220C HW1

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1. Cholesky Decomposition

We generate the matrix as follows.

```r
# matrix generation

x = seq(0,1,length=400)
dens= function(x,y){
  res = exp(-abs(x-y)/2)
  return(res)
}
R = outer(x,x,FUN=dens)
```

1-(a). Inverse matrix

First, we obtain the inverse matrix of R using the 'solve' function in R. Then, we check the required entries.

```r
inv.R = solve(R)
inv.R[1,2]; inv.R[2,3]; inv.R[1,3]; inv.R[1,4]
```

```
## [1] -398.9999

## [1] -398.9999

## [1] 3.308287e-11

## [1] -2.169774e-11
```

```r
round(inv.R[1:15, 1:15], 3)
```

```
##          [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
##   [1,]  399.5 -399    0    0    0    0    0    0    0     0     0     0     0
##   [2,] -399.0  798 -399    0    0    0    0    0    0     0     0     0     0
##   [3,]    0.0 -399  798 -399    0    0    0    0    0     0     0     0     0
##   [4,]    0.0    0 -399  798 -399    0    0    0    0     0     0     0     0
##   [5,]    0.0    0    0 -399  798 -399    0    0    0     0     0     0     0
##   [6,]    0.0    0    0    0 -399  798 -399    0    0     0     0     0     0
##   [7,]    0.0    0    0    0    0 -399  798 -399    0     0     0     0     0
##   [8,]    0.0    0    0    0    0    0 -399  798 -399     0     0     0     0
##   [9,]    0.0    0    0    0    0    0    0 -399  798  -399     0     0     0
##  [10,]    0.0    0    0    0    0    0    0    0 -399   798  -399     0     0
##  [11,]    0.0    0    0    0    0    0    0    0    0  -399   798  -399     0
##  [12,]    0.0    0    0    0    0    0    0    0    0     0  -399   798  -399
##  [13,]    0.0    0    0    0    0    0    0    0    0     0     0  -399   798
```

```
## [14,]    0.0    0    0    0    0    0    0    0    0     0     0     0  -399
## [15,]    0.0    0    0    0    0    0    0    0    0     0     0     0     0
##       [,14] [,15]
##  [1,]     0     0
##  [2,]     0     0
##  [3,]     0     0
##  [4,]     0     0
##  [5,]     0     0
##  [6,]     0     0
##  [7,]     0     0
##  [8,]     0     0
##  [9,]     0     0
## [10,]     0     0
## [11,]     0     0
## [12,]     0     0
## [13,]  -399     0
## [14,]   798  -399
## [15,]  -399   798
```

We can see that this matrix is an example of tridiagonal matrix.

1-(b). Cholesky decomposition and eigendecomposition

```
## cholesky decomposition
system.time({
  L = t(chol(R))
  })
```

```
##    user  system elapsed
##    0.02    0.00    0.02
```

```
## eigendecomposition
system.time(
  {
    E = eigen(R)
  }
)
```

```
##    user  system elapsed
##    0.16    0.00    0.16
```

We can see that eigendecomposition requires more time than Cholesky decomposition does.

1-(c). Quadratic form computation

```
y = seq(0,2,length=400)

# first: inversion
system.time(
  {
    res1 = t(y)%*%solve(R)%*%y
    res1
```

```
  }
)
```

```
##    user  system elapsed
##    0.08    0.00    0.08
```

```
# second: Cholesky decomposition
# and backward/forward substitution

system.time(
  {
    res2 = t(y)%*%(backsolve(t(L), forwardsolve(L,y)))
    res2
  }
)
```

```
##    user  system elapsed
##       0       0       0
```

We can see that both results are the same, and Cholesky decomposition is faster than the naive inversion.

1-(d). Determinant

First, we tried to obtain the $\log|\mathbf{R}|^{-1/2} = -\frac{1}{2}\log|\mathbf{R}|$ based on the direct inversion. Using the determinant function in R, we can directly get the log of the determinant.

```
## first: direct inversion
log.R = determinant(R)$modulus
-.5*log.R
```

```
## [1] 1195.048
## attr(,"logarithm")
## [1] TRUE
```

We got 1195.048 for the required value. Next, we try to compute based on Cholesky decomposition. Denoting Cholesky decomposition of $\mathbf{R}$ as $\mathbf{R} = \mathbf{L}\mathbf{L}^T$ where $\mathbf{L}_{400\times 400} = [L_{ij}]$, we realize that

$$\log|\mathbf{R}|^{-1/2} = -\frac{1}{2}\sum_{i=1}^{400}\log L_{ii}^2 = -\sum_{i=1}^{400}\log L_{ii}.$$

```
## second: Cholesky decomposition
-sum(log(diag(L)))
```

```
## [1] 1195.048
```

Now, we see a finite value for the result, which implies nonsingularity of $\mathbf{R}$.