Phenomenology of TMD distributions in Drell-Yan and Z_0 boson production with the Hadron Structure Oriented approach

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OLD DOMINION UNIVERSITY

Based on

- Phenomenology of TMD parton distributions in Drell-Yan and Z^0 boson production in a hadron structure oriented approach

(ArXiv:2401.14266)

- (F. Aslan, M. Boglione, J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers, A. Simonelli)

- The resolution to the problem of consistent large transverse momentum in TMDs (PhysRevD.107.094029)

- (J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers)

- Combining nonperturbative transverse momentum dependence with TMD evolution (PhysRevD.106.034002)

- (J. O. Gonzalez-Hernandez, T. C. Rogers, N. Sato)

Why TMDs?

Drell-Yan

SIDIS

Studying the role of intrinsic or **nonperturbative effects** in hadrons

$e^+ e^-$ --> H_a H_b X

Predicting transverse momentum distributions in **cross sections** after evolution to **high energies**

Factorization theorems

Universality

Evolution equations

What we know

Drell-Yan

At small q_T << Q the cross section is determined solely by TMD factorization (TMD pdfs and/or TMD FFs)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{q}_T\ldots} \stackrel{q_T \ll Q}{\sim} \sum_j H_{j\bar{j}} \int \mathrm{d}^2\boldsymbol{k}_{T,1}\mathrm{d}^2\boldsymbol{k}_{T,2} f_j(x, k_{T,1};\mu,\zeta) f_{\bar{j}}(x, k_{T,1};\mu,\zeta)\delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{T,1} - \boldsymbol{k}_{T,2})$$

At large $q_T \sim$ Q the cross section is determined solely by fixed order collinear factorization (SIDIS, Drell-Yan, e^+e^- --> back-to-back hadrons,…)

Collinear PDFs

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{q}_T\ldots} \stackrel{q_T \sim Q}{\sim} H(q_T) \otimes f \otimes f$$

What we know

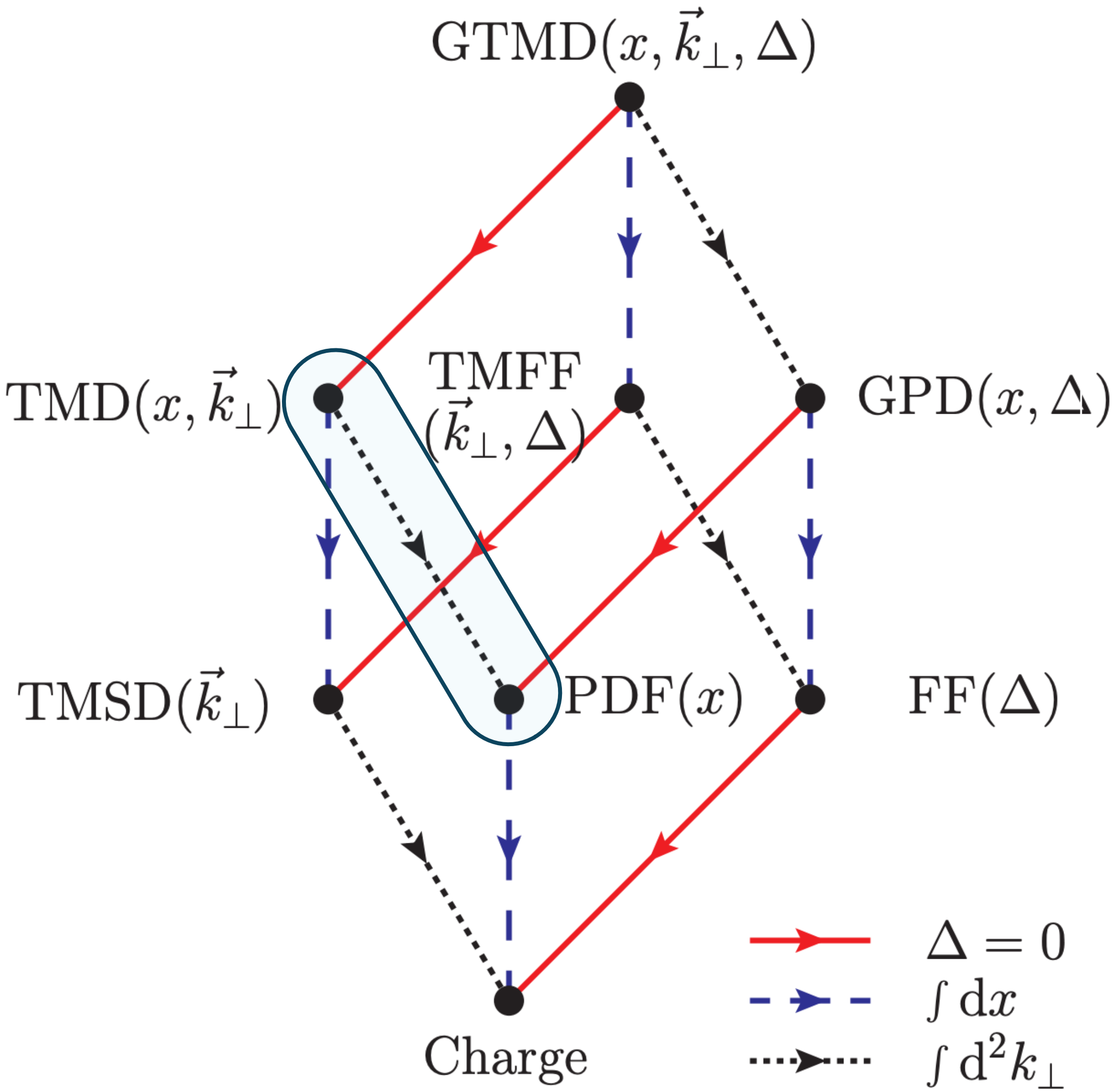
Similarly, at large TM (k_T)/ small b_T the TMDs are **uniquely determined** by an OPE expansion in terms of collinear PDFs/FFs

$$f_{i/H}(x, b_T; \mu, \zeta) = \widetilde{C}_{ij}(x, b_T; \mu, \zeta) \otimes f_{j/H}(x; \mu) + \mathcal{O}(m b_T)$$

Perturbatively calculable

Usual PDFs

What we know



Most of these integrals are divergent.
A more careful treatment is necessary

Credits: Lorcé, Pasquini and Vanderhaeghen

Conventional approach

Final parametrization of a TMD

$$\tilde{f}_{j/p}(x;\boldsymbol{b}_{\mathrm{T}};\mu_Q,Q) = \boxed{\tilde{f}^{\mathrm{OPE}}_{j/p}(x;\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*})} \times$$

$$\times \boxed{\exp\left\{\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma(\alpha_S(\mu');1)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_S(\mu'))\right]+\ln\left(\frac{Q}{\mu_{b_*}}\right)\tilde{K}(\boldsymbol{b}_*;\mu_{b_*})\right\}}$$

$$\times \boxed{\exp\left\{-g_{j/p}(x,\boldsymbol{b}_{\mathrm{T}})-g_K(\boldsymbol{b}_{\mathrm{T}})\ln\left(\frac{Q}{Q_0}\right)\right\}}$$

Nonperturbative

Perturbatively calculable

Drop this

$$\tilde{f}^{\mathrm{OPE}}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}) = \widetilde{C}_{j/j'}(x/\xi,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*})\otimes\tilde{f}_{j'/p}(\xi;\mu_{b_*}) + \cancel{\mathcal{O}\left(m^2 b^2_{\max}\right)}$$

Same for FF

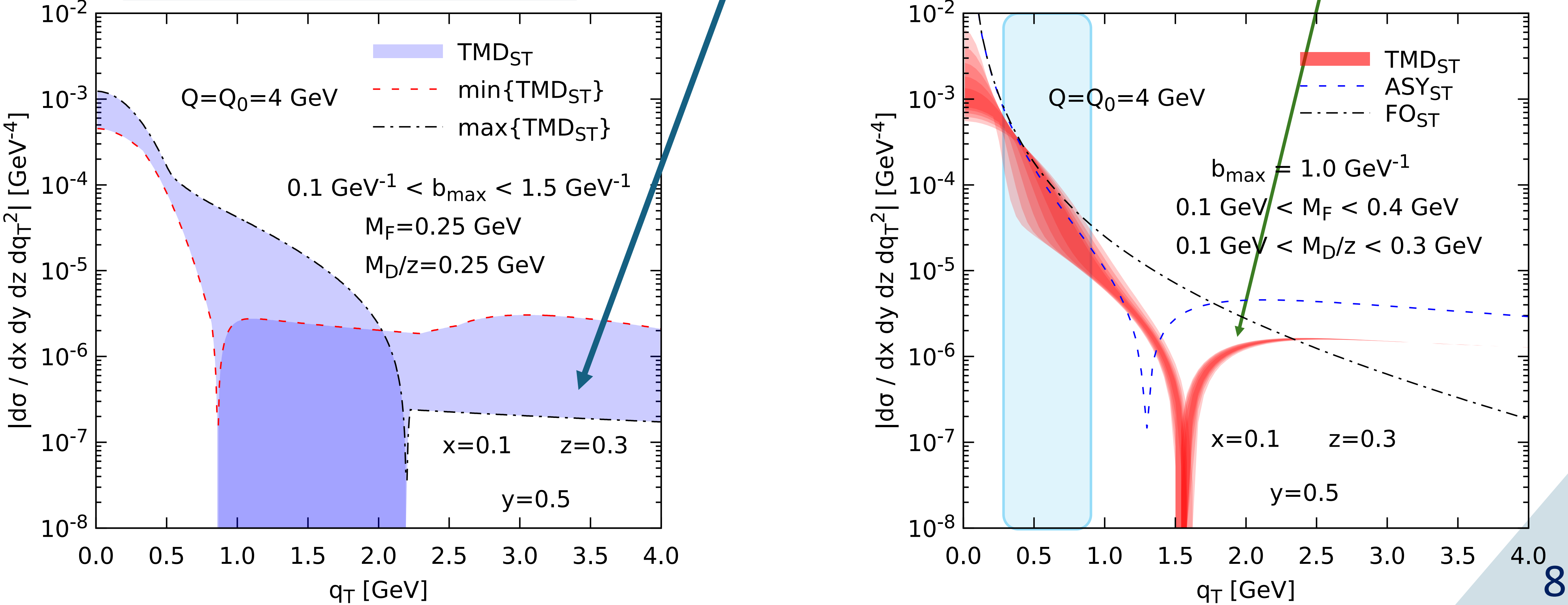
Fixed order collinear factorization

(Some) Issues with conventional approach

Large b_{max} dependence

What is going on?

Large q_T inconsistency



(Some) Questions

- Do we have control over how well the theoretical constraints are satisfied?
- Is the extracted TMD really what we expect it to be?
- How much sensitivity to collinear functions do the TMDs have?
- How can we test the soundness of our model?

Create a framework that facilitates the answers:
HSO approach

Hadron Structure Oriented approach

TMD PDF HSO parametrization at input scale

Fixed order collinear factorization

$\mathcal{O}\left(\alpha_S\right)$

Large k_T OPE coefficients

$$f_{\mathrm{inpt,i/p}}(x,\boldsymbol{k}_T;\mu_{Q_0};Q_0^2)=\frac{1}{2\pi}\frac{1}{k_T^2+m^2}\left[A^f_{i/p}(x;\mu_{Q_0})+B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_T^2+m^2}\right]+\frac{1}{2\pi}\frac{1}{k_T^2+m^2}A^{f,g}_{i/p}(x;\mu_{Q_0})$$
$$+\boxed{C^f_{i/p}}f_{\mathrm{core,i/p}}(x,\boldsymbol{k_T};Q_0^2)$$

Small k_T model

NP parameters

Such that

$$f^c_{j/p}\left(x;\mu_Q\right)\equiv 2\pi\int_0^{k_c}\mathrm{d}k_{\mathrm{T}}\,k_{\mathrm{T}}f_{j/p}\left(x,\boldsymbol{k}_{\mathrm{T}};\mu_Q,\sqrt{\zeta}\right)$$
$$=f_{j/p}\left(x;\mu_Q\right)+\Delta_{j/p}\left(x;\mu_Q,k_c\right)+\mathrm{p.s.}$$

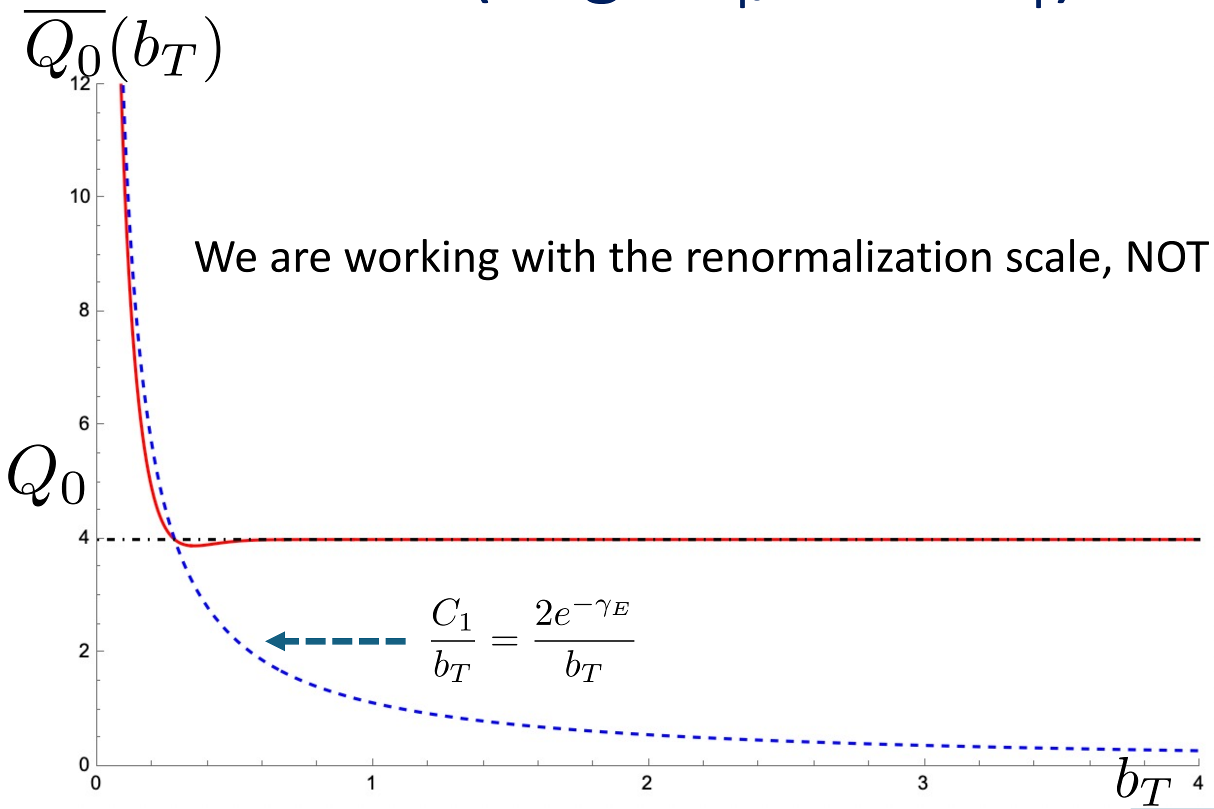
Evolution?

HSO Collins-Soper kernel at the input scale and RG improvement with $\overline{Q_0}(b_T)$ prescription.

Match small b_T/large k_T with OPE and assign "core" model (large b_T/small k_T)

We need to change scheme

$$\overline{Q_0}\left(b_T, a\right) = Q_0 \left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$



We are working with the renormalization scale, NOT b_T

Choose “core” models (examples)

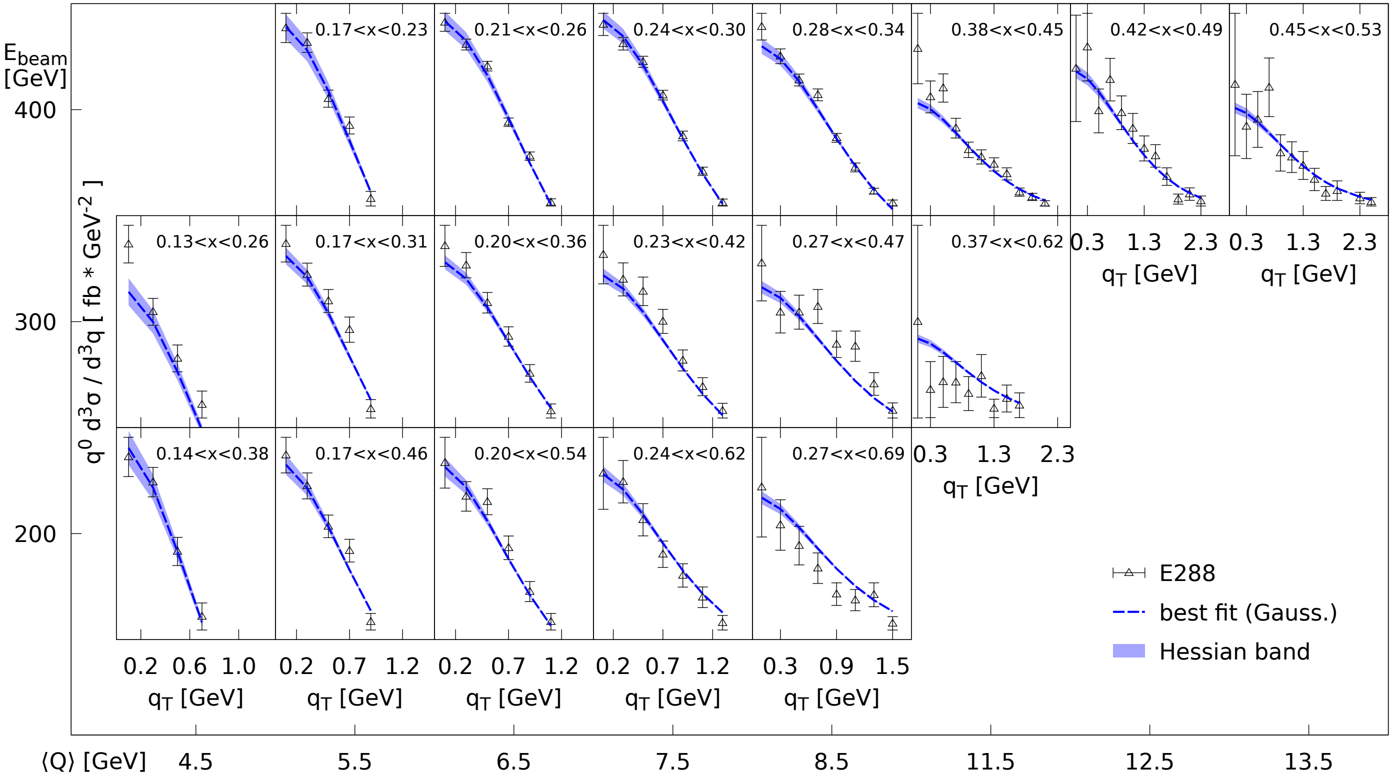
$$f^{\mathrm{Gauss}}_{\mathrm{core},i/p}\left(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2\right)=\frac{e^{-k_{\mathrm{T}}^2/M_F^2}}{\pi M_F^2}$$

Gaussian “core” models

Spectator-like “core” models

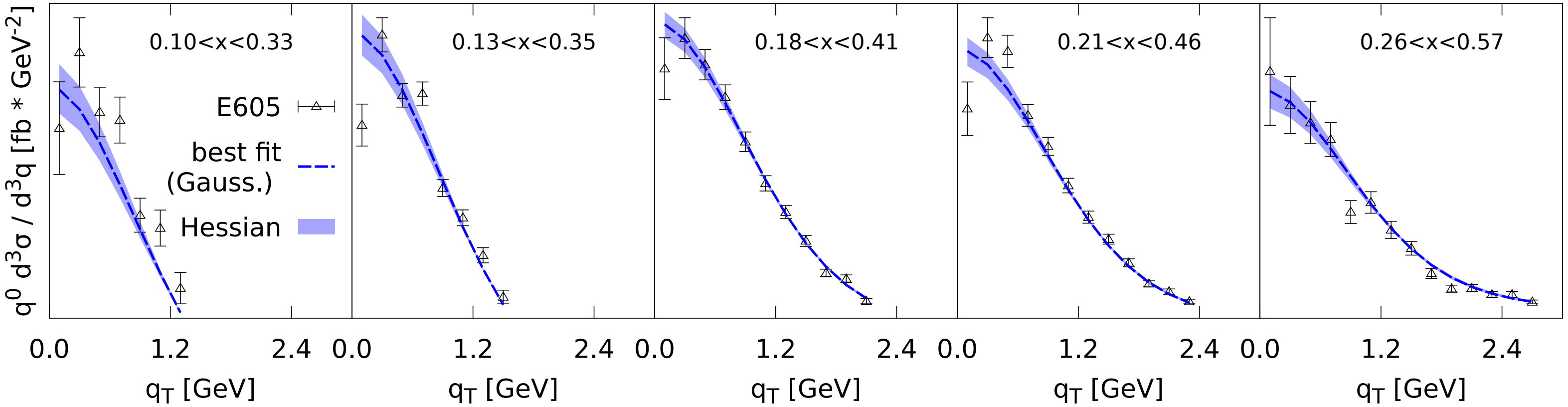
$$f^{\mathrm{Spect}}_{\mathrm{core},j/p}\left(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2\right)=\frac{6M_{0F}^6}{\pi\left(2M_F^2+M_{0F}^2\right)}\frac{M_F^2+k_{\mathrm{T}}^2}{\left(M_{0F}^2+k_{\mathrm{T}}^2\right)^4}$$

Low Q fit results



	Gaussian fits	
	E288 (130 pts.)	E605 (52 pts.)
χ^2_{dof}	1.04	1.68

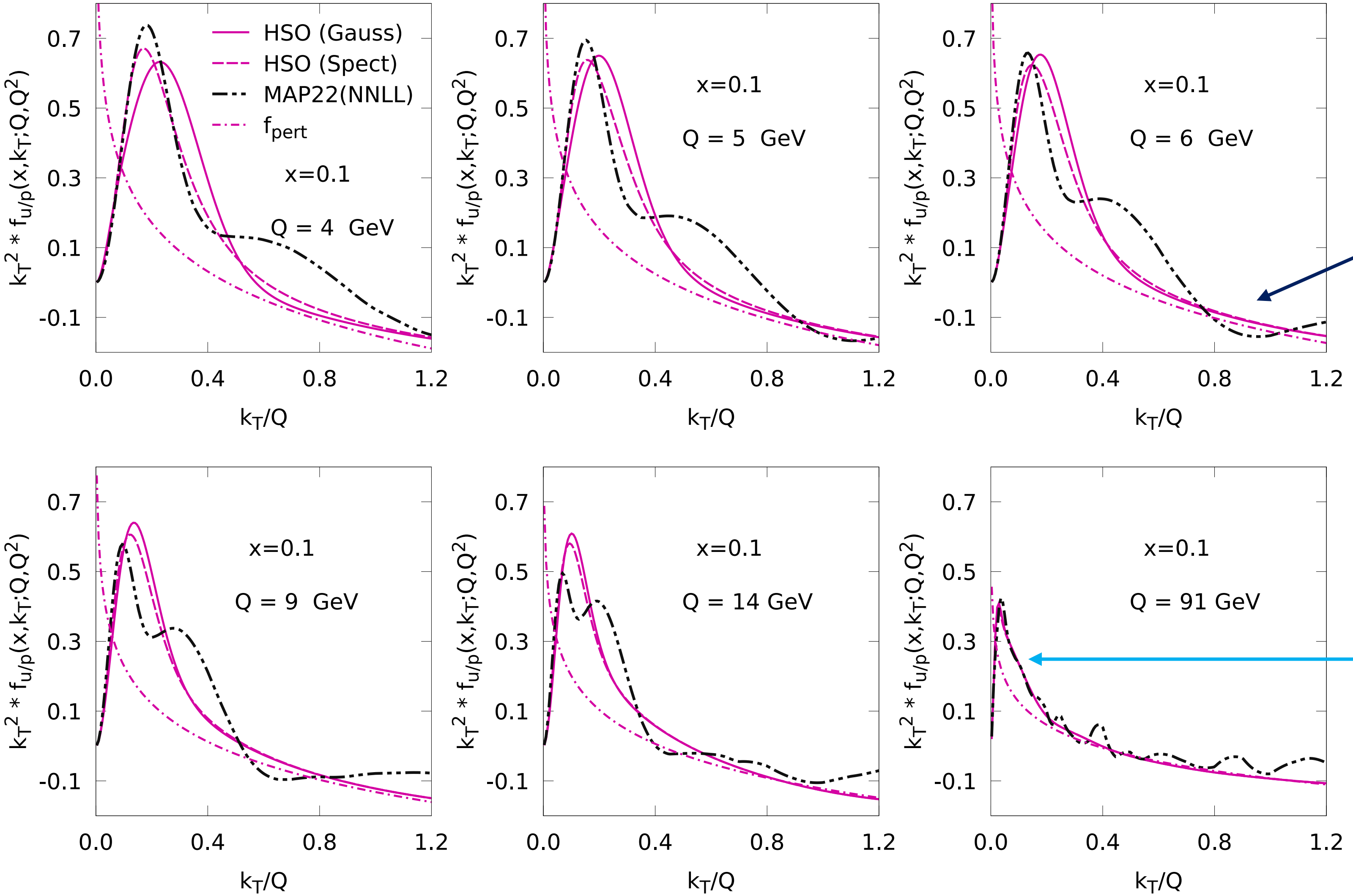
Just 4 parameters for now



Spectator model too:

	Spectator model fit
	E288 (130 pts.)
χ^2_{dof}	1.04

Comparison with MAP22



Observations:

No tail matching

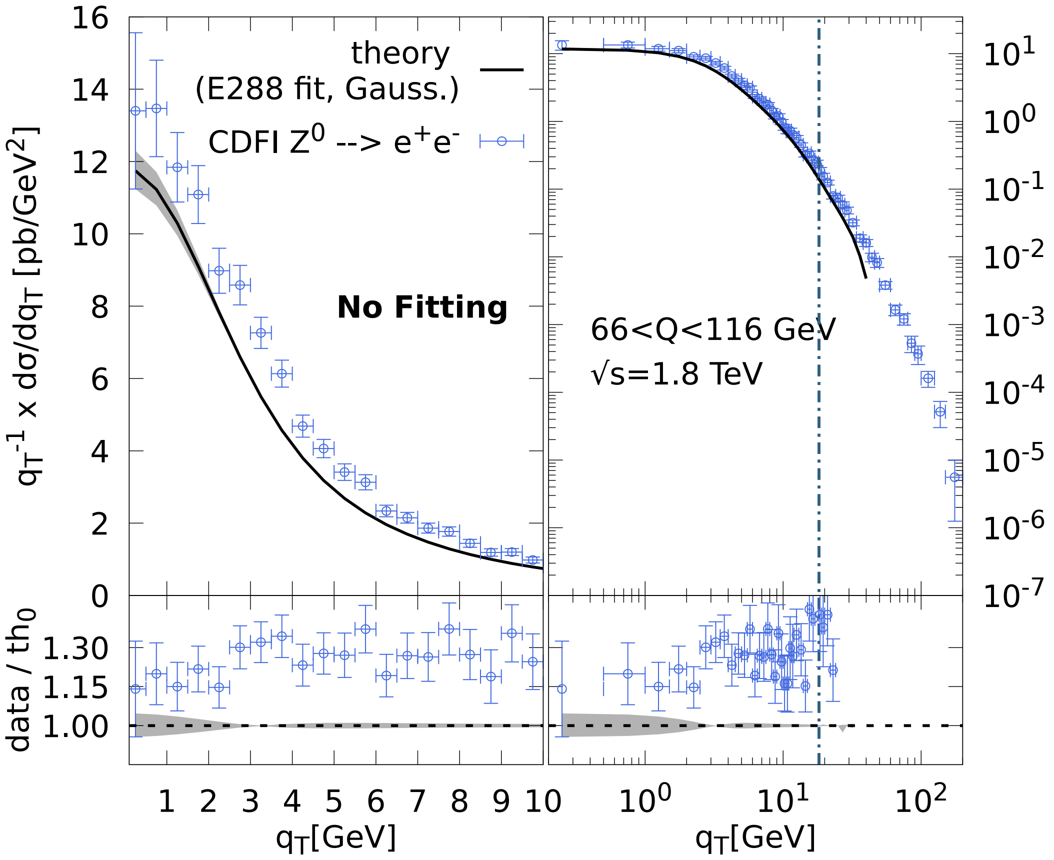
Different models can describe the small k_T region at low Q

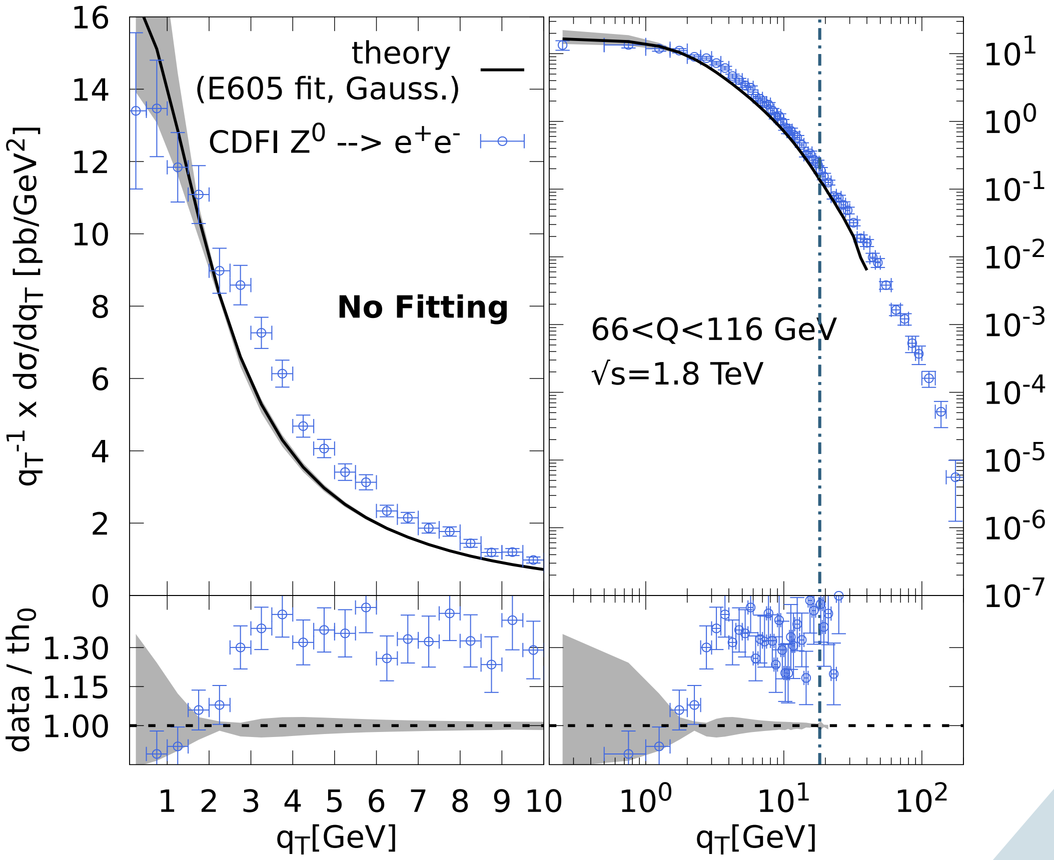
Model dependence washes out at large Q

How do we choose?

Higher Q postdictions: test different fits on the same experiment

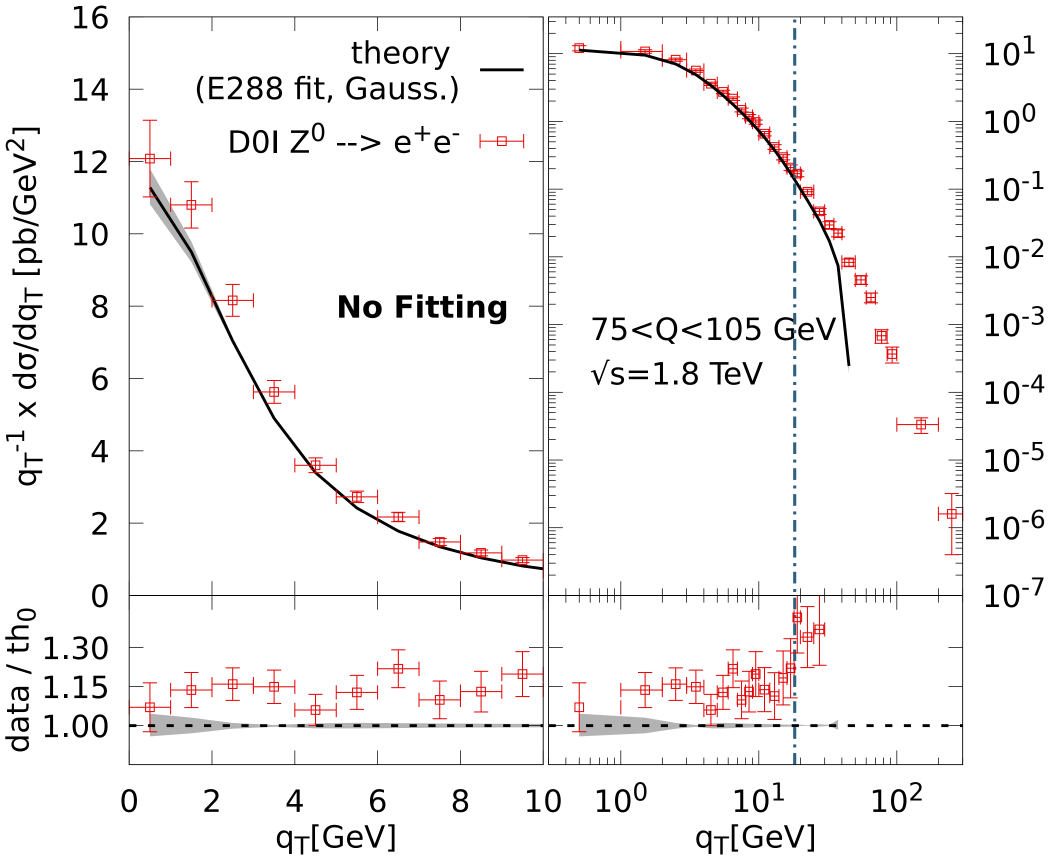
A postdiction of CDFI with just E288 or E605 data → Proof of principle for the methodology

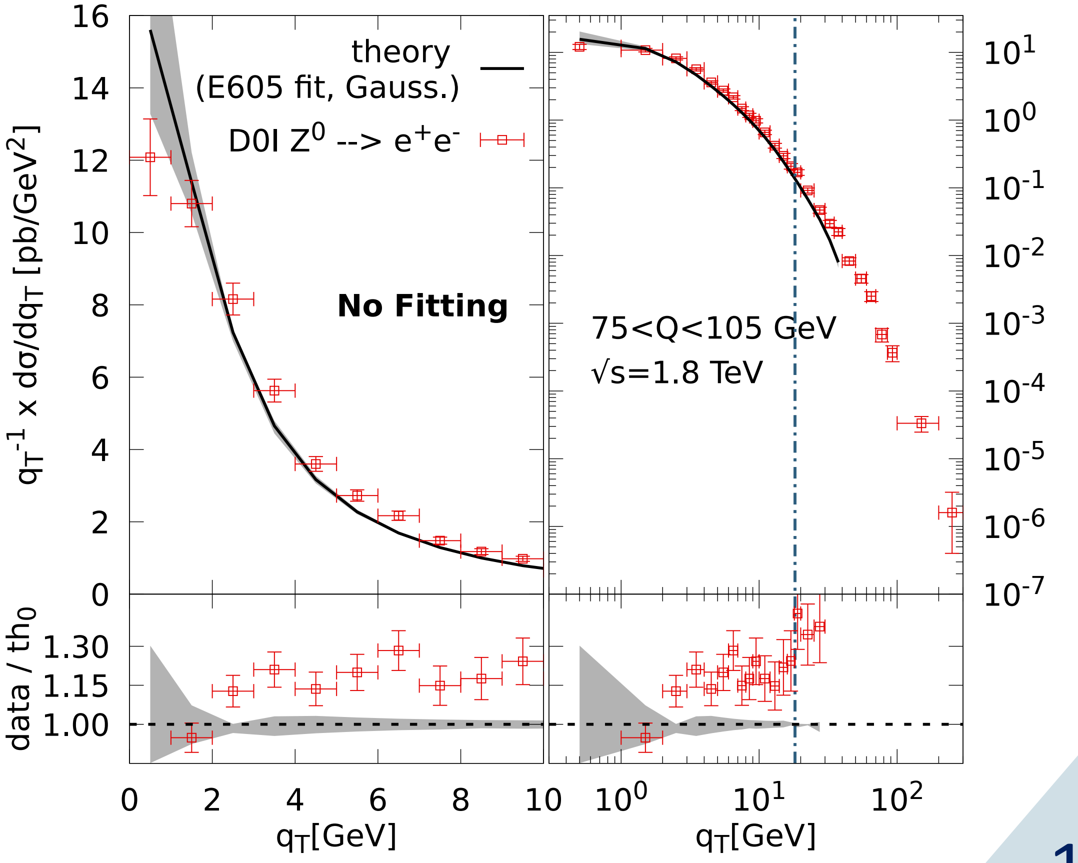




Higher Q postdictions: test different fits on the same experiment

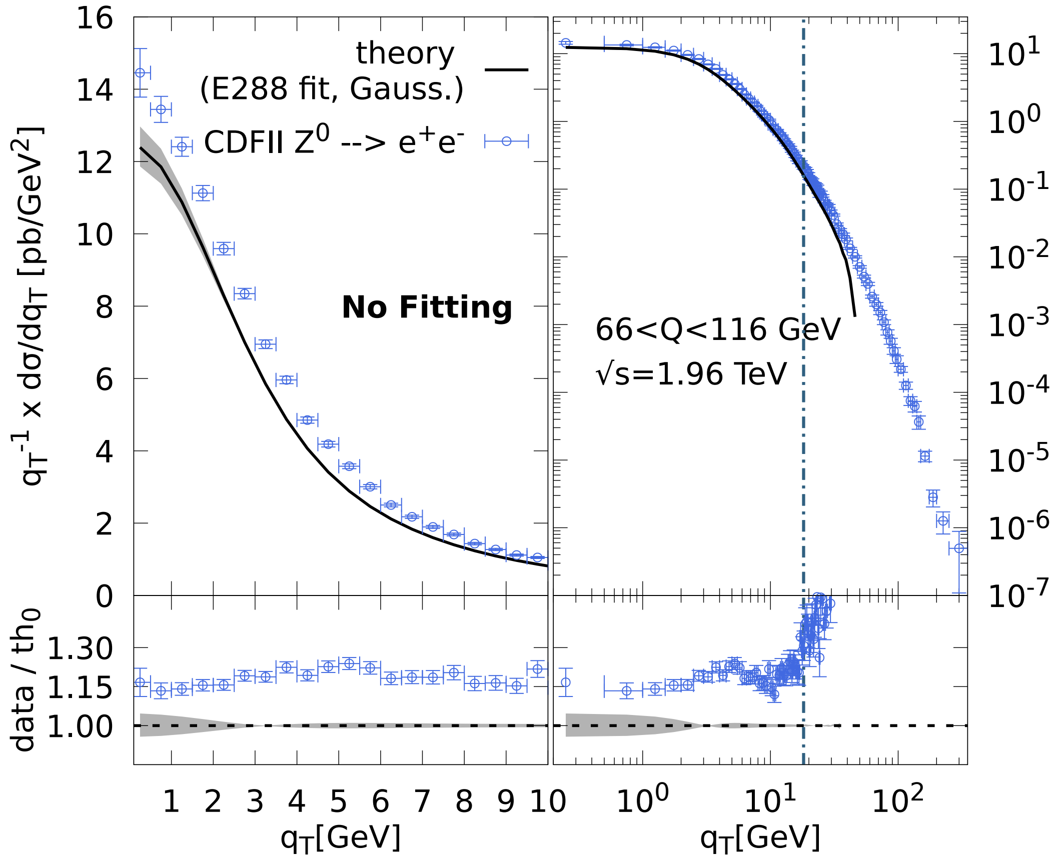
A postdiction of D0I with just E288 or E605 data → Proof of principle for the methodology



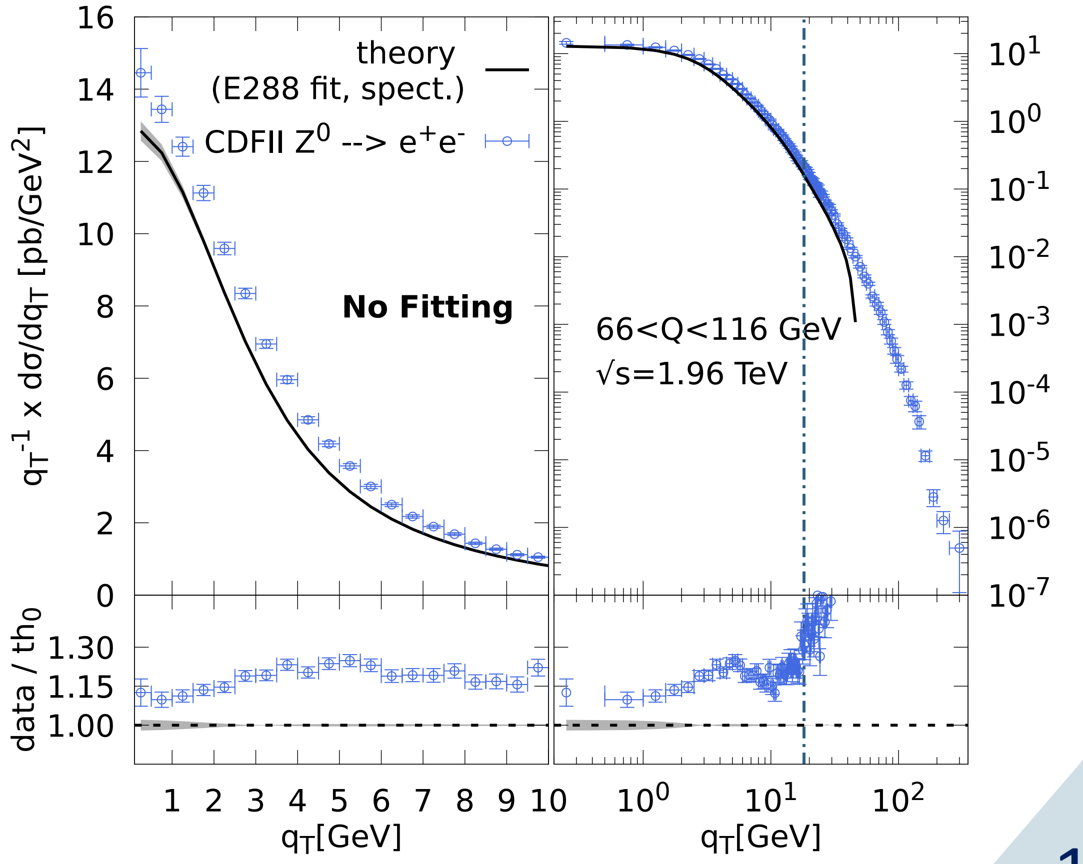


Higher Q postdictions:
test different models on the same experiment

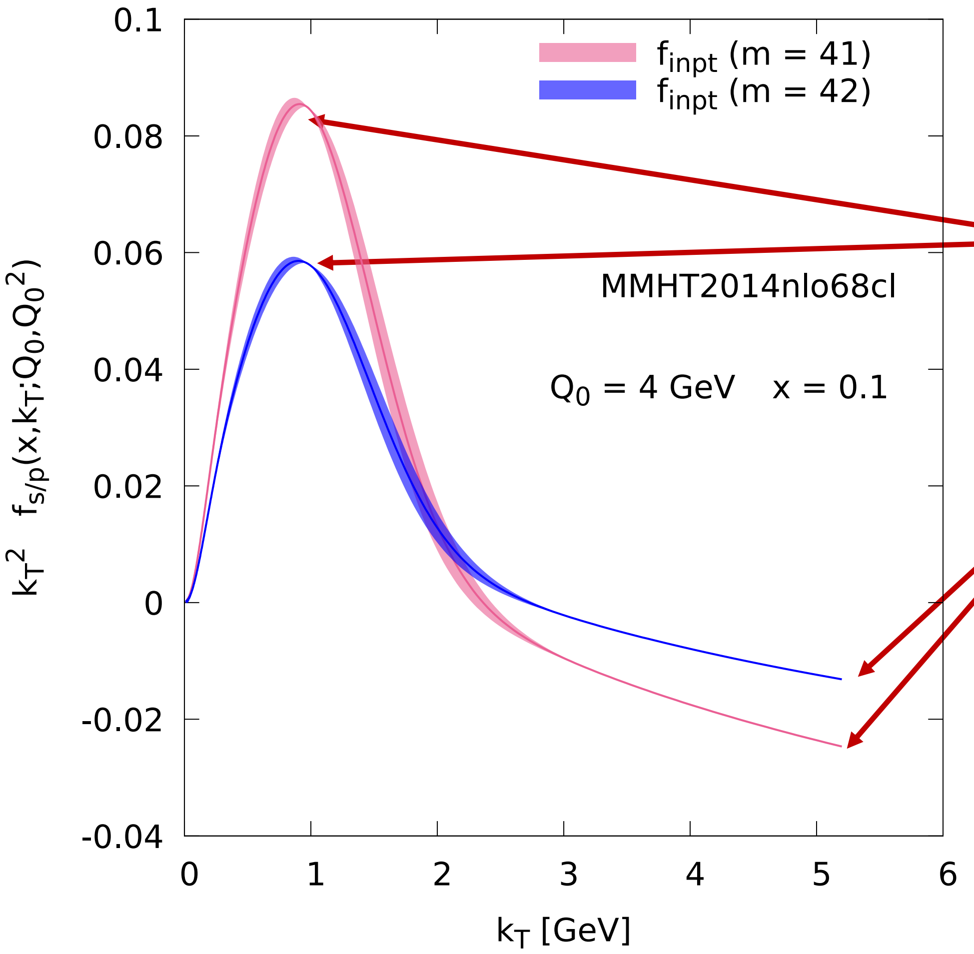
A postdiction of CDFII with E288 GAUSSIAN fit



A postdiction of CDFII with E288 SPECTATOR fit



TMDs are affected by collinear distributions



Example: take two pdfs associated with the same flavor (s here) and compute the input TMD

Maybe unexpected **different small k_T behavior** because of integral relation

Expected **different tails** because of the OPE expansion

Simply:

Changing the integral **necessarily** changes the integrand

Why is this important?

- We can **quantitatively** and **conclusively** answer the question:

  How much collinear dependence do my TMD extractions carry?

TMD operator $\xrightarrow{\int \mathrm{d}^2\boldsymbol{k}_T}$ Collinear operator + O(α_s)

TMD parametrization $\xrightarrow{\int \mathrm{d}^2\boldsymbol{k}_T}$ Collinear operator + O(α_s)

Summary

Usual CSS formalism but the HSO approach

1. Is consistent with the large k_T tail from theory (where it should)
2. Satisfies an integral relation: pseudo probabilistic interpretation
3. No b_{max} or b_{min} dependance: all errors are under control
4. NP (core) models are very easily swappable and testable

Pheno methodology: Fit low Q, test against higher Q (not mandatory)

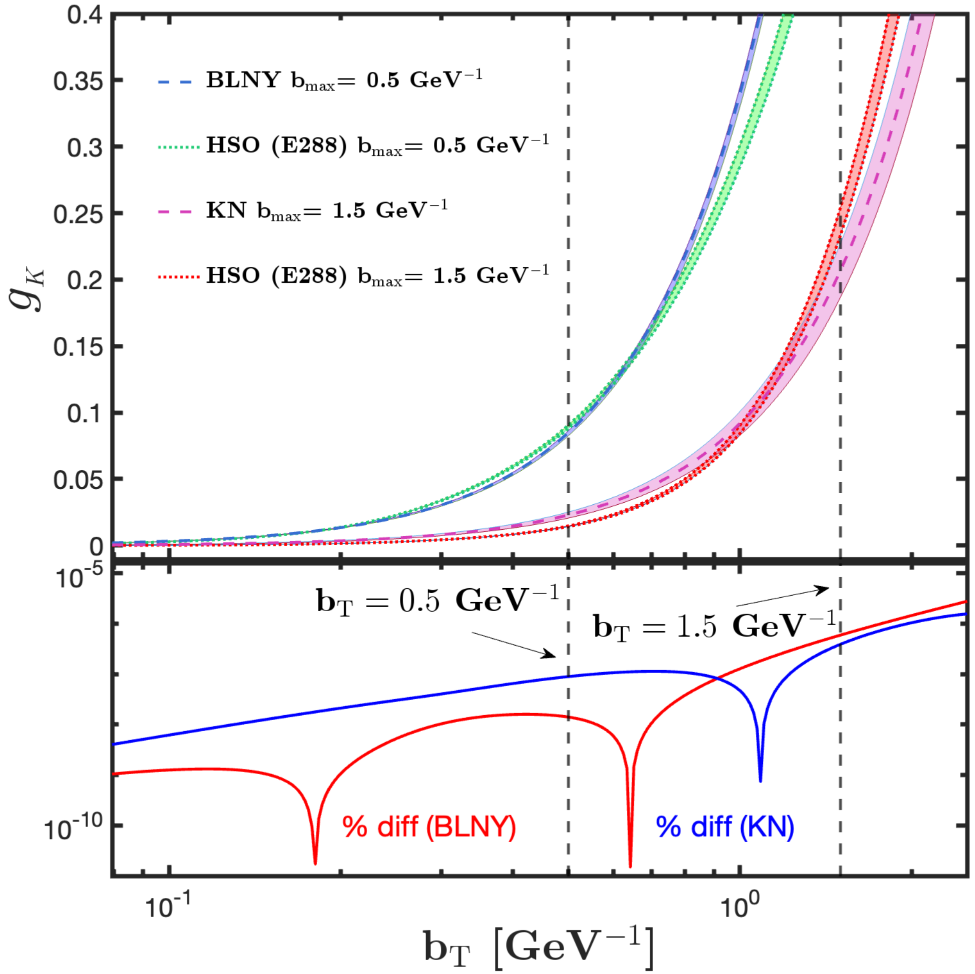
NEXT/SOON:

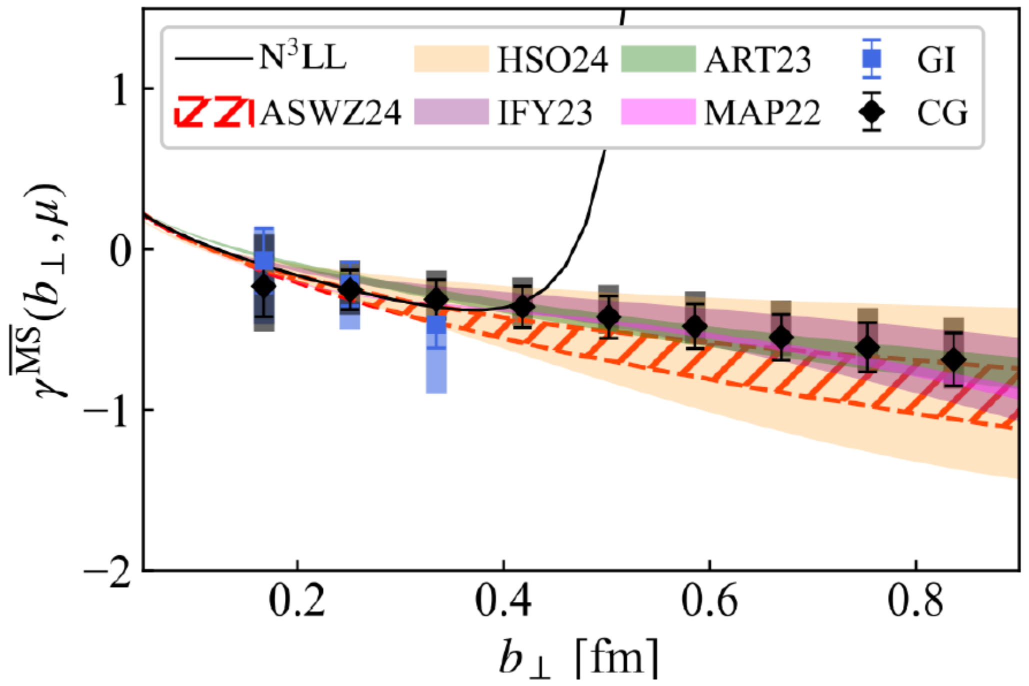
SIDIS large q_T issue, more refined models, input from Lattice?, higher orders…

Thank you

Backup slides

The NP Collins-Soper kernel





Lattice calculation from

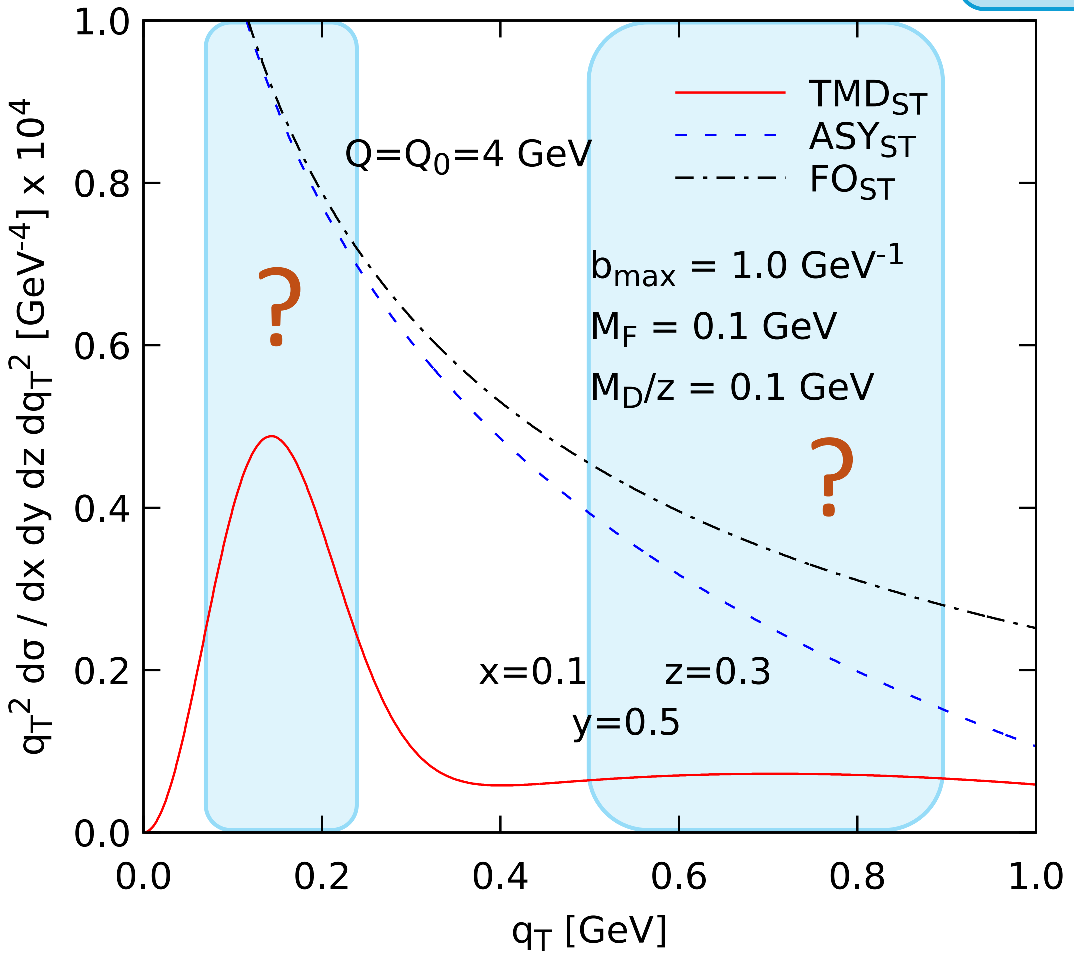
Bollweg, Gao, Mukherjee, Zhao, (2024), 2403.00664 [hep-lat]

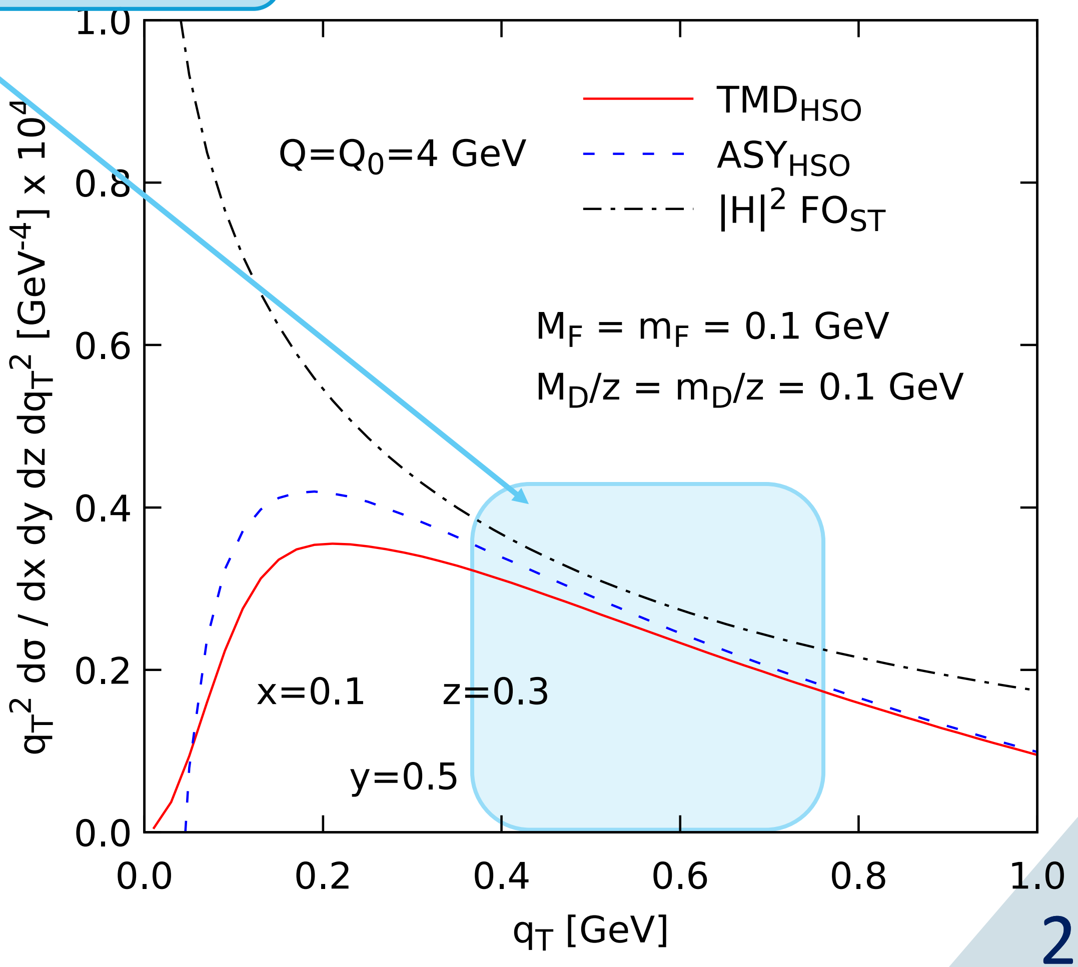
Conventional vs HSO - SIDIS cross section (not a fit)

Conventional

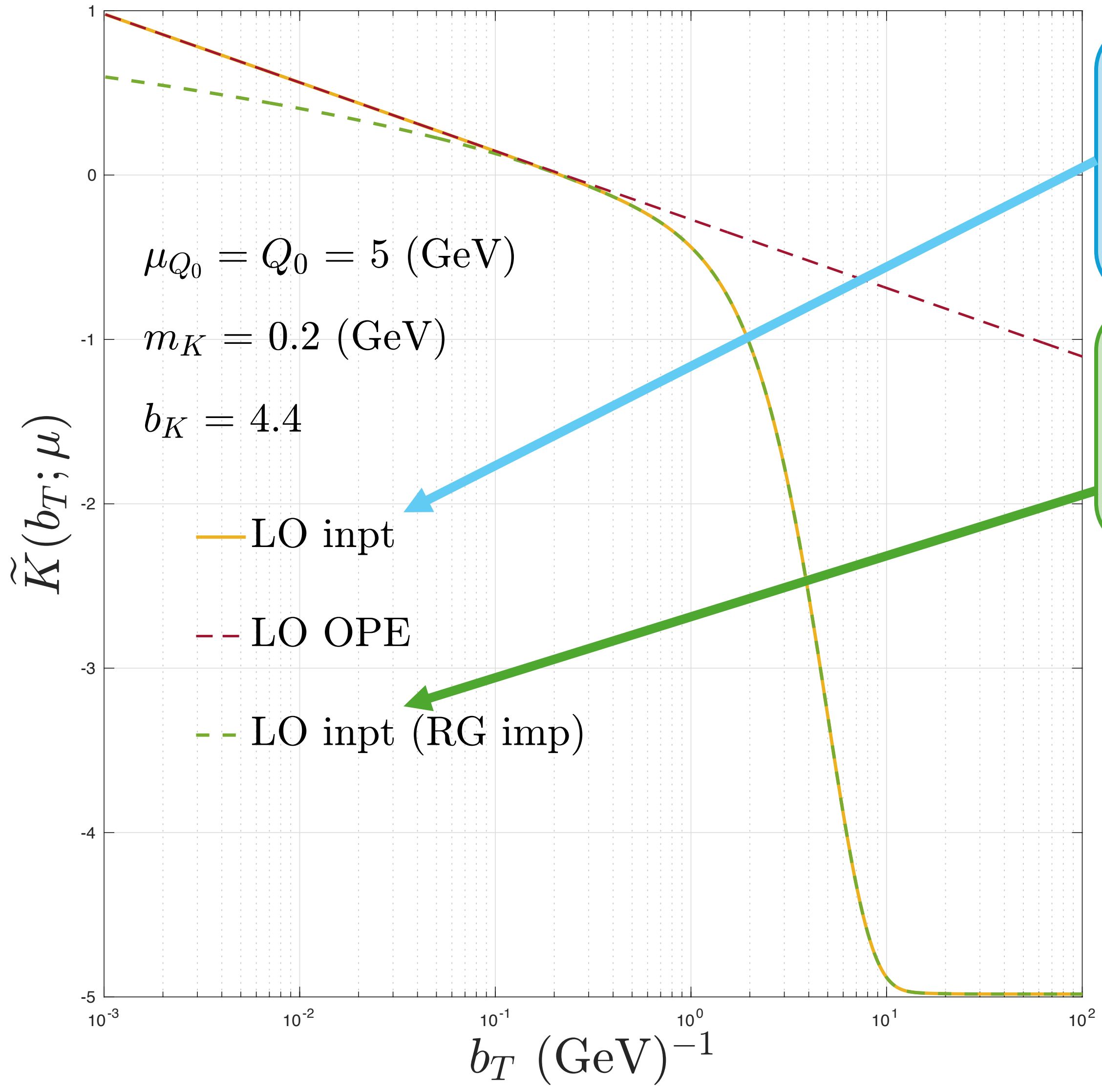
Matching region !!!

HSO (Gaussian)





RG improvements for CS-kernel (LO example)



$$\widetilde{K}^{(LO)}_{\text{input}}(b_T;\mu_{Q_0}) = 2\pi A^{(1)}_K(\mu_{Q_0}) K_0(m_K b_T) + b_K\left(e^{-m_K^2 b_T^2} - 1\right) + D_K(\mu_{Q_0})$$

$$\underline{\widetilde{K}}(b_T;\mu_{Q_0}) \equiv \widetilde{K}(b_T;\mu_{\overline{Q_0}}) - \int_{\mu_{\overline{Q_0}}}^{\mu_{Q_0}} \frac{\mathrm{d}\mu'}{\mu'}\gamma_K\left(a_S(\mu')\right)$$

A good approximation even for $b_T < 1/Q_0$

NO b_* and/or b_{max} / b_{min} necessary