Lotka-Volterra Equations[1, 2] are couples of ordinary differential equations as follows

$$\frac{dp_1}{dt} = ap_1 - bp_1p_2, \tag{1}$$

$$\frac{dp_2}{dt} = -cp_2 + dp_1p_2, \tag{2}$$

where $p_1, p_2 \in \mathbb{R}^+$ are dependent valiables of t, and $a, b, c, d \in \mathbb{R}^+$ are constant parameters. Eliminating t from Eqs.(1) and (2), conserved quantity of the system is derived as

$$V = -a\log(p_2) + bp_2 - c\log(p_1) + dp_1. \tag{3}$$

Solving Eq.(3) for p_2,

$$p_2 = -\frac{a}{b}\,W\left(-\frac{b}{a}\exp\left(\frac{1}{a}\left(-c\log p_1 + dp_1 - V\right)\right)\right), \tag{4}$$

where W is the Lambert W function[3]. W is multivalued function with two branches (W_{-1}, W_0). We choose W_0 when $-b/a\,p_2 \leq -1$ and W_{-1} for other case.

References

[1] Wikipedia, Lotka-Volterra equations.
https://en.wikipedia.org/wiki/Lotka-Volterra_equations. Accessed on June 2016.

[2] Wolfram MathWorld, Lotka-Volterra equations.
http://mathworld.wolfram.com/Lotka-VolterraEquations.html. Accessed on June 2016.

[3] Wikipedia, Lambert W function.
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