## THE TIME SERIES APPROACH TO SHORT TERM LOAD FORECASTING

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Abstract — The application of time series anal-
ysis methods to load forecasting is reviewed. It is
shown than Box and Jenkins time series models, in
particular, are well suited to this application. The
logical and organized procedures for model development
using the autocorrelation function and the partial
autocorrelation function make these models partic-
ularly attractive.

One of the drawbacks of these models is the
inability to accurately represent the nonlinear rela-
tionship between load and temperature. A simple
procedure for overcoming this difficulty is intro-
duced, and several Box and Jenkins models are compared
with a forecasting procedure currently used by a
utility company.

### I. INTRODUCTION

Load forecasting is an integral part of electric
power system operations. Long lead time forecasts of
5 to 20 years ahead are needed for scheduling con-
struction of new generating capacity as well as the
determination of prices and regulatory policy. Inter-
mediate term forecasts of a few months to 5 years
ahead are needed for maintenance scheduling, coordina-
tion of power sharing arrangements and setting of
prices, so that demand can be met with fixed capac-
ity. Short term forecasts of a few hours to a few
weeks ahead are needed for economic scheduling of
generating capacity, scheduling of fuel purchases,
security analysis and short term maintenance schedul-
ing. Very short term forecasts of a few minutes to an
hour ahead are needed for real-time control and real-
time security evaluation. This paper is concerned
with the short term load forecasts of one hour to one
day ahead.

There has been a wide variety of procedures for
short term load forecasting reported in the literature
(see [1]-[4], [18] for overviews of the field). Some
of these procedures have included: multiple regression
[5], spectral decomposition [6], exponential smoothing
[7], and state space (Kalman filter) methods [8].

One of the most commonly discussed procedures for
load forecasting is time series analysis (or system
identification), normally using procedures outlined by
Box and Jenkins [9] or Kashyap and Rao [10]. This
paper will concentrate on the application of time
series analysis methods to short term load forecast-
ing.

Section II of this paper presents the basic time
series models and describes the historical evolution

of the application of these models to load
forecasting.

Section III describes the procedures for develop-
ing, checking and updating Box and Jenkins' models.
It is the organization and the logic of these proce-
dures which makes the Box and Jenkins models so use-
ful. We feel that this point has been overlooked in
some previous work. Section III also compares the
forecasting abilities of the various models.

In section IV a procedure which incorporates a
nonlinear load-temperature model into the standard
time series model is presented. Forecasting results
for this model are compared with results from the
linear models.

It is, in general, very difficult to compare load
forecasting procedures unless they are tested on the
same data sets. In order to provide this type of
benchmark we have tested our procedures against those
currently used by a local utility. Section V de-
scribes this material and compares the accuracy of the
utility's forecasting methods with our own.

Section VI summarizes our results and presents a
few conclusions.

### II. TIME SERIES MODELS IN LOAD FORECASTING

#### A. The Models

The most fundamental time series models are the
autoregressive model and the moving average model. In
the autoregressive model AR(p) the current value of
the process is expressed as a linear combination of p
previous values of the process and a random shock.

$$z_t = \phi_1 z_{t-1} + \ldots + \phi_p z_{t-p} + a_t \qquad (1)$$

In order to write this in a more convenient form the
following operators are introduced.

$$B\, z_t = z_{t-1} \qquad B^m z_t = z_{t-m}$$

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

So that equation (1) can be written

$$\phi(B)\, z_t = a_t \qquad (2)$$

In the moving average model MA(q) the current value of
the process is expressed as a linear combination of q
previous random shocks.

$$z_t = a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

$$z_t = \theta(B)\, a_t \qquad \theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q \qquad (3)$$

The more general mixed autoregressive-moving average
model ARMA (p,q) is a combination of (2) and (3).

$$z_t - \phi_1 z_{t-1} - \ldots - \phi_p z_{t-p} = a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \qquad (4)$$

$$\phi(B)\, z_t = \theta(B)\, a_t$$

Equation (4) can be used to model stationary processes with finite variance, and it is assumed that the roots of $\phi(B)$ and $\theta(B)$ lie outside the unit circle. One can model some types of nonstationary processes by differencing the original process, $z_t$, to obtain a stationary process, $w_t$.

$$w_t = \nabla^d z_t \quad (\nabla z_t = z_t - z_{t-1}; \nabla_k z_t = z_t - z_{t-k}) \quad (5)$$

This results in an <u>autoregressive-integrated-moving average model</u> ARIMA (p,d,q).

$$\phi(B) \nabla^d z_t = \theta(B) a_t \quad (6)$$

Because of the periodic nature of the load curve (eg. the load at 10 A.M. Tuesday is related to the load at 10 A.M. Monday), it is advantageous to use seasonal ARIMA (p,d,q)x(P,D,Q) models.

$$\phi_p(B) \Phi_P(B^{24}) \nabla^d \nabla^D_{24} z_t = \theta_q(B) \Theta_Q(B^{24}) a_t \quad (7)$$

In some cases it is also useful to recognize the weekly periodicity (Sundays are not like Mondays), and a two period ARIMA $(p,d,q)\times(P,D,Q)_{24}\times(P',D',Q')_{168}$ model could be used.

$$\phi_o(B)\Phi_P(B^{24})\Phi'_{P'}(B^{168})\nabla^d\nabla^D_{24}\nabla^{D'}_{168}z_t =$$
$$\theta_q(B)\Theta_Q(B^{24})\Theta'_{Q'}(B^{168})a_t \quad (8)$$

The ARIMA model forecasts are essentially extrapolations of previous load history and have problems when there is a sudden change in the weather. The <u>transfer function model</u> allows for the inclusion of some independent variables, such as temperature. The transfer function model TRFU (r,s) is

$$y_t = \frac{\omega(B)}{\delta(B)} x_{t-b} + n_t \quad (9)$$

where $\omega(B)$ is a polynomial in B of order s and $\delta(B)$ is a polynomial in B of order r. The disturbance process $n_t$ is not white but can be represented by an ARIMA model.

$$\nabla^d\nabla^D_{24}\nabla^{D'}_{168}y_t = \frac{\omega(B)}{\delta(B)}\nabla^d\nabla^D_{24}\nabla^{D'}_{168}x_{t-b} +$$
$$\frac{\theta(B)\Theta(B^{24})\Theta'(B^{168})}{\phi(B)\Phi(B^{24})\Phi'(B^{168})}a_t$$

## B.  Summary of Time Series Applications to Load Forecasting

Box and Jenkins models were first introduced for intermediate term load forecasts by Vemuri, Balasubramanian and Hill [11], [12] who used an ARIMA $(0,1,1)\times(0,1,1)_{12}$ model with a twelve month period to forecast monthly peak loads for lead times of up to 40 months. Uri [13] used the same model (with different parameter values) to forecast monthly average loads for lead times of up to 2 years.

Very short term forecasts were first addressed by Keyhani and El-Abiad [14] using ARMA models. They used an ARMA (1,0) model on 1 minute load data, ARMA (1,1) and (2,1) models on 5 minute data and an ARMA (2,0) model on hourly load data. Later Keyhani and Rad [15] used models which combined AR models with some weather inputs and trigonimetric trend functions to forecast hourly loads up to one week ahead. A typical model is

$$z_t = 0.026+1.36 z_{t-1}-0.45 z_{t-2}-0.33z_{t-167}+$$
$$0.0008z_{t-168}+0.239X_{t-4} \quad (10)$$

where $x_t$ is hourly temperature.

Hagan and Klein [16] used Box and Jenkins methods to develop ARIMA models with a daily period to forecast hourly loads with one to four hour lead times. Different models were developed for each season. They later extended the models to include a weekly as well as a daily period and also to include temperature inputs in a transfer function model [17]. Using load and temperature data at three hour intervals, the following was a typical model.

$$\nabla\nabla_{56}z_t = \frac{\omega_0 - \omega_1 B}{1-\delta_1 B-\delta_2 B^2}\nabla\nabla_{56} x_t + n_t$$
$$n_t = (1 - \theta B)(1 - \theta' B^{56}) a_t \quad (11)$$

They also introduced a method for adaptively updating the parameters in an ARIMA or a TRFU type model [18], [19], so that the load models can adapt continuously to changing seasonal patterns. With this method Box and Jenkins models become well suited to on-line applications.

Meslier [20] later used ARIMA $(1,0,0)\times(0,1,1)\times_7 (0,1,1)_{365}$ models to forecast daily energy consumption one day ahead. Correction factors were added to the model to adjust for the holidays. These corrected models had up to 138 parameters. Meslier also proposed using a transfer function model of the form

$$(1 - \delta_1 B)v_t = W_o(1-W_7 B^7)(1-W_{365}B^{365})x_t+N_t$$
$$(1-\Phi_{365}B^{365})\nabla\nabla_7 N_t = (1-\theta_1 B)(1-\theta_7 B^7) a_t \quad (12)$$

where $v_t$ is consumption which has been corrected to account for holidays, and $x_t$ is temperature.

Uri [21] and Maybee and Uri [22] used periodic ARIMA models to forecast load demand one week ahead and the load duration curve one year ahead.

Abu-El-Magd and Sinha [23], [24] have used multivariate AR models for forecasting load demands of a multinode system. They used an ARI (6,1) model to forecast the load demand of four loading stations at five minute intervals and an AR (2) model with 24 hour differencing to forecast at 1 hour intervals.

Vemuri, Huang and Nelson [25] developed a method for on-line identification of AR models using sequential least squares. They used AR (10) models on 3 hour load data to forecast up to 21 hours ahead.

## C.  The Contribution of This Paper

One of the disadvantages of the Box and Jenkins transfer function model is that, because it is a linear model, it does not accurately reflect the load/temperature relationship. Hagan and Klein [19] were able to circumvent this problem somewhat by continuously updating the model parameters, which effectively relinearized the model after each measurement.

In this paper a further improvement is gained by subjecting the temperature to a nonlinear transformation before using it in the transfer function model. The forecasting ability of this new procedure is compared with that for the standard transfer function and ARIMA models. In addition, all of these methods are compared with a procedure currently used by a local utility.

## III. LINEAR MODEL DEVELOPMENT AND FORECASTING

### A. Model Development Procedures

Techniques for preliminary identification of time series models rely on the analysis of the autocorrelation and partial autocorrelation function. These methods are very systematic and are extremely helpful in the determination of model order, in the preliminary estimation of model parameters, and in diagnostic checking and model refinement.

The autocorrelation function (acf) describes inherent correlation between observations of a time series which are separated in time by some lag k.

$$\gamma_k = E[z_t \, z_{t+k}]$$
$$\rho_k = \gamma_k / \gamma_0 \qquad (12)$$

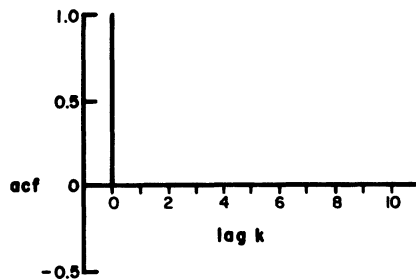For a white noise process, in which there is no correlation in time, the acf would be as shown in Figure 1.



Figure 1. Autocorrelation function of a white noise process.

For an autoregressive process (Eq. (1)) the autocorrelation function satisfies the following equation

$$\rho_k = \phi_1 \rho_{k-1} + \cdots + \phi_p \rho_{k-p} \qquad (13)$$
$$\text{or}$$
$$\phi(B) \, \rho_k = 0$$

For example, Figure 2 illustrates the autocorrelation function of a first order autoregressive process.
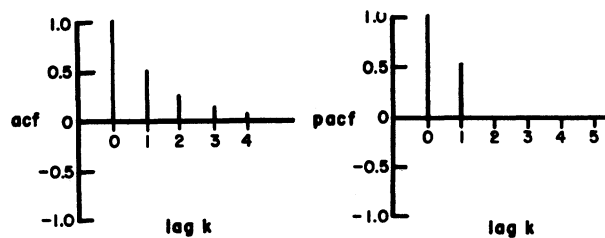


Figure 2. ACF and PACF of $z_t = 0.5 \, a_{t-1} + a_t$

The partial autocorrelation function (pacf) is also useful in determining the order of autoregressive processes. It is defined with the use of equation (13). Notice that this equation can be used to solve for $\phi_1, \ldots, \phi_p$ if the autocorrelation function is known, and if the order, p, is known. Define the partial autocorrelation at lag j, $\phi_{jj}$, to be the solution for $\phi_j$ if the order (p) is equal to j. It is clear that the pacf will be identically zero for all lags, j, greater than the true order of the process, p. For example, Figure 2 shows the pacf for a first order autoregressive process.

For moving average processes (Equation (3)) it can be shown that the autocorrelation function is identically zero for lags greater than the order of the process, q. Figure 3 illustrates the acf and pacf of a first order moving average process.
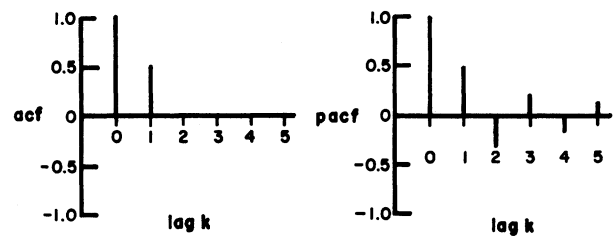


Figure 3. ACF and PACF of $z_t = a_t + 0.8 \, a_{t-1}$

Using the acf and pacf it is possible to determine potentially useful model structures. The ACF and PACF are also very useful in determining appropriate model adjustments when diagnostic checks indicate model inadequacy.

Once the model structure is determined, the parameters are estimated using the method of maximum likelihood [9]. Then the residuals (one step ahead forecasting errors), $a_t$ are calculated. The residual sequence should be white. If it is not, the autocorrelation function can be investigated to suggest additions to the model.

These procedures will be illustrated in the following sections.

### B. Data Set Description

The preceding model development methods were tested on data provided by a moderately sized southwestern utility with 450,000 customers and a summer peak of approximately 3000 MW. The authors were supplied with hourly net system loads and temperature readings for 1983 and 9 months of 1984. The 1983 data were used to develop ARIMA, TRFU, and nonlinear models for each season of the year. Four weeks of data were used to develop each model and then the models were used to forecast for the following three weeks. For the TRFU models actual temperatures were used in the load forecasts, because of the difficulty of obtaining temperature forecasts from the weather bureau.

Utility load forecasts for the 9 months of 1984 were also provided. The 1983 ARIMA, TRFU and non-linear models, which were developed on 1983 data, were used for forecasting on the 1984 data in order to provide comparisons with utility forecasts. The following section will present the technique and steps used to develop the models. A later section will discuss the comparisons.

### C. Model Development and Forecasting Results

Four weeks of summer data (July 11-August 7, 1983) were used to build the summer models. The first step in developing the model is to examine the autocorrelation function (Figure 4). This oscillatory, nondecaying function indicates a nonstationary process. In order to obtain a stationary process, a number of differencing schemes were tested (Figure 5). The differencing $\nabla_1 \nabla_{24}$ was finally chosen. The acf and pacf of the resulting stationary process, $w_t = \nabla_1 \nabla_{24} z_t$, were then examined in order to determine model order. The large spikes at k=1 and k=2 in the

pacf (Figure 6) suggest a model with a hourly AR(2) component.

To facilitate the identification of the daily (P,Q) order, multiples of 24 are pulled out of the acf and pacf and are inspected separately. Figure 7 suggests adding a daily MA(2) to the model because of the large correlations in the acf at k = 24, 48. Hence, our tentative model is of the form $(2,1,0)_1 \times (0,1,2)_{24}$.
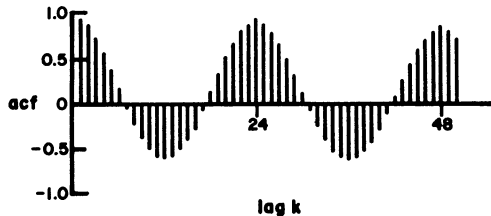


Figure 4. ACF of the Summer Load Series, $z_t$



a) $\nabla_1$
b) $\nabla_{24}$
c) $\nabla_1 \nabla_{24}$
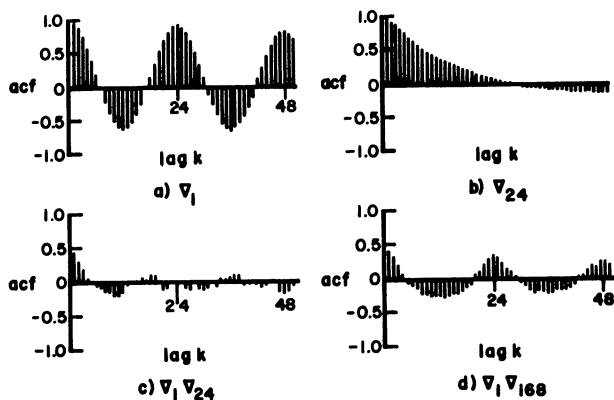d) $\nabla_1 \nabla_{168}$

Figure 5. Autocorrelation Functions of the Summer Load Series Differenced wrt. Hours, Days, and Weeks
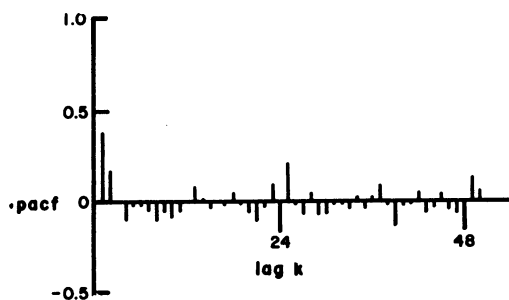


Figure 6. PACF of the Summer Loads Differenced $\nabla_1 \nabla_{24}$
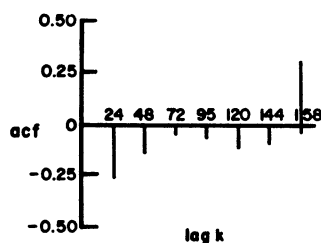


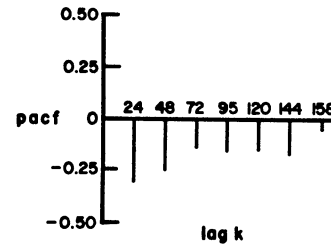Figure 7. Daily Components of Figure 5c (Enlarged)



Figure 8. Daily Components of Figure 6 (Enlarged)

The parameters in the initial model were estimated using maximum likelihood, and the model was checked for goodness of fit. An accurate model will produce residuals that are white and will therefore have an acf as in Figure 1. If, however, the model is a poor fit, the residuals will not be white. The residual acf is very useful in revealing information about parameters that were overlooked.

Figure 9 is the residual acf of our tentative model. The large spike at lag 168 represents correlation that was neglected in the original model. This was corrected by adding a weekly AR(1) factor, and the following model was obtained.

$$(1-0.26B-0.18B^2)(1-0.39B^{168})\nabla_1 \nabla_{24} z_t =$$

$$(1-0.73B^{24}-0.09B^{48})a_t \qquad (14)$$

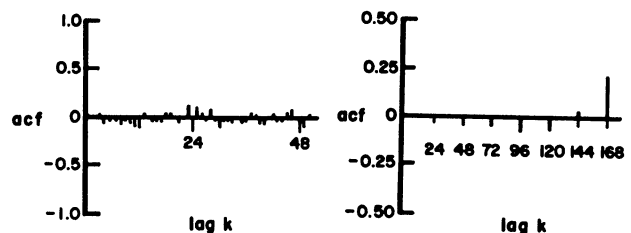A test for goodness of fit for this model was performed and gave no indication of model inadequacy.



Figure 9. ACF of the Residuals, $a_t$, in Summer Model $(2\ 1\ 0)_1 \times (0\ 1\ 2)_{24}$

A series of steps similar to those outlined above were employed in building a summer TRFU model. The resulting model was

$$Y_t = \frac{4.5 + 1.5B}{1-0.85B+0.23B^2} X_t +$$

$$\frac{1-0.70B^{24} -0.11B^{48}}{\nabla_1 \nabla_{24}(1-0.14B-0.18B^2)(1-0.47B^{168})} a_t \qquad (15)$$

where $Y_t$ is the power load and $X_t$ is the temperature at hour t.

In all, 4 ARIMA and 4 TRFU models were developed, one for each season of the year. Each model was used to forecast 24 hour load curves every day for a three week period. The average percent errors for those time periods are shown in Table I. From this table it is clear that the TRFU model, with its explicit temperature inputs, does provide a better forecast, on the average, than the ARIMA model.

Table I

Comparison of Mean Absolute Percentage Errors for
the ARIMA and Transfer Function Models (1983)

Mean Absolute % Forecasting Error

| Season | ARIMA Model | Transfer Function Model |
|--------|-------------|-------------------------|
| Summer | 4.17 | 3.82 |
| Fall | 4.68 | 4.49 |
| Winter | 3.85 | 3.35 |
| Spring | 4.11 | 4.30 |

## IV. A NONLINEAR EXTENSION

Although the TRFU models do provide better fore-
casts than the ARIMA models, they do not accurately
reflect he nonlinear relationship between temperatures
and loads. This causes forecasting errors, partic-
ularly in the spring and fall seasons when there is a
good deal of temperature variation with small effects
on the load.

The goal of this section is to find a simple
model for the load/temperature relationship and to use
it in combination with the TRFU model for forecast-
ing. If this method provides significant improvement
over the linear TRFU model it will indicate that
nonlinear model development is a fruitfull area for
future research.

The fact that the relationship between tempera-
ture and load is nonlinear can be easily seen from the
scatter diagram in Figure 10. This is a plot of 1983
peak daily power load readings vs. 1983 peak daily
temperature readings for the Tulsa area. To quantify
this relationship we fit a third order polynomial to
the data [27].

$$Y = 1810.0 + 21.4\ T - 1.04\ T^2 + 0.0093\ T^3 \qquad (16)$$

This curve is shown in Figure 10.

There have been other approaches to fitting the
load/temperature curve. The most common is to break
the curve into 3 straight line segments [26]: 1) the
left negative slope (winter), 2) the middle flat
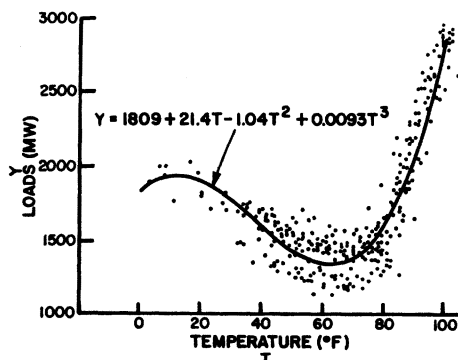portion (spring and fall), and 3) the right positive
slope (summer).



Figure 10. Scatter Diagram of Hourly Loads vs
Hourly Temperatures for 1983

To use this noniinear relationship in load fore-
casting we first transform the temperatures using

Equation (16). We then take the transformed tempera-
tures as input to a TRFU model. Table II compares the
results of the nonlinear method with those for the
standard TRFU model and for the ARIMA model.

Table II

Comparison of Mean Absolute Percentage Error for
the Three Different Time Series Models (1983)

Mean Absolute % Forecasting Error

| Season | ARIMA | Standard Transfer Function | Nonlinear |
|--------|-------|----------------------------|-----------|
| Summer | 4.17 | 3.82 | 3.55 |
| Fall | 4.68 | 4.49 | 3.41 |
| Winter | 3.85 | 3.35 | 2.84 |
| Spring | 4.11 | 4.30 | 5.16 |

From Table II it is clear that the nonlinear
approach offers some promise for improved load fore-
casting. Averaged over the four seasons, the ARIMA
model had 4.2 average percent error, the TRFU model
had 4.0 average percent error, and the nonlinear model
had 3.74 average percent error. This is a significant
improvement, given that a very simple procedure was
used. With further refinements in the nonlinear
modeling process we can expect to obtain even better
performance.

## V. COMPARISON WITH A CONVENTIONAL FORECASTING METHOD

### A. Description of the Forecasting Procedure

The time series methods were compared with a
conventional utility forecasting procedure. The
procedure begins with an historicaly averaged load
curve. This curve is obtained by specifying three
variables: (1) the season of the year, (2) the day of
the week, and (3) the cloud cover (cloudy or sunny)
(see Table III). The dispatcher selects these param-
eters and inputs them into a computer which produces a
load curve describing that kind of day. This curve is
generated by averaging actual hourly loads from pre-
vious years over a predesignated period of time. For
example, suppose that tomorrow was forecasted to be a
sunny Friday, August 14th. The dispatcher would
specify Season #5, Day Type #3, and Cloud Cover #1.
The computer averages all the load curves for sunny
Fridays in the summer season for the past few years,
and generates a representative load curve for tomor-
row. The dispatcher may use his/her own discretion
and experience to use this curve or to select another
one.

After the load curve has been chosen the dis-
patcher predicts the maximum and minimum loads for
tomorrow. These critical points are used to propor-
tionately stretch the shape of the representative load
curve to match the high and low values without losing
the basic pattern of the curve. Clearly the dis-
patcher's experience is the key to the success of this
method.

### B. Comparing Forecasting Results

Time series model forecasts were compared with
the conventional forecasts for three 20 day periods in
1984 (winter, spring and summer). Average percent
forecasting errors within each season for each type of
model are listed in Table IV. All the time series

models (which were developed with 1983 data) do very well in predicting 1984 loads when compared to the conventional forecasting method. The nonlinear extension had the lowest percent errors of all models, except in the summer season where it did just as well as the standard transfer function model. Of course, the nonlinear and the standard transfer function model used actual temperature readings, not forecasted ones.

By averaging the three seasonal percent errors for each forecasting method, we obtain a general idea of how each model faired in predicting future loads. The ARIMA model had a total average error of 4.92%, the transfer function error was 4.25%, the nonlinear extension - 3.73%, and the conventional method, 5.75%. Thus, all three types of time series models performed better than the conventional method during those seasons. This translates into improvements of 14%, 26%, and 35%, respectively. Conclusively, the nonlinear extension model is the best of the models and proves itself to be an efficient way of predicting power loads.

### Table III

### Seasons, Day Types and Cloud Cover Types

| Season | Time Period |
|---|---|
| 1. Winter | December 1 - February 15 |
| 2. Late Winter-Early Spring | February 16 - DST |
| 3. Spring | DST - May 31 |
| 4. Late Spring-Early Summer | June 1 - June 30 |
| 5. Summer | July 1 - September 15 |
| 6. Late Summer-Early Fall | September 16 - CST |
| 7. Fall | CST - November 30 |

| Day Type | Cloud Cover |
|---|---|
| 1. Monday | 1. Clear-Partly Cloudy |
| 2. Tuesday-Thursday | 2. Party Cloudy-Cloudy |
| 3. Friday | |
| 4. Saturday | |
| 5. Sunday | |

### Table IV

### Mean Absolute % Forecasting Error

| 1984 Season | ARIMA | Tr. Fu. | Nonlin. | Convent. |
|---|---|---|---|---|
| Spring | 4.92 | 5.10 | 4.29 | 7.03 |
| Summer | 5.25 | 3.93 | 4.02 | 5.20 |
| Winter | 4.58 | 3.71 | 2.87 | 5.03 |
| Ave. Error | 4.92 | 4.25 | 3.73 | 5.75 |

### VI. CONCLUSIONS

Box and Jenkins time series models (ARIMA, Periodic ARIMA and Transfer Function) are very well suited to load forecasting applications. They have been used for long term forecasts of more than a year ahead, as well as for very short term forecasts of less than five minutes.

One of the drawbacks of these models for short term forecasts is their inability to accurately describe the nonlinear relationship between loads and temperatures. This paper has demonstrated that a simple polynomial regression analysis, when combined with a Box and Jenkins transfer function model, can provide more accurate forecasts.

This nonlinear model has been compared with the standard transfer function model, the periodic ARIMA model and with a utility procedure which uses heavy dispatcher input. All of the Box and Jenkins models provide accurate forecasts, but the simple nonlinear extension to the transfer function model provides the best results. These results suggest that continued development of the nonlinear model of the load/temperature relationship should provide further improvements in short term load forecasts.

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