YZ = XY$
(D) $\frac{1}{2}(XY + YZ) = XZ$

37 ABCD are the four points on a line.

If AD = BC, which of the following has its length the same as BD?

(A) AD (B) BC
(C) AC (D) CD

38 How many common points do two distinct lines in a plane have?

(A) Two points (B) Three points
(C) One point (D) Four points

39 Two distinct parallel lines have how many common points?

(A) No (B) One
(C) Two (D) Three

40 Things which are _____ of the same things are equal to one another.

(A) Parallel (B) Not equal
(C) Halves (D) Triple

Previous Contest Questions

1 If a point R is the mid-point of a line MN, which axiom states that $MR = NR = \frac{MN}{2}$?

(A) Axiom 4 (B) Axiom 6
(C) Axiom 5 (D) Axiom 7

2 If a point A lies in between B and C, which of the following is true?

(A) $BC = \frac{1}{2}AC$ (B) $BC = \frac{1}{2}AB$
(C) $AC = BC$ (D) $AB + AC = BC$

3 Which of the following is true about two distinct lines?

(A) Always intersect.
(B) Always either intersect or are parallel.
(C) Always have two common points.
(D) Always parallel.

4 If C is the mid-point of the line segment AB, D and E are the mid-points of the segments AC and BC respectively, what is the length of AC?

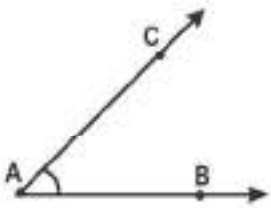
(A) 4 AD (B) 3 AD
(C) 2 AD (D) AD

CHAPTER 6

Lines and Angles

Synopsis

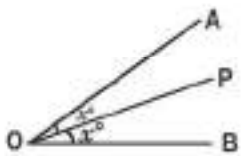
- **Angle:** An angle is the union of two rays with a common initial point. An angle is denoted by symbol $\angle$. It is measured in degrees.



The angle formed by the two rays $\overline{AB}$ and $\overline{AC}$ is $\angle BAC$ or $\angle CAB$.

$\overline{AB}$ and $\overline{AC}$ are called the arms and the common initial point 'A' is called the vertex of the angle.

- **Bisector of an angle:** A line which divides an angle into two equal parts is called the bisector of the angle.

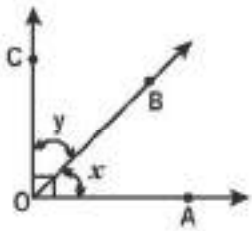


e.g., In the adjacent figure, the line OP divides $\angle AOB$ into two equal parts.

i.e., $\angle AOP = \angle POB = x°$.

So, the line OP is called the bisector of $\angle AOB$.

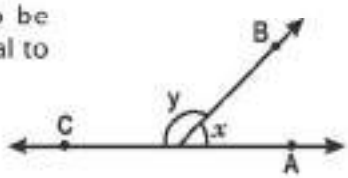
- **Pairs of angles:**



(i) **Complementary angles:** Two angles are said to be complementary if the sum of their measures is equal to $90°$.

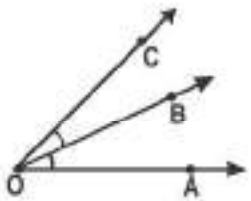
Here $\angle x + \angle y = 90°$, therefore $\angle x$ and $\angle y$ are complementary angles.

(ii) **Supplementary angles:** Two angles are said to be supplementary if the sum of their measures is equal to $180°$.



Here $\angle x + \angle y = 180°$, therefore $\angle x$ and $\angle y$ are supplementary angles.

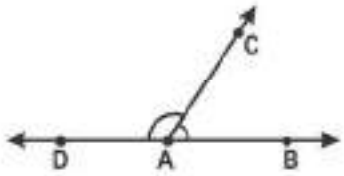
- **Adjacent angles:** Angles having the same vertex and a common arm, and the non-common arms lie on the opposite sides of the common arm are called adjacent angles.



$\angle AOB$ and $\angle COB$ with common vertex O and common arm OB are adjacent angles.

Note: $\angle AOC = \angle AOB + \angle BOC$

- **Linear pair of angles:** Two adjacent angles make a linear pair of angles, if the non-common arms of these angles are two opposite rays (with same end point).



In the adjacent figure, $\angle BAC$ and $\angle CAD$ form a linear pair of angles because the non-common arms AB and AD of the two angles are two opposite rays.

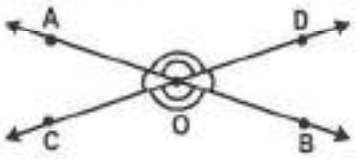
Moreover, $\angle BAC + \angle DAC = 180°$

Note: **(i) Linear pair of angles $\xrightarrow[\text{always}]{\text{are}}$ Adjacent angles.**

Adjacent angles $\xrightarrow[\text{always}]{\text{are not}}$ Linear pair of angles.

(ii) Linear pair of angles are supplementary.

- **Vertically opposite angles:** Two angles having the same vertex are said to form a pair of vertically opposite angles, if their arms form two pairs of opposite rays.



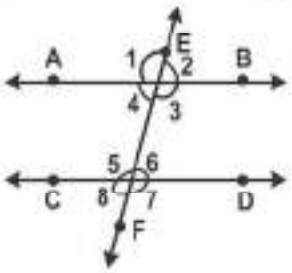
In the above figure, $\angle BOD$ and $\angle AOC$ is a pair of vertically opposite angles because they have a common vertex O and also OB, OA; OC, OD are two pairs of opposite rays.

Similarly, we find that $\angle BOD$ and $\angle AOC$ is a pair of vertically opposite angles because they have common vertex at O and also OB, OA; OC; OD are two pairs of opposite rays.

Note: **If two lines intersect each other, the vertically opposite angles are equal.**

- **Transversal:** A line intersecting two or more distinct lines at distinct points is called a transversal.

In the adjacent figure, transversal EF intersects lines AB and CD at points P and Q respectively.



Here $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are called exterior angles whereas, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are called interior angles.

- **Corresponding angles:** Two angles are said to be a pair of corresponding angles if they are on the same side of the transversal with one angle interior and the other angle exterior and the angles are not adjacent angles. In the above figure, the pairs of corresponding angles are:

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$ (iii) $\angle 3$ and $\angle 7$ and (iv) $\angle 4$ and $\angle 8$

- **Co-interior angles or interior angles on the same side of the transversal:** Two angles are said to be co-interior angles if they are interior angles and lie on the same side of the transversal. In the above figure, the pairs of co-interior angles are:

(i) $\angle 3$ and $\angle 6$ and (ii) $\angle 4$ and $\angle 5$

- **Alternate angles:** Two angles are said to be a pair of alternate angles if the angles are interior angles, which lie on either side of the transversal and are not adjacent angles.

  In the above figure, the pairs of alternate angles are:

  (i) $\angle 4$ and $\angle 6$ and (ii) $\angle 3$ and $\angle 5$

- **Properties of corresponding angles:**

  (i) If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

  (ii) **Converse:** If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.

- **Properties of alternate angles:**

  (i) If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

  (ii) **Converse:** If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

- **Properties of co-interior angles:**

  (i) If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal (i.e., co-interior angles) is supplementary.

  (ii) **Converse:** If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

- Lines parallel to a given line are parallel to each other. In other words, if l, m and n be three lines such that $l \parallel m$ and $l \parallel n$, then $m \parallel n$.

- **Angle sum property of a triangle:**

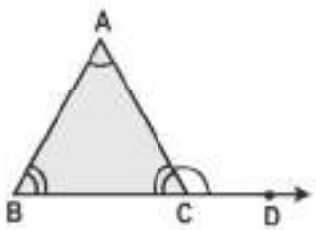
  The sum of the three angles of a triangle is 180°.

  In ΔABC, $\angle A + \angle B + \angle C = 180°$.

- In a right angled triangle, the sum of the two acute angles is 90°.

- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

  In ΔABC, $\angle ACD = \angle BAC + \angle ABC$.



Note : **An exterior angle of a triangle is greater than each of its interior angles. In the above figure, $\angle ACD$ is greater than $\angle BAC$ as well as $\angle ABC$.**

Multiple Choice Questions A B C D

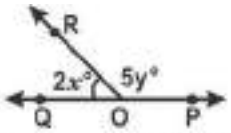
1 The measure of an angle is four times the measure of its supplement. Identify the angles.

(A) 36°, 144° (B) 40°, 160°
(C) 18°, 72° (D) 50°, 200°

2 If two supplementary angles are in the ratio 4 : 5, find the angles.

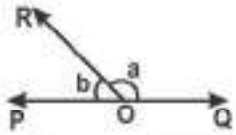
(A) 80°, 100° (B) 85°, 95°
(C) 40°, 50° (D) 60°, 120°

3 From the given figure, find the value of y° when $x° = 30°$.



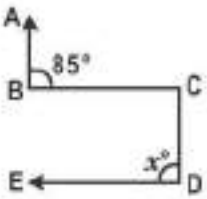
(A) 25° (B) 24°
(C) 36° (D) 45°

4 From the figure given, if ∠POR and ∠QOR form a linear pair and $a - b = 40°$, what are the respective values of a and b?



(A) 110°, 70° (B) 70°, 100°
(C) 80°, 120° (D) 120°, 80°

5 From the figure given, find the value of x, if BC || ED

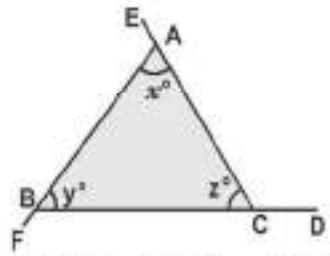


(A) 85° (B) 90°
(C) 95° (D) 80°

6 In ΔABC, $\angle A = \frac{\angle B}{2} = \frac{\angle C}{6}$. Find the measure of ∠A.

(A) 60° (B) 30° (C) 40° (D) 20°

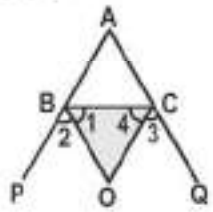
7 The sides BC, CA and AB of ΔABC are produced in order to form exterior angles ∠ACD, ∠BAE and ∠CBF.



Find ∠ACD + ∠BAE + ∠CBF.

(A) 180° (B) 270°
(C) 360° (D) 540°

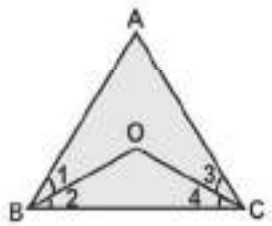
8 In the given figure, BO and CO are angle bisectors of external angles of ΔABC. Find ∠BOC.



(A) $90° - \frac{1}{2}\angle A$ (B) $90° + \frac{1}{2}\angle A$

(C) $180° - \frac{1}{2}\angle A$ (D) $180° + \frac{1}{2}\angle A$

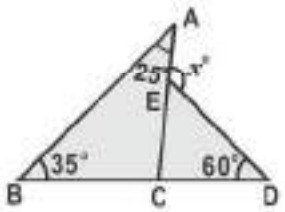
9 In the figure, the bisectors of B and C meet at O. Find the measure of ∠BOC.



(A) $90° + \frac{1}{2}\angle A$ (B) $90° + \frac{1}{2}\angle B$

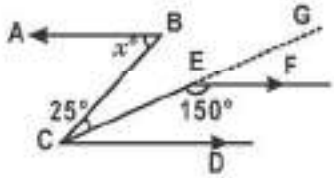
(C) $90° + \frac{1}{2}\angle C$ (D) $90° + \angle A$

10 From the given figure, find the value of x.



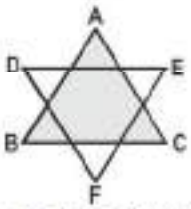
(A) 60° (B) 75° (C) 90° (D) 120°

11 From the figure, find x if AB || CD.



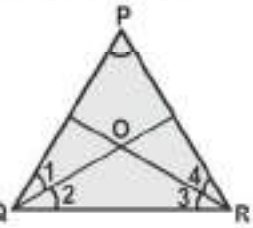
(A) 45° (B) 55° (C) 60° (D) 70°

12 In the given figure, what is the value of $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$?



(A) 360° (B) 270° (C) 540° (D) 180°

13 In Δ PQR, the angle bisectors of $\angle$ PQR and $\angle$ PRQ meet at O.



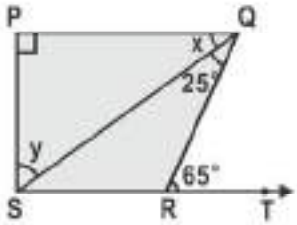
If $\angle QPR = 80°$, find the measure of $\angle QOR$.

(A) 80° (B) 130° (C) 100° (D) 90°

14 If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3, what is the smaller of the two angles?

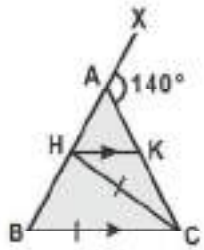
(A) 72° (B) 108° (C) 54° (D) 36°

15 In the given figure, PQ || SR, $\angle SQR = 25°$, $\angle QRT = 65°$, find y.



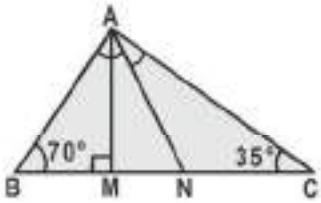
(A) 40° (B) 150° (C) 50° (D) 80°

16 In the figure, AB = AC, CH = CB and HK || BC. If the exterior angle CAX is 140°, find the measure of the angle HCK.



(A) 45° (B) 55° (C) 50° (D) 30°

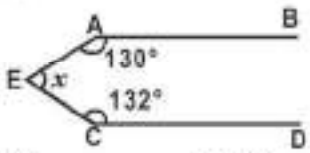
17 In the adjoining figure, AM $\perp$ BC and AN is the bisector of $\angle$ BAC.



If $\angle B = 70°$ and $\angle C = 35°$, find $\angle MAN$.

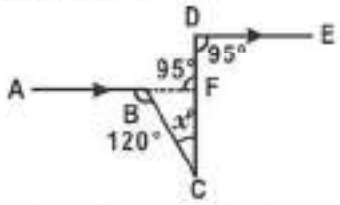
(A) 17.5° (B) 27.5° (C) 37.5° (D) 47.5°

18 In the figure given, what is the value of x if AB || CD?



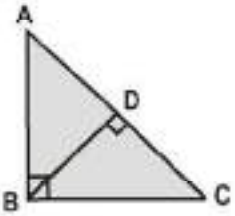
(A) 80° (B) 88°
(C) 90° (D) 98°

19 From the given figure, if AB || DE, what is the value of $x°$?



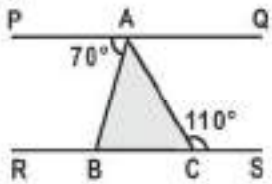
(A) 25° (B) 35° (C) 45° (D) 55°

20 ∠ABC and ∠BDC are right angles. If AD = 9 cm, DC = 16 cm and AB = 15 cm, find length of BD.



(A) 12 cm (B) 16 cm (C) 15 cm (D) 25 cm

21 In the given figure, if PQ || RS, ∠PAB = 70° and ∠ACS = 110°, find the measure of ∠BAC.

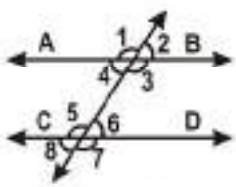


(A) 40° (B) 70° (C) 110° (D) 30°

22 In a right angled triangle, the square of the hypotenuse is equal to twice the product of the other two sides. Which of the following is one of the acute angles of the triangle?

(A) 60° (B) 45° (C) 30° (D) 75°

23 In the given figure, AB || CD and ∠1 : ∠2 = 3 : 2. Find ∠6.

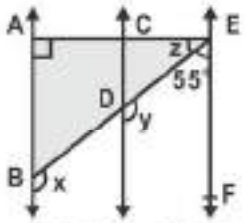


(A) 72° (B) 36° (C) 108° (D) 144°

24 If D is the midpoint of the hypotenuse AC of a right triangle ABC, find the length of BD.

(A) $\frac{1}{2}$AC (B) AC (C) $\frac{3}{2}$AC (D) 2AC

25 In the given figure, AB || CD and CD || EF. Also EA ⊥ AB. If ∠BEF = 55°, find the value of $x - z$.



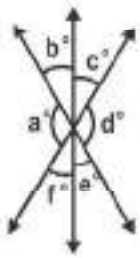
(A) 0° (B) 50° (C) 80° (D) 90°

26 In Δ ABC, if ∠A = 45° and ∠B = 70°, find the shortest and the largest sides of the triangle respectively.

(A) AB, BC (B) BC, AC
(C) AB, AC (D) Either (A) or (C)

27 In the given figure which of the following statements must be true?

(i) a + b = d + c
(ii) a + c + e = 180°
(iii) b + f = c + e



(A) (i) only (B) (ii) only
(C) (iii) only (D) (ii) and (iii) only

28 In a Δ ABC, if 2∠A = 3∠B = 6∠C, find the measures of ∠A, ∠B and ∠C.

(A) 30°, 60°, 90° (B) 90°, 60°, 30°
(C) 30°, 90°, 60° (D) 45°, 90°, 45°

29 A, B and C are the three angles of a triangle. If $A - B = 15°$ and $B - C = 30°$, find $\angle A$, $\angle B$ and $\angle C$.

(A) 80°, 65°, 35° (B) 65°, 80°, 35°
(C) 35°, 80°, 65° (D) 80°, 35°, 65°

30 Consider the following statements.

When two straight lines intersect:
(i) adjacent angles are complementary
(ii) adjacent angles are supplementary
(iii) opposite angles are equal
(iv) opposite angles are supplementary

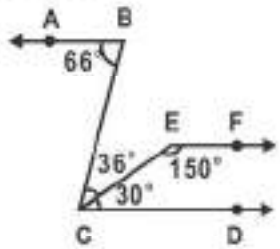
Which of the given statements is correct?

(A) (i) and (iii) only (B) (ii) and (iii) only
(C) (i) and (iv) only (D) (ii) and (iv) only

31 Two straight lines AB and CD intersect one another at point O. If $\angle AOC + \angle COB + \angle BOD = 274°$, find $\angle AOD$.

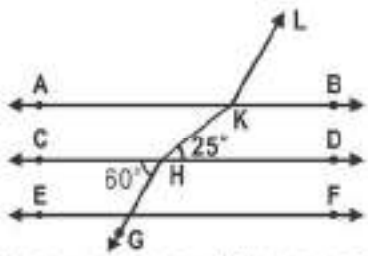
(A) 86° (B) 90° (C) 94° (D) 137°

32 In the given figure, identify the pair of parallel lines.



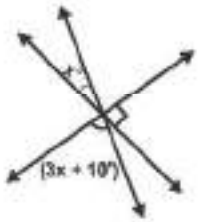
(A) AB || EF (B) BC || CF
(C) EF || BC (D) EF || CE

33 In the given figure, AB || CD || EF and GH || KL. Find the measure of $\angle HKL$.



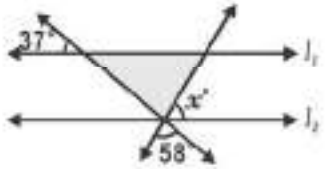
(A) 85° (B) 135° (C) 145° (D) 215°

34 In the given figure, find the value of x.



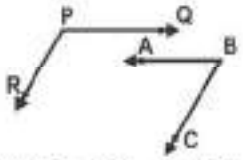
(A) 12° (B) 15° (C) 20° (D) 30°

35 In the given figure if $l_1 \parallel l_2$, what is the value of x?



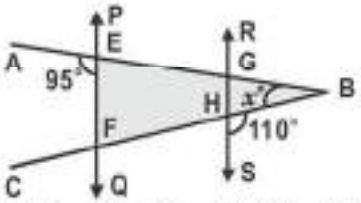
(A) 37° (B) 57° (C) 95° (D) 85°

36 In the given figure, if AB || PQ, PR || BC and $\angle QPR = 102°$, determine $\angle ABC$.



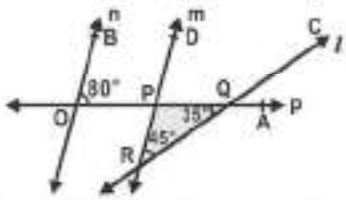
(A) 102° (B) 180° (C) 78° (D) 120°

37 In the given figure, PQ || RS, $\angle AEF = 95°$, $\angle BHS = 110°$, and $\angle ABC = x°$. Find the value of x.



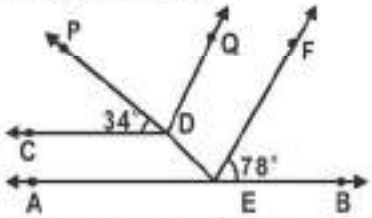
(A) 15° (B) 25° (C) 70° (D) 35°

38 In the given figure, which two lines are parallel?



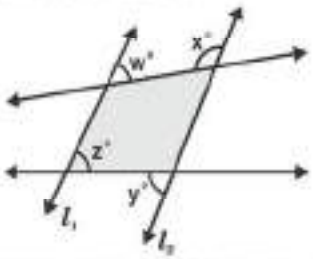
(A) l, m (B) l, n (C) m, n (D) n, p

39 In the given figure, AB || CD and EF || DQ. Determine ∠PDQ.



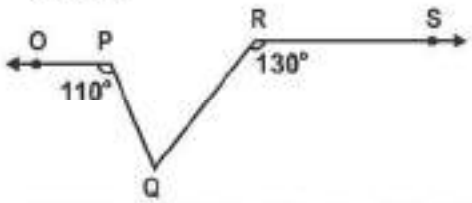
(A) 78° (B) 68° (C) 34° (D) 54°

40 In the given figure if $l_1 \| l_2$, what is $x + y$ in terms of w° and z°?



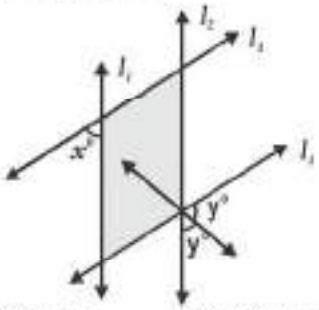
(A) $180° - w° + z°$ (B) $180° + w° - z°$
(C) $180° - w° - z°$ (D) $180° + w° + z°$

41 In the given figure, OP ∥ RS. Determine ∠PQR.



(A) 75° (B) 50° (C) 40° (D) 60°

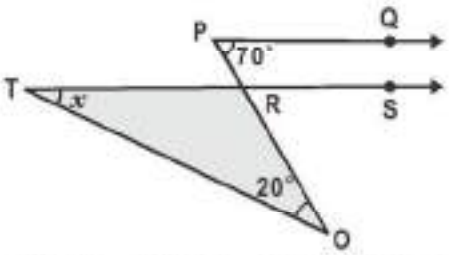
42 In the given figure, if $l_1 \| l_3$ and $l_3 \| l_4$, what is y in terms of x?



(A) $90° + x$ (B) $90° + 2x$

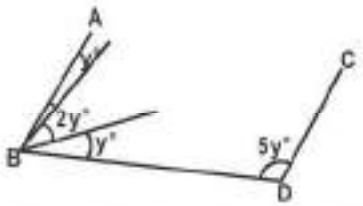
(C) $90° - \frac{x}{2}$ (D) $90° - 2x$

43 In the given figure, PQ || RS, ∠QPR = 70°, ∠ROT = 20°, find the value of x.



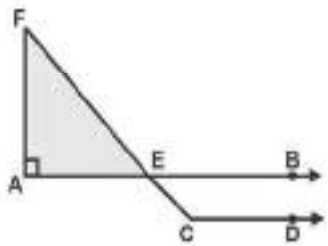
(A) 20° (B) 70° (C) 110° (D) 50°

44 In the given figure, if $\overline{AB} \| \overline{CD}$ is parallel to the line segment CD, what is the value of y?



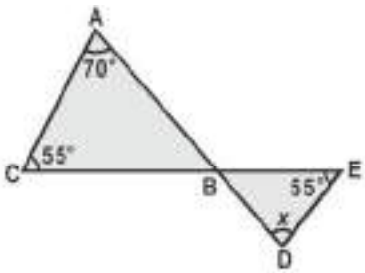
(A) 12 (B) 15 (C) 18 (D) 20

45 In the given figure, AB || CD and ∠F = 30°. Find ∠ECD.



(A) 60° (B) 90° (C) 120° (D) 30°

46 In the given figure, if AC || ED, find the degree measure of x.

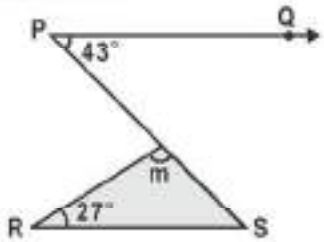


(A) 55° (B) 70° (C) 45° (D) 60°

47 An exterior angle of a triangle is 115°. If one of the interior opposite angles is 35°, what are the other two angles?

(A) 80°, 65° (B) 75°, 65°
(C) 90°, 55° (D) 45°, 100°

48 In the given figure, if PQ || RS, find the measure of m.



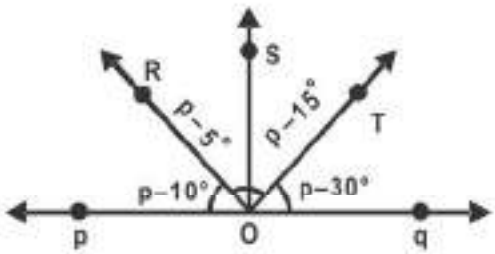
(A) 110° (B) 100° (C) 90° (D) 137°

Previous Contest Questions

1 If OP is a ray standing on a line QR such that ∠POQ = ∠POR, what is the measure of ∠POQ?

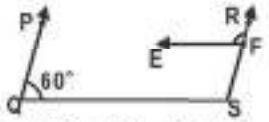
(A) 45° (B) 60° (C) 75° (D) 90°

2 In the figure given, what is the value of ∠p?



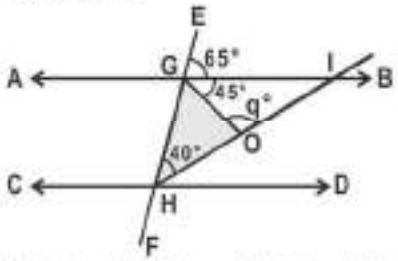
(A) 30° (B) 40° (C) 50° (D) 60°

3 In the given figure, PQ || RS and EF || QS. If ∠Q = 60°, find the measure of ∠RFE.



(A) 115° (B) 120° (C) 60° (D) 180°

4 In the given figure, AB || CD. What is the value of 'q'?

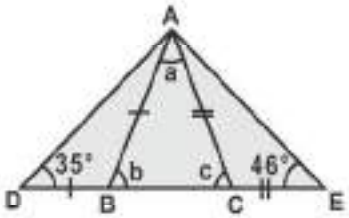


(A) 120° (B) 65° (C) 90° (D) 110°

5 The angles of a triangle are $(x + 10°)$, $(x + 40°)$ and $(2x - 30°)$. What is the value of x?

(A) 30° (B) 40° (C) 20° (D) 10°

6 In the figure given, what are the values of ∠b, ∠c and ∠a respectively?

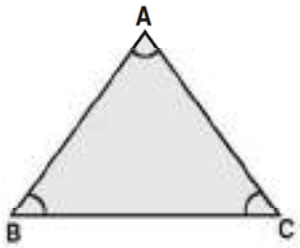


(A) 18°, 70° and 92° (B) 92°, 70° and 18°
(C) 70°, 92° and 18° (D) 70°, 18° and 92°

CHAPTER 7

Triangles

Synopsis



- A triangle is a closed figure bounded by three straight lines. It is denoted by the symbol Δ.

  ΔABC has three sides denoted by AB, BC and CA; three angles denoted by $\angle A$, $\angle B$ and $\angle C$; and three vertices denoted by A, B and C.

- Two geometrical figures are said to be congruent if they have exactly the same shape and size. Congruence is denoted by the symbol $\equiv$.
- Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.
- The congruence of two triangles ABC and PQR under the correspondence $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$ is symbolically expressed as $\Delta ABC \cong \Delta PQR$.
- Two congruent figures are equal in area, but two figures having the same area need not be congruent.
- Congruence relation is an equivalence relation:

  (i) Congruence relation is reflexive.
  $\Delta ABC \cong \Delta ABC$.

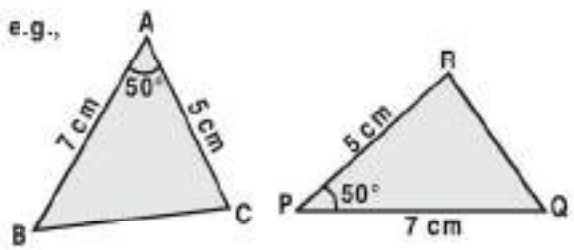
  (ii) Congruence relation is symmetric.
  If $\Delta ABC \equiv \Delta DEF$, then $\Delta DEF \cong \Delta ABC$.

  (iii) Congruence relation is transitive.
  If $\Delta ABC \equiv \Delta DEF$ and $\Delta DEF \equiv \Delta XYZ$ then $\Delta ABC \equiv \Delta XYZ$.

- **Criteria for congruence of triangles:**

  (i) **S.A.S. congruence rule:** Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

  e.g.,

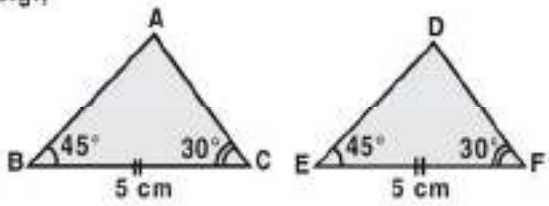


  $\Delta ABC \equiv \Delta PQR$

  since $AB = PQ = 7$ cm, $AC = PR = 5$ cm and $\angle A = \angle P = 50°$.

(ii) **A.S.A. congruence rule:** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.

e.g.,

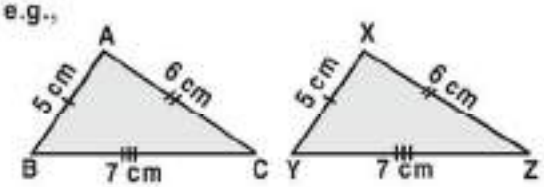


$\Delta ABC \equiv \Delta DEF$

since $\angle B = \angle E = 45°$, $\angle C = \angle F = 30°$ and $BC = EF = 5$cm.

(iii) **S.S.S. congruence rule:** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

e.g.,

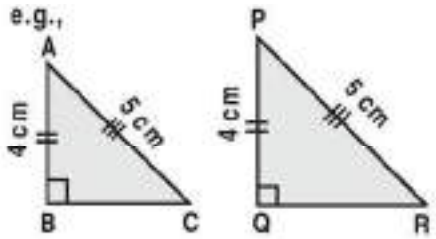


$\Delta ABC \equiv \Delta XYZ$

since $AB = XY = 5$ cm, $BC = YZ = 7$ cm and $CA = ZX = 6$ cm.

(iv) **R.H.S. congruence rule:** If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

e.g.,



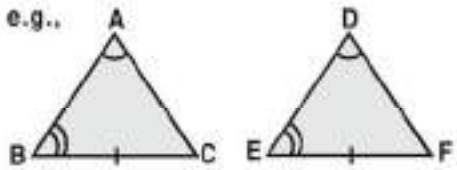
> ***Note:*** **R.H.S. stands for Right angle-Hypotenuse-Side**

$\Delta ABC \equiv \Delta PQR$

since $\angle B = \angle Q = 90°$, $AC = PR = 5$ cm and $AB = PQ = 4$ cm.

(v) **A.A.S. or S.S.A. congruence rule:** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

e.g.,



$\Delta ABC \equiv \Delta DEF$

since $\angle A = \angle D$, $\angle B = \angle E$ and $BC = EF$.

◆ **Inequalities in a triangle:**

(i) If two sides of a triangle are unequal, then the angle opposite to the longer side is greater than that opposite to the shorter side.

> ***Note:*** **In any triangle, the side opposite to the larger angle is longer.**

(ii) In a right triangle, hypotenuse is longer than the other two sides.

(iii) The sum of any two sides of a triangle is greater than the third side.

(iv) An exterior angle of a triangle is greater than either of its interior angles.

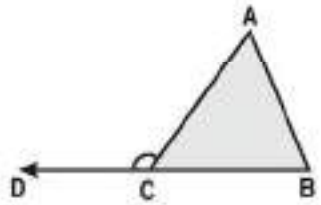
Multiple Choice Questions

A B C D

1 Given 'p' is the exterior angle of ΔPQR and 'q + r' is the sum of interior angles opposite to 'p'. Which of the following is true?

(A) $p + r = q$ (B) $p = q + r$
(C) $p + q = r$ (D) $r = q - p$

2 Identify the interior opposite angles of the exterior angle $\angle ACD$ of ΔABC.

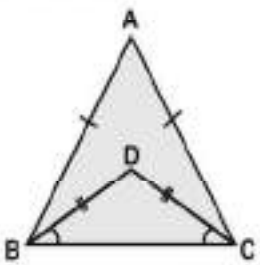


(A) $\angle B, \angle C$ (B) $\angle A, \angle C$
(C) $\angle A, \angle B$ (D) $\angle B, \angle D$

3 A student is asked to draw a triangle whose angles are 90°, 90° and 20°. Which of the following is possible with the given measures?

(A) He can draw a right angled triangle.
(B) He can draw an equilateral triangle.
(C) He cannot draw any triangle.
(D) He can draw an obtuse angled triangle.

4 In the given figure, find the ratio $\angle ABC : \angle ACD$.

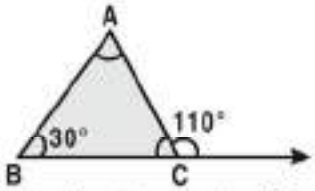


(A) 1 : 1 (B) 2 : 1
(C) 1 : 2 (D) 3 : 1

5 In ΔABC, $AD \perp BC$ and $\angle B = \angle C$ By which property is $\Delta ADB \cong \Delta ADC$?

(A) S.A.S. property (B) S.S.S. property
(C) R.H.S. property (D) A.S.A. property

6 An exterior angle of a triangle is 110° and one of the interior opposite angles is 30°.



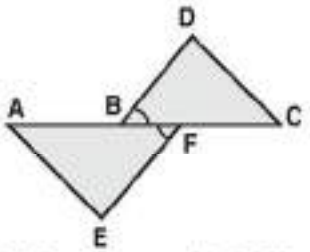
Find the other two angles of the triangle.

(A) 80°, 70° (B) 70°, 40°
(C) 110°, 40° (D) 110°, 75°

7 One of the angles of a triangle is 75°. If the difference of the other two angles is 35°, what is the measure the largest angle of the triangle?

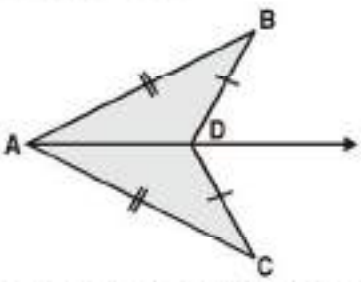
(A) 80° (B) 75°
(C) 100° (D) 100°

8 In the figure, if AB = CF, EF = BD and $\angle AFE = \angle DBC$, by which condition is ΔAFE congruent to ΔCBD?



(A) S.A.S. (B) S.S.S.
(C) A.S.A. (D) R.H.S.

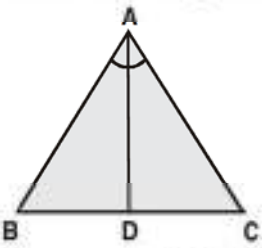
9 In the given figure, $\Delta ABD \cong \Delta ACD$, AB = AC, BD = CD.



By which criterion are the triangles congruent?

(A) S.S.S. (B) R.H.S. (C) S.A.S. (D) A.S.A.

10 In the given figure, AD is the bisector of $\angle A$ and AB = AC. By which criterion are ΔABD and ΔACD congruent?



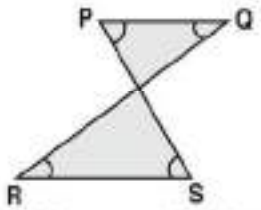
(i) S.S.S. (ii) S.A.S.
(iii) A.S.A. (iv) R.H.S.

(A) (i) only
(B) (ii), (iii) and (iv) only
(C) (i) and (iii)
(D) (ii) and (iv) only

11 In quadrilateral ACBD, AB is a diagonal. If AC = AD and AB bisects $\angle A$, by which congruence property is $\Delta ACB \equiv \Delta ADB$?

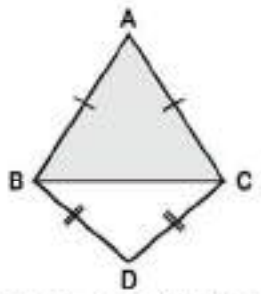
(A) S.A.S. (B) A.S.A.
(C) S.S.S. (D) R.H.S.

12 In the given figure, $\angle Q < \angle P$ and $\angle R < \angle S$. Which of the following is true?



(A) PS < QR (B) PS > QR
(C) QR = PS (D) Either (B) or (C).

13 In the given figure, if AB = AC and BD = DC, by which criterion are Δ ABD and Δ ACD congruent?



(A) S.S.S. (B) A.S.A. (C) S.A.S. (D) R.H.S.

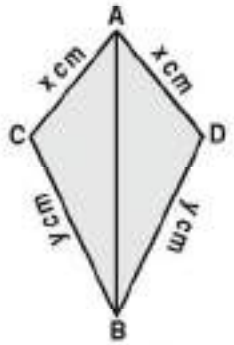
14 In Δ ABC, AB = AC and AD is perpendicular to BC. State the property by which Δ ADB $\equiv$ Δ ADC.

(A) S.A.S. property (B) S.S.S. property
(C) R.H.S. property (D) A.S.A. property

15 Δ ABC is an isosceles triangle with AB = AC. If AD $\perp$ BC, which of the following is true?

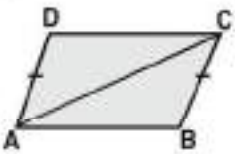
(A) $\angle B > \angle C$ (B) $\angle B = \angle C$
(C) $\angle B < \angle C$ (D) $\angle A = \angle B = \angle C$

16 By which congruence property, are the two triangles in the figure congruent?



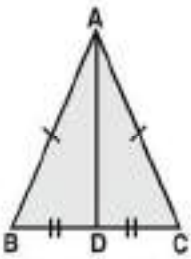
(A) S.A.S. property (B) S.S.S. property
(C) R.H.S. property (D) A.S.A. property

17 In the given figure, if AD = BC and AD $\parallel$ BC, which of the following holds good?



(A) AB = AD (B) AB = DC
(C) BC = CD (D) AB = AC

18 In the given figure, if AB = AC and BD = DC, find the measure of $\angle$ ADC.



(A) 60° (B) 120° (C) 90° (D) 110°

19 If $\Delta ABC \cong \Delta PQR$, which of the following is true?

(A) AB = RP (B) CA = RP
(C) AC = RQ (D) CB = QP

20 Which of the following is incorrect?

(A) If two triangles have their corresponding sides equal, then they are congruent.
(B) If two triangles have their corresponding angles equal, then they are congruent.
(C) If two circles have the same radii, then they are congruent.
(D) If two squares have the same length of their sides, then they are congruent.

21 A : Two triangles are said to be congruent if two sides and an angle of one triangle are respectively equal to the two sides and an angle of the other.

R : Two triangles are congruent if two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other. Which of the following statements is correct?

(A) A is false and R is the correct explanation of A.
(B) A is true and R is the correct explanation of A.
(C) A is true and R is false.
(D) A is false and R is false.

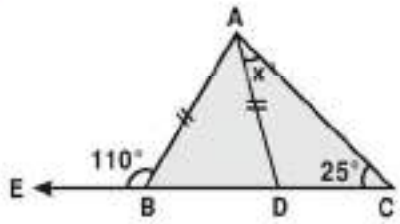
22 Which of the following statements is true?

(A) Two line segments having the same length are congruent.
(B) Two squares having the same side length are congruent.
(C) Two circles having the same radius are congruent.
(D) All the above.

23 Which of the following is true about the difference of any two sides of a triangle?

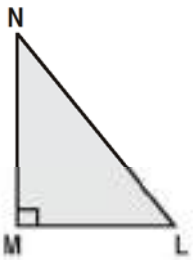
(A) It is greater than the third side.
(B) It is zero.
(C) It is lesser than the third side
(D) It is lesser than zero.

24 In the figure given, find $\angle x$.



(A) 40° (B) 110°
(C) 45° (D) 65°

25 In ΔLMN, if $\angle M = 90°$, which of the following is true?



(A) NL is the longest side.
(B) LM is the longest side.
(C) MN is the longest side.
(D) NL is the shortest side.

26 Which of the following statement(s) is/are false?

(A) Two triangles having the same area are congruent.
(B) If two sides and one angle of a triangle are equal to the corresponding two sides and the angle of another triangle, then the two triangles are congruent.
(C) If the hypotenuse of one right triangle is equal to the hypotenuse of another triangle, then the triangles are congruent.
(D) All the above.

27 It is not possible to construct a triangle with which of the following sides?

(A) 8.3 cm, 3.4 cm, 6.1 cm
(B) 5.4 cm, 2.3 cm, 3.1 cm
(C) 6 cm, 7 cm, 10 cm
(D) 3 cm, 5 cm, 5 cm

28 In ΔABC, if $AB = BC$, which of the following statements is necessarily true?

(A) $\angle B = \angle C$ (B) $\angle A = \angle C$
(C) $\angle A = \angle B$ (D) $\angle A < \angle C$

29 If $\Delta ABC \cong \Delta DEF$ by S.A.S. congruence rule, which of the following could be the equivalent measures?

(A) $AB = DE, BC = EF, \angle B = \angle E$
(B) $AB = DE, BC = EF, \angle D = \angle A$
(C) $AB = DE, BC = EF, \angle C = \angle F$
(D) $AB = DE, BC = EF, CA = FD$

30 In ΔABC, $AB = 5$ cm, $BC = 6$ cm and $CA = 7$ cm. Identify the relation between the angles.

(A) $\angle B > \angle A > \angle C$
(B) $\angle A > \angle B > \angle C$
(C) $\angle B > \angle C > \angle A$
(D) $\angle B > \angle A > \angle C$

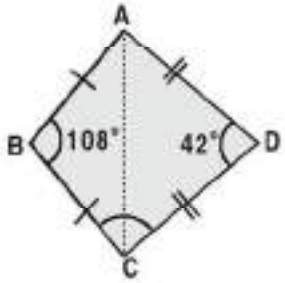
31 In triangles ABC and DEF, $AB = FD$ and $\angle A = \angle D$. Which of the following is true if the two triangles are congruent by S.A.S. axiom?

(A) $BC = EF$ (B) $AC = DE$
(C) $AC = EF$ (D) $BC = DE$

32 If $\Delta ABC \cong \Delta DEF$ by S.S.S. congruence rule, which of the following options holds true?

(A) $\angle A > \angle D$
(B) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$
(C) $\angle B < \angle E$
(D) $\angle C = \angle F$

33 In the given figure, ABCD is a quadrilateral in which $AB = BC$ and $AD = DC$.



Find the measure of $\angle BCD$.

(A) 150° (B) 30° (C) 105° (D) 72°

34 If $\Delta ABC \cong \Delta DEF$ by S.S.S. congruence rule, which of the following statements is true?

(A) $AB = EF, BC = FD, CA = DE$
(B) $AB = FD, BC = DE, CA = EF$
(C) $AB = DE, BC = EF, CA = FD$
(D) $AB = DE, BC = EF, \angle C = \angle F$

35 In ΔABC, if $\angle A = 50°$ and $\angle B = 60°$, what is the greatest side?

(A) AB
(B) BC
(C) AC
(D) Cant be determined

36 In ΔABD, if $\angle B = \angle C$, which of the following is true?

(A) $BC = AB$ (B) $AC = BC$
(C) $AB = AC$ (D) $AB > AC$

37 In ΔABC, $\angle A = 50°$, $\angle B = 60°$. What is the ascending order of the sides of the triangle?

(A) $AB < BC < CA$ (B) $CA < AB < BC$
(C) $BC < CA < AB$ (D) $BC < AB < CA$

38 In ΔDEF, which of the following statements is true?

(A) $DE + EF = FD$ (B) $DE + EF < FD$
(C) $DE > EF + FD$ (D) $DE + EF > FD$

39 $\Delta ABC \cong \Delta PQR$, if $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$, which of the following is true?

(A) $QP = 5$ cm, $\angle P = 60°$
(B) $QP = 5$ cm, $\angle R = 60°$
(C) $QP = 5$ cm, $\angle R = 60°$
(D) $QP = 5$ cm, $\angle Q = 40°$

40 In ΔABC, if AD is the perpendicular bisector of BC, which of the following is true?

(A) $AB > AC$ (B) $AD = AB$
(C) $AB = AC$ (D) $AD = AC$

41 In ΔABC, if AD is a median, what is the relation between the sides?

(A) $AB + AC > AD$ (B) $AB + AC < 2AD$
(C) $AB + AC = 2AD$ (D) $AB + AC > 2AD$

42 Which of the following statements is correct?

(A) A triangle can have two right angles.
(B) A triangle can have two obtuse angles.
(C) A triangle can have all angles more than 60°.
(D) A triangle can have two acute angles.

43 An exterior angle of a triangle is 120° and the two interior opposite angles are equal. Find the value of each interior angle.

(A) 60° (B) 100° (C) 120° (D) 180°

44 In a right triangle, the two acute angles are in the ratio 4 : 5. Find them respectively.

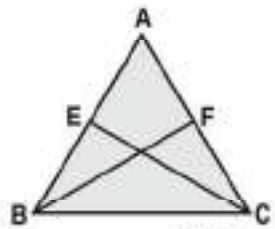
(A) 50°, 40° (B) 40°, 50°
(C) 45°, 55° (D) 30°, 60°

45 In Δ ABC, if AB > BC, which of the following is true?

(A) $\angle C < \angle A$ (B) $\angle C = \angle A$
(C) $\angle C > \angle A$ (D) $\angle A = \angle B$

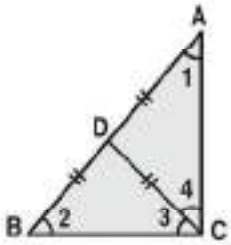
Previous Contest Questions

1 In the figure given, AB = AC, E is midpoint of AB and F is midpoint of AC. What is the length of BF?



(A) AE (B) CF
(C) BE (D) CE

2 In the figure given, if DA = DB = DC, find ∠ACB.



(A) 70° (B) 90° (C) 100° (D) 120°

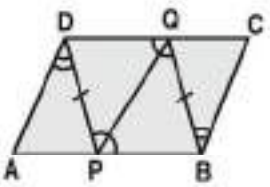
3 ΔABC is right angled at A. AB = 60 units, AC = 80 units, and BC = 100 units. D is a point between B and C such that triangles ADB and ADC have equal perimeters. What is the length of BD?

(A) 10 units (B) 20 units
(C) 40 units (D) 60 units

4 If the length of the largest side of a triangle is 12 cm, find the possible values of its other two sides.

(A) 4.8, 8.2 (B) 3.2, 6.5
(C) 2.9, 7.2 (D) 4.1, 3.9

5 In the figure given, DP = BQ, and ∠ADP = ∠CBQ. To which triangle is ∠ADP congruent to?



(A) ΔCBQ (B) ΔPDQ
(C) ΔQBC (D) ΔPQB

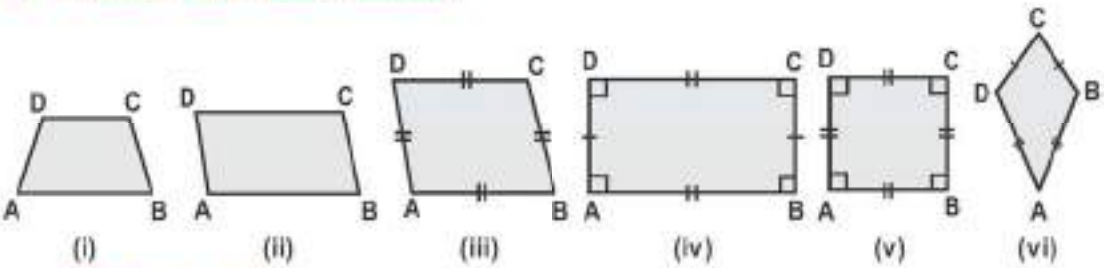
This page is intentionally left blank.

CHAPTER 8

Quadrilaterals

Synopsis

- A quadrilateral in which the measure of each angle is less than 180° is called a convex quadrilateral.
- A quadrilateral in which the measure of at least one of the angles is more than 180° is known as a concave quadrilateral.
- The sum of the angles of a quadrilateral is 360° (or) 4 right angles.
- When the sides of a quadrilateral are produced, the sum of the four exterior angles so formed is 360°.
- **Various types of quadrilaterals:**



(i) Trapezium:

(a) A quadrilateral having exactly one pair of parallel sides is called a trapezium.

(b) A trapezium is said to be an isosceles trapezium if its non-parallel sides are equal.

ABCD is a trapezium in which AB || DC.

This trapezium is said to be an isosceles trapezium if AB || DC and AD = BC.

(ii) Parallelogram: A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram.

ABCD is a parallelogram in which AB || DC and AD || BC.

Properties:

(a) In a parallelogram, any two opposite sides are equal.

(b) In a parallelogram, any two opposite angles are equal.

(c) In a parallelogram, the diagonals bisect each other.

(d) In a parallelogram, each diagonal divides it into two congruent triangles.

(e) In a parallelogram, any two adjacent angles have their sum equal to 180° i.e., the adjacent angles are supplementary.

(iii) **Rhombus:** A quadrilateral having all sides equal is called a rhombus.

ABCD is a rhombus in which AB || DC, AD || BC and AB = BC = CD = DA.

Properties:

(a) The diagonals of a rhombus bisect each other at right angles.

(b) Each diagonal of a rhombus divides it into two congruent triangles.

(c) Opposite angles of a rhombus are equal and the sum of any two adjacent angles is 180°.

(d) The opposite sides of a rhombus are parallel.

(e) All the sides of a rhombus are equal.

(iv) **Rectangle:** A parallelogram whose angles are all right angles is called a rectangle.

ABCD is a rectangle in which,

AD||BC and AB||CD and

$\angle A = \angle B = \angle C = \angle D = 90°$

Properties:

(a) Opposite sides of a rectangle are equal and opposite angles of a rectangle are equal.

(b) The diagonals of a rectangle bisect each other.

(c) Each diagonal divides the rectangle into two congruent triangles.

(d) The diagonals of a rectangle are equal.

(v) **Square:** A parallelogram having all sides equal and each angle equal to a right angle is called a square.

ABCD is a square in which AB || DC, AD || BC,

AB = BC = CD = DA and $\angle A = \angle B = \angle C = \angle D = 90°$.

Properties:

(a) All sides are equal.

(b) All angles are equal.

(c) The diagonals are equal and bisect each other at right angles.

(d) Each diagonal divides the square into two congruent right angled isosceles triangles.

(vi) **Kite:** A quadrilateral having two pairs of equal adjacent sides but unequal opposite sides is called a kite.

ABCD is a kite in which AB = AD and BC = CD.

◆ **Conditions for a quadrilateral to become a parallelogram:**

(a) Both pairs of opposite sides should be equal (or)

(b) One pair of opposite sides should be equal and parallel (or)

(c) Both pairs of opposite angles should be equal (or)

(d) Its diagonals should bisect each other.

- **Conditions for a quadrilateral to become a rectangle:**

(a) All its angles should be right angles (or)

(b) Its diagonals should be equal and bisect each other (or)

(c) Both pairs of opposite sides should be equal and one angle must be 90° (or)

(d) Both pairs of opposite sides and its diagonals should be equal.

- **Conditions for a quadrilateral to become a rhombus:**

(a) All its sides should be equal (or)

(b) Its diagonals should bisect each other at right angles (or)

(c) Both pairs of its opposite sides should be equal and the diagonals should intersect at right angles.

- **Conditions for a quadrilateral to become a square:**

(a) All its sides should be equal and one angle must be 90° (or)

(b) All its sides and the diagonals should be equal (or)

(c) All its angles should be equal and the diagonals should intersect at right angles.

- **Number of measurements required to construct different geometrical figures:**

(i) To construct a quadrilateral, 5 independent measurements are needed.

(ii) To construct a trapezium, 4 independent measurements are required.

(iii) To construct a parallelogram, 3 independent measurements are required.

(iv) To construct a rhombus, 2 independent measurements are required.

(v) To construct a rectangle, 2 independent measurements are required.

(vi) To construct a square, 1 measurement is required.

- **Midpoint theorem for a triangle:**

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it.

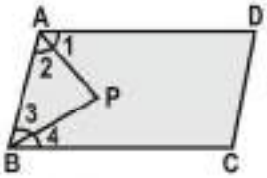
Converse: The line drawn through the midpoint of one side of a triangle parallel to another side of the triangle, bisects the third side of the triangle.

- The quadrilateral formed by joining the midpoints of the sides of a quadrilateral, in order, is a parallelogram.

Multiple Choice Questions

A B C D

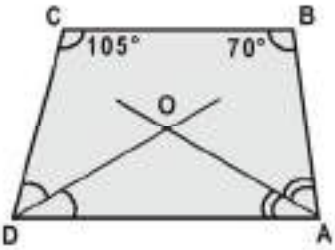
1 In the parallelogram ABCD, AP and BP are bisectors of $\angle A$ and $\angle B$ which meet at P.



What is $2\angle APB$ equivalent to?

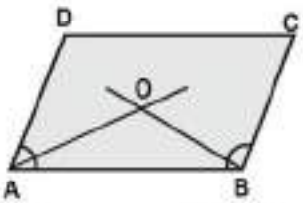
(A) $\angle A + \angle B$ (B) $\angle A + \angle C$
(C) $\angle B + \angle D$ (D) $\angle A - \angle D$

2 In the given figure, AO and DO are the bisectors of $\angle A$ and $\angle D$ of the quadrilateral ABCD. Find $\angle AOD$.



(A) 67.5° (B) 77.5°
(C) 87.5° (D) 99.75°

3 In a parallelogram find the sum of the bisected angles of two adjacent angles.

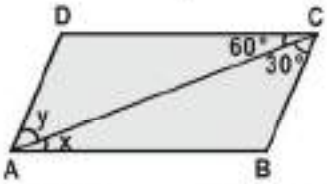


(A) 30° (B) 45°
(C) 60° (D) 90°

4 The diameter of the circumcircle is the diagonal of a rectangle which is 10 cm and breadth of the rectangle is 6 cm. Find its length.

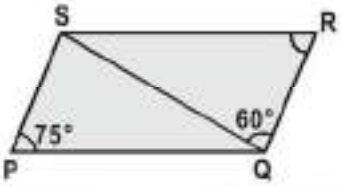
(A) 6 cm (B) 5 cm
(C) 8 cm (D) 7 cm

5 In the parallelogram ABCD, what are the measures of the angles x and y?



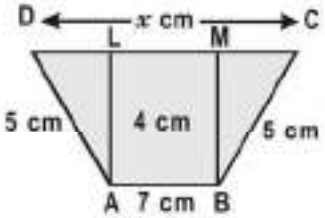
(A) 60°, 30° (B) 30°, 60°
(C) 45°, 45° (D) 90°, 90°

6 In parallelogram PQRS, what are the values of $\angle QSP$ and $\angle SPQ$?



(A) 45°, 60° (B) 60°, 45°
(C) 70°, 35° (D) 35°, 70°

7 In the given figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm and the distance between AB and DC is 4 cm. What is the value of x?



(A) 13 cm (B) 16 cm
(C) 19 cm (D) 12 cm

8 In a square ABCD, the diagonals bisect at O. What type of a triangle is ΔAOB?

(A) An equilateral triangle.
(B) An isosceles but not a right angled triangle.
(C) A right angled but not an isosceles triangle.
(D) An isosceles right angled triangle.

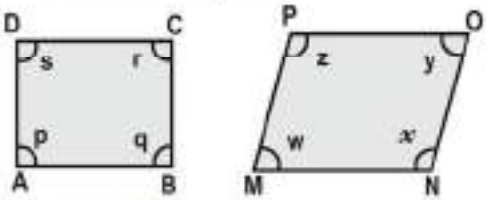
9 ABCD is a quadrilateral. If AC and BD bisect each other, what is ABCD?

(A) A square (B) A rectangle
(C) A parallelogram (D) A rhombus

10 In a parallelogram ABCD, $\angle D = 60°$. Find the measure of $\angle A$.

(A) 120° (B) 65°
(C) 90° (D) 75°

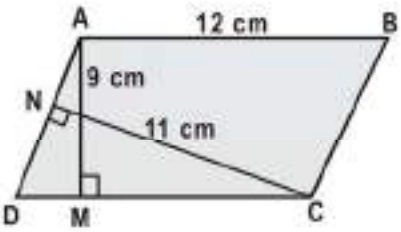
11 ABCD and MNOP are quadrilaterals as shown in the figure.



Which of these is correct?

(A) $p + q + r + s = w + x + y + z$
(B) $p + q + r + s < w + x + y + z$
(C) $p + q + r + s > w + x + y + z$
(D) $p + q + x + y = r + s + w + z$

12 In parallelogram ABCD, AB = 12 cm. The altitudes corresponding to the sides AB and AD are respectively 9 cm and 11 cm. Find AD.



(A) $\frac{108}{11}$ cm (B) $\frac{108}{10}$ cm

(C) $\frac{99}{10}$ cm (D) $\frac{108}{17}$ cm

13 In quadrilateral ABCD, if the diagonals AC and BD are equal and perpendicular to each other, what is ABCD?

(A) A square (B) A parallelogram
(C) A rhombus (D) A trapezium

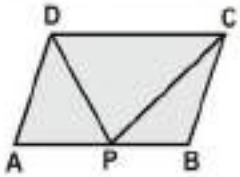
14 One of the diagonals of a rhombus is equal to its side. Find the angles of the rhombus.

(A) 60° and 80° (B) 60° and 120°
(C) 120° and 240° (D) 100° and 120°

15 The perimeter of a parallelogram is 180 cm. One side exceeds another by 10 cm. What are the sides of the parallelogram?

(A) 40 cm and 50 cm
(B) 45 cm each
(C) 50 cm each and 60 cm
(D) Cannot be determined.

16 ABCD is a parallelogram as shown in the figure. If AB = 2AD and P is the mid-point of AB, find the measure of $\angle CPD$.



(A) 90° (B) 60°
(C) 45° (D) 135°

17 In a parallelogram ABCD, if $AB = 2x + 5$, $CD = y + 1$, $AD = y + 5$ and $BC = 3x - 4$, what is the ratio of AB and BC?

(A) 71 : 21 (B) 12 : 11
(C) 31 : 35 (D) 4 : 7

18 When is the quadrilateral formed by joining the midpoints of the sides of a quadrilateral PQRS, taken in order, a rectangle?

(A) PQRS is a rectangle.
(B) PQRS is a parallelogram.
(C) Diagonals of PQRS are perpendicular.
(D) Diagonals of PQRS are equal.

19 The diagonals of a parallelogram ABCD intersect at O. If $\angle BOC = 90°$ and $\angle BDC = 50°$, find $\angle OAB$.

(A) 10° (B) 40°
(C) 50° (D) 90°

20 If ABCD is an isosceles trapezium, what is $\angle C$ equal to?

(A) $\angle B$ (B) $\angle A$
(C) $\angle D$ (D) $90°$

21 When is the quadrilateral formed by joining the midpoints of the sides of a quadrilateral PQRS, taken in order, a rhombus?

(A) PQRS is a rhombus.
(B) PQRS is a parallelogram.
(C) Diagonals of PQRS are per-pendicular.
(D) Diagonals of PQRS are equal.

22 If angles P, Q, R and S of the quadrilateral PQRS, taken in order, are in the ratio 3 : 7 : 6 : 4, what is PQRS?

(A) A rhombus (B) A parallelogram
(C) A trapezium (D) A kite

23 If PQ and RS are two perpendicular diameters of a circle, what is PRQS?

(A) A rectangle
(B) A trapezium
(C) A square
(D) A rhombus but not a square

24 What is the number of measurements required to construct a quadrilateral?

(A) 5 (B) 4
(C) 3 (D) 2

25 If the lengths of two diagonals of a rhombus are 12 cm and 16 cm, find the length of each side of the rhombus.

(A) 10 cm (B) 14 cm
(C) 16 cm (D) 8 cm

26 In a rhombus ABCD, $\angle A = 60°$ and AB = 6 cm. Find the diagonal BD.

(A) $2\sqrt{3}$ cm (B) 6 cm
(C) 12 cm (D) Insufficient data

27 In a parallelogram ABCD, AB = 4 cm and BC = 7 cm. Each of its diagonals is less than which of the following?

(A) 3 cm (B) 4 cm
(C) 7 cm (D) 11 cm

28 Which of the following is not true for a parallelogram?

(A) Opposite sides are equal.
(B) Opposite angles are equal.
(C) Opposite angles are bisected by the diagonals.
(D) Diagonals bisect each other.

29 APB and CQD are parallel lines and a transversal PQ cuts them at P and Q. Which of the following is formed by the bisectors of angles APQ, BPQ, CQP and PQD ?

(A) A rectangle
(B) A rhombus
(C) A square
(D) Any other parallelogram

30 PQRS is a quadrilateral. PR and QS intersect each other at O. In which of the following cases is PQRS a parallelogram?

(A) $\angle P = 100°, \angle Q = 80°, \angle R = 100°$
(B) $\angle P = 85°, \angle Q = 85°, \angle R = 95°$
(C) PQ = 7 cm, QR = 7 cm, RS = 8 cm, OS = 5.2 cm
(D) OP = 6.5 cm, OQ = 6.5 cm, OR = 5.2 cm, OS = 5.2 cm

31 In a quadrilateral, the angles, A, B, C and D are in the ratio 1 : 2 : 3 : 4. What is the measure of the largest angle of the quadrilateral?

(A) 135° (B) 144°
(C) 154° (D) 108°

32 ABCD is a rhombus such that one of its diagonals is equal to its side. Find the measure of the angles of rhombus ABCD.

(A) 45°, 135°, 45°, 135°
(B) 100°, 80°, 100°, 80°
(C) 120°, 60°, 120°, 60°
(D) 60°, 60°, 60°, 60°

33 Which of the following is not a property of a rhombus?

(A) All four sides are equal.
(B) Diagonals bisect each other.
(C) Diagonals bisect opposite angles.
(D) One angle between the diagonals is 60°.

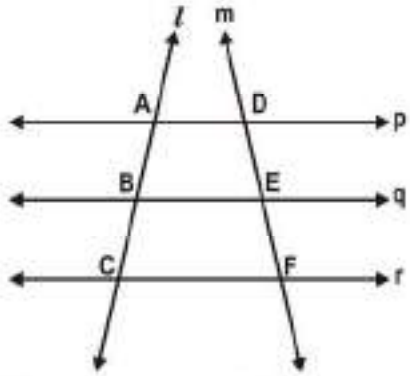
34 All the angles of a convex quadrilateral are congruent. However, not all sides are congruent. What type of a quadrilateral is it?

(A) Parallelogram (B) Square
(C) Rectangle (D) Trapezium

35 In quadrilateral ABCD, $\angle A = 38°$, $\angle C = 3\ \angle A$ and $\angle D = 4\ \angle A$. Find the value of $\angle B$.

(A) 56° (B) 304°
(C) 52° (D) 114°

36 In the given figure, p, q and r are three parallel lines. l and m are two transversals, such that DE = 1.5 cm, EF = 3 cm. Find BC if AB = 2.5 cm.



(A) 4 cm (B) 5.5 cm
(C) 5 cm (D) 6 cm

37 ABCD is a rectangle with $\angle ABD = 40°$. Find $\angle DBC$.

(A) 45° (B) 30°
(C) 50° (D) 35°

38 Find the measure of each angle of a parallelogram, if one of its angles is 30° less than twice the smallest angle.

(A) 70°, 110°, 70°, 110°
(B) 70°, 110°, 80°, 100°
(C) 120°, 60°, 120°, 60°
(D) 100°, 80°, 100°, 80°

39 Which of the following pairs of angles are opposite angles of a cyclic quadrilateral?

(A) 131°, 28° (B) 95°, 55°
(C) 123°, 57° (D) 64°, 52°

40 Two adjacent angles of rhombus are $3x - 40°$ and $2x + 20°$. Find the measurement of the greater angle.

(A) 160° (B) 100°
(C) 80° (D) 120°

41 If PQRS is a parallelogram, find the value of $\angle Q - \angle S$.

(A) 90° (B) 120°
(C) 180° (D) 0°

42 If the degree measures of the angles of a quadrilateral are $4x$, $7x$, $9x$ and $10x$, what is the sum of the measures of the smallest angle and the largest angle?

(A) 140° (B) 150°
(C) 168° (D) 180°

43 In a quadrilateral ABCD, diagonals bisect each other at right angles. If AB = BC = AD = 6 cm, find the length of CD.

(A) 3 cm (B) 6 cm
(C) $6\sqrt{2}$ cm (D) 12 cm

44 In a ΔABC, $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of ΔABC.

(A) 40°, 60°, 80° (B) 35°, 65°, 80°
(C) 40°, 70°, 70° (D) 50°, 60°, 70°

45 In a quadrilateral three angles are in the ratio 3 : 3 : 1 and the fourth angle is 80°. Find the measure of the other angles.

(A) 120°, 120°, 40° (B) 100°, 100°, 80°
(C) 110°, 110°, 60° (D) 90°, 90°, 30°

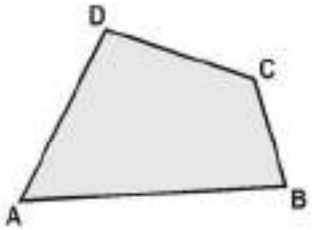
46 JUMP is a square with $\angle JUP = 45°$. Find $\angle PUM$.

(A) 45° (B) 90° (C) 35° (D) 15°

47 In ΔDEF, PQ is a line segment drawn through mid-points of DE and DF respectively. If EF = 7.6 cm, find the length of PQ.

(A) 3.8 cm (B) 4.5 cm
(C) 7.2 cm (D) 2.6 cm

48 In quadrilateral ABCD, $\angle B = 90°$, $\angle C - \angle D = 60°$ and $\angle A - \angle C - \angle D = 10°$. Find $\angle A$, $\angle C$ and $\angle D$.



(A) 140°, 95°, 35° (B) 95°, 70°, 65°
(C) 25°, 45°, 105° (D) 150°, 20°, 35°

49 Two angles of a quadrilateral are 60° are 70° and other two angles are in the ratio of 8 : 15. Which of the following are the remaining two angles?

(A) 80°, 150° (B) 90°, 140°
(C) 100°, 130° (D) 110°, 120°

50 In quadrilateral ABCD. $\angle A + \angle C = 140°$, $\angle A : \angle C = 1 : 3$ and $\angle B : \angle D = 5 : 6$. Find the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

(A) 35°, 100°, 105°, 120°
(B) 60°, 45°, 75°, 100°
(C) 20°, 60°, 105°, 135°
(D) 45°, 35°, 90°, 110°

Previous Contest Questions

1 If an angle of a parallelogram is four-fifths of its adjacent angle, find the angles of the parallelogram.

(A) 60°, 120°, 60°, 120°
(B) 90°, 90°, 90°, 90°
(C) 80°, 100°, 80°, 100°
(D) 30°, 150°, 30°, 100°

2 ABCD is a rhombus in which AC = 16 cm and BC = 10 cm. Find the length of the diagonal BD.

(A) 16 cm (B) 8 cm
(C) 12 cm (D) 10 cm

3 ABCD is a parallelogram. The angle bisectors of $\angle A$ and $\angle D$ intersect at O. Find the measure of $\angle AOD$.

(A) 75° (B) 45°
(C) 90° (D) 180°

4 In quadrilateral PQRS, $\angle P + \angle R = 140°$, and $\angle Q : \angle S = 5 : 6$. Find $\angle Q$.

(A) 100° (B) 120°
(C) 105° (D) 35°

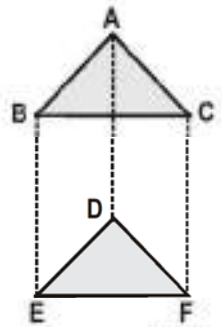
5 E and F are the midpoints of the sides AB and AC respectively of Δ ABC; G and H are midpoints of the sides AE and AF respectively of Δ AEF. If GH = 1.8 cm, find BC.

(A) 0.9 cm (B) 3.6 cm
(C) 7.2 cm (D) 5.4 cm

6 Two opposite angles of a parallelogram are $(3p - 4)°$ and $(48 - p)°$. What is the value of p?

(A) 13 (B) 15
(C) 17 (D) 20

7 In the given figure, Δ ABC and Δ DEF are such that AB = DE, BC = EF, AB // DE and BC // EF. What is ADFC?



(A) A square (B) A rectangle
(C) A parallelogram (D) A rhombus

CHAPTER 9

Areas of Parallelograms and Triangles

Synopsis

- A polygonal region is the union of a polygon and its interior. For e.g., the union of a triangle and its interior is called the triangular region.
- Every polygonal region has area. Area of a figure is a number associated with the part of the plane enclosed by that figure.
- Two congruent figures have equal areas but figures with equal areas need not be congruent.
- If ABCD is a rectangle with AB = l m and BC = b m, then the area of the rectangular region ABCD is lb sq. m or lb m^2.
- If A and B are two regions having at the most a line segment common between them two, then the area of their combined region S is equal to the sum of their areas taken separately. i.e., ar(S) = ar(A) + ar(B).
- The area of a parallelogram is the product of any of its sides and the corresponding altitude.
- Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices, (or the vertex) opposite to the common base of each figure lie on the same line parallel to the base.
- A diagonal of a parallelogram divides it into two triangles of equal areas.
- Parallelograms on the same (or equal) base and between the same parallel lines are equal in area.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
- Parallelograms on equal bases and between the same parallel lines are equal in area.
- Area of a triangle is half the product of any of its sides and the corresponding altitude.
- Triangles on the same base and between the same parallel lines are equal in area.
- Two triangles having the same base (or equal bases) and equal areas lie between the same parallel lines.
- If a triangle and a parallelogram are on the same base and between the same parallel lines, then the area of the triangle is equal to half that of the parallelogram.
- The area of a trapezium is half the product of its height and the sum of the parallel sides.
- Triangles with equal areas and having one side of one triangle, equal to one side of the other, have their corresponding altitudes equal.
- A rectangle and a parallelogram on the same base and between the same parallels have the same area.
- A median of a triangle divides it into two triangles of equal area.

Multiple Choice Questions

A B C D

1. Two parallelograms are on the same base and between the same parallels. What is the ratio of their areas?

(A) 2 : 1 (B) 1 : 2
(C) 1 : 1 (D) 3 : 1

2. ABCD is a parallelogram and 'O' is the point of intersection of its diagonals $\overline{AC}$ and $\overline{BD}$. If the area of $\Delta AOD = 8\ cm^2$, what is the area of the ΔBOC?

(A) $2\ cm^2$ (B) $4\ cm^2$
(C) $8\ cm^2$ (D) $32\ cm^2$

3. The area of a trapezium is $24\ cm^2$. The distance between its parallel sides is 4 cm. If one of the parallel sides is 7 cm, what is the measure of the other parallel side?

(A) 5 cm (B) 8 cm
(C) 12 cm (D) 7 cm

4. The area of a square is $36\ cm^2$. If the side of the square is doubled, what is the ratio of area of the original square to that of the new square formed?

(A) 4 : 1 (B) 4 : 3
(C) 1 : 4 (D) 1 : 2

5. The area of a rhombus is $60\ cm^2$. If one of its diagonals is 15 cm, find the other diagonal.

(A) 4 cm (B) 8 cm
(C) 10 cm (D) 16 cm

6. The area of a triangle is $16\ cm^2$. If its base is 8 cm, what is its corresponding altitude?

(A) 4 cm (B) 8 cm
(C) 18 cm (D) 12 cm

7. One of the diagonals of a quadrilateral is 16 cm. The perpendiculars drawn to it from its opposite vertices are 2.6 cm and 1.4 cm. Find its area.

(A) $32\ cm^2$ (B) $40\ cm^2$
(C) $26\ cm^2$ (D) $28\ cm^2$

8. The ratio of the bases of two triangles is a : b. If the ratio of their corresponding altitudes is c : d, find the ratio of their areas (in the same order).

(A) ac : bd (B) ad : bc
(C) bd : ac (D) bc : ad

9. The base and corresponding altitude of a parallelogram are 18 cm and 6 cm respectively. What is its area?

(A) $36\ cm^2$ (B) $40\ cm^2$
(C) $44\ cm^2$ (D) $108\ cm^2$

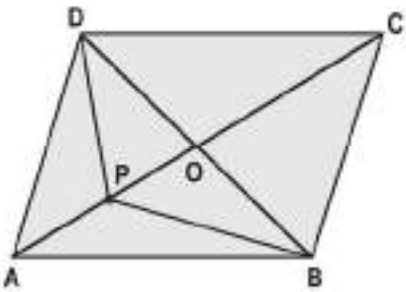
10. In ΔABC, AB = 8 cm. If the altitudes corresponding to AB and BC are 4 cm and 5 cm respectively, find the measure of BC.

(A) 6.4 cm (B) 4.6 cm
(C) 5.4 cm (D) 4.5 cm

11. ABCD is a trapezium whose area is $a^2 - b^2$. If AB = a, DC = b and $\overline{AB} \parallel \overline{CD}$, what is the distance between the parallel sides?

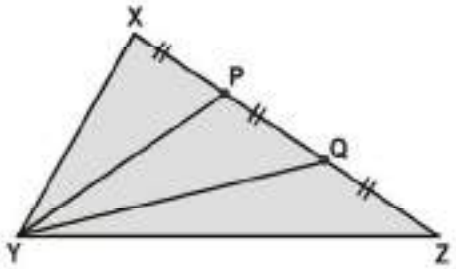
(A) (a – b) (B) (a + b)
(C) 2(a – b) (D) 2(a + b)

12. The diagonals AC and BD of a parallelogram ABCD intersect at O. P is a point on AC such that $AP = \frac{1}{4}AC$. Which of the following is true?



(A) ar(ΔADP) = ar(ΔAPB)
(B) ar(ΔADP) = ar(ΔDOC)
(C) ar(ΔADP) = ar(ΔBCD)
(D) ar(ΔADP) = ar(ΔADB)

13 In Δ XYZ, P and Q are two points on side XZ such that XP = PQ = QZ.



Which of the following is correct?

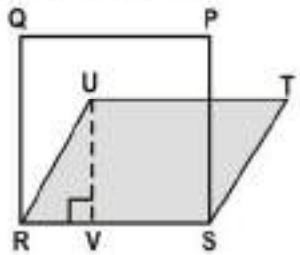
(A) ar(ΔPXY) = ar(ΔPYZ)

(B) ar(ΔPXY) = ar(ΔPYQ) = ar(ΔQYZ)

(C) ar(ΔPYQ) = ar(ΔXYQ)

(D) ar(ΔQYZ) = ar(ΔXYQ)

14 A square PQRS and a rhombus RSTU lie on the same base RS.



What is the relation between their areas?

(A) ar(PQRS) = ar(RSTU)

(B) ar(PQRS) < ar(RSTU)

(C) ar(PQRS) > ar(RSTU)

(D) $\text{ar(PQRS)} = \frac{1}{2}\text{ ar(RSTU)}$

15 If E, F, G and H are respectively the mid points of the sides of a parallelogram ABCD and ar (EFGH) = 40 cm^2, find the ar (parallelogram ABCD) .

(A) 40 cm^2 (B) 20 cm^2

(C) 80 cm^2 (D) 60 cm^2

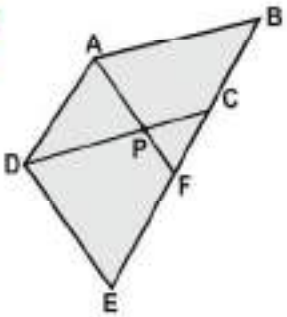
16 If a triangle and a square are on the same base and between the same parallels, What is the ratio of their areas in order?

(A) 1 : 3 (B) 1 : 2

(C) 3 : 1 (D) 1 : 4

(17-20): Observe the figures given and choose the incorrect statements.

17



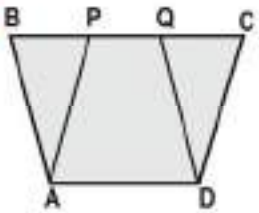
(A) ar(ABCD) = ar(ADEF)

(B) ar(ABCD) = ar(ΔADP) + ar(DEFP)

(C) $\text{ar}(\Delta\text{ADP}) = \frac{1}{2}[\text{ar(ABCD)}]$

(D) ar(ABCP) = ar(DEFP)

18



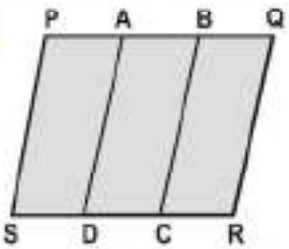
(A) ar(ΔAPB) = ar(ΔQCD)

(B) ar(ABQD) = 2ar(APQD)

(C) ar(ABQD) = ar(Δ ABP) + ar(APQD)

(D) ar(ADCP) = ar(Δ ABP) + ar(APQD)

19



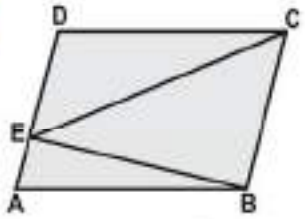
(A) ar(ABCD) = ar(PBCS) – ar(PSDA)

(B) ar(ABCD) = ar(AQRD) – ar(QRCB)

(C) ar(ABCD) = ar(PQRS) + [ar(PSDA) – ar(QRCB)]

(D) ar(ABCD) = ar(PQRS) – [ar(PSDA) + ar(QRCB)]

20



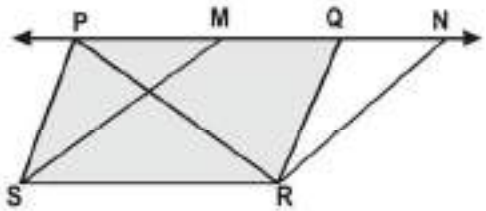
(A) $ar(\Delta BCE) = \frac{1}{2} ar(ABCD)$

(B) $ar(\Delta BCE) = 2ar(ABCD)$

(C) $ar(\Delta BCE) = ar(ABCD) - ar(\Delta EDC) - ar(\Delta EBA)$

(D) $ar(\Delta BCE) = ar(ABCD) - [ar(\Delta EDC) + ar(\Delta EBA)]$

(21-22): Based on the figure given, identify the true statements.



21 (A) $ar(PQRS) = 2ar(\Delta PNR)$

(B) $ar(PQRS) = 2ar(MNRS)$

(C) $ar(PQRS) = ar(MNRS)$

(D) $ar(MNRS) = 2ar(PQRS)$

22 (A) $ar(\Delta PRS) = ar(\Delta QRN)$

(B) $ar(\Delta PQR) = ar(\Delta PSR)$

(C) $ar(\Delta QRN) = \frac{1}{2} ar(PNR)$

(D) $ar(\Delta PMS) = ar(\Delta QNR)$

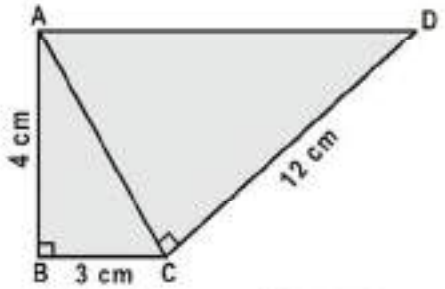
23 In a ΔABC, G is the centroid. What is the ratio of area of ΔAGC to area of ΔABC?

(A) 1 : 2 (B) 1 : 3 (C) 1 : 4 (D) 1 : 6

24 A square and a rhombus are on the same base and between the same parallels. Which of the following is the ratio of their areas?

(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

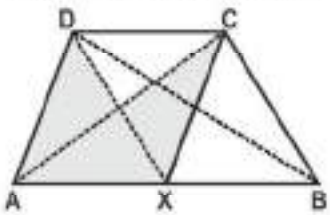
25 Find the area of the given quadrilateral ABCD.



(A) 36 cm² (B) 24 cm²

(C) 48 cm² (D) 32 cm²

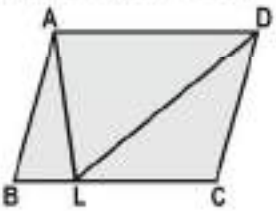
26 In the given figure, AB || DC. Identify the triangles that have equal areas.



(A) $\Delta ADX, \Delta ACX$ (B) $\Delta ADX, \Delta XCB$

(C) $\Delta ACX, \Delta XDB$ (D) All the above

27 In the given figure, ABCD is a parallelogram. If area of ΔABL is 15 cm² and area of ΔDCL is 32 cm², find the area of the parallelogram ABCD.



(A) 94 cm² (B) 86 cm²

(C) 72 cm² (D) 65 cm²

28 Which of the following figures is obtained by joining mid-points of adjacent sides of a rectangle of sides 8 cm and 6 cm ?

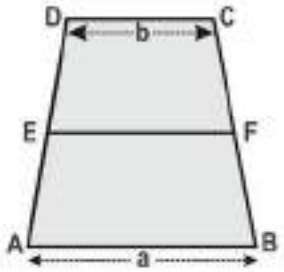
(A) A rectangle of area 24 cm².

(B) A square of area 25 cm².

(C) A trapezium of area 24 cm².

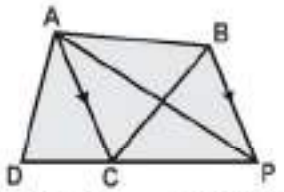
(D) A rhombus of area 24 cm².

29 In the given figure, ABCD is a trapezium with parallel sides AB = a cm, and CD = b cm. E and F are the mid points of non parallel sides. What is the ratio of ar (EFCD) and ar (ABFE)?



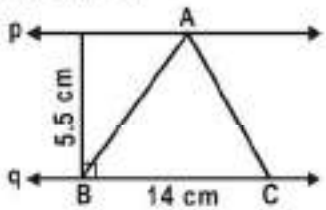
(A) b : a
(B) (a + 3b) : (3a + b)
(C) (1 + 3b) : (3a + b)
(D) (2a + b) : (3a + b)

30 In the given figure, ABCD is a quadrilateral. BP || AC and BP meets DC (produced) in P. If area of the quadrilateral ABCD is 145 square units, find the area of the Δ ADP.



(A) 126 sq. units (B) 145 sq. units
(C) 135 sq. units (D) 112 sq. units

31 In the given figure, if p // q, what is the area of Δ ABC ?

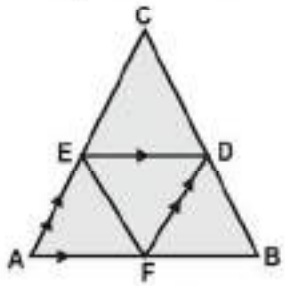


(A) 77 cm² (B) 38.5 cm²
(C) 40 cm (D) 19.25 cm²

32 AD is the median of a Δ ABC and the area of Δ ADC = 15 cm². Find the ar (Δ ABC).

(A) 15 cm² (B) 22.5 cm²
(C) 30 cm² (D) 37.5 cm²

33 If D, E and F are the mid-points of the sides BC, CA and AB of a Δ ABC respectively, what is the quadrilateral AFDE ?



(A) A square (B) A rectangle
(C) A parallelogram (D) A trapezium

34 A triangle and a rhombus are on the same base and between the same parallels. What is the ratio of area of triangle to that of rhombus?

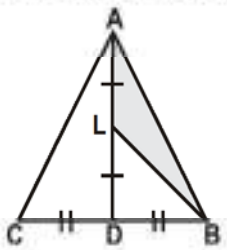
(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

35 P is a point in the interior of parallelogram ABCD.

If ar (||gm ABCD) = 18 cm², what is the value of [ar (Δ APD) + ar (Δ CPB)] ?

(A) 9 cm² (B) 12 cm²
(C) 18 cm² (D) 15 cm²

36 In the given figure, D is the mid-point of BC and L is the mid-point of AD. If ar (Δ ABL) = x ar (Δ ABC), what is the value of x ?



(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 4

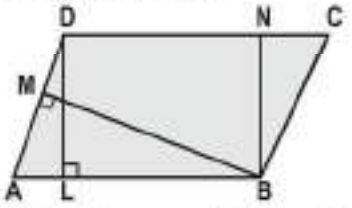
37 Area of a parallelogram ABCD is 432 cm². If BC // AD and the distance between BC and AD is 20 cm, what is the measure of the side BC of parallelogram ABCD?

(A) 43.2 cm (B) 10.8 cm
(C) 18.2 cm (D) 21.6 cm

38 Which of the following statements is true about a median of a triangle?

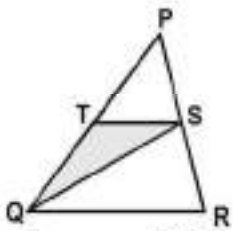
(A) It divides the triangle into two triangles of equal area.
(B) It divides the triangle into two congruent triangles.
(C) It divides the triangle into two right triangles.
(D) It divides the triangle into two isosceles triangles.

39 In the given figure, what is the area of parallelogram ABCD ?



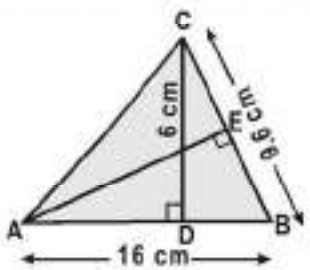
(A) AB × BM (B) BC × BN
(C) DC × DL (D) AD × DL

40 In the given figure, S and T are the mid-points of PR and PQ respectively of Δ PQR. If ar (Δ PQR) = 48 cm², find the area of (Δ TSQ).



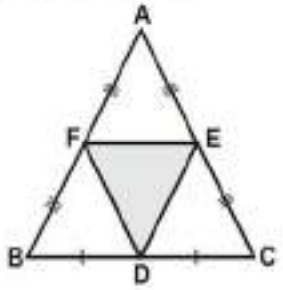
(A) 48 cm² (B) 24 cm²
(C) 12 cm² (D) 6 cm²

41 In Δ ABC, AB = 16 cm, BC = 9.6 cm, CD ⊥ AB. If CD = 6 cm, find AE.



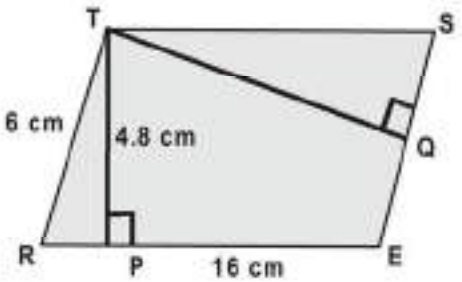
(A) 10 cm (B) 48 cm
(C) 8.4 cm (D) 9.6 cm

42 In the given figure, D, E and F are the mid-points of the sides BC, CA and AB respectively of a Δ ABC. If ar (Δ DEF) = 14 cm², find the area of Δ ABC.



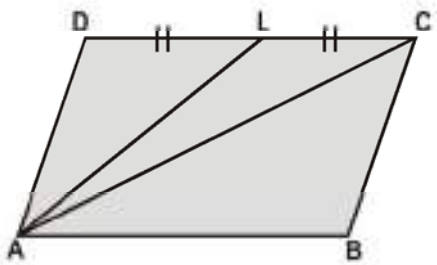
(A) 14 cm² (B) 28 cm²
(C) 7 cm² (D) 56 cm²

43 In the given figure, REST is a parallelogram. TP ⊥ RE and TQ ⊥ ES. If TR = 6 cm, RE = 16 cm and TP = 4.8 cm, what is the length of TQ?



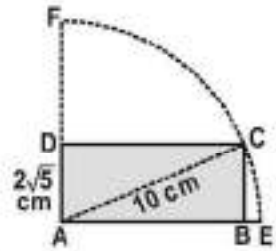
(A) 12.8 cm (B) 15 cm
(C) 8 cm (D) 10.2 cm

44 In the given figure, ABCD is a parallelogram and L is the mid-point of DC. If ar (ABCL) is 72 cm², find ar (Δ ADC).



(A) 24 cm² (B) 48 cm²
(C) 36 cm² (D) 54 cm²

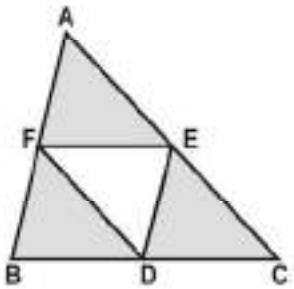
45 In the figure ABCD is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If AD = $2\sqrt{5}$ cm, find the area of the rectangle.



(A) 30 cm^2 (B) 50 cm^2
(C) 40 cm^2 (D) 35 cm^2

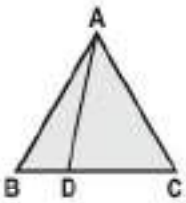
Previous Contest Questions

1 In the given figure, D, E and F are the mid-points of the sides BC, CA and AB respectively. If ar (BDEF) = x ar (Δ AFE), what is the value of x ?



(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4

2 In the figure given, D divides the side BC of Δ ABC in the ratio 3 : 5. What is the area of Δ ABD ?



(A) $\frac{2}{5} \times$ ar(ΔABC) (B) $\frac{3}{5} \times$ ar(ΔABC)
(C) $\frac{5}{8} \times$ ar(ΔABC) (D) $\frac{3}{8} \times$ ar(ΔABC)

3 ABCD is a parallelogram, AL $\perp$ CD and AM $\perp$ BC. If AB = 12 cm, AD = 8 cm and AL = 6 cm, find the measure of AM.

(A) 9 cm (B) 10 cm
(C) 12 cm (D) 15 cm

4 ABCD is a parallelogram and P is the midpoint of AB. If ar(APCD) = 36 cm^2, find ar (ΔABC).

(A) 36 cm^2 (B) 48 cm^2
(C) 24 cm^2 (D) 42 cm^2

5 In Δ ABC, AB = 7.2 cm, BC = 4.8 cm, AM $\perp$ BC and CL $\perp$ AB. If CL = 4 cm, find the measure of AM.

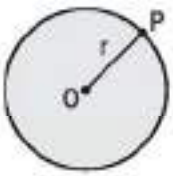
(A) 12 cm (B) 8 cm
(C) 4 cm (D) 6 cm

CHAPTER 10

Circles

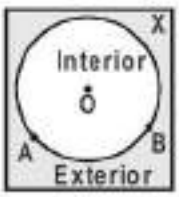
Synopsis

- A circle is a closed figure in a plane formed by the collection of all the points in the plane which are at a constant distance from a fixed point in the plane. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

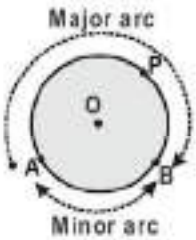


O = centre
OP = radius
r = length of radius

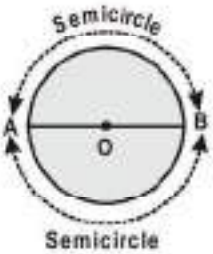
- The plane region inside the circle is called the interior of the circle.
- If a circle is drawn in the plane X (infinite dimensions), then the part of the plane region outside the circular region is called the exterior of the circle.



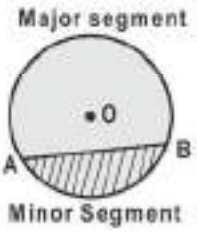
- The circumference of a circle is the length of the complete circular curve constituting the circle, given by circumference, $C = 2\pi r$ where r is the radius of the circle.
- Any two points A and B of a circle, divide the circle into two parts. The smaller part is called a minor arc of the circle denoted by $\overset{\frown}{AB}$ (read as arc AB). The larger part is called a major arc denoted by $\overset{\frown}{APB}$ or $\overset{\frown}{BA}$ (read as arc BA).



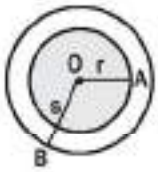
- A line segment joining two points on the circumference of the circle, is called a chord of the circle. In the figure, AB is a chord.
- A chord passing through the centre O of the circle is called a diameter of the circle.
- A diameter of a circle is the longest chord of the circle and its length is twice that of the radius of the circle. Diameter, $d = 2r$ where r is the radius of the circle.



- A diameter of a circle divides it into two equal arcs. Each of these two arcs is called a semicircle. In the figure $\overset{\frown}{AB}$ is a semicircle.
- A chord AB divides a circular region into two parts. The smaller part is called the minor segment and the larger part is called the major segment. In case chord AB is a diameter of the circle, the two segments are equal and each is called a semicircular region.

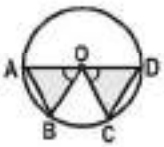


- Circles which have the same centre O and different radii OA (= r) and OB (= s) are called concentric circles.



- Two circles C(O, r) and C(O', s) are said to be congruent if and only if r = s.
- One-fourth of a circle is called a quadrant. In the figure the shaded part is called a quadrant.

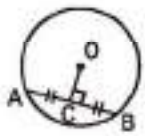
- The angle subtended by a chord AB at the centre O of a circle is $\angle AOB$.
- The angle subtended by chord AB, at a point (the centre) is the same as the angle subtended by the arc AB at the same point (the centre).
- Equal chords of a circle subtend equal angles at the centre of the circle. Conversely, if two chords subtend equal angles at the centre of a circle, then the chords are equal.



If AB = CD, then $\angle AOB = \angle COD$. Conversely, if $\angle AOB = \angle COD$, then AB = CD.

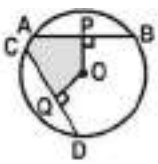
Note: **If two chords subtend angles of unequal measure at the centre of a circle, then the chords are unequal.**

- The perpendicular drawn from the centre to a chord of the circle bisects the chord. Conversely, the line segment joining the centre of a circle to the mid-point of a chord of the circle is perpendicular to the chord.



If $OC \perp AB$, then AC = CB. Conversely, if AC = CB, then $OC \perp AB$.

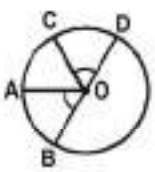
- Equal chords of a circle are equidistant from the centre of the circle. Conversely, chords equidistant from the centre of a circle are equal.



If AB = CD, then OP = OQ. Conversely, if OP = OQ, then AB = CD.

Note : **Equal chords of congruent circles are equidistant from their respective centres. Conversely, chords equidistant from the corresponding centres of congruent circles are equal.**

- One and only one circle can be made to pass through three given non- collinear points in a plane.
- If two chords of a circle are equal, then their corresponding arcs are congruent. Conversely, if two arcs are congruent, then their corresponding chords are equal.



If AB = CD, then $\overset{\frown}{AB} = \overset{\frown}{CD}$. Conversely, if $\overset{\frown}{AB} = \overset{\frown}{CD}$, then AB = CD.

- Congruent arcs of a circle subtend equal angles at the centre of the circle.

If $\overset{\frown}{AB} = \overset{\frown}{CD}$, then $\angle AOB = \angle COD$. The converse is also true.

- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\angle AOB = 2\angle ACB = 2\angle ADB$

The angle subtended in a semicircle is a right angle.

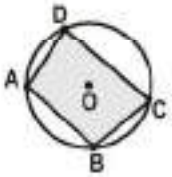
$\angle ACB = 90°$

- Angles in the same segment of a circle are equal.
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

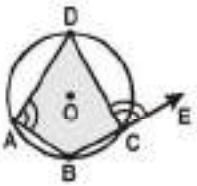
$\angle ACB$ and $\angle ADB$ are angles in the same segment.

$\Rightarrow \angle ACB = \angle ADB$

- If all the four vertices of a quadrilateral lie on a circle, then it is called a cyclic quadrilateral. ABCD is a cyclic quadrilateral. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°. In the figure, $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$. If the sum of a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

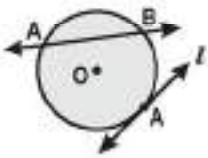


- A cyclic parallelogram is a rectangle.
- If one side of a cyclic quadrilateral is produced, then the exterior angle so formed is equal to the interior opposite angle.



Here, $\angle DCE = \angle DAB$.

- A line that touches a circle at a point is called a tangent. l is a tangent to the circle with centre O. A is called the point of contact.
- A line segment that intersects a circle at two distinct points is called a secant. $\overleftrightarrow{AB}$ is the secant to the circle.



Multiple Choice Questions

A B C D

1 AB is a chord of a circle with centre O and radius 17 cm. If OM ⊥ AB and OM = 8 cm, find the length of chord AB.

(A) 12 cm (B) 30 cm
(C) 15 cm (D) 24 cm

2 AB is a chord of length 24 cm of a circle with centre O and radius 13 cm. Find the distance of the chord from the centre.

(A) 5 cm (B) 6 cm
(C) $\sqrt{407}$ cm (D) 25 cm

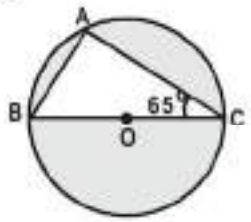
3 48 cm long chord of a circle is at a distance of 7 cm from the centre. Find the radius of the circle.

(A) 5 cm (B) 17 cm
(C) 25 cm (D) 16 cm

4 A chord of a circle is 12 cm in length and its distance from the centre is 8 cm. Find the length of the chord of the same circle which is at a distance of 6 cm from the centre.

(A) 30 cm (B) 24 cm
(C) 16 cm (D) 18 cm

5 In the given figure, Δ ABC is inscribed in a circle with centre O. If ∠ ACB = 65°, find ∠ ABC.

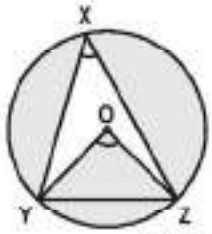


(A) 25° (B) 35° (C) 90° (D) 65°

6 An equilateral Δ PQR is inscribed in a circle with centre O. Find ∠ QOR.

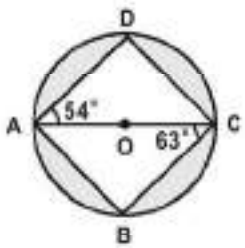
(A) 60° (B) 120°
(C) 30° (D) 90°

7 In the given figure, Δ XYZ is inscribed in a circle with centre O. If the length of the chord YZ is equal to the radius of the circle OY, find the measure of ∠ YXZ.



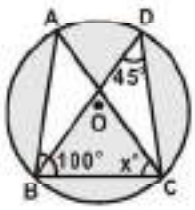
(A) 60° (B) 30°
(C) 80° (D) 100°

8 In the given figure, O is the centre of a circle. If ∠ DAC = 54° and ∠ ACB = 63°, find ∠ DAB.



(A) 72° (B) 54°
(C) 81° (D) 27°

9 Find the value of x in the following figure.

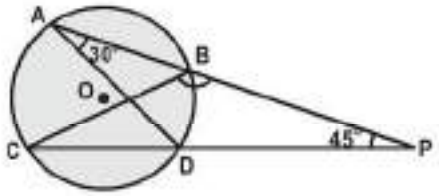


(A) 45° (B) 35°
(C) 60° (D) 55°

10 What is the least number of noncollinear points required to draw a circle passing through them?

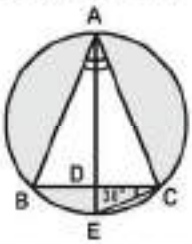
(A) Two (B) Three
(C) Four (D) Nine

11 Two chords AB and CD of a circle cut each other when produced outside the circle at P. AD and BC are joined. If $\angle BAD = 30°$ and $\angle CPA = 45°$, find $\angle CBP$.



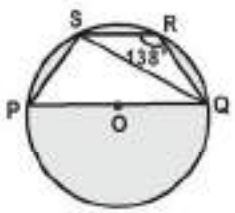
(A) 105° (B) 115°
(C) 135° (D) 75°

12 In the given figure, ΔABC is inscribed in a circle. The bisector of $\angle BAC$ meets BC at D and the circle at E. If EC is joined and $\angle ECD = 30°$, find $\angle BAC$.



(A) 30° (B) 40° (C) 50° (D) 60°

13 In the given figure, POQ is a diameter of a circle with centre O and PQRS is a cyclic quadrilateral. SQ is drawn. If $\angle R = 138°$, find $\angle PQS$.

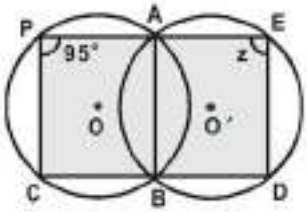


(A) 90° (B) 42° (C) 48° (D) 38°

14 The circumference of a circle is 60 cm. An arc subtends 90° at the centre. Find its length.

(A) 10 cm (B) 15 cm
(C) 20 cm (D) 25 cm

15 Two circles intersect in A and B. Quadrilaterals PCBA and ABDE are inscribed in these circles such that PAE and CBD are line segments. If $\angle P$ is 95° find the value of z.



(A) 65° (B) 105° (C) 95° (D) 85°

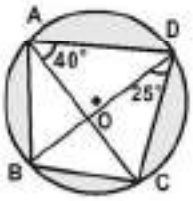
16 A circle is divided into 12 equal parts. What is the measure of central angle in each arc?

(A) 30° (B) 60° (C) 45° (D) 48°

17 In a circle, the major arc is twice the minor arc. What are the corresponding central angles and the degree measures of the two arcs?

(A) 90° and 270° (B) 60° and 300°
(C) 240° and 120° (D) 140° and 220°

18 In the given figure, ABCD is a quadrilateral inscribed in a circle. Diagonals AC and BD are drawn. If $\angle CAD = 40°$ and $\angle BDC = 25°$, find $\angle BCD$.

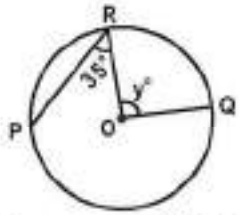


(A) 85° (B) 120°
(C) 115° (D) 95°

19 A circle with radius 2 units is intersected by a line at points R and T. Find the maximum possible distance between R and T.

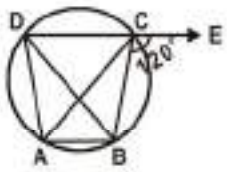
(A) 1 unit (B) 2π units
(C) 4π units (D) 4 units

20 O is the centre of the circle as shown in the figure. $\angle ORP = 35°$ and the distance between P and Q through 'O' is 4 cm. What is the measure of $\angle ROQ$?



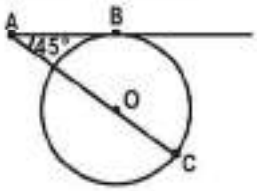
(A) 55° (B) 35°
(C) 105° (D) 70°

21 In the figure given, $\overline{DC}$ is produced to E and if $\overline{AC}$ is the bisector of $\angle A$, find the measure of $\angle BCD$.



(A) 120° (B) 160°
(C) 60° (D) 240°

22 A line AB intersects a circle with centre O and radius 2 cm at a point B. C is any other point on the circle in line with O and A. Find the measure of AC.

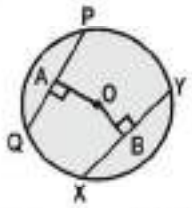


(A) 4 cm (B) $2+\sqrt{2}$ cm
(C) $4+\sqrt{2}$ cm (D) $2+2\sqrt{2}$ cm

23 The length of a chord of a circle is equal to the radius of the circle. Determine the angle which this chord subtends on the major segment of the circle.

(A) 30° (B) 45° (C) 60° (D) 90°

24 PQ and XY are two chords of a circle with centre O. If OA = OB, which of the following is correct?



(A) PQ > XY (B) PQ < XY
(C) PQ || XY (D) PQ = XY

25 A triangle ABC is inscribed in a circle, the bisectors of whose angles meet the circumference at X, Y and Z. Determine the angles X, Y and Z respectively.

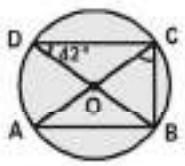
(A) $90°-\frac{A}{2}, 90°-\frac{B}{2}, 90°-\frac{C}{2}$

(B) 90°, 60°, 30°

(C) $\frac{A}{2}, \frac{B}{2}, \frac{C}{2}$

(D) $\frac{B}{2}, \frac{A}{2}, \frac{A}{2}-\frac{B}{2}$

26 In the given circle ABCD, O is the centre and $\angle BDC = 42°$. Find the measure of $\angle ACB$.



(A) 42° (B) 45° (C) 48° (D) 60°

27 Two parallel chords on the same side of the centre of a circle are 10 cm and 24 cm long and their distance apart is 7 cm. Determine the radius of the circle.

(A) 13 cm (B) 8 cm
(C) 5 cm (D) 7 cm

28 Into how many parts does a circle divide a plane including itself?

(A) 2 parts (B) 3 parts
(C) 4 parts (D) 5 parts

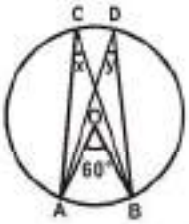
29 Given two concentric circles with centre O. A line cuts the circles at A, B, C and D respectively. If AB = 10 cm, find the length of CD.

(A) 5 cm (B) 10 cm
(C) 7.5 cm (D) 15 cm

30 A chord of a circle divides the circular region into two parts. What is the region that contains the centre?

(A) Minor Arc (B) Major Arc
(C) Minor Segment (D) Major Segment

31 In the given figure, find the value of y.



(A) 35° (B) $60° + x°$
(C) $60° - x°$ (D) 120°

32 An equilateral ΔABC is inscribed in a circle with centre O. Find the measure of $\angle BOC$.

(A) 110° (B) 100° (C) 120° (D) 130°

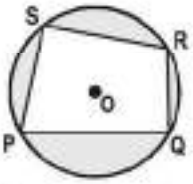
33 In the given figure, ABDC is a cyclic quadrilateral and AB = AC. If $\angle ACB = 70°$, find $\angle BDC$.

(A) 100° (B) 70° (C) 140° (D) 40°

34 When are two circles said to be concentric?

(A) If they have the same radius.
(B) If they have different radii.
(C) If they have the same centre.
(D) If their centres are collinear.

35 In the given figure, $\overset{\frown}{PQR} = \overset{\frown}{SRQ}$.



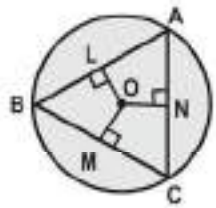
Which of the following is correct?

(A) $PQ \neq SR$ (B) $PS = QR$
(C) $PQ = SR$ (D) $PS \neq QR$

36 'O' is the centre of a circle, $\angle BOA = 90°$ and $\angle COA = 110°$. Find the measure of $\angle BAC$.

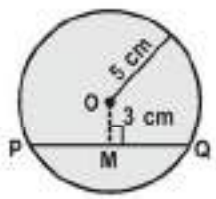
(A) 70° (B) 80°
(C) 120° (D) 50°

37 In the given figure, O is the centre of the circle, $OM \perp BC$, $OL \perp AB$, $ON \perp AC$ and OM = ON = OL. What is ΔABC?



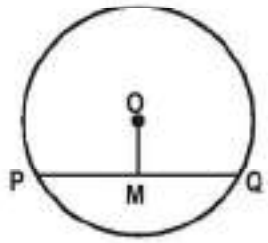
(A) Right angled triangle
(B) Scalene triangle
(C) Isosceles triangle
(D) Equilateral triangle

38 In the given figure, O is the centre of a circle having radius 5 cm. If $OM \perp PQ$ and OM = 3 cm, find the length of chord PQ.



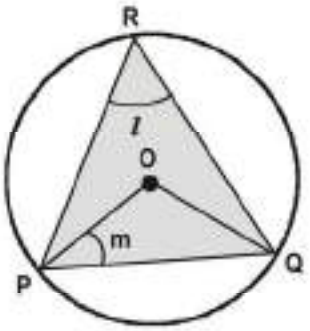
(A) 5 cm (B) 8 cm
(C) 10 cm (D) 6 cm

39 In the given figure, O is the centre of the circle and OM ⊥ PQ. If PQ = 6 cm and OM = 4 cm, find the radius of the circle.



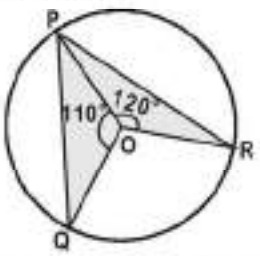
(A) 6 cm (B) 3 cm
(C) 2 cm (D) 5 cm

40 In the given figure, O is the centre of the circle. PQ is a chord of the circle and R is any point on the circle. If ∠PRQ = l and ∠OPQ = m, what is the value of l + m?



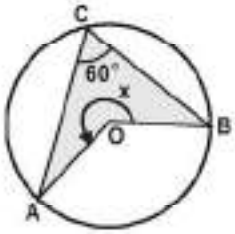
(A) Greater than 90°
(B) Less than 90°
(C) Equal to 180°
(D) Equal to 90°

41 In the given figure, O is the centre of the circle. If ∠POQ = 110° and ∠POR = 120°, find ∠QPR.



(A) 65° (B) 55°
(C) 50° (D) 60°

42 If O is the centre of the given circle with ∠ACB = 60°, find the value of x.

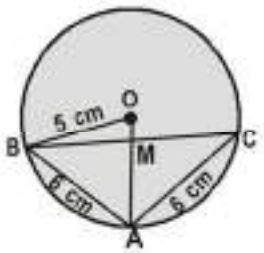


(A) 120° (B) 100°
(C) 240° (D) 180°

43 AD is a diameter of a circle and AB is a chord. If AD = 34 cm and AB = 30 cm, find the distance of AB from the centre of the circle.

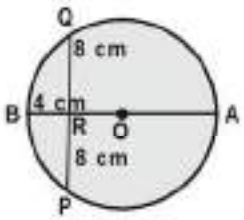
(A) 17 cm (B) 15 cm
(C) 4 cm (D) 8 cm

44 In the given figure, O is the centre of the circle of radius 5 cm. AB and AC are two chords, such that AB = AC = 6 cm. If OA meets BC at M, find OM.



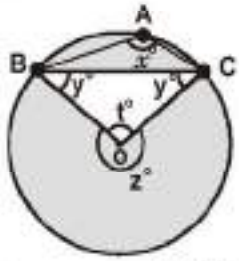
(A) 2 cm (B) 3 cm
(C) 1.4 cm (D) 3.6 cm

45 The given figure shows a circle with centre O in which a diameter AB bisects the chord PQ at the point R. If PR = RQ = 8 cm and RB = 4 cm, find the radius of the circle.



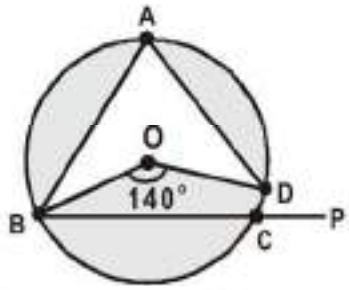
(A) 10 cm (B) 8 cm
(C) 16 cm (D) 6 cm

46 In the given figure, O is the centre of the circle. A is any point on minor arc BC. Find the value of $\angle BAC - \angle OBC$.



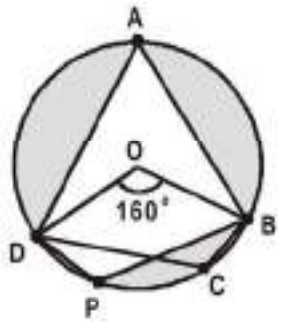
(A) 90° (B) 120°
(C) 60° (D) 45°

47 In the given figure, O is the centre of the circle. The angle subtended by the arc BCD at the centre is 140°. If BC is produced to P, find $\angle DCP$.



(A) 60° (B) 110°
(C) 70° (D) 80°

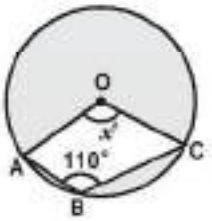
48 In the given figure, ABCD and ABPD are cyclic quadrilaterals.



If $\angle BOD = 160°$, find the difference of $\angle BPD$ and $\angle BCD$.

(A) 80° (B) 90° (C) 0° (D) 110°

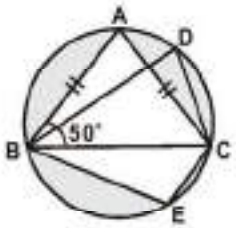
49 In the given figure, O is the centre of the circle.



If $\angle ABC = 110°$, find x.

(A) 70° (B) 140° (C) 60° (D) 120°

50 In the given figure, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 50°$. Find the sum of $\angle BDC$ and $\angle BEC$.



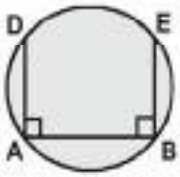
(A) 30° (B) 60°
(C) 50° (D) 180°

Previous Contest Questions

1 If the radius of a circle is 10 cm, find the length of a chord of the circle which is at a distance of 6 cm from its centre.

(A) 16 cm (B) 10 cm
(C) 12 cm (D) 14 cm

2 Given a chord AB in a circle as shown.



If two more chords AD and BE are drawn perpendicular to AB which of the following is correct?

(A) $AD = BE$ (B) $AD = 2BE$
(C) $2AD = BE$ (D) $AD = 3BE$

3 In a circle of radius 5 cm, AB and AC are two chords such that AB = AC = 6 cm. What is the distance of the centre of the circle from BC ?

(A) 1.4 cm (B) 2.1 cm
(C) 2.4 cm (D) 2.7 cm

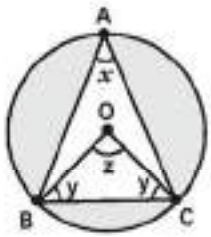
4 Angles subtended by chords AC and BC at the centre O of the circle are 55° and 155° respectively. What is the measure of ∠ACB ?

(A) 65° (B) 75°
(C) 105° (D) 135°

5 ABCD is a cyclic quadrilateral in which ∠DBC = 55° and ∠BAC = 55°. Find ∠BCD.

(A) 60° (B) 80°
(C) 100° (D) 120°

6 BC is a chord of a circle with centre O. A is a point on major arc BC. Find the total measure of ∠BAC and ∠OBC



(A) 90° (B) 100°
(C) 120° (D) 150°

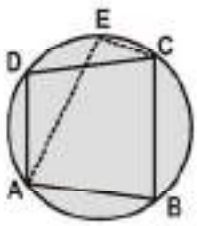
7 AB is a chord of a circle of radius 4.3 cm and P is a point on AB which divides it into two parts in the ratio 7 : 10. If P is 2.7 cm away from the centre O, find the length of AB.

(A) 5 cm (B) 6.8 cm
(C) 6.4 cm (D) 6.1 cm

8 The radius of a circle is 2.5 cm. AB and CD are two parallel chords 2.7 cm apart. If AB = 4.8 cm, find the measure of CD.

(A) 4.8 cm (B) 2.4 cm
(C) 3 cm (D) 4 cm

9 A quadrilateral ABCD is inscribed in a circle as shown.



If ∠B=125° and E is a point on the circle, find ∠AEC.

(A) 55° (B) 125°
(C) 130° (D) 62.5°

10 ABCD is a quadrilateral whose vertices are on a semicircle such that AB = BC = CD = 10 cm and AD is diameter of the circle with centre O. Find the perimeter of the quadrilateral ABCD.

(A) 80 cm (B) 70 cm
(C) 60 cm (D) 50 cm

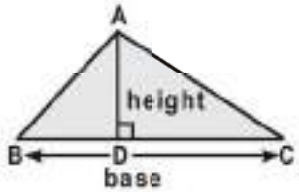
This page is intentionally left blank.

CHAPTER 11

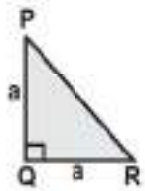
Heron's Formula

Synopsis

- Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

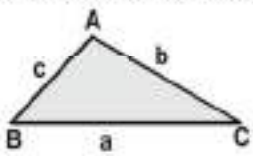


- Area of a right angled isosceles triangle with perpendicular sides each equal to 'a' units $= \frac{1}{2} a^2$ sq. units.



- **Heron's formula:**

  Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ where a, b, c are the sides of the triangle and s = semiperimeter i.e., half the perimeter of the triangle $= \frac{a+b+c}{2}$.



> ***Note:*** **Heron's formula can be used when three sides of triangle are given and can be applied to any triangle.**

- Area of an equilateral triangle with each side equal to 'a' units $= \frac{\sqrt{3}}{4} a^2$ sq. units.
- Area of an equilateral triangle $= \frac{h^2}{\sqrt{3}}$ sq. units where altitude $h = \frac{\sqrt{3}}{2} a$ units.
- Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using the Heron's formula.
- Three positive integers a, b and c such that $c^2 = a^2 + b^2$ is called a pythagorean triplet.

Multiple Choice Questions

A B C D

1. The perimeter of a triangular field is 144 m and the ratio of its sides is 3 : 4 : 5. Find the area of the field.

(A) 864 sq.m (B) 764 sq.m
(C) 854 sq.m (D) 754 sq.m

2. One side of an equilateral triangle is 8 cm. What is its area?

(A) $12\sqrt{3}$ cm^2 (B) $16\sqrt{3}$ cm^2
(C) $8\sqrt{3}$ cm^2 (D) $48\sqrt{3}$ cm^2

3. The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. What is its area?

(A) 12 sq.cm (B) 36 sq.cm
(C) 24 sq.cm (D) 48 sq.cm

4. The edges of a triangular board are 6 cm, 8 cm and 10 cm. What is the cost of painting it at the rate of 9 paise per cm^2?

(A) ₹ 2.00 (B) ₹ 2.16
(C) ₹ 2.48 (D) ₹ 3.00

5. What is the area of a triangle whose sides are 13 cm, 14 cm and 15 cm?

(A) 84 sq.cm (B) 64 sq.cm
(C) 82 sq.cm (D) 48 sq.cm

6. Two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm. Find the area of parallelogram.

(A) $5\sqrt{3}$ cm^2 (B) $10\sqrt{3}$ cm^2
(C) $15\sqrt{3}$ cm^2 (D) $20\sqrt{3}$ cm^2

7. The sides of a triangle are in the ratio of 13 : 14 : 15 and its perimeter is 84 cm. Determine the area of the triangle.

(A) 136 cm^2 (B) 236 cm^2
(C) 336 cm^2 (D) 436 cm^2

8. What is the area of a parallelogram whose diagonal is 6.8 cm and the perpendicular distance of this diagonal from an opposite vertex is 7.5 cm?

(A) 25.5 cm^2 (B) 11.9 cm^2
(C) 12.5 cm^2 (D) 51 cm^2

9. The adjacent sides of a parallelogram are 4 cm and 9 cm. What is the ratio of its altitudes?

(A) 16 : 81 (B) 9 : 4
(C) 2 : 3 (D) 3 : 2

10. The perimeter of a rhombus is 52 cm and one of its diagonals is 24 cm. Determine the length of the other diagonal.

(A) 24 cm (B) 10 cm
(C) $2\frac{1}{6}$ cm (D) 12 cm

11. The area of a rhombus is 28 cm^2 and one of its diagonals is 4 cm. What is its perimeter?

(A) $4\sqrt{53}$ cm (B) 36 cm
(C) $2\sqrt{53}$ cm (D) 52 cm

12. The adjacent sides of a parallelogram are 8 cm and 9 cm. The diagonal joining the ends of these sides is 13 cm. Find its area.

(A) 72 cm^2 (B) $12\sqrt{35}$ cm^2
(C) $24\sqrt{35}$ cm^2 (D) 150 cm^2

13. The sides of a triangle are 11 cm, 15 cm and 16 cm. Find the measure of the altitude to the largest side.

(A) $30\sqrt{7}$ cm (B) $\frac{15\sqrt{7}}{2}$ cm
(C) $\frac{15\sqrt{7}}{4}$ cm (D) 30 cm

14. The perimeter of a triangle is 60 m and its sides are in the ratio 5 : 12 : 13, Find the length of the altitude of the triangle corresponding to the longest side.

(A) $9\frac{3}{13}$m (B) $5\frac{4}{12}$m
(C) $6\frac{9}{11}$m (D) $7\frac{2}{15}$m

15 If the altitude of an equilateral triangle is $\sqrt{6}$ cm, what is its area?

(A) $2\sqrt{3}$ cm^2 (B) $2\sqrt{2}$ cm^2

(C) $3\sqrt{3}$ cm^2 (D) $6\sqrt{2}$ cm^2

16 The sides of an equilateral triangle are $(2a - b + 5)$, $(a + b)$ and $(2b - a + 2)$. What is the area of the triangle?

(A) $\frac{\sqrt{3}}{4} \times a^2$ (B) $\frac{\sqrt{3}}{4} \times b^2$

(C) $\frac{\sqrt{3}}{4} \times 49$ (D) $\frac{\sqrt{3}}{4} \times 81$

17 In Δ ABC, AB = 6 cm, BC = 7 cm and AC = 5 cm. What is the area of Δ ABC?

(A) $6\sqrt{6}$ cm^2 (B) $6\sqrt{3}$ cm^2

(C) $6\sqrt{2}$ cm^2 (D) $9\sqrt{6}$ cm^2

18 If the angles of a triangle are in the ratio 5 : 3 : 7, what is such a triangle called?

(A) An acute angled triangle
(B) An obtuse angled triangle
(C) A right triangle
(D) An isosceles triangle

19 The base of an isosceles right triangle is 30 cm. Find its area.

(A) 225 cm^2 (B) $225\sqrt{3}$ cm^2

(C) $225\sqrt{2}$ cm^2 (D) 450 cm^2

20 Which of the following is the area of an isosceles triangle with equal sides b and base a?

(A) $\frac{a}{2}\sqrt{b^2 - a^2}$ (B) $\frac{1}{2}a\sqrt{4b^2 - a^2}$

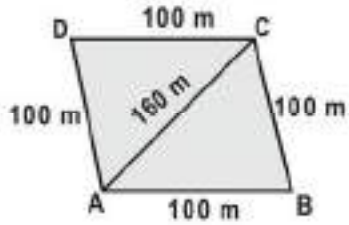
(C) $\frac{a}{4}\sqrt{b^2 - a^2}$ (D) $\frac{a}{4}\sqrt{4b^2 - a^2}$

21 An isosceles right triangle has area 8 cm^2. What is the length of its hypo-tenuse?

(A) $\sqrt{32}$ cm (B) $\sqrt{16}$ cm

(C) $\sqrt{48}$ cm (D) $\sqrt{24}$ cm

22 A farmer divided his field which is in the shape of a rhombus of side 100 m between his two sons equally. If the line of division is 160 m, what area of the field did each son get?



(A) 4840 sq m (B) 1400 m^2

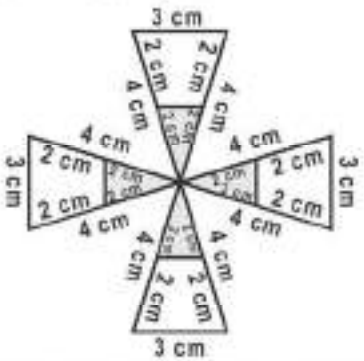
(C) 4800 m^2 (D) 3600 m^2

23 The sides of a triangle are 50 cm, 78 cm and 112 cm. Find the smallest altitude.

(A) 20 cm (B) 30 cm

(C) 40 cm (D) 50 cm

24 Suman made an arrangement with shaded and unshaded paper sheets as shown in the figure.



Find the total area of the shaded paper sheets used in making the arrangement.

(A) $\frac{9}{4}\sqrt{55}$ cm^2 (B) $\frac{8}{5}\sqrt{11}$ cm^2

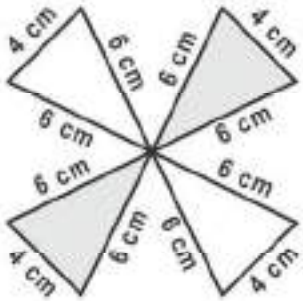
(C) $\frac{16}{9}\sqrt{55}$ cm^2 (D) $\frac{6}{5}\sqrt{11}$ cm^2

25 The base and hypotenuse of a right triangle are respectively 5 cm and 13 cm long. Find its area.

(A) 25 cm^2 (B) 28 cm^2

(C) 30 cm^2 (D) 40 cm^2

26 White and grey coloured triangular plastic sheets are used to make a toy as shown in figure. Find the difference in areas of shaded and unshaded coloured sheets used for making the toy.



(A) 1 cm^2 (B) 0 cm^2

(C) $5\sqrt{2}$ cm^2 (D) $16\sqrt{2}$ cm^2

27 What is the area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm?

(A) $\sqrt{15}$ cm^2 (B) $\sqrt{\frac{15}{2}}$ cm^2

(C) $2\sqrt{15}$ cm^2 (D) $4\sqrt{15}$ cm^2

28 ΔABC is an equilateral triangle where each side is of length 'a' units. Find its area if its perimeter is 150 m.

(A) $1000\sqrt{3}$ m^2 (B) $250\sqrt{3}$ m^2

(C) $500\sqrt{3}$ m^2 (D) $625\sqrt{3}$ m^2

29 If the area of an isosceles right triangle is 8 cm^2, what is its perimeter?

(A) $(8+\sqrt{2})$ cm (B) $(8+4\sqrt{2})$ cm

(C) $(4+8\sqrt{2})$ cm (D) $12\sqrt{2}$ cm

30 The perimeter of an equilateral triangle is 60 m. What is its area?

(A) $10\sqrt{3}$ m^2 (B) $15\sqrt{3}$ m^2

(C) $20\sqrt{3}$ m^2 (D) $100\sqrt{3}$ m^2

31 A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, what is the area of the triangle?

(A) $24\sqrt{2}$ cm^2 (B) $24\sqrt{3}$ cm^2

(C) $48\sqrt{3}$ cm^2 (D) $64\sqrt{3}$ cm^2

32 A rhombus shaped field has green grass for 48 cows to graze. If each side of the rhombus is 50 m and its longer diagonal is 80 m, how much area of the grass field will each cow be able to graze in?

(A) 150 cm^2 (B) 120 cm^2

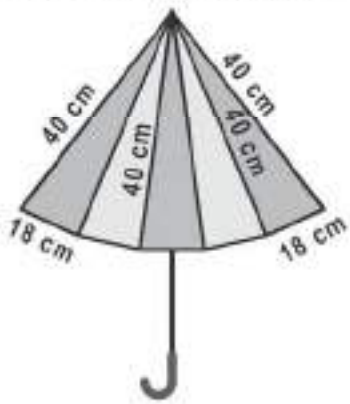
(C) 50 cm^2 (D) 100 cm^2

33 The length of the sides of ΔABC are consecutive integers. If ΔABC has the same perimeter as an equilateral triangle with a side of length 9 cm, what is the length of the shortest side of ΔABC?

(A) 4 cm (B) 6 cm

(C) 8 cm (D) 10 cm

34 An umbrella is made by stitching 12 triangular pieces of cloth of two different colours as shown in figure. Each piece measures 40 cm, 40 cm and 18 cm.



How much cloth of each colour is required for the umbrealla?

(A) $378\sqrt{3}$ cm^2 (B) $756\sqrt{3}$ cm^2

(C) $252\sqrt{3}$ cm^2 (D) $567\sqrt{3}$ cm^2

35 If the area of an equilateral triangle is $16\sqrt{3}\,cm^2$, what is the perimeter of the triangle?

(A) 48 cm (B) 24 cm
(C) 12 cm (D) 306 cm

36 Which of the following should we have in order to find the area of a triangle by Heron's formula?

(A) All the angles
(B) Altitudes
(C) Two sides and the including angle
(D) All the sides

37 If the length of a median of an equilateral triangle is x cm, find its area.

(A) $x^2\ cm^2$ (B) $\frac{\sqrt{3}}{2}x^2\ cm^2$
(C) $\frac{x^2}{\sqrt{3}}\ cm^2$ (D) $\frac{x^2}{2}\ cm^2$

38 A floral design on a floor is made up of 16 triangular tiles of sides 26 cm, 20 cm and 10 cm. The tiles are polished at the rate of 20 p per cm^2.

Find the cost of polishing the tiles.

(Take $\sqrt{14} = 3.74$.)

(A) ₹ 195.64 (B) ₹ 216.52
(C) ₹ 287.23 (D) ₹ 325.13

39 A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 15 cm, 14 cm and 13 cm and the parallelogram stands on the side of 15 cm, find the height of the parallelogram.

(A) 4.2 cm (B) 5.6 cm
(C) 8.4 cm (D) 2.1 cm

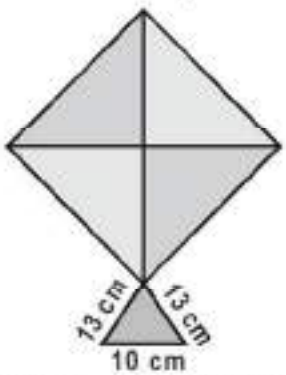
40 What is the length of each side of an equilateral triangle having an area of $9\sqrt{3}\,cm^2$?

(A) 8 cm (B) 36 cm
(C) 4 cm (D) 6 cm

41 What is the area of a triangle having two sides as 18 cm and 10 cm and perimeter is 42 cm?

(A) $3\sqrt{11}\,cm^2$ (B) $7\sqrt{11}\,cm^2$
(C) $33\sqrt{7}\,cm^2$ (D) $21\sqrt{11}\,cm^2$

42 A kite in the shape of a square with a diagonal 40 cm and an isosceles triangle of base 10 cm and side 13 cm each is to be made of three different shades as shown in the figure. Find the ratio of each shade used in making the kite.



(A) 20 : 20 : 3 (B) 10 : 10 : 3
(C) 25 : 35 : 3 (D) 9 : 11 : 1

43 What is the semiperimeter of a scalene triangle of sides k, 2k and 3k?

(A) k (B) 2k
(C) 3k (D) 4k

Previous Contest Questions

1 A triangle has perimeter 32 cm, one side is 11 cm and difference of other two sides is 5 cm. Determine its area.

(A) $4\sqrt{30}$ cm^2 (B) $8\sqrt{30}$ cm^2

(C) $6\sqrt{30}$ cm^2 (D) $5\sqrt{30}$ cm^2

2 A triangular park in a city has dimensions 10 m × 90 m × 110 m. A contract is given to a company for planting grass in the park at the rate of ₹ 4000 per hectare. What is the amount to be paid to the company?

(Take $\sqrt{2} = 1.414$ and
One hectare = 10000 m^2.)

(A) ₹ 1968.60 (B) ₹ 1698.60
(C) ₹ 1986.60 (D) ₹ 1696.80

3 Area of a given triangle is x_1 square units. If the sides of this triangle are doubled, the area of the new triangle becomes x_2 square units. Calculate the percentage increase in the area.

(A) 100 % (B) 200 %
(C) 300 % (D) 400 %

4 In a quadrilateral ABCD if AB = 5 m, BC = 5 m, CD = 6 m, AD = 6 m and diagonal AC = 6 m, what is its area?

(A) $2(4+3\sqrt{3})$ m^2 (B) $3(4+3\sqrt{3})$ m^2

(C) $5(4+3\sqrt{3})$ m^2 (D) $7(4+3\sqrt{3})$ m^2

5 In a quadrilateral ABCD if AB = 9 m, BC = 40 m, CD = 28 m, AD = 15 m and $\angle ABC = 90°$, find its area.

(A) 136 m^2 (B) 163 m^2
(C) 360 m^2 (D) 306 m^2

6 A rhombus has perimeter 100 m and one of its diagonals is 40 m. Find the area of the rhombus.

(A) 600 m^2 (B) 500 m^2
(C) 400 m^2 (D) 300 m^2

7 A field is in the shape of a trapezium whose parallel sides are 50 m and 15 m. The non-parallel sides are 20 m and 25 m. What is the area of the trapezium?

(A) $\frac{900\sqrt{6}}{7}$ m^2 (B) $\frac{1100\sqrt{6}}{7}$ m^2

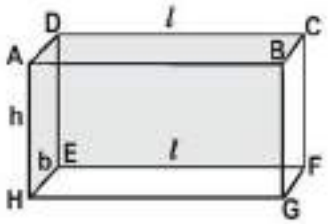
(C) $\frac{1300\sqrt{6}}{7}$ m^2 (D) $\frac{1500\sqrt{6}}{7}$ m^2

CHAPTER 12

Surface Areas and Volumes

Synopsis

- **Cuboid:** Let 'l' be the length, 'b' the breadth and 'h' the height of a cuboid, then



  (i) Sum of the lengths of the 12 edges of a cuboid $= 4(l + b + h)$

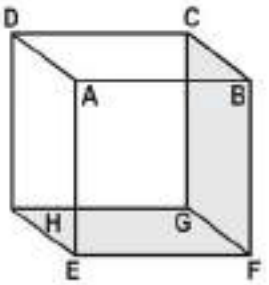
  (ii) Lateral surface area $= 2(l + b) \times h$

  (iii) Total surface area $= 2(lb + bh + hl)$

  (iv) Diagonal $= \sqrt{l^2 + b^2 + h^2}$

  (v) Volume $= lbh$

- **Cube:** If 'a' is the edge of a cube, then



  (i) Sum of the lengths of the 12 edges of a cube $= 12a$

  (ii) Lateral surface area $= 4a^2$

  (iii) Total surface area $= 6a^2$

  (iv) Diagonal $= \sqrt{3}a$

  (v) Volume $= a^3$

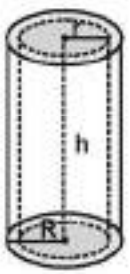
- **Cylinder:** If 'r' is the radius and 'h' is the height of a cylinder, then

  (i) Curved or lateral surface area $= 2\pi rh$

  (ii) Total surface area $= 2\pi r(h + r)$

  (iii) Volume $= \pi r^2 h$

- **Hollow cylinder:** A solid bounded by two co-axial cylinders of the same height but with different radii is called a hollow cylinder.



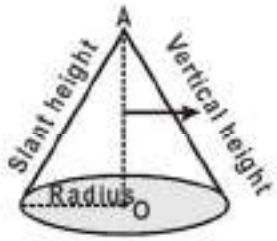
  If 'r' is the radius of the inner cylinder, 'R' is the radius of the outer cylinder and 'h' is the height of the hollow cylinder, then

  (i) Curved or lateral surface area $= 2\pi(R + r)h$

  (ii) Total surface area $= 2\pi(R + r)(h + R - r)$

  (iii) Volume $= \pi h(R^2 - r^2)$

◆ **Cone:** If 'l' is the slant height, 'r' is the radius of the base and 'h' is the vertical height of a cone, then



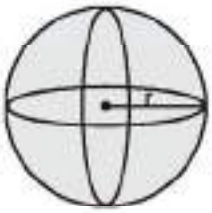
(i) Curved surface area = πrl

(ii) Total surface area = $\pi r(l+r)$

(iii) Volume = $\frac{1}{3}\pi r^2 h$

(iv) Slant height, $l = \sqrt{h^2 + r^2}$

◆ **Sphere:** A sphere is a solid which can be defined as the set of all points in space which are equidistant from a fixed point.



If 'r' is the radius of a sphere, then

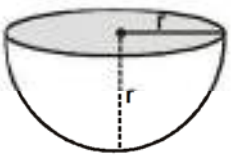
(i) Surface area = $4\pi r^2$ sq. units

(ii) Volume = $\frac{4}{3}\pi r^3$ cu. units

Note : **L.S.A and T.S.A are the same for a sphere.**

◆ **Hemisphere:** A plane passing through the centre of a sphere divides the sphere into two equal parts, each of which is called a hemisphere.

If 'r' is the radius of the sphere from which a hemisphere is cut out, then



(i) Curved surface area = $2\pi r^2$ sq. units

(ii) Total surface area = $3\pi r^2$ sq. units

(iii) Volume = $\frac{2}{3}\pi r^3$ cu. units.

Multiple Choice Questions A B C D

1. If volume and surface area of a sphere are numerically equal, find its radius.
   (A) 2 units (B) 3 units
   (C) 4 units (D) 5 units

2. The base radius of a cylinder is $1\frac{2}{3}$ times its height. The cost of painting its C.S.A. at 2 paise/cm² is ₹ 92.40. What volume of the paint is required?
   (A) 80850 cm³ (B) 80580 cm³
   (C) 80508 cm³ (D) 85800 cm³

3. The total surface area of a cylinder of height 6.5 cm is 220 sq cm. Find its volume.
   (A) 25.025 cm³ (B) 2.5025 cm³
   (C) 2502.5 cm³ (D) 250.25 cm³

4. The height of a cylinder is 15 cm. The curved surface area is 660 sq. cm. Find its radius. (Take π as $\frac{22}{7}$)
   (A) 7 cm (B) 9 cm
   (C) 6 cm (D) 11 cm

5. A cylindrical vessel contains 49.896 litres of liquid. Cost of painting its C.S.A. at 2 paise/sq cm is ₹ 95.04. What is its total surface area?
   (A) 5724 cm² (B) 7524 cm²
   (C) 5742 cm² (D) 7254 cm²

6. What is the ratio of volumes of two cones with the same radii?
   (A) $h_1 : h_2$ (B) $r_1 : r_2$
   (C) $s_1 : s_2$ (D) 1 : 2

7. What is the length of the longest pole that can be put in a room of dimensions 10 m × 10 m × 5 m?
   (A) 15 m (B) 16 m (C) 10 m (D) 12 m

8. The cost of painting the T.S.A. of cone at 5 ps/cm² is ₹ 35.20. Determine the volume of the cone if its slant height is 25 cm.
   (A) 1223 cm² (B) 1232 cm²
   (C) 1323 cm² (D) 1332 cm²

9. A vessel is conical in shape. If its volume is 33.264 litres and height is 72 cm, what is the cost of repairing its C.S.A. at ₹ 12/sq. m?
   (A) ₹ 5.94 (B) ₹ 6.94
   (C) ₹ 7.95 (D) ₹ 7.59

10. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the areas of the sheet required to make 10 such caps.
    (A) 5000 cm² (B) 6200 cm²
    (C) 5500 cm² (D) 6000 cm²

11. A hemispherical bowl is made of steel of 0.25 cm thickness. The inner radius of the bowl is 5 cm. Find the volume of steel used.
    (A) 42.15 cm³ (B) 41.52 cm³
    (C) 41.28 cm³ (D) 45.21 cm³

12. How many litres of water flows out through a pipe having an area of cross-section of 5 cm² in one minute, if the speed of water in pipe is 30 cm/sec?
    (A) 9 litres (B) 15 litres
    (C) 30 litres (D) 3 litres

13. A cylindrical vessel of diameter 9 cm has some water in it. A cylindrical iron piece of diameter 6 cm and height 4.5 cm is dropped in it. After it was completely immersed, find the rise in the level of water.
    (A) 0.8 cm (B) 2 cm
    (C) 1 cm (D) 0.4 cm

14. A well, 14 m deep is 2 m in radius. Find the cost of cementing the inner curved surface at the rate of ₹ 2 per square metre.
    (A) ₹ 242 (B) ₹ 352 (C) ₹ 464 (D) ₹ 294

15. The radius of a sphere is increased by P%. What is the increase in its surface area?
    (A) P % (B) P² %
    (C) $\left(2P + \frac{P^2}{100}\right)\%$ (D) $\frac{P^2}{2}\%$

16 The diameter of two cones are equal. If their slant heights are in the ratio 5 : 4, find the ratio of their curved surface areas.

(A) 2 : 3 (B) 4 : 5
(C) 5 : 4 (D) 3 : 2

17 A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. Find the capacity of the cup.

(A) 622 cm^3 (B) 504 cm^3
(C) 645 m^3 (D) 592 m^3

18 The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions? (Take $\pi = \frac{22}{7}$.)

(A) 11 sq. m (B) 16 sq. m
(C) 28 sq. m (D) 44 sq. m

19 The volumes of two spheres are in the ratio 64 : 27. Find the difference of their surface areas, if the sum of their radii is 7 units.

(A) 28 π sq. units (B) 64 π sq. units
(C) 88 π sq. units (D) 4 π sq. units

20 Into a conical tent of radius 7 m and vertical height 4.5 m, how many full bags of rice can be emptied, if volume of each bag is 1.5 m^3.

(A) 120 bags (B) 144 bags
(C) 154 bags (D) 172 bags

21 Find the largest volume (in cm^3) of a cube that can be enclosed in a sphere of diameter 2 cm.

(A) 1 (B) $2\sqrt{2}$ (C) π (D) $\frac{8}{3\sqrt{3}}$

22 Find the curved surface of a right circular cone, whose slant height and the base radius are 25 cm and 7 cm respectively.

(A) 420 cm^2 (B) 550 cm^2
(C) 460 cm^2 (D) 580 cm^2

23 A right circular cylinder has a height of 21 cm and base radius of 5 cm. Find the curved surface area of the cylinder.

(A) 230 cm^2 (B) 660 cm^2
(C) 550 cm^2 (D) 450 cm^2

24 The area of the curved surface of a right circular cylinder is 4400 cm^2 and the circumference of its base is 110 cm. Find the height of the cylinder.

(A) 30 cm (B) 10 cm
(C) 20 cm (D) 40 cm

25 The circumference of the base of a 9 m high wooden solid cone is 44 m. Find the slant height of the cone.

(A) $\sqrt{120}$ m (B) $\sqrt{130}$ m
(C) $\sqrt{150}$ m (D) $7\sqrt{5}$ m

26 If A_1, A_2 and A_3 denote the areas of three adjacent faces of a cuboid, find its volume.

(A) $A_1 A_2 A_3$ (B) $2A_1 A_2 A_3$
(C) $\sqrt{A_1 A_2 A_3}$ (D) $\sqrt[3]{A_1 A_2 A_3}$

27 If 5 men are in a room of dimensions 3 m × 4 m × 10 m, what is the amount of air available for each of them?

(A) 48 m^3 (B) 36 m^3
(C) 24 m^3 (D) 120 m^3

28 If l is the length of a diagonal of a cube of volume V, what is the relation between l and V?

(A) $3V = l^3$ (B) $\sqrt{3V} = l^3$
(C) $3\sqrt{3}V = 2l^3$ (D) $3\sqrt{3}V = l^3$

29 If each edge of a cube is increased by 50%, what is the percentage increase in its surface area?

(A) 50% (B) 75%
(C) 100% (D) 125%

30 A cylinder of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm. Find the radius of the ball. (Take $\pi = \frac{22}{7}$.)

(A) 8 cm (B) 9 cm
(C) 10 cm (D) 11 cm

31 Find the volume of a cube whose surface area is 150 cm^2.

(A) $25\sqrt{5}\ cm^3$ (B) 64 cm^3
(C) 125 cm^3 (D) 27 cm^3

32 The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the volume of the cylinder. $\left(\text{Take } \pi = \frac{22}{7}.\right)$

(A) 22 cm^3 (B) 44 cm^3
(C) 88 cm^3 (D) 11 cm^3

33 A spherical ball of diameter 21 cm, is melted and recasted into cubes, each of side 1 cm. Find the number of cubes thus formed. (Use $\pi = 22/7$.)

(A) 2057 (B) 1962
(C) 4851 (D) 3272

34 If the areas of the adjacent faces of a rectangular block are in the ratio 2 : 3 : 4 and its volume is 9000 cm^3, what is the length of the shortest edge?

(A) 30 cm (B) 20 cm
(C) 15 cm (D) 10 cm

35 If each edge of a cube, of volume, V, is doubled, find the volume of the new cube.

(A) 2V (B) 4V (C) 6V (D) 8V

36 Two cubes of side 6 cm each are joined end to end. Find the surface area of the resultant cuboid.

(A) 200 cm^2 (B) 420 cm^2
(C) 360 cm^2 (D) 270 cm^2

37 If each edge of a cube of surface area S is doubled, what is the surface area of the new cube?

(A) 2S (B) 4S
(C) 6S (D) 8S

38 Find the cost of constructing a wall 8 m long, 4 m high and 20 cm thick at the rate of ₹ 25 per m^2.

(A) ₹ 16 (B) ₹ 80
(C) ₹ 160 (D) ₹ 320

39 The circumference of the base of 9 m high wooden solid cone is 44 m. Find its volume. (Use $\pi = \frac{22}{7}$.)

(A) 235 m^3 (B) 456 m^3
(C) 365 m^3 (D) 462 m^3

40 If 10 cubic metres of clay is uniformly spread on a land of area 10 ares, what is the rise in the level of the ground?

(A) 1 cm (B) 10 cm
(C) 100 cm (D) 1000 cm

41 Volume of a cuboid is 12 cm^3. Find the volume (in cm^3) of the cuboid if its sides are double.

(A) 24 (B) 48
(C) 72 (D) 96

42 The largest sphere is carved out of a cube of side 7 cm. Find the volume of the sphere. (Take $\pi = 3.14$.)

(A) 152.74 cm^3 (B) 243.41 cm^3
(C) 179.67 cm^3 (D) 195.01 cm^3

43 On a particular day, the rain fall recorded on a terrace 6 m long and 5 m broad is 15 cm. Find the quantity of water collected on the terrace.

(A) 300 litres (B) 450 litres
(C) 3000 litres (D) 4500 litres

44 The height of sand in a cylindrical box drops 3 inches when 1 cubic foot of sand is poured out. What is the diameter, in inches, of the cylinder?

(A) $\frac{24}{\sqrt{\pi}}$ (B) $\frac{48}{\sqrt{\pi}}$
(C) $\frac{32}{\sqrt{\pi}}$ (D) $\frac{48}{\pi}$

45 If the diameter of the base of a closed right circular cylinder is equal to its height h, find its total surface area.

(A) $2\pi h^2$ (B) $\frac{3}{2}\pi h^2$
(C) $\frac{4}{3}\pi h^2$ (D) πh^2

46 The radius of a wire is decreased to its one-third. If its volume remains the same, by how many times will its length increase?

(A) 3 times (B) 6 times
(C) 9 times (D) 27 times

Previous Contest Questions

1 If the lateral surface area of a cube is 1600 cm^2, what is its edge?

(A) 15 cm (B) 18 cm
(C) 20 cm (D) 25 cm

2 A cylinder and a cone have equal base radii and equal heights. If their curved surface areas are in the ratio 8 : 5, what is the ratio of their radii to heights?

(A) 8 : 5 (B) 4 : 3
(C) 3 : 4 (D) 5 : 8

3 If a square card board piece of side 4 cm is rotated 360° about one of its sides, what is the volume of the solid so formed?

(A) 64π cm^3 (B) 16π cm^3
(C) 32π cm^3 (D) 128π cm^3

4 The radius of a cylinder is doubled and its height is halved. What is the change in its curved surface area?

(A) Halved
(B) Doubled
(C) Remains the same
(D) Becomes four times

5 A right circular cylindrical tunnel of diameter 2 m and length 40 m is to be constructed from a sheet of iron. What is the area of the iron sheet required in m^2?

(A) 40π (B) 60π
(C) 80π (D) 100π

6 A cube of edge 'k' is divided into 'n' equal cubes. Determine the edge of the new cube.

(A) $\sqrt{nk}$ (B) $\frac{k}{\sqrt[3]{n}}$
(C) $\sqrt[3]{nk}$ (D) $\frac{\sqrt[3]{n}}{k}$

7 Two similar spherical drops each of radius 'r' cm are combined to form a bigger drop. Find the radius of the bigger drop.

(A) $\sqrt[3]{2r}$ cm (B) $\sqrt{2r}$ cm
(C) $\frac{r}{\sqrt[3]{2}}$ cm (D) $\frac{\sqrt[3]{2}}{r}$ cm

8 What is the total surface area of a hollow hemisphere of external and internal radii R and r?

(A) $\pi(R^2 + 3r^2)$ (B) $\pi(3R^2 + r^2)$
(C) $\pi(3r^2 + 2R^2)$ (D) $\pi(2r^2 + 3R^2)$

9 If the surface area of a cube increases by 1%, what is the percentage increase in its volume?

(A) 0.5 % (B) 1 %
(C) 1.5 % (D) 2 %

10 Find the ratio of volume of a cylinder to the volume of a cone to the volume of a hemisphere of the same radius and height.

(A) 1 : 1 : 1 (B) 3 : 2 : 1
(C) 1 : 2 : 3 (D) 3 : 1 : 2

This page is intentionally left blank.

CHAPTER 13

Statistics

Synopsis

- Statistics, a branch of mathematics is useful in the collection, classification and interpretation of data.

  The word statistics is used in two different senses:

  (i) In plural sense, statistics means data.

  (ii) In singular sense, statistics is the science which deals with the collection, presentation, analysis and interpretation of some numerical data.

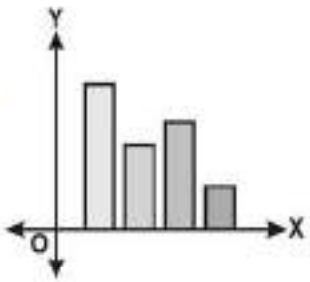
- **Data:** The word data means information in the form of numerical figures or a set of given facts.
- A statistical data is collected by a single person or by a group of persons having uniform approach. Someone who carries out the job of collection of data is called an investigator.
- **Primary data:** The data collected for a definite purpose by an investigator or a group of investigators directly, is called primary data.
- **Secondary data:** The data collected from a source which already has the information stored, is called secondary data.
- **Raw data:** An information of facts and figures collected for a definite purpose in any manner is termed as raw data.
- **Tabulation:** Arranging the data in a systematic way in tabular form is called tabulation.
- **Observation:** Each numerical figure in a data is called an observation.
- **Frequency:** The number of times a particular observation occurs in the data is called its frequency.
- **Range:** The difference of the greatest and the least values of the given data is called its range.
- To determine the frequency of each distinct entry of the data, we draw tally marks in the form of small vertical lines, called bars.
- **Grouped frequency distribution:** When a data has a large number of values (entries) and most of them are distinct, it becomes inconvenient to present it in the form of ungrouped frequency distribution. The data is condensed into a finite number of groups called classes of the data. Presenting data in this form is called a grouped frequency distribution.
- **Frequency of a grouped data:** The number of entries of the data having their values lying in a class is defined as the frequency of the class. The table, in which the corresponding frequencies are written against each class, is called a frequency distribution of the given data.
- **Types of frequency distribution:** There are two types of grouped frequency distributions of a data. They are:

(i) Inclusive method or discrete form.

(ii) Exclusive method or continuous form.

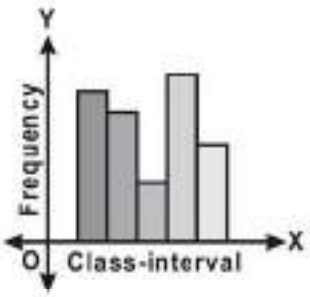
- **Lower limit and upper limit:** In the classes 0-9, 10-19, 20-29 and 30-39, the values 0, 10, 20 and 30 are called the lower limits of the classes and the values 9, 19, 29 and 39 are called the upper limits of the classes.
- Presentation of a data in the form of continuous grouped frequency distribution has some advantage in comparison to the discrete grouped frequency distribution.
- **Class mark (or Midmark or Midvalue) of a class:** In a grouped frequency distribution, the class mark or the midmark of a class a - b is equal to the value $\frac{a+b}{2}$.
- **Width of a class:** The difference of the upper limit and the lower limit of the class is called the width of a class.
- **Cumulative frequency table:** The total of the frequency of a class and the frequencies of all the classes proceding that class is called the cumulative frequency of that class. The table showing the cumulative frequencies recorded against each class is called cumulative frequency table.
- **Graphical representation of data:** The main features of a frequency distribution can be easily presented with the help of graphical representation such as bar graphs, histograms, frequency polygons, etc.,

(i) **Bar graph:** A bar graph (diagram) is a pictorial representation of the numerical data by a series of bars or rectangles of uniform width standing on the same horizontal (or vertical) base line with equal spacing between the bars. Each rectangle or bar represents only one value of the data.



(ii) **Histogram:** A histogram is a graphical representation of grouped frequency distribution for continuous classes in the form of rectangles with class intervals as bases and corresponding frequencies as heights.

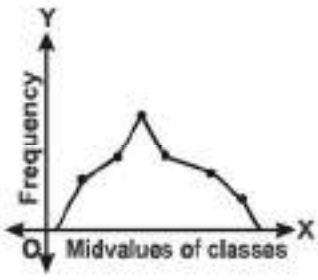
Note : **In a histogram, rectangles are drawn leaving no gap in between consecutive rectangles.**



(iii) **Frequency polygon:** If the points pertaining to the midvalues of the classes of a frequency distribution, and the corresponding frequencies are plotted on a graph sheet and these points are joined by straight lines, then the figure formed is called a frequency polygon.

Frequency polygons are useful for large and continuous data.

It is also useful for comparing two different sets of data of the same nature.



◆ **Measures of central tendency (of ungrouped data):**

(i) **Arithmetic Mean (A.M.) or Mean :** Arithmetic mean or simply mean is the most common and widely used measure of central tendency. It is found by adding all the values of the observations and dividing it by the total number of observations. It is denoted by $\bar{x}$ (Read as x bar.).

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

If $x_1, x_2, \ldots, x_n$ are 'n' observations, then Arithmetic Mean

$$\text{A.M. } (\bar{x}) = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}.$$

(ii) **Median :** If the observations are arranged in increasing or decreasing order, then median is defined as the middle-most observation.

If the number of observations is odd, median is the value of the $\left(\frac{n+1}{2}\right)^{th}$ observation.

If the number of observations is even, median is the mean of the values of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

(iii) **Mode:** An observation with the highest frequency is called the mode.

Multiple Choice Questions

A B C D

1. Which of the following is the number of times a particular item occurs in a class interval?

(A) Mean
(B) Frequency
(C) Cumulative frequency
(D) Median

2. What are the groups into which large data is condensed called?

(A) Class limits (B) Classes
(C) Class size (D) Class width

3. Find the mean of first 5 whole numbers.

(A) 2.5 (B) 3
(C) 1.5 (D) 2

4. The relative humidity (in %) of a city for 10 days is given in the box.

92.1	97.1	95.7	93.3	89
96.2	94.9	97.3	92.1	98.3

Determine its range.

(A) 9.3 (B) 9.6
(C) 9.5 (D) 9.8

5. The demand for different shirt sizes, as obtained from a survey, is given in the table.

Size	38	39	40	41	42	43	44
No. of persons	26	36	20	15	13	7	5

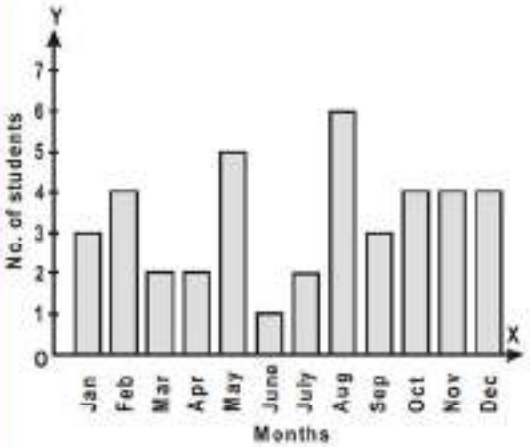
Find the modal shirt size.

(A) 39 (B) 40
(C) 44 (D) 42

6. The class marks of a frequency distribution are given as 15, 20, 25, ... Find the class corresponding to the class mark 20.

(A) 12.5 – 17.5 (B) 17.5 – 22.5
(C) 18.5 – 21.5 (D) 19.5 – 20.5

(7-10): The bar graph given shows the months of birthdays of 40 students of a class.



Answer the following questions based on the graph.

7. How many students were born in August?

(A) 6 (B) 4
(C) 5 (D) 3

8. In which month were the minimum number of students born?

(A) February (B) June
(C) December (D) May

9. In which months were at least 5 students born?

(A) June and October
(B) March and November
(C) February and September
(D) May and August

10. In which months was the difference in the number of students born the same as that in October and November?

(A) February and January
(B) May and July
(C) March and April
(D) August and September

11 Find the median of the first ten prime numbers.

(A) 24 (B) 14
(C) 12 (D) 22

12 In a school 90 boys and 30 girls appeared for a public examination. The mean marks of boys was found to be 45% whereas the mean marks of girls was 70%. What is the average marks % of the school?

(A) 61.50% (B) 51.25%
(C) 40.50% (D) 51.52%

13 What is a graph drawn with the midpoints of the top sides of the rectangles forming the histogram of a frequency distribution called?

(A) Bar graph
(B) Ogive
(C) Frequency polygon
(D) Frequency curve

14 What do you call the value in a data around which the values of all the other observations tend to concentrate?

(A) Common value
(B) Range
(C) Measure of central tendency
(D) Midvalue

15 Of the class intervals 10 – 20 and 20 – 30, the number 20 is included in which of the following?

(A) 10 – 20 (B) 20 – 30
(C) 15 – 20 (D) Both (A) and (B)

16 Find the arithmetic mean of 30, 36, 39, 23 and 27.

(A) 28 (B) 20
(C) 31 (D) 35

17 The median of given observations arranged in ascending order is 25.

11, 13, 15, 19, p + 2, p + 4, 30, 35, 39, 46.

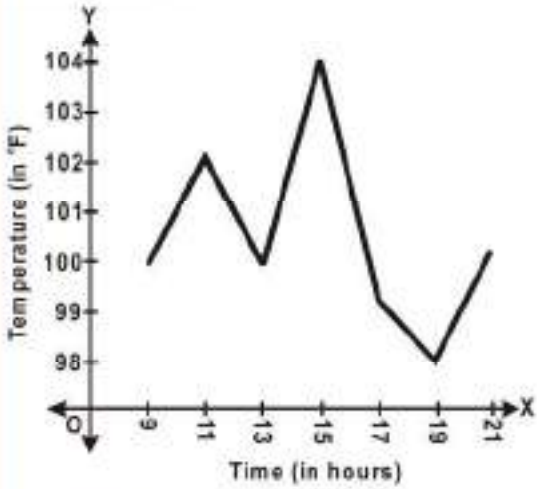
Find p.

(A) 22 (B) 24
(C) 21 (D) 26

18 Which one of the following is not a measure of central tendency?

(A) Mean (B) Range
(C) Median (D) Mode

(19-22): The following is a chart showing the temperature of a patient recorded at different times.



Read the temperature chart and answer the given questions.

19 What is the temperature of the patient at 21 hrs?

(A) 100°F (B) 101°F
(C) 102°F (D) 103°F

20 What is the percentage increase in temperature between 9 hrs and 15 hrs?

(A) 4% (B) 3% (C) 2% (D) 1%

21 What is the percentage decrease in temperature between 17 hrs and 19 hrs?

(A) 1.02% (B) 1.03%
(C) 1.04% (D) 1.01%

22 What is the average temperature of the patient between 13, 15 and 17 hrs?

(A) 103 °F (B) 102 °F
(C) 101 °F (D) 100 °F

23 The mean of first 8 observations is 18 and last 8 observations is 20. If the mean of all 15 observations is 19, find the 8^{th} observation.

(A) 18 (B) 12 (C) 19 (D) 20

24 A grouped frequency distribution table with classes of equal sizes using 63 – 72 (72 included) as one of the classes is constructed for the following data.

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 96, 102, 110, 88, 74, 112, 14, 34, 44

Find the number of classes in the distribution.

(A) 9 (B) 11
(C) 10 (D) 12

25 The mid-point of a class is m and l is the upper class limit in a continuous frequency distribution. Which of the following would be the lower class limit of the class?

(A) $2m + l$ (B) $2m - l$
(C) $m - l$ (D) $m - 2l$

26 The width of each of nine classes in a frequency distribution is 2.5 and the lower class boundary of the lowest class is 10.6. What is the upper class boundary of the highest class?

(A) 35.6 (B) 33.1
(C) 30.3 (D) 28.1

27 In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. What is the lower limit of the class?

(A) 6 (B) 7 (C) 8 (D) 12

28 In a frequency distribution, ogives are graphical representation of which of the following?

(A) Frequency
(B) Relative frequency
(C) Cumulative frequency
(D) Raw data

29 Apart from plotting frequencies of the class intervals, which of the following are used to construct a frequency polygon?

(A) Upper limits of the classes
(B) Lower limits of the classes
(C) Any values of the classes
(D) Mid values of the classes

30 The mean wage of 150 labourers working in a factory running three shifts with 60, 40 and 50 labourers is ₹ 114. The mean wage of 60 labourers working in the first shift is ₹ 121.50 and that of 40 labourers working the second shift is 107.75. Find the mean wage of those who are working in the third shift.

(A) ₹ 110 (B) ₹ 100
(C) ₹ 120 (D) ₹ 115.75

31 The mean of n observations is $\bar{X}$. If each observation is multiplied by k, what is the mean of new observations?

(A) $k\bar{X}$ (B) $\frac{\bar{X}}{k}$
(C) $\bar{X} + k$ (D) $\bar{X} - k$

32 The mean of 75 numbers is 25. If each number is divided by 5, find the new mean.

(A) 5 (B) 20
(C) 8 (D) 15

33 For which set of numbers do the mean, median and mode have the same value?

(A) 2, 2, 2, 4 (B) 1, 3, 3, 3, 5
(C) 1, 1, 2, 5, 6 (D) 1, 1, 1, 2, 5

34 Find the difference between arithmetic means of all even and odd numbers between 50 and 60.

(A) 2 (B) 0
(C) 1 (D) 3

35 For the set of numbers 2, 2, 4, 5 and 12 which of the following statements is true?

(A) Mean = Median (B) Mean > Mode
(C) Mean < Mode (D) Mode = Median

36 A cricketer has a mean score of 60 runs in ten innings. Find the number of runs that are to be scored in the eleventh inning to raise the mean score to 62.

(A) 62 (B) 78
(C) 58 (D) 82

37 Which of the following is the empirical relation between mean, mode and median?

(A) Mode = 3 Median – 2 Mean
(B) Mode = 2 Median – 3 Mean
(C) Median = 3 Mode – 2 Mean
(D) Mean = 3 Median – 2 Mode

38 Find the median of the data given.

0, 2, 2, 2, –3, 5, –1, 5, 5, –3, 6, 6, 5, 6

(A) 0 (B) –1.5
(C) 2 (D) 3.5

39 The mean of 20 numbers is 40. If 5 is subtracted from every number, what will be the new mean?

(A) 45 (B) 40
(C) 20 (D) 35

40 The mean of a, b, c, d and e is 28. If the mean of a, c, and e is 24, what is the mean of b and d?

(A) 31 (B) 32
(C) 33 (D) 34

41 What is the algebraic sum of the deviations of a set of n values from their mean?

(A) 0 (B) $n-1$
(C) n (D) $n+1$

42 The mean of 50 observations was 250. It was detected on checking that the value of 165 was wrongly copied as 115 for computation of mean. Find the correct mean.

(A) 215 (B) 151
(C) 156 (D) 251

43 The mean of the data $x_1, x_2, x_3, \ldots, x_n$ is 'a'. Find the mean of the data $x_1 + \alpha, x_2 + \alpha, x_3 + \alpha, \ldots, x_n + \alpha$

(A) $a + \alpha$ (B) $a\ \alpha$
(C) $a + \alpha$ (D) $a - \alpha$

44 If the mean of 9 observations p, p + 2, p + 4, p + 6, p + 8, p – 2, p – 4, p – 6 and p – 8 is 10. Find the mean of the least 5 observations.

(A) 6 (B) 25
(C) 10 (D) 9

45 The median of the data 26, 56, 32, 33, 60, 17, 34, 29, 45, is 33. If 26 is replaced with 62, what is the new median?

(A) 34 (B) 29
(C) 32 (D) 33

46 The mean of 6 numbers is 20. If one number is deleted, their mean is 15. Find the deleted number.

(A) 45 (B) 52
(C) 20 (D) 36

47 The number of children in 10 families of a locality are 1, 4, 3, 3, 4, 2, 2, 3, 3 and 5. Find the mean number of children per family.

(A) 3 (B) 4
(C) 1 (D) 2

Previous Contest Questions

1 The mean of some observations is 50. If their sum is 600, what is the number of observations?

(A) 8 (B) 12
(C) 16 (D) 10

2 Find the class mark of the class 90 – 120.

(A) 90 (B) 105
(C) 115 (D) 120

3 If the mean of the given data is 5.3, find the missing frequency 'p'.

x	4	8	6	7
f	11	2	3	p

(A) 1.7 (B) 3
(C) 6.8 (D) 4

CHAPTER 14

Probability

Synopsis

- **Random experiment:**

  An experiment in which all possible outcomes are known and the exact outcome cannot be predicted in advance is called a random experiment.

  e.g., (1) Tossing a coin. (2) Rolling an unbiased die.

- **Sample space:**

  The set S of all possible outcomes of a random experiment is called the sample space.

  e.g., (1) In tossing a coin, sample space (S) = {H, T}

  (2) In rolling a die, sample space (S) = {1, 2, 3, 4, 5, 6}

- **Probability :**

  Probability is a concept which numerically measures the degree of certainty of the occurrence of events.

- **Definition of probability :**

  In a random experiment, let S be the sample space and let E be the event. Then probability of occurence of E = $P(E) = \frac{n(E)}{n(S)}$, where

  n(E) is the number of elements favourable in E, and
  n(S) is the number of distinct elements in S.

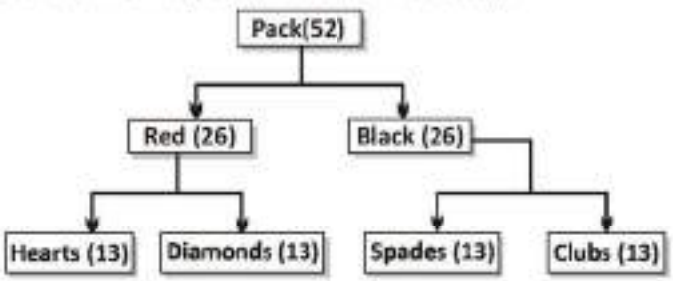
> ***Note*** : **(1)** $0 \leq P(E) \leq 1$
>
> **(2) If P(E) = 1, then the event E is called a certain event and if P(E) = 0, then the event E is called an impossible event.**

- **Important points:**

  (a) A coin has 2 sides – one side is head (H) and the other side is tail (T).

  (b) A die is a cube with 6 faces - with numbers (or dots) 1 to 6 on each face.

  (c) Description of a normal pack (or deck) of cards (52):



The cards in each suit are Ace(A), King(K), Queen(Q), Jack(J), 10, 9, 8, 7, 6, 5, 4, 3 and 2.

The cards A, J, Q and K are called honours and the cards 2, 3, 4, 5, 6, 7, 8, 9 and 10 are called numbered cards. The cards J, Q and K are called face cards.

Multiple Choice Questions

A B C D

1. In an experiment, what is the sum of probabilities of all events?

(A) 0.5 (B) 1 (C) – 2 (D) $\frac{3}{8}$

2. In a throw of a die, what is the probability of getting a prime number?

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 6

3. Find the probability that a vowel selected at random in the English alphabet is an "i".

(A) $\frac{1}{5}$ (B) $\frac{1}{26}$ (C) $\frac{1}{6}$ (D) $\frac{4}{5}$

4. Determine the probability of three coins falling all heads up when tossed simultaneously.

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{1}{8}$ (D) $\frac{7}{8}$

5. The probability of happening of an event is 45%. Find the probability of that event.

(A) 45 (B) 4.5 (C) 0.45 (D) 0.045

6. When two dice are thrown, what is the probability of getting a number always greater than 4 on the second dice?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{36}$ (D) $\frac{2}{3}$

7. Determine the probability of getting an even number when a die is rolled.

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

8. Two numbers are chosen from 1 to 5. Find the probability for the two numbers to be consecutive.

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{1}{10}$ (D) $\frac{3}{5}$

9. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing a '10' of a black suit.

(A) $\frac{1}{13}$ (B) $\frac{1}{26}$ (C) $\frac{5}{13}$ (D) $\frac{1}{52}$

10. Two dice are thrown at a time. What is the probability that the difference of the numbers shown on the dice is 1?

(A) $\frac{5}{18}$ (B) $\frac{1}{36}$ (C) $\frac{1}{6}$ (D) $\frac{7}{36}$

11. A bag contains 3 white and 5 red balls. If a ball is drawn at random, find the probability that it is red.

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{3}{15}$ (D) $\frac{5}{15}$

12. From a normal pack of cards, a card is drawn at random. What is the probability of getting a jack or a king?

(A) $\frac{1}{26}$ (B) $\frac{1}{52}$ (C) $\frac{2}{13}$ (D) $\frac{1}{13}$

13. A card is drawn from a packet of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a square.

(A) $\frac{1}{10}$ (B) $\frac{9}{100}$ (C) $\frac{1}{100}$ (D) $\frac{1}{50}$

14. Find the probability for a randomly selected number out of 1, 2, 3, 4, ..., 25 to be a prime number.

(A) $\frac{2}{25}$ (B) $\frac{23}{25}$ (C) $\frac{2}{5}$ (D) $\frac{9}{25}$

15. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, find the number of blue balls in the bag.

(A) 15 (B) 12 (C) 5 (D) 10

16. A book containing 100 pages is opened at random. Find the probability that a doublet page is found.

(A) $\frac{9}{100}$ (B) $\frac{9}{10}$ (C) $\frac{1}{10}$ (D) $\frac{1}{5}$

17. If a coin is tossed twice, what is the probability of getting atleast one head?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

18 What is the probability of getting a number greater than 2 or an even number in a single throw of a fair die?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{1}{6}$

19 Find the probability that in a family of 3 children, there will be at least one boy.

(A) $\frac{3}{4}$ (B) $\frac{1}{8}$ (C) $\frac{3}{8}$ (D) $\frac{5}{8}$

20 What is the probability that a non-leap year contains 53 Saturdays?

(A) $\frac{2}{7}$ (B) $\frac{1}{7}$ (C) $\frac{2}{365}$ (D) $\frac{1}{365}$

21 A bag contains 12 balls out of which x are white. If one ball is drawn at random, what is the probability that it will be a white ball?

(A) x (B) $\frac{x}{2}$ (C) $\frac{x}{12}$ (D) $12x$

22 A coin is tossed 150 times and data about the outcomes is given in the table.

Outcomes	H	T
Frequency	90	60

What is the probability of getting a head in a trial?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

(23-24): Two coins are tossed simultaneously 80 times and data is recorded in the table given.

Number of heads in a trial	0	1	2
Frequency	20	35	25

23 What is the probability of getting two heads?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{7}{16}$ (D) $\frac{5}{16}$

24 What is the probability of getting atleast one head?

(A) $\frac{3}{4}$ (B) $\frac{5}{16}$ (C) $\frac{1}{2}$ (D) $\frac{2}{9}$

(25-26): A die is tossed 100 times and the data is recorded in the table.

Outcomes	1	2	3	4	5	6
Frequency	20	15	20	15	20	10

25 What is the probability that we get an even number in a trial?

(A) 0.4 (B) 0.5
(C) 0.3 (D) 0.35

26 What is the probability of getting a number less than 3?

(A) 0.2 (B) 0.25
(C) 0.35 (D) 0.5

27 A dental survey of 400 students in a school revealed that 80 of them had two or more cavities. If the total school enrolment is 1500, about how many students would you expect to have two or more cavities?

(A) 300 (B) 800
(C) 500 (D) 600

28 The most recent freshman class at Loyola consists of 880 students. Of these, 500 identified themselves as "smokers". Compute the empirical probability that a randomly selected freshman student from this class is not a "smoker".

(A) $\frac{8}{22}$ (B) $\frac{19}{44}$ (C) $\frac{15}{44}$ (D) $\frac{9}{44}$

29 If the probability of winning a game is 0.35, what is the probability of losing it?

(A) 0.55 (B) 0.75
(C) 0.65 (D) 0.56

30 There are 50 students in a class and their annual result is presented in the given table.

Result	Fail	Pass
No. of Students	7	43

If a student is selected at random, find the probability that the student has passed the examination.

(A) 0.85 (B) 0.68
(C) 0.86 (D) 0.76

31 If P (E) = 0.37, find P (not E).

(A) −0.37 (B) 0.73
(C) −0.73 (D) 0.63

32 1000 tickets of a lottery were sold and there are 5 prizes on these tickets. If Monu has purchased one lottery ticket, what is her probability of winning a prize?

(A) 0.005 (B) 0.75
(C) 0.055 (D) 0.5

33 When a thumbtack is tossed, there are two possible outcomes. If the empirical probability of "point up" is fixed to be 0.73, what should be the probability of "point down"?

(A) 0.37 (B) 0.20
(C) 0.27 (D) 0.51

34 A die is thrown once. What is the probability of getting a number 4 or 5?

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$
(C) $\frac{1}{6}$ (D) $\frac{1}{2}$

35 A boy tosses a coin 1000 times and finds that it comes up tail 453 times. Find the probability of getting up head.

(A) 0.457 (B) 0.453
(C) 0.574 (D) 0.547

36 In a cricket match, a batsman hits a sixer 8 times out of 32 balls played. Find the probability that a sixer is not hit in a ball.

(A) 0.75 (B) 0.25
(C) − 0.25 (D) 0.5

37 A coin is tossed 15 times and observed that head comes up 11 times. What is the probability for a tail to come up?

(A) $\frac{11}{15}$ (B) $\frac{4}{15}$ (C) $\frac{15}{4}$ (D) $\frac{15}{11}$

38 A die is tossed once. Which of the following is the probability of getting 3?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{6}$ (D) $\frac{4}{6}$

(39-41): Two coins are tossed 90 times and data recorded as given in the table.

Number of heads in a trial	0	1	2
Frequency	10	25	55

39 What is the probability of getting one head?

(A) 0 (B) $\frac{1}{9}$ (C) $\frac{11}{18}$ (D) $\frac{5}{18}$

40 What is the probability of getting two heads?

(A) $\frac{8}{9}$ (B) $\frac{11}{18}$ (C) $\frac{5}{18}$ (D) $\frac{1}{9}$

41 Find the proability of getting atleast one head.

(A) $\frac{8}{9}$ (B) $\frac{1}{9}$ (C) $\frac{5}{18}$ (D) $\frac{11}{18}$

42 Find the probability of not getting 1 or 6 in a single toss of a die.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

43 A child has a block in the shape of a cube with one letter written on each face as shown.

A	B	C	D	A	E

If the cube is thrown once, what is the probability of getting A?

(A) $\frac{1}{3}$ (B) $\frac{2}{8}$
(C) $\frac{2}{5}$ (D) $\frac{2}{3}$

44 A bag contains 3 red balls, 5 black balls and 4 white balls. A ball is drawn at random from the bag. What is the probability of getting a black ball?

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$
(C) $\frac{7}{12}$ (D) 1

45 A die is rolled 100 times and the data is recorded as given in the table.

Outcomes	1	2	3	4	5	6
Frequency	22	13	20	10	25	10

Which of the following is the probability of getting an odd number in a trial?

(A) $\frac{11}{50}$ (B) $\frac{1}{5}$

(C) $\frac{67}{100}$ (D) $\frac{1}{4}$

46 A card is drawn at random from a pack of well shuffled 52 cards. What is the probability of getting a queen of red suit?

(A) $\frac{1}{26}$ (B) $\frac{1}{13}$

(C) $\frac{1}{52}$ (D) $\frac{1}{14}$

47 Find the probability that a number selected at random from the numbers 1 to 25 is not a prime number when each of the given numbers is equally likely to be selected.

(A) $\frac{9}{25}$ (B) $\frac{16}{25}$

(C) $\frac{11}{25}$ (D) $\frac{6}{25}$

48 Find the probability of throwing an even number with an ordinary six faced die.

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Previous Contest Questions

1 In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

(A) $\frac{1}{5}$ (B) $\frac{4}{5}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

2 400 students of class IX of a school appeared for a test of 100 marks in the subject of mathematics and the data about the marks secured is presented in the table.

Marks secured	0-25	26-50	51-75	Above 75
Number of students	50	220	100	30

If the result card of a student is picked up at random, what is the probability that the student has secured more than 50 marks?

(A) 0.523 (B) 0.532
(C) 0.325 (D) 0.352

3 A die is rolled 120 times and the outcomes are recorded in the given table.

Outcome	1	Even number less than 6	Odd number greater-than 1	6
Frequency	30	35	30	25

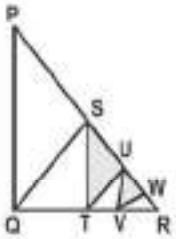
Determine the probability of getting an odd number greater than 1 in a trial.

(A) 0.167 (B) 0.125
(C) 0.29 (D) 0.25

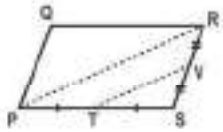
This page is intentionally left blank.

Questions@stimulating-minds

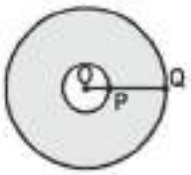
1 In the right-angled triangle PQR, PQ = QR. The segments QS, TU and VW are perpendicular to PR, and the segments ST and UV are perpendicular to QR, as shown. What fraction of Δ PQR is shaded?



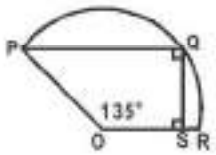
2 PQRS is a parallelogram with area 40. If T and V are the midpoints of sides PS and RS respectively, find the area of PRVT is.



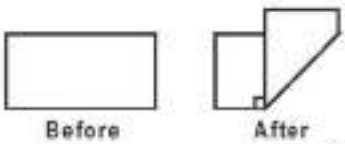
3 In the diagram, each of the two circles has centre O. Also, OP : PQ = 1 : 2. If the radius of the larger circle is 9, what is the area of the shaded region?



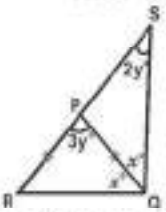
4 In the diagram, points P, Q and R lie on a circle with centre O and radius 12, and point S lies on OR. If $\angle POR = 135°$, what is the area of trapezoid OPQS ?



5 A rectangular piece of paper measures 17 cm by 8 cm. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is that area of the new figure?

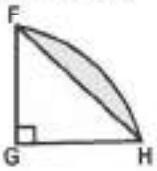


6 In the given figure, P is on RS so that QP bisects $\angle SQR$. PQ = PR, $\angle RSQ = 2y°$, $\angle RPQ = 3y°$. What is the measure of $\angle RPQ$?

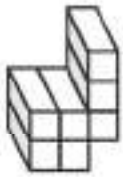


7 Two cylindrical tanks sit side by side on a level surface. The first tank has a radius of 4 metres, a height of 10 metres, and is full of water. The second tank has a radius of 6 metres, a height of 8 metres, and is empty. Water is pumped from the first tank to the second until the depth of water the both tanks is the same. What is the depth of water in each tank (in metres) ?

8 In right angled, isosceles triangle FGH, FH = $\sqrt{8}$. Arc FH is part of the circumference of a circle with centre G and radius GH, as shown. What is the area of the shaded region?

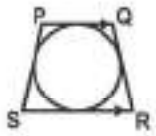


9 In the diagram, the object is made up of seven $1 \times 1 \times 2$ solids. What is the total surface area of the object?



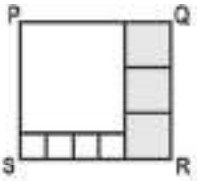
Find the numerical value of $a + b + c$.

10 A circle is inscribed in trapezoid PQRS. If PS = QR = 25 cm, PQ = 18 cm and SR = 32 cm, what is the length of the diameter of the circle?



11 Six consecutive integers are written on a blackboard. When one of them is erased the sum of the remaining five integers is 2012. What is the sum of the digits of the integer that was erased?

12 Rectangle PQRS is divided into eight squares, as shown. The side length of each shaded square is 10. What is the length of the side of the largest square?

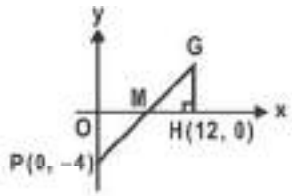


13 Suppose that a, b and c are three numbers with

$$\begin{aligned} a + b &= 3 \\ ac + b &= 18 \\ bc + a &= 6 \end{aligned}$$

Find the value of c.

14 The line from G through the midpoint M of OH intersects the y-axis at P(0, – 4). What are the coordinates of G?



15 Point P is on the line $y = 5x + 3$. The coordinates of point Q are $(3, -2)$. If M is the midpoint of PQ, the M must lie on the line

(A) $y = \frac{5}{2}x - \frac{7}{2}$ (B) $y = 5x + 1$ (C) $y = -\frac{1}{5}x - \frac{7}{5}$ (D) $y = 5x - 7$

Model Test Paper

Score 25

1 JUMP is a square with $\angle JUP = 45°$. Find $\angle PUM$.
(A) 45° (B) 90° (C) 35° (D) 15°

2 The sides of an equilateral triangle are $(2a - b + 5)$, $(a + b)$ and $(2b - a + 2)$. What is the area of the triangle?
(A) $\frac{\sqrt{3}a^2}{4}$ (B) $\frac{\sqrt{3}b^2}{4}$ (C) $\frac{49\sqrt{3}}{4}$ (D) $\frac{81\sqrt{3}}{4}$

3 The ratio of base radius and height of a cone is 3 : 4. If the cost of smoothening the C.S.A. at 5 paise/sq cm is ₹ 115.50, find the capacity of the cone.
(A) 12963 cm^3 (B) 12693 cm^3
(C) 12936 cm^3 (D) 12369 cm^3

4 In the given figure, AOB and COD are two diameters of a circle with centre O. If $\angle OAC = 60°$, find $\angle ABD$.

(A) 40° (B) 60° (C) 50° (D) 80°

5 A river 3 m deep and 40 m wide flows river at the rate at 2 km/hour. How much of the river water will fall into the sea in one minute?
(A) 3000 litres (B) 3500 litres
(C) 2500 litres (D) 4000 litres

6 The width of each of five continuous classes in a frequency distribution is 5 and the lower class-limit of the lowest class is 10. Find the upper class-limit of the highest class.
(A) 15 (B) 25 (C) 35 (D) 40

7 In quadrilateral PQRS, PQ and RS are the shortest and the longest sides. Which of the following is true?
(A) $\angle P < \angle R$ and $\angle Q < \angle S$
(B) $\angle P > \angle R$ and $\angle Q < \angle S$
(C) $\angle P < \angle S$ and $\angle Q < \angle R$
(D) $\angle P < \angle Q$ and $\angle R < \angle S$

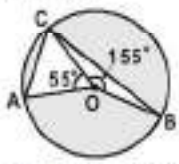
8 Identify the point at which the graph of the equation $2x + 3y = 9$ cuts y – axis.
(A) $\left(\frac{9}{2}, 0\right)$ (B) (0, 9)
(C) (0, 3) (D) (3, 1)

9 The average monthly consumption of petrol for first 3 months is 86 litres and for next 9 months is 152 litres. Find the average consumption for the whole year.
(A) 162.6 litres (B) 136.5 litres
(C) 153.6 litres (D) 135.5 litres

10 If a triangle and a square are on the same base and between the same parallels, What is the ratio of their areas in order?
(A) 1 : 3 (B) 1 : 2 (C) 3 : 1 (D) 1 : 4

11 Which of the following statements is false about $\frac{-2}{3}$?
(A) It lies to the left side on the number line.
(B) It lies to the right side on the number line.
(C) It is not possible to represent on the number line.
(D) Negative rational numbers lie on the left of 0.

12 In the given figure angles subtended by chords AC and BC at the centre O of the circle are 55° and 155° respectively. Determine $\angle ACB$.



(A) 80° (B) 110° (C) 60° (D) 75°

13 Which of the following is an irrational number?
(A) 0.40140014000140... (B) $0.\overline{1416}$
(C) $0.1\overline{416}$ (D) 0.14

14 Find the sum of the deviations of the variates, 6, 8, 10, 16, 20 and 24 from their mean.

(A) 4 (B) 2 (C) 0 (D) 1

15 The total value of a collection of coins of denominations ₹ 1.00, 50 paise, 25 paise, 10 paise and 5 paise, is ₹ 380. If the number of coins of each denomination is the same, find the number of one-rupee coins

(A) 160 (B) 180 (C) 200 (D) 220

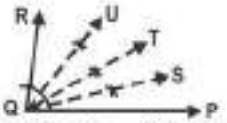
16 Let R_1 and R_2 be the remainders when the polynomials $f(x) = 4x^3 + 3x^2 - 12ax - 5$ and $g(x) = 2x^3 + ax^2 - 6x + 2$ are divided by $(x - 1)$ and $(x - 2)$ respectively. If $3R_1 + R_2 + 28 = 0$, find the value of 'a'.

(A) 0 (B) –1 (C) 1 (D) 32

17 A telegraph wire spans 20 m with a dip of 10 cm at the centre. Assuming the wire is in the form of a circular arc, find its radius.

(A) 45 m (B) 116 m
(C) 208 m (D) 500.05 m

18 In the following figure, $\angle PQR = 60°$, $\angle RQU = \angle UQT = \angle TQS = \angle SQP$. Find $\angle SQT$.



(A) 20° (B) 10° (C) 15° (D) 18°

19 Which of the following figures is formed by plotting the points (–1, 0), (1, 0), (1, 1), (0, 2), (–1, 1) on a graph sheet and joining them in order?

(A) A square (B) A rhombus
(C) A parallelogram (D) A pentagon

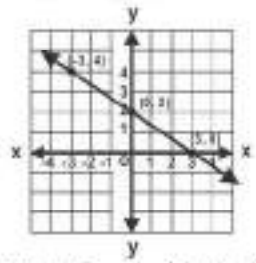
20 A class has two sections. The mean marks of one section of size 40 is 60 and mean marks of other section of size 60 is 80. Find the combined mean of the students of the school.

(A) 48 (B) 28 (C) 72 (D) 66

21 In a quadrilateral ABCD if AB = 5 m, BC = 5 m, CD = 6 m, AD = 6 m and diagonal AC = 6 m, what is its area?

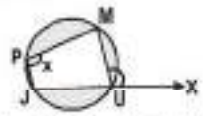
(A) $2(4 + 3\sqrt{3})$ m² (B) $3(4 + 3\sqrt{3})$ m²
(C) $5(4 + 3\sqrt{3})$ m² (D) $7(4 + 3\sqrt{3})$ m²

22 Which of the following equations represents the given graph?



(A) $2x + y = 6$ (B) $y + 2x + 4$
(C) $2(x - 1) + 3y = 4$ (D) $2x - 3y = 6$

23 In the given figure, JUMP is a cyclic quadrilateral.



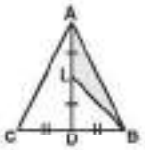
If $\angle MUX = 105°$, what is the measure of $\angle x$?

(A) 105° (B) 75° (C) 100° (D) 90°

24 Identify the exponential form of $\sqrt[3]{\sqrt[3]{2^2}}$.

(A) $2^{-1/6}$ (B) 2^{-6} (C) $2^{1/6}$ (D) 2^6

25 In the given figure, D is the mid-point of BC and L is the mid-point of AD. If ar (Δ ABL) = x ar (Δ ABC), what is the value of x?



(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 4

Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	C	B	D	C	A	C	D	B	B	D	A	C	C	C	D	C	D	C

21	22	23	24	25
B	C	A	C	C

Explanatory Answers

1. Number Systems

Multiple Choice Questions

1. (A) $\sqrt{2}$ is not a rational number.
Since, 2 cannot be expressed as a square of any rational number.
$\sqrt{4} = \sqrt{2^2} = 2$; $\sqrt{9} = \sqrt{3^2} = 3$;
$\sqrt{16} = \sqrt{4^2} = 4$

2. (D) 3. (C) 4. (A) 5. (B) 6. (B)

7. (B) Rationalizing factor of $\sqrt[n]{a^p}$ is $\sqrt[n]{a^{n-p}}$ or $a^{1-\frac{p}{n}}$ where $n > p$.
Rationalizing factor of $5^{\frac{1}{3}} = \sqrt[3]{5^2}$

8. (A) Rationalizing factor of $\sqrt[n]{a^p}$ is $\sqrt[n]{a^{n-p}}$ or $a^{1-\frac{p}{n}}$ where $n > p$.
$\therefore$ R.F. of $\sqrt[3]{a^2b^3c^4} = \sqrt[3]{a^1b^0c^1}$

9. (C) $\sqrt[3]{9} = 3^{\frac{2}{3}}$
Rationalizing factor of $\sqrt[n]{a^p}$ is $\sqrt[n]{a^{n-p}}$ or $a^{1-\frac{p}{n}}$ where $n > p$.
Here $\sqrt[3]{9} = \sqrt[3]{3^2}$ i.e., $n = 3, p = 2$
R.F. of $\sqrt[3]{9} = 3^{1-\frac{2}{3}} = 3^{\frac{1}{3}}$ or $\sqrt[3]{3}$
$\therefore$ The denominator $\sqrt[3]{9}$ when rationalised is 3.

10. (D) $\left(\sqrt[3]{27} - \sqrt{6\frac{3}{4}}\right)^2 = \frac{\left(3^{\frac{1}{2}}\right)^2}{4}[2-3]^2 = \frac{3}{4}$

11. (A) $\sqrt[3]{4} \times \sqrt[3]{22} = \sqrt[3]{2^2 \times 2 \times 11} = 2\sqrt[3]{11}$

12. (B) 13. (B) 14. (D) 15. (A) 16. (C) 17. (C)

18. (D) $x^{-\frac{1}{2}} = 8x^{-1}$
squaring on both sides, we get
$x = 8^2 = 64$

19. (C) $4^x - 4^{x-1} = 24 \Rightarrow 4^x - \frac{4^x}{4} = 24$
$\Rightarrow 4^x = 8 \times 4 \Rightarrow 2^{2x} = 2^5 \Rightarrow x = \frac{5}{2}$
$\therefore (2x)^x = \left(2 \times \frac{5}{2}\right)^{\frac{5}{2}} = 5^{\frac{5}{2}} = 5^2\, 5^{\frac{1}{2}}$
$= 25\sqrt{5}$

20. (B) $x = \sqrt{5} + 2 \Rightarrow \frac{1}{x} = \sqrt{5} - 2$
$x - \frac{1}{x} = \sqrt{5} + 2 - \sqrt{5} + 2 = 4$

21. (D) $(x-3)^2 = \left[\frac{2}{3-\sqrt{7}} - 3\right]^2 = \left[\frac{2\sqrt{7}}{2}\right]^2 = 7$

22. (C) $x = \frac{1}{y} = 7 + 4\sqrt{3}$, $y = \frac{1}{x} = 7 - 4\sqrt{3}$
$\frac{1}{x^2} + \frac{1}{y^2} = x^2 + y^2 = 194$

23. (C) $\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
$= \frac{\sqrt{5}+\sqrt{3}}{2} = \frac{3.968}{2} = 1.984$

24. (A) 0 is the only rational number among the given options between –5 and 5.

25. (D) The resulting number is always divisible by 9.
e.g., Consider 47. Sum of its digits is $4 + 7 = 11$. So, $47 - 11 = 36$ which is exactly divisible by 9

26. (C) $1.\bar{4} = \frac{13}{9}$, $0.\bar{2} = \frac{2}{9}$
So, $1.\bar{4} - 0.\bar{2} = \frac{13}{9} - \frac{2}{9} = \frac{11}{9}$

27. (A) 28. (B) 29. (C) 30. (A)

31. (B) We have $x = \frac{1}{2-\sqrt{3}}$
$\Rightarrow x = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$
$\Rightarrow x - 2 = \sqrt{3}$

On squaring both sides

$x^2 - 4x + 4 = 3$ (or) $x^2 - 4x + 1 = 0$

Now on dividing

$x^3 - 2x^2 - 7x + 10$ by $x^2 - 4x + 1$,

we get remainder 8.

Hence, by using division algorithm we find

$x^3 - 2x^2 - 7x + 10 =$

$(x^2 - 4x + 1)$ (quotient) + 8

On putting $x^2 - 4x + 1 = 0$ we get the value of $x^3 - 2x^2 - 7x + 10$ as 8.

32. (D) Since a = 3 and b = 5, $a^3 + b^3 = 3^3 + 5^3$
$= 27 + 125 = 152$

33. (B) $\left(\frac{1}{p} + \frac{1}{q}\right)^3 = \left(\frac{1}{3} + \frac{1}{5}\right)^3 = \left(\frac{5+3}{15}\right)^3$

$= \left(\frac{8}{15}\right)^3 = \frac{512}{3375}$

34. (B) $\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \frac{\sqrt{5}+2}{(\sqrt{5})^2 - (2)^2} = \sqrt{5} + 2$ or $= \sqrt{5} + \sqrt{4}$

35. (C) Let $x = 0.\overline{5} = 0.555...$ ...(i)

Multiply (i) by 10 on both sides

$10x = 5.555$ ...(ii)

Subtract (i) from (ii). to get $x = \frac{5}{9}$

36. (B) 37. (C) 38. (D) 39. (D) 40. (C)

Previous Contest Questions

1. (C) $\frac{\sqrt{a} + \frac{1}{\sqrt{a}}}{1-a} + \frac{1 - \frac{1}{\sqrt{a}}}{1 + \sqrt{a}}$

$= \frac{a+1}{\sqrt{a}(1-a)} + \frac{\sqrt{a}-1}{\sqrt{a}(1+\sqrt{a})}$

$= \frac{2\sqrt{a}(\sqrt{a}+1)}{\sqrt{a}(1-a)(1+\sqrt{a})} = \frac{2}{1-a}$

2. (A) The set of all positive integers is the set of natural numbers. It is denoted by the symbol N.

3. (D) Let $x = 0.9999$ .....

$\Rightarrow 10 \times x = 10 \times 0.9999$ .....

$\Rightarrow x = 1$

$\therefore 0.9999$ ..... $= 0.\overline{9} = 1$

4. (B) $x = \sqrt[3]{2+\sqrt{3}}$

$\Rightarrow x^3 = 2 + \sqrt{3}$ and $\frac{1}{x^3} = 2 - \sqrt{3}$

$\therefore x^3 + \frac{1}{x^3} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

5. (C) $5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$ is in the form of $(a + b)$.

We know that $(a + b)(a^2 - ab + b^2) = a^3 + b^3$.

So, the simplest R.F. of $5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$ is

$\left(5^{\frac{1}{3}}\right)^2 - \left(5^{\frac{1}{3}}\right)\left(5^{\frac{-1}{3}}\right) + \left(5^{\frac{-1}{3}}\right)^2$

$= 5^{\frac{2}{3}} - 1 + 5^{\frac{-2}{3}}$

2. Polynomials

Multiple Choice Questions

1. (D) The degree of the constant polynomial '0' is not defined.

2. (D) 3. (A) 4. (D) 5. (A) 6. (B)

7. (C) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

8. (B) $a^2 + b^2 + 2ab + 2bc + 2ca + c^2 - c^2$
$= (a + b + c)^2 - c^2$
$= (a + b)(a + b + 2c)$
On comparing with $(a + b + m)(a + b + nc)$, we get $m = 0$ and $n = 2$.
$\Rightarrow m + n = 0 + 2 = 2$

9. (C) $(x^2 + 3x + 5)(x^2 - 3x + 5)$
$= (x^2 + 5)^2 - (3x)^2$
$\therefore m = x^2 + 5$

10. (D) Given $\frac{x}{y} + \frac{y}{x} = -1 \Rightarrow \frac{x^2 + y^2}{xy} = -1$
$\Rightarrow x^2 + y^2 + xy = 0$ .....(i)
$x^3 - y^3 = (x + y)(x^2 + xy + y^2) = 0$

11. (C) $x^2 + ax + bx + cx + ab + bc$
$= (x^2 + bx) + (ax + ab) + (cx + bc)$
$= (x + b)(x + a + c)$

12. (D) $a^4 + 4$
$= a^4 + 4a^2 - 4a^2 + 4$
$= (a^2 + 2a + 2)(a^2 - 2a + 2)$

13. (A) 14. (C) 15. (C) 16. (A) 17. (B)

18. (A) $x+\frac{1}{x}=a+b$ and $x-\frac{1}{x}=a-b$

$\Rightarrow 2x=2a \Rightarrow x=a$

Similarly, $\frac{2}{x}=2b \Rightarrow \frac{1}{x}=b$

$\therefore x\times\frac{1}{x}=a\times b \Rightarrow ab=1$

19. (B) $2^{\frac{1}{4}}.4^{\frac{1}{8}}.16^{\frac{1}{16}}.256^{\frac{1}{32}}$

$=2^{\frac{1}{4}}.2^{\frac{1}{4}}.2^{\frac{1}{4}}.2^{\frac{1}{4}}=2^{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}=2^1=2$

20. (C) $\frac{(0.96)^3+(0.04)^3}{(0.96)^2-(0.96)(0.04)+(0.04)^2}$

$=0.96+0.04=1$

21. (A) $x-8xy^3=x[1-8y^3]=x[1^3-(2y)^3]$

$=x(1-2y)(1+2y+4y^2)$

22. (A) 23. (B) 24. (D) 25. (C) 26. (C)

27. (B) $2(1)^3+a(1)^2+3(1)-5=(1)^3+(1)^2-4(1)-a$

$2+a+3-5=1+1-4-a$

$\Rightarrow 2a=-2 \Rightarrow a=-1$

28. (C) $ab(a+b)^2-3ab(a+b)$

Here $(a+b)^2$ is a common factor.

So, $ab(a+b)^2-3ab(a+b)$

$=ab(a+b)(a+b-3)$

29. (D) If $x-1$ is a factor then $f(1)=0$. [Factor theorem]

30. (B) If $y-2$ is a factor of py^2+5y+r,

$\Rightarrow 4p+r+10=0$.....(1)

If $y-\frac{1}{2}$ is a factor of py^2+5y+r,

$\Rightarrow \frac{p}{4}+\frac{5}{2}+r=0$ ....(2)

By solving (1) and (2), we get

$p=r=-2$

31. (B) 32. (B) 33. (A) 34. (A) 35. (C)

36. (A) If $x-1$ is a factor of $p(x)$.

$p(1)=0 \Rightarrow k=-3$

37. (B) Let $p(x)=3x^3+8x^2+8x+3+5k$

$\Rightarrow -2+5k=0$ (Since, it is given that x^2+x+1 is a factor of $3x^3+8x^2+8x+3+5k$, the remainder is equal to 0)

$\Rightarrow k=\frac{2}{5}$

38. (A) Given $(x-1)$ is a factor of $f(x)$, but not of $g(x)$

$\Rightarrow f(1)=0$ and $g(1)\neq 0$

$\therefore$ Of the given options $(x-1)$ must be a factor of $f(x)\,g(x)$.

Since, $f(1)\,g(1)=0$ as $f(1)=0$

39. (B) $(x-a)(x-b)$ are the factors of $g(x)$.

$\Rightarrow g(a)=0, g(b)=0$

40. (D) Let $p(x)=x^{2014}+2014$

$\Rightarrow p(-1)=(-1)^{2014}+2014$

$=1+2014=2015$

41. (C) $x^2+kx+6=(x+2)(x+3)$

$\Rightarrow x^2+kx+6=x^2+5x+6$

By comparing L.H.S. and R.H.S. the value of k is 5.

42. (C) $\left(x+\frac{1}{x}\right)^2=3^2 \Rightarrow x^2+\frac{1}{x^2}=7$

43. (C) $(x-1)-(x^2-1)=-x(x-1)$

Hence, one of the factors of $(x-1)-(x^2-1)$ is $(x-1)$.

44. (C) 45. (D) 46. (A) 47. (A) 48. (C)

49. (D) 50. (C) 51. (A) 52. (C) 53. (A)

54. (A) Given $(x-1)^7=a_7x^7+a_6x^6+.....+a_1x+a_0$

Since, the given polynomial terms are in expression, form Pascal's triangle we get the coefficients of 1st four terms symmetrical to those of the last four terms.

$\Rightarrow (x-1)^7=x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1=0$

$\Rightarrow a_7+a_6+a_5.....+a_1+a_0$

$=1-7+21-35+35-21+7-1=0$

55. (A) $1+a+b+c+ab+bc+ca+abc$

$=(1+a)+(b+ab)+(c+ac)+(bc+abc)$

$=1(1+a)+b(1+a)+c(1+a)+bc(1+a)$

$=(1+a)(1+b+c+bc)$

$=(1+a)[1(1+b)+c(1+b)]$

$=(1+a)(1+b)(1+c)$

Previous Contest Questions

1. (D) $4x^2-y^2+2x-2y-3xy$

$=(x-y)(4x+y+2)$

2. (B) Let $p(x)=x^2-x-1$,

$p(-1)=2-1=1$

$\therefore$ Value of the given polynomial $p(x)$ at $x=-1$ is 1.

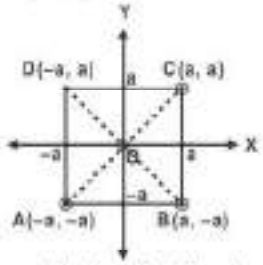
3. (B) 4. (A) 5. (C)

3. Co-ordinate Geometry

Multiple Choice Questions

1. (A) 2. (C) 3. (B) 4. (C) 5. (D) 6. (A)

7. (A) Two ordered pairs are equal if their corresponding coordinates are equal.
8. (D) The point (0, –3) is 3 units away from the origin.
9. (B) 10. (D) 11. (A) 12. (C)
13. (C) Given points are A(–a, –a), B(a, –a), C (a, a) and D(–a, a)



Hence, it is clear that the given points form a square and the origin lies at the point where the diagonals of the square intersect.

14. (A) 15. (D) 16. (B) 17. (C) 18. (B)
19. (C) Given, the point lies on y-axis at a distance of 4 units in –ve direction of x-axis.

$\therefore$ The required coordinates of the point are (0, –4).

20. (A) 21. (C) 22. (A) 23. (C) 24. (A)
25. (B) 26. (C) 27. (D) 28. (C) 29. (B)
30. (D) 31. (A) 32. (D) 33. (B) 34. (D)
35. (B) The coordinates of the point M in the given graph are $\left(-3, \frac{3}{2}\right)$.
36. (C) 37. (B) 38. (A) 39. (C) 40. (D)

The quadrilateral formed by joining the given points is a trapezium as shown.

41. (B) Hence, the point (–2, 5) represents 2 units towards left and 5 units upwards.

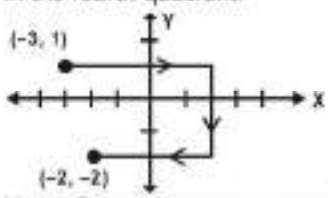
Previous Contest Questions

1. (A) The point P(–a, b) usually represents second quadrant but given $a < 0$, $b < 0$, has positive.

The point P(–a, b) has positive x-coordinate and negative y-coordinate.

$\therefore$ The point P(–a, b) for $a < 0$, $b < 0$ will be in the fourth quadrant.

2. (A)



Hence, Reshma's starting point is (–3, 1).

3. (D) 4. (A) 5. (B)

4. Linear Equations in two Variables

Multiple Choice Questions

1. (C) An infinite number of lines pass through the given point (0, 0).
2. (C) $y = 4x - 3$ is a linear equation in two variables. A linear equation in two variables has infinitely many solutions. So, $y = 4x - 3$ has infinitely many solutions.
3. (A) Given that, $x = -2$ and $y = 3$ is the solution of $3x - 5y = k$, we have

$k = 3(-2) - 5(3) = -21$

4. (A) 5. (D) 6. (C) 7. (D) 8. (A)
9. (C) The equation of the line passing through the origin is $y = mx$.

Hence $y = 5x$ is the required equation.

10. (A) $y - 2 = 0 \Rightarrow y = 2$

It represents a line parallel to X-axis.

11. (A) Given (3, 4) is a solution of $5x - 2y = k$

$\Rightarrow 5(3) - 2(4) = k \Rightarrow k = 7$

12. (D) Given (2, –3) lies on $ay = 7x - 26$

$\Rightarrow a(-3) = 7(2) - 26 \Rightarrow a = \frac{-12}{-3} = 4$

13. (C) Cost of one shirt = ₹ 800

Cost of x shirts = ₹ (800 × x)

Also ₹ y is the total cost.

$\therefore y = 800 \times x$ i.e., $y = 800x$

14. (C) The points of the form (p, p) $\forall$ $p \neq 0$ always lies on the line $y = x$.

e.g., (5, 5) (16, 16) etc., have their x and y coordinates same. So, they lie on the line $y = x$.

15. (D) $\frac{5}{3} - \frac{2}{x} = \frac{8}{x} \Rightarrow x = 6$

Hence, the solution set for $\frac{5}{3} - \frac{2}{x} = \frac{8}{x}$ is {6}.

16. (A) Given equation is

$\sqrt{k + 64} - 8 = -2$

$\Rightarrow k = -28$

17. (C) 18. (C) 19. (A) 20. (C) 21. (A)
22. (C) The given equation of the line parallel to y – axis and 5 – units away from the origin is $x = 5$.
23. (D) 24. (A) 25. (A) 26. (C) 27. (D)
28. (B) Given, cost of an adult ticket = A

cost of a child ticket = S

$\Rightarrow A = S + 3$

Vihan spent 132 to buy tickets for 20 children and 4 adults

$\Rightarrow 4A + 20S = 132$

29. (B) 30. (C) 31. (B) 32. (D) 33. (C)
34. (B) Given, amount earned by school by selling tickets for a play = ₹ 5100
Cost of each ticket = ₹ 12
No. of tickets sold = t
Cost of t tickets = $12t$
$\therefore$ The equation used to determine the number tickets were sold is $12t = 5100$.
35. (C) Amount = 35 + 15(25) = ₹ 410
36. (A) Let the speed of one plane be x km/hour.
Then the speed of other plane is $(x + 40)$ km/hour.
Distance travelled by first plane in 5 hours = Speed × Time = $x \times 5 = 5x$
Distance travelled by second plane in 5 hours = $(x + 40)5$.
Distance travelled by first plane + Distance travelled by the other plane = 3400 km
$5x + 5(x + 40) = 3400$
$\Rightarrow x = \frac{3200}{10} = 320 \text{km/hour}$
Sum of speeds = (320 + 360) km/h = 680 km/hour
37. (B) 38. (B) 39. (B) 40. (D) 41. (C)
42. (D) 43. (A) 44. (B) 45. (D) 46. (B)

Previous Contest Questions

1. (C) $b = l - 2$ ...(i)
$2(l + b) = 14$
$\Rightarrow l + b = 7$ (or) $b = 7 - l$ ...(ii)
From (i) & (ii)
$l - 2 = 7 - l$ (Since $b = l - 2$)
$\Rightarrow l = 4.5$ m and $b = 2.5$ m
2. (C) $x(100 + 50 + 25 + 10 + 5) = 38000$
$\Rightarrow x = 200$
3. (C) Substituting $x = 3$ and $y = -1$ in $x + 2y = 1$
$\Rightarrow 3 + 2(-1) = 1$
$\Rightarrow 3 - 2 = 1 \Rightarrow 1 = 1$ (True)
Hence $(3, -1)$ is the solution of $x + 2y = 1$.
4. (A) $y = x + 10$
put $x = 5$, $y = 15$
$\Rightarrow 15 = 5 + 10$
$15 = 15$
Similarly, $x = 10$, $y = 20$ also satisfies the line $y = x + 10$
Hence, $y = x + 10$ is the required line.
5. (C) A straight line models a linear equation.

5. Introduction to Euclid's Geometry

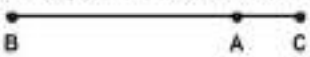
Multiple Choice Questions

1. (C) 2. (B) 3. (A) 4. (C) 5. (A)
6. (C) 7. (A) 8. (D) 9. (B) 10. (D)
11. (C) 12. (A) 13. (B) 14. (B) 15. (C)
16. (A) 17. (D) 18. (D) 19. (A) 20. (C)
21. (C) 22. (B)
23. (C) It is evident that 6 lines can be drawn through three non-collinear points.
24. (C) A straight line is formed when two planes intersect with each other.
25. (A) The required 3 steps from solids to point are given as solids - surfaces - lines - points.
26. (B) 27. (C) 28. (C) 29. (D) 30. (A)
31. (D)

From the figure,
$CF + FN = CN$ .....(1),
and $MC + CF + FN = MN$
$\therefore MC + CN = MN$ (From eq. .. 1)
Since, C is the mid-point of MF, $MC = MF$.
$\therefore CF + CN = MN$
32. (B) 33. (A) 34. (C) 35. (D) 36. (A)
37. (C) 38. (C) 39. (A) 40. (C)

Previous Contest Questions

1. (D) According to Axiom 7, if a point R is the mid-point of line MN, $= MR = NR = \frac{MN}{2}$
2. (D) Given point A lies in between B and C.



$\Rightarrow BA + AC = BC$
3. (B) 4. (C)

6. Lines and Angles

Multiple Choice Questions

1. (A) Given that, $x^\circ + 4x^\circ = 180^\circ$
$\Rightarrow x = 36^\circ$ and $4x = 4 \times 36^\circ = 144^\circ$
2. (A) One angle is $\frac{4}{9} \times 180^\circ = 80^\circ$
Second angle is $\frac{5}{9} \times 180^\circ = 100^\circ$
3. (B) $2x^\circ + 5y^\circ = 180^\circ$
$\Rightarrow 2(30^\circ) + 5y^\circ = 180^\circ \Rightarrow y = 24^\circ$
4. (A) $b + a = 180^\circ$ (Linear pair)
$a - b = 40^\circ \Rightarrow a = 110^\circ, b = 70^\circ$
5. (C) 6. (D) 7. (C) 8. (A) 9. (A) 10. (D)

11. (B) $\angle ABC = \angle BCD \Rightarrow x = \angle BCD$
But $\angle BCD = \angle BCE + \angle ECD$
$\therefore \angle ABC = 25° + 30° = 55°$
$\Rightarrow x° = 55°$

12. (A) From ΔABC and ΔDEF,
$\angle A + \angle B + \angle C = 180°$ and
$\angle D + \angle E + \angle F = 180°$.
$\therefore \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$

13. (B) Refer the question figure
$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$
In ΔPQR, $80° + \angle Q + \angle R = 180°$
$\Rightarrow \angle Q + \angle R = 100°$
$\Rightarrow \angle 2 + \angle 3 = 50°$
In ΔQOR, $50° + \angle QOR = 180°$
$\Rightarrow \angle QOR = 180° - 50° = 130°$

14. (A) 15. (C) 16. (D)

17. (A) $\angle NAB = \angle NAC = \frac{1}{2}\angle A$
In ΔABC, $\angle A = 75° \therefore \frac{1}{2}\angle A = 37.5°$
From ΔABN, $\angle BAM = 20°$.
$\therefore \angle MAN = 37.5° - 20° = 17.5°$.

18. (D) 19. (B) 20. (A) 21. (A)
22. (B) 23. (A) 24. (A)

25. (D) $x + 55° = 180°$ [Since EF || AB]
$x = 125°$
and $z = 90° - 55° = 35°$
$x - z = 125° - 35° = 90°$

26. (B) Side opposite to the greatest angle is the largest side and side opposite to the least angle is the smallest side.
Hence, BC and AC are the shortest and the largest sides respectively.

27. (D) $a + f + e = 180°$ [Straight angle]
$\angle f = \angle c$ [Vertically opposite angles]
$\Rightarrow a + c + e = 180°$
Hence, statement (ii) is true.
Also, $b + a + f = c + d + e$
and $\angle a = \angle d$ [Vertically opposite angles]
$\Rightarrow b + f = c + e$
Hence, statement (iii) is also true.

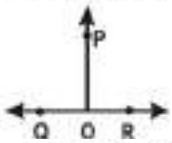
28. (B) $\angle A : \angle B : \angle C = \frac{1}{2} : \frac{1}{3} : \frac{1}{6}$
$\Rightarrow \angle A : \angle B : \angle C = 3 : 2 : 1$
$\therefore \angle A = 90°, \angle B = 60°, \angle C = 30°$

29. (A) $A + B + C = 180°$
$\Rightarrow (15° + B) + B + (B - 30°) = 180°$
$\Rightarrow B = 65°$
$\therefore A = 80°, C = 35°$

30. (B) 31. (A) 32. (A) 33. (C) 34. (C)
35. (D) 36. (C) 37. (B) 38. (C) 39. (B)
40. (A) 41. (D) 42. (C) 43. (D) 44. (D)
45. (C) 46. (B) 47. (A) 48. (A)

Previous Contest Questions

1. (D) Given OP is a ray on line QR, and
$\angle POQ = \angle POR$ ....(1)



$\Rightarrow \angle POQ = \angle POR = 90°$
Since, $\angle POQ + \angle POR = 180°$.

2. (D) Here $p - 10° + p - 5° + p - 15° + p - 30° = 180°$
$\Rightarrow p = \frac{240°}{4} = 60°$

3. (B) $\angle PQS + \angle FSQ = 180°$
$\Rightarrow \angle FSQ = 180° - 60° \quad \angle FSQ = 120°$
and $\angle RFE = \angle FSQ = 120°$
[Corresponding angles since EF || QS]

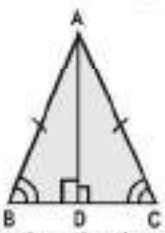
4. (D) 5. (B) 6. (C)

7. Triangles

Multiple Choice Questions

1. (B) 2. (C) 3. (C) 4. (B)

5. (D) In ΔABD and ΔADC, $\angle B = \angle C$ (Given)
$\Rightarrow AB = AC$
$\angle ADB = \angle ADC$ and $BD = DC$ (Since, $AD \perp BC$.)
$\therefore \Delta ADB \cong \Delta ADC$ (By A.S.A. property)



6. (A) Let $x°$ be the other interior angle.
Then $30° + x = 110°$
$\Rightarrow x° = 80°$
$\therefore$ The angles are 80°, 70°.
(Since, $80° + 30° + 70° = 180°$)

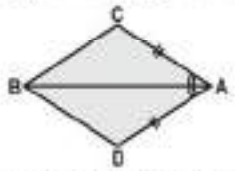
7. (B) Let x, y, z be three angles of the Δ
Given $x = 75°$ and $y - z = 35°$ ....(1)
$x + y + z = 180°$ ....(2)
$\Rightarrow 75° + y + z = 180°$
$y + z = 105°$ .... (3)
From (1) and (3) $y = 70°$, $z = 35°$
From (2), we get $x = 75°$
$\therefore$ The measure of largest angle is 75°.

8. (A) $AB = CF$ (Side) $\Rightarrow AF = CB$.
$EF = BD$ (Side)
$\angle AFE = \angle DBC$ (Angle)
$\therefore \Delta AFE \cong \Delta CBD$ (By S.A.S. criterion)

9. (A) $AB = AC$ (Given)
$BD = DC$ (Given)
$AD = AD$ (Common side)
Hence, the triangles are congruent by S.S.S. criterion.

10. (B) In ΔABC, $AB = AC \Rightarrow \angle B = \angle C$
$AB = AC \Rightarrow \angle B = \angle C$
$AD \perp BC \Rightarrow BD = DC$
In ΔABD and ΔACD,
$AB = AC$ (side)
AD is the common side $\angle DAB = \angle DAC$
(Since AD bisects $\angle A$.)
$\therefore \Delta ABD \cong \Delta ACD$ (By A.S.A. criterion)

11. (A) In ΔACB and ΔADB, $AC = AD$ (Given)



$\angle CAB = \angle DAB$ (AB bisects $\angle A$)
and $AB = AB$ (Common side)
$\therefore \Delta ACB \cong \Delta ADB$ (S.A.S. rule)

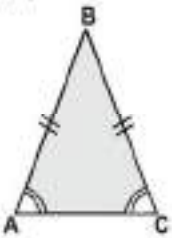
12. (A) 13. (A) 14. (C) 15. (B) 16. (B)
17. (B) 18. (C) 19. (B) 20. (B) 21. (A)
22. (D) All the given statements are correct.
23. (C) The difference of two sides is less than the third side [Since, in ΔABC, $AB + AC > BC$
$\Rightarrow AB > BC - AC$ or $BC - AC < AB$]
24. (C) From the figure, we have
$\angle ADB = \angle ABD$. (Since $AB = AD$.)
$\therefore \angle ADB = 180° - 110° = 70°$
But $\angle ADB = \angle DAC + \angle ACD$
(Exterior angle of ΔADC.)
$\Rightarrow 70° = x° + 25°$
$\Rightarrow x° = 70° - 25° = 45°$

25. (A) Since $\angle M$ is the greatest angle,
$\Rightarrow$ NL is the longest side.

26. (D) Geometric figures having the same area need be congruent though congruent figures have equal area.
The angle must be included within the sides considered in the two triangles. Only then, the triangles are congruent.
Along with the hypotenuse and the right angle, the third side must also be correspondingly equal for the triangles to be congruent.

27. (B) For any triangle sum of any two sides is greater than the third side. But in option B,
$2.3 + 3.1 \not> 5.4$
Hence, it is not possible to construct a triangle.

28. (B) Since, $AB = BC \Rightarrow \angle A = \angle C$
(Equal sides contain equal angles.)



29. (A) Given $\Delta ABC \cong \Delta DEF$ by S.A.S. criterion
$AB = DE$, $BC = EF$, $\angle B = \angle E$

30. (A) $\angle A > \angle B > \angle C$
$AC > BC \Rightarrow \angle B > \angle A$ ....(i)
Also, $BC > AB \Rightarrow \angle A > \angle C$ ....(ii)
From (i) and (ii), we have
$\angle B > \angle A > \angle C$

31. (B) An angle is included between two sides. Hence, $AC = DE$ for the two triangles to be congruent.

32. (B) Given, $\Delta ABC \cong \Delta DEF$ by S.S.S.
Congruence $\Rightarrow \angle A = \angle D$
$\angle B = \angle E$
$\angle C = \angle F$ [Corresponding parts of congruent triangles.]

33. (C) 34. (C) 35. (A) 36. (C) 37. (C)

38. (D) Sum of two sides of a triangle is greater than its third side.
$\therefore DE + EF > FD$

39. (D) $\Delta ABC \cong \Delta PQR$ (Given)

$\Rightarrow \angle A = \angle P;\ \angle B = \angle Q;\ \angle C = \angle R$

and $AB = PQ;\ BC = QR;\ AC = PR$

Since $AB = 5$ cm,

$PQ = 5$ cm, and $\angle B = \angle Q = 40°$.

40. (C) Given, $AD \perp BC \Rightarrow BD = DC$

In ΔABD and ΔACD

$BD = DC$ (Given)

$AD = AD$ (Common side)

$\angle ADB = \angle ADC = 90°$

$\Delta ABD \cong \Delta ADC$ [S.A.S. congruence]

$\Rightarrow AB = AC$

41. (D) 42. (D) 43. (A) 44. (B) 45. (C)

Previous Contest Questions

1. (D) In ΔABC, $AB = AC$ (Given)

E and F are respectively the mid-points of the sides AB and AC.

$\Rightarrow AE = \frac{1}{2}AB$ and $AF = \frac{1}{2}AC$

We know that halves of the equal sides are equal

$\Rightarrow AE = AF$ ...(1)

Now, in ΔABF and ΔACE,

$AB = AC$ (Given)

$\angle BAF = \angle CAE$ (Each = $\angle A$)

$AE = AF$ [By (1)]

Thus, $\Delta ABF \cong \Delta ACE$ (By S.A.S. congruence)

$\therefore BF = CE$ (c.p.c.t.)

2. (B) 3. (D) 4. (A) 5. (A)

8. Quadrilaterals

Multiple Choice Questions

1. (A) $\angle 2 + \angle 3 + \angle APB = 180°$

$\Rightarrow \angle 1 + \angle 4 + \angle APB = 180°$

(Since $\angle 2 = \angle 1$ and $\angle 3 = \angle 4$.)

$\Rightarrow 2\angle 1 + 2\angle 3 + 2\angle APB = 360°$

$\Rightarrow \angle APB = \angle A + \angle B$

2. (C) $\angle A + \angle D = 185°$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle D = \frac{1}{2}(185°)$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle D\ (92.5°)$

From ΔAOD

3. (D) 4. (C) 5. (A) 6. (B) 7. (A) 8. (B)

9. (C) Since diagonals of a parallelogram bisect each other, ABCD must be a parallelogram.

10. (A) 11. (A)

12. (A) Area of parallelogram with base AB and attitude AM

$= 12 \times 9 = 108\ cm^2$

$108\ cm^2 = AD \times 11$ cm

$\Rightarrow AD = \frac{108}{11}$ cm

13. (A) 14. (B) 15. (A)

16. (A) As shown in the figure, since P is the midpoint of AB and $AB = 2AD$,

we have $AB = 2AP = 2AD$.

or $AP = AD$.

i.e., triangle ADP is an isosceles triangle.

If $\angle ADP = x$

and $\angle APD = x$, then,

$\angle A = 180° - 2x$

$\Rightarrow \angle B = 2x$

$\angle CPB = \angle PCB = 90° - x$

Since $\angle APB = 180°$ $\angle DPC = 90°$

17. (C) 18. (C) 19. (B) 20. (C) 21. (D)

22. (C) Let the angles be $3x$, $7x$, $6x$ and $4x$.

$\therefore 3x + 7x + 6x + 4x = 360°$ or

$20x = 360°$ or $x = 18°$. The angles are $54°$, $126°$, $108°$, $72°$. We see that adjacent angles are supplementary but opposite angles are not equal. Clearly, it is a trapezium.

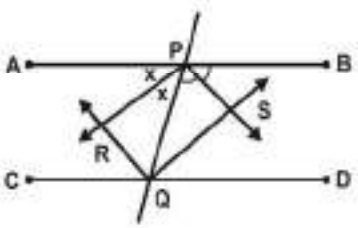
23. (C) Let the diagonals meet at O as shown in the figure.

$\angle POS = \angle ROQ = 90°$

Also $OP = OQ = OS = OR$, i.e., the diagonals are equal and bisect at right angles. Clearly, PRQS is a square.

24. (A) 25. (A) 26. (B) 27. (D) 28. (C)

29. (A)



$\angle APQ + \angle CQP = 180°$

$\Rightarrow \frac{\angle APQ}{2} + \frac{\angle CQP}{2} = 90°$

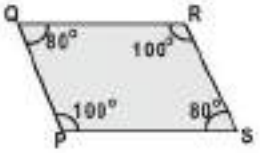
$\Rightarrow \angle RPQ + \angle PQR = 90°$

$\Rightarrow$ ΔPQR is a right angled triangle

Similarly, ΔPQS is a right angled triangle

$\Rightarrow$ PQRS is rectangle.

30. (A)



$\Rightarrow \angle P = \angle R = 100°$ and

$\angle Q = \angle S = 80°$

Adjacent sides of a parallelogram are not equal and sum of the adjacent angles is supplementary.

31. (B) Given that,

$\angle A : \angle B : \angle C : \angle D = 1 : 2 : 3 : 4$

Let $\angle A = x°$,

$\angle B = 2x°, \angle C = 3x°, \angle D = 4x°$.

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

Thus, the angles are

$\angle A = 36°$,

$\angle B = (2 \times 36)° = 72°$,

$\angle C = (3 \times 36)° = 108°$ and

$\angle D = (4x)° = (4 \times 36)° = 144°$

32. (C) Adjacent sides of a rhombous are equal and since, one of the diagonals is equal to the side, each of the angles in the triangle with one of the sides as diagonal is 60°.

Hence, the angles must be 60°, 120°, 60°, 120°.

33. (C) In a rhombus, diagonals do not bisect opposite angles.

34. (C) 35. (A)

36. (C) Refer question figure,

$p \parallel q \parallel r \Rightarrow p \parallel r$

$\Rightarrow \frac{AB}{BC} = \frac{DE}{EF} \Rightarrow \frac{2.5}{BC} = \frac{1.5}{3}$

$\Rightarrow BC = \frac{2.5 \times 3}{1.5} \Rightarrow BC = 5$ cm

37. (C) 38. (A) 39. (C) 40. (B) 41. (D)

42. (C) 43. (B) 44. (D) 45. (A) 46. (A)

47. (A) 48. (A) 49. (A) 50. (A)

Previous Contest Questions

1. (C) Let one angle be x.

$\therefore \frac{4}{5}x =$ another angle

Now, $x + \frac{4x}{5} = 180°$

$\Rightarrow x = 100°$

$\therefore \frac{4x}{5} = \frac{4}{5} \times 100° = 80°$

2. (C) 3. (C) 4. (A) 5. (C)

6. (A) Given two opposite angles of a parallelogram are $(3p - 4)°$ and $(48 - p)°$.

$\Rightarrow 3p - 4 = 48 - p$

$\Rightarrow p = 13$

7. (C) Given AB = DE and AB // DE

$\Rightarrow$ ABED is a parallelogram.

$\therefore$ BE || AD and BE = AD ....(1)

Given BC = EF and BC // EF

$\Rightarrow$ BEFC is a parallelogram.

$\therefore$ BE // CF and BE = CF ....(2)

From (1) and (2), we get

AD // CF and AD = CF

$\therefore$ ADFC is a parallelogram.

9. Areas of Parallelograms and Triangles

Multiple Choice Questions

1. (C) 2. (C) 3. (A) 4. (C) 5. (B) 6. (A)

7. (A) Given diagonal d = 6 cm,

$h_1 = 2.6$ cm and $h_2 = 1.4$ cm.

$\text{Area} = \frac{1}{2} d(h_1 + h_2)$

$= \frac{1}{2} \times 16 \times (2.6 + 1.4) = 8 \times 4 = 32$ cm^2

8. (A) Given ratio of the bases of two triangles, $b_1 : b_2$ is a : b and the ratio of their corresponding altitudes, $h_1 : h_2 = c : d$

$\therefore$ Ratio of areas

$= \frac{\frac{1}{2} b_1 h_1}{\frac{1}{2} b_2 h_2} = \frac{b_1}{b_2} \times \frac{h_1}{h_2}$

$= \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} = ac : bd$

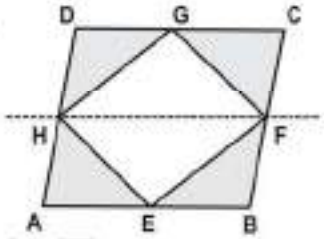
9. (D) Given base of parallelogram is 18 cm and corresponding altitude, h = 6 cm.

$\therefore$ Area of parallelogram = b × h

= 18 × 6 cm^2 = 108 cm^2

10. (A) 11. (C) 12. (A) 13. (B) 14. (C)

15. (C)



Join HF.

Then $HF \parallel AB \parallel CD$

$\Rightarrow \text{ar}(\Delta FGH) = \frac{1}{2}(HFCD)$ ..... (1)

$\Rightarrow \text{ar}(\Delta EFH) = \frac{1}{2}(ABFH)$ .....(2)

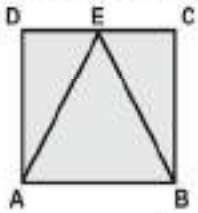
From (1) and (2), we get

$\text{ar}(EFGH) = \frac{1}{2}(ABCD)$

$\Rightarrow 40 = \frac{1}{2}(ABCD)$

$\Rightarrow 80 \text{ cm}^2 = ABCD$

16. (B)



$\text{ar}(\Delta AEB) = \frac{1}{2}(ABCD)$

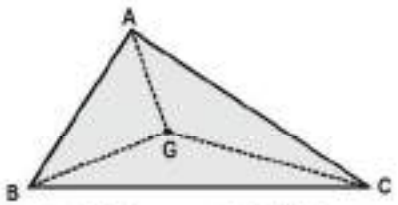
[Area of triangle is half the area of square if they are on the same base and between the same parallels.]

$\Rightarrow \frac{\text{ar}(\Delta AEB)}{\text{ar}(ABCD)} = \frac{1}{2}$

17. (C) ΔADP is on the same base as parallelogram ABCD, but not between the same parallels AD and BE.

18. (B) 19. (C) 20. (B) 21. (C) 22. (B)

23. (B)



ar of ΔAGB = ar of ΔBGC = ar of ΔAGC

$\Rightarrow 3[\Delta AGC] = \Delta ABC$

$\Rightarrow \frac{\Delta AGC}{\Delta ABC} = \frac{1}{3} = 1:3$

24. (A) Quadrilaterals on the same base and between the same parallels are equal in area.

Hence, the ratio of their areas is 1 : 1.

25. (A) ΔABC is a right triangle.

$\therefore AC^2 = AB^2 + BC^2 = 16 + 9 = 25$

$\Rightarrow AC = 5 \text{ cm}$

Area of the quad. ABCD

= Area of rt. ΔABC + Area of rt. ΔACD

$= \frac{1}{2} \times 4 \times 3 + \frac{1}{2} \times 5 \times 12 = 6 + 30 = 36 \text{ cm}^2$

26. (A) Refer the question figure.

Triangles on the same base and in between the same parallels are equal in area.

ΔAXD and ΔAXC are on the same base and between same parallels.

27. (A) Area of $\parallel^{gm}$ ABCD

$= \text{ar}(\Delta ABL) + \text{ar}(\Delta DCL) + \text{ar}(\Delta ADL)$

$= 15 + 32 + \text{ar}(\Delta ADL)$

$= 47 + \frac{1}{2}$ ar of $\parallel^{gm}$ ABCD

[Since, $\parallel^{gm}$ ABCD and ΔADL are on the same base AD and between the same parallels AD and BC.]

$\Rightarrow$ Area of $\parallel^{gm}$ ABCD $- \frac{1}{2}$ ar of $\parallel^{gm}$ ABCD $= 47$

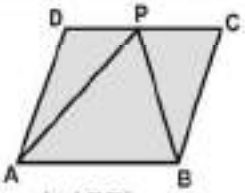
$\Rightarrow \frac{1}{2}$ ar of $\parallel^{gm}$ ABCD $= 47$

$\therefore$ ar of $\parallel^{gm}$ ABCD $= 94 \text{ cm}^2$

28. (D) 29. (B) 30. (B) 31. (B) 32. (C)

33. (C) AFDE is a parallelogram, as ED // AB and DF // AC.

34. (B)



$\frac{\text{ar}(\Delta ABP)}{\text{ar}(ABCD)} = \frac{1}{2}$

35. (A) 36. (C) 37. (D) 38. (A) 39. (C)

40. (C) 41. (A) 42. (D) 43. (A) 44. (B)

45. (C)

Previous Contest Questions

1. (C) Here, $\Delta AFE \cong \Delta BDF \cong \Delta CDE \cong \Delta DEF$

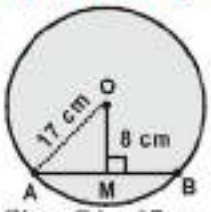
$\therefore$ ar (BDEF)
$= ar(\Delta BDF) + ar(\Delta DEF)$
$= 2\ ar(\Delta AFE)$
$\Rightarrow x = 2$

2. (D) 3. (A) 4. (C) 5. (D)

10. Circles

Multiple Choice Questions

1. (B)



Given OA = 17 cm and OM = 8 cm.

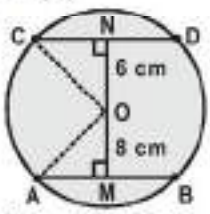
In ΔOMA, $OA^2 = OM^2 + AM^2$

$\Rightarrow AM = \sqrt{225} = 15$ cm

So, AB = 2 × AM = 2 × 15 = 30 cm.

2. (A) 3. (C)

4. (C)



Given AB = 12 cm $\Rightarrow$ AM = MB = 6 cm

In ΔOMA, $OA^2 = OM^2 + AM^2$

$\Rightarrow OA = \sqrt{100} = 10$ cm

$\therefore$ Radius = OA = OC = 10 cm

In ΔONC, $OC^2 = ON^2 + NC^2$

$\Rightarrow NC = \sqrt{64} = 8$ cm

$\therefore$ Length of the chord = 2×NC = 16 cm

5. (A) 6. (B) 7. (B) 8. (C) 9. (B) 10. (B)

11. (A) Angles in the same segment are equal.

$\Rightarrow \angle BAD = \angle BCD = 30°$

In ΔCBP, $\angle C + \angle B + \angle P = 180°$

$\Rightarrow \angle CBP = 105°$

12. (D) Since angles in the same segment are equal, $\angle BCE = \angle BAE = 30°$.

Since, AD is a bisector,

$\angle BAE = \angle CAE = 30°$

$\therefore \angle BAC = 30° + 30° = 60°$

13. (C) PQRS is a cyclic quadrilateral.

$\Rightarrow \angle P + \angle R = 180°$

$\Rightarrow \angle P = 180° - 138° = 42°$

In ΔPQS, $\angle P + \angle Q + \angle S = 180°$

$\therefore \angle PQS = 48°$

14. (B)

15. (D) 16. (A) 17. (C) 18. (C) 19. (D)

20. (D) Since PQ = 4 cm = 2 × OQ
= 2 × radius, PQ is the diameter of circle. Join RQ. $\angle PRQ = 90°$.
(Angle in a semicircle.)

$\therefore \angle ORQ = 90° - 35° = 55°$

But OR = OQ.

$\therefore \angle ORQ = \angle OQR = 55°$

$\therefore y = 180° - (55° + 55°) = 70°$

21. (C) Since ABCD is a cyclic quadrilateral, $\angle DAB = \angle BCE = 120°$.

Since AC is the bisector of $\angle DAB$,

$\angle DAC = \frac{120°}{2} = 60°$.

Since angles in the same segment are equal, $\angle DBC = \angle DAC = 60°$.

22. (D) Join OB, Then $\angle OBA = 90°$

$AB^2 + OB^2 = AO^2$.

$\therefore$ OB = AB = 2 cm

Hence $AO^2 = 2^2 + 2^2$ or $AO = 2\sqrt{2}$ cm

$\therefore AC = AO + OC = (2\sqrt{2} + 2)$ cm

23. (A) 24. (D)

25. (A) Clearly, $\angle BYX = \angle BAX = \frac{\angle A}{2}$.

Also $\angle ZYB = \angle ZCB = \frac{\angle C}{2}$. (Angles in the same segment.)

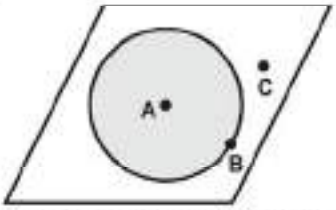
Hence, $\angle ZYX = \angle ZYB + \angle BYX$

$\Rightarrow \angle ZYX = \frac{\angle C}{2} + \frac{\angle A}{2} = \frac{\angle A + \angle C}{2}$

$= \frac{180° - \angle B}{2} = 90° - \frac{\angle B}{2}$

Similarly, the other angles are $90° - \frac{A}{2}$ and $90° - \frac{C}{2}$.

26. (C) 27. (A)
28. (B)



A circle divides a plane in three parts.
Point A refers to the interior region.
Point B refers to the point on the boundary (i.e. the circle).
Point C refers to the exterior region.

29. (B) 30. (D) 31. (C) 32. (C) 33. (C) 34. (C)

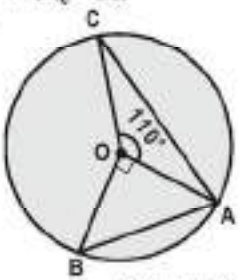
35. (C) Since, $\overset{\frown}{PQR} = \overset{\frown}{SRQ}$

$\Rightarrow \overset{\frown}{PQR} - \overset{\frown}{QR} = \overset{\frown}{SRQ} - \overset{\frown}{QR}$

$\Rightarrow \overset{\frown}{PQ} = \overset{\frown}{SR}$

$\therefore$ PQ = SR

36. (B)



$\angle OAC = \frac{180° - 110°}{2} = 35°$

$\angle OAB = \frac{180° - 90°}{2} = 45°$

$\angle BAC = 35° + 45° = 80°$

37. (D) Since the chords are equidistant from the centre, AB = BC = CA

Hence, ΔABC is an equilateral triangle.

38. (B) OM $\perp$ PQ

$\Rightarrow$ M is the mid-point of PQ. Join OQ.

Now, in rt. Δ OMQ, we have
$MQ^2 = OQ^2 - OM^2$

$\Rightarrow MQ = \sqrt{16} = 4$ cm

$\therefore PQ = 2 \times MQ = 2 \times 4 = 8$ cm

39. (D) Refer the question figure,
OM $\perp$ PQ

$\Rightarrow$ M is the mid-point of PQ

$MQ = \frac{1}{2} \times 6 = 3$ cm

Join OQ, In rt. Δ OMQ, we have
$OQ^2 = OM^2 + MQ^2 = 4^2 + 3^2 = 25$

$\Rightarrow OQ = \sqrt{25} = 5$ cm

40. (D) $\angle POQ = 2\angle R \Rightarrow \angle POQ = 2l$

In Δ PQO, OP = OQ = r

$\Rightarrow \angle OQP = \angle OPQ = m$

Also, $\angle OPQ + \angle OQP + \angle POQ = 180°$

$\Rightarrow m + m + 2l = 180°$

$\Rightarrow m + l = 90°$

41. (A) $\angle QOR = 360° - 110° - 120° = 130°$

$\angle QPR = \frac{1}{2}\angle QOR = \frac{1}{2} \times 130° = 65°$

42. (C) $\angle AOB = 2\angle C = 2 \times 60° = 120°$

Now, $\angle AOB + x = 360°$

$\Rightarrow x = 360° - \angle AOB$

$= 360° - 120° = 240°$

43. (D) 44. (C) 45. (A) 46. (A)

47. (C) $\angle BAD = \frac{1}{2}\angle BOD = \frac{1}{2} \times 140° = 70°$

$\angle DCP = \angle BAD$ [Since exterior angle of a cyclic quadrilateral is equal to the interior opp.angle.]

$\therefore \angle DCP = \angle BAD = 70°$

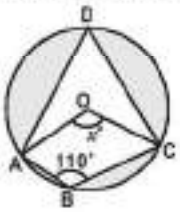
48. (C) $\angle DAB = \frac{1}{2} \times 160° = 80°$

$\angle BPD = 180° - 80° = 100°$

$\angle BCD = \angle BPD = 100°$

[Angles in the same segment are equal in measure.]
Difference between the required angles
$= 100° - 100° = 0°$

49. (B) Take any point D on the circle another than A, B and C. Join DA and DC.



Now, ABCD is a cyclic quadrilateral.

$\therefore \angle B + \angle D = 180^\circ$

$\Rightarrow \angle D = 180^\circ - \angle B = 70^\circ$

Now, $x = 2\angle D = 2 \times 70^\circ = 140^\circ$

Hence, $x = 140^\circ$

50. (D) ΔABC is isosceles with $AB = AC$

$\Rightarrow \angle BCA = \angle ABC = 50^\circ$ (Base angles)

$\Rightarrow \angle BAC = 180^\circ - 2 \times 50^\circ = 80^\circ$

$\angle BDC = \angle BAC = 80^\circ$ (Angles in the same segment are equal.)

BECD is a cyclic quadrilateral in which opposite angles are supplementary.

$\Rightarrow \angle BEC = 180^\circ - \angle BDC$

$= 180^\circ - 80^\circ = 100^\circ$

Hence, the required sum

$= BDC + BEC = 80^\circ + 100^\circ = 180^\circ$

Previous Contest Questions

1. (A) 2. (A) 3. (A) 4. (B) 5. (B)

6. (A) Refer the question figure.

In ΔOBC, $OB = OC$ (= radius)

$\Rightarrow \angle OBC = \angle OCB = y$

Now, $z + y + y = 180^\circ$

$\Rightarrow z = 180^\circ - 2y$ ..... (1)

Also, $\angle BOC = 2\angle BAC$

$\Rightarrow z = 2x$ ..... (2)

From (1) and (2),

$\Rightarrow 2x + 2y = 180^\circ \Rightarrow x + y = 90^\circ$

$\therefore \angle BAC + \angle OBC = 90^\circ$

7. (B) 8. (C) 9. (A) 10. (D)

11. Heron's Formula

Multiple Choice Questions

1. (A) Let the length of the sides be $3x$, $4x$, $5x$ metres.

$\Rightarrow 144 = 12x \Rightarrow x = 12$

$\therefore$ Sides of the triangular field are 36 m, 48 m, 60 m.

$\Delta = \sqrt{72(72-36)(72-48)(72-60)}$

$= 864$ sq. m.

2. (B) If 8 cm is the length of a side of an equilateral triangle then its area is

$= \frac{\sqrt{3}}{4} \times (8)^2 = 16\sqrt{3}$ cm^2

3. (D) Let the equal sides of an isosceles triangle be x cm. Its perimeter is

$x + x + 12 = 32 \Rightarrow x = 10$ cm

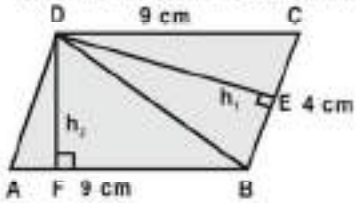
Area of the triangle is

$\Delta = \sqrt{16(16-10)(16-10)(16-12)}$

$= 8 \times 6 = 48$ sq. cm

4. (B) 5. (A) 6. (B) 7. (C) 8. (D)

9. (B) A diagonal divides a parallelogram into two triangles of equal area.



Area $\Delta ABC = \frac{1}{2} \times BC \times DE$

$= \frac{1}{2} \times 4 \times h_1 = 2h_1$

Area $\Delta BAD = \frac{1}{2} \times AB \times DF$

$= \frac{1}{2} \times 9 \times h_2 = \frac{9h_2}{2}$

Since ΔBDC and ΔBAD are equal in area,

$2h_1 = \frac{9h_2}{2} \Rightarrow 4h_1 = 9h_2$ or $h_1 : h_2 = 9 : 4$

10. (B) Given perimeter of a rhombus is 52 cm,

each side of the rhombus $= \frac{52}{4} = 13$ cm.

Area of rhombus

$= 2\sqrt{25(25-24)(25-13)(25-13)}$

$= 120$ sq.cm.

Area $= \frac{1}{2} \times$ product of diagonals

$\Rightarrow 120 = \frac{1}{2} \times 24 \times d \Rightarrow d = 10$ cm

$\therefore$ The other diagonal is 10 cm.

11. (A) Let $AC = x$ cm and $BD = 4$ cm (Given)

Given area of $ABCD = 28$ cm^2

$\Rightarrow \frac{1}{2} \times x \times 4 = 28 \Rightarrow x = 14$ cm

Clearly, $AO = \frac{14}{2} = 7$ cm.

By Pythagoras' theorem,

$AO^2 + BO^2 = AB^2$

or $7^2 + 2^2 = 53$ or $AB = \sqrt{53}$

$\therefore$ Perimeter $= 4AB = 4\sqrt{53}$

12. (B) 13. (C) 14. (A) 15. (A) 16. (C)
17. (A) 18. (A) 19. (D) 20. (D) 21. (A)
22. (C) The line of division is along the diagonal of the rhombus shaped field. So, each son gets an equal area.
Area of ΔABC
$= \sqrt{s(s-a)(s-b)(s-c)}$
where $s = \frac{a+b+c}{2}$.
Given a = 100 m, b = 100 m and c = 160 m.
$\therefore$ Area of ΔABC
$= \sqrt{180(80)(80)(20)} = 4800\ m^2$
$\therefore$ Hence each son gets an area of 4800 m^2.
23. (B) 24. (A) 25. (C) 26. (B) 27. (A) 28. (D)
29. (B) Sides of an isosceles right triangle are a, a and $\sqrt{2}a$.
Area of isosceles right triangle
$= \frac{1}{2}a^2 \Rightarrow 8\ cm^2 = \frac{1}{2}a^2 \Rightarrow a = 4\ cm$
Perimeter of the triangle
$= (a + a + \sqrt{2}a)\ cm$
$= (8 + 4\sqrt{2})\ cm$
30. (D) Perimeter of an equilateral triangle = 60 m
$\Rightarrow$ 3a = 60 m $\Rightarrow$ a = 20 m
Area of an equilateral triangle
$= \frac{\sqrt{3}}{4}a^2 = 100\sqrt{3}\ m^2$
31. (D) Diagonal of a square is $12\sqrt{2}$ cm
$\Rightarrow 2\sqrt{a} = 12\sqrt{2}\ cm \Rightarrow a = 12\ cm$
$\therefore$ Perimeter of the square = 4a
$= 4 \times 12 = 48\ cm$
Perimeter of an equilateral triangle is 3a
$\Rightarrow 48\ cm = 3a \Rightarrow a = \frac{48}{3}cm = 16\ cm$
$\therefore$ Area of equilateral triangle
$= \frac{\sqrt{3}}{4}a^2 = 64\sqrt{3}\ cm^2$
32. (C) Area of ΔPQR
$= \sqrt{s(s-a)(s-b)(s-c)}$
where $s = \frac{50+50+80}{2} = 90$
Area $= \sqrt{90(90-50)(90-50)(90-80)}$
$= 1200\ cm^2$
Area of ΔPQR = Area of ΔPSR
$= 1200\ cm^2$
$\therefore$ Area of Rhombus =
Area of ΔPQR + Area of ΔPSR
$= (1200 + 1200)\ cm^2 = 2400\ cm^2$
The area of grass field for 48 cows to graze = 2400 cm^2
$\therefore$ Area of the grass field for each cow to graze $= \frac{2400}{48} = 50\ cm^2$
33. (C) 34. (A) 35. (B) 36. (D) 37. (C)
38. (C) Sides of each tile are 26 cm, 20 cm and 10 cm.
Semiperimeter s = 28 cm
Area of each tile
$= \sqrt{s(s-a)(s-b)(s-c)}$
$= 89.76\ cm^2$
Area of 16 tiles $= 89.76 \times 16\ cm^2$
$= 1436.16\ cm^2$
Cost of polishing the tiles at the rate of 20 p per cm^2 = 1436.16 × ₹ 0.2
= ₹ 287.23
39. (B) 40. (D) 41. (D) 42. (A) 43. (C)

Previous Contest Questions

1. (B) Let a, b and c be the three sides of ΔABC.
Given a = 11 cm and
a + b + c = 32 cm
$\Rightarrow$ 11 + b + c = 32 cm
$\Rightarrow$ b + c = 21 cm ..... (1)
Also, we are given that
b − c = 5 cm .....(2)
Adding (1) and (2),
2b = 26 cm
$\Rightarrow$ b = 13 cm and c = 8 cm
Now, $s = \frac{a+b+c}{2} = \frac{32}{2} = 16\ cm$
Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{16\times(16-11)\times(16-13)\times(16-8)}$
$= \sqrt{16\times5\times3\times8} = \sqrt{64\times30} = 8\sqrt{30}\ cm^2$
2. (D) 3. (C) 4. (B) 5. (D) 6. (A) 7. (C)

12. Surface Areas and Volumes

Multiple Choice Questions

1. (B) 2. (A) 3. (D) 4. (A) 5. (B)

6. (A) Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ ratio $= \frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$

$h_1 : h_2$ (Given $r_1 = r_2$)

7. (A) Dimensions of a room $= 10\text{ m} \times 10\text{ m} \times 5\text{ m}$

Length of the diagonal

$= \sqrt{(10)^2 + (10)^2 + (5)^2} = 15\text{ m}$

$\therefore$ The length of the longest pole that can be put in a room = 15 m

8. (B) T.S.A. $= \frac{3520}{5} = 704\text{ cm}^2$

$\Rightarrow \pi r(l + r) = 704 \Rightarrow r = 7$

$\Rightarrow h = \sqrt{l^2 - r^2} = 24$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 = 1232\text{ cm}^3$

9. (A) $V = 33264\text{ cm}^3$ (Since $1000\text{ cm}^3 = 1$ litre.)

$\frac{1}{3}\pi r^2 h = 33264 \Rightarrow r = 21\text{ cm}$

$\therefore l = \sqrt{h^2 + r^2} = 75\text{ cm}$

C.S.A. $= \pi r l = \frac{22}{7} \times 21 \times 75\text{ cm}^2$

1 sq. m $= 10000\text{ cm}^2$

$\therefore$ cost $= \frac{12 \times 22 \times 21 \times 75}{7 \times 10000} = ₹\ 5.94$

10. (C) Here, $r = 7$ cm and $h = 24$ cm

We know that $l^2 = r^2 + h^2$

$l^2 = (7)^2 + (24)^2 = 49 + 576$

$l = \sqrt{(7)^2 + (24)^2} = 25\text{cm}$

$\therefore$ C.S.A $= \pi r l = \frac{22}{7} \times 7 \times 25 = 550\text{ cm}^2$

Area of sheet required to make 10 such caps $= 550 \times 10\text{ cm}^2 = 5500\text{ cm}^2$

11. (C) 12. (A) 13. (B) 14. (B) 15. (C)

16. (C) Diameter of first cone = Diameter of second cone

$\frac{\text{Slant height of first cone } (l_1)}{\text{Slant height of second cone } (l_2)} = \frac{5}{4} \Rightarrow \frac{l_1}{l_2} = \frac{5}{4}$

$\Rightarrow \frac{\text{Curved surface area of first cone}}{\text{Curved surface area of the second cone}}$

$= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \left(\frac{r_1}{r_2}\right)\left(\frac{l_1}{l_2}\right) = 1 \times \frac{5}{4} = \frac{5}{4}$

Hence, the ratio of their curved surface areas is 5 : 4.

17. (A) 18. (D) 19. (A) 20. (C) 21. (B)

22. (B) Slant height $l = 25$ cm

Radius of the base of the cone, $r = 7$ cm

C.S.A cone $= \pi r l$

$= \frac{22}{7} \times 7 \times 25\text{cm}^2 = 550\text{cm}^2$

23. (B) C.S.A $= 2\pi r h = 660\text{ cm}^2$

Curved surface area $= 660\text{ cm}^2$

24. (D) 25. (B) 26. (C) 27. (C) 28. (D)

29. (D) The edge of cube is 'a' units. The edge is increased by 50%

$= a + \frac{a}{2} = \frac{3a}{2}$

$\therefore$ Surface area $= 6a^2 = \frac{27a^2}{2}$

$\therefore$ Percentage increase of surface area

$= \frac{\left(\frac{27a^2}{2} - 6a^2\right)}{6a^2} \times 100\% = 125\%$

30. (B) 31. (C) 32. (B) 33. (C) 34. (C)

35. (D) Volume of a cube = V

Edge of a cube is doubled, $a = 2a$

$\therefore$ Volume of new cube $= a^3 = 8V$

36. (C) Consider two cubes of side 6 cm joined end to end.

The cuboid formed is of dimensions 12 cm $\times$ 6 cm $\times$ 6 cm.

$\therefore$ Surface area of a cuboid

$= 2(lb + bh + hl) = 360\text{ cm}^2$

37. (B) Surface area of a cube = S

Edge of a new cube $= 2a$

$\therefore$ Surface area of new cube

$= 6a^2 = 4S$

38. (C) The dimensions of wall $= 8\text{ m} \times 4\text{ m} \times 20\text{ cm} = 8\text{ m} \times 4\text{ m} \times 0.2\text{ m}$

Volume of the wall $= (8 \times 4 \times 0.2)\text{m}^3 = 6.4\text{ m}^3$

$\therefore$ Cost of constructing a wall at a rate of ₹ 25 per $\text{m}^3 = 6.4 \times ₹\ 25 = ₹\ 160$

39. (D) Circumference of the base of cone = circumference of circle = $2\pi r$

$\Rightarrow 44 = 2\pi r \Rightarrow r = 7$ m

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(7)^2\times 9 = 462$ m^3

Hence, the volume of cone is 462 m^3.

40. (A) Volume of clay = 10 m^3

Area of land = 10 ares

$= 10 \times 100$ m^2 = 1000 m^2

$\therefore$ Rise in the level of ground

$= \frac{10}{1000} = 0.01$ m = 1 cm

41. (D) Volume of a cuboid = 12 cm^3

$\Rightarrow l \times b \times h = 12$ cm^3

When the sides are doubled, the volume of cuboid = $2l \times 2b \times 2h$ = 96 cm^3

42. (C) 43. (D) 44. (B) 45. (B) 46. (C)

Previous Contest Questions

1. (C) 2. (C) 3. (A) 4. (C) 5. (C)

6. (B) Edge of big cube = k units

Let the edge of small cube be 'a' units.

$\Rightarrow$ Volume of each small cube = a^3 cu.units;

$\Rightarrow$ Volume of big cube = k^3

Given there are 'n' small cubes

$\Rightarrow k^3 = n \cdot a^3 \Rightarrow a^3 = \frac{k^3}{n} \Rightarrow a = \frac{k}{\sqrt[3]{n}}$

$\therefore$ Length of the edge of the new cube is $\frac{k}{\sqrt[3]{n}}$.

7. (A) 8. (B) 9. (C) 10. (D)

13. Statistics

Multiple Choice Questions

1. (B) 2. (B) 3. (D) 4. (A) 5. (A)

6. (B) Class mark = $\frac{\text{upper limit} + \text{lower limit}}{2}$

From the given options, 17.5 – 22.5 is the required class

since, $\frac{17.5+22.5}{2} = \frac{40}{2} = 20$

7. (A) 8. (B) 9. (D) 10. (C)

11. (C) First ten prime numbers in ascending order are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Here, n = 10, which is even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{ item} + \left(\frac{n}{2}+1\right)^{th}\text{ item}}{2}$$

$$= \frac{5^{th}\text{ item} + 6^{th}\text{ item}}{2}$$

$$= \frac{11+13}{2} = \frac{24}{2} = 12$$

12. (B) No. of boys in a school = 90

Mean marks of boys = 45%

Total marks of 90 boys = $90 \times 45 = 4050$

No. of girls in a school = 30

Mean marks of girls = 70%

$\therefore$ Total marks of 30 girls = $30 \times 70 = 2100$

Total marks secured by 90 boys and 30 girls = 4050 + 2100 = 6150

No. of boys and girls in a school = 90 + 30 = 120

$\therefore$ Average marks % of school

$= \frac{6150}{120} = 51.25\%$

13. (C) The mid-point of the top sides of rectangles represent the mid values of classes for which the histogram is drawn. So, the required graph is a frequency polygon.

14. (C) Measure of central tendency is the central value of the data around which the values of the other observations tend to concentrate.

15. (B) The number 20 will be included in the class interval 20 – 30.

16. (C) Mean = $\frac{30+36+39+23+27}{5} = 31$

17. (A) Given:

No. of observations = 10 (even)

Median = 25

We know that, for n = 10 (even)

$$\text{Median} = \text{Value of } \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th}\text{ item} + \left(\frac{n}{2}+1\right)^{th}\text{ item}\right]$$

$$25 = \text{Value of } \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th}\text{ item} + \left(\frac{10}{2}+1\right)^{th}\text{ item}\right]$$

$$25 = \text{Value of } \frac{1}{2}\left[5^{th}\text{ item} + 6^{th}\text{ item}\right]$$

$$25 = \frac{1}{2}[x+2+x+4] \Rightarrow x = 22$$

18. (B) 19.(A) 20.(A) 21.(D) 22.(C) 23. (C)

24. (B) Given,
Class interval = 63 – 72 (72 included)
Class size (width) = 9
Maximum observation = 112
Minimum observation = 14

$$\therefore \text{No. of classes} = \frac{\text{Max} - \text{Min}}{\text{width}}$$

$$= \frac{112-14}{9} = \frac{98}{9} = 10.8 \equiv 11$$

Hence, the no. of classes in the distribution is 11.

25. (B) Given, mid point of the class = m
The upper class limit = l
We know that,

$$\frac{\text{upper limit} + \text{lower limit}}{2} = \text{mid point}$$

$$\Rightarrow m = \frac{l + \text{lower limit}}{2}$$

$$\Rightarrow \text{lower limit} = 2m - l$$

Hence, the required lower class limit is $2m - l$.

26. (B) Given:
Width of each of 9 classes = 2.5
Total width of 9 classes = $9 \times 2.5 = 22.5$
Lower class boundary of lowest class = 10.6
$\Rightarrow$ The upper class boundary of highest class = lower class boundary of lowest class + Total width = 10.6 + 22.5 = 33.1
Hence, the required upper class boundary = 33.1.

27. (B) Mid value of a class = 10
Width of class = 6
Let x_1 be the lower limit and x_2 be the upper limit of the class

$$\Rightarrow \frac{x_1 + x_2}{2} = 10 \text{ (mid point)}$$

$$\Rightarrow x_1 + x_2 = 20 \quad \ldots\ldots (i)$$

and $x_2 - x_1 = 6$ (width) ..... (ii)
Solving (i) and (ii), we get
$x_1 = 7, x_2 = 13$
Hence, the lower limit of the class is 7.

28. (C) 29. (D) 30. (A) 31. (A)

32. (A) Mean of 75 numbers = 25
$\therefore$ Sum of 75 numbers = 75×25
If each number is divided by 5, the sum is also divided by 5.

$$\text{New sum} = \frac{75 \times 25^{5}}{\cancel{5}} = 75 \times 5$$

$$\text{New mean} = \frac{75 \times 5}{75} = 5$$

33. (B) 34. (B) 35. (B) 36. (D) 37. (A)

38. (D) Given data = 0, 2, 2, 2, –3, 5 –1, 5, 5, –3, 6, 6, 5, 6
Their ascending order is
–3, –3, –1, 0, 2, 2, 2, 5, 5, 5, 5, 6, 6, 6.
No. of observations = 14
for n = 14 (even)
Median = value of

$$\frac{1}{2}\left[\left(\frac{n}{2}\right)^{\text{th}} \text{item} + \left(\frac{n}{2}+1\right)^{\text{th}}\right]$$

$$\Rightarrow \text{Value of } \frac{1}{2}\left[7^{\text{th}} \text{ item} + 8^{\text{th}} \text{ item}\right]$$

$$\Rightarrow \text{Median} = \frac{1}{2}[2 + 5] = \frac{7}{2} = 3.5$$

$\therefore$ The required median is 3.5.

39. (D) Mean, $\bar{x} = 40$, n = 20. (Given)

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_{20}}{20}$$

$$x_1 + x_2 + \ldots\ldots + x_{20} = 40 \times 20 = 800$$

New numbers are $(x_1 - 5), (x_2 - 5), \ldots\ldots, (x_{20} - 5)$
Then mean of new numbers =

$$\frac{(x_1 - 5) + (x_2 - 5) + \ldots\ldots + (x_{20} - 5)}{20}$$

$$= \frac{800 - 100}{20} = \frac{700}{20} = 35$$

40. (D) Given, mean of a, b, c, d, e = 28

$$\Rightarrow a + b + c + d + e = 140 \quad \ldots\ldots (i)$$

Mean of a, c, e = 24

$$\Rightarrow a + c + e = 72 \quad \ldots\ldots (ii)$$

By substituting (ii) in (i), we get

$$b + d + 72 = 140$$

$$\Rightarrow b + d + 72 = 140$$

$$\Rightarrow b + d = 140 - 72$$

$$\Rightarrow b + d = 68 \quad \ldots\ldots (iii)$$

$$\text{Mean of b and d} = \frac{b+d}{2} = \frac{68}{2} = 34$$

$\therefore$ Mean of b and d is 34.

41. (A) The algebraic sum of the deviations of a set of n values from their mean is always zero (0).

42. (D) 43.(C) 44.(A) 45.(A) 46.(A) 47.(A)

Previous Contest Questions

1. (B) Number of observations

$$= \frac{\text{Sum of observations}}{\text{Mean}} = \frac{600}{50} = 12$$

2. (B) Given,
Class = 90 – 120

$$\text{Class mark} = \frac{90+120}{2} = \frac{210}{2} = 105$$

Hence, the required class mark is 105.

3. (D) Given mean = 5.3

$$\text{We know that mean} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$\therefore \frac{4\times11+8\times2+6\times3+7p}{11+2+3+p} = 5.3$$

$$\Rightarrow p = \frac{6.8}{1.7} = 4$$

14. Probability

Multiple Choice Questions

1. (B) 2. (B) 3. (A) 4. (C) 5. (C)

6. (B) E = {(1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)}

$\Rightarrow$ n(E) = 12; n(S) = 6 × 6 = 36

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

7. (C) E = 2, 4, 6
n(E) = 3, n(S) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

8. (B) E = 1, 2 ; 2, 3 ; 3, 4; 4, 5

$\Rightarrow$ n(E) = 4

$n(S) = {}^{5}C_{2} = 10$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{10} = \frac{2}{5}$$

9. (B) There are two '10's of black suit (i.e., spade and club)

$\therefore$ Number of favourable outcomes = 2
Total number of possible outcomes = 52

$$\therefore P(\text{'10' of a black suit}) = \frac{2}{52} = \frac{1}{26}$$

10. (A) 11. (B)

12. (C) n(E) = 4 + 4 = 8
n(S) = 52

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

13. (A) 14. (D) 15. (D) 16. (A) 17. (C)

18. (C) E = {2, 3, 4, 5, 6} $\Rightarrow$ n(E) = 5
S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ n(S) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{6}$$

19. (A) S = {BBB, BBG, BGG, GGG}
$\Rightarrow$ n(S) = 4
E = {BBB, BBG, BGG}
$\Rightarrow$ n(E) = 3

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

20. (B) A non-leap year contains 365 days, i.e., 52 weeks + 1 day.
S = {S, M, T, W, Th, F, Sa}
$\Rightarrow$ n(S) = 7
E = {Saturday}
$\Rightarrow$ n(E) = 1

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$$

21. (C) Random drawing of balls ensures equally likely outcomes.
Total number of balls = 12
$\therefore$ Total number of possible outcomes = 12
Number of white balls = x
$\Rightarrow$ Out of total 12 outcomes, favourable outcomes = x
P (white ball) =

$$\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x}{12}$$

22. (C) No. of coins tossed = 1
No. of times the coin is tossed = 150
Probability of getting a head

$$= \frac{90}{150} = \frac{3}{5}$$

23. (D) No. of coins tossed = 2
No. of times coins are tossed = 80
From the data given, the probability of getting two heads

$$= \frac{25}{80} = \frac{5}{16}$$

24. (A) From the data given, the probability of getting atleast one head = $\frac{35+25}{80}$

$$= \frac{60}{80} = \frac{3}{4}$$

($\because$ Atleast one head means one or more than one head.)

25. (A) No. of times a die is tossed = 100
From the data given, the probability of getting an even number in a trial
$= \frac{15+15+10}{100} = \frac{40}{100} = 0.4$

26. (C) From the given data, the probability of getting a number less than 3
$= \frac{20+15}{100} = \frac{35}{100} = 0.35$

27. (A) Number of students having two or more cavities
$= \frac{80}{400} \times 1500 = 300$

28. (B) Total students = 880
No. of smokers = 500
∴ Non - smokers = 880 – 500 = 380
Now, required probability
$= \frac{380}{880} = \frac{19}{44}$

29. (C) 30. (C) 31. (D) 32. (A) 33. (C)

34. (B) Probability (getting a no. 4 or 5)
$= \frac{2}{6} = \frac{1}{3}$

35. (D) Probability (getting up head)
$= \frac{1000-453}{1000} = \frac{547}{1000} = 0.547$

36. (A) Total no. of balls = 32
No. of sixer hits = 8
∴ Probability that a sixer is not hit in a ball
$= \frac{32-8}{32} = \frac{24}{32} = 0.75$

37. (B) 38. (A) 39. (D) 40. (B) 41. (A)

42. (D) Probability (not getting 1 or 6)
= 1 – probability of getting 1 or 6.
$\Rightarrow 1 - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

43. (A) Random drawing of blocks ensures equally likely outcomes.
The total number of blocks = 6
The number of blocks representing A = 2
Total number of possible outcomes = 6
P(A) =
$$\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$= \frac{2}{6} = \frac{1}{3}$

44. (B) Total balls = 12
No. of black balls = 5
∴ P (a black ball) = $\frac{5}{12}$

45. (C) In 100 trials, total number of odd numbers is 22 + 20 + 25 = 67
P(an odd no.) = $\frac{67}{100}$

46. (A) No. of red queens = 2
P (a red queen) = $\frac{2}{52} = \frac{1}{26}$

47. (B) S = {1, 2, 3, 4, ..., 25}
Let E = event of getting a prime number = {2, 3, 5, 7, 11, 13, 17, 19, 23}.
Then, n(E) = 9.
∴ $P(E) = \frac{n(E)}{n(S)} = \frac{9}{25}$
Required probability
= 1 – P(E)
$= \left(1 - \frac{9}{25}\right) = \frac{16}{25}$

48. (C) Total number of outcomes of throwing a die = 6
Number of outcomes that are even numbers i.e., 2, 4, 6 = 3
∴ The required probability
$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$= \frac{3}{6} = \frac{1}{2}$

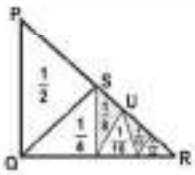
Previous Contest Questions

1. (B) Total number of balls = 30
Number of times the boundary is not hit = 30 – 6 = 24
∴ P (boundary is not hit)
$= \frac{24}{30} = \frac{4}{5}$

2. (C) Total number of students who appeared for the test = 400
Number of students who scored more than 50 marks = 100 + 30 = 130
∴ The required probability
$= \frac{130}{400} = \frac{13}{40} = \frac{1.3}{4} = 0.325$

3. (D) Total number of trials = 120
Chances or trials which favour the outcome (odd number greater than 1) = 30
Probability (Odd number greater than 1) $= \frac{30}{120} = \frac{1}{4} = 0.25$

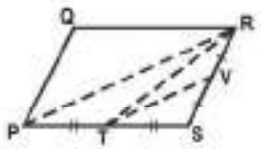
Questions@stimulating-minds

1. 

The required fraction = $\frac{1}{8}+\frac{1}{32}=\frac{5}{32}$

2. Diagonal PR divides ||gm PQRS into two equal areas.

$\therefore$ Area of $\Delta PRS = \frac{40}{2} = 20$



Draw a median RT which divides Δ PRS into two equal areas.

$\Rightarrow$ Area of $\Delta TRS = \frac{20}{2} = 10$

Now draw a median TV which divides Δ TRS into two equal areas.

$\Rightarrow$ Area of Δ TSV = $\frac{10}{2} = 5$

$\therefore$ Area of PRVT = Area of Δ PRS – Area of Δ STV
= 20 – 5
= 15

3. Given OQ = 9.
The area of the larger circle is $\pi 9^2 = 81\pi$
OP : PQ = 1 : 2 and OQ = 9,

$\Rightarrow$ OP = $\frac{1}{3}$ OQ = 3

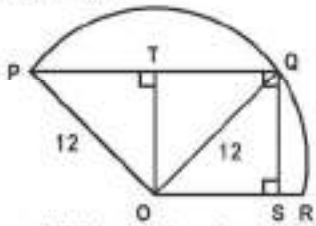
Thus, the radius of the smaller circle is 3 and so the area of the smaller circle is $\pi 3^2 = 9\pi$.

The area of the shaded region = Area of the large circle = Area of the small circle,
$\Rightarrow 81\pi - 9\pi = 72\pi$

4. Join O to Q and draw a perpendicular from O to T on PQ.
Since the radius of the circle is 12, then OP = OQ = 12.

Consider Δ OTP and Δ OTQ. Each is right-angled, they share side OT, and they have hypotenuses (OP and OQ) of equal length. Therefore, these triangles are congruent.

Consider quadrilateral TQSO. Since the quadrilateral has three right angles, then it must be a rectangle so its fourth angle, $\angle$ TOS, is 90°.



Thus, $\angle$ TOP = 135° – 90° = 45°.

Since the angles in Δ OTP add to 180°, then $\angle$ OPT = 180° – 90° – 45° = 45°.

Therefore, Δ OTP is isosceles and right-angled with hypotenuse 12.

Since Δ OTQ is congruent to Δ OTP, it is also isosceles and right-angled with hypotenuse 12.

Since $\angle$ TOP = $\angle$ TOQ = 45°, then $\angle$ QOS = 135° – 45° – 45° = 45°, which tells us that Δ OQS, which is right-angled, has one 45° angle and so must have a second. Therefore, Δ OQS is also isosceles and right-angled, and also has hypotenuse OQ = 12.

So Δ OQS is congruent of Δ OTQ.

Therefore, the area of trapezoid OPQS equals three times the area of an isosceles right-angled triangle with hypotenuse 12.

We calculate the area of Δ OTP, which is one of these triangles.

Suppose that OT = TP = a.

Since Δ OTP is right-angled and isosceles, then OP = $\sqrt{2a}$.

(We can see this by using the Pythagorean Theorem to obtain

$OP = \sqrt{OT^2 + TP^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2a}$

since a > 0.)

Since OP = 12, then $\sqrt{2a} = 12$

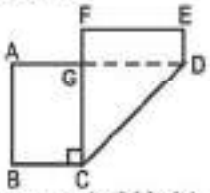
$\Rightarrow a = \frac{12}{\sqrt{2}}$

The area of Δ OTP is $\frac{1}{2}$(OT)(TP)

$= \frac{1}{2}a^2 = \frac{1}{2}\left(\frac{12}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{144}{2}\right) = 36.$

Thus, the area of trapezoid OPQS is 3 × 36 = 108.

5. The area of the original piece of paper is $17(8) = 136\ cm^2$.



When the paper is folded in this way, the portion of the original bottom face of the paper that is visible has the same area as the original portion of the top side of the paper to the right of the fold. (This is quadrilateral CDEF).

Of the portion of the original sheet to the left of the fold, the part that is hidden (and thus not included in the area of the new figure) is the triangular portion under the folded part. (This is the section under Δ CDG.) The hidden triangle is congruent to Δ CDG.

Thus, the area of the portion of the original top face of the paper that is visible is the area to the left of the fold, minus the area of the hidden triangle.

Therefore, the area of the new figure equals the area of the original rectangle minus the area of Δ CDG.

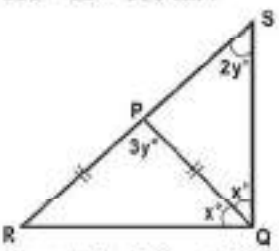
Δ CDG has height GC = 8 cm (the height of the rectangle), is right-angled (since the folded portion of the original bottom edge is perpendicular to the top and bottom edges), and has base GD = 8 cm.

Therefore, Δ CDG has area $\frac{1}{2}(8)(8)$

$= 32\ cm^2$.

In summary, the area of the new figure is $136 - 32 = 104\ cm^2$.

6.



Given, PR = PQ = $\angle$ PRQ = $\angle$ RPQ = x°

$\Rightarrow x^\circ + x^\circ + 3y^\circ = 180^\circ$

$\Rightarrow 2x + 3y = 180^\circ$ .... (i)

and $3y^\circ = 2y^\circ + x^\circ$

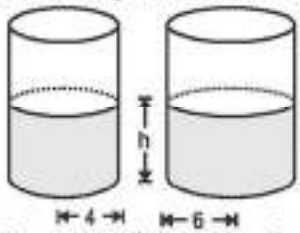
$\Rightarrow x^\circ = y^\circ$ .... (ii)

From (i) and (ii) we get

$x^\circ = y^\circ = 36^\circ$

$\therefore \angle$ RPQ = $3y^\circ = 3 \times 36^\circ = 108^\circ$

7. Let the final depth of the water be h.



The total initial volume of water

$= \pi(4\,m)^2(10\,m) = 160\pi\,m^3$

When the depths of water are equal,

$\pi(4\,m)^2 h + \pi(6\,m)^2 h = 160\pi m^3$

$(52\pi\,m^2)h = 160\pi\,m^3$

$h = \frac{160}{52}\,m$

$h = \frac{40}{13}\,m$

8. In ΔFGH, $FH = \sqrt{8}$ FG = GH = x (since they are both radii of the same circle.)

By the Pythagorean Theorem, $FH^2 = FG^2 + GH^2 = x^2 + x^2 = 2x^2 \Rightarrow \left(\sqrt{8}\right)^2 = 2x^2$

$\Rightarrow x^2 = 4 \Rightarrow x = 2$ (since $x > 0$)

Since $\angle$FGH = 90°, which is $\frac{1}{4}$ of 360°, then the area of sector FGH is one quarter of the area of the circle with centre G and radius GH = FG = 2.

Shaded area = Area of sector FGH − Area of Δ FGH.

The area of sector FGH

$= \frac{1}{4}\pi(4) = \pi$

The area of FGH $= \frac{FG \times GH}{2} = \frac{2 \times 2}{2} = 2$

Therefore, the area of the shaded region is $\pi - 2$.

9. The object has 7 "front" face, each of which is 1 × 1. Therefore, the surface area of the front is 7 × 1 × 1 = 7.

Similarly, the surface area of the "back" is 7.

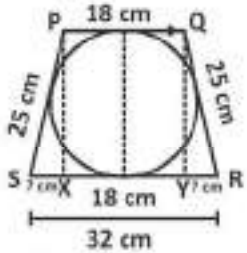
Now consider the faces on the left, top, right and bottom. Each of these faces is 1 × 2, so each has an area of 2.

If we start at the bottom left and travel clockwise around the figure, we have 2 left faces, 2 top faces, 2 left faces, 1 top face, 3 right faces, 1 bottom face, 1 right face, and 2 bottom faces, or 14 faces in total.

Therefore, the surface area accounted for by these faces is $14 \times 2 = 28$.

Therefore, the total surface area of the object is $7 + 7 + 28 = 42$.

10. Draw a perpendicular line from P and Q.



Here, PQ = XY = 18 cm and SX = YR

SX + YR = SR – XY

= (32 – 18) cm = 14 cm

$\Rightarrow$ 2SX = 14 cm

$\Rightarrow$ SX = 7cm = YR ($\because$ SX = YR)

$PS^2 = SX^2 + PX^2$ (Pythagorean theorem)

$$\Rightarrow PX = \sqrt{(PS)^2 - (SX)^2}$$

$$= \sqrt{(25)^2 - (7)^2}$$

$$= \sqrt{625 - 49} = \sqrt{576} = 24$$

$\therefore$ PX = 24 cm

PX is equal to the diameter of the circle.

Hence, the length of the diameter is 24 cm.

11. After one of the six integers is erased, there are five integers remaining which add to 2012.

Since the original six integers are consecutive, them we can treat them as roughly equal.

Since there are five roughly equal integers that add to 2012, then each is roughly equal to $\frac{2012}{5}$, which is roughly 400.

We finish our solution by trial and error.

Suppose that the original six integers were 400, 401, 402, 403, 404, 405.

The sum of these integers is 2415. If one of the integers is to be removed to obtain a total of 2012, then the integer removed must be 2415 – 2012 = 403.

Is there another possible answer?

Suppose that the original six integers were larger, say 401, 402, 403, 404, 405, 406. In this case, the smallest that the sum of five of these could be is 401 + 402 + 403 + 404 + 405 = 2015, which is too large for the given sum. Any larger set of integer only makes the smallest possible sum of five integers larger.

Suppose that the original six integers were smaller, say 399, 400, 401, 402, 403, 404. In this case, the largest that the sum of five of these could be is 400 + 401 + 402 + 403 + 404 = 2010, which is too small for the given sum. Any smaller set of integers only makes the largest possible sum of five integers smaller.

Therefore, the possibility found above is the only possibility, and so the sum of the digits of the integer that was erased is 4 + 0 + 3 = 7.

12. Suppose that the side length of each of the small squares is x. (Since the 4 small squares have the same height, they must have the same side length).

Then the side length of the largest square is 4x.

Since the side length of each of the shaded squares is 10, then QR = 3(10) = 30 = PS.

Thus, PS = 30 = 4x + x or 5x = 30 or x = 6.

Therefore, the side length of the largest square is 4x = 24.

13. If we add the second and third equations, we obtain

ac + b + bc + a = 18 + 6

c(a + b) + (a + b) = 24

(c + 1)(a + b) = 24

From the first given equation, a + b = 3 and so we get (c + 1)(3) = 24 or c + 1 = 8.

Thus, c = 7.

14. Since $\angle$ OMP and $\angle$ GMH are opposite angles, then $\angle$ OMP = $\angle$ GMH.

The x-axis and the y-axis are perpendicular, so $\angle$ POM = 90°, so $\angle$ POM = $\angle$ GHM.

Since M is the midpoint of OH, then OM = HM.

Therefore Δ POM and Δ GHM are congruent by Angle-Side-Angle.

Thus, GH = OP = 4.

Since GH is perpendicular to the x-axis, then the x-coordinate of G is 12, so G has coordinates (12, 4).

15. If M is the midpoint of PQ,

then M andQ lie on the same line

Of the given options,

Q (3, –2) lies on line $y = -\frac{1}{5}x + -\frac{7}{5}$

Hence, M also lie on the same line.

SALIENT FEATURES

- Suitable for curricula of major boards like CBSE / ICSE / State Boards.
- Fundamental concepts are thoroughly revised
- Extensive range of questions that stimulate the interests of the students while testing their knowledge
- Application/skill/knowledge/understanding oriented questions
- Suitable for International/National/Regional Olympiads and Talent exams like NSTSE, Maths Olympiad, RMO, NMO, IMO etc.
- Hundreds of objective questions

UNIQUE FEATURES

- **Synopsis:** To provide the essence of a chapter in a nutshell
- **Previous Contest Problems:** Problems appeared in various talent & olympiad exams
- **Crossword Puzzles:** To stimulate the mind to think beyond regular preparation & to offer fun
- **Solutions:** Solutions and explanations for maximum no. of questions
- **Questions @ Stimulating Minds:** Selected questions to challenge your higher order thinking

BRAIN MAPPING ACADEMY
Mapping Your Future

www.bmatalent.com

ISBN 978-93-82058-53-3