Matrix Calculation: Homework #7

Due on Nov 7, 2022 at 3:10pm

Professor Jun Lai Monday

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Problem 1

(Page 232,Problem 4.8.8)
For a point P whose discretrize stencil is totally inside the domain Ω, the equation below is true:

$$\frac{u(N)+u(S)+u(E)+u(W)-4u(P)}{h^2}=F(P). \tag{1}$$

Assume the identifier of grid is the same as Figure 4.8.1, then for boundary cells near the left-side, a.g. u_{n_1+1}, we can write the following discreterize equation:

$$4u_{n_1+1}-u_1-u_2-u_{2n_1+1}=h^2F_{n_1+1}. \tag{2}$$

while the equation is derived by (1) and boundary condition $u|_{x=0}=0$.
For the bottom side of the problem domain, we can use totally the same method to discuss this problem. Then, for the Neumann-Boundary side points, such as u_2, it's a good idea to use so-called **Ghost Cell** to handle it. But we can also write the discreterize function like this:

$$\begin{cases}\dfrac{u_1+u_3+u_{n_1+2}+u_G-4u_2}{h^2}=F(2)\\ \dfrac{u_G-u_{n_1+2}}{2h}=0\end{cases} \tag{3}$$

This discreterized equation is derived by the homogenious Neumann boundary condition. (1), (2) and (3) makes the coefficient matrix A. And in the same time, the right-hand vector b can be derived by the function F, which is:

$$b=h^2\begin{bmatrix}F(1)\\ \vdots\\ F(n_1n_2).\end{bmatrix} \tag{4}$$

Problem 2

(Page 245,Problem 5.1.8)
First, I derive that Q is an orthogonal matrix.

$$\begin{aligned}QQ^t&=(I+S)(I-S)^{-1}(I-S^T)^{-1}(I+S^T)\\&=(I+S)(I-S)^{-1}(I+S)^{-1}(I-S)\\&=(I+S)(I+S)^{-1}(I-S)^{-1}(I-S)\\&=I.\end{aligned} \tag{5}$$

It means that Q is an orthogonal matrix.
Assume the Householder transformation from x to $\|x\|e_1$ is

$$H=I-\beta vv^t, \tag{6}$$

then:

$$\begin{aligned}(I+S)(I-S)^{-1}&=I-\beta vv^t\\(I+S)&=(I-S)(I-\beta vv^t)\\S(2I-\beta vv^t)&=-\beta vv^t\\S&=-\beta(2I-\beta vv^t)^{-1}vv^t.\end{aligned} \tag{7}$$

This is exactly the rank-2 anti-symmetric matrix S.