M/M/s queueing system

Assumptions

- Inter-arrival times: Exponential distribution with parameter λ
- Service times: Exponential distribution with parameter μ (expected service time is $\frac{1}{\mu}$)
- Number of servers: s

The following formulas are only valid when $\boldsymbol{\rho = \frac{\lambda}{s \cdot \mu} < 1}$ (traffic intensity less than 1)

- Probability that a new customer arriving will find 0 customers already in the system (either waiting or being served):

$$P_0 = \frac{1}{\sum_{k=0}^{s-1} \frac{1}{k!} \cdot \left(\frac{\lambda}{\mu}\right)^k + \frac{1}{s!} \cdot \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{1}{1-\rho}}$$

- Expected number of customers in the queue:

$$L_q = \frac{1}{s!} \cdot \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{(1-\rho)^2} \cdot P_0$$

- Expected time spent by a customer waiting in the queue:

$$W_q = \frac{L_q}{\lambda}$$

- Expected time spent by a customer in the system (queue + service):

$$W = W_q + \frac{1}{\mu}$$

- Expected number of customers in the system (either queueing or being served):

$$L = \lambda \cdot W$$

- Probability that a new customer arriving will find N customers already in the system (either waiting or being served):

$$P_N = \begin{cases} \frac{1}{N!} \cdot \left(\frac{\lambda}{\mu}\right)^N \cdot P_0, & N = 1,2,\ldots,s-1 \\ \frac{1}{s! \cdot s^{N-s}} \cdot \left(\frac{\lambda}{\mu}\right)^N \cdot P_0, & N = s, s+1, \ldots \end{cases}$$