状態方程式

$$x_{t/t-1} = Fx_{t-1/t-1} + Gu_{t-1}$$

観測方程式

$$z_{t/t-1} = Hx_{t/t-1}$$

$$R_{t/t-1} = H^2 P_{t/t-1}$$

x_t の分散共分散行列

$$
\begin{aligned}
P_{t/t-1} &= E[\Delta x_{t/t-1}^2] \\
&= E[(F\Delta x_{t-1/t-1} + G\Delta u_{t-1})^2] \\
&= F^2 E[\Delta x_{t-1/t-1}^2] + 2E[F\Delta x_{t-1/t-1}G\Delta u_{t-1}] + G^2 E[\Delta u_{t-1}^2] \\
&= F^2 E[\Delta x_{t-1/t-1}^2] + G^2 E[\Delta u_{t-1}^2] \\
&= F^2 P_{t-1/t-1} + G^2 Q_{t-1}
\end{aligned}
$$

計測更新

$$z_{t/t} = w_1 z_{t/t-1} + w_2 z_t$$

計測更新: 分散

$$
\begin{aligned}
\sigma^2 &= E[\Delta z_{t/t}^2] \\
&= w_1^2 R_{t/t-1} + w_2^2 R_t
\end{aligned}
$$

最小分散推定

$$\frac{\partial \sigma^2}{\partial w_1} = \frac{\partial}{\partial w_1}\{w_1^2 R_{t/t-1} + (1-w_1)^2 R_t\} = 0$$

$$w_1 = \frac{R_t}{R_{t/t-1} + R_t}, \ w_2 = \frac{R_{t/t-1}}{P_{t/t-1} + R_t}$$

計測更新: 状態の更新

$$
\begin{aligned}
z_{t/t} &= w_1 z_{t/t-1} + w_2 z_t \\
&= \frac{R_t}{R_{t/t-1} + R_t} z_{t/t-1} + \frac{R_{t/t-1}}{R_{t/t-1} + R_t} z_t \\
&= z_{t/t-1} + \frac{R_{t/t-1}}{R_{t/t-1} + R_t}(z_t - z_{t/t-1})
\end{aligned}
$$

$$
\begin{aligned}
Hx_{t/t} &= Hx_{t/t-1} + \frac{H^2 P_{t/t-1}}{H^2 P_{t/t-1} + R_t}(z_t - z_{t/t-1}) \\
x_{t/t} &= x_{t/t-1} + \frac{H P_{t/t-1}}{H^2 P_{t/t-1} + R_t}(z_t - z_{t/t-1})
\end{aligned}
$$

カルマンフィルタ: 計測更新

$$\begin{aligned}
\tilde{z} &= z_t - z_{t/t-1} \\
S &= H^2 P_{t/t-1} + R_t \\
K &= H P_{t/t-1} S^{-1}
\end{aligned}$$