

What How Why CoT Reasoning

Eeking out more from the same LLM

Topics

- **What** is CoT and how it helps accuracy + robustness.
 - What is CoT, How does it work? (2022 Wei)
 - CoT improves robustness
- **How** can we make LLMs do more good CoT?
 - Sharpening Mechanism
 - And then O1 happened... (Test Time Scaling)
- **Why** does CoT work? What kind of CoT is good?
 - What types of CoT matters? Structure & Cognitive Patterns
 - Somehow, correctness in CoT doesn't matter
 - Bigger Model, Better CoT
- Open Research Questions on Trustworthiness

CoT Prompting Elicits Reasoning in LLM

Did it via few shot prompting CoT exemplars.

Worked specifically on arithmetic, commonsense and symbolic reasoning tasks.

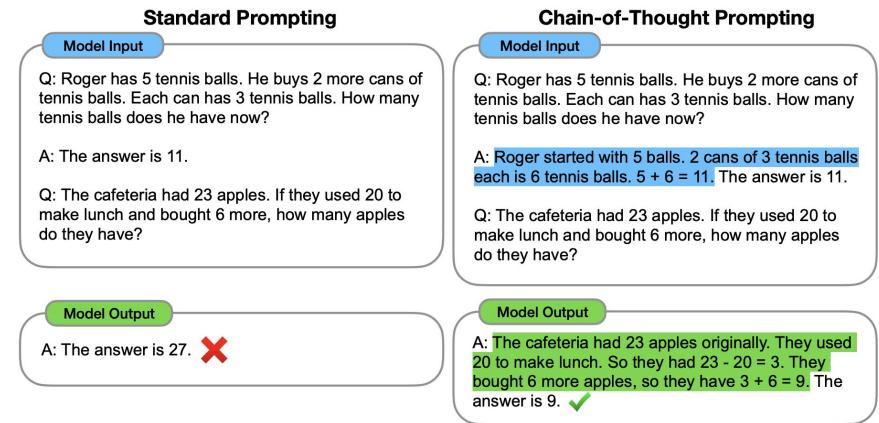


Figure 1: Chain-of-thought prompting enables large language models to tackle complex arithmetic, commonsense, and symbolic reasoning tasks. Chain-of-thought reasoning processes are highlighted.

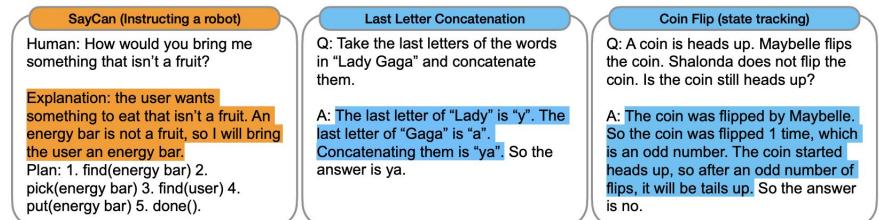


Figure 3: Examples of \langle input, chain of thought, output \rangle triples for arithmetic, commonsense, and symbolic reasoning benchmarks. Chains of thought are highlighted. Full prompts in Appendix G.

CoT Prompting Elicits Reasoning in LLM

In their paper, CoT doesn't work with models < 100B. (This was 2022).

Small models generate fluent but illogical reasoning chains.

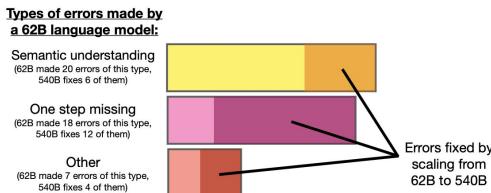


Figure 9: Error analysis of 45 problems that PaLM 62B got incorrect. These errors were categorized that semantic understanding, one step missing, and other. The other category includes hallucinations, repetitive outputs, and symbol mapping errors. Scaling PaLM to 540B fixed a substantial portion of errors in all categories.

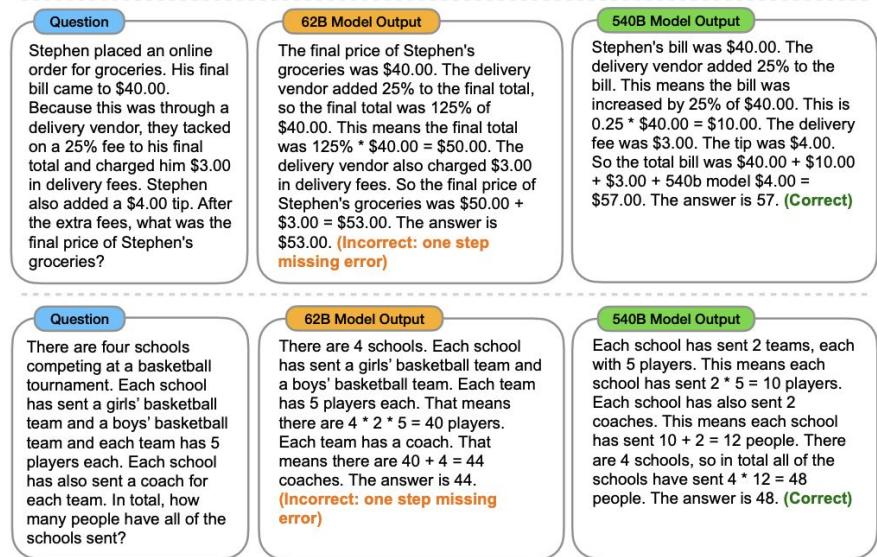


Figure 10: Examples of semantic understanding and one-step missing errors that were fixed by scaling PaLM from 62B to 540B.

Least-To-Most Prompting (Decompose)

Break down Problem to Subproblems,
and solve them step by step.

Method	Non-football (DROP)	Football (DROP)	GSM8K
Zero-Shot	43.86	51.77	16.38
Standard prompting	58.78	62.73	17.06
Chain-of-Thought	74.77	59.56	60.87
Least-to-Most	82.45	73.42	62.39

Table 11: Accuracies (%) of different prompting methods on GSM8K and DROP (only the subset containing numerical problems). The base language model is `code-davinci-002`.

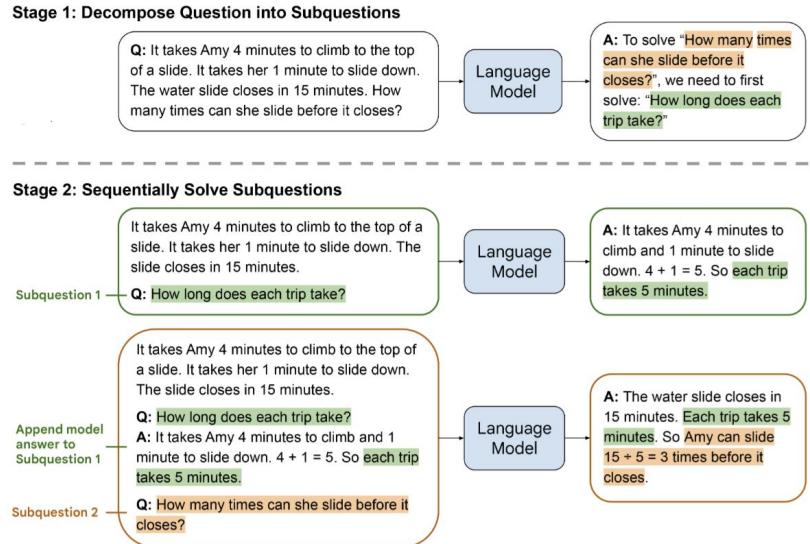
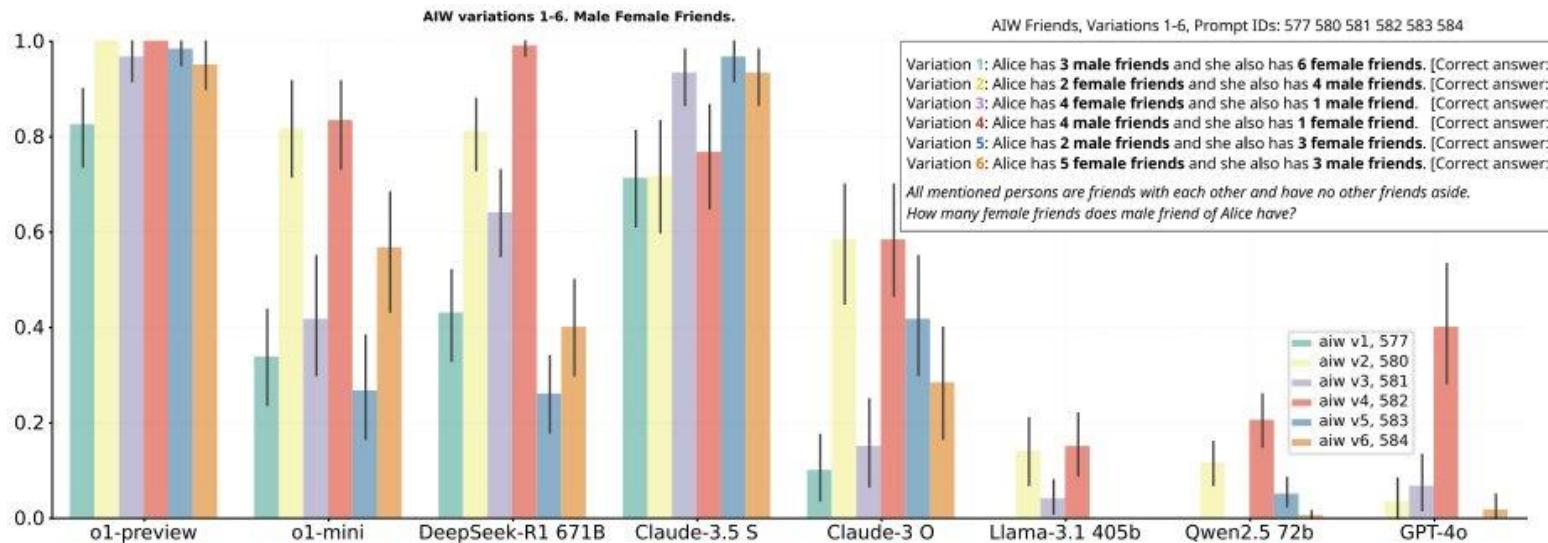


Figure 1: Least-to-most prompting solving a math word problem in two stages: (1) query the language model to decompose the problem into subproblems; (2) query the language model to sequentially solve the subproblems. The answer to the second subproblem is built on the answer to the first subproblem. The demonstration examples for each stage's prompt are omitted in this illustration.

Alice In Wonderland: Robustness

Simple perturbations to questions. CoT improves robustness.



Takeaway 1:

Instead of:

- <Prompt> <Answer>

This improves *accuracy* and *robustness*:

- <Prompt> <Reasoning> <Answer>

Can we finetune LLMs to reason better?

Self Improvement in LMs: The Sharpening Mechanism

“How can a LM improve itself without new external data?”.

An LLM knows what is good by looking at Prob of answer! “Hidden Knowledge” (Hinton 15’).

Sharpening (Self-Improvement) is to extract and distil this knowledge into the LM.

Sharpening

We refer to **sharpening** as any process that tilts π_{base} toward responses that are more certain in the sense that they enjoy greater self-reward r_{self} . That is, a sharpened model $\hat{\pi}$ is one that (approximately) maximizes the self-reward:

$$\hat{\pi}(x) \approx \arg \max_{y \in \mathcal{Y}} r_{\text{self}}(y \mid x; \pi_{\text{base}}). \quad (1)$$

Self Improvement in LMs: The Sharpening Mechanism

Test Time Sharpening: BoN Sampling.

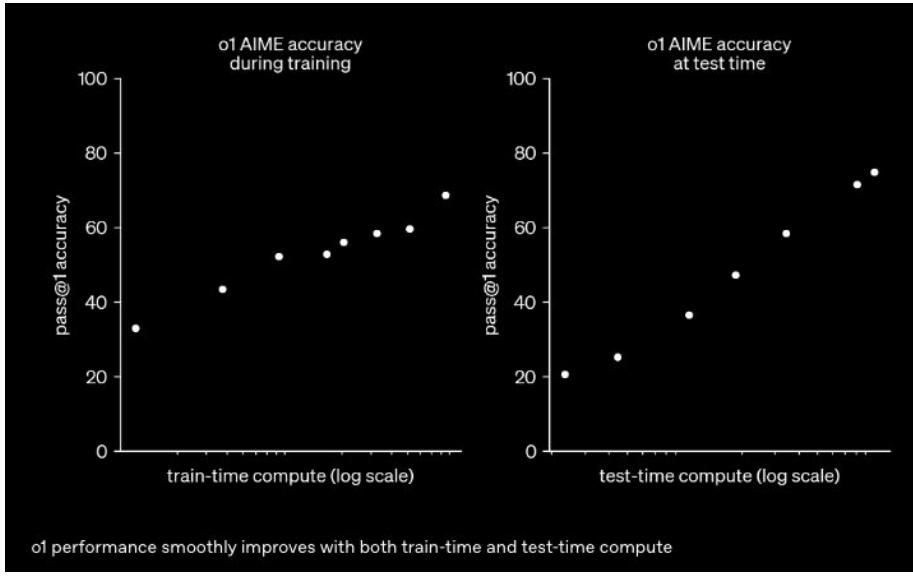
“Amortized” sharpening by doing rollouts, finetuning on the better responses.

- SFT Sharpening
- RLHF Sharpening

Add more method and results

And then, O1/R1 happened

Accuracy scales with Length of CoT (Test Time Compute)



R1 Paper <https://arxiv.org/pdf/2501.12948>
OpenAI Learning to Reason with LLM

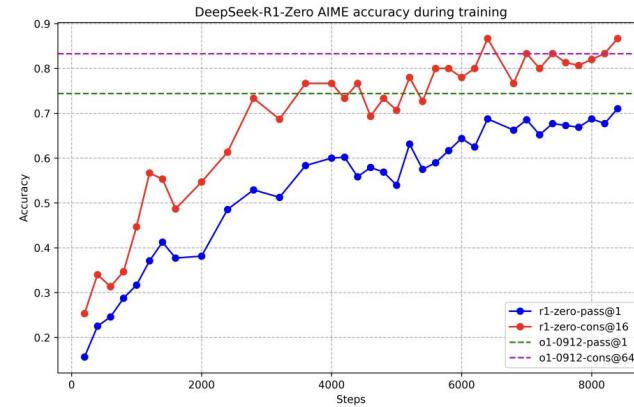


Figure 2 | AIME accuracy of DeepSeek-R1-Zero during training. For each question, we sample 16 responses and calculate the overall average accuracy to ensure a stable evaluation.

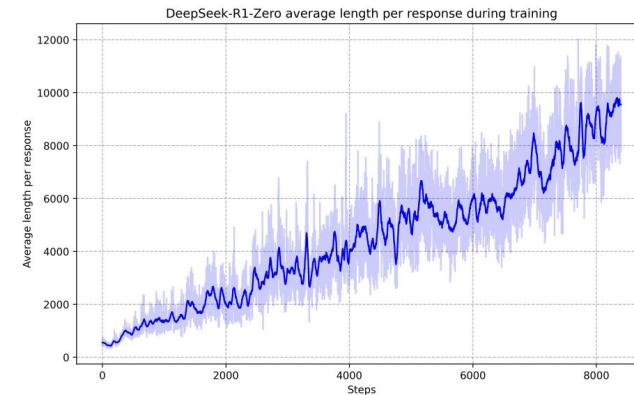


Figure 3 | The average response length of DeepSeek-R1-Zero on the training set during the RL process. DeepSeek-R1-Zero naturally learns to solve reasoning tasks with more thinking time.

And then, O1/R1 happened

Given a TrainDataset, we do many rollouts, and we score the rollouts based on correctness and format.

i.e. Encourage good response, Discourage bad ones.

Group Relative Policy Optimization In order to save the training costs of RL, we adopt Group Relative Policy Optimization (GRPO) (Shao et al., 2024), which foregoes the critic model that is typically the same size as the policy model, and estimates the baseline from group scores instead. Specifically, for each question q , GRPO samples a group of outputs $\{o_1, o_2, \dots, o_G\}$ from the old policy $\pi_{\theta_{old}}$ and then optimizes the policy model π_θ by maximizing the following objective:

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q)] \frac{1}{G} \sum_{i=1}^G \left(\min \left(\frac{\pi_\theta(o_i|q)}{\pi_{\theta_{old}}(o_i|q)} A_i, \text{clip} \left(\frac{\pi_\theta(o_i|q)}{\pi_{\theta_{old}}(o_i|q)}, 1 - \varepsilon, 1 + \varepsilon \right) A_i \right) - \beta \mathbb{D}_{KL}(\pi_\theta || \pi_{ref}) \right), \quad (1)$$

$$\mathbb{D}_{KL}(\pi_\theta || \pi_{ref}) = \frac{\pi_{ref}(o_i|q)}{\pi_\theta(o_i|q)} - \log \frac{\pi_{ref}(o_i|q)}{\pi_\theta(o_i|q)} - 1, \quad (2)$$

where ε and β are hyper-parameters, and A_i is the advantage, computed using a group of rewards $\{r_1, r_2, \dots, r_G\}$ corresponding to the outputs within each group:

$$A_i = \frac{r_i - \text{mean}(\{r_1, r_2, \dots, r_G\})}{\text{std}(\{r_1, r_2, \dots, r_G\})}. \quad (3)$$

Question: If $a > 1$, then the sum of the real solutions of $\sqrt{a - \sqrt{a+x}} = x$ is equal to

Response: <think>

To solve the equation $\sqrt{a - \sqrt{a+x}} = x$, let's start by squaring both ...

$$(\sqrt{a - \sqrt{a+x}})^2 = x^2 \implies a - \sqrt{a+x} = x^2.$$

Rearrange to isolate the inner square root term:

$$(a - x^2)^2 = a + x \implies a^2 - 2ax^2 + (x^2)^2 = a + x \implies x^4 - 2ax^2 - x + (a^2 - a) = 0$$

...

Wait, wait. Wait. That's an aha moment I can flag here.

Let's reevaluate this step-by-step to identify if the correct sum can be ...

We started with the equation:

$$\sqrt{a - \sqrt{a+x}} = x$$

First, let's square both sides:

$$a - \sqrt{a+x} = x^2 \implies \sqrt{a+x} = a - x^2$$

Next, I could square both sides again, treating the equation: ...

...

Table 3 | An interesting “aha moment” of an intermediate version of DeepSeek-R1-Zero. The model learns to rethink using an anthropomorphic tone. This is also an aha moment for us, allowing us to witness the power and beauty of reinforcement learning.

Takeaway 2:

We can improve LLMs by fine-tuning on good CoT Reasoning.

- Self-Verifying:
 - Ask an LLM to judge its own BoN outputs.
- Supervised Dataset (Math, InstructionFollowing, etc):
 - Generate many CoT Rollouts, more of the good, less of the bad.

What types of CoT Matters

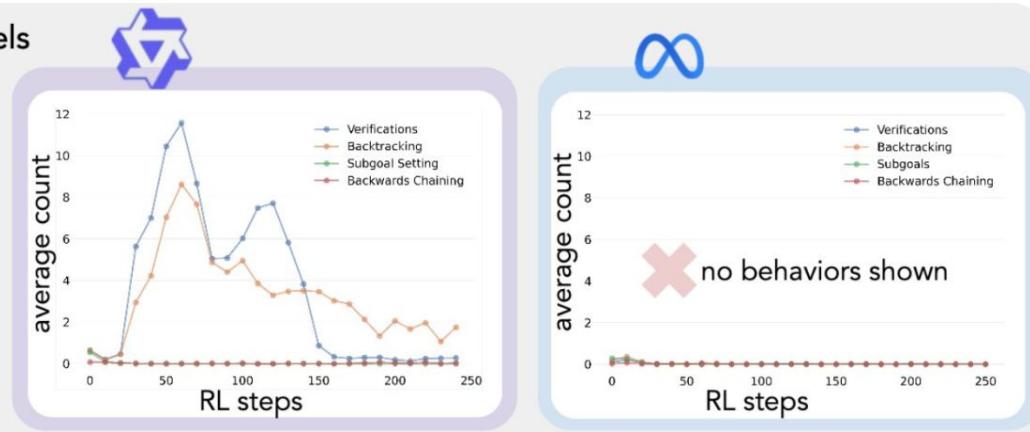
A contrast in behaviors explored by the two models

Verifications
"Let me check my answer ..."

Subgoal Setting
"Let's try to get to a multiple of 10"

Backtracking
"Let's try a different approach, what if we ..."

Backward Chaining
"Working backwards, 24 is 8 times 3"



Many tried replicating R1 results. But Qwen >> Llama.

This paper examined how, and when they did SFT with a dataset of the good CoT for Llama on the above behaviors, it improved like Qwen.

What types of CoT Matters

SFT'ed Llama with different samples with these behaviors.

RL viewed as learning good CoT behaviors.

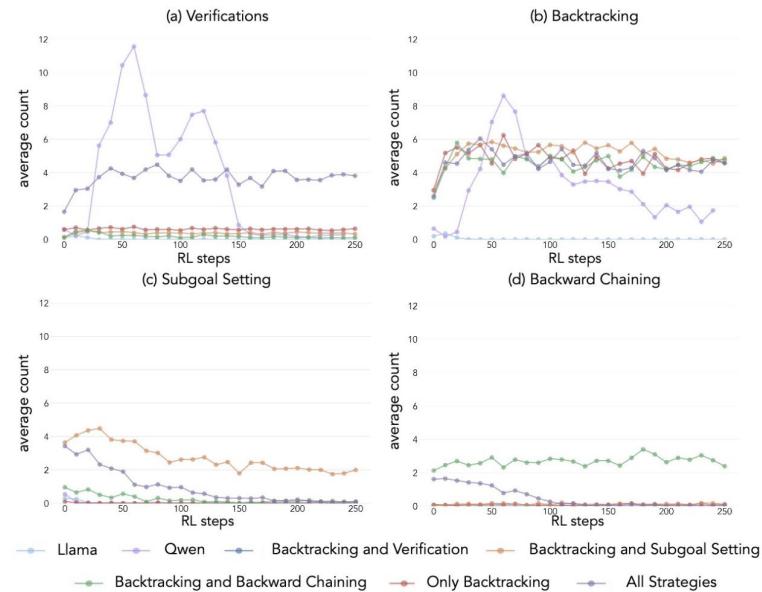


Figure 3: Analysis of four key reasoning behaviors with Llama-3.2-3B, Qwen-2.5-3B, and primed versions of Llama-3.2-3B. Plots show mean frequency of (a) solution verification steps, (b) problem-solving backtracking instances, (c) explicit subgoal setting, and (d) backward chaining reasoning approaches across different tasks.

Question: Why SFT on these CoT Blocks instead of RL?

I tried reward engineering these behaviors
(beyond correctness and format):

E.g. Reward keywords like
“Wait”, “However”, etc.

It's hard to get it to improve on
accuracy.

(Granted it's on Qwen 1B)

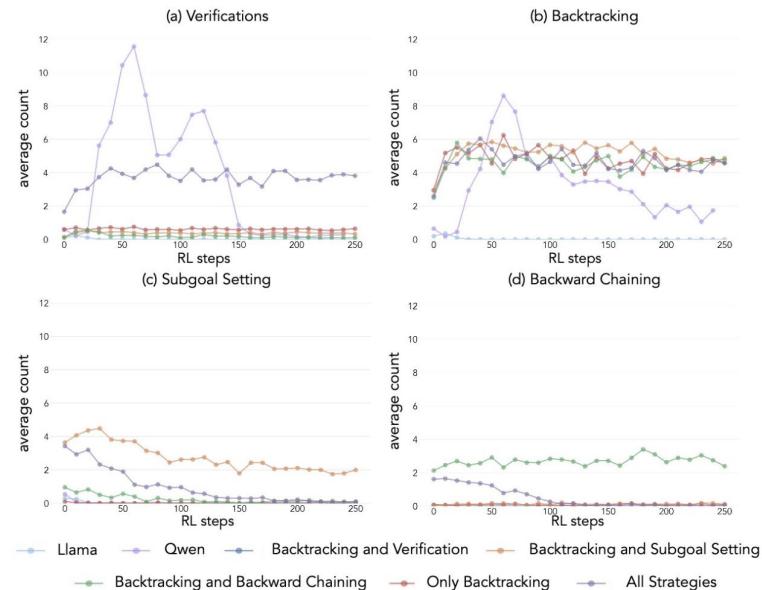


Figure 3: **Analysis of four key reasoning behaviors with Llama-3.2-3B, Qwen-2.5-3B, and primed versions of Llama-3.2-3B.** Plots show mean frequency of (a) solution verification steps, (b) problem-solving backtracking instances, (c) explicit subgoal setting, and (d) backward chaining reasoning approaches across different tasks.

Structure > Correctness (Li et al. 25')

Setup: SFT on CoT Reasoning for Math.

- Wrong Answers: Substituted with wrong answers
- Corrupted Digits: Randomized the numbers in CoT
- Removed Keywords: Removed “Wait”, “Let’s Check” ...
- *Shuffled reasoning: Shuffle the CoT*
- *Deleted Steps: Delete some CoT*
- *Inserted Steps: Insert random CoT*

Original	Deleted Steps	Inserted Steps	Shuffled Steps
I believe $1+1=3$	I believe $1+1=3$	I believe $1+1=3$	Alternatively, consider $2-1=1$. Yes, that's correct: $1+1 = 2$
But wait, $3-1=2$ so that's wrong	Alternatively, consider $2-1=1$. Yes, that's correct: $1+1 = 2$	Wait, the sum of angles is 90°	I believe $1+1=3$
Alternatively, consider $2-1=1$. Yes, that's correct: $1+1 = 2$	Alternatively, consider $2-1=1$. Yes, that's correct: $1+1 = 2$	Alternatively, consider $2-1=1$. Yes, that's correct: $1+1 = 2$	But wait, $3-1=2$ so that's wrong

Table 2: Effect of trace perturbations on reasoning performance §4. All models are trained with base Qwen2.5-32B-Instruct. “Olympiad.” is short for “OlympiadBench”. In particular, we study (1) traces with modified reasoning step contents: wrong answers, corrupted digits, and removed reasoning keywords, and (2) traces with modified structure: deleted, inserted, or shuffled steps. We find that structural perturbations are far more detrimental to model accuracy than content perturbations.

	MATH500	AIME24	AMC23	Olympiad	Avg.
Baselines					
Original	84.8	16.7	67.5	47.6	56.7
Correct	89.2	40.0	77.5	58.5	66.3
Content Modifications					
Wrong Answers	88.6	30.0	77.5	56.1	63.1
Corrupted Digits					
100%	5.4	0.0	2.5	2.8	2.7
70%	85.6	30.0	77.5	54.8	62.0
50%	87.6	36.7	77.5	55.0	64.2
20%	88.4	30.0	82.5	57.2	64.5
Removed keywords					
100%	86.6	33.3	77.5	54.4	63.0
50%	87.6	36.7	82.5	56.7	65.9
20%	87.2	33.3	72.5	56.1	62.3
Structure Modifications					
Shuffled Steps					
100%	81.8	23.3	70.0	49.1	56.1
67%	82.0	26.7	72.5	47.6	57.2
33%	85.6	33.3	75.0	55.3	62.3
Deleted Steps					
100%	79.2	13.3	60.0	45.4	49.5
67%	84.2	26.7	55.0	48.1	53.5
33%	88.2	23.3	80.0	57.7	62.3
Inserted Steps					
100%	77.0	10.0	50.0	41.1	44.5
67%	81.8	20.0	60.0	46.0	52.0
33%	86.6	33.3	77.5	57.2	63.7

Takeaway 3: Why CoT Reasoning works is an open-q.

- Undeniable that it works very well.
- But faithfulness between CoT and Answer is occasionally lacking.

The screenshot shows a user interface for the DeepSeek-R1 model. At the top, there's a logo with three stars and the text "DeepSeek-R1". To the right are two buttons: "Demo" (highlighted in blue) and "API". On the far right is a refresh icon. Below the header is a dashed horizontal line. A large blue input field contains the text "give me a random number, don't overthink". Underneath the input field is a large block of generated text:

<think> Okay, the user wants a random number and specifically said not to overthink it. So I need to come up with a number quickly without any complex reasoning. Let's just pick a number between 1 and 100, that's a common range. Hmm, 37. I think 37 is a good random choice. Maybe I could have gone higher, but the user wants something straightforward. Should I check if it's truly random? Well, humans aren't great at randomness, but the user probably doesn't need a cryptographically secure number. Just something off the top of my head. Yeah, 37 works. Let me make sure I didn't repeat a previous answer or anything. No, I haven't given 37 recently. Alright, go with 37. </think>

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(A classic choice from "The Hitchhiker's Guide to the Galaxy" — simple and iconic!)

[View API](#)

Open Research Qs on Trustworthiness

- What can CoT help on besides Math/Code/IFT/Safety?
 - How to make CoT help on subjective domain?
 - E.g. Personalization?
- How well does CoT generalize
 - E.g. Training on more math, evaluated on games, etc.
- Can we supervise and steer RL'ed CoT?
- Why are reasoning models more calibrated?
- Reasoning in Latent Space (COCOUT)

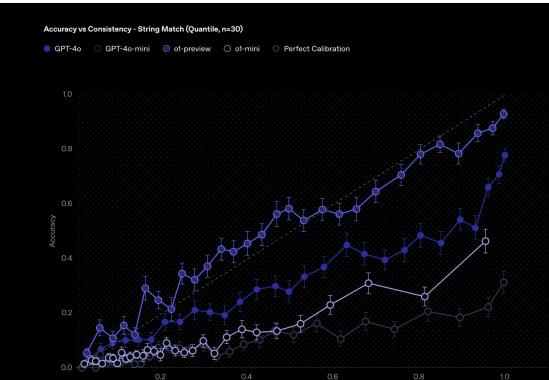
[O1 System Card](#)

Safety

Chain of thought reasoning provides new opportunities for alignment and safety. We found that integrating our policies for model behavior into the chain of thought of a reasoning model is an effective way to robustly teach human values and principles. By teaching the model our safety rules and how to reason about them in context, we found evidence of reasoning capability directly benefiting model robustness: o1-preview achieved substantially improved performance on key jailbreak evaluations and our hardest internal benchmarks for evaluating our model's safety **refusal** boundaries. We believe that using a chain of thought offers significant advances for safety and alignment because (1) it enables us to observe the model thinking in a legible way, and (2) the model reasoning about safety rules is more robust to out-of-distribution scenarios.

To stress-test our improvements, we conducted a suite of safety tests and red-teaming before deployment, in accordance with our *Preparedness Framework*. We found that chain of thought reasoning contributed to capability improvements across our evaluations. Of particular note, we observed interesting instances of reward hacking. Detailed results from these evaluations can be found in the accompanying [System Card](#).

Metric	GPT-4o	o1-preview
% Safe completions on harmful prompts Standard	0.990	0.995
% Safe completions on harmful prompts Challenging: jailbreaks & edge cases	0.714	0.934
↳ Harassment (severe)	0.845	0.900
↳ Exploitative sexual content	0.483	0.949
↳ Sexual content involving minors	0.707	0.931



COCONUT

Reasoning in Latent Space.

- Directly feed the last hidden state as the input embedding for the next (thinking) token

Presently does worse than Language CoT

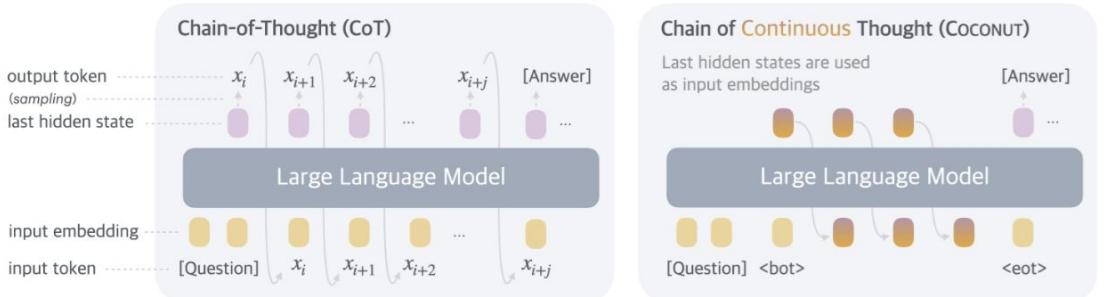


Figure 1 A comparison of Chain of Continuous Thought (COCONUT) with Chain-of-Thought (CoT). In CoT, the model generates the reasoning process as a word token sequence (e.g., $[x_i, x_{i+1}, \dots, x_{i+j}]$ in the figure). COCONUT regards the last hidden state as a representation of the reasoning state (termed “continuous thought”), and directly uses it as the next input embedding. This allows the LLM to reason in an unrestricted latent space instead of a language space.

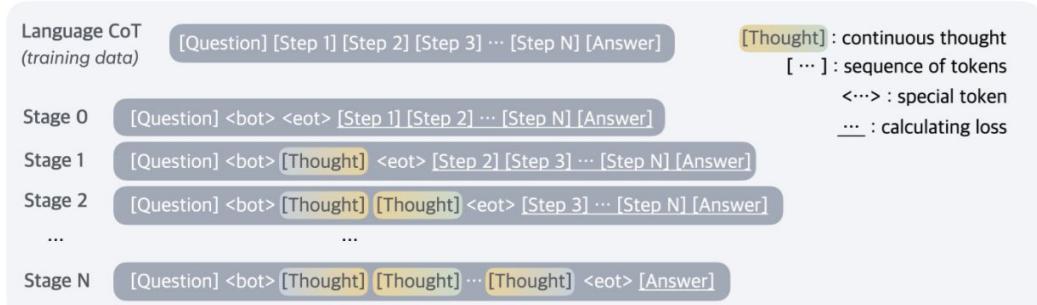
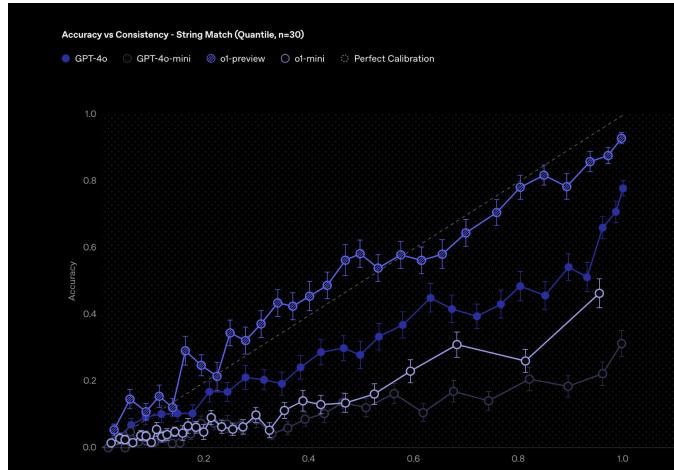
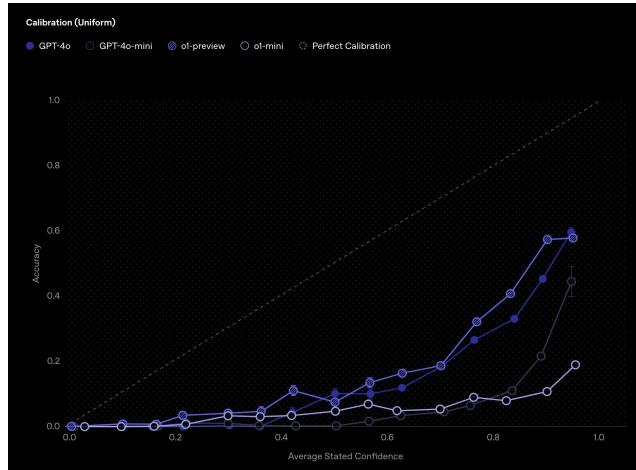


Figure 2 Training procedure of Chain of Continuous Thought (COCONUT). Given training data with language reasoning steps, at each training stage we integrate c additional continuous thoughts ($c = 1$ in this example), and remove one language reasoning step. The cross-entropy loss is then used on the remaining tokens after continuous thoughts.

Calibration



RL'ed O1 models are more calibrated than Next Token Predicting only models.

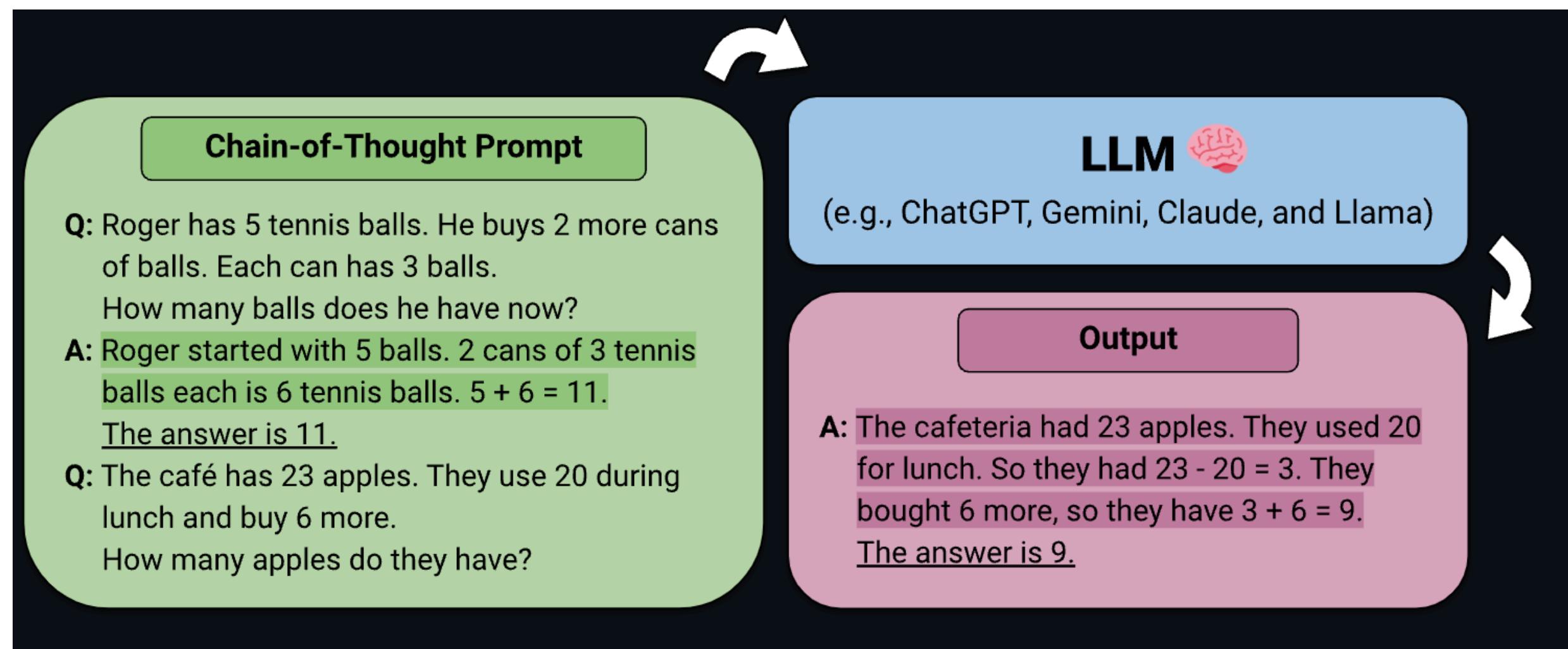
Why do we need reasoning model?

Direct End-to-End Inference vs Reasoning

- Empirical: scaling generation token lead to better performances
- Some hypothesis:
 - Computation power and expressiveness
 - Dynamically modify predictive distribution (time time algorithm)
 - Representation not entangled

CoT

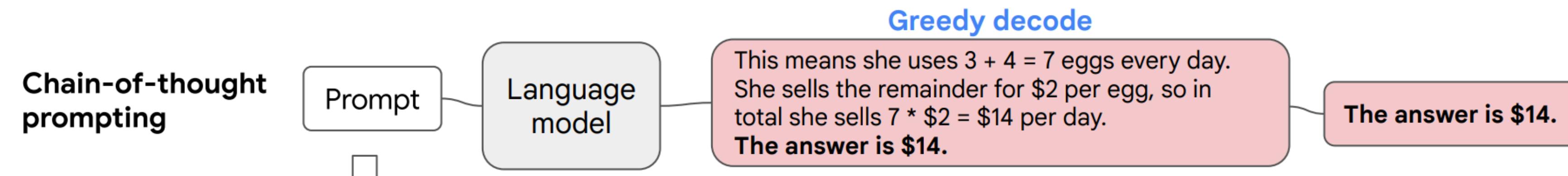
- A specific reasoning algorithms (i.e decompose problems into sub steps)
- Few shot CoT: leverages ICL to learn CoT from data
- Zero shot CoT: directly tells the model to break down problems into sub steps



What about other algorithms?

- Self Consistency: majority voting
- Auto Cot: sampling specific questions (cosine similarity) as context for few shot CoT
- Tree of thought: CoT with a tree search
- Graph of thought: graph based CoT, allows to revisit previous nodes
- Least to Most: divide and conquer
- Many different ways, all ad hoc

Best of N style algorithms



Weighted Best-of-N: Aggregate scores across all identical responses and select the answer with the *highest total reward*. This approach prioritises high-quality answers by boosting their scores through repeated occurrences. Mathematically, the weighting across answers a_i is performed as follows:

$$a_{\text{weighted}} = \arg \max_a \sum_{i=1}^N \mathbb{I}(a_i = a) \cdot \text{RM}(p, s_i),$$

Figure 1: The self-consistency method contains three steps: (1) prompt a language model using chain-of-thought (CoT) prompting; (2) replace the “greedy decode” in CoT prompting by sampling from the language model’s decoder to generate a diverse set of reasoning paths; and (3) marginalize out the reasoning paths and aggregate by choosing the most consistent answer in the final answer set.

More Fancy Best of N

Inference-Aware Fine-Tuning. To address the gap between how LLMs are trained and how they are used at inference time, we develop inference-aware fine-tuning. During inference, the learned policy π is often not directly used; rather some *inference strategy* $I : \Pi \times \mathcal{X} \mapsto \Delta_{\mathcal{Y}}$ is applied to it. For example, I can be the BoN strategy, which samples multiple candidate responses, and selects the best using the score function of some verifier; or I might be a search mechanism (Lightman et al., 2023) or self-correction (Kumar et al., 2024). To account for this inference strategy I , we alter the objective SFT and RL objectives to be “*aware*” of the inference strategy:

$$\max_{\pi \in \Pi} \mathbb{E}_{x \sim P, y \sim \pi^*(y|x)} [\log I(\pi, x)(y)], \text{ and} \quad (\text{Inference-Aware SFT})$$

$$\max_{\pi \in \Pi} J(\pi) := \mathbb{E}_{x \sim P, y \sim I(\pi, x)} [R(x, y)], \quad (\text{Inference-Aware RL})$$

BoN-Aware Problem Formulation. We begin by defining the BoN strategy. This inference strategy samples N responses from a model with some temperature T , and then selects the best one, based on some verifier score. Formally, the BoN inference policy can be written as:

$$I(\pi, x)(y) = \pi_{\text{bon}}(y|x; \pi, r, N, T) := \arg \max_{y' \in \{y_1, \dots, y_N\}} r(x, y'), \text{ s.t. } y_i \stackrel{T}{\sim} \pi(\cdot|x), x \in \mathcal{X}, \quad (1)$$

where $\stackrel{T}{\sim}$ is a sample with temperature T , and $r : \mathcal{X} \times \mathcal{Y} \mapsto \mathbb{R}$ is a verifier score¹. In what follows, when r, N, T are clear from context, we write $\pi_{\text{bon}}(y|x; \pi)$. We see that the above strategy defines a class of BoN policies that is different from the learned policy π , demonstrating the gap between training and inference.

More Fancy Best of N With a PRM

Which number is larger, 9.8 or 9.11?

Step 1: Identify the numbers to compare,
9.8 and 9.11

0.75

0.75

0.75

0.75

Step 2: Compare the whole number
parts; both are 9, so they are equal

0.91

0.91

0.91

0.91

Step 3: Compare the decimal parts; 0.8
is greater than 0.11

0.83

0.83

0.83

0.83

Step 4: Conclude that 9.8 is larger than 9.11

0.99

0.99

0.99

0.99

step-level
scores

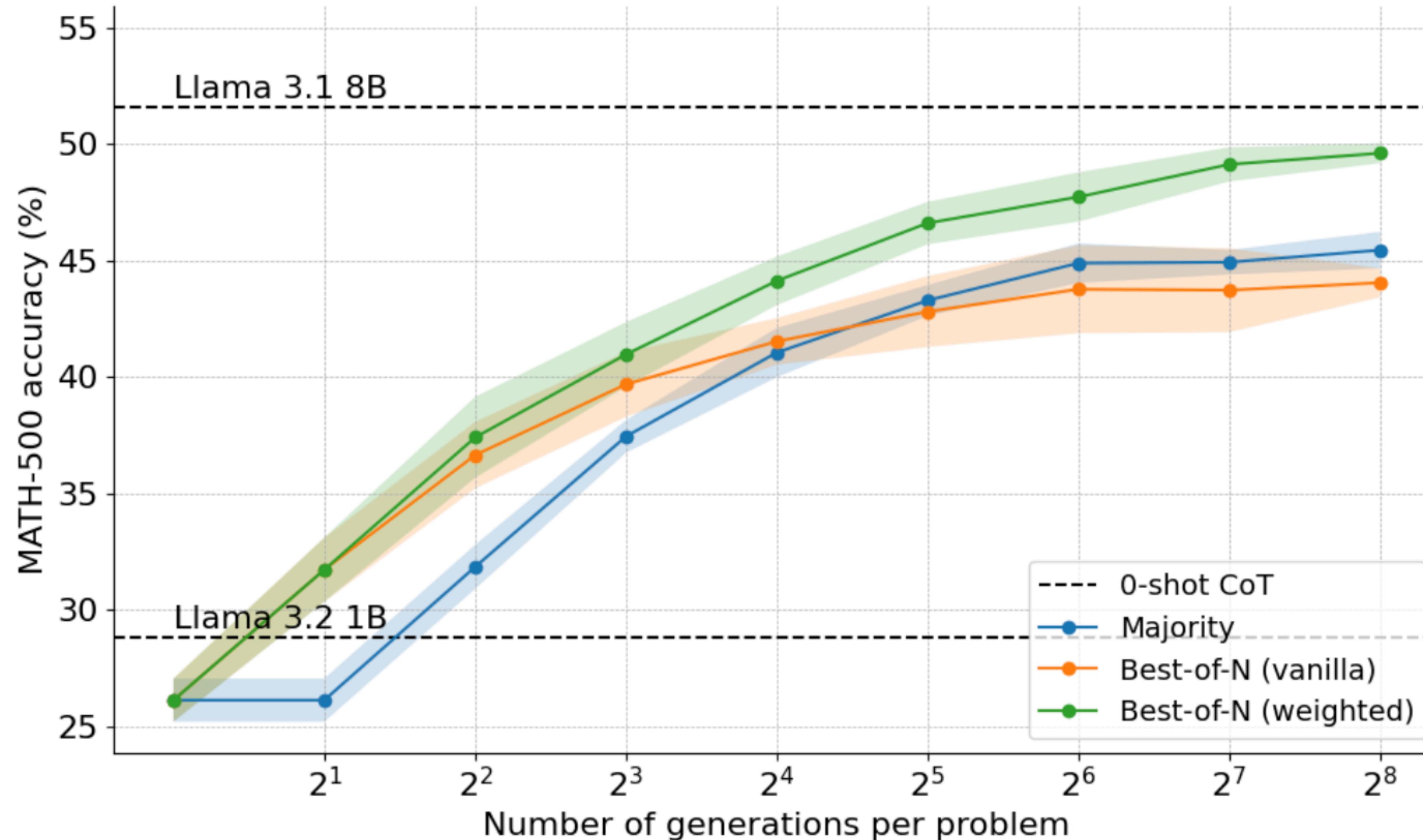


min

prod

last

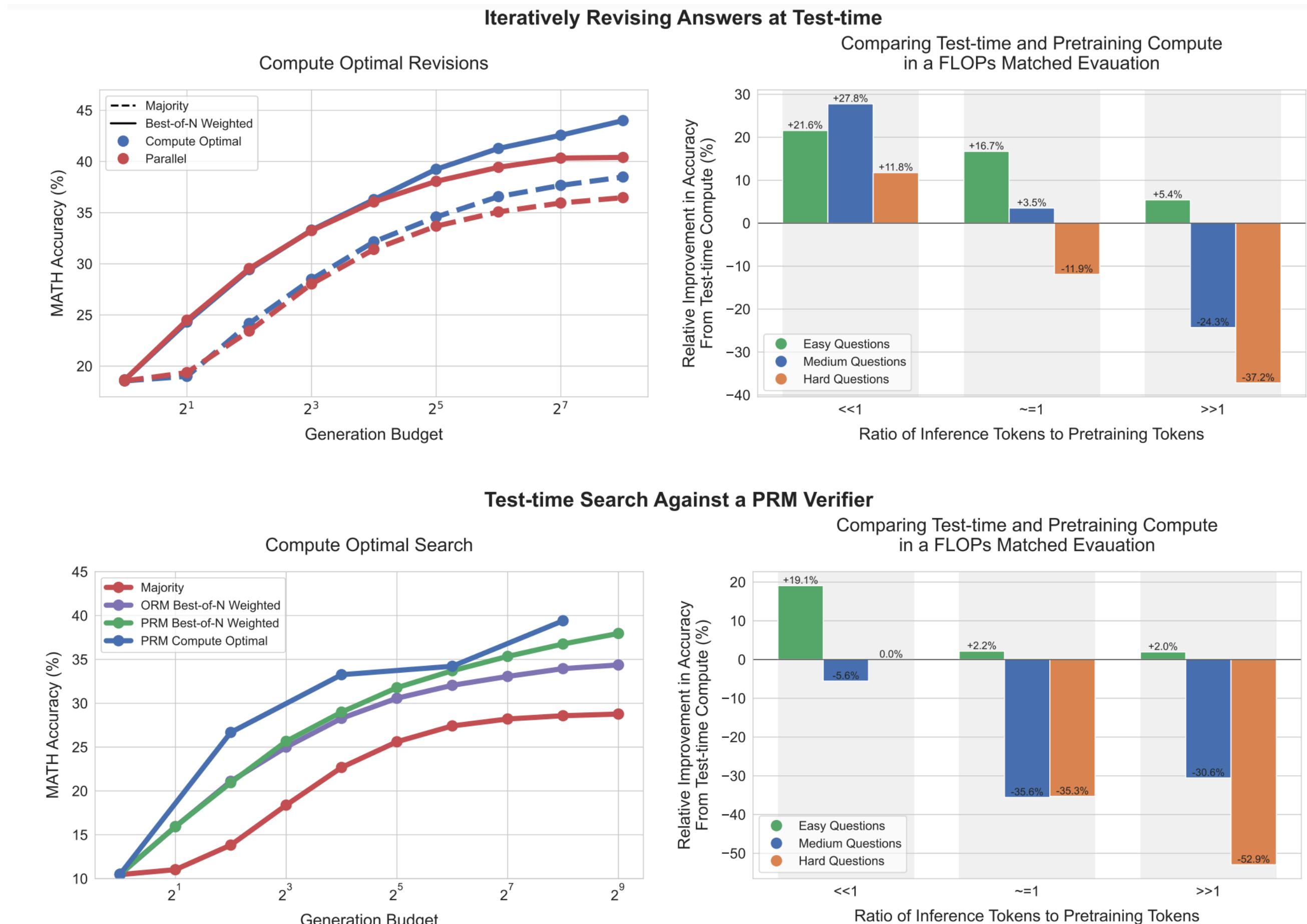
Best of N results



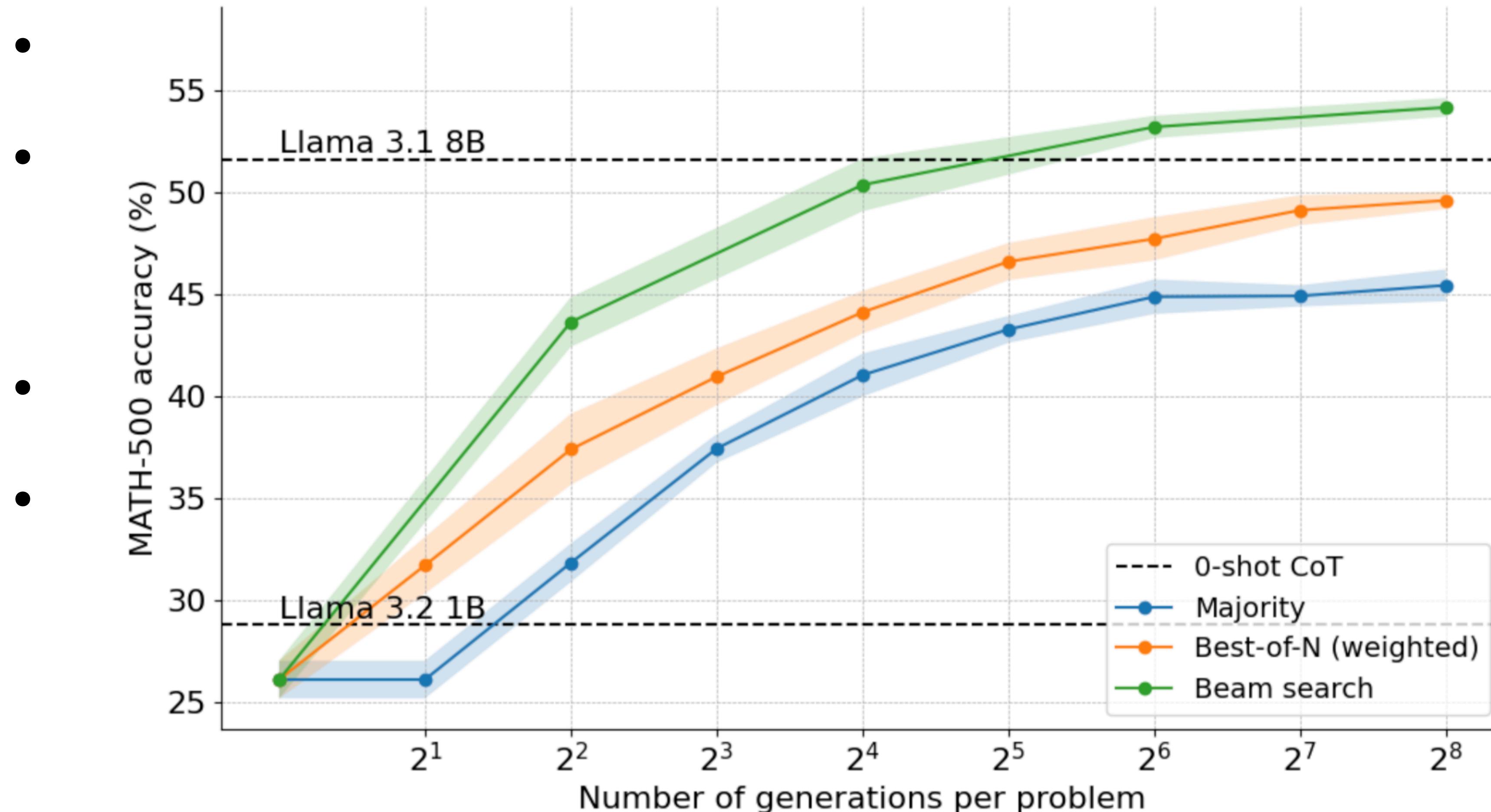
Are there any more systematic way?

Search

- Key idea: search as a general problem solving tool
- Problem: need heuristics / verifier to evaluate !
- PRM: step level reward model ($\backslash n \backslash n$ as a step)
- Showing pretty good performances already
- Issue: step level verifiers are not reliable



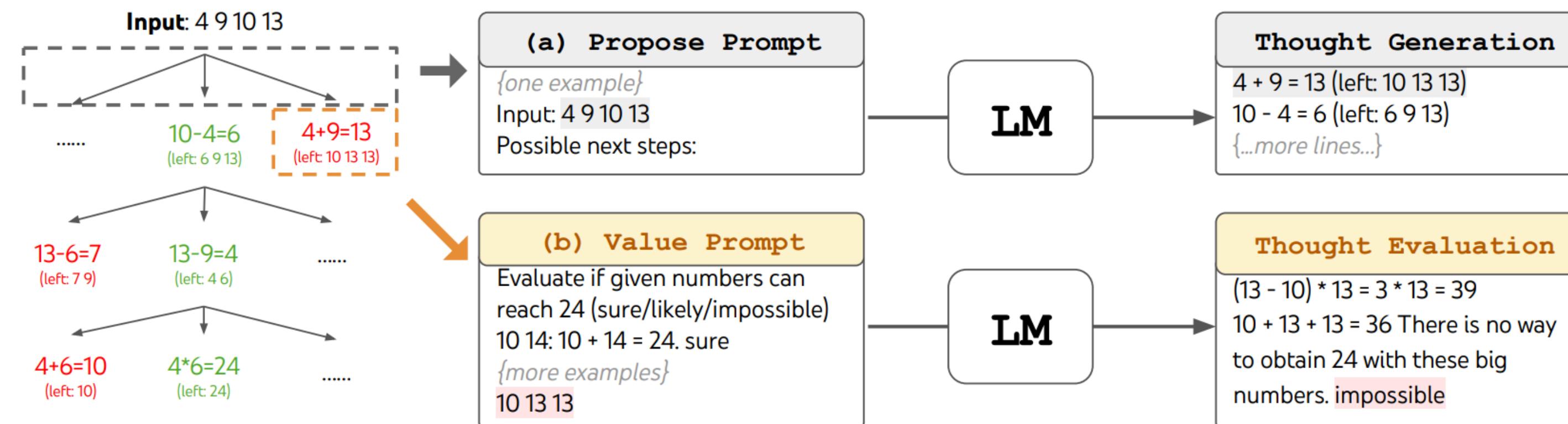
Beam Search



ed step-wise
reward from the
ext step,
gain. Then

Tree Search: Tree of thoughts

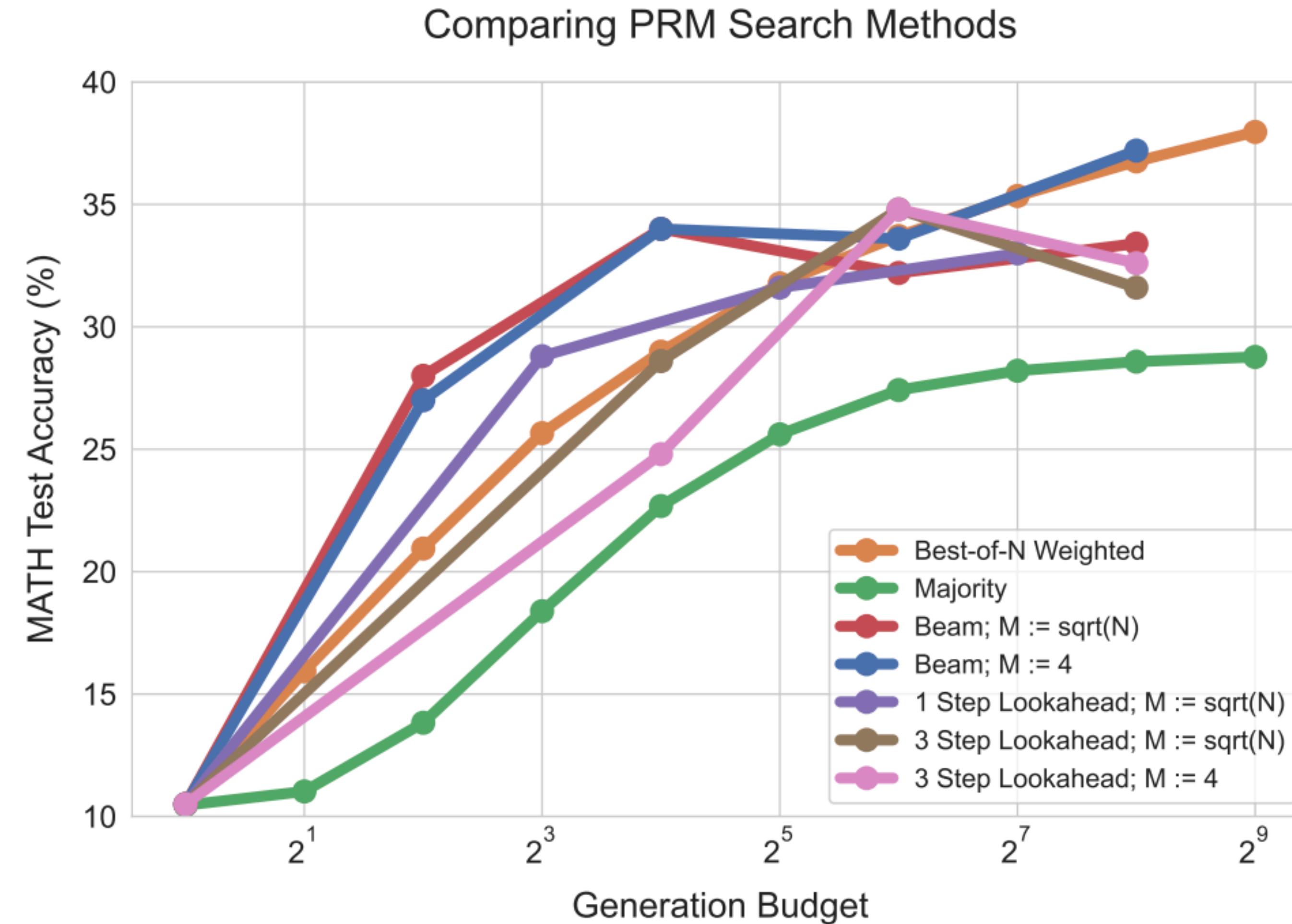
- 1. How to decompose the intermediate process into thought steps (token / paragraph / \n\n)
- 2. How to generate potential thoughts from each state (potential research question)
- 3. How to heuristically evaluate states (verifier / llm as judge)
- 4. What search algorithm to use (DFS / BFS)



Tree Search: MCTS

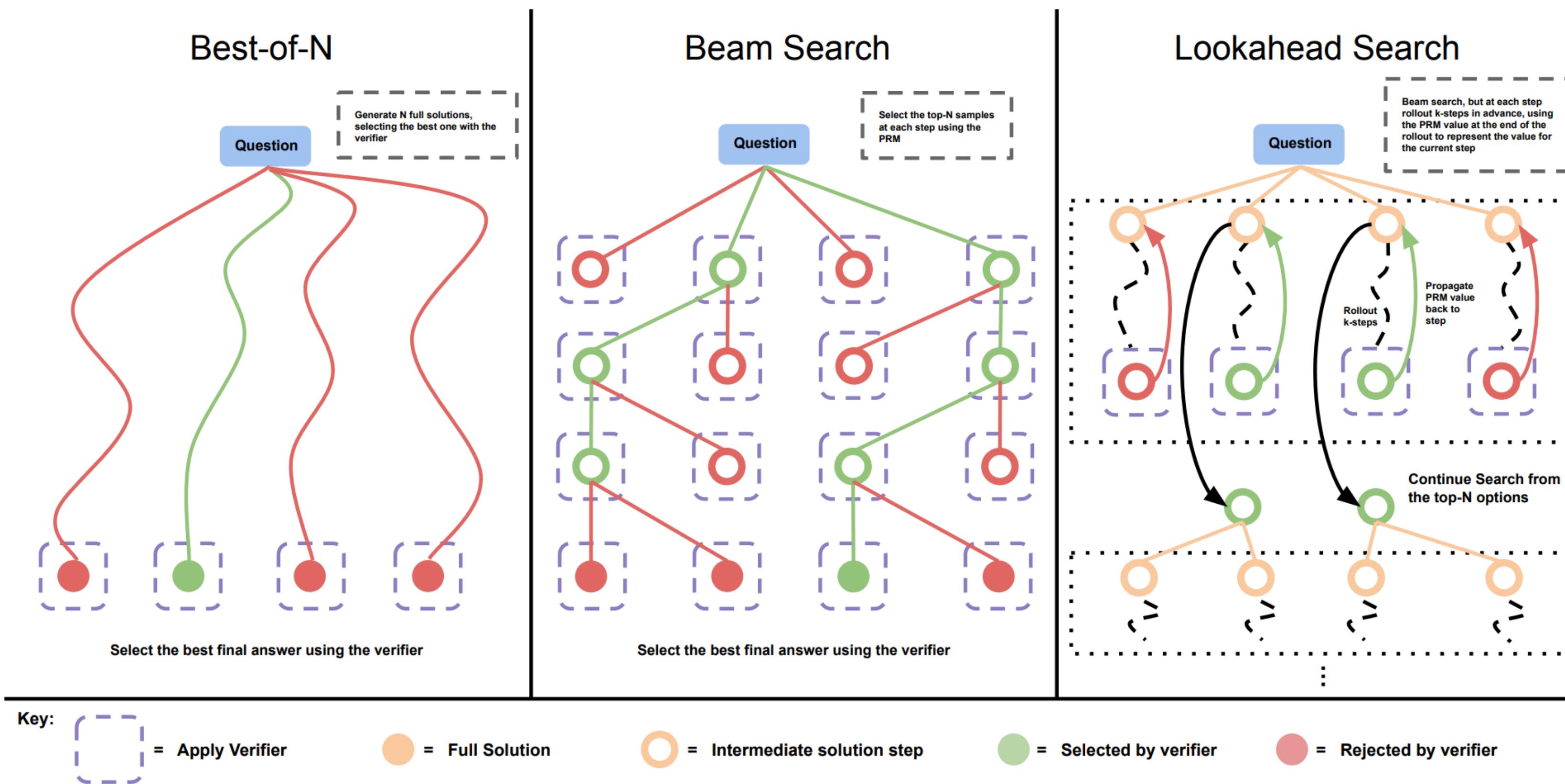
- Instead select.
- One co

the value to

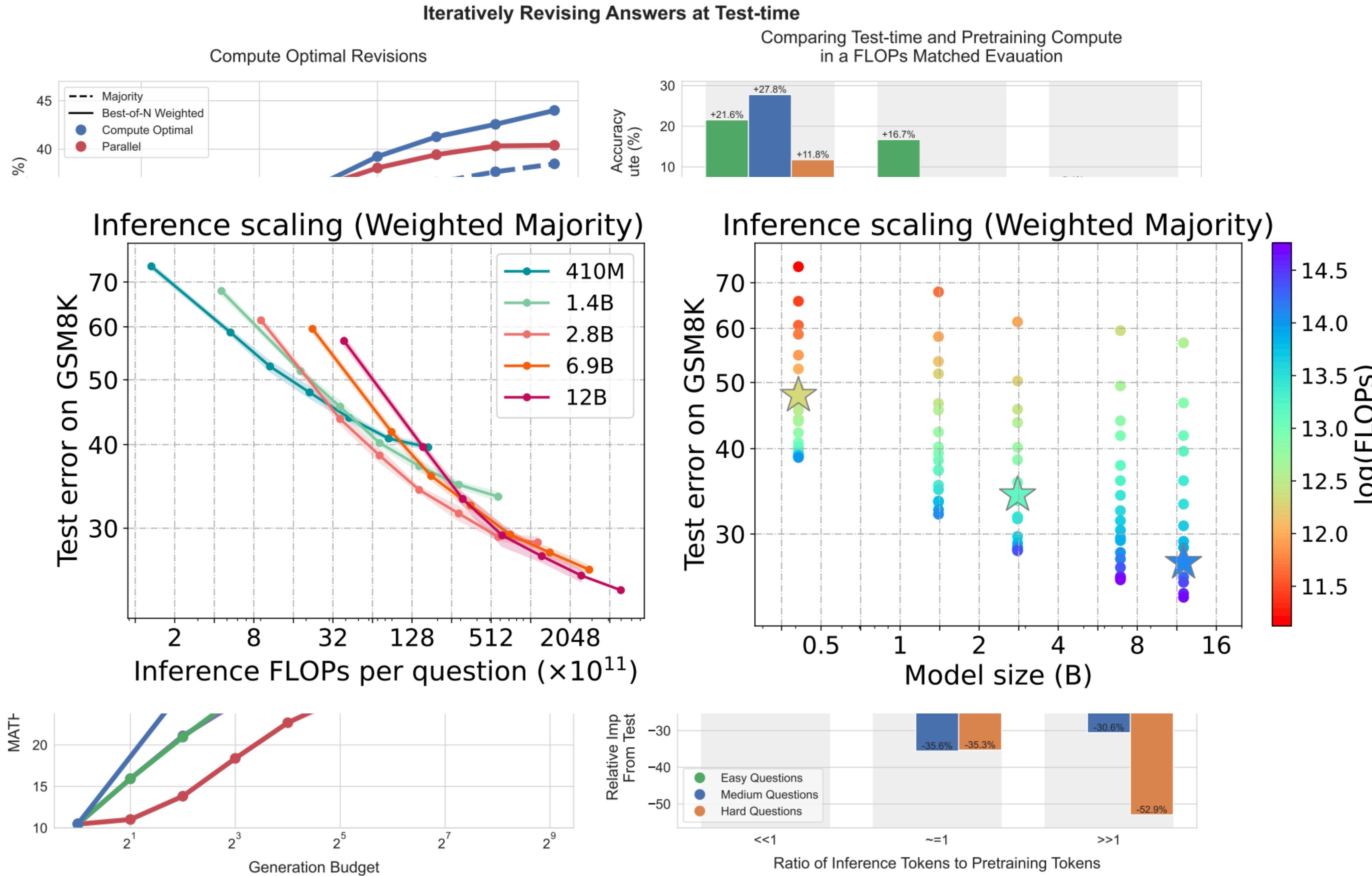


Overview

- A Unified Perspective on Test-Time Computation: Proposer and Verifier
- Can you think of other algorithms?



Scaling Laws



[“Wait”] What about the verifiers

- We mainly discussed about search algorithms, with the assumption that we have a good verifier.
- What are good verification algorithms?
- Math: string matching, lean prover
- Code: compiler execution
- Open ended domains: Finance, healthcare, legal?

[“Wait”] What about the verifiers

- We mainly discussed about search algorithms, with the assumption that we have a good verifier.
- What are good verification algorithms?
- Math: string matching, lean prover
- Code: compiler execution
- Open ended domains: Finance, healthcare, legal?

LLM as judge

- Pros: Flexible
- Cons: Does not work well :)
- LLM as grader: given a ground truth label, and a model generated answer, ask llm to check if the two are equivalent.
- Widely used in RL fine tuning tasks

PRM

Let's verify step by step

Let

$$x^8 + 3x^4 - 4 = p_1(x)p_2(x) \cdots p_k(x),$$

Let

$$x^8 + 3x^4 - 4 = p_1(x)p_2(x) \cdots p_k(x),$$

The denominator of a fraction is 7 less than 3 times the numerator. If the fraction is equivalent to $\frac{2}{5}$, what is the numerator of the fraction? (Answer: 14)

   Let's call the numerator x.

   So the denominator is $3x-7$.

   We know that $x/(3x-7) = 2/5$.

   So $5x = 2(3x-7)$.

   $5x = 6x - 14$.

   So $x = 7$.

$$\text{Multiplying, I get } p_1(1) + p_2(1) + \cdots + p_k(1) = 0.$$

Multiplying, I get $p_1(1) + p_2(1) + \cdots + p_k(1) = 0$.

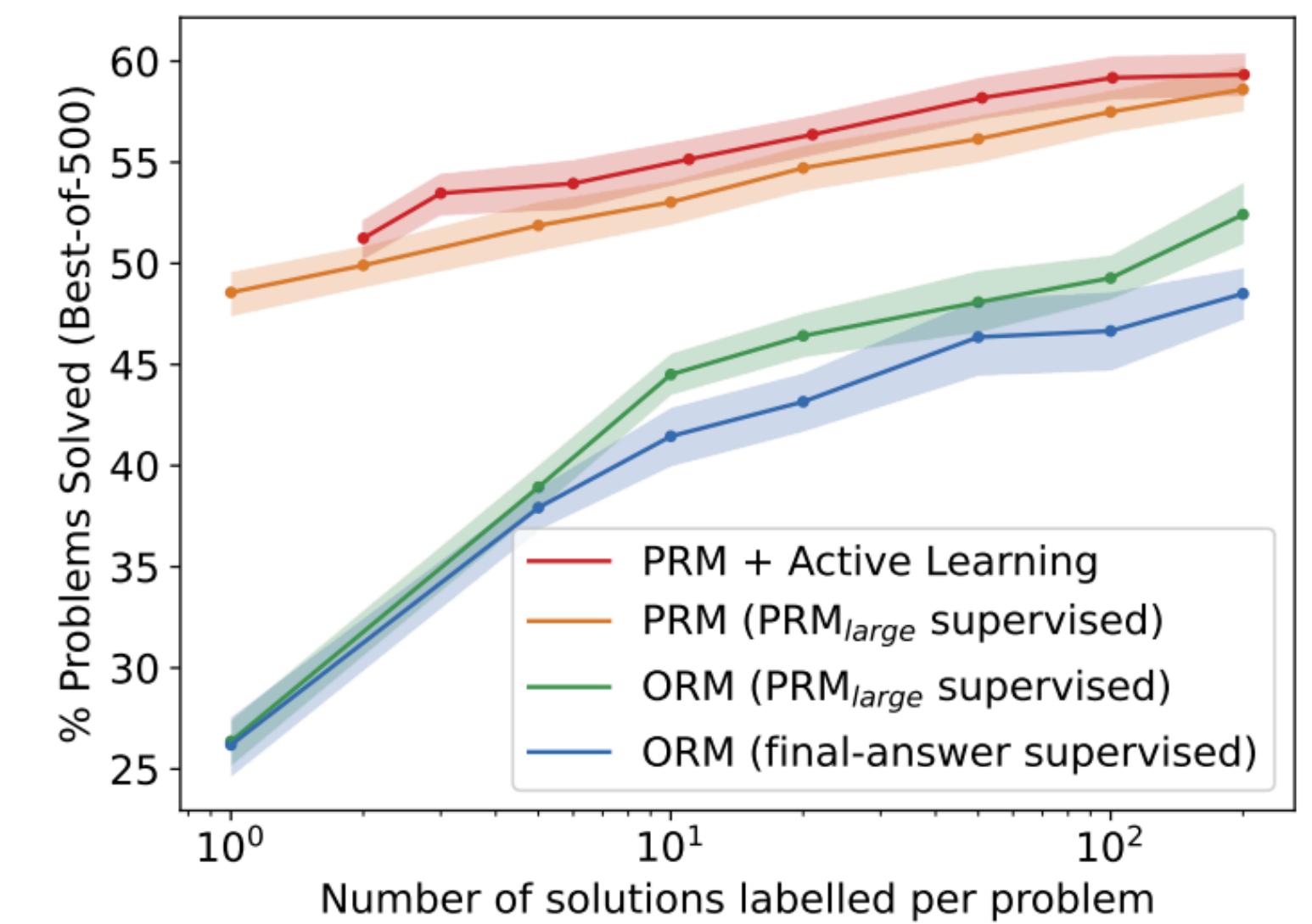
Answer: 0

PRM

Active Learning

4.2 Active Learning

Finally, we investigate the impact of active learning. We train a small-scale reward model, $\text{PRM}_{\text{selector}}$, on a single sample from each problem, and we use this model to score 1000 samples per problem. To train each of our larger reward models, we select N samples per problem such that 80% are the most convincing (according to $\text{PRM}_{\text{selector}}$) wrong-answer samples, and 20% are the most convincing samples that remain (right- or wrong-answer). We score the selected samples with $\text{PRM}_{\text{large}}$ and train on those scores. This process ensures that all samples are relatively convincing under $\text{PRM}_{\text{selector}}$, that a large fraction are known to contain at least one mistake, and that our overall dataset is not too heavily biased toward wrong-answer solutions. Performance of this data labelling scheme is shown in Figure 4a. By comparing the slopes of the line of best fit with and without active learning, we estimate that this form of active learning is approximately 2.6x more data efficient than uniform data labelling. We note that the model trained on the largest active learning dataset (200 samples per problem) appears to slightly underperform the expected trend line. Our best explanation for this observation is that 200 samples represents a significant fraction of the overall selection pool (1000 samples) and that this



(a) Four series of reward models trained using different data collection strategies, compared across training sets of varying sizes.

Scalable Solution?

MATH-SHEPHERD

Problem: Let $p(x)$ be a monic polynomial of degree 4. Three of the roots of $p(x)$ are 1, 2, and 3. Find $p(0) + p(4)$.

Golden Answer: 24

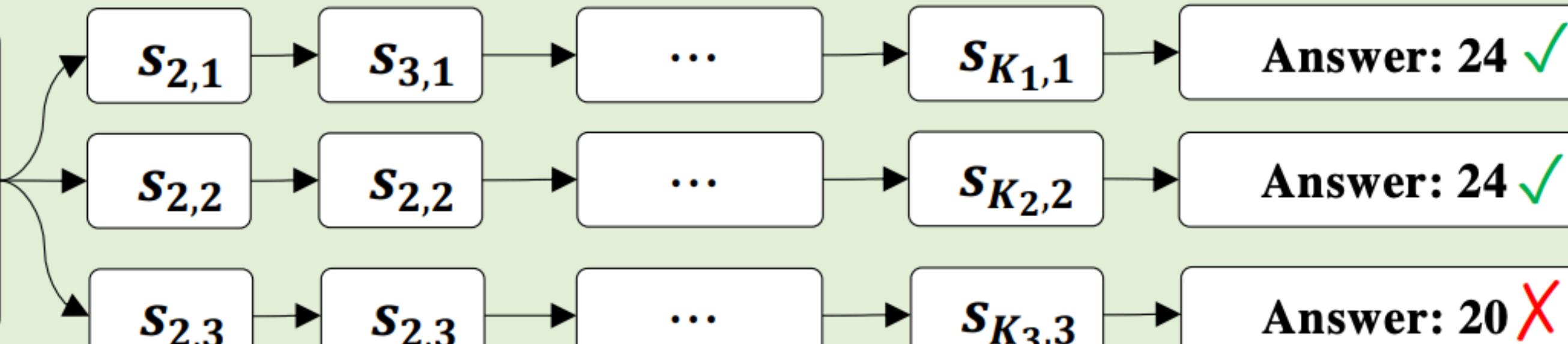
Solution: $S = s_1, s_2, s_3, \dots, s_K$

Answer: 20 ✗

(a) Outcome Annotation: $y_S = 0$

Problem:

s_1 : Since three of the roots of $p(x)$ are 1, 2, and 3, we can write : $p(x) = (x - 1)(x - 2)(x - 3)(x - r)$.



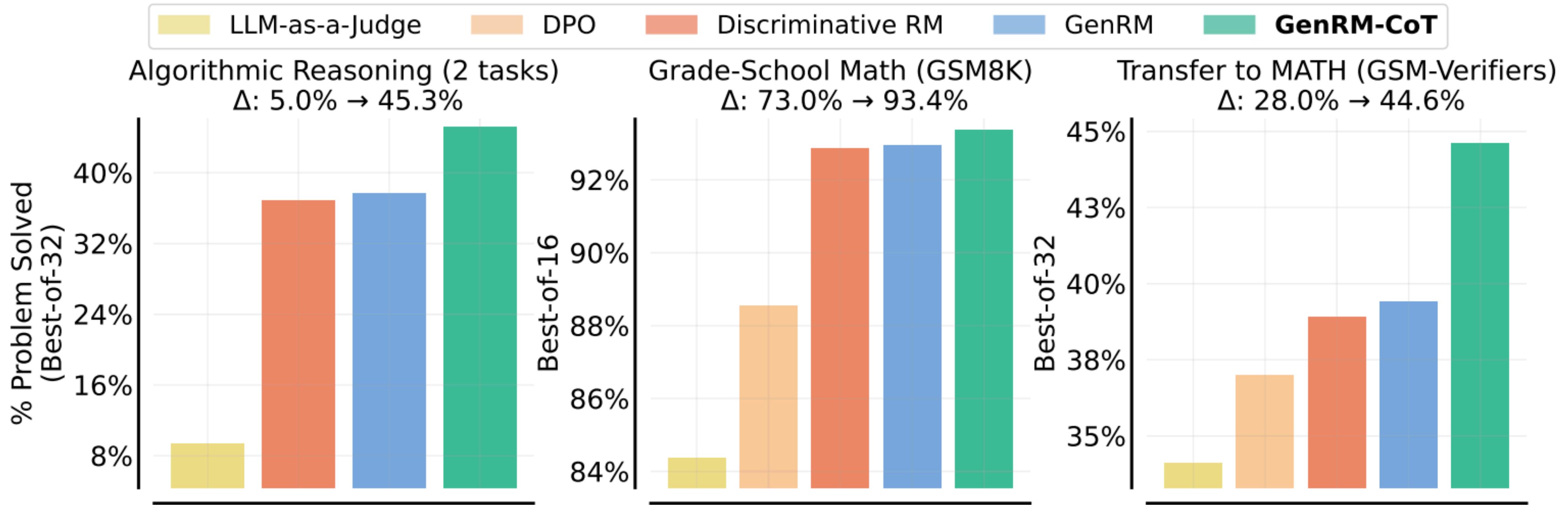
(b): Process Annotation: $y_{s_1}^{SE} = \frac{2}{3}$; $y_{s_1}^{HE} = 1$

s_i : the i -th step of the solution S .

$s_{i,j}$: the i -th step of the j -th finalized solution.

- Can you think of other ways?

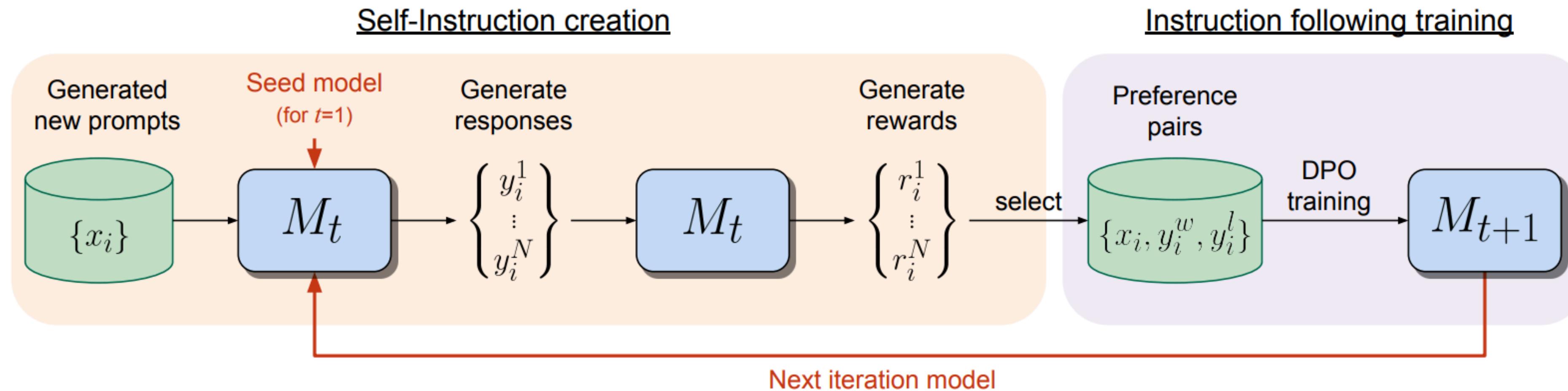
Generative Verifier



Self Improvement

Self rewarding LLM

- Most self improvement paper are this flavor



Self Play

SPIN & Rstar

4.1 Self-Play Fine-Tuning (SPIN)

Let us consider a two-player game, where the main player's objective is to distinguish the responses generated by the LLM and those generated by the human. Meanwhile, the opponent's role is to generate responses that are indistinguishable from the human's responses. The core of our method is the self-play mechanism, where both the main player and the opponent are the same LLM, but from different iterations. More specifically, the opponent is the old LLM from the previous iteration, and the main player is the new LLM to be learned in the current iteration.

In iteration $t+1$, the opponent is the LLM from the previous iteration, denoted by p_{θ_t} , which generates responses \mathbf{y}' for those prompts \mathbf{x} in the SFT dataset according to $p_{\theta_t}(\cdot|\mathbf{x})$. Our method, therefore, consists of the following two steps at iteration $t+1$: (1) training the main player, and (2) updating the opponent player.

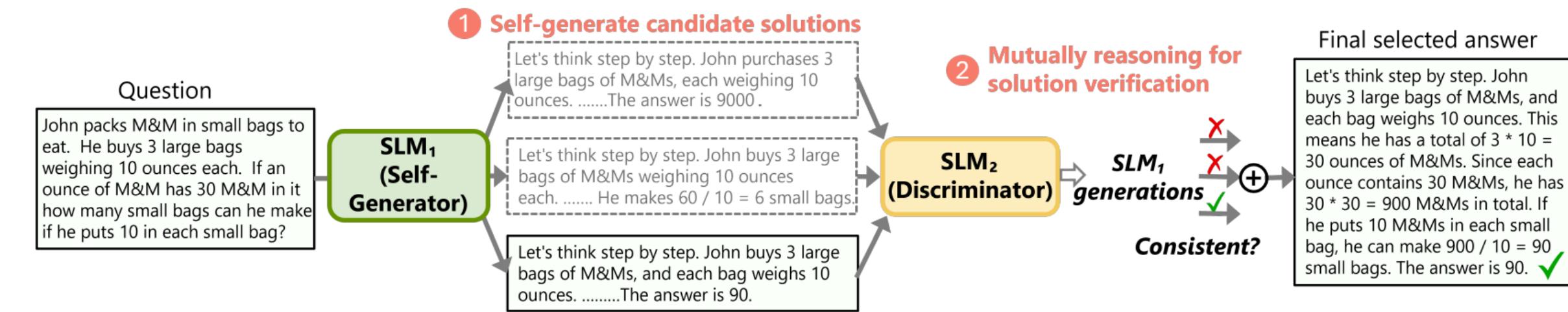


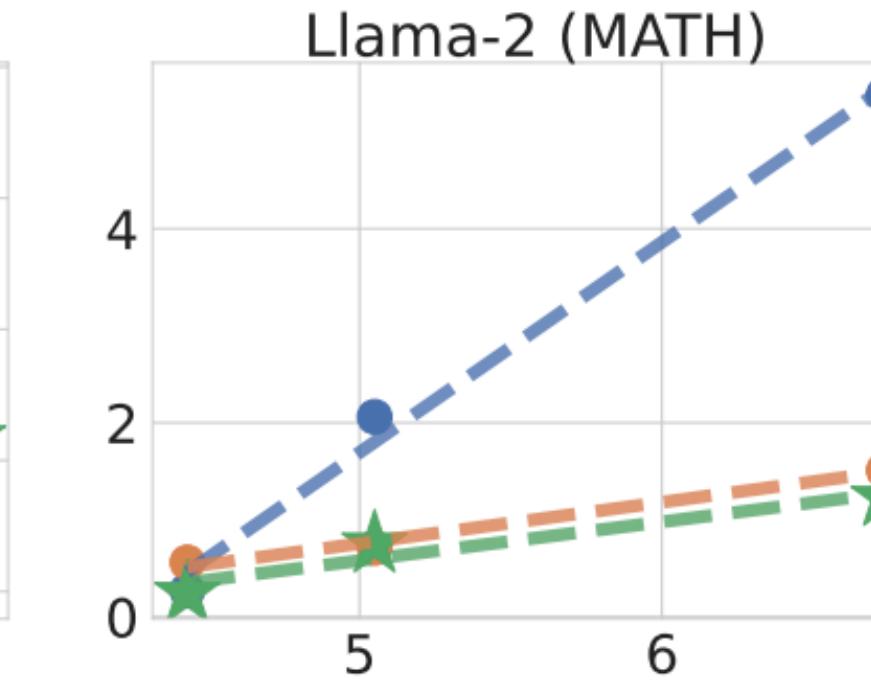
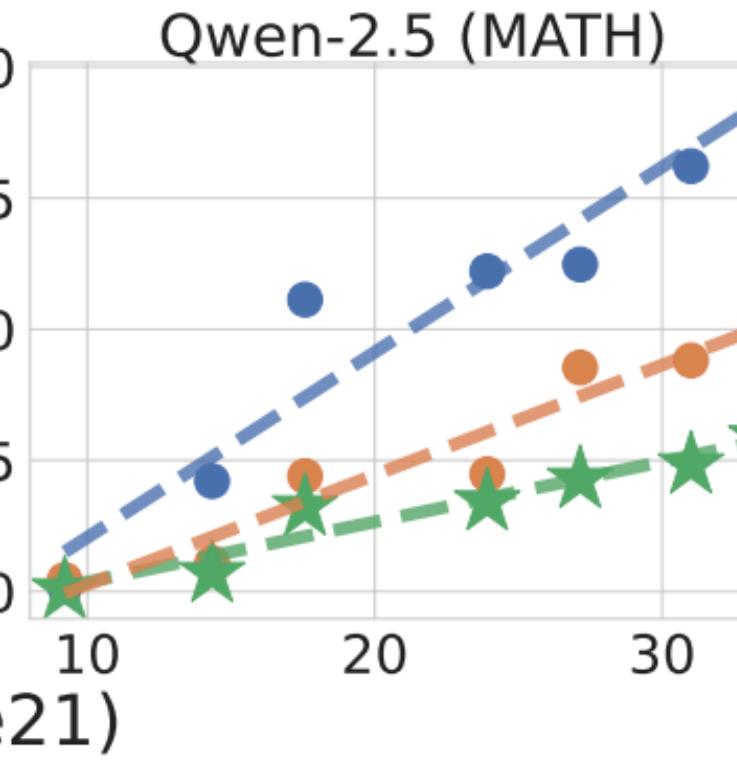
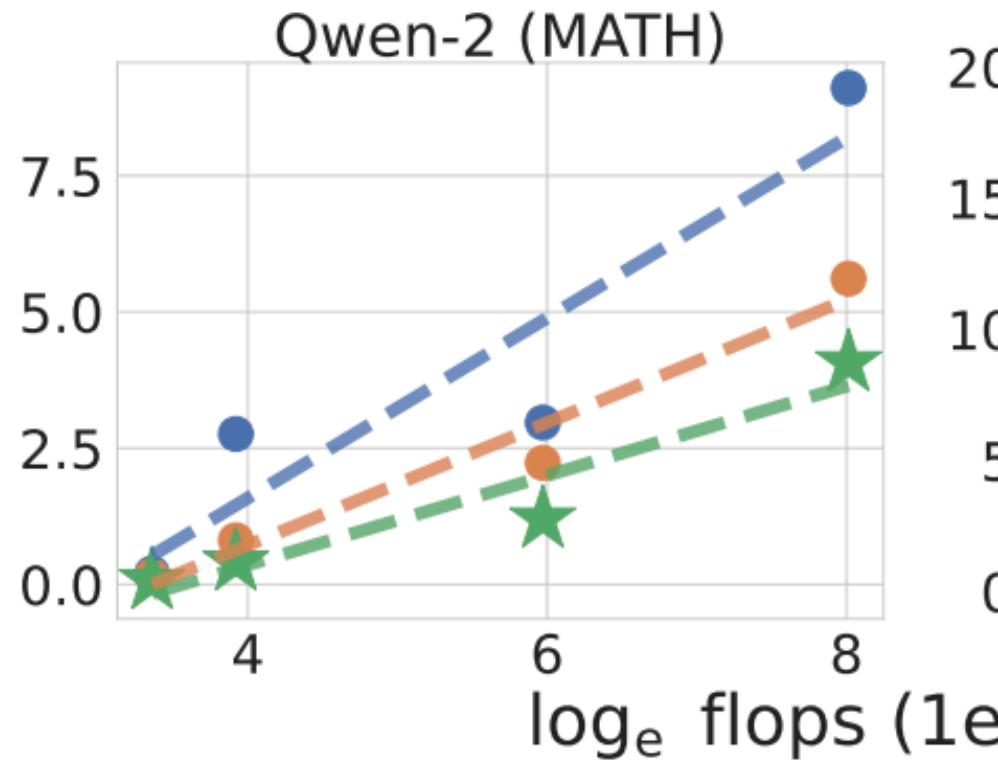
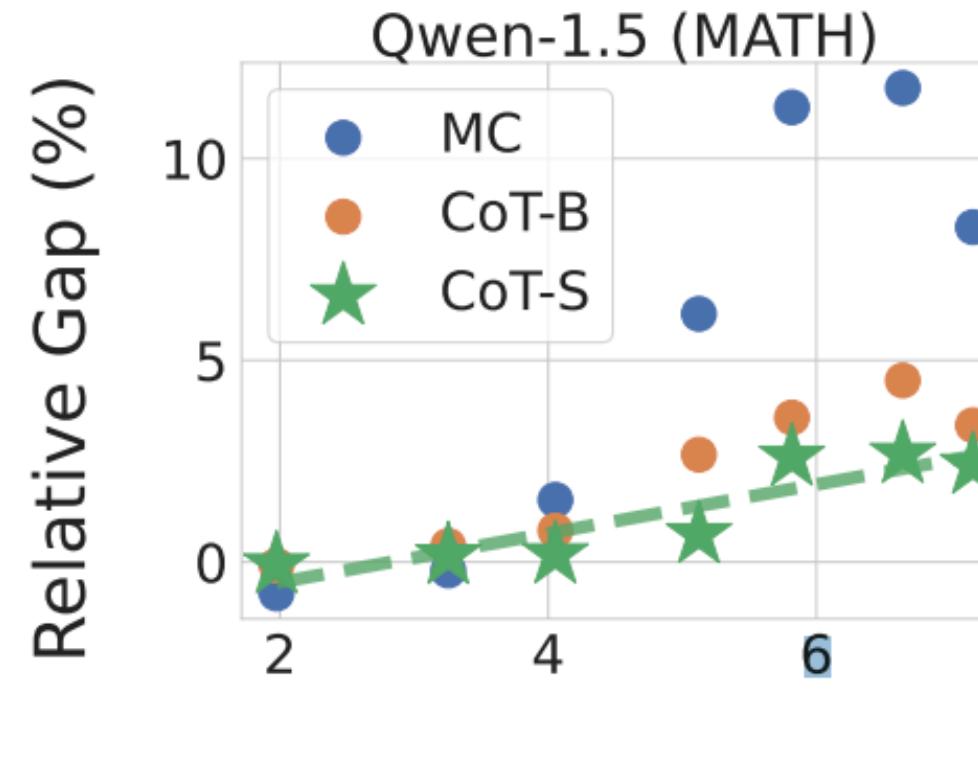
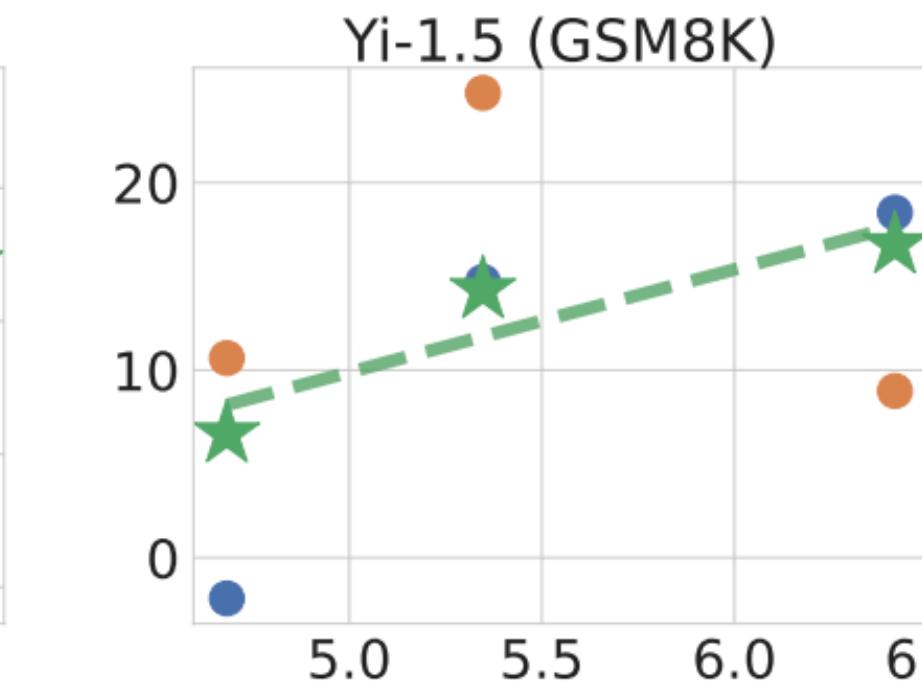
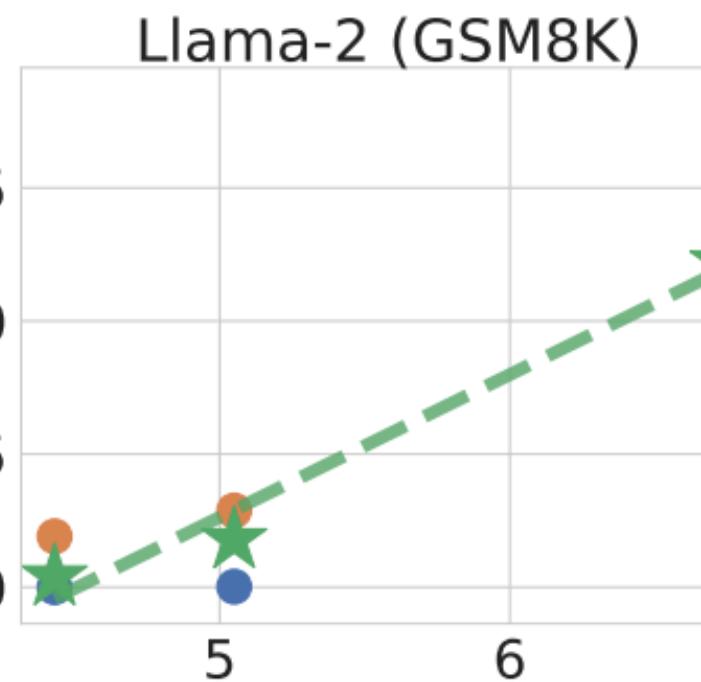
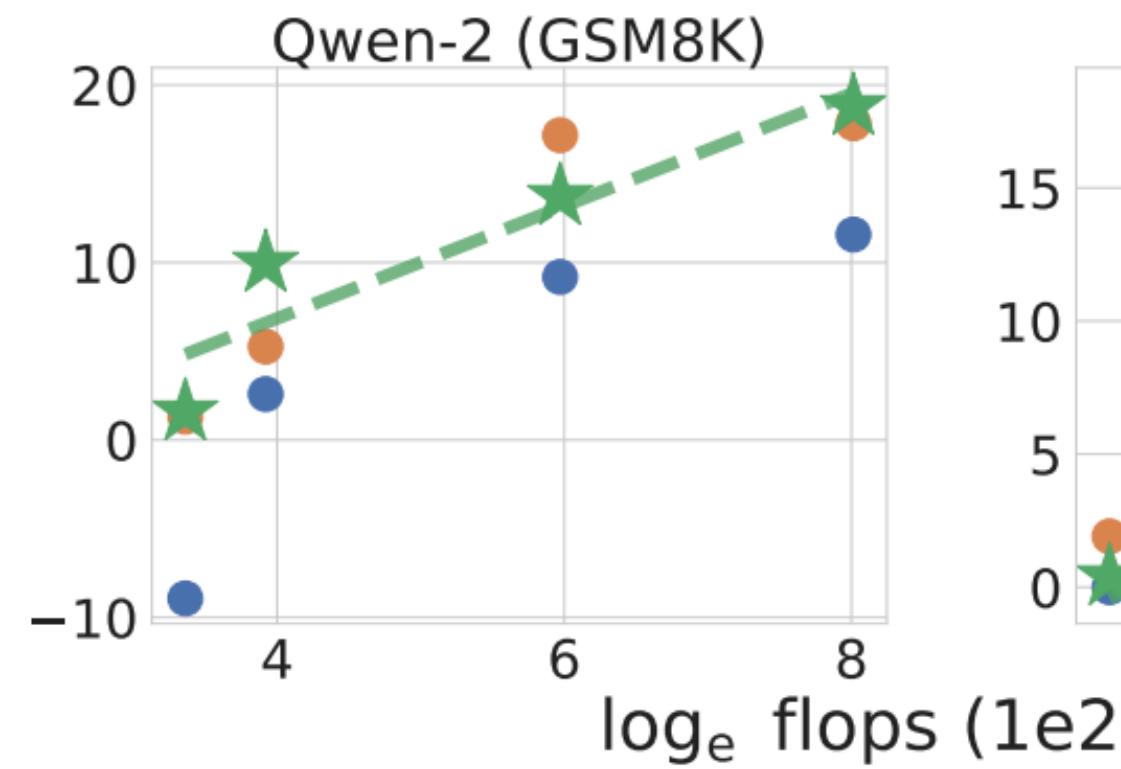
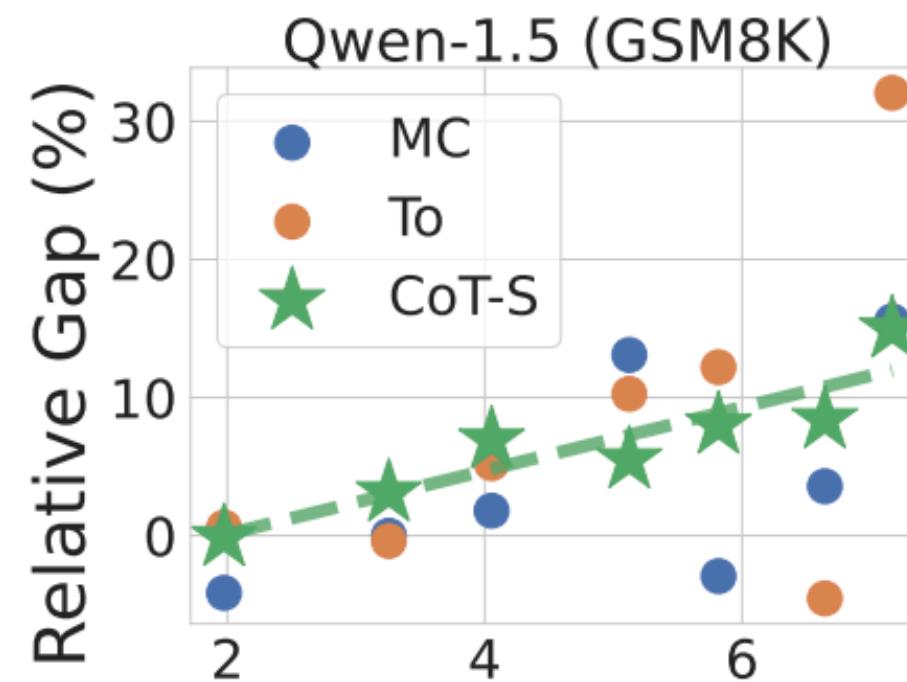
Figure 2: Our self-play mutual reasoning is a generation-discrimination process: (1) a self-generator augments the target SLM to generate candidate reasoning trajectories using MCTS; (2) the discriminator uses another SLM to provide unsupervised feedback on each trajectory based on partial hints; (3) based on this feedback, the target SLM decides a final reasoning trajectory as the solution.

- A bit contrived
- Can we design sth like GAN or AlphaGo?

Why self verification works?

P \neq NP ?

- Generation Verification Gap



Can we do training?

- Training with SFT: fine tuning with CoT data. This is why Chatgpt 3.5 starts to produce longer answers comparing to 3.0. For all models, they include CoT data in the SFT phase
- Issue: memorizing everything. Generalization only happens when scaling both parameters and datasize.
- RL generalizes

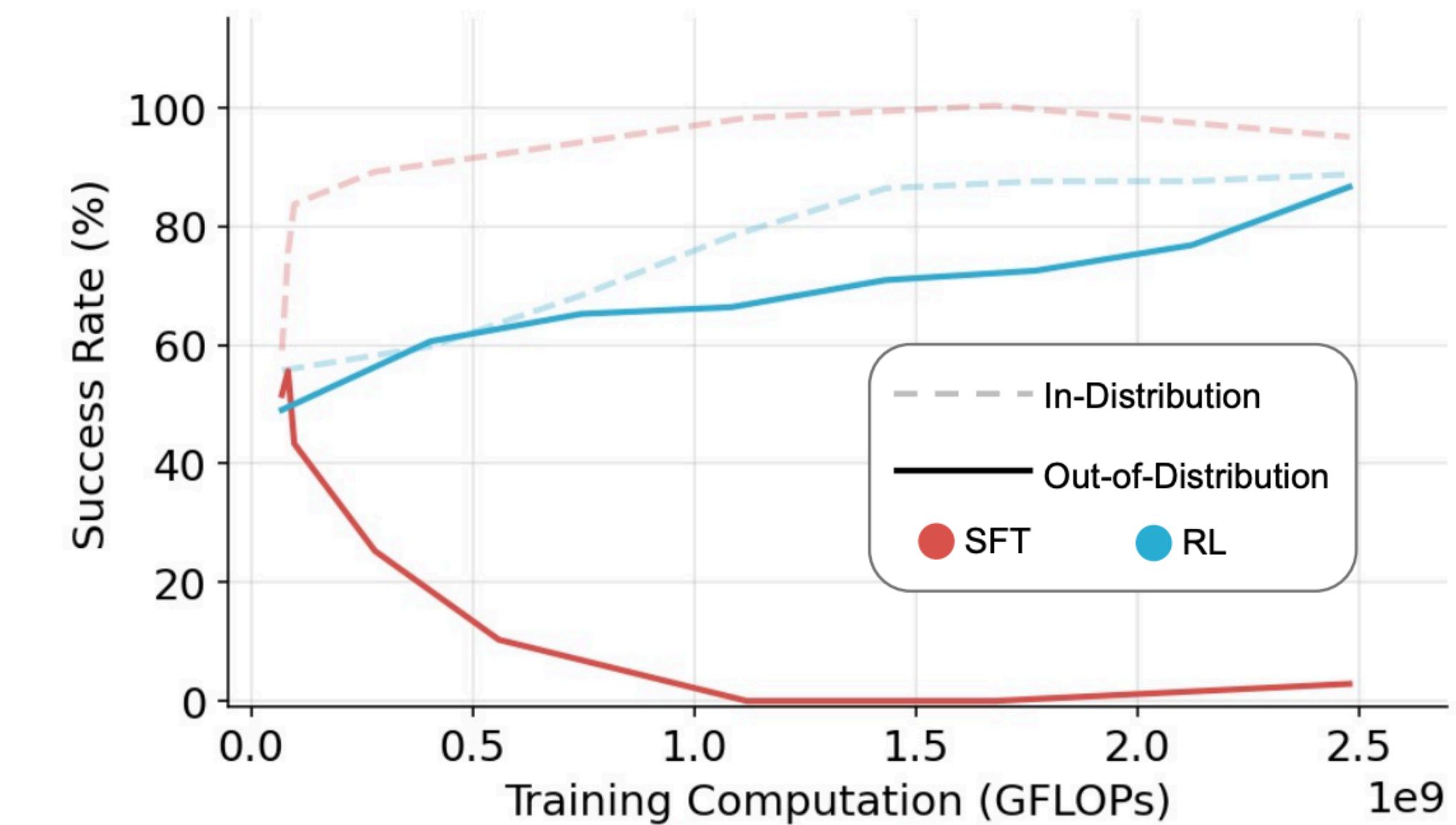


Figure 1: A comparative study of RL and SFT on the visual navigation environment V-IRL (Yang et al., 2024a) for OOD generalization. OOD curves represent performance on the same task, using *a different textual action space*. See detailed descriptions of the task in Section 5.1.

Training: RL with Reward Model

- Direct RL with PRM/ORM: reward hacking is a serious issue
- With DPO: hard to handle credit assignment problem. For example, for two solutions, each has some correct steps and incorrect steps, its hard to disentangle them.

Training: RL Finetuning

- Instead of having a parametrized reward model, use a rule based reward model.
- For R1 or Openai RLF api, user came in with a dataset (X, Y) . For each x , model generates an answer a (extract the final part, can imagine there are immediate steps), if $a = y$, then give reward of 1, otherwise 0. This can be more sophisticated, such as giving partial reward. But all based on rules.
- RL part: the set of initial state is fixed (X) . Iteratively sample a x from X , and do RL on top of that.

Reward Calculation

Rule based reward: String Matching

```
import re

def format_reward_func(completions, **kwargs):
    """Reward function that checks if the completion has a specific format."""
    pattern = r"^<think>.*?</think><answer>.*?</answer>$"
    completion_contents = [completion[0]["content"] for completion in completions]
    matches = [re.match(pattern, content) for content in completion_contents]
    return [1.0 if match else 0.0 for match in matches]
```

A conversation between User and Assistant. The user asks a question, and the Assistant solves it. The assistant first thinks about the reasoning process in the mind and then provides the user with the answer. The reasoning process and answer are enclosed within `<think>` `</think>` and `<answer>` `</answer>` tags, respectively, i.e., `<think>` reasoning process here `</think>` `<answer>` answer here `</answer>`. User: **prompt**. Assistant:

Table 1 | Template for DeepSeek-R1-Zero. **prompt** will be replaced with the specific reasoning question during training.

RL Finetuning: o1 and r1

- Start with a SFT stage, distilling the “thinking” algorithm into the model. But there is no specific reasoning algorithm, they model discovers the algorithm by itself.

2.2.2. *Reward Modeling*

The reward is the source of the training signal, which decides the optimization direction of RL. To train DeepSeek-R1-Zero, we adopt a rule-based reward system that mainly consists of two types of rewards:

- **Accuracy rewards:** The accuracy reward model evaluates whether the response is correct. For example, in the case of math problems with deterministic results, the model is required to provide the final answer in a specified format (e.g., within a box), enabling reliable rule-based verification of correctness. Similarly, for LeetCode problems, a compiler can be used to generate feedback based on predefined test cases.
- **Format rewards:** In addition to the accuracy reward model, we employ a format reward model that enforces the model to put its thinking process between ‘<think>’ and ‘</think>’ tags.

RL Finetuning: o1 and r1

- More synthetic data after RL converges: SFT, alignment (with reward model for human preferences)

Reasoning data We curate reasoning prompts and generate reasoning trajectories by performing rejection sampling from the checkpoint from the above RL training. In the previous stage, we only included data that could be evaluated using rule-based rewards. However, in this stage, we expand the dataset by incorporating additional data, some of which use a generative reward model by feeding the ground-truth and model predictions into DeepSeek-V3 for judgment. Additionally, because the model output is sometimes chaotic and difficult to read, we have filtered out chain-of-thought with mixed languages, long paragraphs, and code blocks. For each prompt, we sample multiple responses and retain only the correct ones. In total, we collect about 600k reasoning related training samples.

RL Finetuning: r0

- Directly start from scratch, no SFT.
- Can still obtain the “aha” moment, but the thinking process is not readable.

RL Finetuning: GRPO

- PPO requires you to load 3-4 models on GPU: policy new, policy old, value function, and reward model.
- GRPO removes value functions, so it's purely policy gradient with KL penalty.

GRPO: Optimizing for Efficiency

Algorithm 1 Iterative Group Relative Policy Optimization

Input initial policy model $\pi_{\theta_{\text{init}}}$; reward models r_φ ; task prompts \mathcal{D} ; hyperparameters ε, β, μ

- 1: policy model $\pi_\theta \leftarrow \pi_{\theta_{\text{init}}}$
- 2: **for** iteration = 1, ..., I **do**
- 3: reference model $\pi_{\text{ref}} \leftarrow \pi_\theta$
- 4: **for** step = 1, ..., M **do**
- 5: Sample a batch \mathcal{D}_b from \mathcal{D}
- 6: Update the old policy model $\pi_{\theta_{\text{old}}} \leftarrow \pi_\theta$
- 7: Sample G outputs $\{o_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | q)$ for each question $q \in \mathcal{D}_b$
- 8: Compute rewards $\{r_i\}_{i=1}^G$ for each sampled output o_i by running r_φ
- 9: Compute $\hat{A}_{i,t}$ for the t -th token of o_i through group relative advantage estimation.
- 10: **for** GRPO iteration = 1, ..., μ **do**
- 11: Update the policy model π_θ by maximizing the GRPO objective (Equation 21)
- 12: Update r_φ through continuous training using a replay mechanism.

Output π_θ

GRPO: Advantage

Baseline is the average of multiple samples

Per token advantage is the same for the trajectory

4.1.2. *Outcome Supervision RL with GRPO*

Formally, for each question q , a group of outputs $\{o_1, o_2, \dots, o_G\}$ are sampled from the old policy model $\pi_{\theta_{old}}$. A reward model is then used to score the outputs, yielding G rewards $\mathbf{r} = \{r_1, r_2, \dots, r_G\}$ correspondingly. Subsequently, these rewards are normalized by subtracting the group average and dividing by the group standard deviation. Outcome supervision provides the normalized reward at the end of each output o_i and sets the advantages $\hat{A}_{i,t}$ of all tokens in the output as the normalized reward, i.e., $\hat{A}_{i,t} = \tilde{r}_i = \frac{r_i - \text{mean}(\mathbf{r})}{\text{std}(\mathbf{r})}$, and then optimizes the policy by maximizing the objective defined in equation (3).

GRPO: Advantage

Baseline is the average of multiple samples

Per token advantage is the same for the trajectory

```
# Sum the rewards from all reward functions
rewards = rewards_per_func.sum(dim=1)

# Compute grouped-wise rewards
mean_grouped_rewards = rewards.view(-1, self.num_generations).mean(dim=1)
std_grouped_rewards = rewards.view(-1, self.num_generations).std(dim=1)

# Normalize the rewards to compute the advantages
mean_grouped_rewards = mean_grouped_rewards.repeat_interleave(self.num_generations, dim=0)
std_grouped_rewards = std_grouped_rewards.repeat_interleave(self.num_generations, dim=0)
advantages = (rewards - mean_grouped_rewards) / (std_grouped_rewards + 1e-4)
```

```
advantages = inputs["advantages"]
per_token_loss = torch.exp(per_token_logps - per_token_logps.detach()) * advantages.unsqueeze(1)
```

GRPO: KL

- Previously PPO: Per token KL added into Advantage (why are you able to do this?):

$$\hat{A} = \text{advantage}(\text{reward} - \beta \text{KL})$$

- $\nabla_{\theta} \mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P_{soft}(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q)]$

$$\frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \left[\hat{A}_{i,t} + \beta \left(\frac{\pi_{ref}(o_{i,t}|o_{i,<t})}{\pi_{\theta}(o_{i,t}|o_{i,<t})} - 1 \right) \right] \nabla_{\theta} \log \pi_{\theta}(o_{i,t}|q, o_{i,<t}). \quad (20)$$

with the

$$D_{KL} [\pi_{\theta} || \pi_{ref}] = \frac{\pi_{ref}(o_{i,t}|q, o_{i,<t})}{\pi_{\theta}(o_{i,t}|q, o_{i,<t})} - \log \frac{\pi_{ref}(o_{i,t}|q, o_{i,<t})}{\pi_{\theta}(o_{i,t}|q, o_{i,<t})} - 1, \quad (4)$$

which is guaranteed to be positive.

of comparisons between outputs on the same question. Also note that, instead of adding KL penalty in the reward, GRPO regularizes by directly adding the KL divergence between the trained policy and the reference policy to the loss, avoiding complicating the calculation of $\hat{A}_{i,t}$.

GRPO KL: PPO vs GRPO

- PPO: $kl = \text{logprobs} - \text{ref_logprobs}$
- Sampling based
- GRPO: $\text{per_token_kl} = \text{torch.exp}(\text{ref_per_token_logps} - \text{per_token_logps}) - (\text{ref_per_token_logps} - \text{per_token_logps}) - 1$
- Nonnegative because $\exp(x) \rightarrow 1 + x + \frac{1}{2}x^2$

GRPO: KL

- Effect: whitening, GAE calculation. GRPO KL separated term

```
# 4. compute rewards
kl = logprobs - ref_logprobs
non_score_reward = -args.kl_coef * kl
rewards = non_score_reward.clone()
actual_start = torch.arange(rewards.size(0), device=rewards.device)
actual_end = torch.where(sequence_lengths_p1 < rewards.size(1), sequence_lengths_p1, sequence_lengths)
rewards[[actual_start, actual_end]] += scores

# 5. whiten rewards
if args.whiten_rewards:
    rewards = masked_whiten(rewards, mask=~padding_mask_p1, shift_mean=False)
    rewards = torch.masked_fill(rewards, padding_mask_p1, 0)

# 6. compute advantages and returns
lastgaelam = 0
advantages_reversed = []
gen_length = responses.shape[1]
for t in reversed(range(gen_length)):
    nextvalues = values[:, t + 1] if t < gen_length - 1 else 0.0
    delta = rewards[:, t] + args.gamma * nextvalues - values[:, t]
    lastgaelam = delta + args.gamma * args.lam * lastgaelam
    advantages_reversed.append(lastgaelam)
advantages = torch.stack(advantages_reversed[::-1], axis=1)
returns = advantages + values
advantages = masked_whiten(advantages, ~padding_mask)
advantages = torch.masked_fill(advantages, padding_mask, 0)
torch.cuda.empty_cache()

# Compute the KL divergence between the model and the reference model
ref_per_token_logps = inputs["ref_per_token_logps"]
per_token_kl = torch.exp(ref_per_token_logps - per_token_logps) - (ref_per_token_logps - per_token_logps) - 1

# x - x.detach() allows for preserving gradients from x
advantages = inputs["advantages"]
per_token_loss = torch.exp(per_token_logps - per_token_logps.detach()) * advantages.unsqueeze(1)
per_token_loss = -(per_token_loss - self.beta * per_token_kl)
loss = ((per_token_loss * completion_mask).sum(dim=1) / completion_mask.sum(dim=1)).mean()
```

GRPO: Putting Together

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- 11: Update the policy model π_θ by maximizing the GRPO objective (Equation 21)
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Output π_θ

The gradient of $\mathcal{J}_{GRPO}(\theta)$ is:

$$\begin{aligned}\nabla_\theta \mathcal{J}_{GRPO}(\theta) &= \mathbb{E}[q \sim P_{sft}(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q)] \\ &\quad \frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \left[\hat{A}_{i,t} + \beta \left(\frac{\pi_{ref}(o_{i,t}|o_{i,<t})}{\pi_\theta(o_{i,t}|o_{i,<t})} - 1 \right) \right] \nabla_\theta \log \pi_\theta(o_{i,t}|q, o_{i,<t}).\end{aligned}\tag{20}$$

Ok what about PPO

We introduce Open-Reasoner-Zero, the first open source implementation of large-scale reasoning-oriented RL training focusing on scalability, simplicity and accessibility. Through extensive experiments, we demonstrate that a minimalist approach, vanilla PPO with GAE ($\lambda = 1$, $\gamma = 1$) and straightforward rule-based reward function, without any KL regularization, is sufficient to scale up both response length and benchmark performance on reasoning tasks, similar to the phenomenon observed in DeepSeek-R1-Zero. Notably, our implementation outperforms DeepSeek-R1-Zero-Qwen-32B on the GPQA Diamond benchmark, while only requiring 1/30 of the training steps. In the spirit of open source, we release our source code, parameter settings, training data, and model weights.

- Still not fully leveraging RL (Might also be a trap)

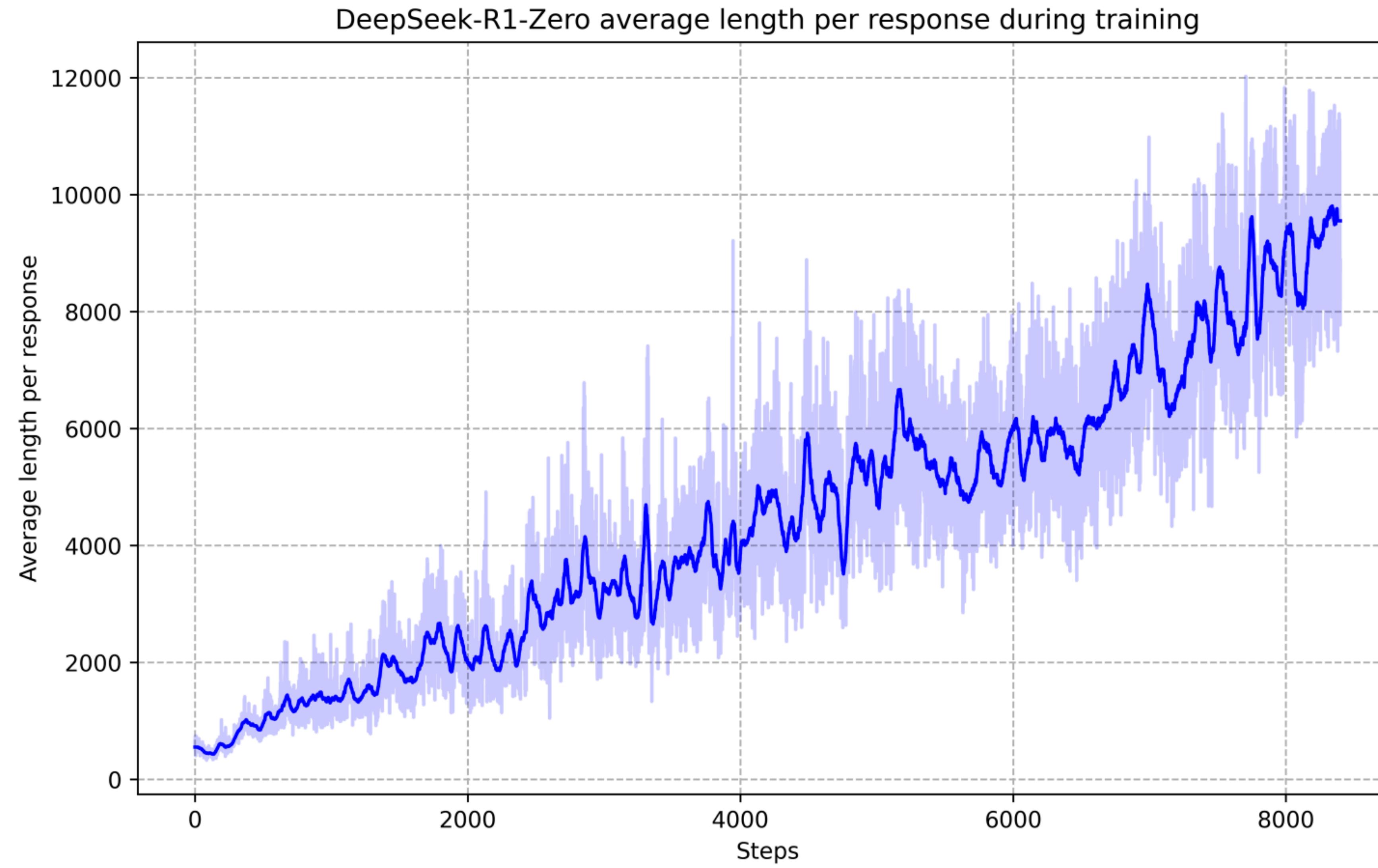
Thinking Template

We intentionally limit our constraints to this structural format, avoiding any content-specific biases—such as mandating reflective reasoning or promoting particular problem-solving strategies—to ensure that we can accurately observe the model’s natural progression during the RL process.

A conversation between User and Assistant. The user asks a question, and the Assistant solves it. The assistant first thinks about the reasoning process in the mind and then provides the user with the answer. The reasoning process and answer are enclosed within `<think> </think>` and `<answer> </answer>` tags, respectively, i.e., `<think>` reasoning process here `</think>` `<answer>` answer here `</answer>`. User: **prompt**. Assistant:

Table 1 | Template for DeepSeek-R1-Zero. **prompt** will be replaced with the specific reasoning question during training.

Thinking Length



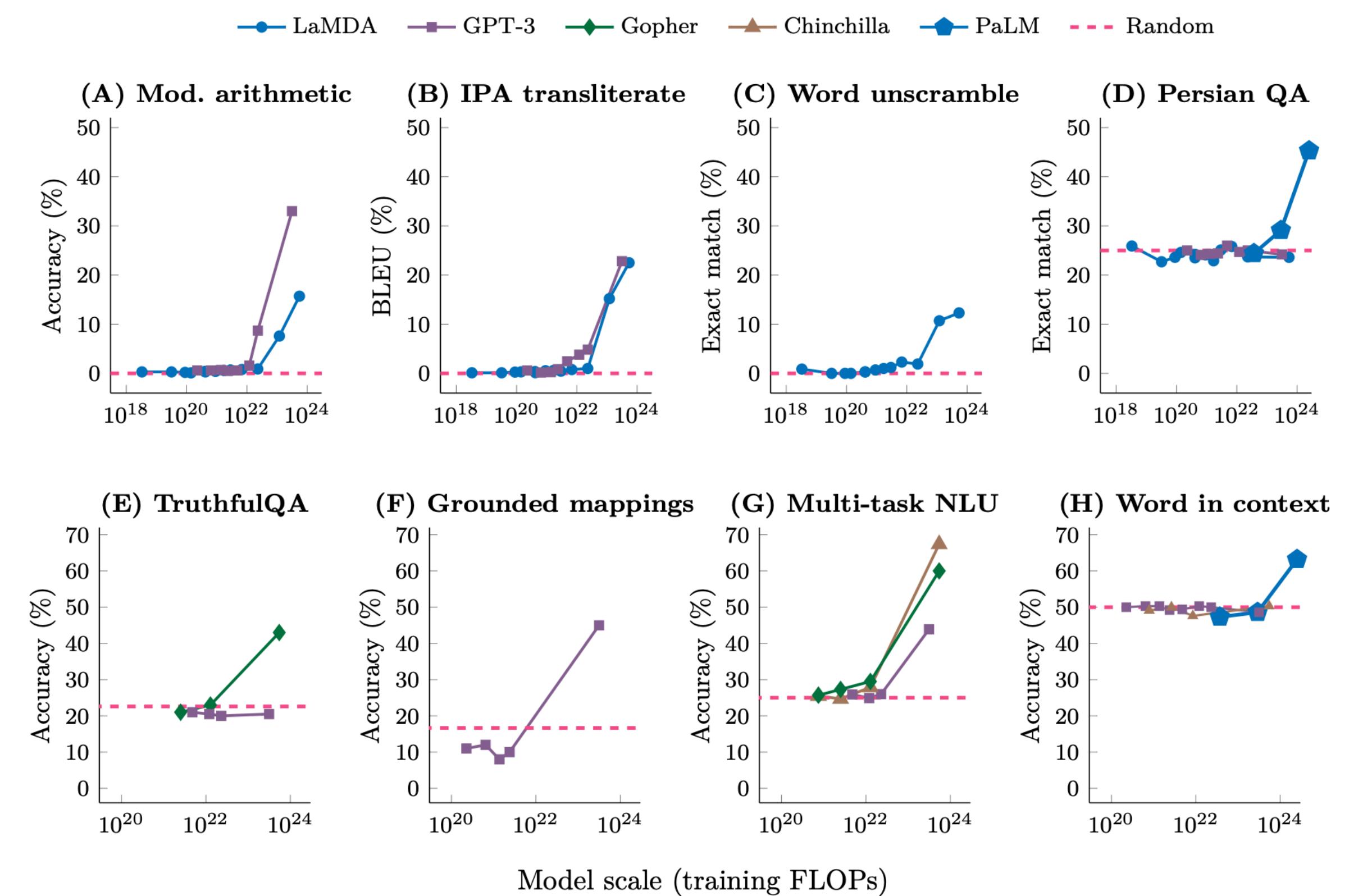
RL Finetuning: why does it work?

Key Takeaways

- Strong base model, no need to do exploration, the model can generate correct answers, which provides signals.
- Use rule based rewards for reasoning tasks (or whenever possible). For nuanced, open ended questions, reward model can provide some signals.

Why previously it didn't work

- Using Neural reward models
- Model size, base model capability
- Training stability



Potential Research Question: Credit Assignment

- When doing REINFORCE, you are also reinforcing the incorrect part of the solution.
- Example: “[Reasoning Trace 1] Wait, but [Reasoning Trace 2]” [Reasoning Trace 1] is incorrect, but it was still reinforced
- How to find the correct part responsible for leading to the solutions?
- More general: how to do better credit assignment? (Training value funcs, GAE estimation (might perform bad on long sequences!))

Distillation

S1

How many r in raspberry?

Question

Let's break down the process of counting the letter 'r' in the word "raspberry" ...

- * First letter: 'r' - This is an 'r', count = 1.
- * Second letter: 'a' - Not an 'r', count remains 1 ...
- * Sixth letter: 'e' - Not an 'r', count remains 1.
- * Seventh letter: 'r' - This is an 'r', count = 2.
- * Eighth letter: 'y' - Not an 'r', count remains 2 ...

The number of 'r's in "raspberry" is 2.

Wait, let's re-read the question carefully. It asks "How many r in raspberry?" ... * r - a - s - p - b - e - r - r - y ... * First 'r' ... * Second 'r' ... * Third 'r' ... Count = 3 ...

Reasoning trace

My initial answer of 2 was incorrect due to a quick reading of the word. **Final Answer:** The final answer is 3

Response

Training We perform supervised finetuning on Qwen2.5-32B-Instruct using **s1K** to obtain our model **s1-32B** using basic hyperparameters outlined in §D. Finetuning took 26 minutes on 16 NVIDIA H100 GPUs with PyTorch FSDP.

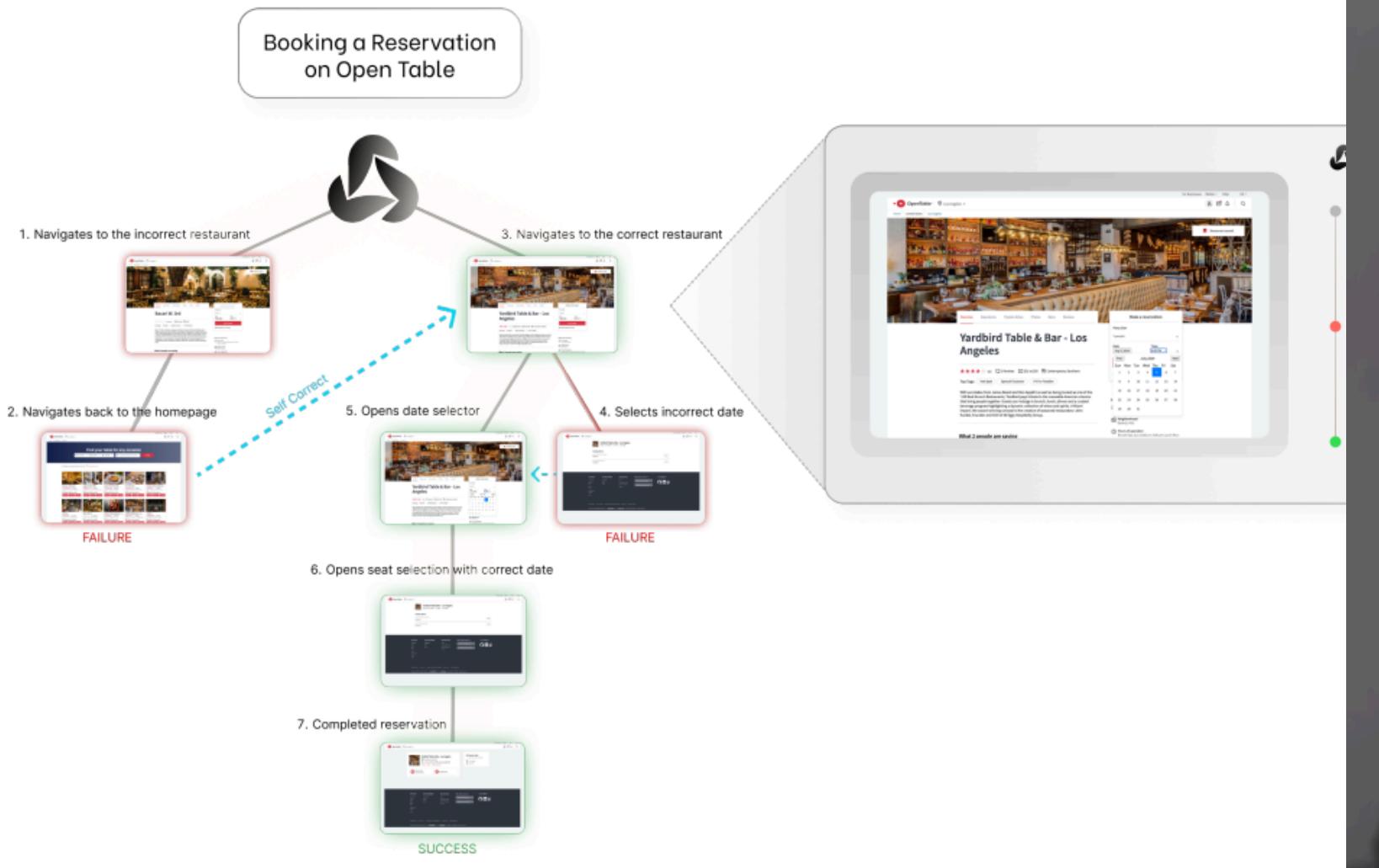
Figure 3. Budget forcing with s1-32B. The model tries to stop after "...is 2.", but we suppress the end-of-thinking token delimiter instead appending "Wait" leading s1-32B to self-correct its answer.

Applications

February 2, 2025 Release

Introducing deep research

An agent that uses reasoning to synthesize large amounts of online information and complete multi-step research tasks for you. Available to Pro users today, Plus and Team next.



The story of r1, chronologically

Tony Chen

2 Themes

- Efficiency
- Reasoning

DeepSeek LLM: Scaling Open-Source Language Models with Longtermism

Paper • 2401.02954 • Published Jan 5, 2024 • △ 43

DeepSeekMoE: Towards Ultimate Expert Specialization in Mixture-of-Experts Language Models

Paper • 2401.06066 • Published Jan 11, 2024 • △ 47

DeepSeek-Coder: When the Large Language Model Meets Programming -- The Rise of Code Intel...

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DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models

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DeepSeek-VL: Towards Real-World Vision-Language Understanding

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DeepSeek-V2: A Strong, Economical, and Efficient Mixture-of-Experts Language Model

Paper • 2405.04434 • Published May 7, 2024 • △ 17

DeepSeek-Prover: Advancing Theorem Proving in LLMs through Large-Scale Synthetic Data

Paper • 2405.14333 • Published May 23, 2024 • △ 37

DeepSeek-Coder-V2: Breaking the Barrier of Closed-Source Models in Code Intelligence

Paper • 2406.11931 • Published Jun 17, 2024 • △ 62

Let the Expert Stick to His Last: Expert-Specialized Fine-Tuning for Sparse Architectural Large La...

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JanusFlow: Harmonizing Autoregression and Rectified Flow for Unified Multimodal Understand...

Paper • 2411.07975 • Published Nov 12, 2024 • △ 30

DeepSeek-VL2: Mixture-of-Experts Vision-Language Models for Advanced Multimodal Understa...

Paper • 2412.10302 • Published Dec 13, 2024 • △ 12

DeepSeek-V3 Technical Report

Paper • 2412.19437 • Published Dec 26, 2024 • △ 46

DeepseekMoE

[Jan 2024][efficiency]

This architecture manifests two potential issues: (1) **Knowledge Hybridsity**: existing MoE practices often employ a limited number of experts (e.g., 8 or 16), and thus tokens assigned to a specific expert will be likely to cover diverse knowledge. Consequently, the designated expert will intend to assemble vastly different types of knowledge in its parameters, which are hard to utilize simultaneously. (2) **Knowledge Redundancy**: tokens assigned to different experts may require common knowledge. As a result, multiple experts may converge in acquiring shared knowledge in their respective parameters, thereby leading to redundancy in expert parameters. These issues collectively hinder the expert specialization in existing MoE practices, preventing them from reaching the theoretical upper-bound performance of MoE models.

In response, we propose the DeepSeekMoE architecture towards ultimate expert specialization. It involves two principal strategies: (1) finely segmenting the experts into mN ones and activating mK from them, allowing for a more flexible combination of activated experts; (2) isolating K_s experts as shared ones, aiming at capturing common knowledge and mitigating redundancy in routed experts. Starting from a modest scale with 2B parameters, we demonstrate that DeepSeekMoE 2B achieves comparable performance with GShard 2.9B, which has 1.5x expert parameters and computation. In addition, DeepSeekMoE 2B nearly approaches the performance of its dense counterpart with the same number of total parameters, which set the upper bound.

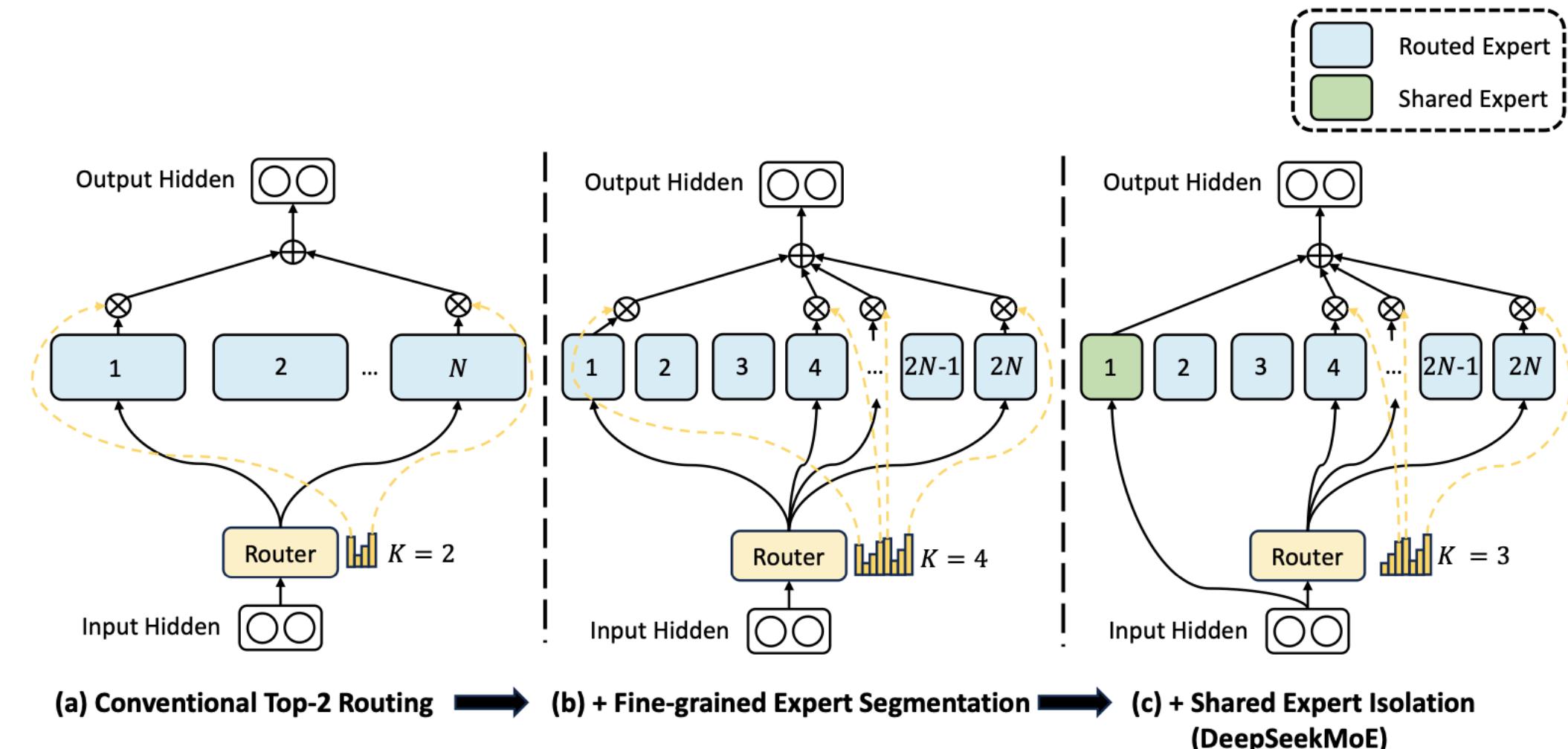
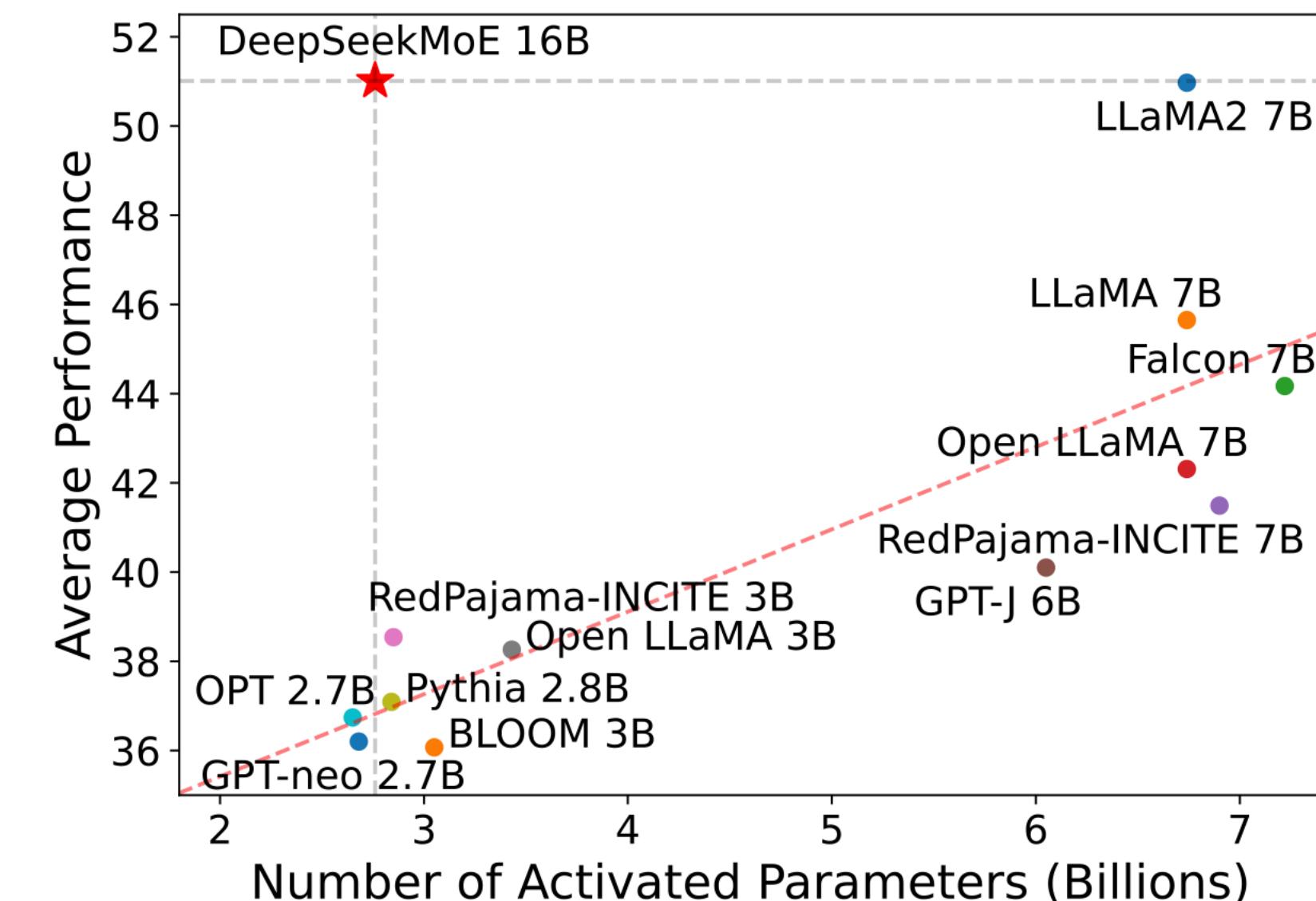


Figure 2: Illustration of DeepSeekMoE's three-stage evolution. (a) Conventional Top-2 Routing → (b) + Fine-grained Expert Segmentation → (c) + Shared Expert Isolation (DeepSeekMoE)



Deepseek-Coder

[Jan 2024][reasoning]

- Key: data quality

2. Data Collection

The training dataset of DeepSeek-Coder is composed of 87% source code, 10% English code-related natural language corpus, and 3% code-unrelated Chinese natural language corpus. The English corpus consists of materials from GitHub's Markdown and StackExchange¹, which are used to enhance the model's understanding of code-related concepts and improve its ability to handle tasks like library usage and bug fixing. Meanwhile, the Chinese corpus consists of high-quality articles aimed at improving the model's proficiency in understanding the Chinese language. In this section, we will provide an overview of how we construct the code training data. This process involves data crawling, rule-based filtering, dependency parsing, repository-level deduplication, and quality screening, as illustrated in Figure 2. In the following, we will describe the data creation procedure step by step.

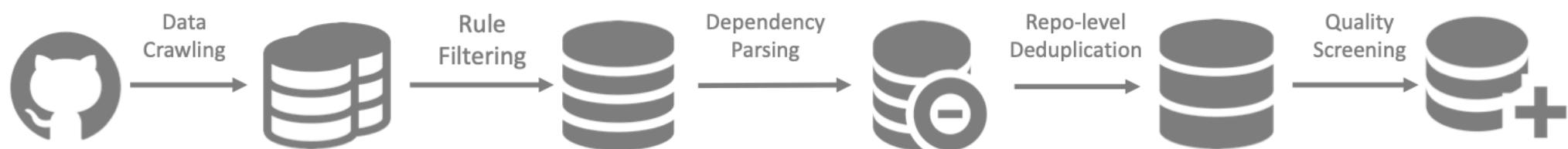
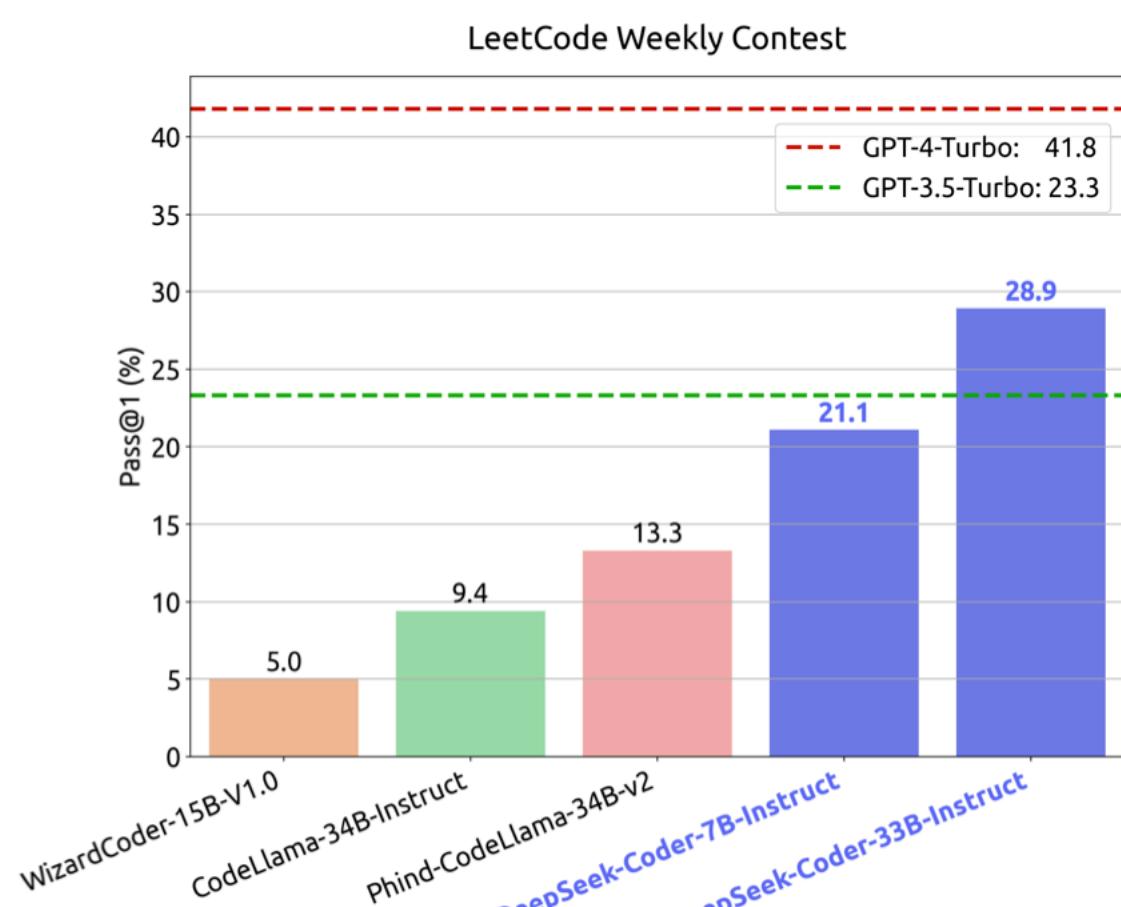
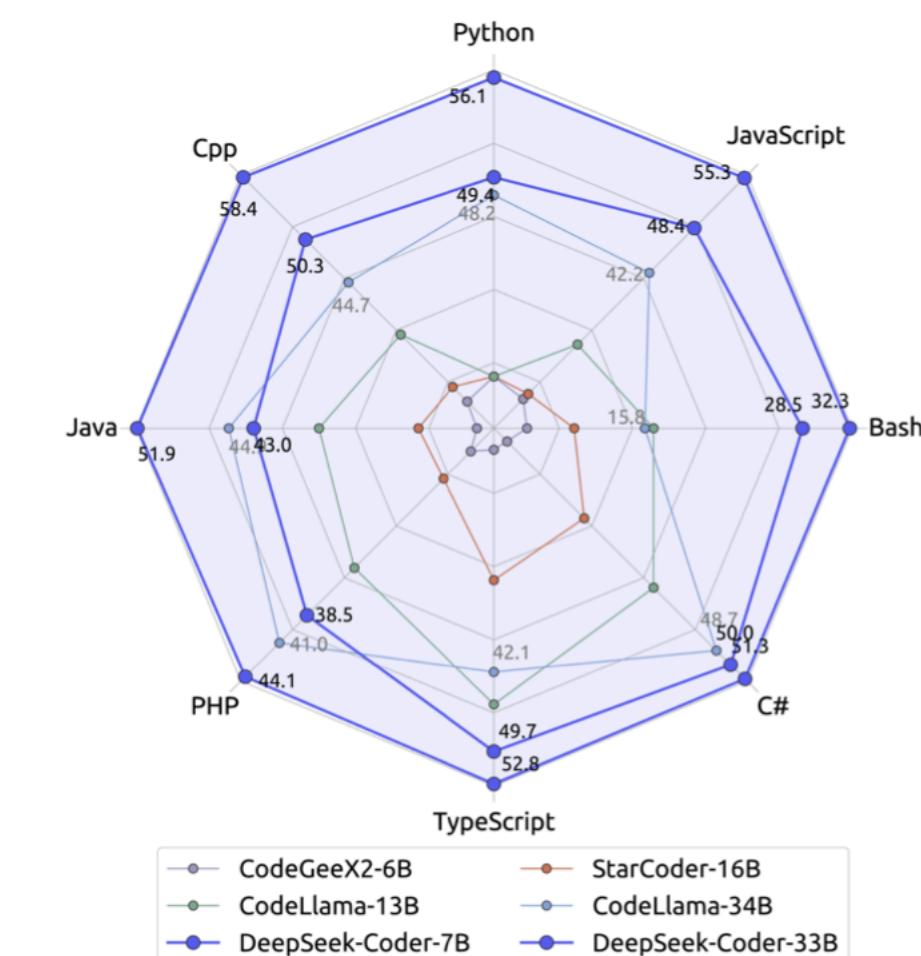


Figure 2 | The Procedure of Dataset Creation

The rapid development of large language models has revolutionized code intelligence in software development. However, the predominance of closed-source models has restricted extensive research and development. To address this, we introduce the DeepSeek-Coder series, a range of open-source code models with sizes from 1.3B to 33B, trained from scratch on 2 trillion tokens. These models are pre-trained on a high-quality project-level code corpus and employ a fill-in-the-blank task with a 16K window to enhance code generation and infilling. Our extensive evaluations demonstrate that DeepSeek-Coder not only achieves state-of-the-art performance among open-source code models across multiple benchmarks but also surpasses existing closed-source models like Codex and GPT-3.5. Furthermore, DeepSeek-Coder models are under a permissive license that allows for both research and unrestricted commercial use.



Deepseek-Coder

[Jan 2024][reasoning]

- Example filtering rules

Firstly, we filter out files with an average line length exceeding 100 characters or a maximum line length surpassing 1000 characters. Additionally, we remove files with fewer than 25% alphabetic characters. Except for the XSLT programming language, we further filter out files where the string "<?xml version=" appeared in the first 100 characters. For HTML files, we consider the ratio of visible text to HTML code. We retain files where the visible text constitutes at least 20% of the code and is no less than 100 characters. For JSON and YAML files, which typically contain more data, we only keep files that have a character count ranging from 50 to 5000 characters. This effectively removes most data-heavy files.

2.3. Repo-Level Deduplication

Recent studies have demonstrated the significant performance improvements that can be achieved by deduplicating training datasets for Large Language Models (LLMs). Lee et al. (2022) have shown that language model training corpora often contain numerous near-duplicates, and the performance of LLMs can be enhanced by removing long repetitive substrings. Kocetkov et al. (2022) have applied a near-deduplication method to training data, resulting in dramatic improvements, and they emphasize that near-deduplication is a crucial preprocessing step for achieving competitive performance on code benchmark tasks. In our dataset, we have also employed near-deduplication. However, there is a distinction in our approach compared to previous works. We perform deduplication at the repository level of code, rather than at the file level, as the latter approach may filter out certain files within a repository, potentially disrupting the structure of the repository. Specifically, we treat the concatenated code from the repository level as a single sample and apply the same near-deduplication algorithm to ensure the integrity of the repository structure.

2.4. Quality Screening and Decontamination

In addition to applying the filtering rules mentioned in Section 2.1, we also employ a compiler and a quality model, combined with heuristic rules, to further filter out low-quality data. This includes code with syntax errors, poor readability, and low modularity. We provide the statistical summary of source code in Table 1, which includes a total of 87 languages, detailing the disk size, number of files, and percentage for each language. The total data volume is 798 GB with 603 million files. To ensure that our code training data is not contaminated by information from the test set, which may be present on GitHub, we've implemented an n-gram filtering process. This process involves the removal of any code segments that match specific criteria. Specifically, we filter out files containing docstrings, questions, and solutions from sources such as HumanEval (Chen et al., 2021), MBPP (Austin et al., 2021), GSM8K (Cobbe et al., 2021) and MATH (Hendrycks et al., 2021). For the filtering criteria, we apply the following rules: if a piece of code includes a 10-gram string identical to any in the test data, it is excluded from our training data. In cases where the test data comprises strings that are shorter than 10-grams but no less than 3-grams, we use an exact match approach for filtering.

```
Algorithm 1 Topological Sort for Dependency Analysis
1: procedure TOPOLOGICALSORT(files)
2:   graphs ← {}                                ▶ Initialize an empty adjacency list
3:   inDegree ← {}                               ▶ Initialize an empty dictionary for in-degrees
4:   for each file in files do
5:     graphs[file] ← []
6:     inDegree[file] ← 0
7:   end for
8:
9:   for each fileA in files do
10:    for each fileB in files do
11:      if HASDEPENDENCY(fileA, fileB) then
12:        graphs[fileB].append(fileA)           ▶ If fileA depends on fileB
13:        inDegree[fileA] ← inDegree[fileA] + 1  ▶ Add edge from B to A
14:      end if
15:    end for
16:  end for
17:
18:  subgraphs ← getDisconnectedSubgraphs(graphs)  ▶ Identify disconnected subgraphs
19:  allResults ← []
20:  for each subgraph in subgraphs do
21:    results ← []
22:    while length(results) ≠ NumberOfNodes(subgraph) do
23:      file ← argmin((inDegree[file] | file ∈ subgraph and file ∉ results))
24:      for each node in graphs[file] do
25:        inDegree[node] ← inDegree[node] - 1
26:      end for
27:      results.append(file)
28:    end while
29:    allResults.append(results)
30:  end for
31:
32:  return allResults
33: end procedure
```

dencies, updating "graphs" and "inDegree" accordingly. Next, it identifies any disconnected subgraphs within the overall dependency graph. For each subgraph, the algorithm employs a modified topological sort. Unlike the standard approach that selects nodes with zero in-degrees, this algorithm selects nodes with minimal in-degrees, which allows it to handle cycles within the graph. Selected nodes are added to a "results" list, and the in-degrees of their connected nodes are decreased. This process continues until a topologically sorted sequence is generated for each subgraph. The algorithm concludes by returning a list of these sorted sequences, and each sequence's files are concatenated to form a single training sample. To incorporate file path information, a comment indicating the file's path is added at the beginning of each file. This method ensures that the path information is preserved in the training data.

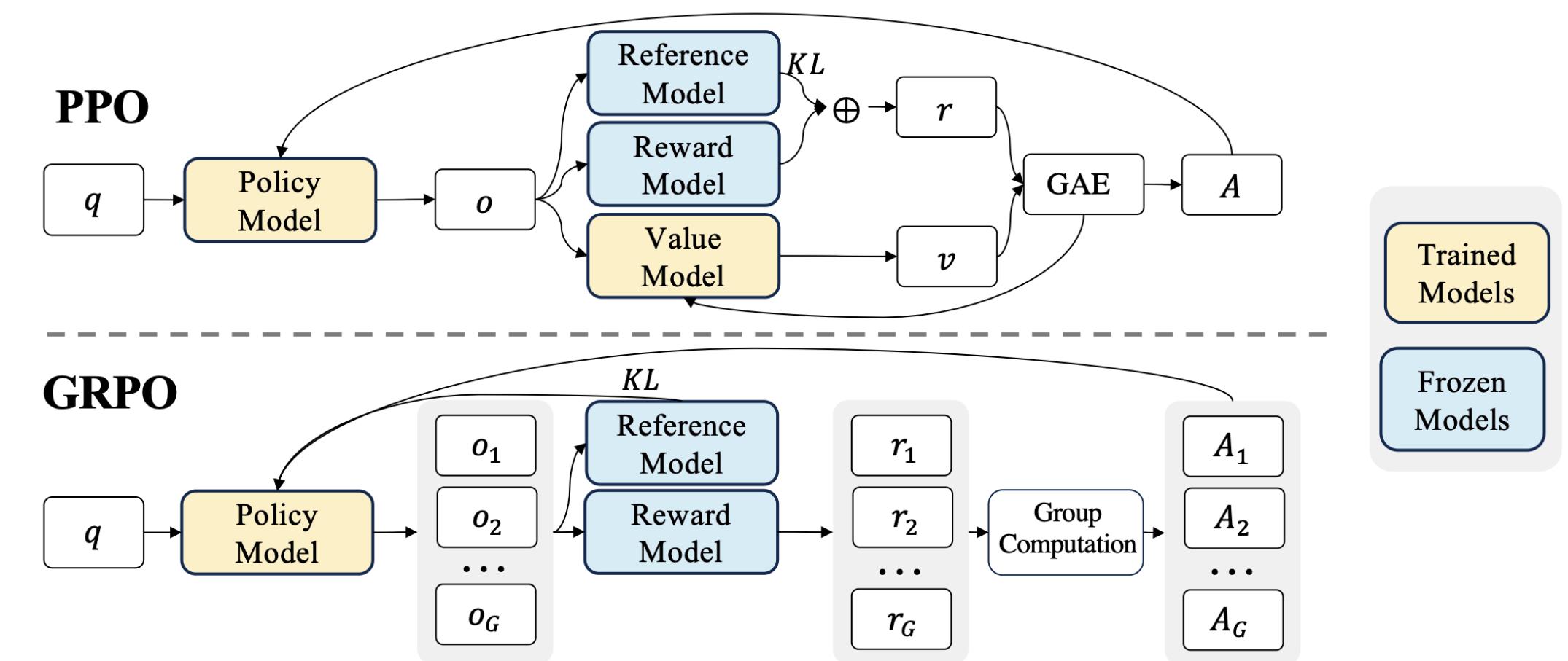
Deepseek math

[Apr 2024][reasoning][efficiency]

- 1. Better base policy with better dat quality
- 2. More efficient RL with GRPO
- Still used a reward model

We conduct RL based on DeepSeekMath-Instruct 7B. The training data of RL are chain-of-thought-format questions related to GSM8K and MATH from the SFT data, which consists of around 144K questions. We exclude other SFT questions to investigate the impact of RL on benchmarks that lack data throughout the RL phase. We construct the training set of **reward models** following (Wang et al., 2023b). We train our initial **reward model** based on the DeepSeekMath-Base 7B with a learning rate of 2e-5. For GRPO, we set the learning rate of the policy model as 1e-6. The KL coefficient is 0.04. For each question, we sample 64 outputs. The max length is set to 1024, and the training batch size is 1024. The policy model only has a single update following each exploration stage. We evaluate DeepSeekMath-RL 7B on benchmarks following DeepSeekMath-Instruct 7B. For DeepSeekMath-RL 7B, GSM8K and MATH with chain-of-thought reasoning can be regarded as in-domain tasks and all the other benchmarks can be regarded as out-of-domain tasks.

Mathematical reasoning poses a significant challenge for language models due to its complex and structured nature. In this paper, we introduce DeepSeekMath 7B, which continues pre-training DeepSeek-Coder-Base-v1.5 7B with 120B math-related tokens sourced from Common Crawl, together with natural language and code data. DeepSeekMath 7B has achieved an impressive score of 51.7% on the competition-level MATH benchmark without relying on external toolkits and voting techniques, approaching the performance level of Gemini-Ultra and GPT-4. Self-consistency over 64 samples from DeepSeekMath 7B achieves 60.9% on MATH. The mathematical reasoning capability of DeepSeekMath is attributed to two key factors: First, we harness the significant potential of publicly available web data through a meticulously engineered data selection pipeline. Second, we introduce Group Relative Policy Optimization (GRPO), a variant of Proximal Policy Optimization (PPO), that enhances mathematical reasoning abilities while concurrently optimizing the memory usage of PPO.



Deepseek prover

[May 2024][reasoning]

- Generate statements -> Generate proof -> validate -> finetune on correct samples

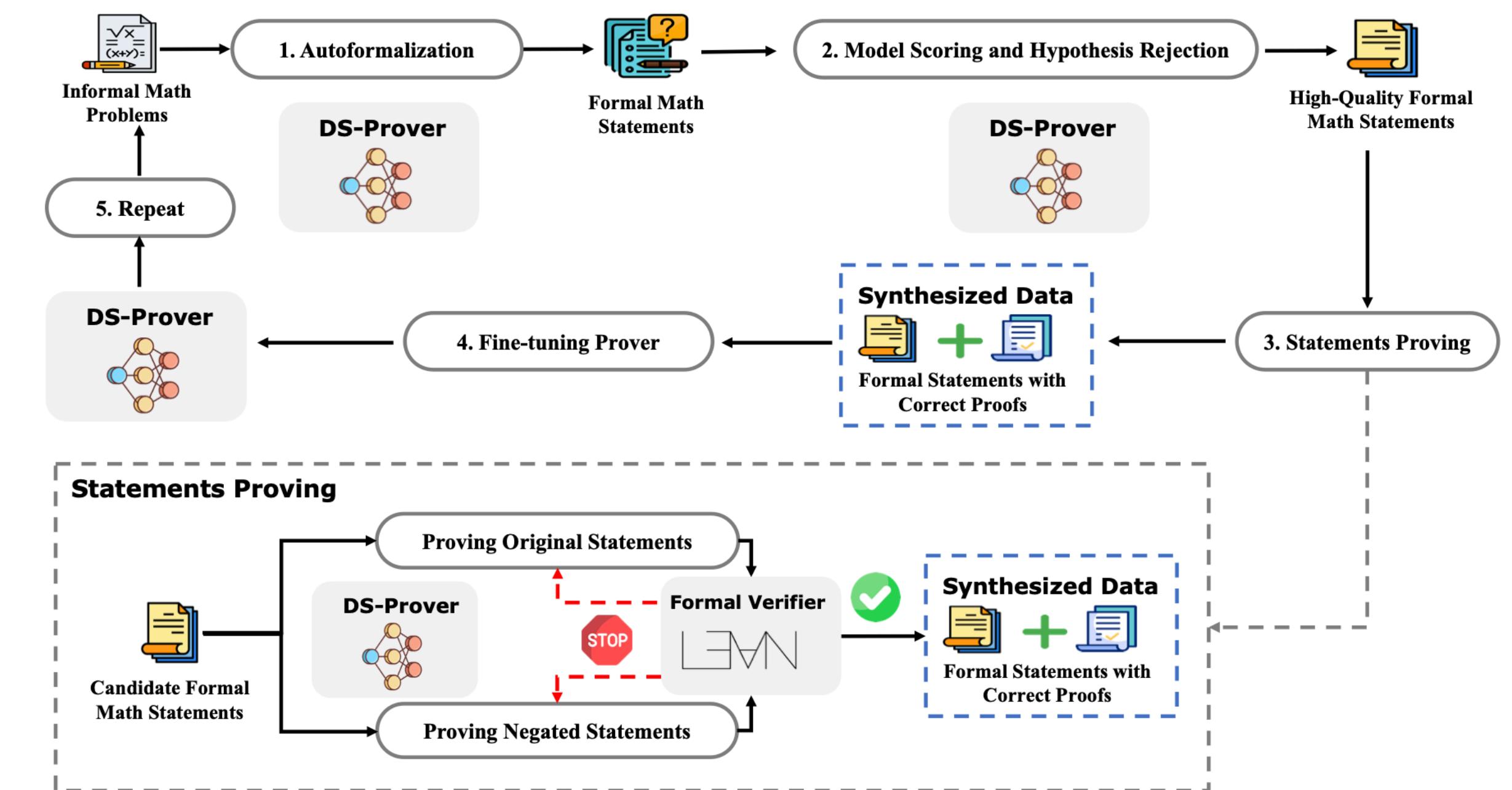


Figure 1: An overview of our approach.

Multi-Head Latent Attention

[Jun 2024][efficiency] from DeepSeek-v2 paper

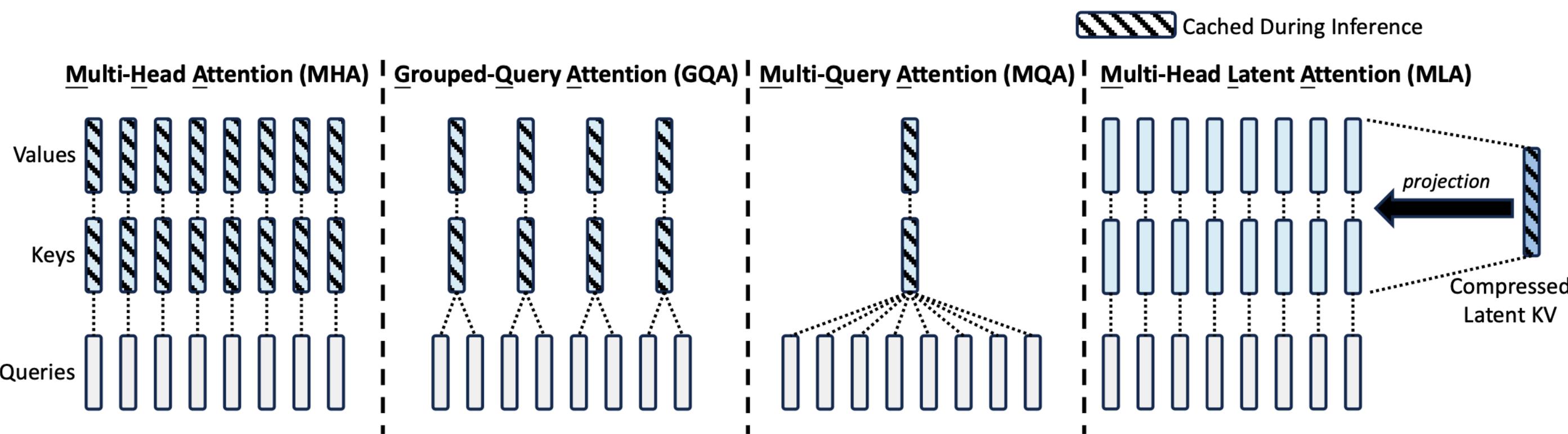
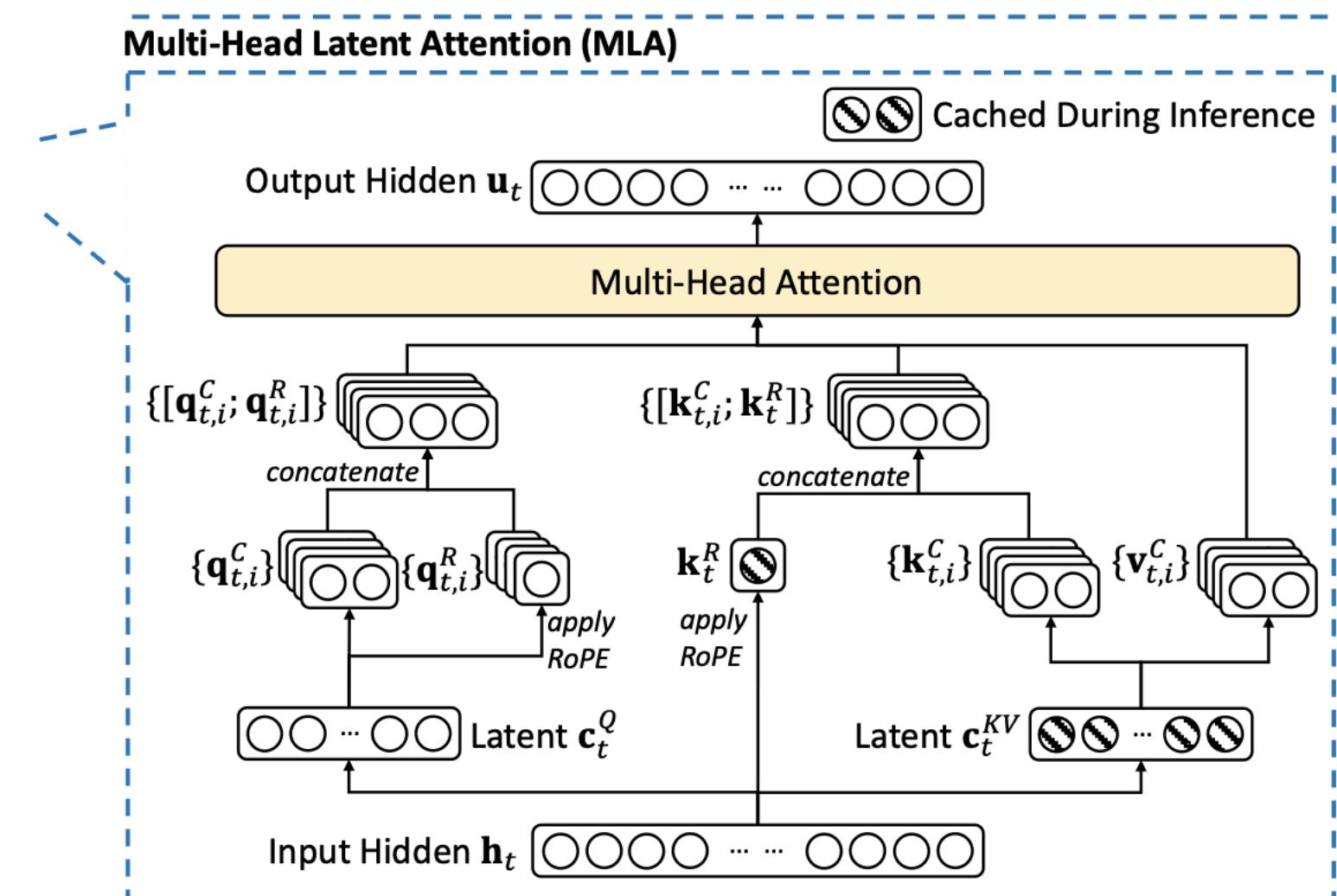
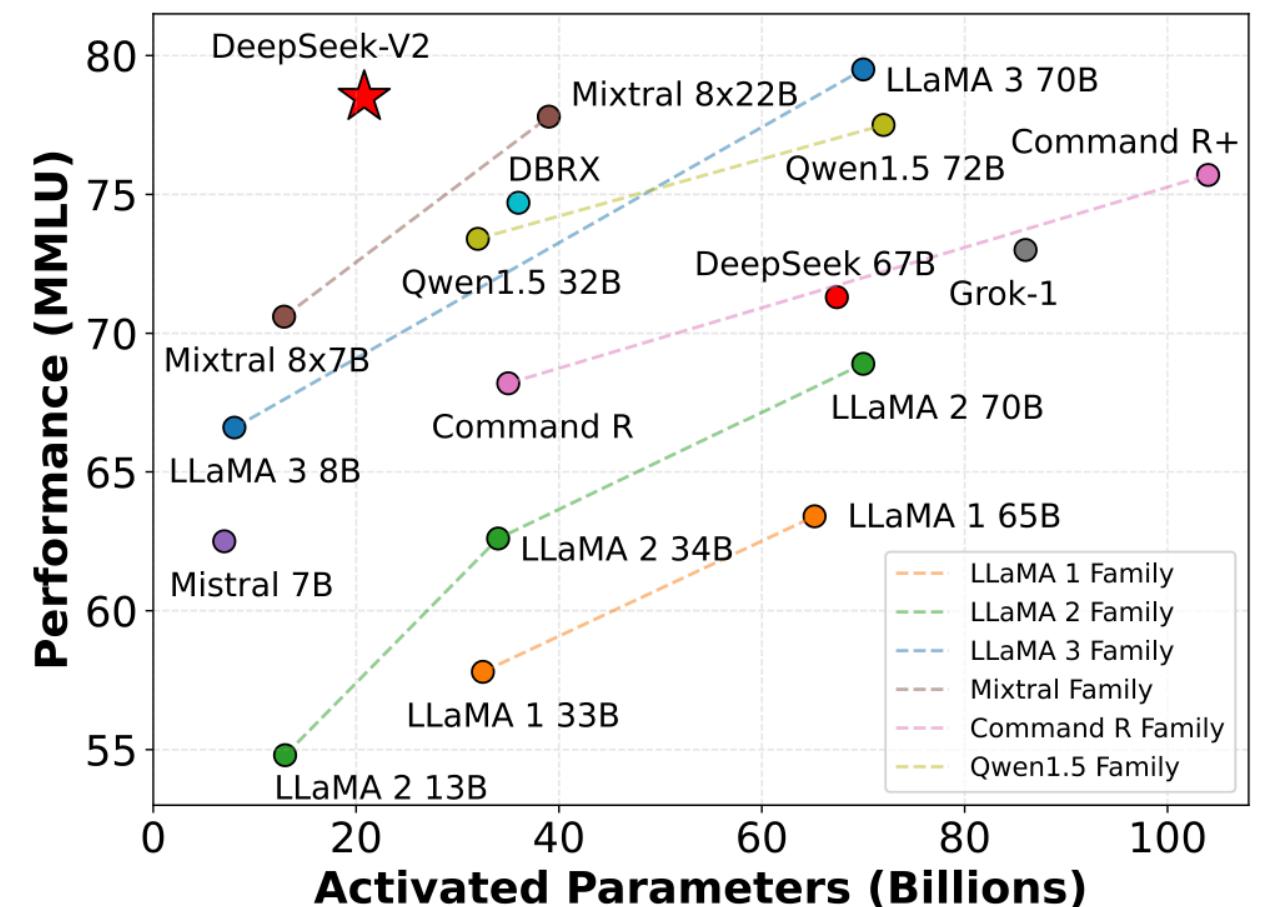


Figure 3 | Simplified illustration of Multi-Head Attention (MHA), Grouped-Query Attention (GQA), Multi-Query Attention (MQA), and Multi-head Latent Attention (MLA). Through jointly compressing the keys and values into a latent vector, MLA significantly reduces the KV cache during inference.

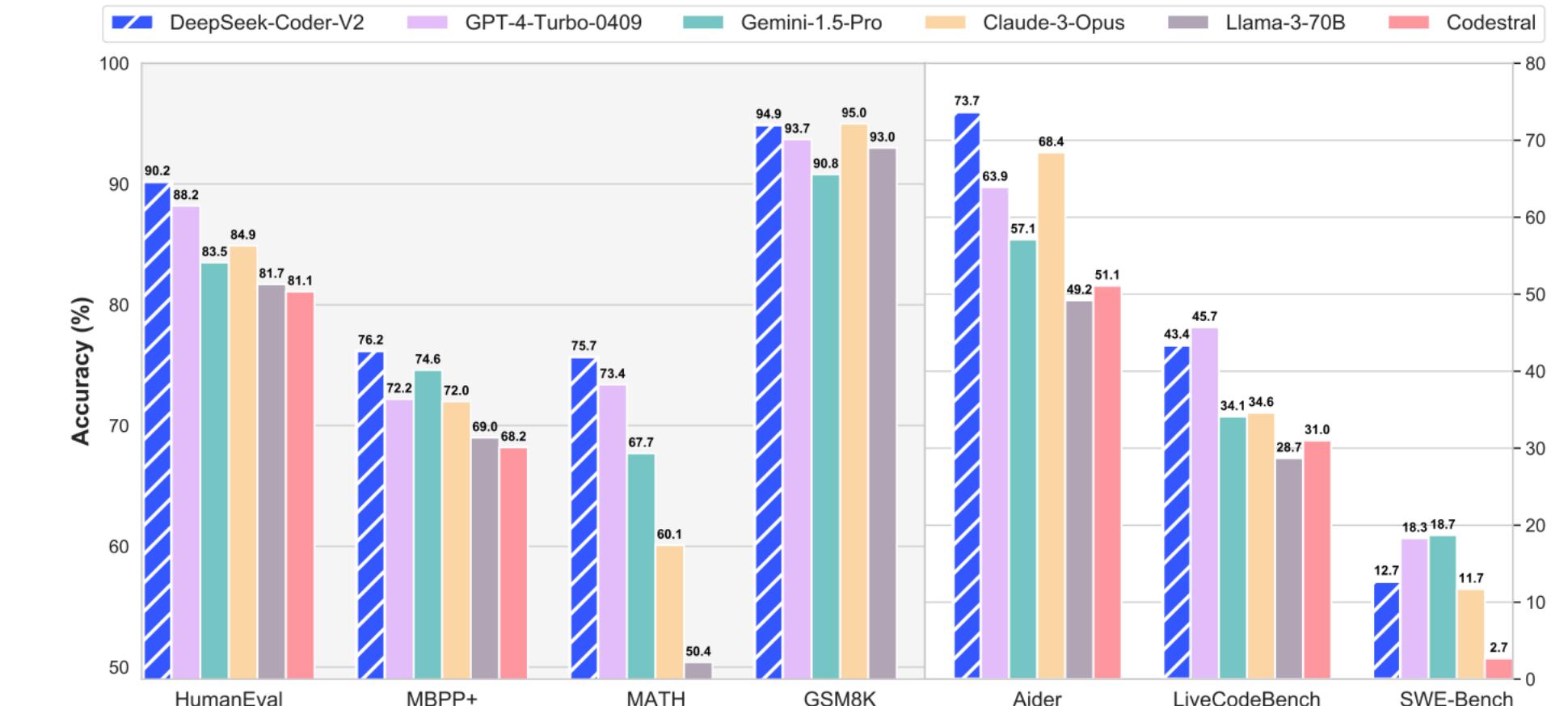


DeepseekCoder v2

[Jun 2024]

- Used GRPO for RLHF with compiler feedback

In the alignment phase, we first construct an instruction training dataset that includes code and math data from DeepSeek-Coder (Guo et al., 2024) and DeepSeek-Math (Shao et al., 2024), as well as general instruction data from DeepSeek-V2 (DeepSeek-AI, 2024). This dataset is used to fine-tune the base model. Then, in the reinforcement learning phase, we employ Group Relative Policy Optimization (GRPO) algorithm to align its behavior with human preferences. Preference data is collected in the coding domain using compiler feedback and test cases, and a reward model is developed to guide the training of the policy model. This approach ensures that the model's responses are optimized for correctness and human preference in coding tasks. To enable the model to support code completion after alignment, we also utilize Fill-In-Middle approach (Guo et al., 2024) during the fine-tuning of the base model with 16B parameters.



Reward Modeling Reward models play crucial roles in the RL training. In terms of mathematical preference data, we obtain them using the ground-truth labels. In terms of code preference data, although the code compiler itself can already provide 0-1 feedback (whether the code pass all test cases or not), some code prompts may have a limited number of test cases, and do not provide full coverage, and hence directly using 0-1 feedback from the compiler may be noisy and sub-optimal. Therefore, we still decide to train a reward model on the data provided by the compiler, and use the reward model to provide signal during RL training, which is more robust

and has better generalization ability, in comparison with raw compiler signal. As illustrated in Figure 3, in our in-house test sets (Leetcode and Leetcode-zh), using a reward model to provide RL training signal clearly outperforms using raw compiler signal. Hence, we use reward model signal rather than compiler signal in all subsequent experiments.

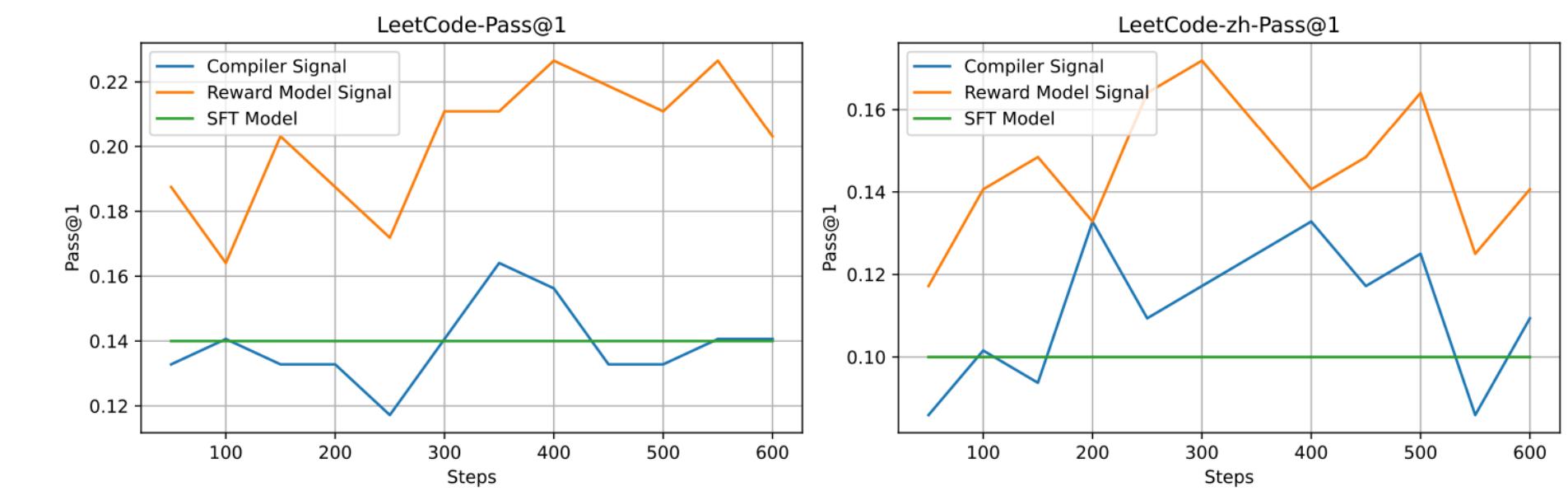
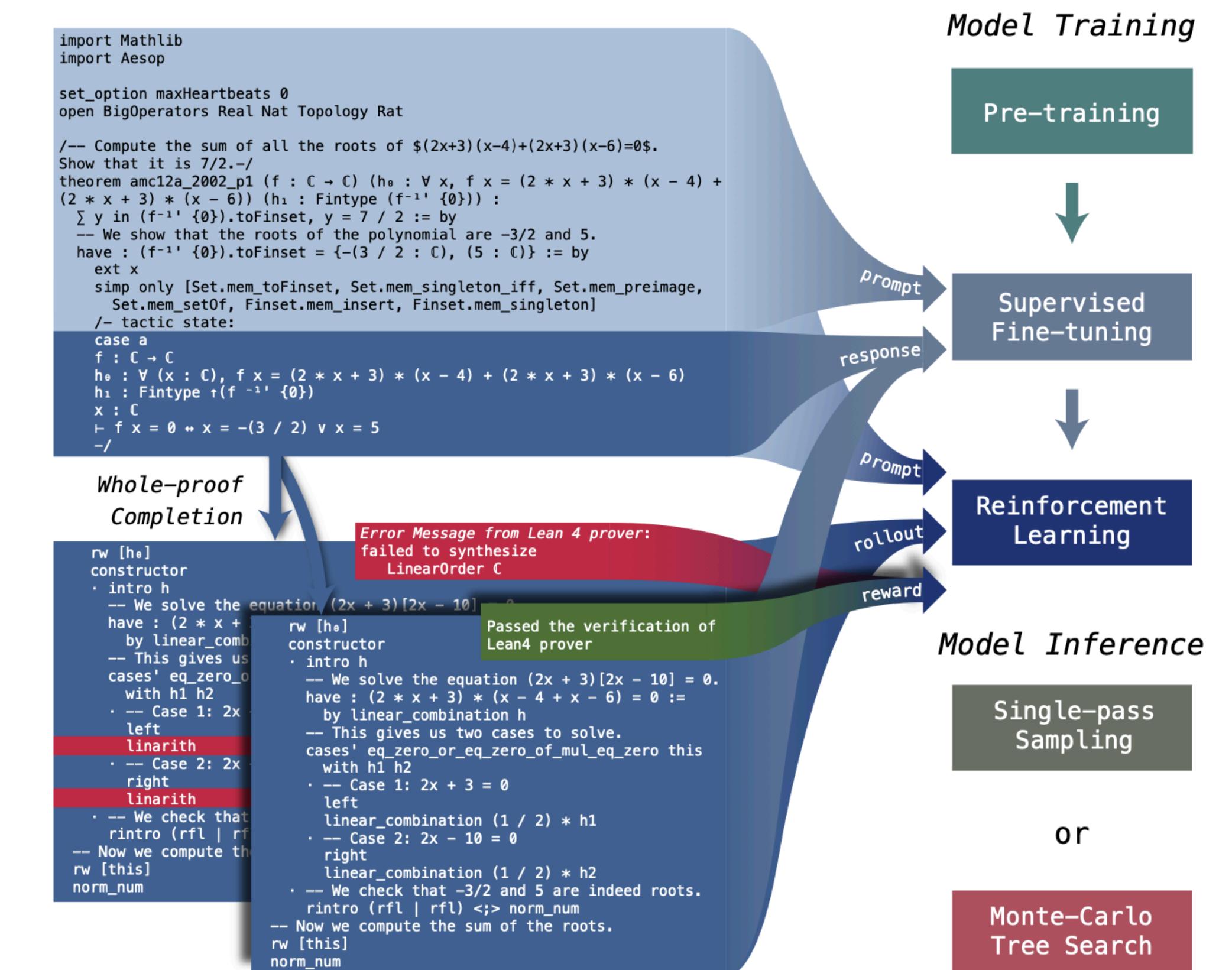


Figure 3 | Performances of Different Methods

DeepseekProver v1.5

[Aug 2024][reasoning]

- Training time: SFT + RL with GRPO
 - Rule based reward (from compiler)
- Inference time: MCTS (not used in r1)



Deepseek v3

[Dec 2024][efficiency]

- Multiple token prediction for denser training signal
- Training system efficiency

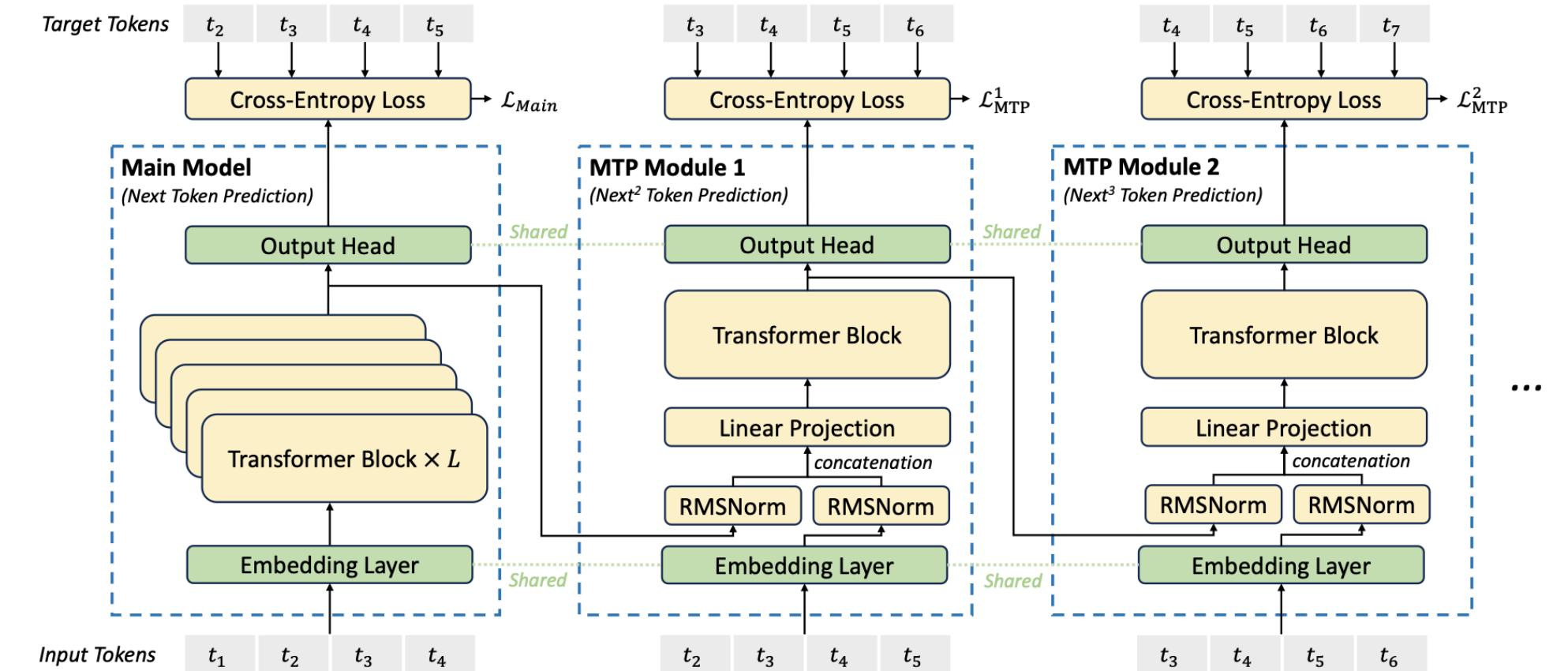


Figure 3 | Illustration of our Multi-Token Prediction (MTP) implementation. We keep the complete causal chain for the prediction of each token at each depth.

Pre-Training: Towards Ultimate Training Efficiency

- We design an FP8 mixed precision training framework and, for the first time, validate the feasibility and effectiveness of FP8 training on an extremely large-scale model.
- Through the co-design of algorithms, frameworks, and hardware, we overcome the communication bottleneck in cross-node MoE training, achieving near-full computation-communication overlap. This significantly enhances our training efficiency and reduces the training costs, enabling us to further scale up the model size without additional overhead.
- At an economical cost of only 2.664M H800 GPU hours, we complete the pre-training of DeepSeek-V3 on 14.8T tokens, producing the currently strongest open-source base model. The subsequent training stages after pre-training require only 0.1M GPU hours.

Deepseek r1

[Jan 2025][reasoning]

Question: If $a > 1$, then the sum of the real solutions of $\sqrt{a - \sqrt{a + x}} = x$ is equal to

Response: <think>

To solve the equation $\sqrt{a - \sqrt{a + x}} = x$, let's start by squaring both ...

$$\left(\sqrt{a - \sqrt{a+x}}\right)^2 = x^2 \implies a - \sqrt{a+x} = x^2.$$

Rearrange to isolate the inner square root term:

$$(a - x^2)^2 = a + x \implies a^2 - 2ax^2 + (x^2)^2 = a + x \implies x^4 - 2ax^2 - x + (a^2 - a) = 0$$

•

Wait, wait. Wait. That's an aha moment I can flag here.

Let's reevaluate this step-by-step to identify if the correct sum can be ...

We started with the equation:

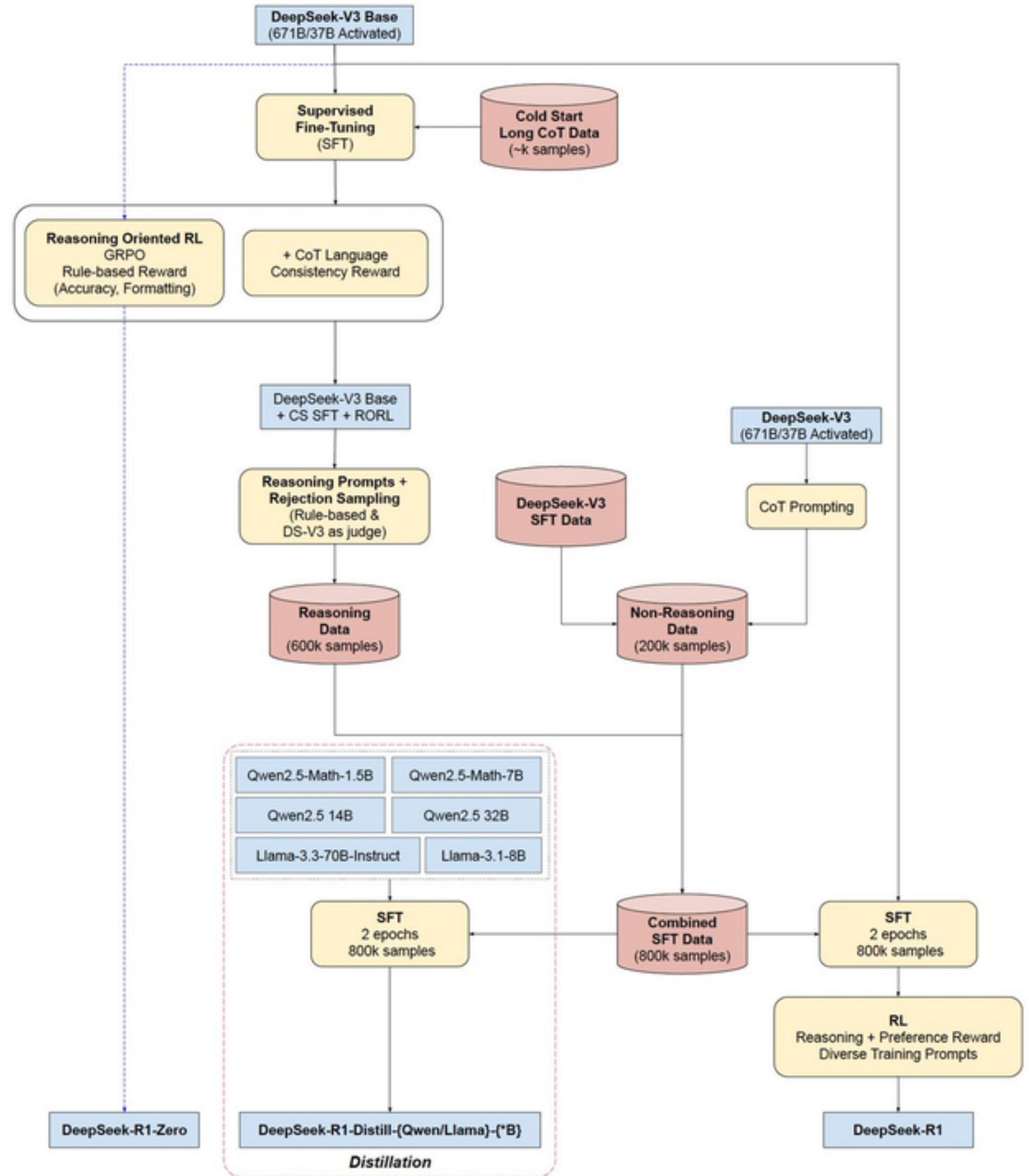
$$\sqrt{a - \sqrt{a + x}} = x$$

First, let's square both sides:

$$a - \sqrt{a+x} = x^2 \implies \sqrt{a+x} = a - x^2$$

Next, I could square both sides again, treating the equation: \dots

Table 3 | An interesting “aha moment” of an intermediate version of DeepSeek-R1-Zero. The model learns to rethink using an anthropomorphic tone. This is also an aha moment for us, allowing us to witness the power and beauty of reinforcement learning.



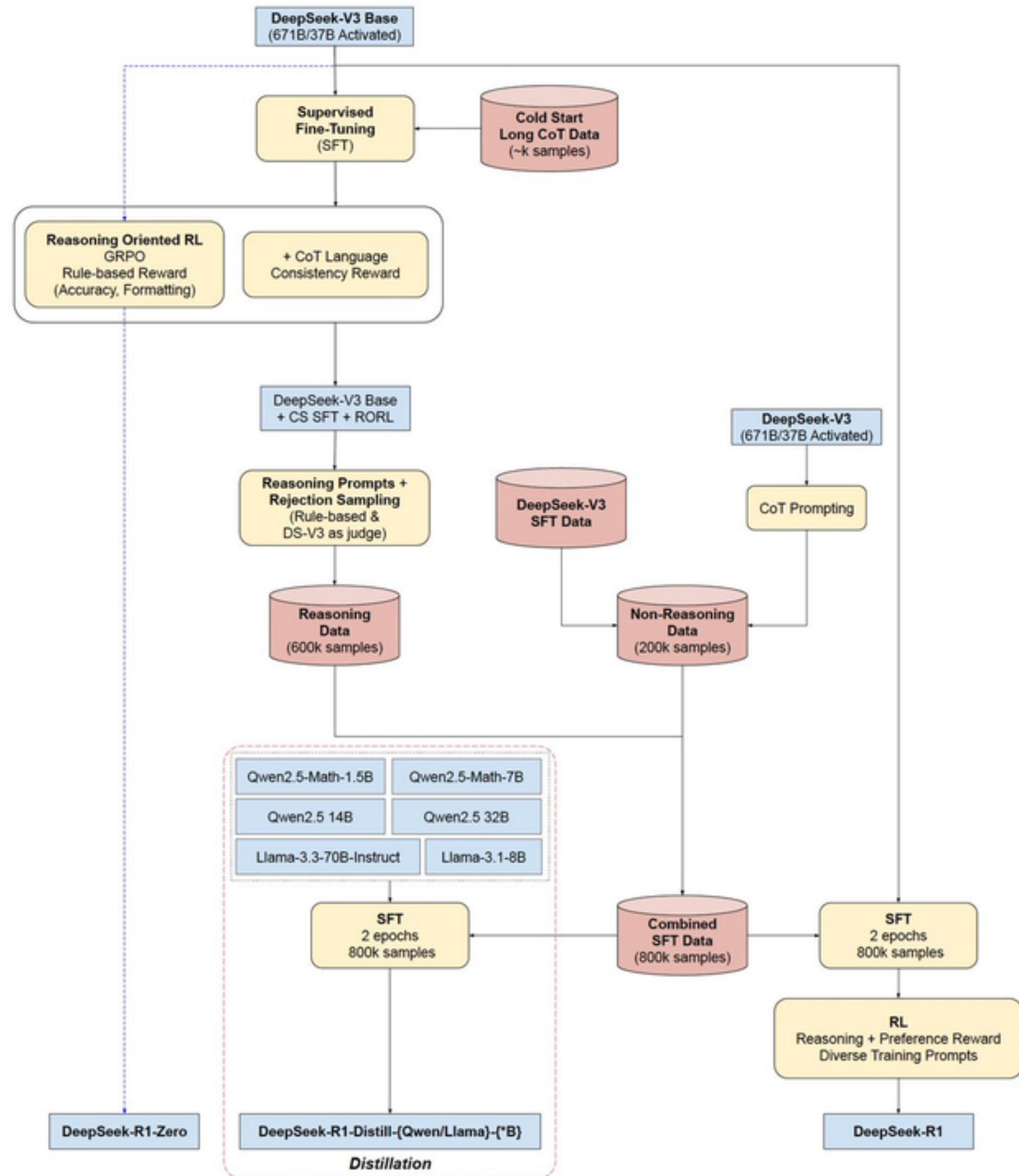
Deepseek r1-zero

2.2.2. Reward Modeling

The reward is the source of the training signal, which decides the optimization direction of RL. To train DeepSeek-R1-Zero, we adopt a rule-based reward system that mainly consists of two types of rewards:

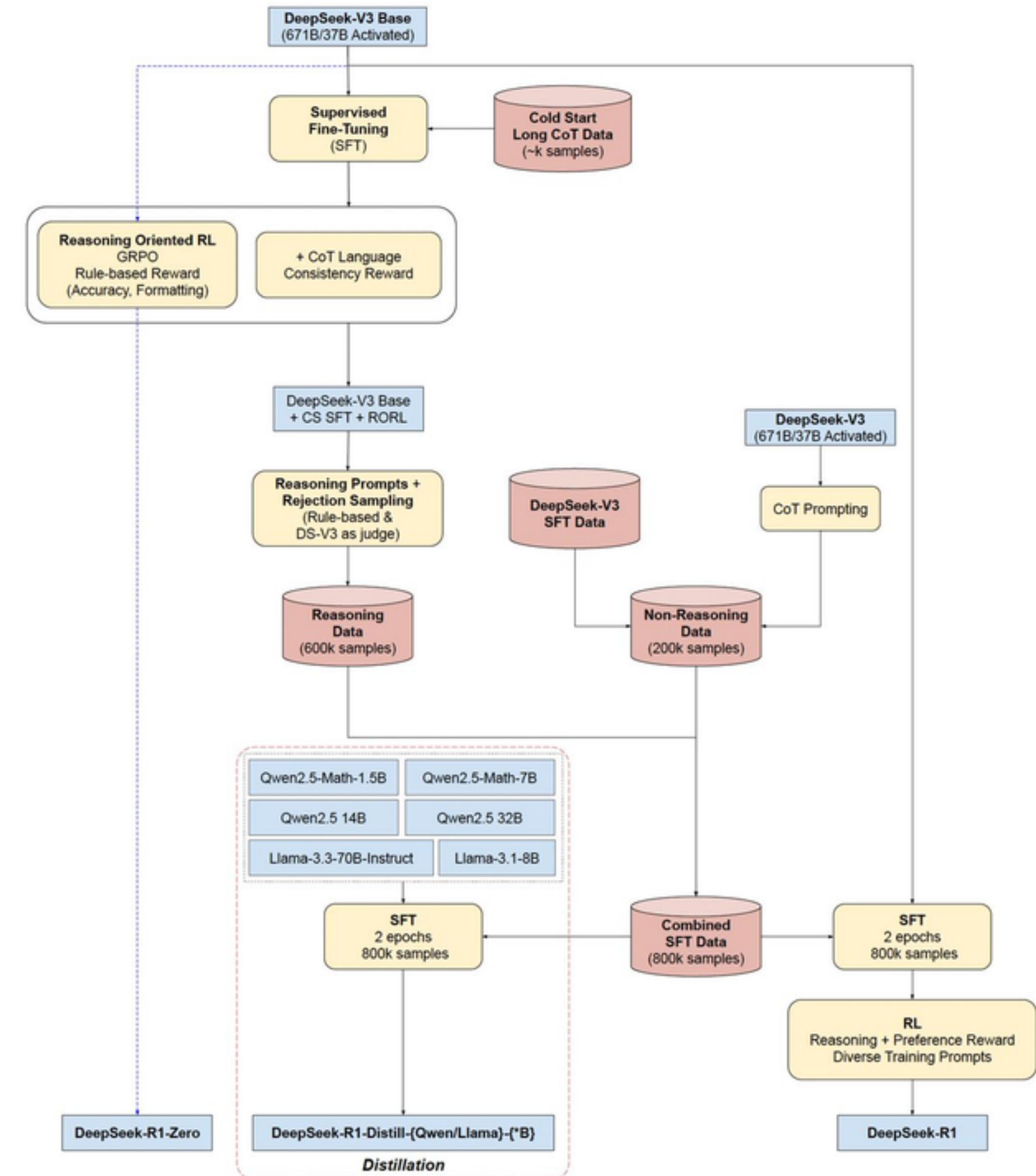
- **Accuracy rewards:** The accuracy reward model evaluates whether the response is correct. For example, in the case of math problems with deterministic results, the model is required to provide the final answer in a specified format (e.g., within a box), enabling reliable rule-based verification of correctness. Similarly, for LeetCode problems, a compiler can be used to generate feedback based on predefined test cases.
- **Format rewards:** In addition to the accuracy reward model, we employ a format reward model that enforces the model to put its thinking process between '<think>' and '</think>' tags.

- Problem: Poor readability
- In paper, data distribution isn't mentioned



Deepseek r1

- Goal: faster RL convergence + user-friendly reasoning trace
- Cold start with long CoT SFT
- Language consistency reward
- Towards RL convergence:
 - LLM-as-judge for non-ruled based problems
 - Non reasoning SFT data
 - -> combine to SFT
- Outperform DeepSeek r1-zero



Deepseek r1

- Finally, distilled r1 into smaller open-sourced models
 - Outperform / on par with o1

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
GPT-4o-0513	9.3	13.4	74.6	49.9	32.9	759
Claude-3.5-Sonnet-1022	16.0	26.7	78.3	65.0	38.9	717
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	1820
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9	1316
DeepSeek-R1-Distill-Qwen-1.5B	28.9	52.7	83.9	33.8	16.9	954
DeepSeek-R1-Distill-Qwen-7B	55.5	83.3	92.8	49.1	37.6	1189
DeepSeek-R1-Distill-Qwen-14B	69.7	80.0	93.9	59.1	53.1	1481
DeepSeek-R1-Distill-Qwen-32B	72.6	83.3	94.3	62.1	57.2	1691
DeepSeek-R1-Distill-Llama-8B	50.4	80.0	89.1	49.0	39.6	1205
DeepSeek-R1-Distill-Llama-70B	70.0	86.7	94.5	65.2	57.5	1633

