NIHR | Applied Research Collaboration South West Peninsula

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Module 5 : Geographic Modelling & Visualisation
Session 5B : Combinatorial Optimisation for Geospatial Problems
Elliott Coyne

“ahiti Nui Mare'are'a”

HSMA 5

#hsma5isalive

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Facility Location Problems

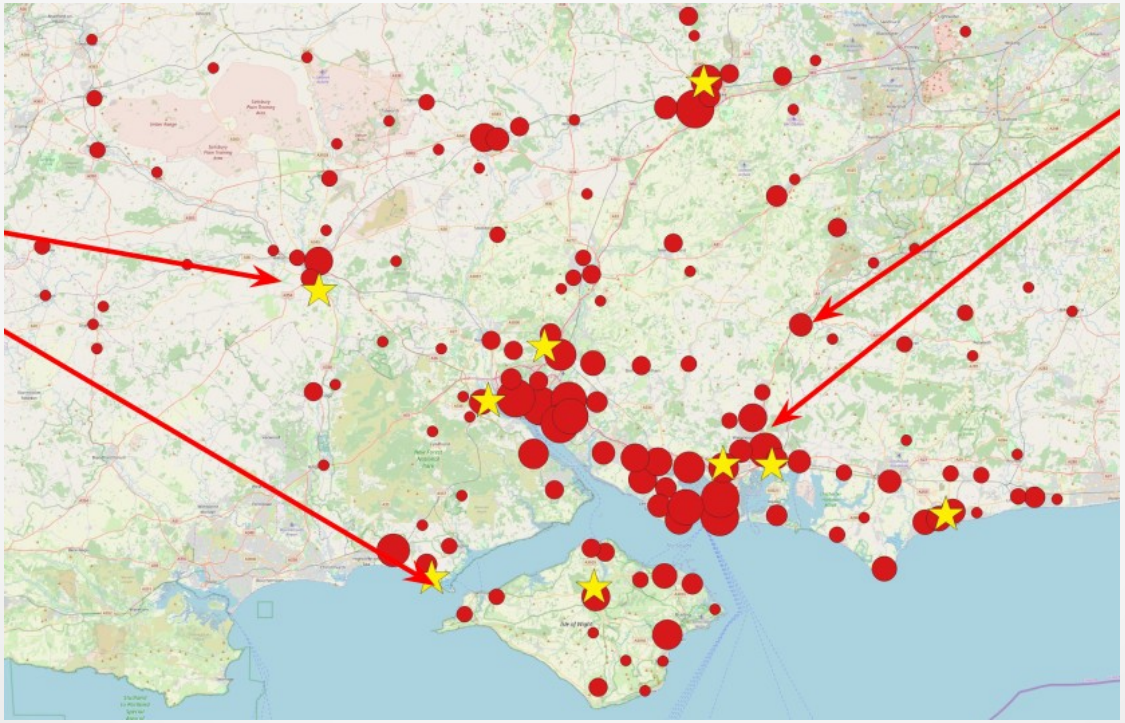
One job of budding Data Scientists is to help their organisations in making decisions when it comes to deciding about the locations for services.

Real life healthcare examples may include reviewing (and rationalising) existing service locations.

Understanding how to approach these problems will then given you data to use with maps. Great for Modules **5C - QGIS** and **5D – GeoPandas**.

Example...

☆ Location of Dialysis treatment centres (i.e., hospitals)



Home ‘location’ of patients using dialysis service (size represents density)

Data Requirements

For these types of problems, you'll usually encounter…

Travel times/ distances – From different areas/ locations to existing/ proposed services or facilities (Postcodes, LSOAs)

Demand (aggregated) – Total numbers of service users originating from different locations

Demand (granular) – Details of individual service users and their origin (i.e., home address); these are required to calculate Max or 95th percentile travel times/ distances

Facility details – generally including their name, location and possibly other details.

Integer Representation - Chromosome

Integer *aka variable length array*

- A chromosome is of length *f*: number of facilities in a feasible solution
- Each element is encoded 1, 2, …, *m*-1, *m*

Integer as opposed to binary integer

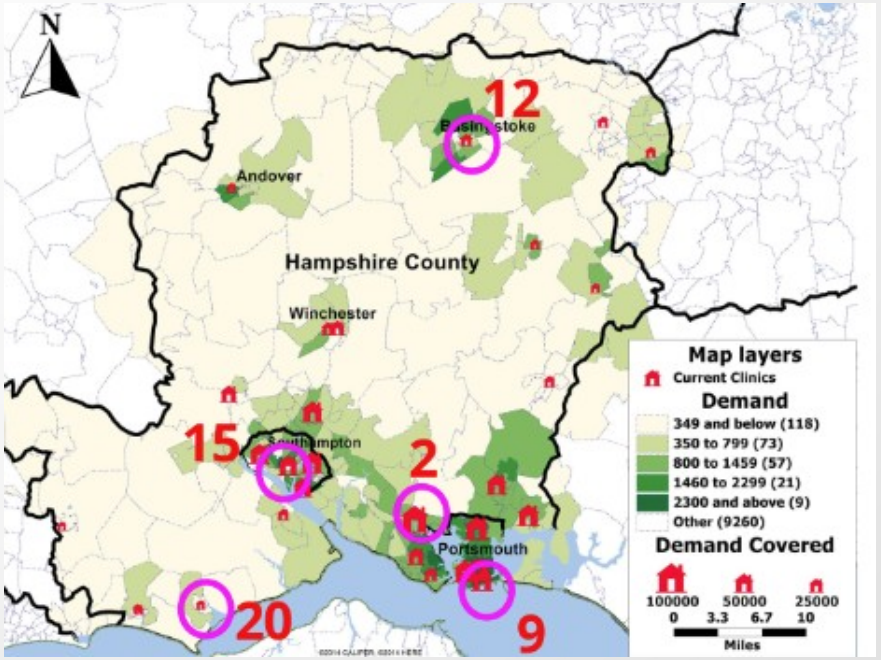
m = 28

f = 5

12	15	2	9	20

=

20	15	12	2	9



‘fitness’ Function

- In an EA, the quality of a solution is called its fitness
  - This is the target of the optimising algorithm *c.f.* cost
- In a single objective facility location problem fitness may be:
  - Weighted average travel time (p-median problem), or
  - Proportion of demand within a target travel time (maximal covering location problem).
- Likely requirements:
  - Candidate solution (your representation of a solution)
  - Travel cost matrix (e.g. travel time in minutes)
  - Demand by geographic location (e.g. stroke admissions by LSOA)

What We'll Cover….

During this session you'll be shown a live demo of the following steps. You'll then work in your groups to tackle a different problem, using similar steps for yourselves.

- Library Imports
- Data Imports
- Representing a Solution
- Constructing a Random Solution
- Evaluating a Solution
- Small Problem: Enumerating all Possible Combinations
- Bruteforce Solution
- Graphical Representation of Bruteforce Solution
- Medium to Large: Using random restarts (Demo Only)

What You'll Need...

Please open the following Jupyter Notebooks for **Location Problem** and **Folium Map**:

- *Code Alongs…*
- *Group Exercises…*

Demonstration only:

- *Evolutionary and Genetic Algorithms*

When Size Matter Even More!

Evolutionary algorithms (EAs)

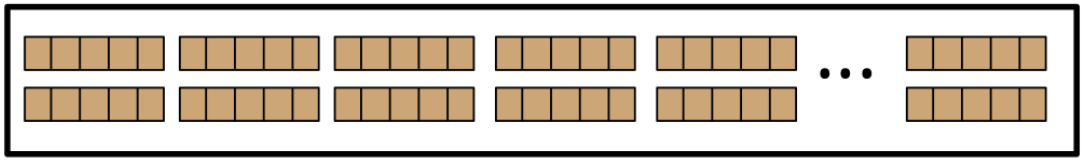
Not always possible to compute all possible combinations!

- Seeded with a **population** of possible solutions
- **Breeds and evolves** the population to find ‘good’ or ‘optimal’ solutions
- *In the context of facility location, each solution represents a possible combination of locations*
- The approach can be also be viewed as:
  - reinforcement learning or stochastic optimisation
- Common EAs are
  - Evolutionary strategies: **(μ, λ)** and **(μ + λ)**
  - Genetic Algorithms (GAs)
- Single or multi-objective
- **Can be computationally expensive!**

Evolutionary Algorithms

Evolutionary Algorithms

(λ) "lambda" represents initial population size (i.e. how many solutions)
(μ) "mu" represents the number of best solutions to keep, remainder are deleted



Initial pop: size **λ**

General EA Pseudo-Code

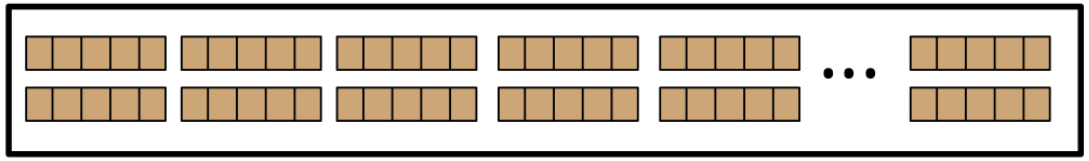
```
pop ← initial_population()
best = []

repeat:
    evaluate_fitness(pop)
    for individual in pop:
        if best = [] or fitness(individual) > fitness(best):
            best ← copy(individual)
     pop ← Join(pop, breed(pop))
until time_limit or max_iter
return best
```

(μ, λ) and $(\mu + \lambda)$

(μ, λ)

There are **3** hyper parameters to consider
Lamda: size of initial population
Mu: truncation selection operator
Mutation Function: May code yourself

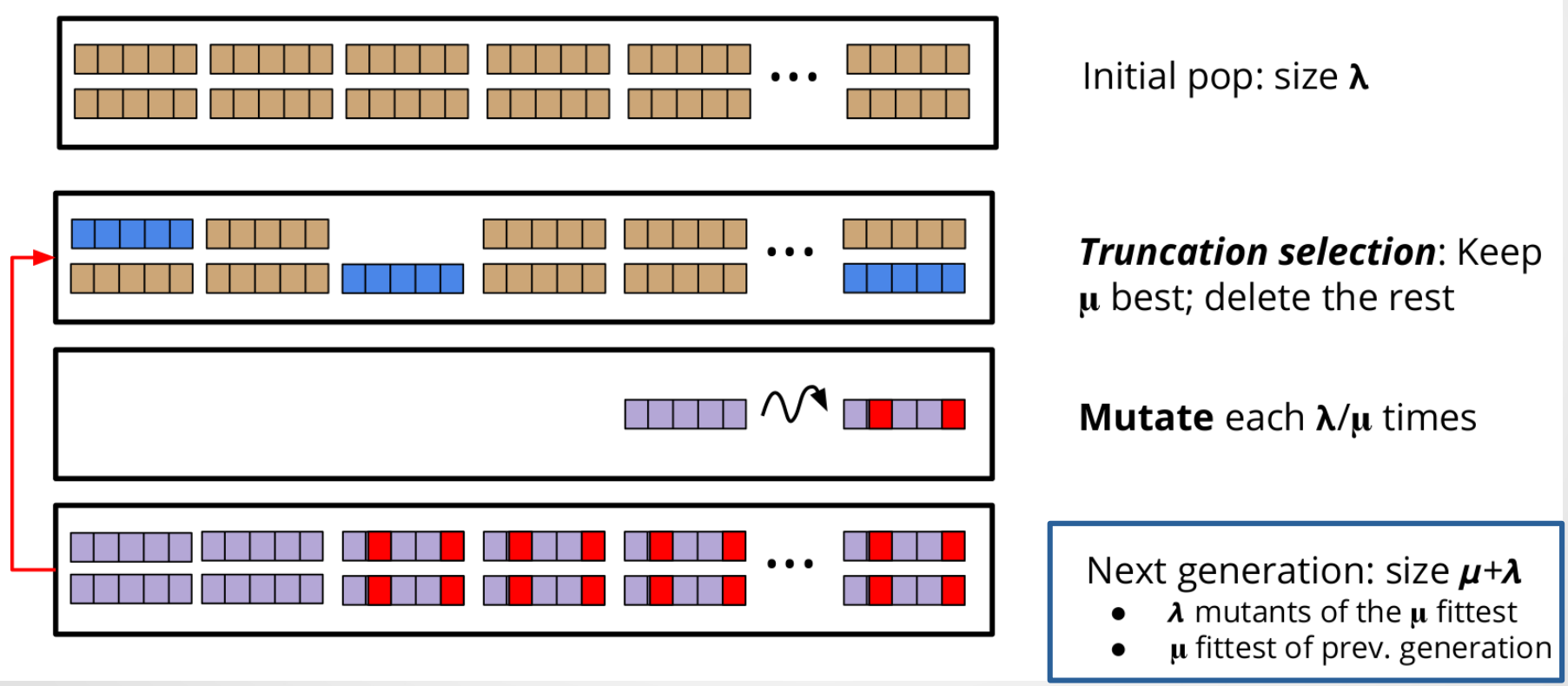
(λ) "lambda" represents initial population size (i.e. how many solutions)
(μ) "mu" represents the number of best solutions to keep, remainder are deleted



Initial pop: size **λ**

$(\mu + \lambda)$

(λ) "lambda" represents initial population size (i.e. how many solutions)
(μ) "mu" represents the number of best solutions to keep, remainder are deleted



Comparing (μ, λ) and $(\mu + \lambda)$

The main difference between the two algorithms is **ELITISM**

- The fittest parents from the previous generation persist

Benefits of ELITISM

- High performing solutions have opportunity to breed with other new high performing solutions

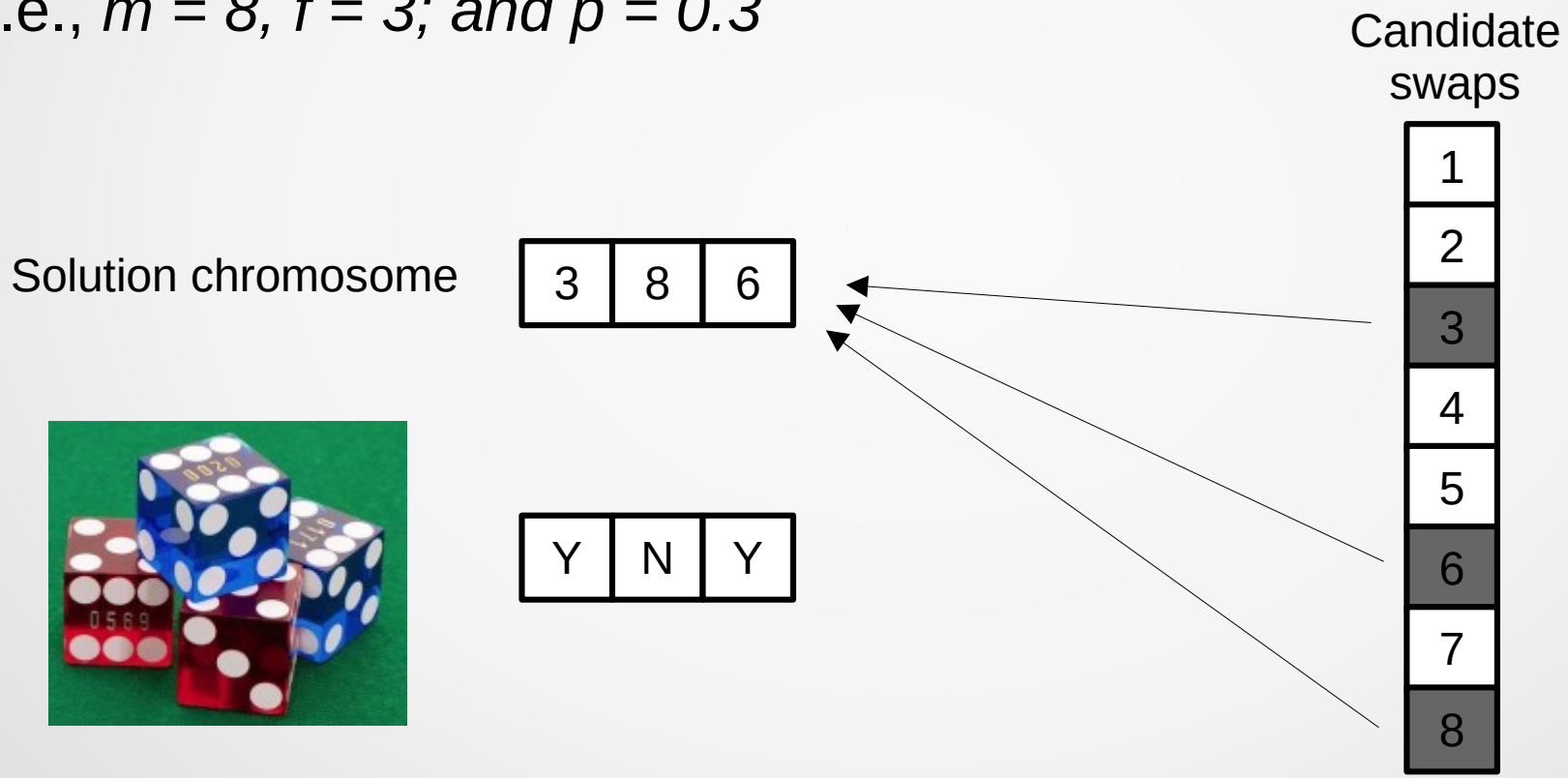
Risk

- Potentially reduces EX**PLOR**ATION in favour of more EX**PLOIT**ATION.
- Failure to learn and getting stuck in a local optimum.

Mutation Operator

Mutate each element within chromosome with probability *p*

i.e., *m = 8, f = 3; and p = 0.3*



Sample “Y” (Yes!) with probability *p*

Mutation Operator

Mutate each element within chromosome with probability *p*

i.e., *m = 8, f = 3; and p = 0.3*

Solution chromosome

3	8	6

Y	N	Y

Swap if “higher” than 0.3

Sample “Y” (Yes!) with p

Mutant

2	8	7

Candidate swaps

1
~~2~~
3
4
5
6
~~7~~
8

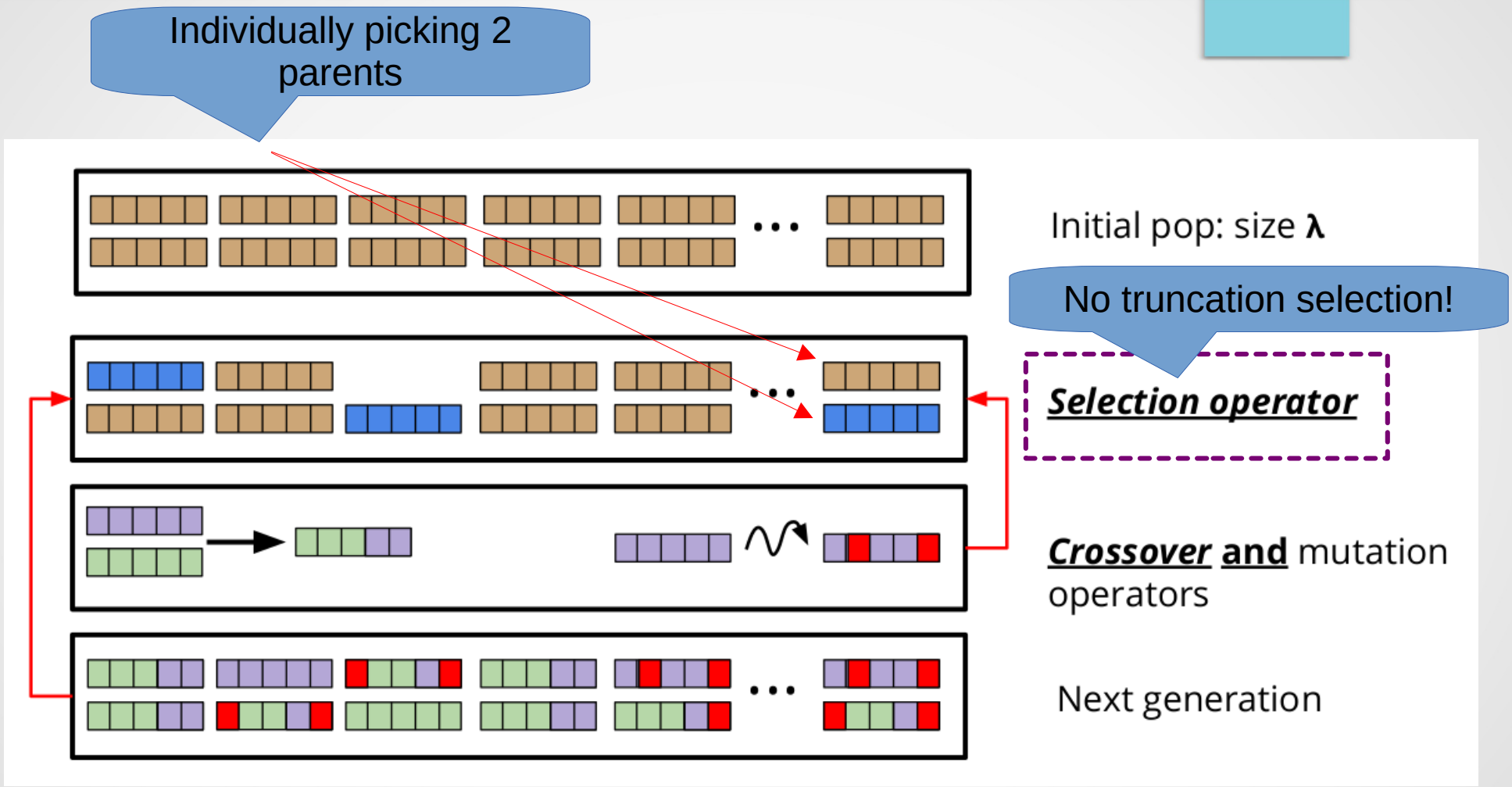
2

7

Sample without replacement

Genetic Algorithms

Genetic Algorithms



Selection operator

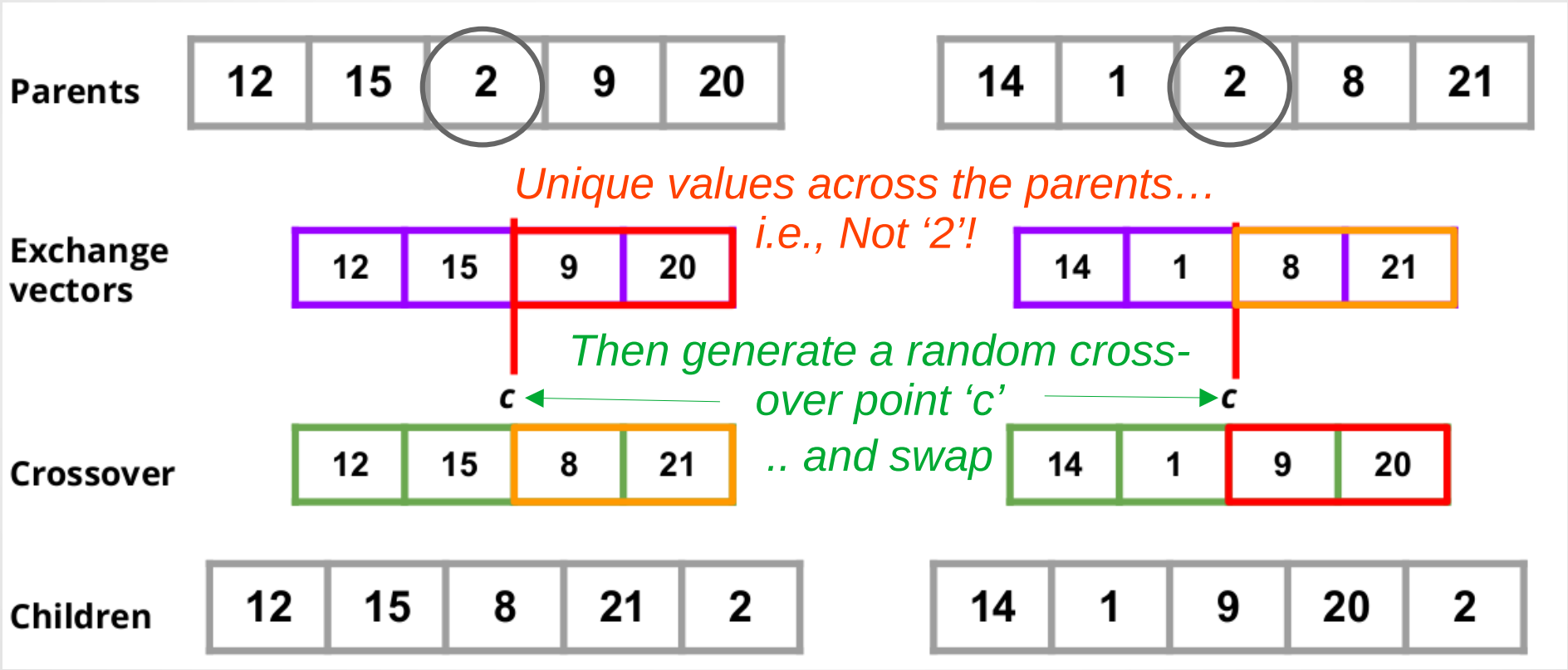
Classic approaches

- Roulette Selection
  - Not used much these days
  - Use random sampling, where probability for selection is proportional to its fitness. More here.
- **Tournament Selection**
  - Pick a ‘Champion’ at random then pick some ‘Challengers’
  - ‘Challengers’ try and beat that ’Champion’.

```
def tournament_selection(pop, tournament_size=2):
    """
    Stochastic tournament to select chromosome for breeding
    Parameters
    ---------
    pop: array, population of chromosomes
    tournament_size: int, number of rounds

    Returns:
    -------
    selected individual chromosome
    """
    best ← random_individual(pop, replace=True)
    for i in range(2, tournament_size):
        challenger ← random_individual(pop, replace=True)
        if fitness(challenger) > fitness(best):
            best ← challenger
    return best
```

Crossover: Single Point

(As opposed to multi-point...)



Advice on practical use of EAs

Remember the approach is powerful and also computationally expensive

- Only use for medium to large problems
- Preliminary work should involve setting a benchmark (similar to forecasting!)
  - A **greedy method** (i.e., only considering the next step rather than overall problem) and or **random search** are easy to implement and quick to run
    - Also easy for a health service stakeholders to understand (compare to GAs)
  - Only use if it evident that EAs offer improvement
- GAs vs. Evolutionary strategies
  - GAs need **more** tuning and have **longer** runtimes (as more to “do”)
- Often a bottleneck in execution is the fitness function
  - Large populations, lots of generations and complex objectives → careful implementation (Implementation in Python Vs. Numpy Vs. C; using a dictionary to cache values of chromosomes to save unnecessarily running fitness function)
- The state-of-the-art literature will have *ideas*, but **unlikely** to include code

Demonstration

Demonstration only:

- *Evolutionary and Genetic Algorithms*

Summary

- Evolutionary strategies versus full Genetic Algorithm
- Key steps:
  - Decide how to represent the problem
  - Code a function to evaluate the fitness of a solution
  - Choose and code: selection, crossover and mutation operators
  - Bring it all together in a EA or GA framework
  - Tune parameters