Mathematics of AI

Square Root

With a random walk problem, A is the adjacency matrix of a directed weighted graph where, $0 \le a_{i,j} \le 1$ is the probability of a move from the vertex v_i to the vertex v_j.

$$\forall v_i: \sum_{j=1}^{N} a_{i,j} = 1$$

, where N is the number of vertices. You know that $B = A^2$ represents the probabilities of moves with length two. In other words, $b_{i,j}$ is the probability of starting from v_i and reaching to v_j after two steps.

$$b_{i,j} = \sum_{k} probability\ of\ v_i \to v_k \to v_j = \sum_{k=1}^{N} a_{i,k} a_{k,j} = \overrightarrow{a_i}^T \overrightarrow{A_j}$$

$$\Rightarrow B = A \times A = A^2$$

But it is a forward problem. Now the matrix B is given to you. It is desired to decompose it into two equal matrices $B = A \times A$. The matrix B may be an incorrect adjacency matrix ($\nexists A: A \times A = B$), however the probability constraints are held:

$$0 \le b_{i,j}$$

$$\forall i: \sum_{j=1}^{N} b_{i,j} = 1$$

Try to find A in order to minimize the error $\|B - A \times A\|_F$.

Report your average error for 100 random matrices of dimension 5×5. More reports are also valuable.

This inverse problem may be solved with different approaches and I have no idea about them to help you. Just try!

Good Luck!

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