Matrix Calculation: Homework #6

Due on Oct 31, 2022 at 3:10pm

Professor Jun Lai Monday

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Problem 1

(Page41 Problem1.4.2)
Set

$$G = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \tag{1}$$

By the definition of $F_n(:, 1:3:n-1)$, we can see that:

$$F_{11} = F_{21} = F_{31} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega_{3m}^3 & \cdots & \omega_{3m}^{3(m-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_{3m}^{3(m-1)} & \cdots & \omega_{3m}^{3(m-1)^2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega_m & \cdots & \omega_m^{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_m^{m-1} & \cdots & \omega_m^{(m-1)^2} \end{bmatrix} = F_m. \tag{2}$$

Consider F_{12}, we can see:

$$F_{12} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \omega_{3m} & \omega_{3m}^4 & \cdots & \omega_{3m}^{3m-2} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{3m}^{m-1} & \omega_{3m}^{4(m-1)} & \cdots & \omega_{3m}^{(3m-2)(m-1)} \end{bmatrix} \tag{3}$$

which means that

$$F_{12} = \Omega_m F_m, \Omega_m = \mathrm{diag}(1, \omega_{3m}, \cdots, \omega_{3m}^{m-1}). \tag{4}$$

Similarly, we can write:

$$F_{22} = \omega_3 \Omega_m F_m, F_{32} = \omega_3^2 \Omega_m F_m. \tag{5}$$

and:

$$F_{31} = \Omega_m^2 F_m, F_{32} = \omega_3^2 \Omega_m^2 F_m, F_{33} = \omega_3^4 \Omega_m^2 F_m. \tag{6}$$

Set $y_a = x(1:3:n-1), y_b = x(2:3:n-1), y_c = x(3:3:n-1)$, we can see:

$$F_n y = G(y_a^t, y_b^t, y_c^t)^t = \begin{bmatrix} F_m y_a + \Omega_m F_m y_b + \Omega_m^2 F_m y_c \\ F_m y_a + \omega_3 \Omega_m F_m y_b + \omega_3^2 \Omega_m^2 F_m y_c \\ F_m y_a + \omega_3^2 \Omega_m F_m y_b + \omega_3^4 \Omega_m^2 F_m y_c \end{bmatrix} \tag{7}$$

Then we can derive an induction algorithm by equation (7).