

# **The hadron structure oriented approach to TMD phenomenology**

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Based on:

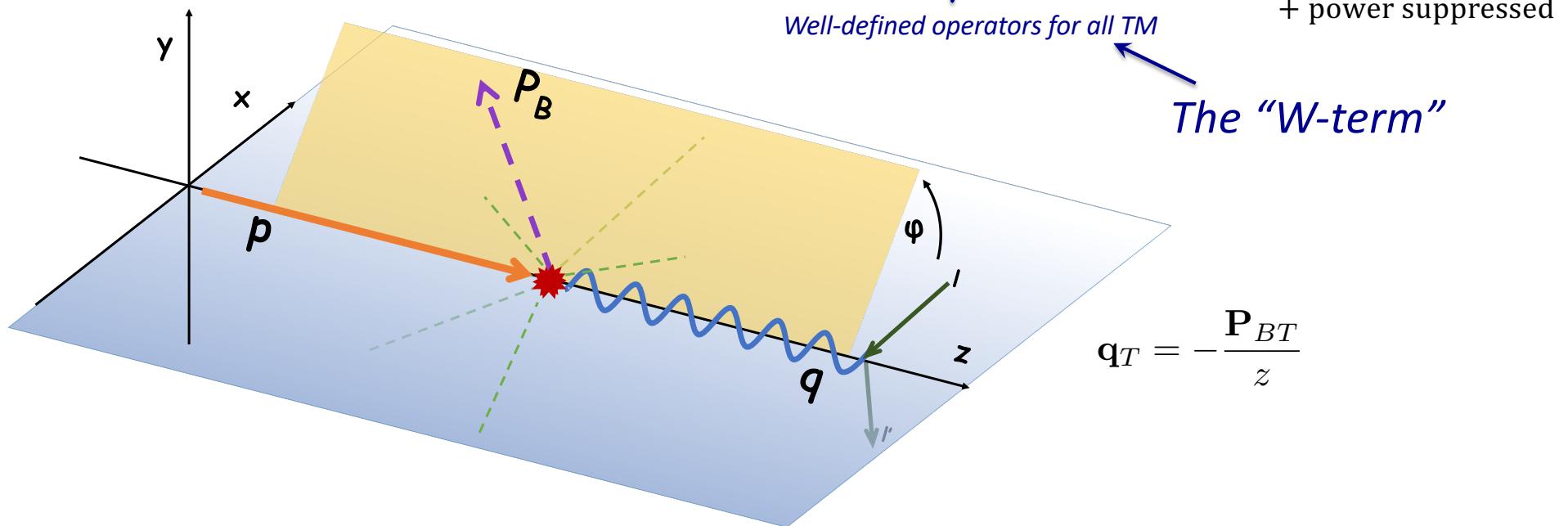
- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- J.O. Gonzalez, T. Rainaldi, TCR (2023), Phys.Rev.D 107 (2023) 9, 094029  
&
- Work in progress with F. Aslan, M. Boglione, J.O. Gonzalez, T. Rainaldi, A. Simonelli

JLab EIC Meeting, October 13, 2023

## Transverse momentum dependent (TMD) factorization & SIDIS $\left(\text{for } \frac{q_T}{Q} \ll 1\right)$

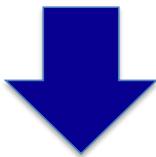
$$\frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} = H(Q/\mu_Q) \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} \underbrace{f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2)}_{\text{Well-defined operators for all TM}} \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

+ power suppressed



## **Simplistic parton model**

$$\frac{d\sigma}{dQ \, dx} \sim \sum_i f_{i/p}(x)$$

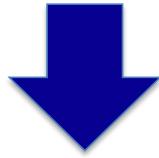


**QCD**

$$\frac{d\sigma}{dQ \, dx} = \sum_i \int_x^1 \frac{d\xi}{\xi} H_i(x/\xi, \alpha_s(Q)) f_{i/p}(\xi; Q)$$

## **Simplistic parton model**

$$\frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} = \sum_j \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}) D_{h/j}(z, z\mathbf{k}_{2T}) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

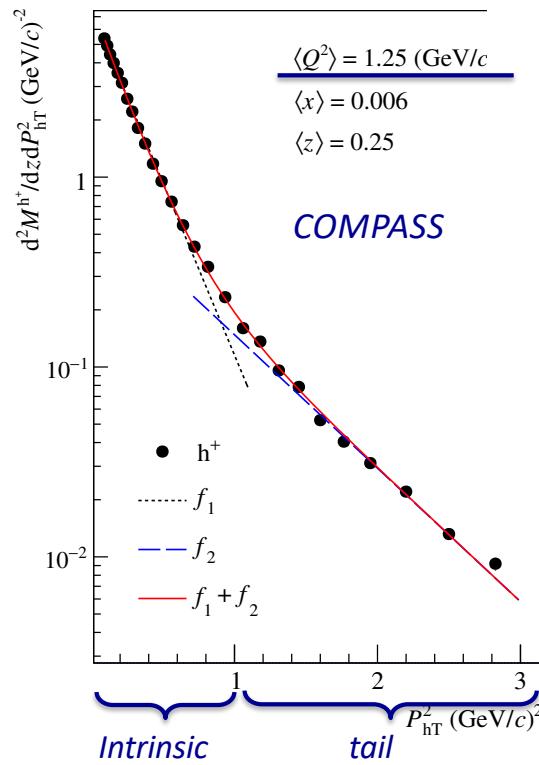


**QCD**

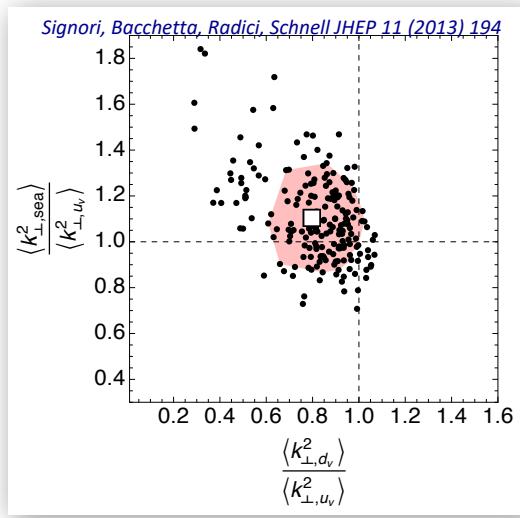
$$\begin{aligned} \frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} &= H(\alpha_s(Q)) \int d^2\mathbf{k}_{1T} \sum_j d^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T}) \\ &\quad + O\left(\frac{q_T^2}{Q^2}\right) \text{ corrections (the "Y-term")} \end{aligned}$$

# Nonperturbative structures in pheno

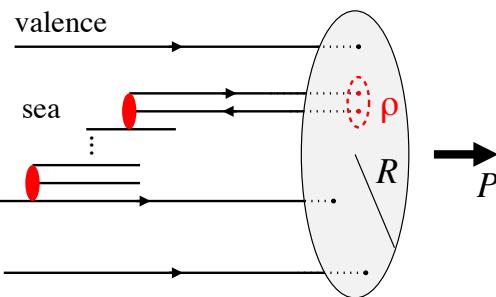
PHYSICAL REVIEW D 97, 032006 (2018)



Transverse-momentum-dependent multiplicities of charged hadrons  
in muon-deuteron deep inelastic scattering

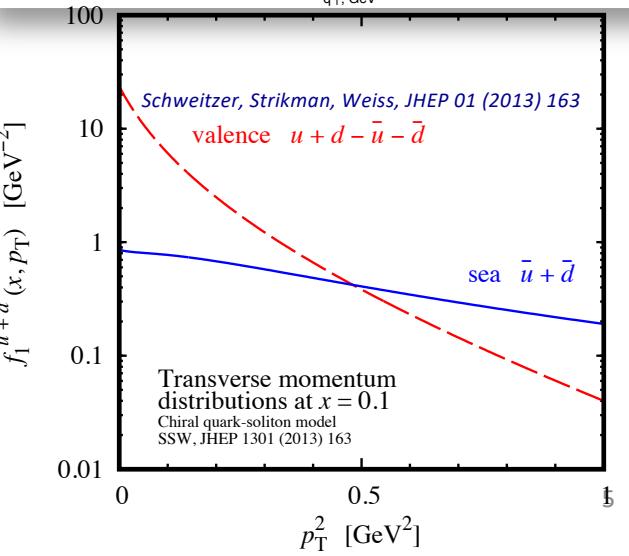
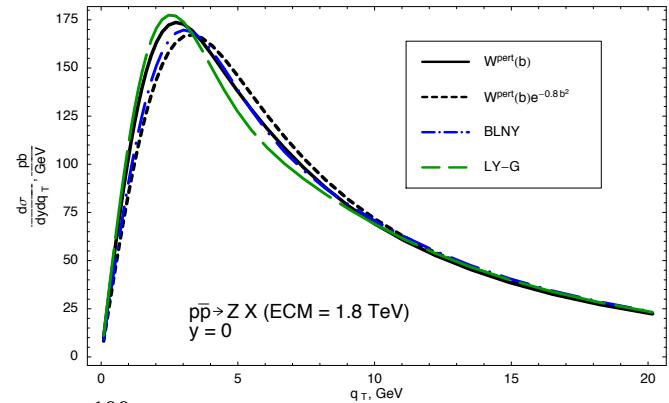


“... the two exponential functions in our parametrizations  $F_1$  can be attributed to two completely different underlying physics mechanisms that overlap in the region  $P_{hT} \simeq 1 \text{ GeV}^2$ .”

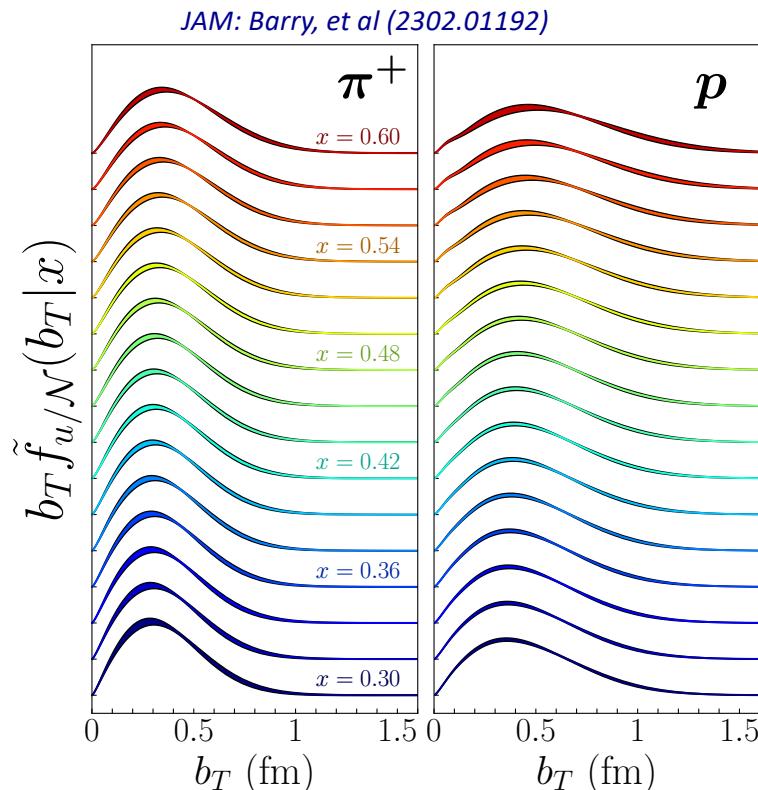


“While significant effort has been put into the study of  $W(b)$  at large  $b$  [36, 42, 43, 44], none ... adequately describe the observed  $Z$  boson distribution without introducing free parameters.”

P. Nadolsky, (2004) Theory of  $W$  and  $Z$  Production



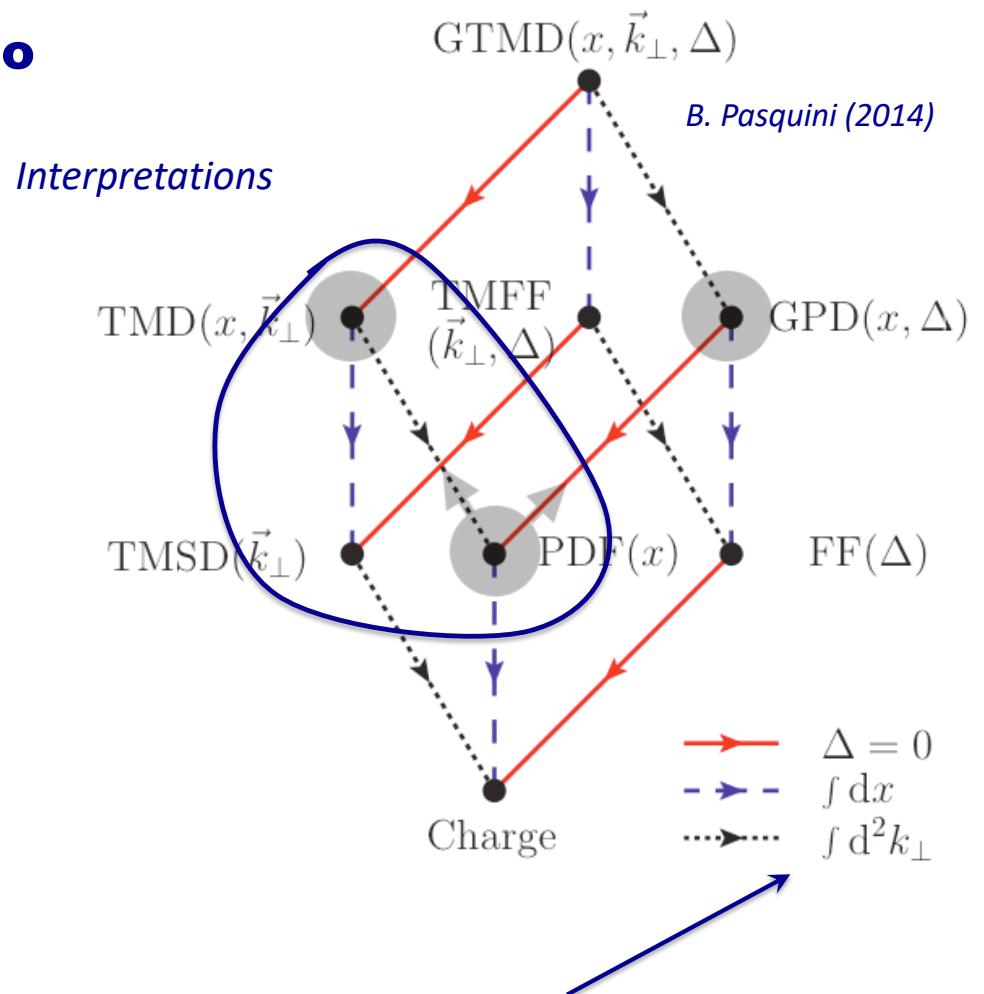
## Nonperturbative structures in pheno



??

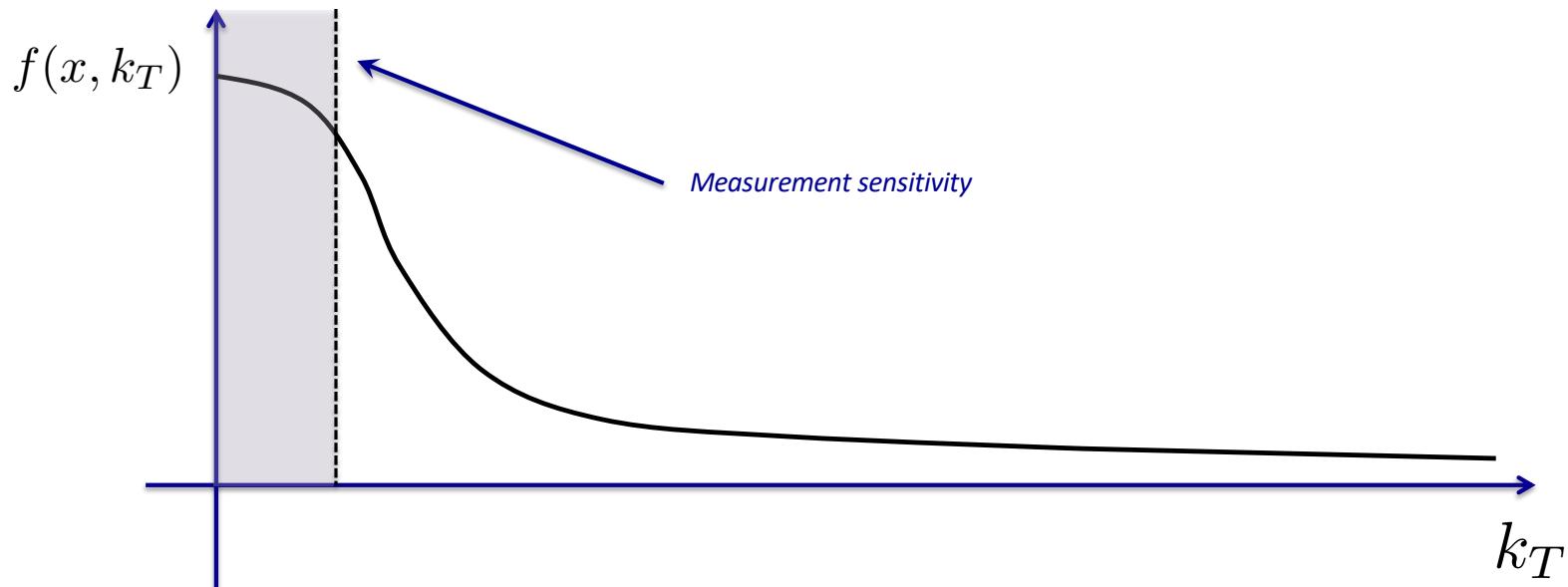
"Importantly, we have checked that the differences between the proton and pion  $\langle b_T | x \rangle$  are completely due to the nonperturbative TMD structure, independent of the collinear PDFs."

??



These integrals are often divergent in real QCD!

## Why worry about “large” $k_T$ ?



- TMD pdf exists for all  $k_T$ . Large/small  $k_T$ , consistency constrains small  $k_T$ !
- How to compare low/moderate Q TMD pdf to large Q pdf?

## Where is the hadron structure!?

$$\begin{aligned}
\frac{d\sigma}{dQ^2 dy dq_T^2} &= \frac{4\pi^2 \alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{d^2 b_T}{(2\pi)^2} e^{i\mathbf{q}_T \cdot \mathbf{b}_T} \\
&\times \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A; \mu_{b_*}) \tilde{C}_{j/j_A}^{\text{CSS1, DY}} \left( \frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*}) \right) \\
&\times \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B; \mu_{b_*}) \tilde{C}_{\bar{j}/j_B}^{\text{CSS1, DY}} \left( \frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*}) \right) \\
&\times \exp \left\{ - \int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[ A_{\text{CSS1}}(a_s(\mu'); C_1) \ln \left( \frac{\mu_Q^2}{\mu'^2} \right) + B_{\text{CSS1, DY}}(a_s(\mu'); C_1, C_2) \right] \right\} \\
&\times \exp \left[ -g_{j/A}^{\text{CSS1}}(x_A, b_T; b_{\max}) - g_{\bar{j}/B}^{\text{CSS1}}(x_B, b_T; b_{\max}) - g_K^{\text{CSS1}}(b_T; b_{\max}) \ln(Q^2/Q_0^2) \right] \\
&+ \text{suppressed corrections.}
\end{aligned}$$

*CSS1 = Collins-Soper-Sterman ( $\approx 1985$ )*

## **What is meant by “hadron structure oriented?”**

- Adhere strictly to logic of TMD factorization
- Preserve parton model interpretation
- Preserve relationships between TMD and collinear factorization
- Allow to compare/contrast different models and descriptions of hadron structure

## The conventional organization

- 1) Solve evolution equations to relate overall SIDIS hard scale ( $\mu_Q = Q$ ) to input scale ( $\mu_{Q_0} = Q_0$ )
- 2) How to use small  $b_T \ll 1/\Lambda_{QCD}$  collinear factorization?
  - Partition small ( $b_T < b_{\max}$ ) & large ( $b_T > b_{\max}$ ) regions with a  $b_*$
  - Define hard scale  $\mu_{b_*} \sim 1/b_*$
- 3) Evolve again to relate  $Q_0$  to  $\mu_{b_*}$
- 4) Place remaining NP parts in an exponent:

$$-g_{j/p}(x, b_T) \equiv \ln \left( \frac{\tilde{f}_{j/p}(x, b_T; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, b_*; \mu_{Q_0}, Q_0^2)} \right)$$

$$g_K(b_T) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_T; \mu)$$

- 5) Perform small- $b_T$  expansions & drop  $O(\Lambda_{QCD} b_{\max})$  errors
- 6) Ansatz for g-functions

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{d \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{d \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$\frac{d \tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O(b_T \Lambda_{QCD})$$

Or

$$f_{j/p}(x, k_T; \mu, \zeta) = \frac{1}{k_T^2} \left[ \int_x^1 \frac{d\xi}{\xi} C_{j/k}(x/\xi, k_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O\left(\frac{\Lambda_{QCD}}{k_T}\right) \right]$$

$$\tilde{K}(b_T; \mu) = \tilde{K}_{\text{pert}}(b_T \mu, \alpha_s(\mu)) + O(b_T \Lambda_{QCD})$$

## The conventional organization

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$$\int d^2k_{1T} d^2k_{2T} f_{j/p}(x, k_{1T}; \mu_Q, Q^2) D_{h/j}(z, z k_{2T}; \mu_Q, Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$



$$\begin{aligned} & \int \frac{d^2 b_T}{(2\pi)^2} e^{-i \mathbf{q}_T \cdot \mathbf{b}_T} \tilde{f}_{j/p}^{\text{OPE}}(x_{bj}, \mathbf{b}_*; \mu_{b_*}, \mu_{b_*}^2) \tilde{D}_{h/j}^{\text{OPE}}(z_h, \mathbf{b}_*; \mu_{b_*}, \mu_{b_*}^2) \\ & \times \exp \left\{ 2 \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[ \gamma(\alpha_s(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*; \mu_{b_*}) \right\} \\ & \times \exp \left\{ -g_{j/p}(x_{bj}, b_T) - g_{h/j}(z_h, b_T) - g_K(b_T) \ln \left( \frac{Q^2}{Q_0^2} \right) \right\} + O(b_{\max} \Lambda_{QCD}) \end{aligned}$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \underbrace{\int_x^1 \frac{d\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu)}_{\tilde{f}_{j/p}^{\text{OPE}}(x, b_T; \mu, \zeta)} + O(b_T \Lambda_{QCD})$$

$$\tilde{f}_{j/p}^{\text{OPE}}(x, b_T; \mu, \zeta)$$

## How to test consistency

- $\int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$

is uniquely determined by its operator definition

- At  $q_T \approx Q$

$$= \underbrace{\frac{1}{q_T^2} \left[ \mathcal{C}(q_T/\mu_Q, \alpha_s(\mu_Q)) \otimes f_{j/p}(x; \mu_Q) \otimes d_{h/j}(z; \mu_Q) + O\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right) \right]}_{q_T \sim Q, Q \rightarrow \infty \text{ asymptote}}$$

- For TMD pdfs & ffs

$$f_{j/p}(x, k_T \approx Q; \mu_Q, Q^2) = \frac{1}{k_T^2} \left[ \int_x^1 \frac{d\xi}{\xi} C_{j/k}(x/\xi, k_T/Q, \alpha_s(Q)) f_{k/p}(\xi; Q) + O\left(\frac{\Lambda_{\text{QCD}}}{k_T}\right) \right]$$

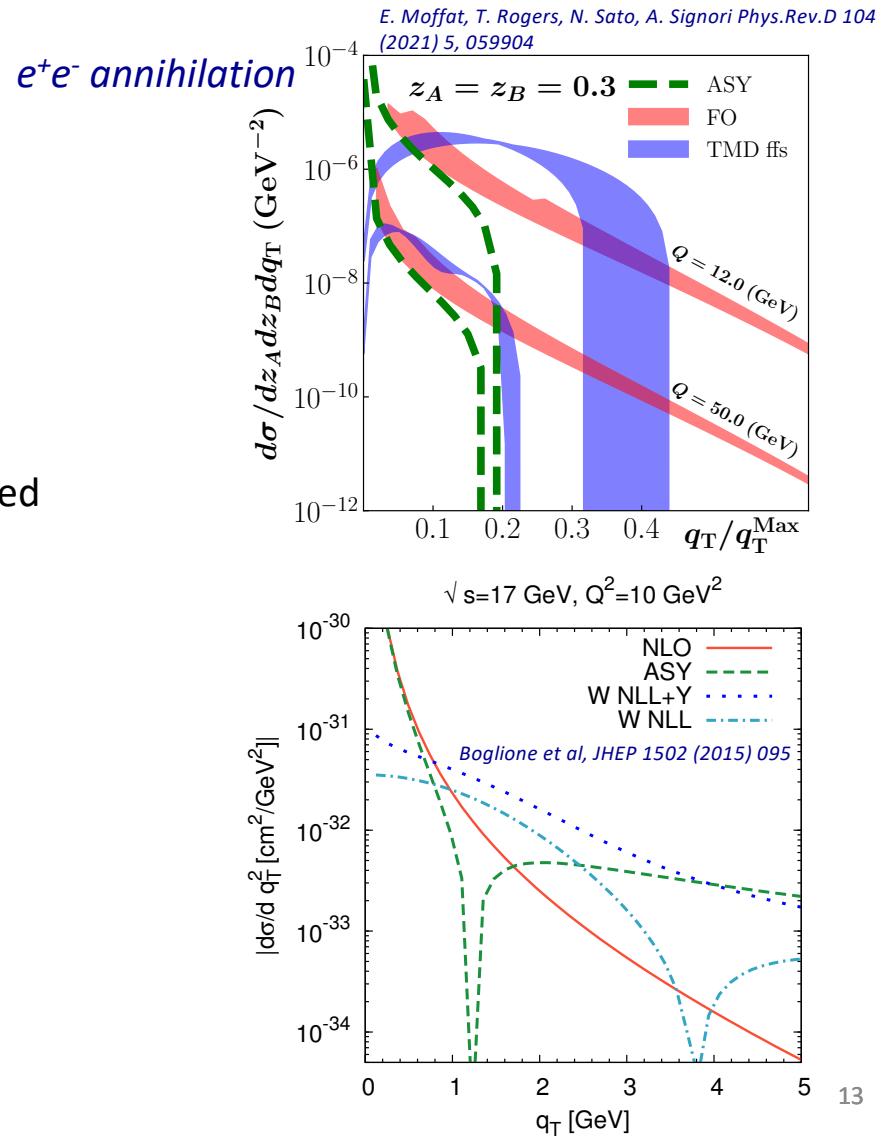
&

$$\int d^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) \approx f_{j/p}(x; \mu_Q)$$

- $\frac{d}{db_{\max}} \left( \frac{d\sigma}{d^2\mathbf{q}_T \dots} \right) = 0 \quad \text{for } b_{\max} \ll 1/\Lambda_{\text{QCD}}$

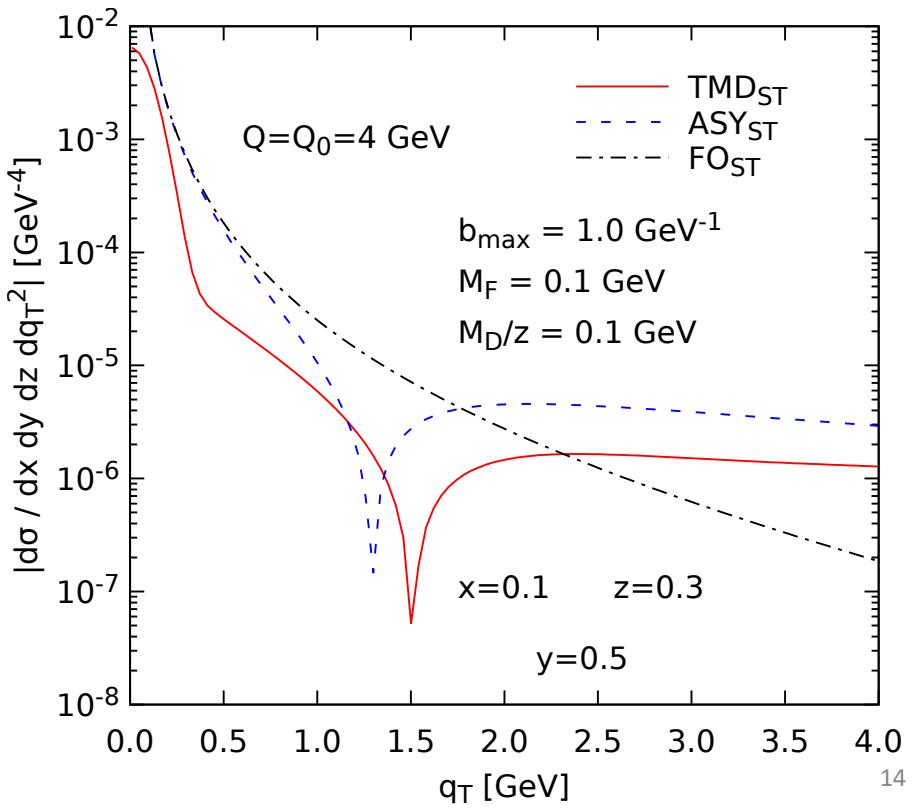
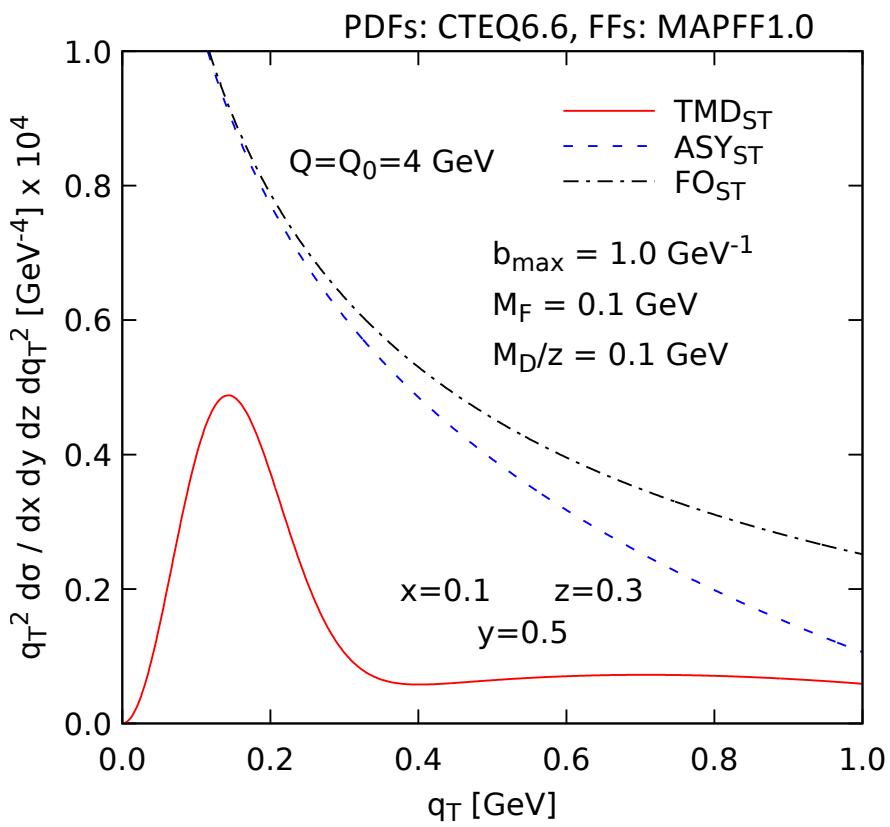
## How to test consistency

- What does  $q_T \approx Q$  mean?
  - No sensitivity to parameters related nonperturbative transverse momentum
  - $\Lambda_{QCD} \ll q_T \ll Q$ ? Look for matching between fixed order x-section and asymptotic term
    - Is there a region where both  $\frac{\Lambda_{QCD}}{q_T}$  &  $\frac{q_T}{Q}$  powers are simultaneously negligible?
  - No large logarithms: Look for node in asymptotic term



## **Conventional organization & complications**

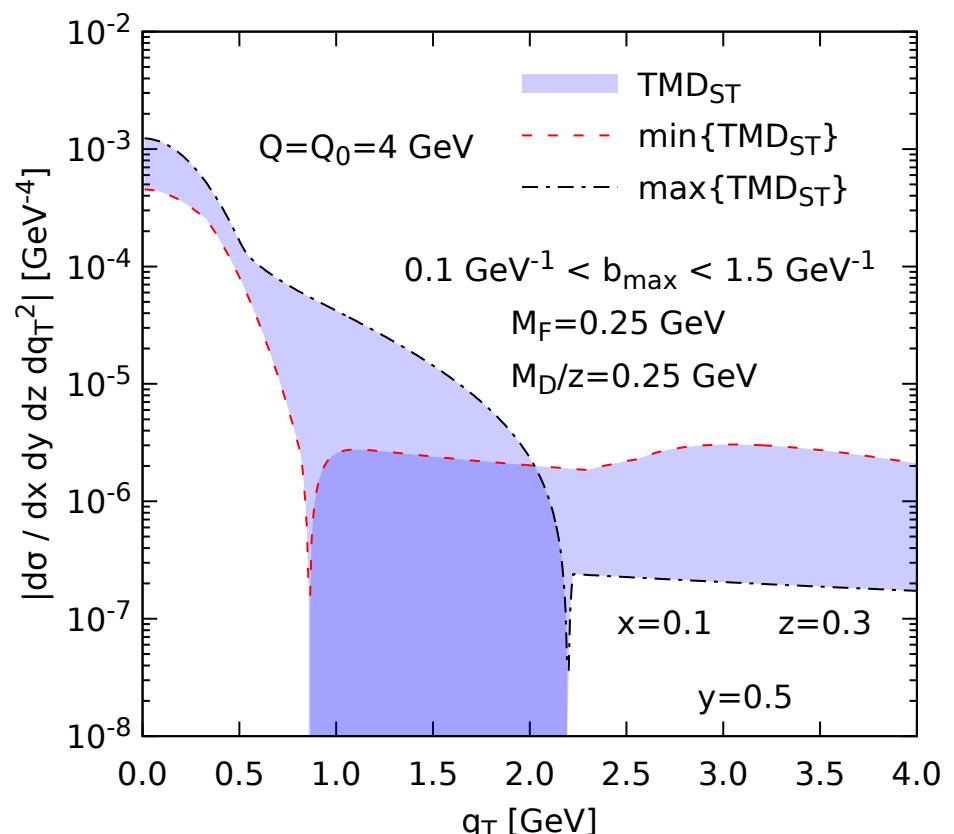
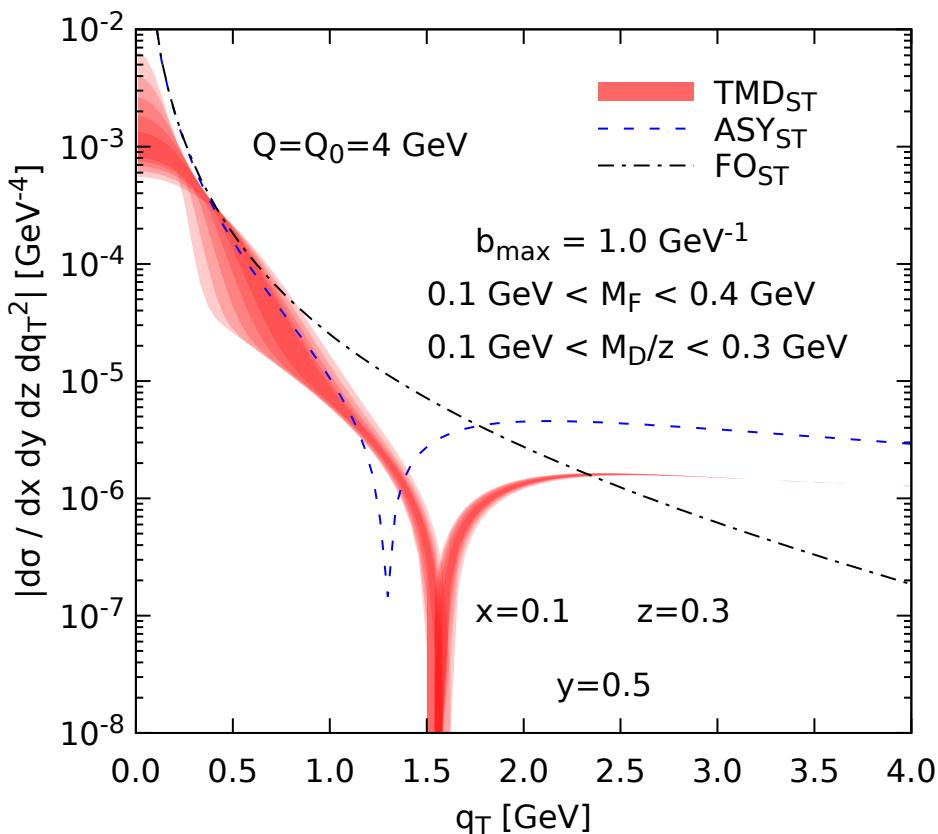
*(An example typical of conventional approach)*



$$g_{j/p}(x, b_T) = \frac{1}{4} M_F^2 b_T^2$$

$$g_{h/j}(z, b_T) = \frac{1}{4z^2} M_D^2 b_T^2$$

## Conventional organization & complications



$$g_{j/p}(x, b_T) = \frac{1}{4} M_F^2 b_T^2$$

$$g_{h/j}(z, b_T) = \frac{1}{4 z^2} M_D^2 b_T^2$$

$$\frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} = H(Q/\mu_Q) \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T}) + \text{power suppressed}$$

## Diagnosis

*Well-defined operators for all TM*

- 1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed
- 2) Fixed order perturbation theory should work fine for  $q_T \approx Q_0$ , but evol. factors have a large effect. What is going on?
- 3)  $\exists$  no region at input scale  $Q = Q_0$  where  $\Lambda_{QCD} \ll q_T \ll Q_0$
- 4) Backwards evolution...  
No large, perturbative  $\ln \frac{Q_0}{q_T}$ .
- 5)  $\int d^2\mathbf{k}_T f_{j/p}(x, \mathbf{k}_T; \mu_Q, Q^2) \approx f_{j/p}(x; \mu_Q)$   
Very badly violated at moderate scales

$$\frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} = H(Q/\mu_Q) \underbrace{\int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})}_{\text{Well-defined operators for all TM}} + \text{power suppressed}$$

## Diagnosis

- 1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

$$\frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} =$$

- 2) Fixed order perturbation theory should work fine for  $q_T \approx Q_0$ , but evol. factors have a large effect. What is going on?

$$H(Q/\mu_Q) \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{q}_T \cdot \mathbf{b}_T} \tilde{f}_{j/p}(x, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) \tilde{D}_{h/j}(z, \mathbf{b}_T; \mu_{Q_0}, Q_0^2)$$

$$\times \exp \left\{ \tilde{K}(\mathbf{b}_T; \mu_{Q_0}) \ln \left( \frac{Q^2}{Q_0^2} \right) + \int_{\mu_{Q_0}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[ 2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

- 3)  $\exists$  no region at input scale  $Q = Q_0$  where  $\Lambda_{QCD} \ll q_T \ll Q_0$

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## Diagnosis

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$$\frac{d\sigma}{dQ d^2\mathbf{q}_T dx dz} =$$

- 2) Fixed order perturbation theory should work fine for  $q_T \approx Q_0$ , but evol. factors have a large effect. What is going on?

$$H(Q/\mu_Q) \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{q}_T \cdot \mathbf{b}_T} \tilde{f}_{j/p}(x, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) \tilde{D}_{h/j}(z, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) \\ \times \exp \left\{ \tilde{K}(\mathbf{b}_T; \mu_{Q_0}) \ln \left( \frac{Q^2}{Q_0^2} \right) + \int_{\mu_{Q_0}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[ 2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

- 3)  $\exists$  no region at input scale  $Q = Q_0$  where  $\Lambda_{QCD} \ll q_T \ll Q_0$

$$\tilde{D}_{h/j}(z, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) = \tilde{D}_{\text{inpt}, h/j}(z, \mathbf{b}_T; \mu_{\bar{Q}_0}, \bar{Q}_0^2) E(\bar{Q}_0/Q_0, b_T)$$

- 4) Backwards evolution...

No large, perturbative  $\ln \frac{Q_0}{q_T}$ .

$$\lim_{b_T \rightarrow 0} \bar{Q}_0 \sim 1/b_T$$

- 5)  $\int d^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) \approx f_{j/p}(x; \mu_Q)$

Very badly violated at moderate scales

*Instead, characterize the full range of  $k_T$  behavior of TMD functions at the input scale*

## A hadron structure oriented (HSO) reorganization

- 1) Use the uniquely determined TMDs for all  $k_T$
- 2) Smoothly interpolate between nonperturbative TM dependence at small TM ( $k_T \approx \Lambda_{QCD}$ ) & perturbative (collinear) TM at large TM ( $k_T \approx Q$ )
- 3) (Approximate) probability interpretation

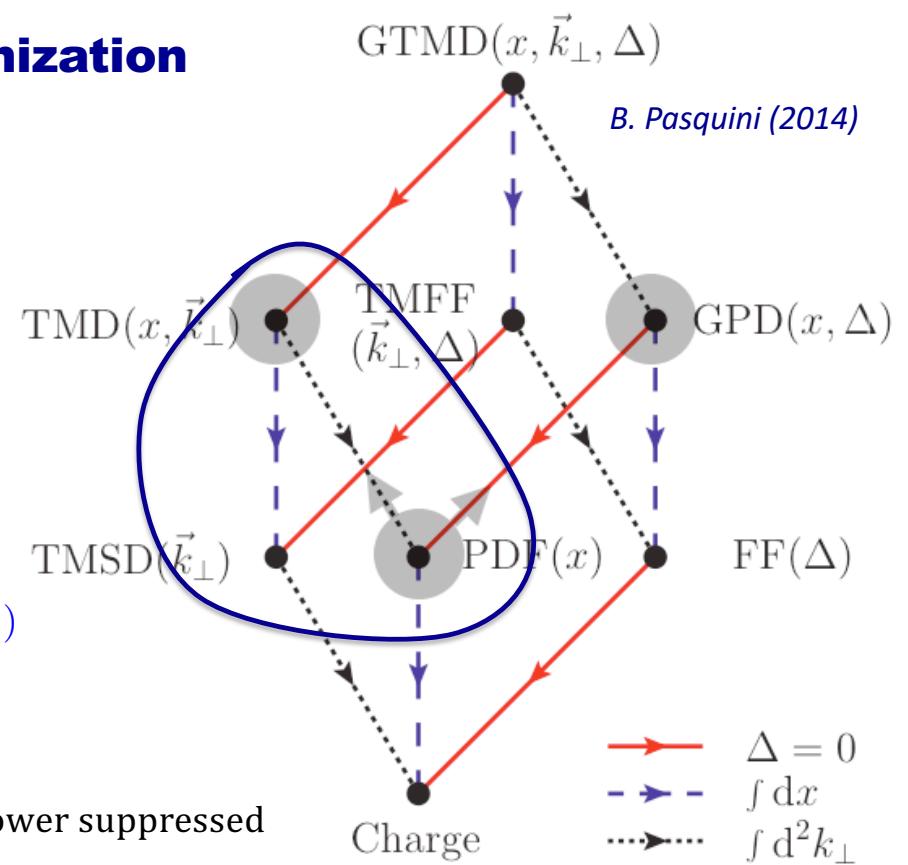
- Parton model:  $\int d^2 k_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$

- QCD:

$$\pi \int^{\mu_Q^2} dk_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q) + \Delta_{j/p} + \text{power suppressed}$$

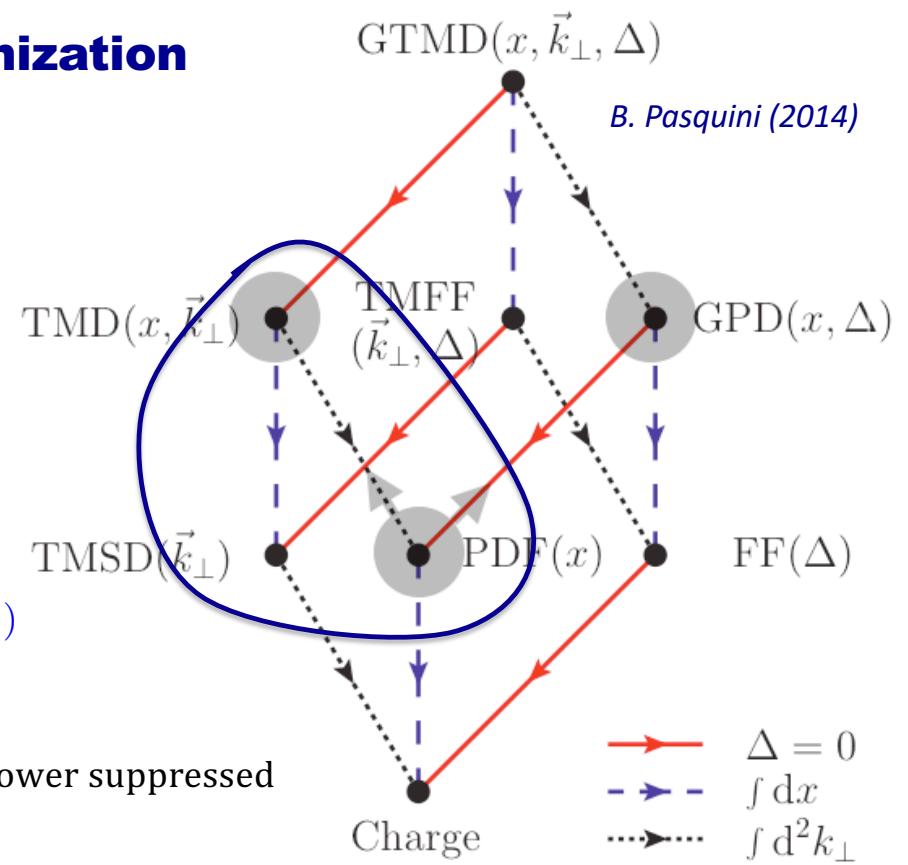
*The "W-term"*

$$\overline{\text{MS}} \quad \sum_{j'} \underbrace{\mathcal{C}_{j/j'}^\Delta \otimes f_{j'/p}}_{\mathcal{O}(\alpha_s)}$$



## A hadron structure oriented (HSO) reorganization

- 1) Use the uniquely determined TMDs for all  $k_T$
- 2) Smoothly interpolate between **nonperturbative** TM dependence at **small** TM ( $k_T \approx \Lambda_{QCD}$ ) & **perturbative** (collinear) TM at **large** TM ( $k_T \approx Q$ )
- 3) (Approximate) probability interpretation
  - Parton model:  $\int d^2k_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$
  - QCD:  $\pi \int_{\mu_Q^2} dk_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q) + \Delta_{j/p}$   
+ power suppressed
- 4) All should apply at an input scale,  $Q_0$
- 5) Pheno requirement: Must be simple to swap one model/parametrization for another while still satisfying 1-4



## A hadron structure oriented (HSO) reorganization

- Provides a recipe to transform a NP TMD parametrization into an evolved parametrization at other scales:
  - *Sec. VI of Phys.Rev.D 106 (2022) 3, 034002*
- No  $b_{\max}$  or  $b_*$  necessary
- HSO approach is equivalent to standard TMD factorization, CSS, etc, just with additional consistency constraints on the g-functions
- It is straightforward to translate between standard treatment and HSO
  - *Sec. IX of Phys.Rev.D 106 (2022) 3, 034002*

Called “bottom up”  
approach here

## An $\mathcal{O}(\alpha_s)$ example with $\overline{\text{MS}}$ pdfs and ffs

- Parametrizing the very small transverse momentum

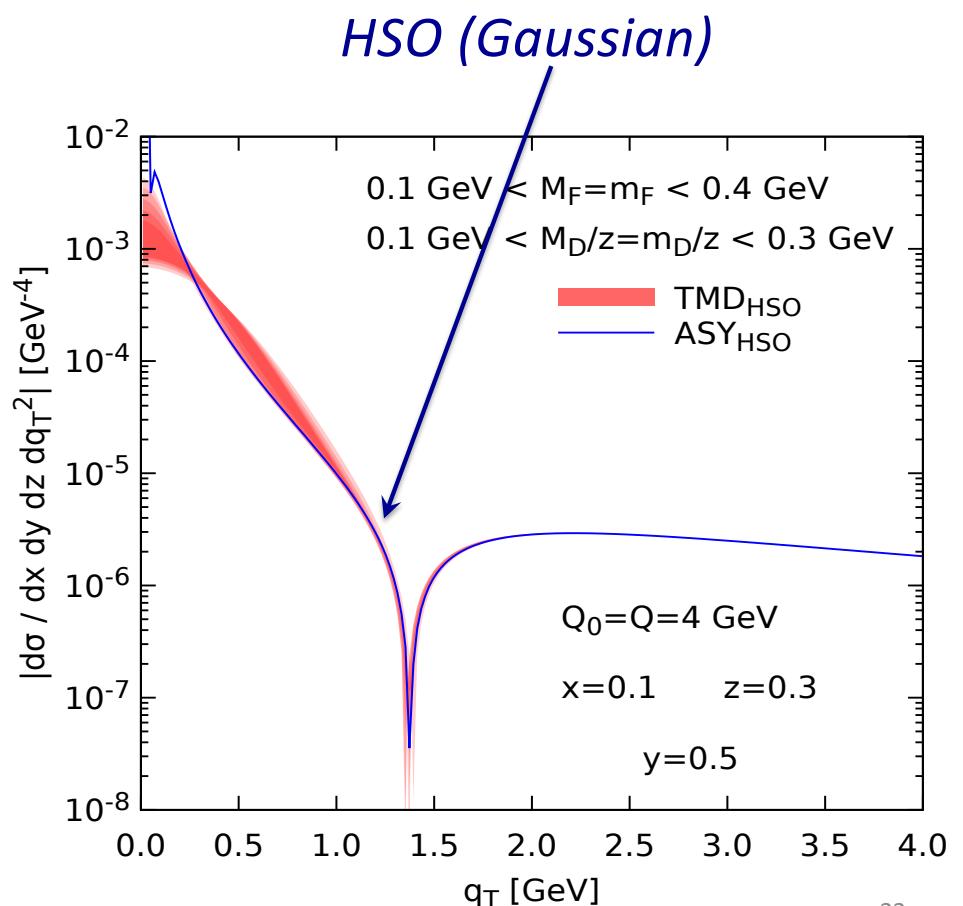
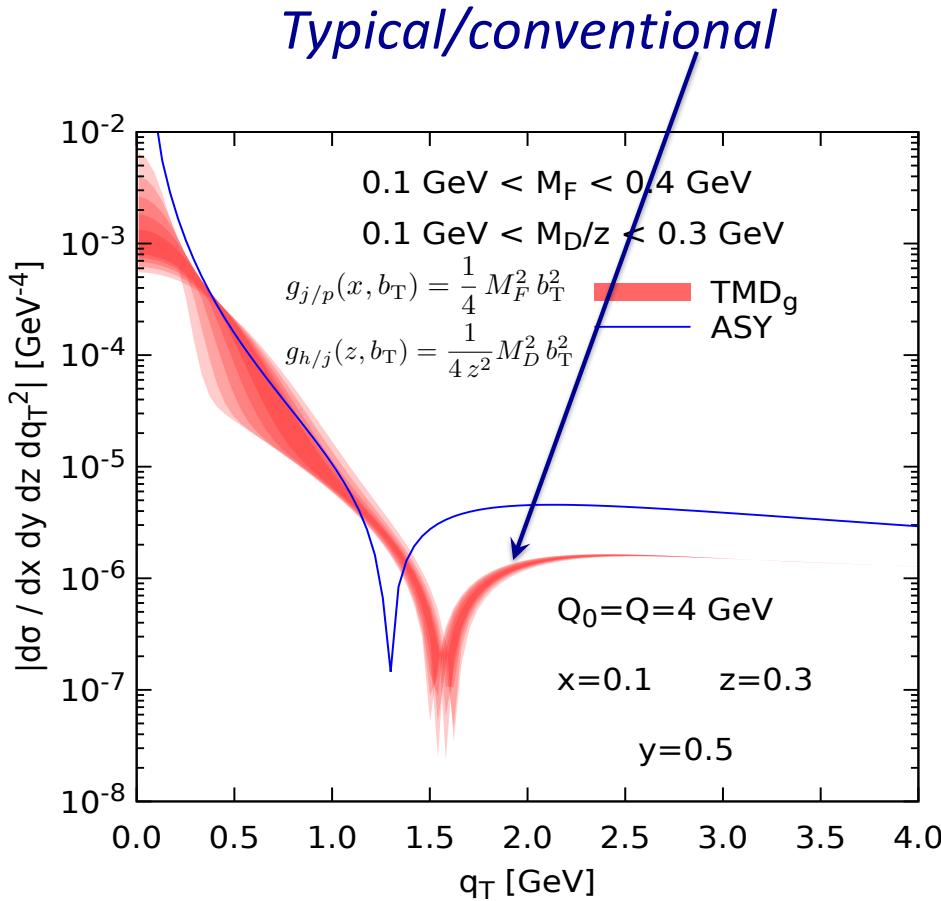
### A. Gaussian model (very commonly used)

$$f_{\text{core},i/p}^{\text{Gauss}}(x, \mathbf{k}_T; Q_0^2) = \frac{e^{-k_T^2/M_F^2}}{\pi M_F^2}, \quad D_{\text{core},h/j}^{\text{Gauss}}(z, z\mathbf{k}_T; Q_0^2) = \frac{e^{-z^2 k_T^2/M_D^2}}{\pi M_D^2}$$

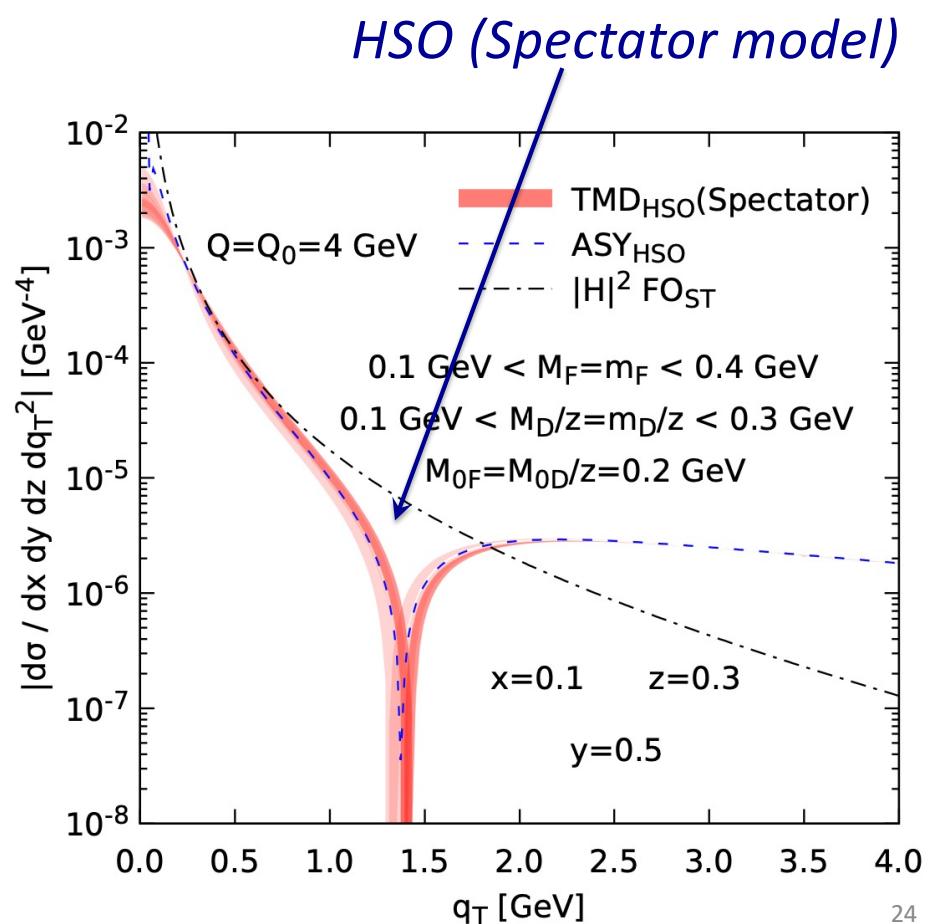
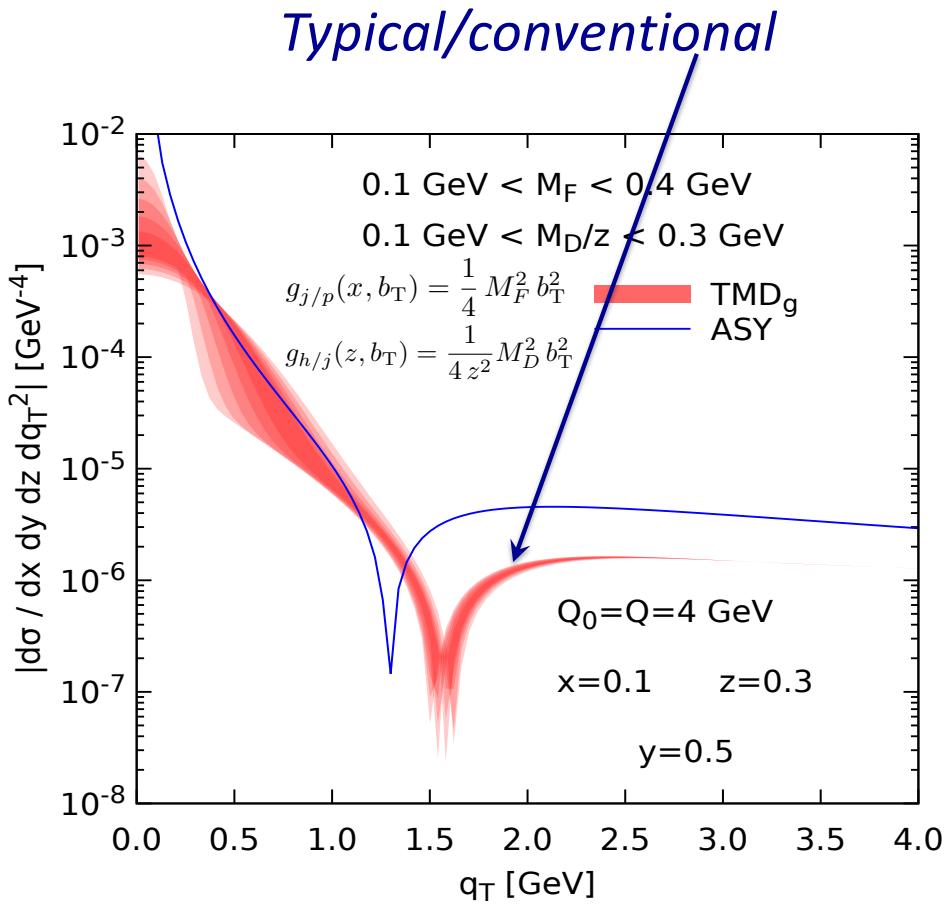
### B. Spectator model

$$f_{\text{core},i/p}^{\text{Spect}}(x, \mathbf{k}_T; Q_0^2) = \frac{6M_{0F}^6}{\pi (2M_F^2 + M_{0F}^2)} \frac{M_F^2 + k_T^2}{(M_{0F}^2 + k_T^2)^4}, \quad D_{\text{core},h/j}^{\text{Spect}}(z, z\mathbf{k}_T; Q_0^2) = \frac{2M_{0D}^4}{\pi (M_D^2 + M_{0D}^2)} \frac{M_D^2 + k_T^2 z^2}{(M_{0D}^2 + k_T^2 z^2)^3}$$

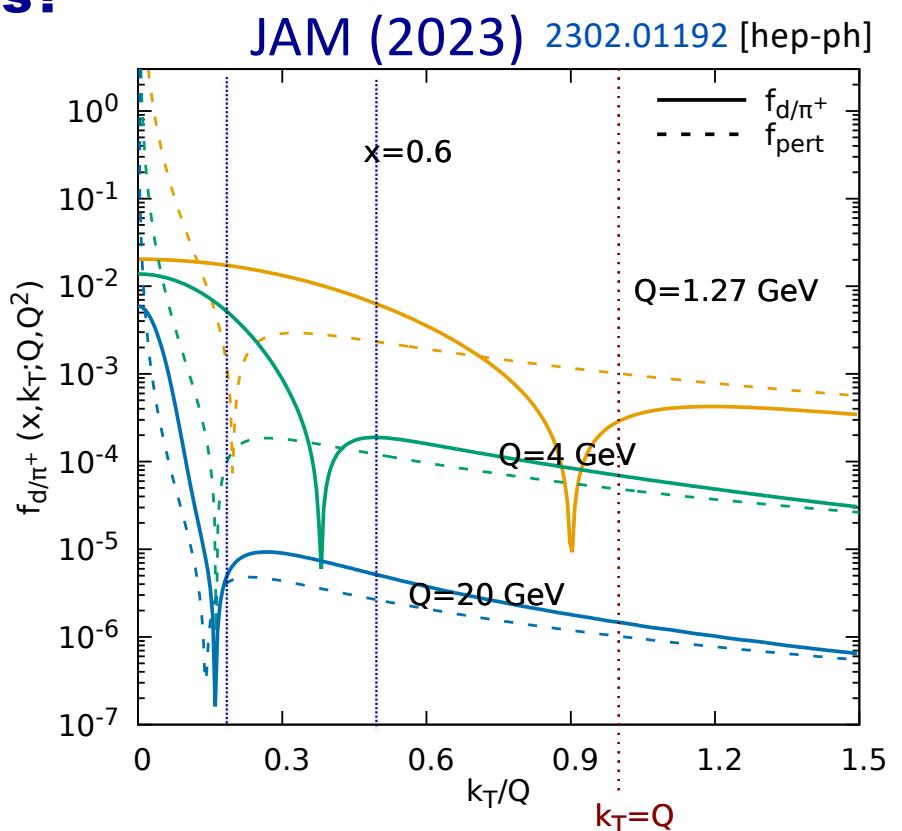
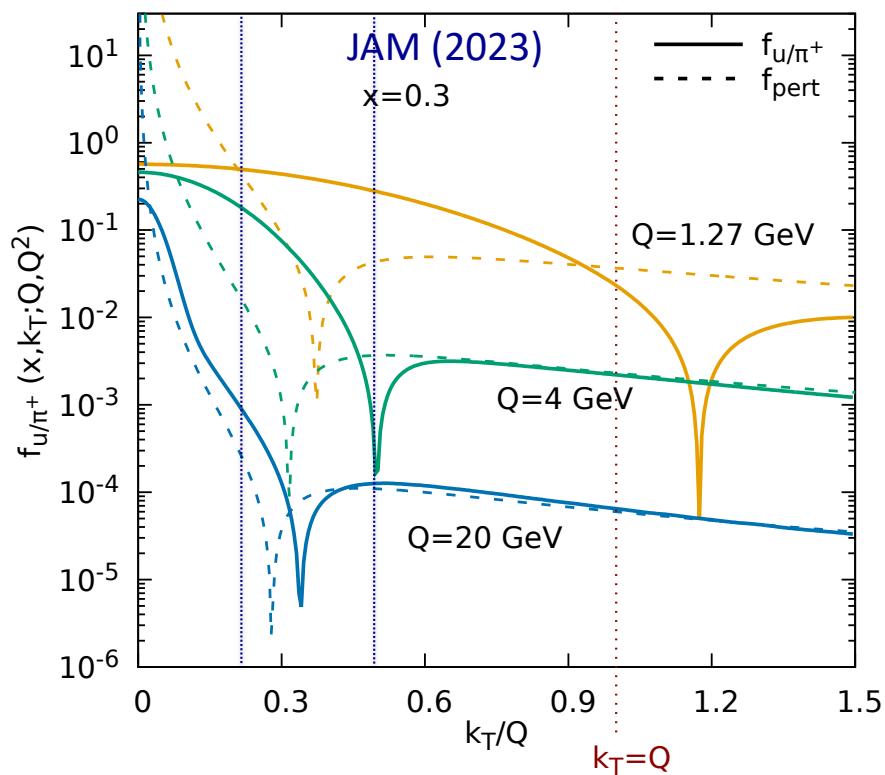
## Compare standard/unconstrained with HSO ( $\mathcal{O}(\alpha_s)$ )



## Compare standard/unconstrained with HSO ( $\mathcal{O}(\alpha_s)$ )



## What about the individual TMD pdfs?

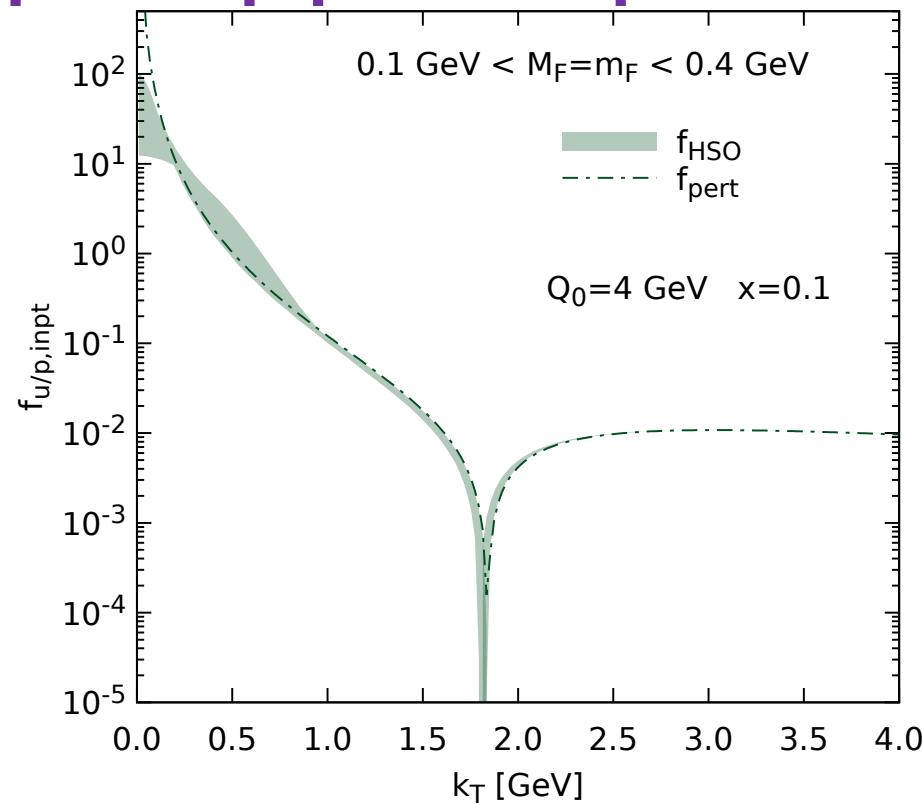


## Summary

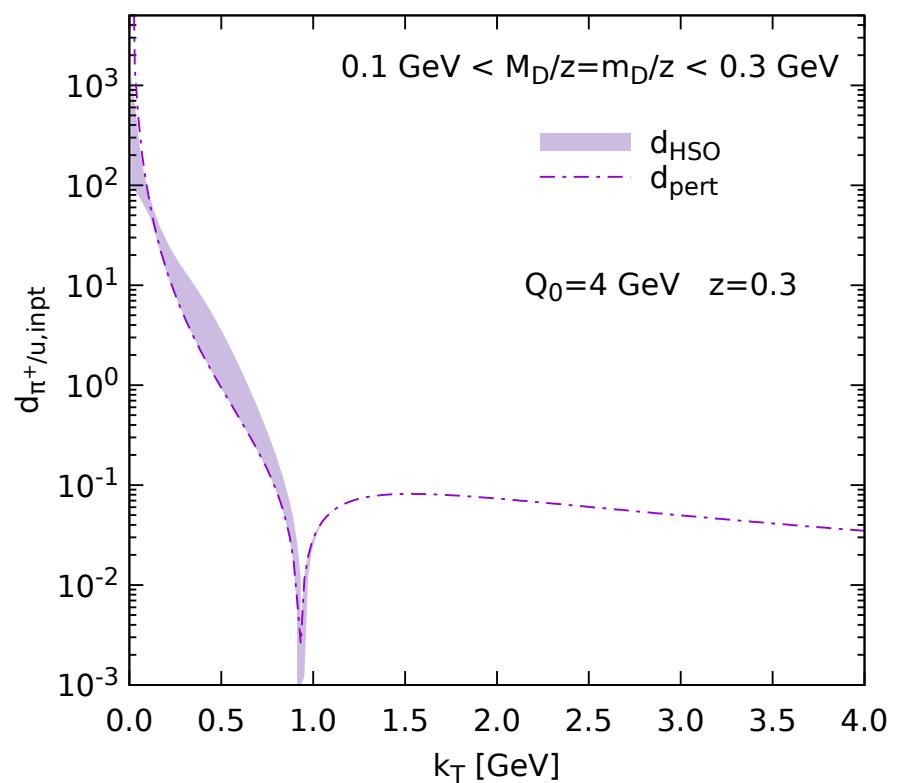
- Switching to a hadron-structure-oriented approach to pheno with TMD factorization improves consistency in the large transverse behavior of TMD correlation functions
- Necessary for understanding the shapes of nonperturbative distributions, separating perturbative and nonperturbative parts, etc
- Necessary for transforming calculations of nonperturbative TM physics into testable/falsifiable predictions
- HSO is not a new formalism; **HSO = “standard CSS”!**
- Next:
  - Applications
  - Higher orders
  - Incorporating NP calculations (lattice, EFTs, models etc)
  - Spin dependent TMDs

## What about the individual TMD pdfs?

### proton up-quark TMD pdfs



### $\pi^+$ from up-quark TMD ff



*HSO (Gaussian)*

## Backup

$$\begin{aligned}
A_{i/p}^f(x; \mu_{Q_0}) &\equiv \sum_{ii'} \delta_{i'i} \frac{\alpha_s(\mu_{Q_0})}{\pi} \left\{ \left[ (P_{i'i} \otimes f_{i'/p})(x; \mu_{Q_0}) \right] - \frac{3C_F}{2} f_{i'/p}(x; \mu_{Q_0}) \right\}, \\
B_{i/p}^f(x; \mu_{Q_0}) &\equiv \sum_{i'i} \delta_{i'i} \frac{\alpha_s(\mu_{Q_0}) C_F}{\pi} f_{i'/p}(x; \mu_{Q_0}), \\
A_{i/p}^{f,g}(x; \mu_{Q_0}) &\equiv \frac{\alpha_s(\mu_{Q_0})}{\pi} \left[ (P_{ig} \otimes f_{g/p})(x; \mu_{Q_0}) \right], \\
C_{i/p}^f &\equiv \frac{1}{N_{i/p}^f} \left[ f_{i/p}(x; \mu_{Q_0}) - A_{i/p}^f(x; \mu_{Q_0}) \ln \left( \frac{\mu_{Q_0}}{m_{f_{i,p}}} \right) - B_{i/p}^f(x; \mu_{Q_0}) \ln \left( \frac{\mu_{Q_0}}{m_{f_{i,p}}} \right) \ln \left( \frac{Q_0^2}{\mu_{Q_0} m_{f_{i,p}}} \right), \right. \\
&\quad \left. - A_{i/p}^{f,g}(x; \mu_{Q_0}) \ln \left( \frac{\mu_{Q_0}}{m_{f_{g,p}}} \right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi} \left\{ \sum_{ii'} \delta_{i'i} [\mathcal{C}_\Delta^{i/i'} \otimes f_{i'/p}](x; \mu_{Q_0}) + [\mathcal{C}_\Delta^{i/g} \otimes f_{g/p}](x; \mu_{Q_0}) \right\} \right].
\end{aligned}$$

$$P_{ig}(x) = T_F [x^2 + (1-x)^2],$$

$$\mathcal{C}_\Delta^{i/i}(x) = C_F(1-x) - C_F \frac{\pi^2}{12} \delta(1-x),$$

$$\mathcal{C}_\Delta^{g/p}(x) = 2T_F x(1-x),$$

$$N_{i/p}^f \equiv 2\pi \int_0^\infty dk_T k_T f_{\text{core},i/p}(x, \mathbf{k}_T; Q_0^2)$$

## Backup

$$\begin{aligned}
A_{h/j}^D(z; \mu_{Q_0}) &\equiv \sum_{jj'} \delta_{j'j} \frac{\alpha_s(\mu_{Q_0})}{\pi} \left\{ [(P_{jj'} \otimes d_{h/j'})(z; \mu_{Q_0})] - \frac{3C_F}{2} d_{h/j'}(z; \mu_{Q_0}) \right\}, \\
B_{h/j}^D(z; \mu_{Q_0}) &\equiv \sum_{jj'} \delta_{j'j} \frac{\alpha_s(\mu_{Q_0}) C_F}{\pi} d_{h/j'}(z; \mu_{Q_0}), \\
A_{h/j}^{D,g}(z; \mu_{Q_0}) &\equiv \frac{\alpha_s(\mu_{Q_0})}{\pi} [(P_{gj} \otimes d_{h/g})(z; \mu_{Q_0})], \\
C_{h/j}^D &\equiv \frac{1}{N_{h/j}^D} \left[ d_{h/j}(z; \mu_{Q_0}) - A_{h/j}^D(z; \mu_{Q_0}) \ln \left( \frac{\mu_{Q_0}}{m_{D_{h,j}}} \right) - B_{h/j}^D(z; \mu_{Q_0}) \ln \left( \frac{\mu_{Q_0}}{m_{D_{h,j}}} \right) \ln \left( \frac{Q_0^2}{\mu_{Q_0} m_{D_{h,j}}} \right), \right. \\
&\quad \left. - A_{h/j}^{D,g}(z; \mu_{Q_0}) \ln \left( \frac{\mu_{Q_0}}{m_{D_{h,g}}} \right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi} \left\{ \sum_{jj'} \delta_{j'j} [\mathcal{C}_\Delta^{j'/j} \otimes d_{h/j'}](z; \mu_{Q_0}) + [\mathcal{C}_\Delta^{g/j} \otimes d_{h/g}](z; \mu_{Q_0}) \right\} \right].
\end{aligned}$$

$$P_{qq}(z) = P_{\bar{q}\bar{q}}(z) = C_F \left[ \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right],$$

$$P_{gq}(z) = C_F \frac{1+(1-z)^2}{z},$$

$$\mathcal{C}_\Delta^{q/q}(z) = 2P_{qq}(z) \ln z + C_F(1-z) - C_F \frac{\pi^2}{12} \delta(1-z),$$

$$\mathcal{C}_\Delta^{g/q}(z) = 2P_{gq}(z) \ln z + C_F z,$$

$$N_{h/j}^D \equiv 2\pi z^2 \int_0^\infty dk_T k_T D_{\text{core}, h/j}(z, z\mathbf{k}_T; Q_0^2).$$

## Backup

$$\begin{aligned}
f_{\text{inpt},i/p}^c(x; \mu_{Q_0}) &= 2\pi \int_0^{\mu_{Q_0}} dk_T k_T f_{\text{inpt},i/p}(x, \mathbf{k}_T; \mu_{Q_0}, Q_0^2) = \\
&C_{i/p}^f f_{\text{core},i/p}^c(x; \mu_{Q_0}) + \frac{1}{2} A_{i/p}^{f,g}(x; \mu_{Q_0}) \ln \left( 1 + \frac{\mu_{Q_0}^2}{m_{f_{g,p}}^2} \right) \\
&+ \frac{1}{2} A_{i/p}^f(x; \mu_{Q_0}) \ln \left( 1 + \frac{\mu_{Q_0}^2}{m_{f_{i,p}}^2} \right) + \frac{1}{4} B_{i/p}^f(x; \mu_{Q_0}) \left[ \ln^2 \left( \frac{m_{f_{i,p}}^2}{Q_0^2} \right) - \ln^2 \left( \frac{\mu_{Q_0}^2 + m_{f_{i,p}}^2}{Q_0^2} \right) \right] \\
&= f_{i/p}(x; \mu_{Q_0}) + O \left( \alpha_s(\mu_0), \frac{m^2}{Q_0^2} \right),
\end{aligned}$$

$$\begin{aligned}
d_{\text{inpt},h/j}^c(z; \mu_{Q_0}) &= 2\pi z^2 \int_0^{\mu_{Q_0}} dk_T k_T D_{\text{inpt},h/j}(z, z\mathbf{k}_T; \mu_{Q_0}, Q_0^2) = \\
&C_{h/j}^D d_{\text{core},h/j}^c(z; \mu_{Q_0}) + \frac{1}{2} A_{h/j}^{D,g}(z; \mu_{Q_0}) \ln \left( 1 + \frac{\mu_{Q_0}^2}{m_{D_{h,g}}^2} \right) \\
&+ \frac{1}{2} A_{h/j}^D(z; \mu_{Q_0}) \ln \left( 1 + \frac{\mu_{Q_0}^2}{m_{D_{h,j}}^2} \right) + \frac{1}{4} B_{h/j}^D(z; \mu_{Q_0}) \left[ \ln^2 \left( \frac{m_{D_{h,j}}^2}{Q_0^2} \right) - \ln^2 \left( \frac{\mu_{Q_0}^2 + m_{D_{h,j}}^2}{Q_0^2} \right) \right] \\
&= d_{h/j}(z; \mu_{Q_0}) + O \left( \alpha_s(\mu_0), \frac{m^2}{Q_0^2} \right),
\end{aligned}$$