# Matrix Calculation: Homework #11

Due on Dec 12, 2022 at 3:10pm

*Professor Jun Lai Monday*

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# Problem 1

(P383 Problem7.4.4)
Denote Hessenberg matrix

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ & \ddots & \ddots & \vdots \\ & & h_{n-1,n} & h_{nn} \end{bmatrix} \tag{1}$$

and the diagonal matrix

$$D = \mathrm{diag}\{d_1, d_2, \cdots, d_n\}. \tag{2}$$

As the subdiagonal elements of $D^{-1}HD$ are all 1, we can see that:

$$\frac{d_{i+1}}{d_i} = h_{i+1,i}, i \in [1, n-1]. \tag{3}$$

It derives the value of matrix $D$. Its condition number:

$$\kappa_2(D) = \frac{\max \prod_i |h_{i,i-1}|}{\min \prod_i |h_{i,i-1}|}. \tag{4}$$

# Problem 2

(P383 Problem7.4.5)
Assume the eigenvector related to $\lambda$ is $X_1 + iX_2$, we can see:

$$(W + iY)(X_1 + iX_2) = \lambda(X_1 + iX_2). \tag{5}$$

i.e.

$$(WX_1 - YX_2) + i(YX_1 + WX_2) = \lambda X_1 + i\lambda X_2. \tag{6}$$

Then:

$$\begin{cases} WX_1 - YX_2 = \lambda X_1, \\ YX_1 + WX_2 = \lambda X_2. \end{cases} \tag{7}$$

i.e.

$$\begin{bmatrix} W & -Y \\ Y & W \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \lambda \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \tag{8}$$

So $\lambda$ is an eigenvalue of $B$, with the eigenvector $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$.