Matrix Calculation: Homework #7

Due on Nov 7, 2022 at 3:10pm

Professor Jun Lai Monday

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Problem 1

(Page 232,Problem 4.8.8)
For a point P whose discretrize stencil is totally inside the domain Ω, the equation below is true:

$$\frac{u(N)+u(S)+u(E)+u(W)-4u(P)}{h^2}=F(P). \tag{1}$$

Assume the identifier of grid is the same as Figure 4.8.1, then for boundary cells near the left-side, a.g. u_{n_1+1}, we can write the following discreterize equation:

$$4u_{n_1+1}-u_1-u_2-u_{2n_1+1}=h^2F_{n_1+1}. \tag{2}$$

while the equation is derived by (1) and boundary condition $u|_{x=0}=0$.
For the bottom side of the problem domain, we can use totally the same method to discuss this problem. Then, for the Neumann-Boundary side points, such as u_2, it's a good idea to use so-called **Ghost Cell** to handle it. But we can also write the discreterize function like this:

$$\begin{cases} \dfrac{u_1+u_3+u_{n_1+2}+u_G-4u_2}{h^2}=F(2) \\ \dfrac{u_G-u_{n_1+2}}{2h}=0 \end{cases} \tag{3}$$

This discreterized equation is derived by the homogenious Neumann boundary condition. (1), (2) and (3) makes the coefficient matrix A. And in the same time, the right-hand vector b can be derived by the function F, which is:

$$b=h^2\begin{bmatrix} F(1) \\ \vdots \\ F(n_1n_2). \end{bmatrix} \tag{4}$$

Problem 2

(Page 244,Problem 5.1.1)
Set the vector

$$v=\frac{\|x\|}{\|y\|}y-x, \tag{5}$$

and the projection matrix

$$P=I-\frac{2}{v^tv}vv^t, \tag{6}$$

P is just the Householder matrix which maps x to a multiple of y. Now I try to varify this equation.

$$\begin{aligned} Px&=(I-\frac{2}{v^tv})vv^tx \\ &=(I-\frac{2}{v^tv}vv^t)(\frac{\|x\|}{\|y\|}y-v) \\ &=v+\frac{\|x\|}{\|y\|}y-\frac{2\|x\|}{v^tv\|y\|}vv^ty \\ &=\frac{2\|x\|}{\|y\|}y-x-\frac{2\|x\|}{v^tv\|y\|}vv^ty \\ &=\frac{2\|x\|}{\|y\|}y-x-\frac{2\|x\|(\|x\|\|y\|-x^ty)}{v^tv\|y\|}(\frac{\|x\|}{\|y\|}y-x) \\ &=\frac{\|x\|}{\|y\|}y. \end{aligned} \tag{7}$$