

J. OSVALDO GONZÁLEZ-HERNÁNDEZ

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# WHY HSO? (AND OTHER QUESTIONS)

# 1) TOO MANY CHOICES IN PHENOMENOLOGY

MODEL  
FOR  
TMD PDF/FF

MODEL FOR COLLINS-SOPER  
KERNEL  
(EVOLUTION)

INTERPLAY BETWEEN PERTURBATIVE  
AND  
NONPERTURBATIVE “INGREDIENTS”

CHOICE OF  
COLLINEAR  
FUNCTIONS

TREATMENT  
OF  
EXPERIMENTAL ERRORS

TREATMENT  
OF  
THEORETICAL ERRORS

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**FITTER**



**FITTER**

**CAN WE RELY ON RESULT?**

# STATISTICAL METHODS HELP

## THE LARGE-SAMPLE DISTRIBUTION OF THE LIKELIHOOD RATIO FOR TESTING COMPOSITE HYPOTHESES<sup>1</sup>

BY S. S. WILKS

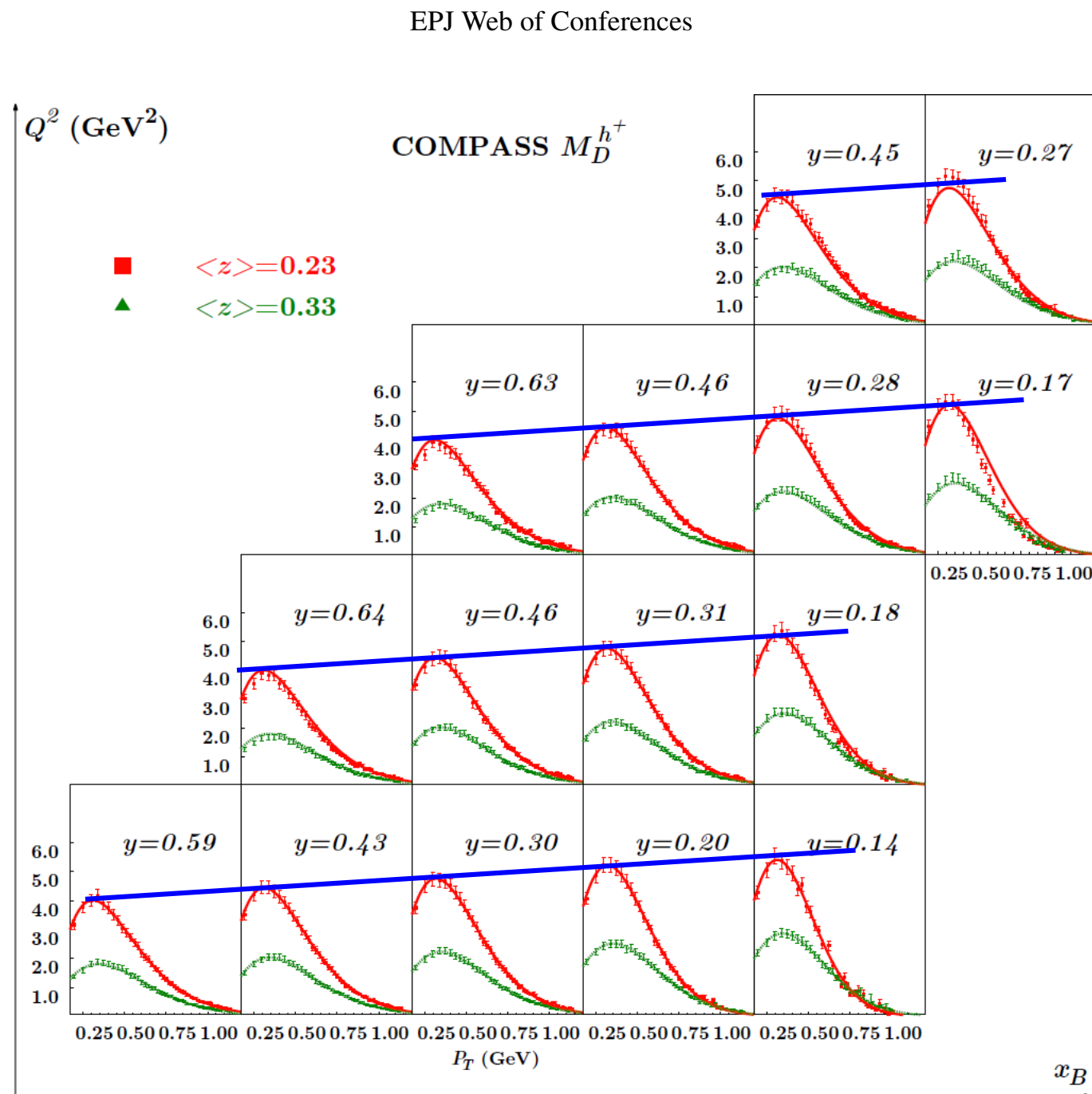
*Theorem: If a population with a variate  $x$  is distributed according to the probability function  $f(x, \theta_1, \theta_2, \dots, \theta_h)$ , such that optimum estimates  $\bar{\theta}_i$  of the  $\theta_i$  exist which are distributed in large samples according to (3), then when the hypothesis  $H$  is true that  $\theta_i = \theta_{0i}$ ,  $i = m + 1, m + 2, \dots, h$ , the distribution of  $-2 \log \lambda$ , where  $\lambda$  is given by (2) is, except for terms of order  $1/\sqrt{n}$ , distributed like  $\chi^2$  with  $h - m$  degrees of freedom.*

$$(3) \quad \frac{|c_{ij}|^{\frac{1}{2}}}{(2\pi)^{h/2}} e^{-\frac{1}{2} \sum_{i,j=1}^h c_{ij} z_i z_j} (1 + \phi) dz_1 \dots dz_h$$

where  $z_i = (\bar{\theta}_i - \theta_i)\sqrt{n}$ ,  $c_{ij} = -E\left(\frac{\partial^2 \log f}{\partial \theta_i \partial \theta_j}\right)$ ,  $E$  denoting mathematical expectation, and  $\phi$  is of order  $1/\sqrt{n}$  and  $||c_{ij}||$  is positive definite. Denoting (3) by

<sup>1</sup>For conditions under which the  $\bar{\theta}$ 's exist which are distributed according to (3), see J. L. Doob, Probability and Statistics, Trans. Amer. Math. Soc. Vol. 36, p. 759-775.

**BUT NOTE: ALL THIS WORKS ONLY IF MODEL IS CORRECT**



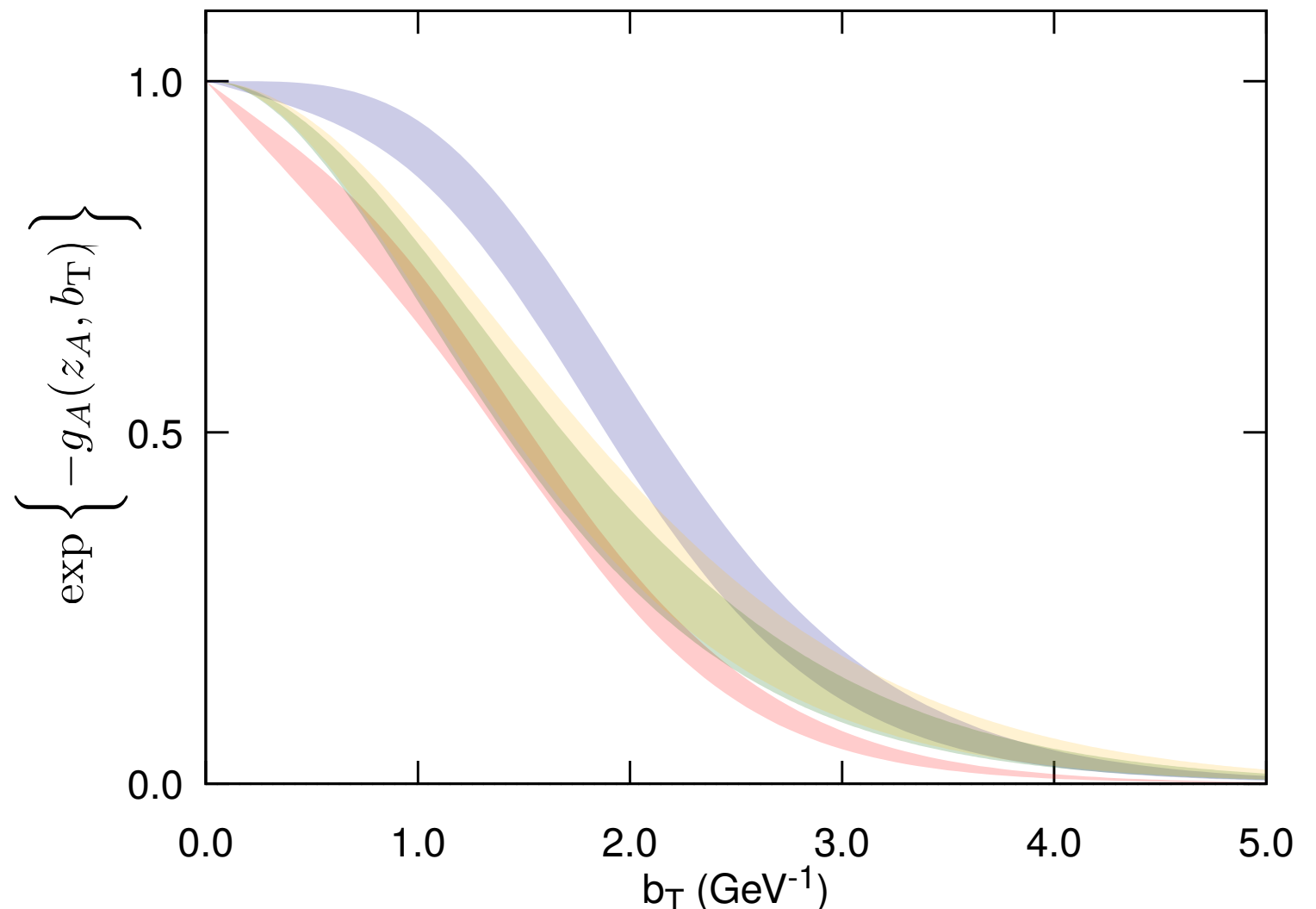
**SIMPLE GENERALIZED  
PARTON MODEL  
(NO CS KERNEL, ETC.)**

**CAN WE RELY ON RESULT?**

# CSS USUAL APPROACH

$$\begin{aligned}
 W(q_T, Q) = & H(\mu_Q; C_2) \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{-i \mathbf{q}_T \cdot \mathbf{b}_T} \tilde{D}_A(z_A, \mathbf{b}_*; \mu_{b_*}, \mu_{b_*}^2) \tilde{D}_B(z_B, \mathbf{b}_*; \mu_{b_*}, \mu_{b_*}^2) \\
 & \times \exp \left\{ 2 \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[ \gamma(\alpha_s(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*; \mu_{b_*}) \right\} \\
 & \times \exp \left\{ -g_A(z_A, b_T) - g_B(z_B, b_T) - g_K(b_T) \ln \left( \frac{Q^2}{Q_0^2} \right) \right\} .
 \end{aligned}$$

**CONSIDER A SET  
OF MODELS THAT  
SATISFY  
 $g_A(b_T=0)=0$**

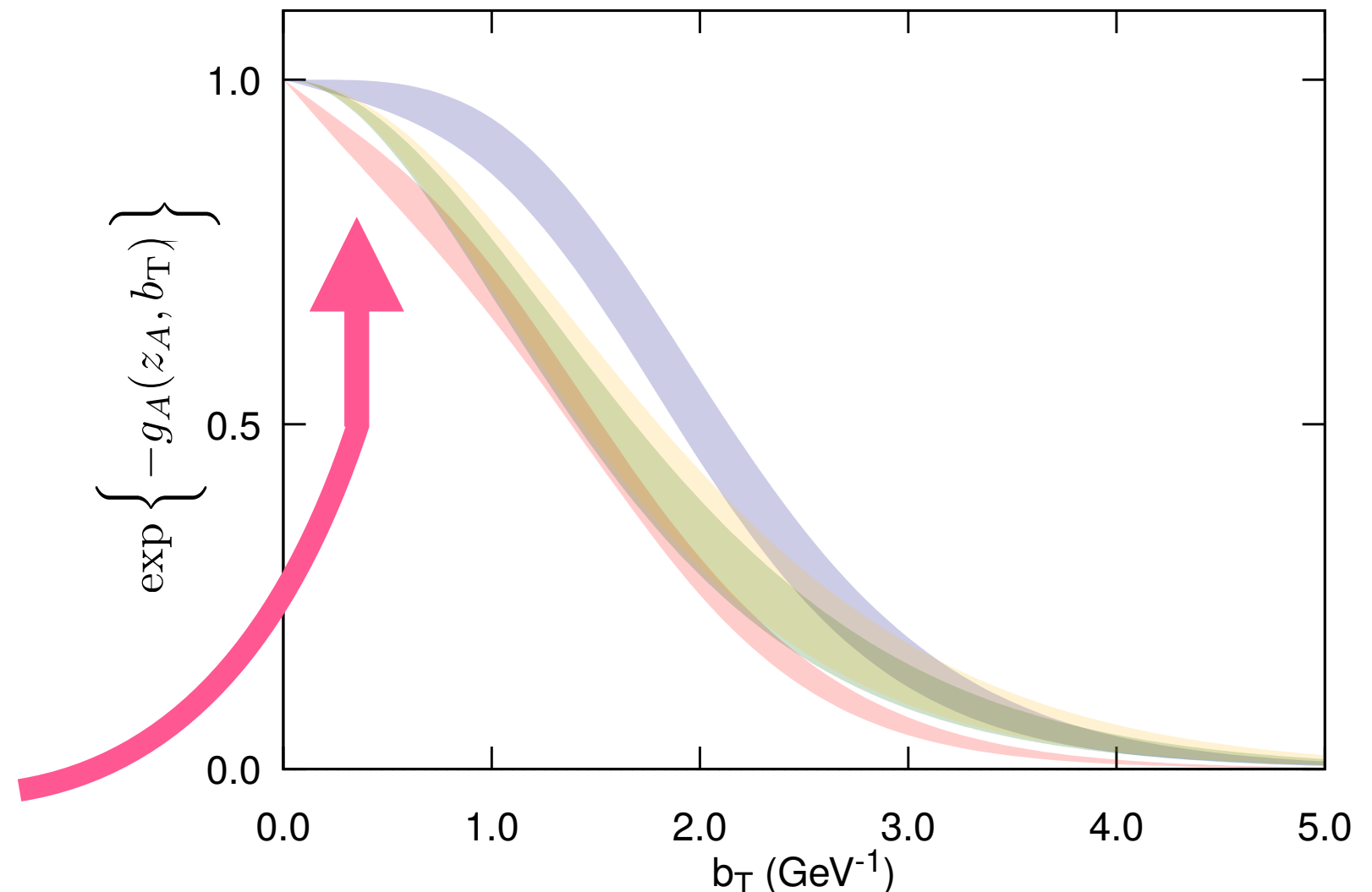


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 \end{aligned}$$

CONSIDER A SET  
OF MODELS THAT  
SATISFY  
 $g_A(b_T=0)=0$

ISSUES AT SMALL  
 $b_T$





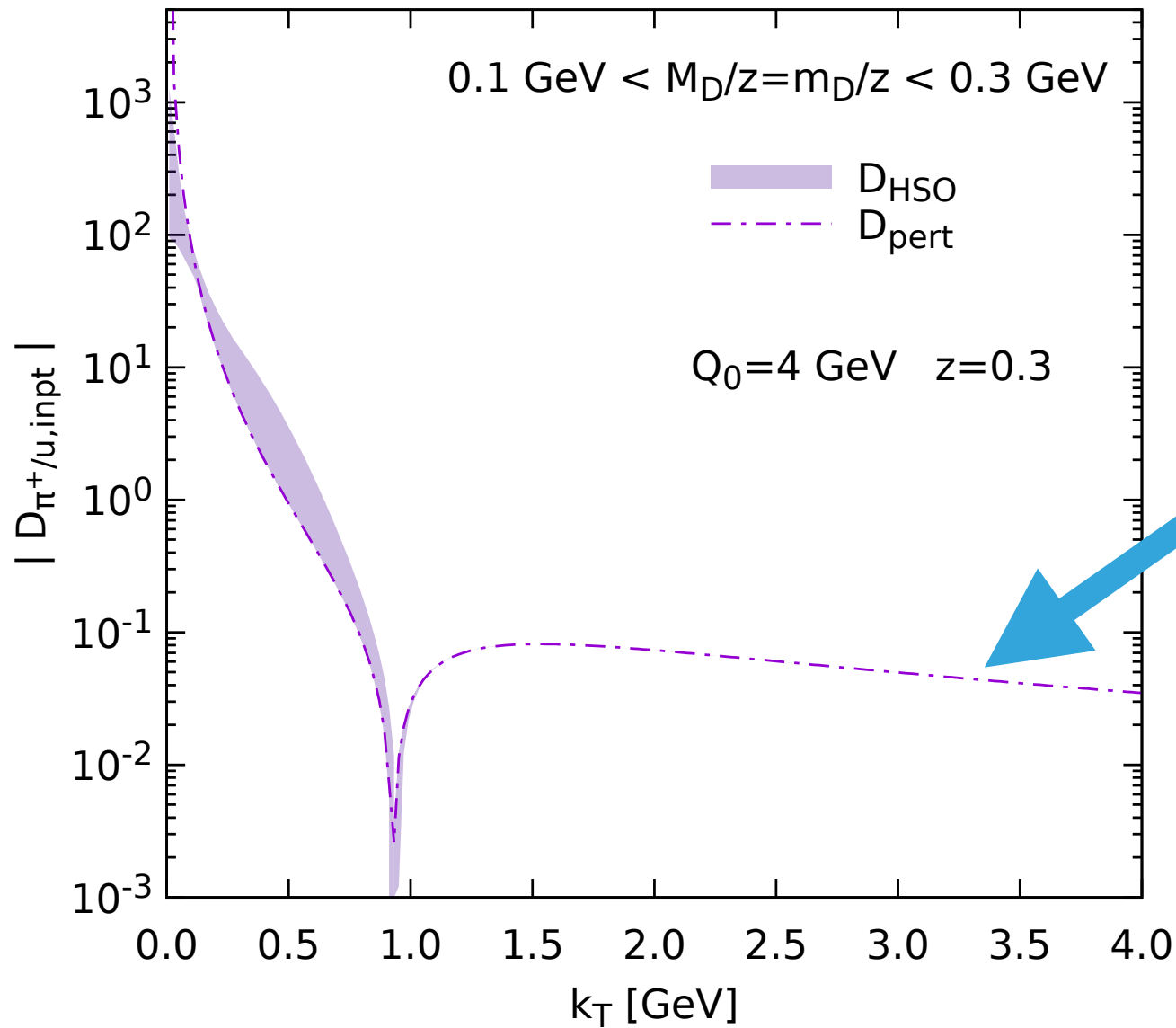
# CSS IN HSO

$$W^{(n)}(q_T, Q) \equiv H^{(n)}(\alpha_s(\mu_Q); C_2) \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{-i \mathbf{q}_T \cdot \mathbf{b}_T} \underline{\tilde{D}}_A^{(n, d_r)}(z_A, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) \underline{\tilde{D}}_B^{(n, d_r)}(z_B, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) \\ \times \exp \left\{ \underline{\tilde{K}}^{(n)}(b_T; \mu_{Q_0}) \ln \left( \frac{Q^2}{Q_0^2} \right) + \int_{\mu_{Q_0}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[ 2\gamma^{(n)}(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K^{(n)}(\alpha_s(\mu')) \right] \right\} .$$

$$\tilde{D}_{h/j}(z, \mathbf{b}_T; \mu_{Q_0}, Q_0^2) = \tilde{D}_{\text{inpt}, h/j}(z, \mathbf{b}_T; \mu_{\overline{Q}_0}, \overline{Q}_0^2) E(\overline{Q}_0/Q_0, b_T) . \quad \textbf{RG IMPROVEMENTS}$$

$$D_{\text{inpt}, h/j}(z, z \mathbf{k}_T; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi z^2} \frac{1}{k_T^2 + m_{D_{h,j}}^2} \left[ A_{h/j}^D(z; \mu_{Q_0}) + B_{h/j}^D(z; \mu_{Q_0}) \ln \frac{Q_0^2}{k_T^2 + m_{D_{h,j}}^2} \right] \\ + \frac{1}{2\pi z^2} \frac{1}{k_T^2 + m_{D_{h,g}}^2} A_{h/j}^{D,g}(z; \mu_{Q_0}) \\ + C_{h/j}^D D_{\text{core}, h/j}(z, z \mathbf{k}_T; Q_0^2) , \quad \textbf{MODEL}$$

# CSS IN HSO



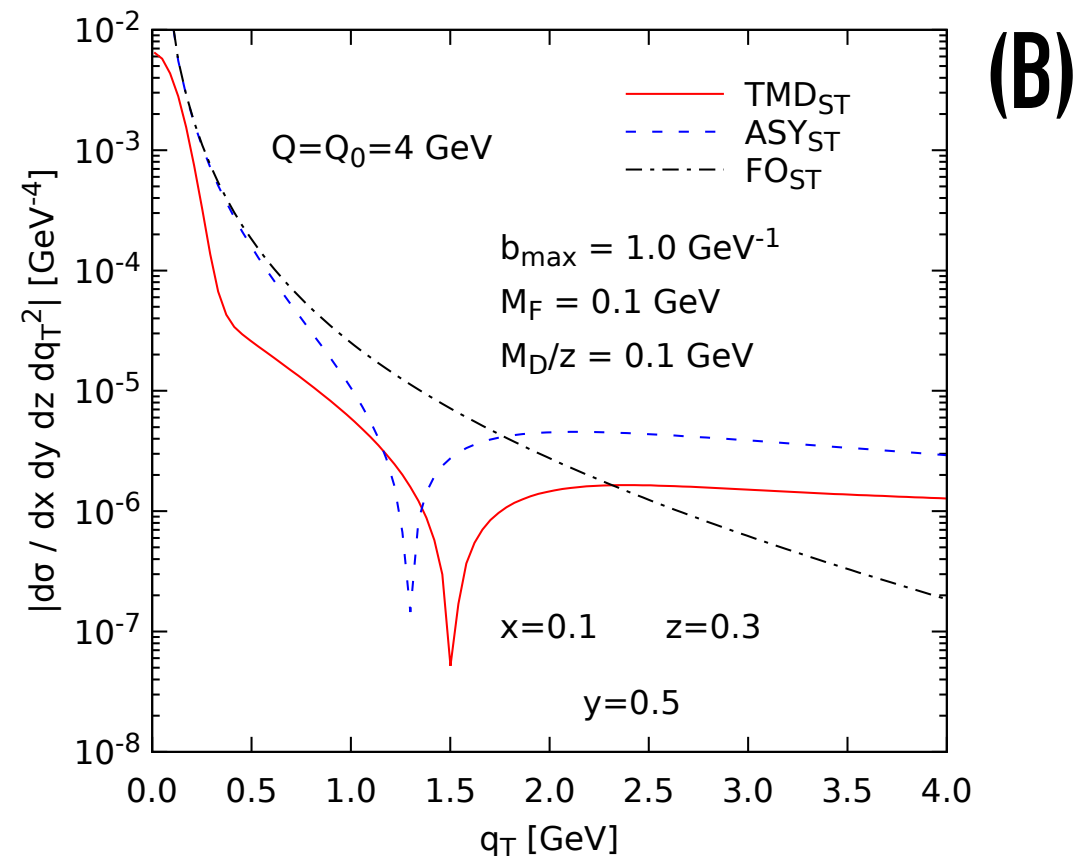
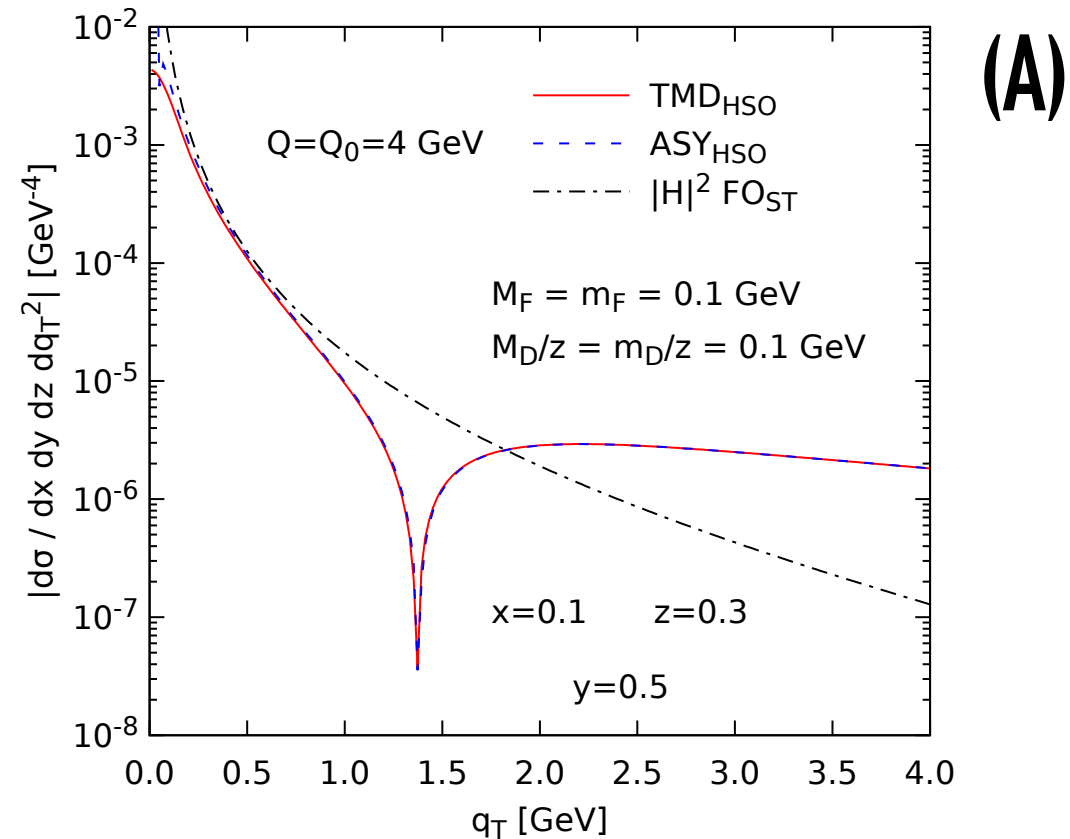
**PQCD TAIL**

$$\begin{aligned}
 D_{\text{inpt}, h/j}(z, z\mathbf{k}_T; \mu_{Q_0}, Q_0^2) = & \frac{1}{2\pi z^2} \frac{1}{k_T^2 + m_{D_{h,j}}^2} \left[ A_{h/j}^D(z; \mu_{Q_0}) + B_{h/j}^D(z; \mu_{Q_0}) \ln \frac{Q_0^2}{k_T^2 + m_{D_{h,j}}^2} \right] \\
 & + \frac{1}{2\pi z^2} \frac{1}{k_T^2 + m_{D_{h,g}}^2} A_{h/j}^{D,g}(z; \mu_{Q_0}) \\
 & + C_{h/j}^D D_{\text{core}, h/j}(z, z\mathbf{k}_T; Q_0^2),
 \end{aligned}$$

**MODEL**

## 2) TMDS HAVE CONCRETE DEFINITIONS IN QCD

**IMPROVEMENT IN  
DESCRIBING/PREDICTING DATA  
BY IMPOSING THE CORRECT  
pQCD TAIL  
(TO BE TESTED)**

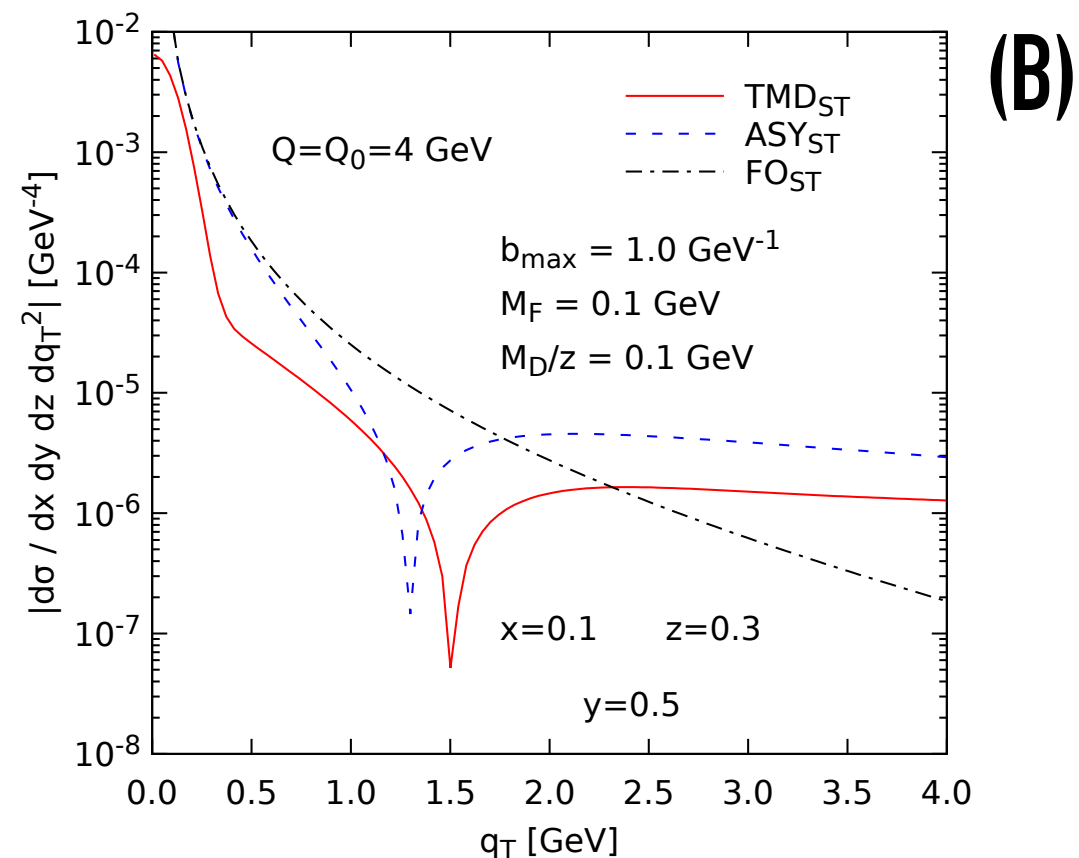
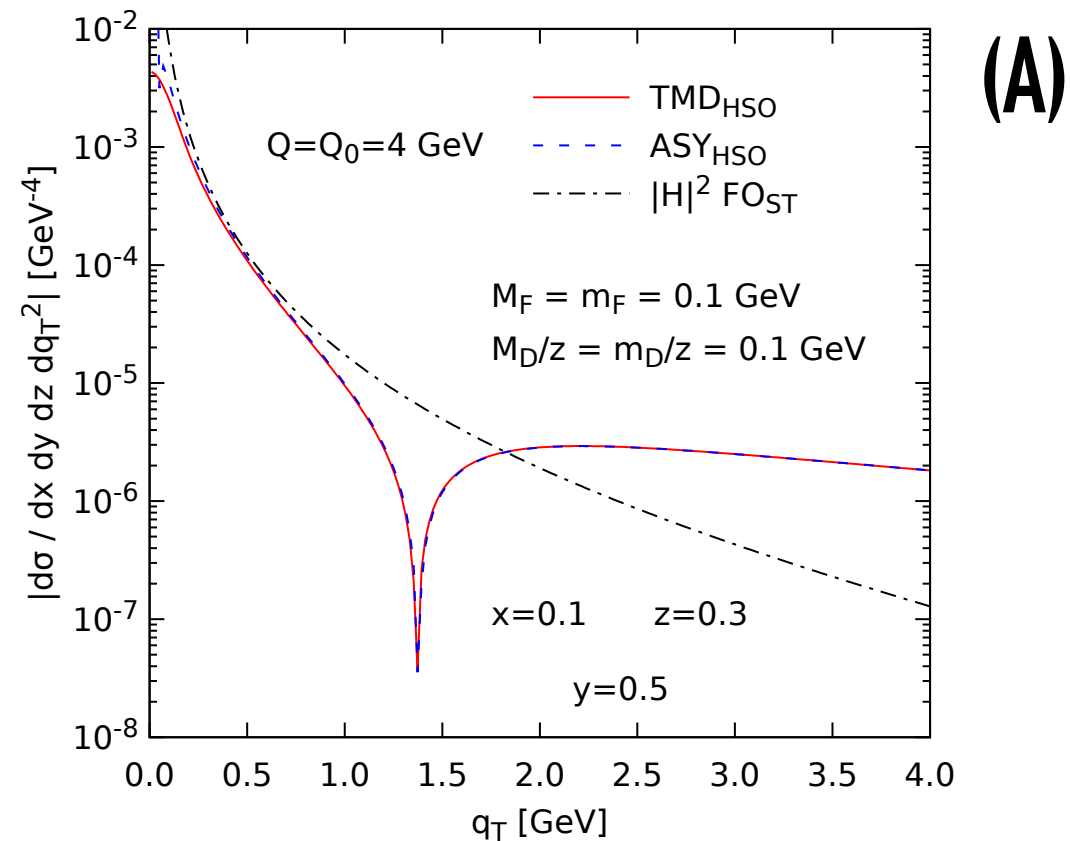


## 2) TMDS HAVE CONCRETE DEFINITIONS IN QCD

ANOTHER SCENARIO :

- EXTRACTION "A" WITH CORRECT PQCD TAIL.
- EXTRACTION "B" WITH INCONSISTENT LARGE- $k_T$  BEHAVIOR

BUT OTHERWISE EQUIVALENT  
(E.G. SAME  $\chi^2$ )



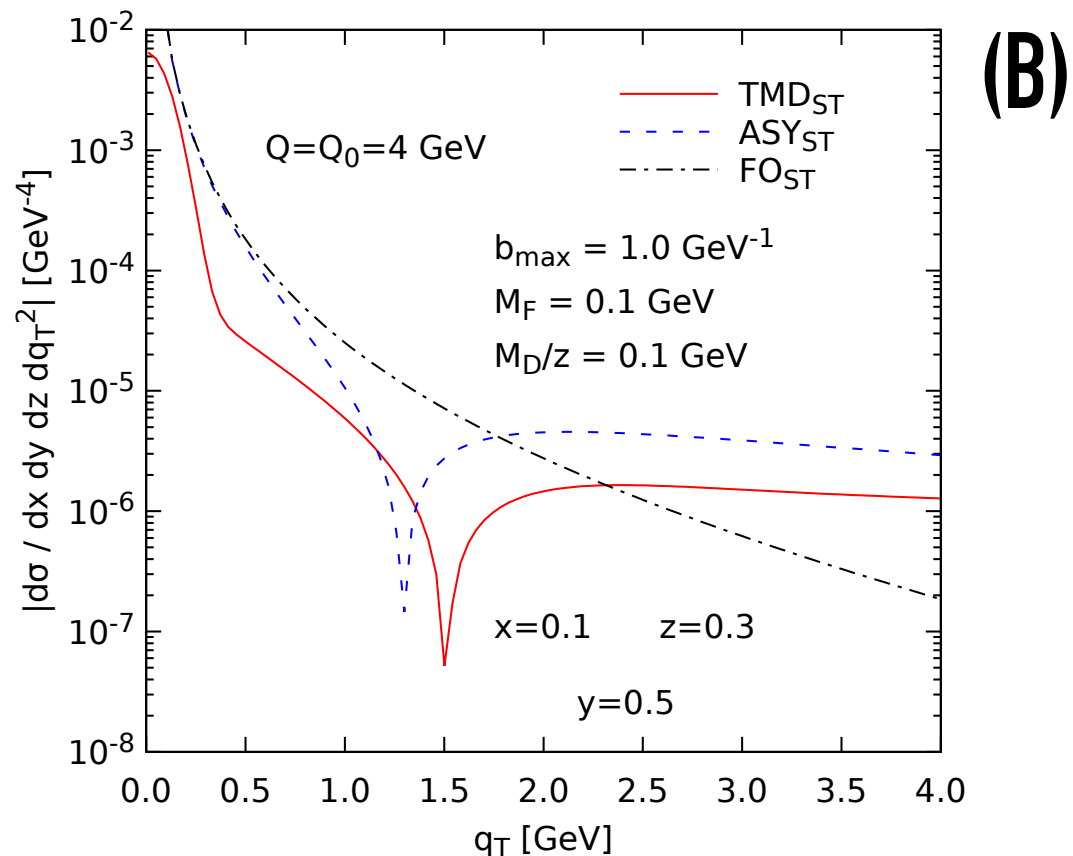
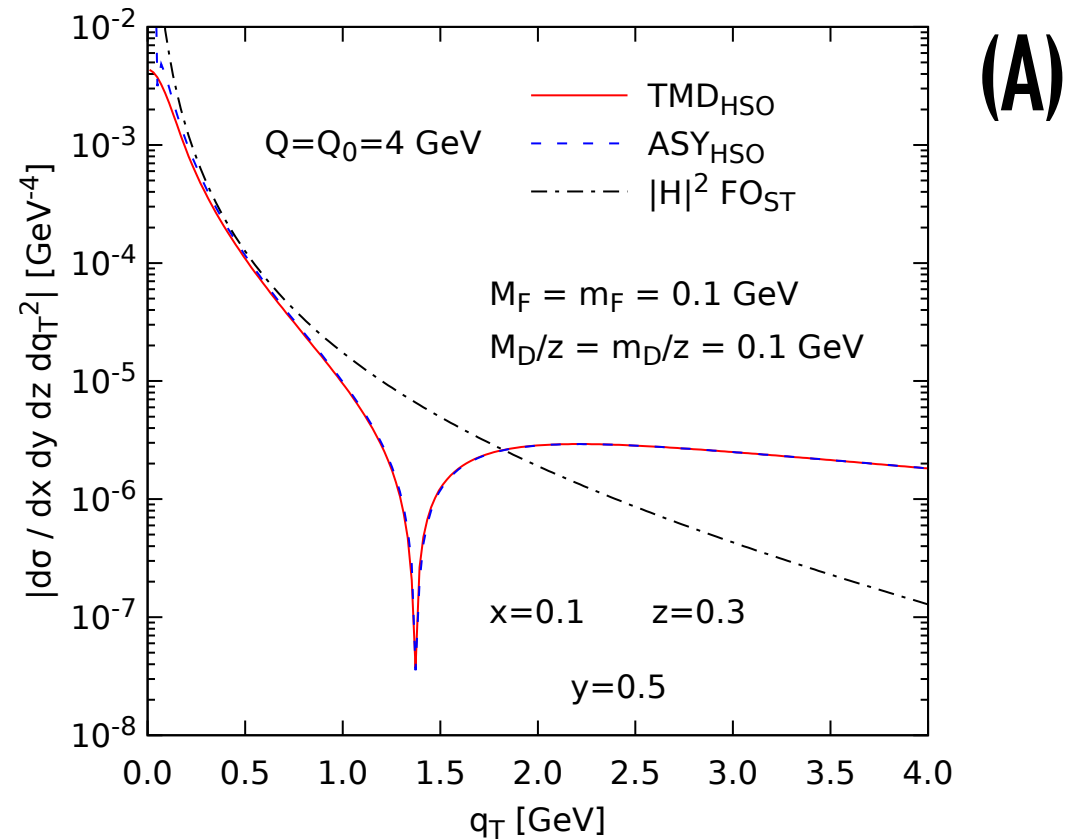
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ANOTHER SCENARIO :

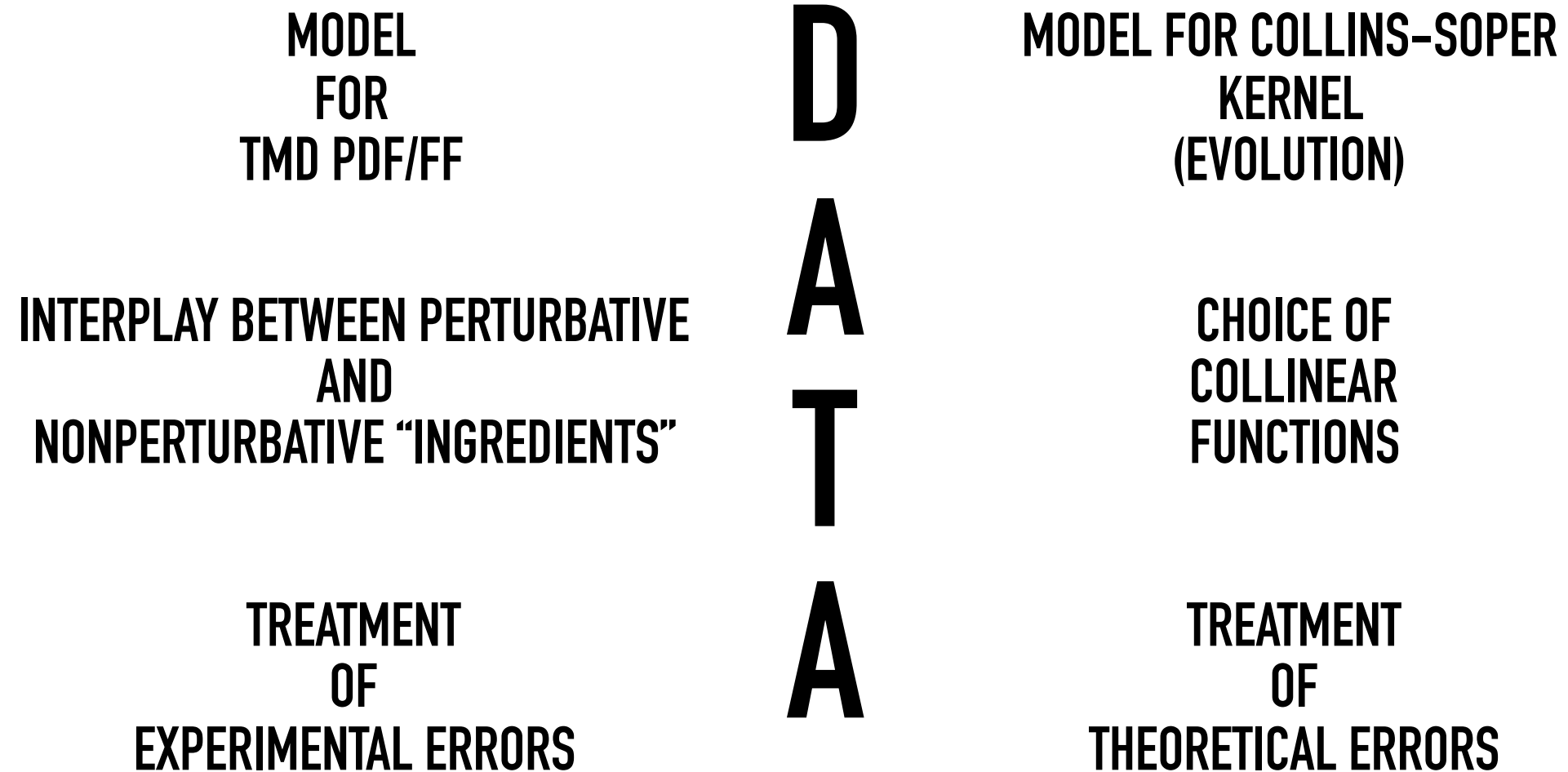
- EXTRACTION “A” WITH CORRECT PQCD TAIL.
- EXTRACTION “B” WITH INCONSISTENT LARGE- $k_T$  BEHAVIOR

BUT OTHERWISE EQUIVALENT  
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“A” IS A STRONGER CANDIDATE  
FOR THE TRUE BEHAVIOR OF  
TMDS

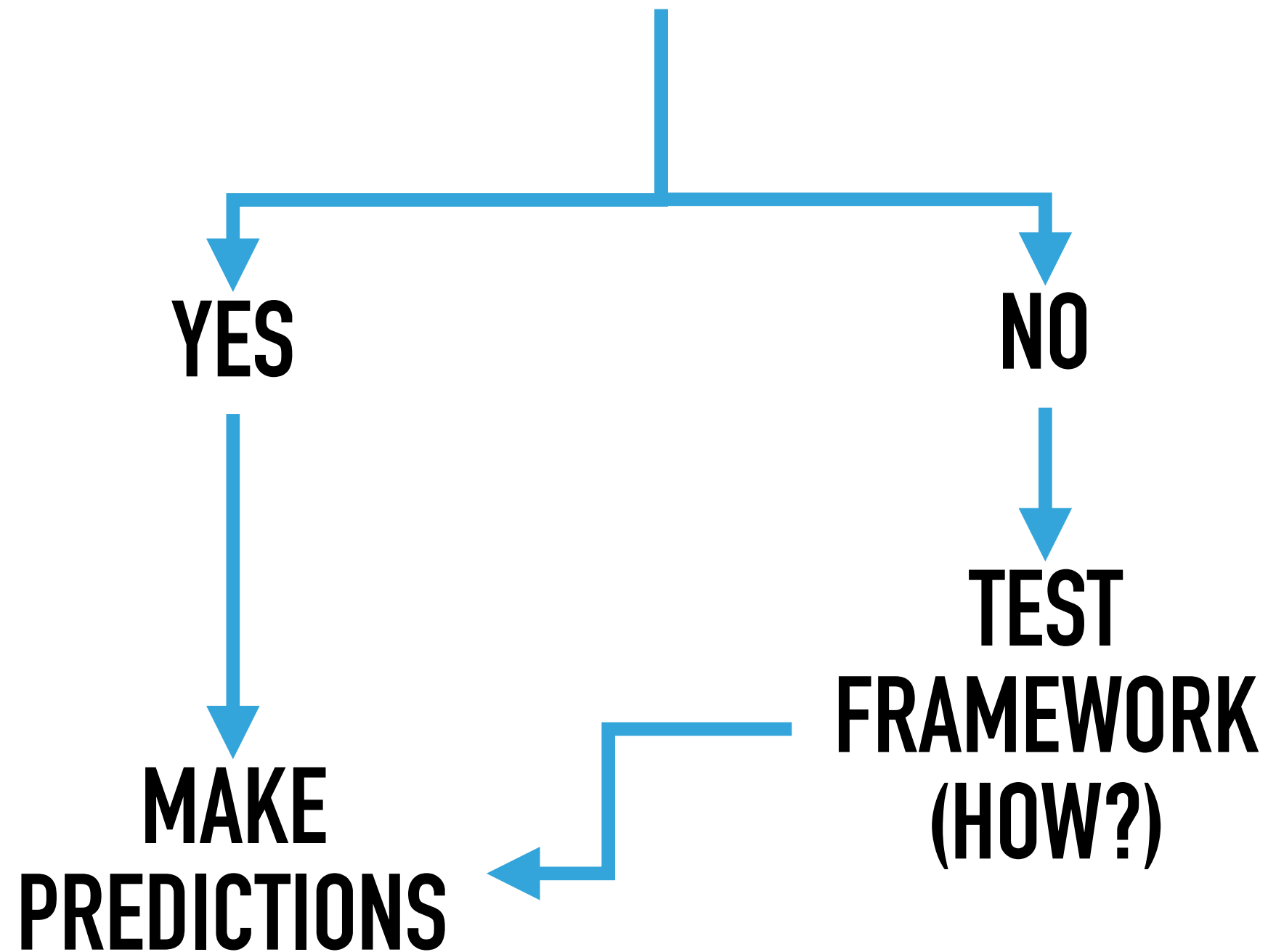


# Q: DO WE TRUST OUR FRAMEWORK?



**STATISTICAL THEOREMS + ADVANCED TOOLS/Frameworks**

**Q: DO WE TRUST OUR FRAMEWORK?**



**Q: POSTDICTIONS = PREDICTIONS?**



**Q: POSTDICTIONS = PREDICTIONS?**

**POSTDICTIONS  $\neq$  PREDICTIONS**

**FITS  $\neq$  PREDICTIONS**

**0 < INTERPOLATING  
LINES < SIMPLE  
FIT < ROBUST  
MODEL FIT ...**

**< THEORY  
COMPLIANT FIT < POSTDICTION < PREDICTION**

**Q: THE FUTURE EIC DATA WILL \_\_\_\_\_**

# Q: THE FUTURE EIC DATA \_\_\_\_\_

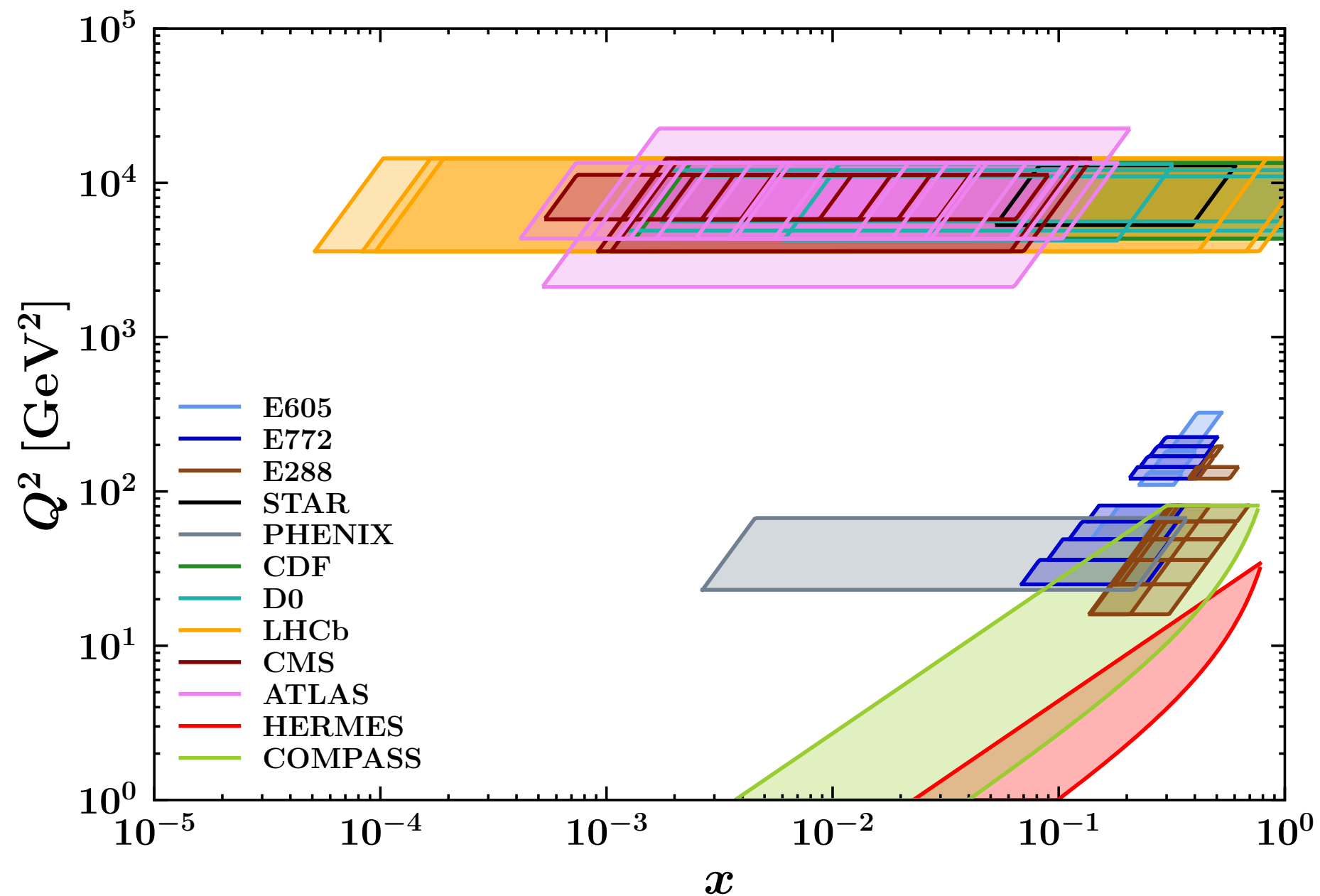
## The case for an EIC Theory Alliance: Theoretical Challenges of the EIC

Guiding and understanding the future experimental measurements will require a laborious and meticulous analysis of the data, new approaches and new methods in the theoretical treatment and in the phenomenological extraction of TMDs. The EIC Theory Alliance will provide an essential framework for guiding and organizing the broad theoretical

- Theoretical and phenomenological exploration of QCD factorization theorems and expanding the region of their applicability, for instance by inclusion of power corrections in  $q_T/Q$ . A crucial ingredient will be matching collinear factorization ( $\Lambda_{\text{QCD}} \ll q_T \sim Q$ ) and TMD factorization ( $\Lambda_{\text{QCD}} \lesssim q_T \ll Q$ ) in the overlap region  $\Lambda_{\text{QCD}} \ll q_T \ll Q$  in a stable and efficient way. Such a matching is needed for our ability to describe the measured quantities, differential in transverse momentum, in the widest possible region of phase space. In turn, this will lead to a much more reliable understanding of both collinear and TMD related functions and uncertainties in their determinations.

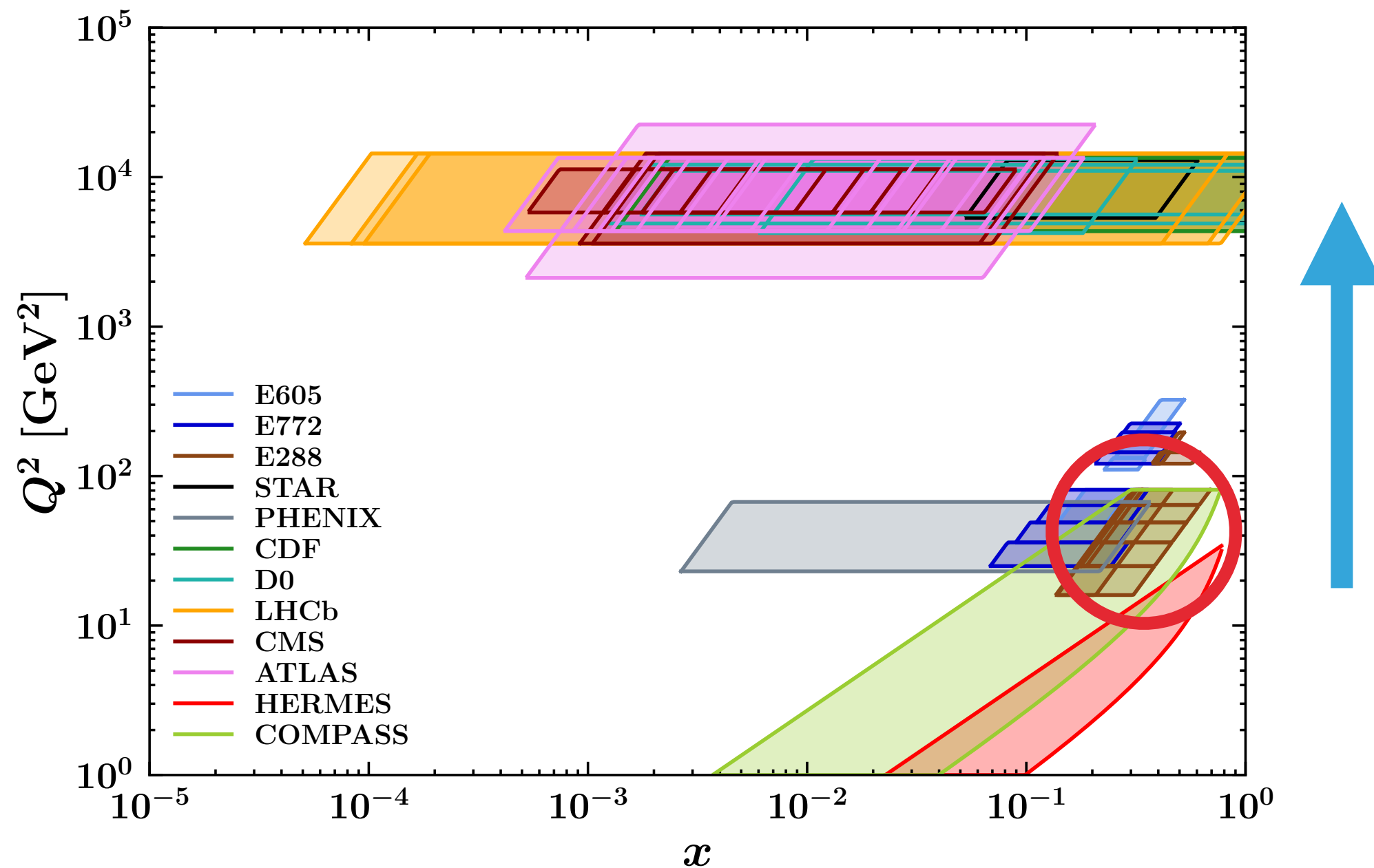
**Q: ARE WE DOING PHENO USING THE HSO?**

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Plot from (MAP collaboration):  
*JHEP* 10 (2022) 127

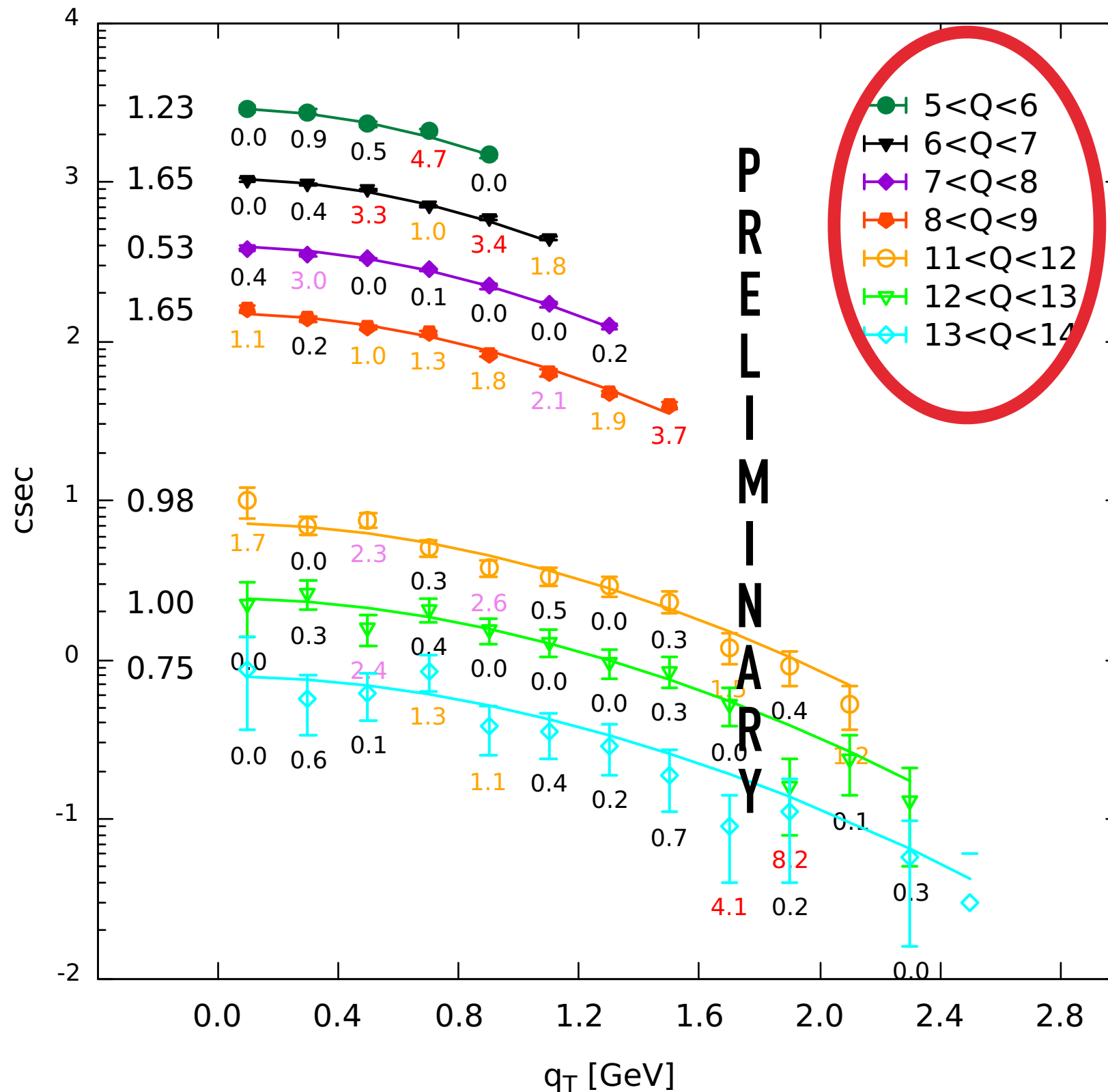
# Q: ARE WE DOING PHENO USING THE HSO?



Plot from (MAP collaboration):  
*JHEP* 10 (2022) 127

# Q: ARE WE DOING PHENO USING THE HSO?

E288: test. E = 400 GeV



# FINAL (PERSONAL) REMARK

( HOPEFULLY, EVENTUALLY)

**A: THE FUTURE EIC DATA WAS SUCCESSFULLY PREDICTED**