Consistent treatment of TMDs in the large-kT limit.

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Based on:

JOGH, T.C. Rogers T., N. Sato
Phys.Rev.D 106 (2022) 3, 034002 • e-Print: 2205.05750 [hep-ph]

JOGH, T. Rainaldi, T.C. Rogers
e-Print: 2303.04921 [hep-ph]
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Work in progress Jlab/ODU/Torino collaboration:
F. Aslan, M. Boglione, JOGH, T.C. Rogers, T. Rainaldi, A. Simonelli

OUTLINE

- ✳ CSS formula, standard treatment.
- ✳ Potential issues in pheno applications.
- ✳ Constraints on TMD models and HSO approach.
- ✳ Standard treatment vs HSO approach.

*CSS formula, standard treatment.

Take the SIDIS cross section as an example

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}q_{\mathrm{T}}^2} = \frac{\pi^2 \alpha_{\mathrm{em}}^2 z}{Q^2\, x\, y} \left[F_1\, x\, y^2 + F_2\,(1-y)\right]$$

$$F = F^{\mathrm{TMD}} + \boxed{O\left(m/Q, q_{\mathrm{T}}/Q\right)}\,, \longleftarrow \text{errors}$$

$$F_1^{\mathrm{TMD}} \equiv 2\, z \sum_j |H|_j^2 \left[f_{j/p}, D_{h/j}\right]\,, \qquad F_2^{\mathrm{TMD}} \equiv 4\, z\, x \sum_j |H|_j^2 \left[f_{j/p}, D_{h/j}\right]$$

$$[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\ e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} \boxed{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},\mu_{Q_0}^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\boldsymbol{b}_\mathrm{T};\mu_{Q_0},\mu_{Q_0}^2)}$$
$$\times \exp\left\{2\int_{\mu_{Q_0}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right]+\ln\frac{Q^2}{Q_0^2}\tilde{K}(\boldsymbol{b}_\mathrm{T};\mu_{Q_0})\right\}.$$

Operator definitions:
Universality, predictive power, true properties of hadrons.

These definitions imply a behavior at small bT (large kT), calculable in pQCD.

$$
\begin{aligned}
\left[f_{j/p}, D_{h/j}\right] \to & \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2} \; e^{-i\boldsymbol{q}_\mathrm{T}\cdot \boldsymbol{b}_\mathrm{T}} \; \tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \; \tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \\
& \times \exp\left\{ 2 \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*; \mu_{b_*}) \right\} \\
& \times \exp\left\{ -g_{j/p}(x, b_\mathrm{T}) - g_{h/j}(z, b_\mathrm{T}) - g_K(b_\mathrm{T}) \ln\left(\frac{Q^2}{Q_0^2} \right) \right\} \; .
\end{aligned}
$$

`Same formula, just reorganized`

$$
-g_{j/p}(x, b_\mathrm{T}) \equiv \ln \left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right) \, , \qquad -g_{h/j}(z, b_\mathrm{T}) \equiv \ln \left(\frac{\tilde{D}_{h/j}(z, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right) \, ,
$$

$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_\mathrm{T}; \mu) \, .
$$

$$
\begin{aligned}
\left[f_{j/p}, D_{h/j}\right] \to \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\; e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; & \tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)\; \tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \\
& \times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*; \mu_{b_*})\right\} \\
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$$

$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_\mathrm{T}; \mu) .
$$

Precise definitions for g functions, $b_*(b_T)$ is a transition function bounded by some b_{max}. Note that b_* dependence cancels exactly.

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\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2} \; e^{-i\boldsymbol{q}_\mathrm{T}\cdot \boldsymbol{b}_\mathrm{T}} \; \tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \; \tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \\
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$$

$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_\mathrm{T}; \mu) .
$$

$$
\boxed{\boldsymbol{b}_*(b_\mathrm{T}) = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2 / b_\mathrm{max}^2}} ,}
$$

Precise definitions for g functions, $b_*(b_\mathrm{T})$ is a transition function bounded by some b_{max}. Note that b_* dependence cancels exactly. **It is really unimportant which b_* we choose.**

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\; e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; & \tilde{f}^{\mathrm{OPE}}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*})\; \tilde{D}^{\mathrm{OPE}}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*}) \\
& \times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu')) \right] + \ln\frac{Q^2}{\mu^2_{b_*}} \tilde{K}(b_*; \mu_{b_*}) \right\} \\
& \times \exp\left\{ -g_{j/p}(x, b_\mathrm{T}) - g_{h/j}(z, b_\mathrm{T}) - g_K(b_\mathrm{T}) \ln\left(\frac{Q^2}{Q_0^2}\right) \right\} \boxed{+\, O(b_{\max}\, m)} \longleftarrow \text{errors}
\end{aligned}
$$

Use of OPE introduces errors. Must keep b_{max} reasonably small.

$$
\frac{\mathrm{d}}{\mathrm{d}b_{\max}} \left[f_{j/p}, D_{h/j} \right] = O\left(m b_{\max} \right)
$$

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\; e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; \tilde{f}^{\mathrm{OPE}}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})\; \tilde{D}^{\mathrm{OPE}}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*}) \\
\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right]+\ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*;\mu_{b_*})\right\} \\
\times \exp\left\{-g_{j/p}(x,b_\mathrm{T})-g_{h/j}(z,b_\mathrm{T})-g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}\boxed{+O(b_{\max}\, m)} \longleftarrow \text{errors}
\end{aligned}
$$

Use of OPE introduces errors. Must keep b_{max} reasonably small.

$$
\frac{\mathrm{d}}{\mathrm{d}b_{\max}}\left[f_{j/p}, D_{h/j}\right] = O\left(m b_{\max}\right)
$$

One should also have small sensitivity to the choice of b_*

$$
\boldsymbol{b}_*(b_\mathrm{T}) = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1+b_\mathrm{T}^2/b_{\max}^2}}\,,
$$

$$
\left[f_{j/p}, D_{h/j}\right] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\; e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\;\tilde{f}^{\mathrm{OPE}}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})\;\tilde{D}^{\mathrm{OPE}}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})
$$

$$
\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right]+\ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*;\mu_{b_*})\right\}
$$

Models

$$
\boxed{\times \exp\left\{-g_{j/p}(x,b_\mathrm{T})-g_{h/j}(z,b_\mathrm{T})-g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}} + O(b_{\max}\, m)
$$

Definitions:
Smooth transition to small-b_T region by construction

$$
-g_{h/j}(z,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)
$$

$$
-g_{j/p}(x,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)
$$

$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_\mathrm{T};\mu)\,.
$$

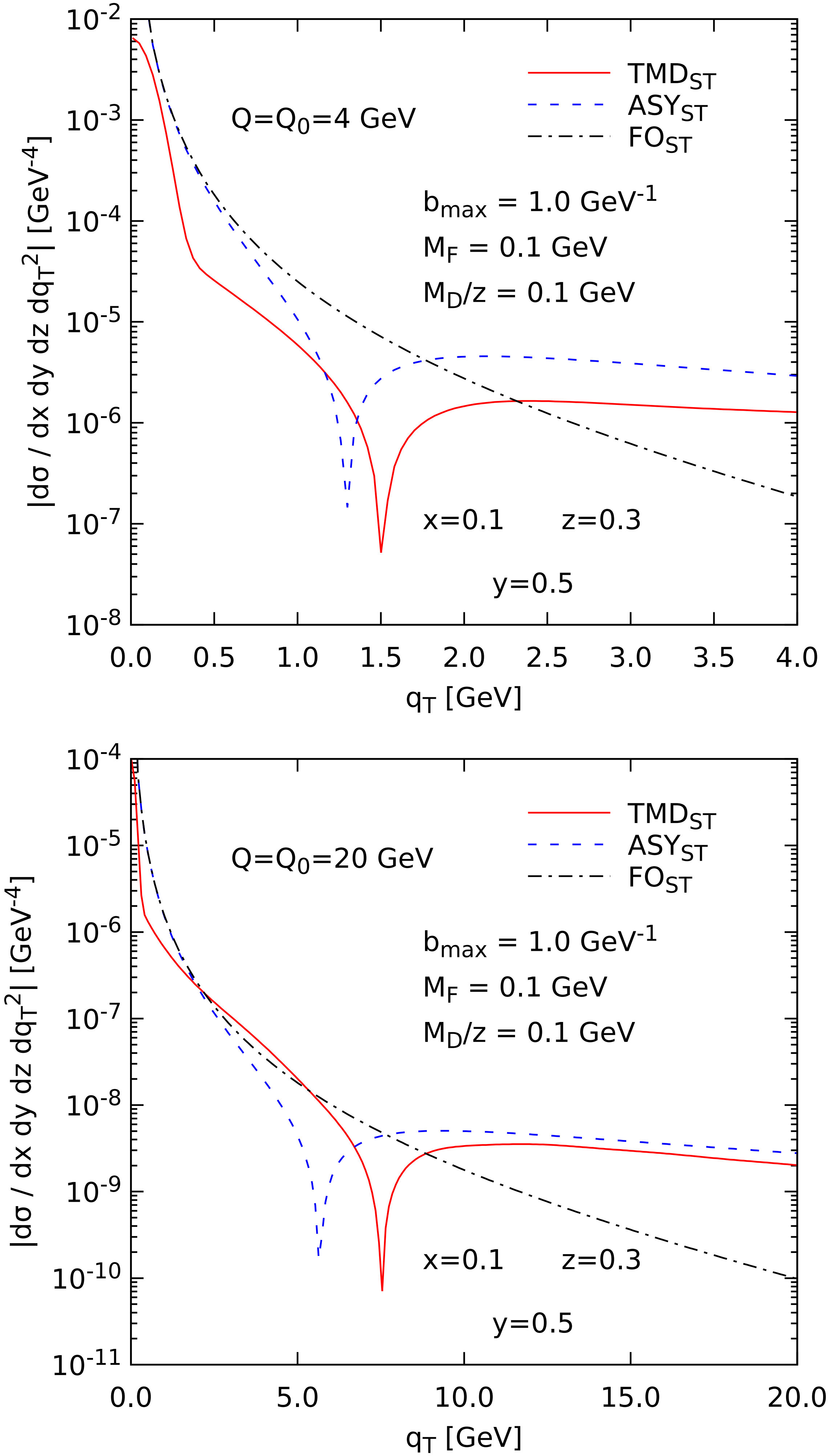
Typical choices:
generally unconstrained

$$
g_{h/j}(z,b_\mathrm{T}) = \frac{1}{4\,z^2}M_D^2\,b_\mathrm{T}^2
$$

$$
g_{j/p}(x,b_\mathrm{T}) = \frac{1}{4}\,M_F^2\,b_\mathrm{T}^2
$$

$$
g_K(b_\mathrm{T}) = \frac{g_2}{2\,M_K^2}\ln\left(1+M_K^2 b_\mathrm{T}^2\right)
$$

✳Potential issues in pheno applications.



Issues:

Typical choices:

generally unconstrained

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\,z^2} M_D^2\, b_{\mathrm{T}}^2$$

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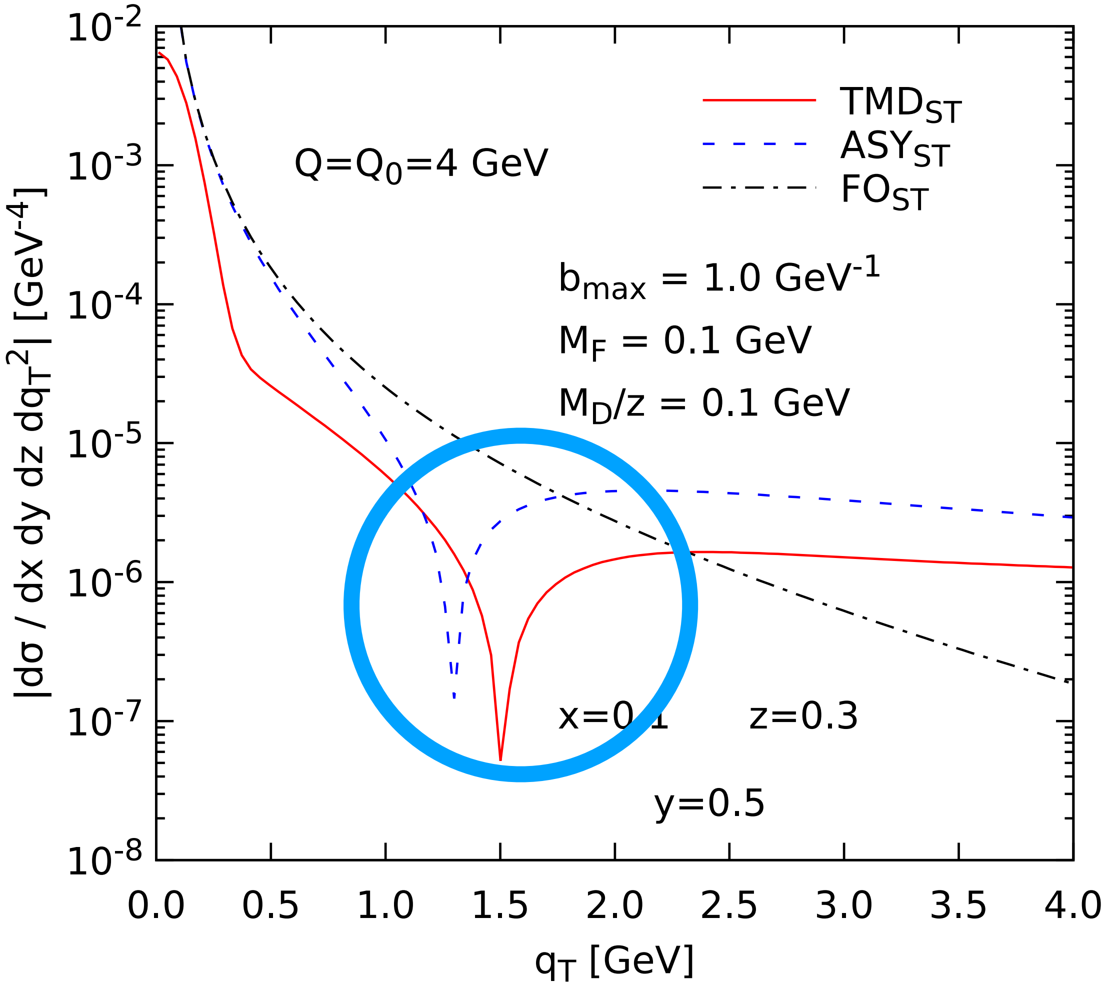

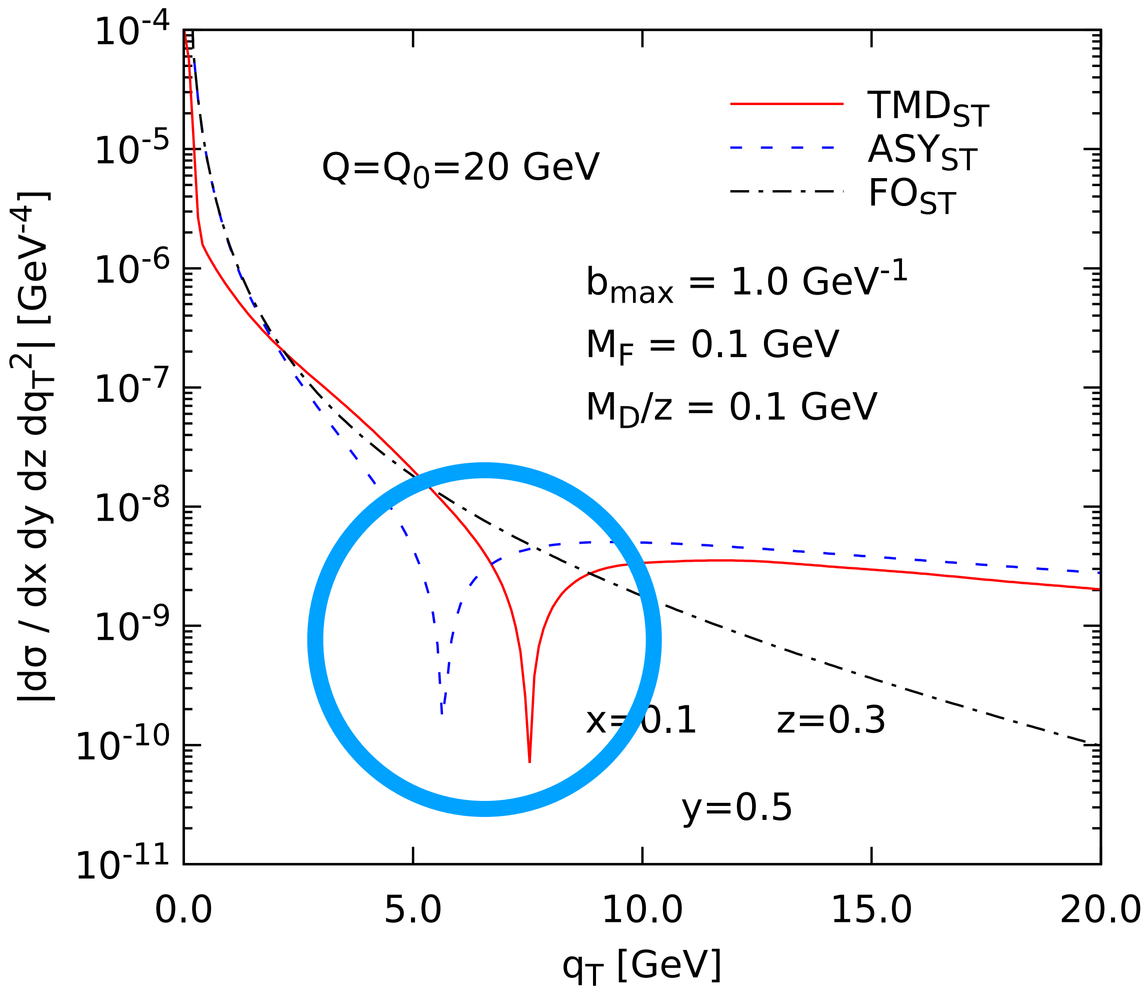

Issues:

Asymptotic term does not approximate well the TMD term, even at a scale of **Q_0=20 GeV**

Typical choices:
generally unconstrained

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\,z^2} M_D^2\, b_{\rm T}^2$$

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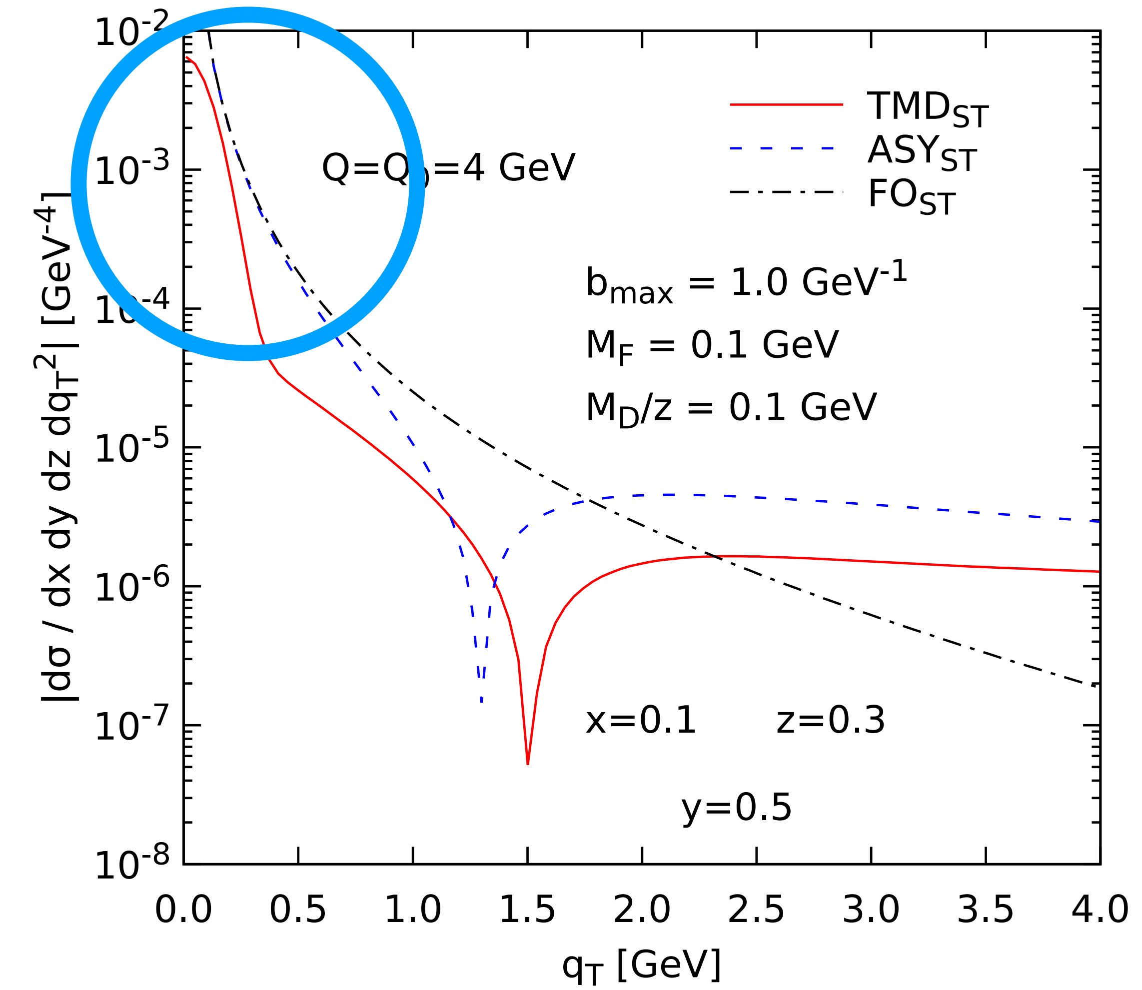

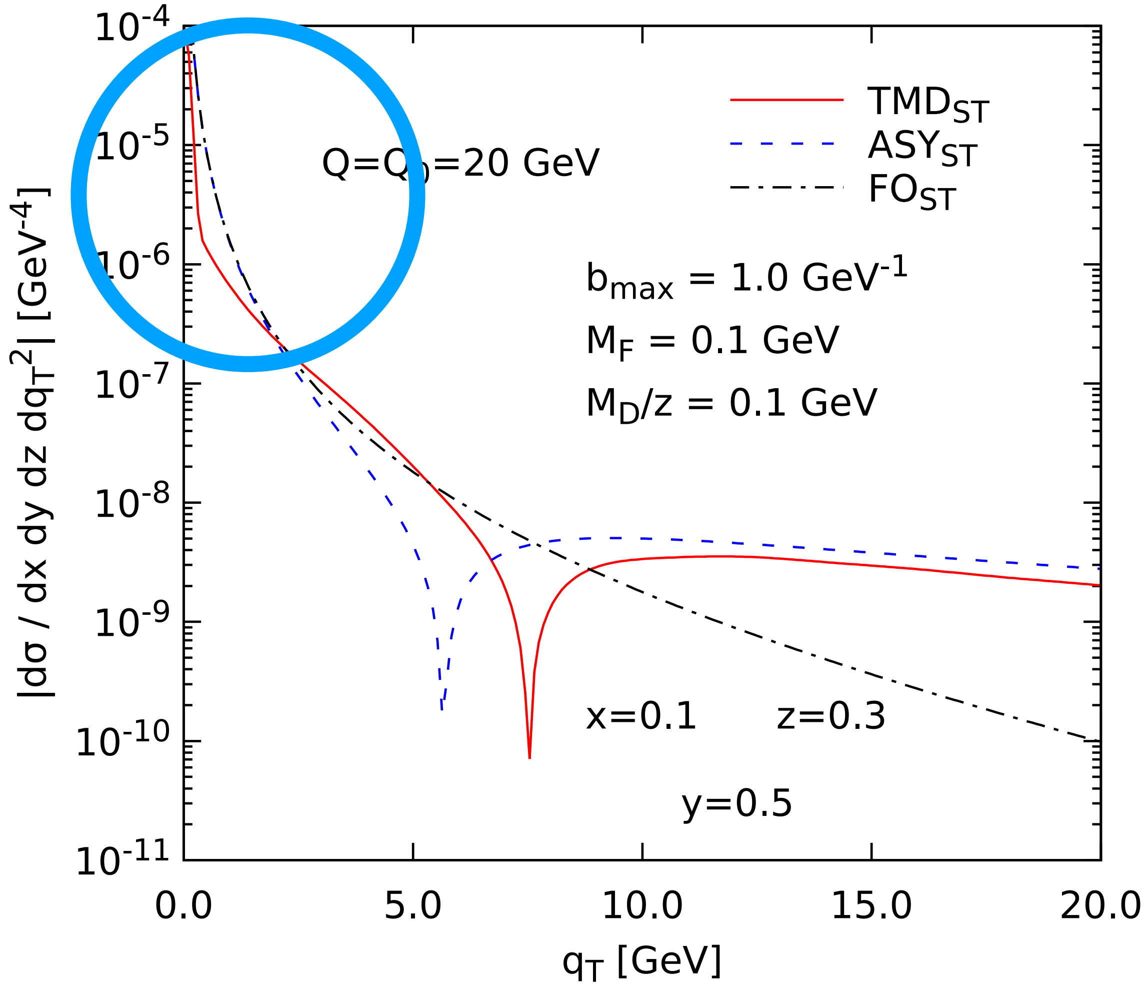

Issues:

No region of "overlap" between TMD term and FO. This means smooth matching is not possible

Typical choices:
generally unconstrained

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\,z^2} M_D^2\, b_{\mathrm{T}}^2$$

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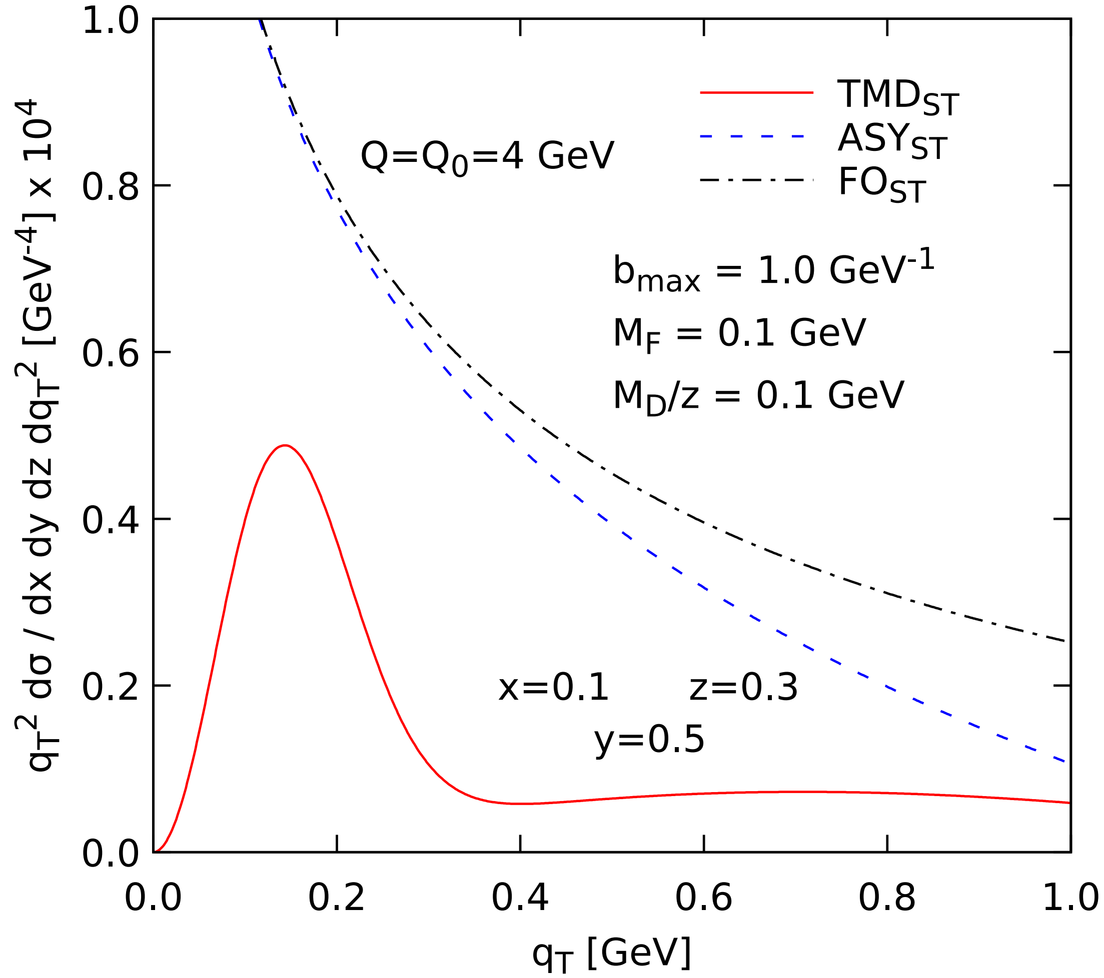

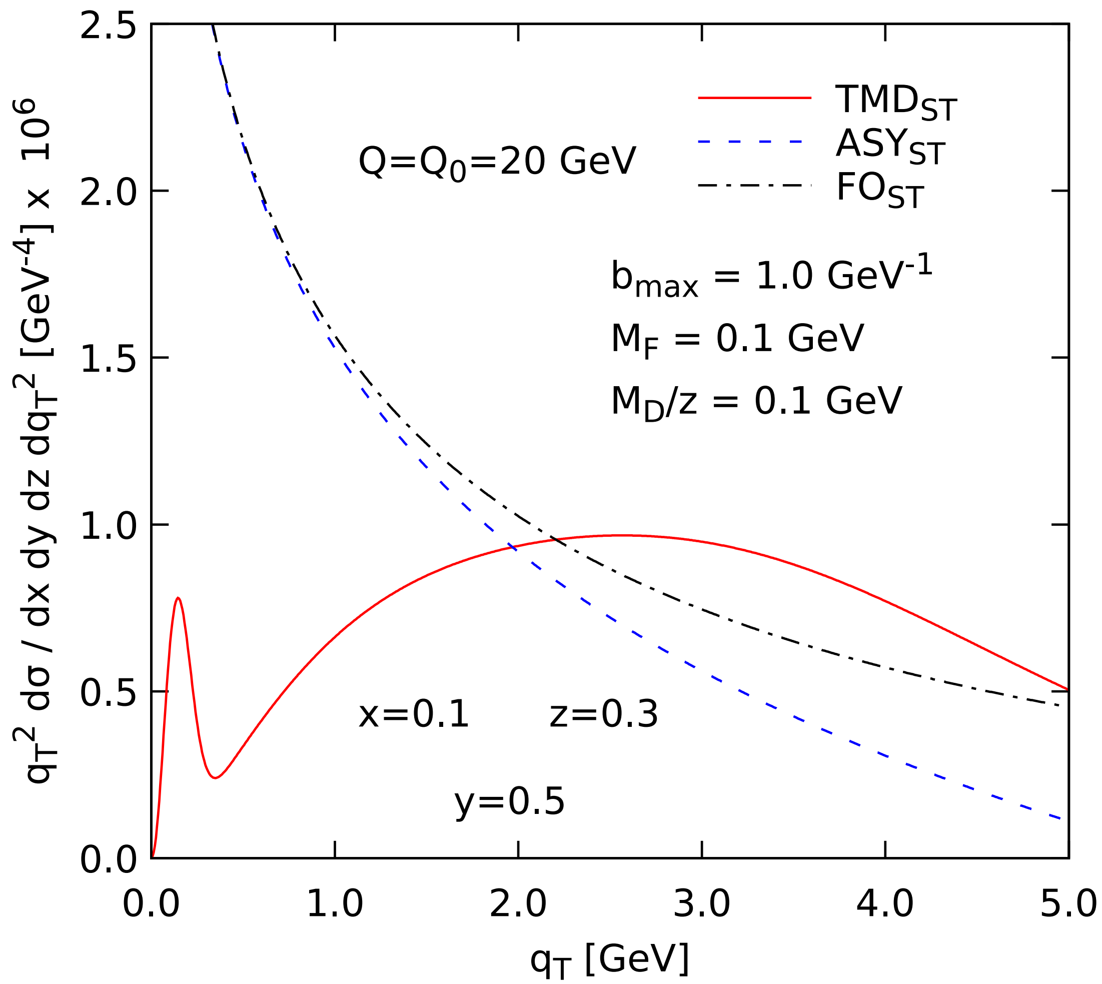

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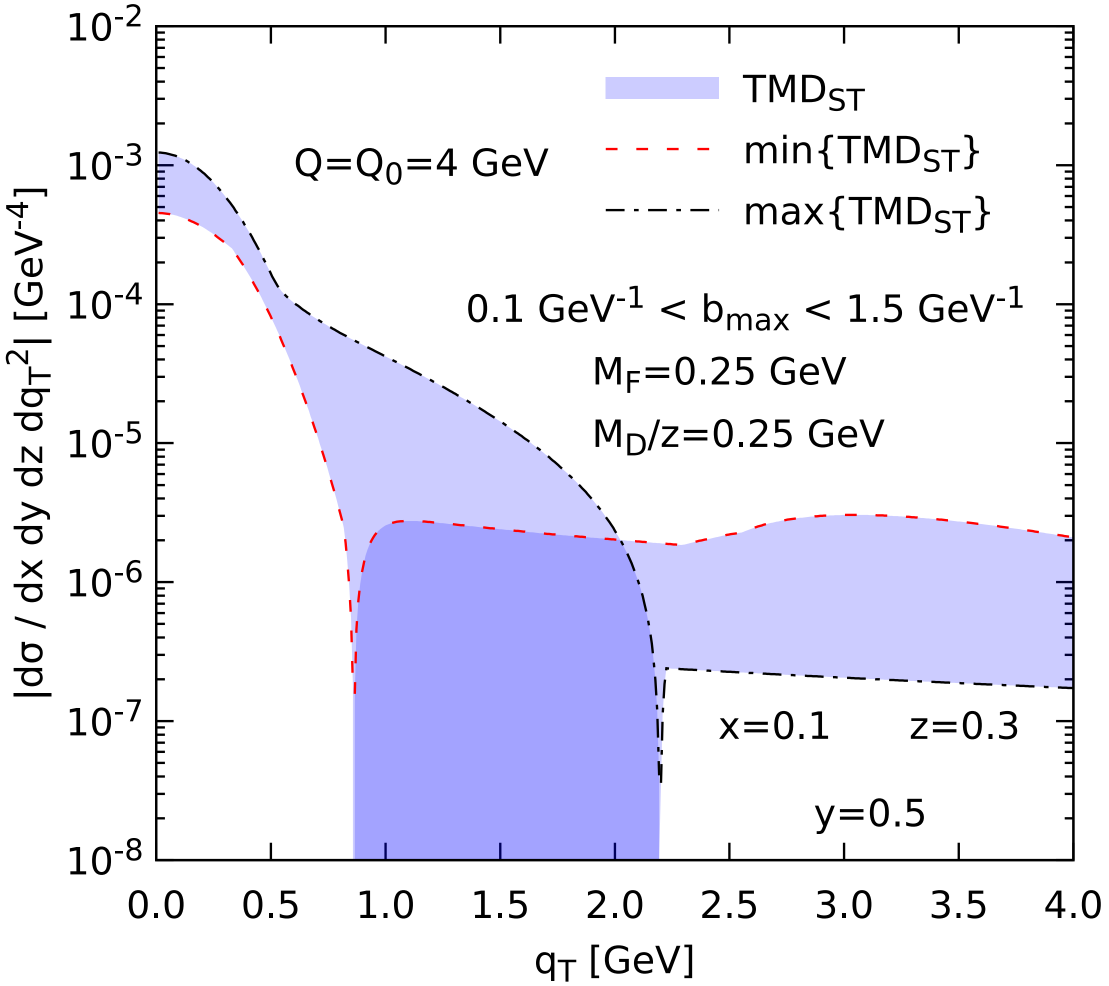

Issues:

Values of b_{max} in pheno applications are often too large. Strong dependence b_{max} or the functional form of b_* signals a problem.

Typical choices:
generally unconstrained

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\,z^2} M_D^2\, b_{\mathrm{T}}^2$$

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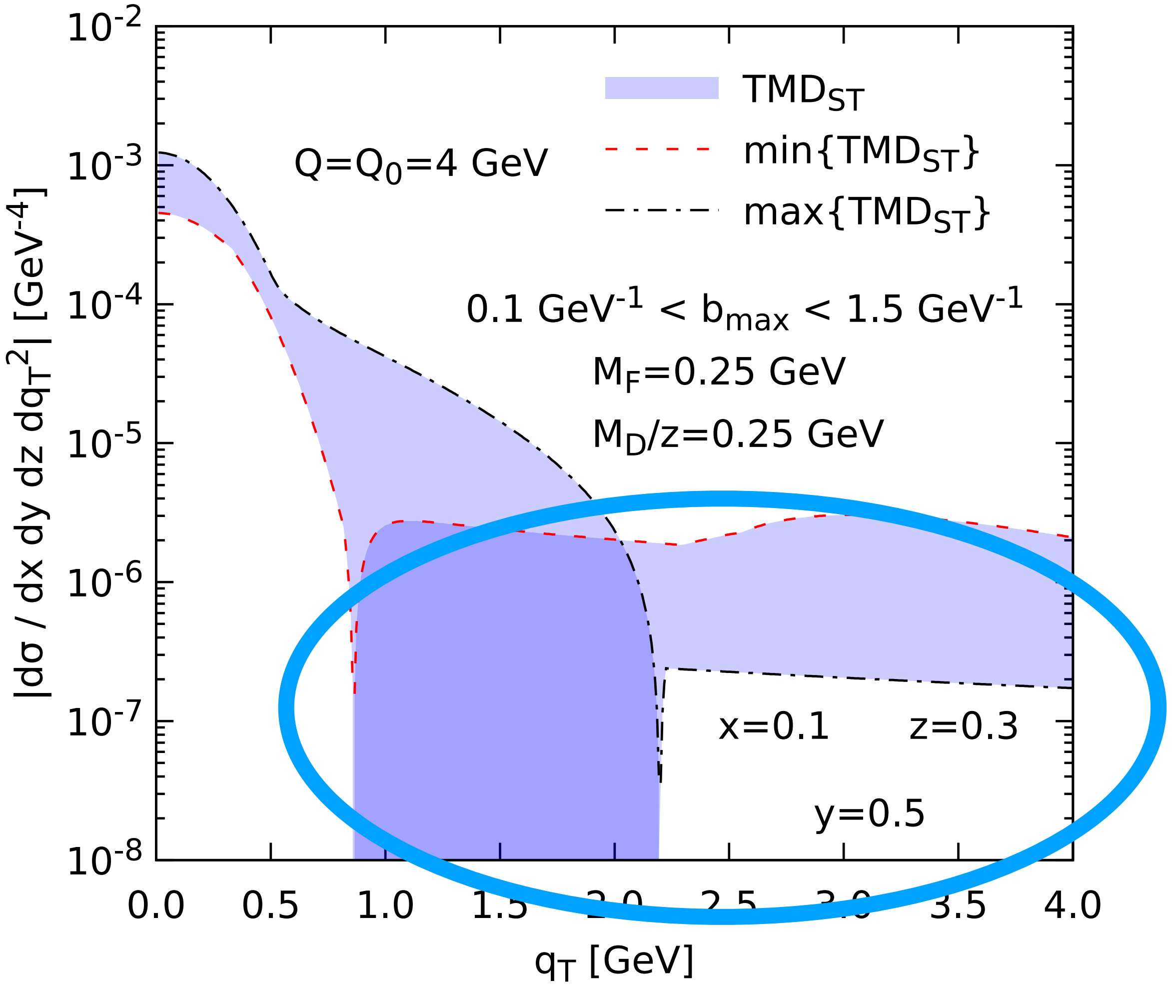

Issues:

Note the large-q_T (small-b_T) region should be determined by the OPE.

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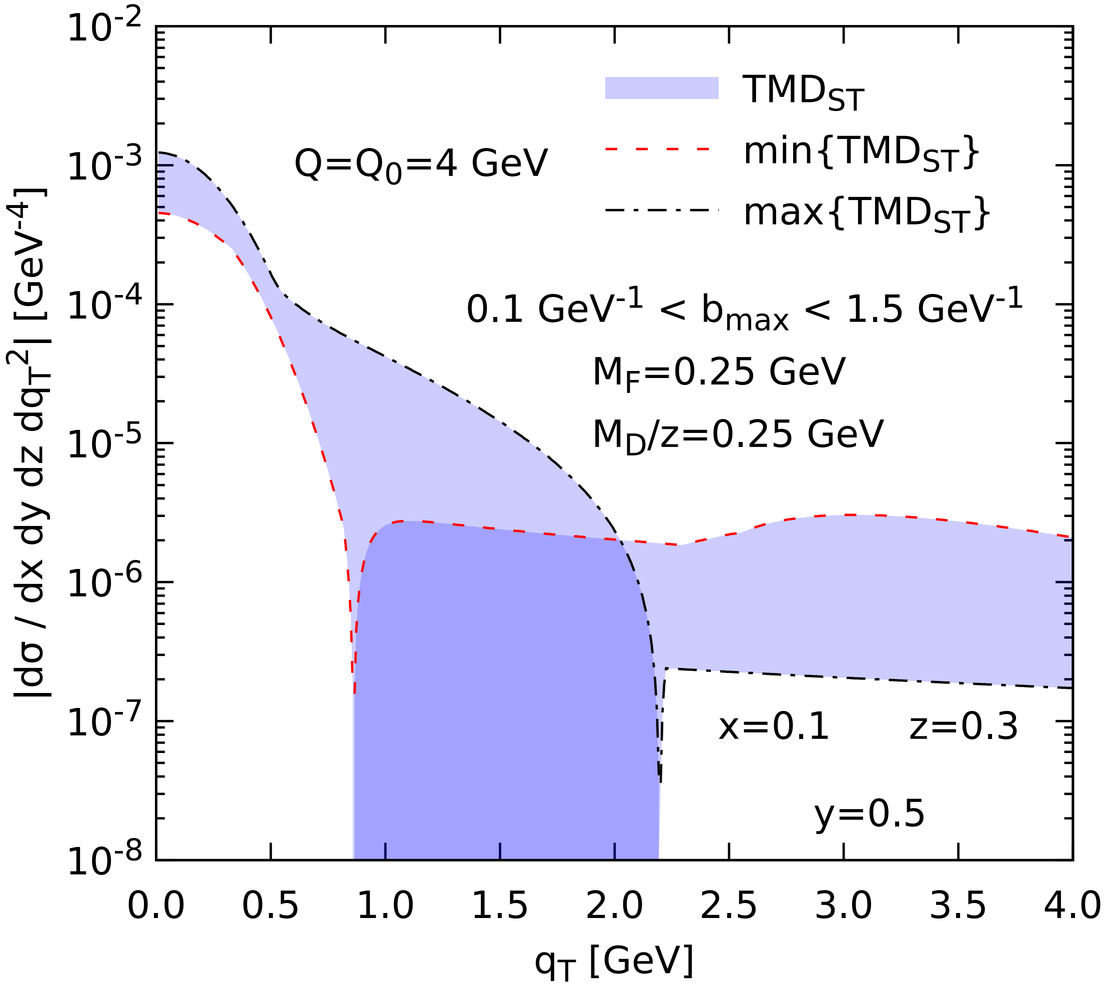
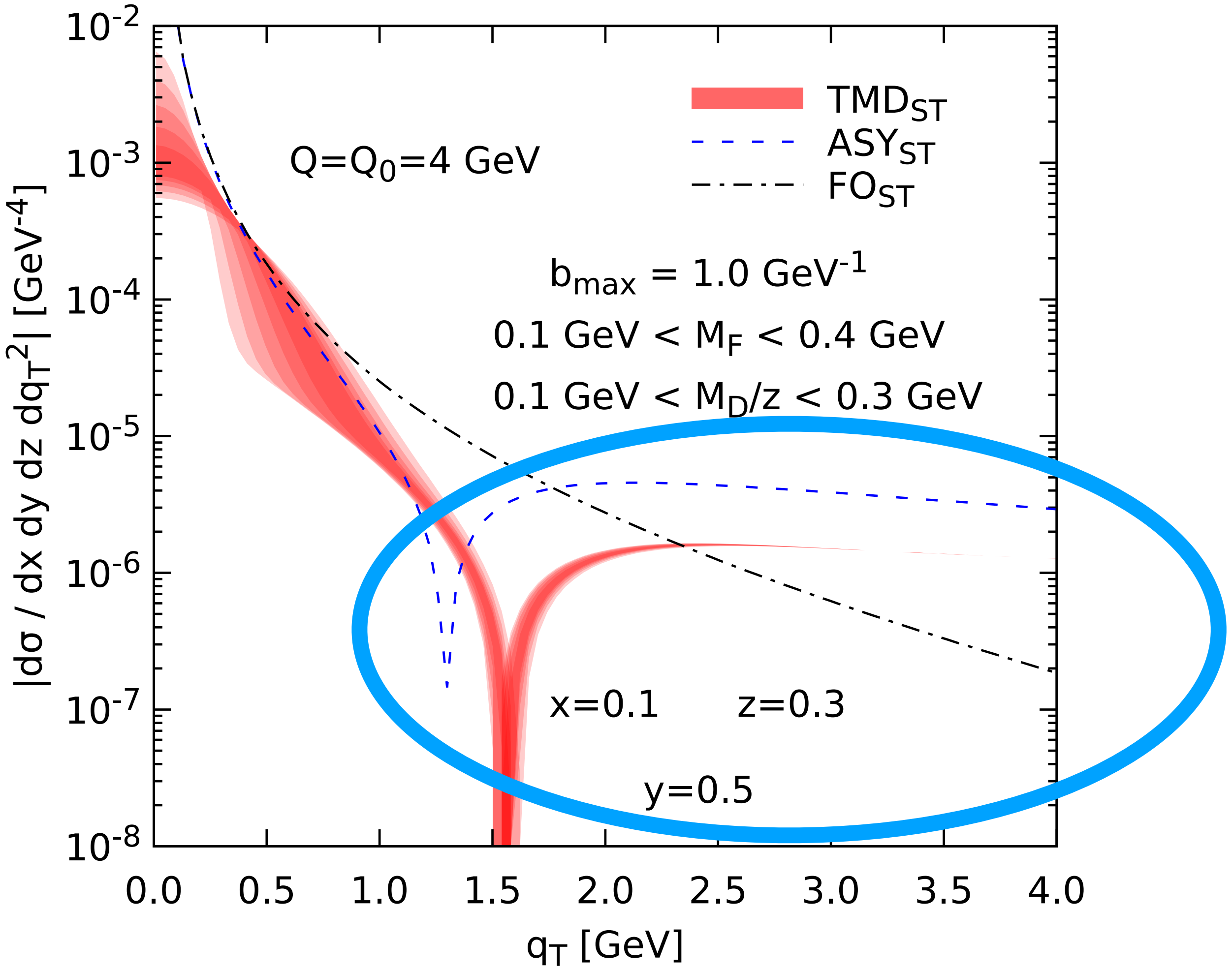


Issues:

Note the large-q_T (small-b_T) region should be determined by the OPE.

Small mass parameters can't really compensate for this b_{max} dependence.



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* These issues can be resolved by carefully constraining the TMD models.

* It turns out to be more natural to do so in momentum space.

- ✳These issues can be resolved by carefully constraining the TMD models.
- ✳It turns out to be more natural to do so in momentum space.
- ✳Constraints are ultimately equivalent to those that one **attempts** to implement by means of the OPE (although, as we saw, this is not automatic):

* These issues can be resolved by carefully constraining the TMD models.

* It turns out to be more natural to do so in momentum space.

* Constraints are ultimately equivalent to those that one **attempts** to implement by means of the OPE (although, as we saw, this is not automatic):

1) pQCD tail

$$f^{\text{pert}}_{\text{inpt},i/p}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\text{T}}^2}\right] + \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2}A^{f,g}_{i/p}(x;\mu_{Q_0})\,,$$

2) Integral relations

$$f^c(x;\mu) \equiv \pi\int_0^{\mu^2} \mathrm{d}k_{\text{T}}^2\, f_{i/p}(x,\boldsymbol{k}_{\text{T}};\mu;\zeta)$$

Note collinear function defined with a cutoff in the k_T integral. This retains a parton model interpretation.

More details follow, first spoilers…
(from J.O.G.H., T. Rainaldi, T.C. Rogers
e-Print: 2303.04921 [hep-ph]
Accepted in Phys. Rev. D)

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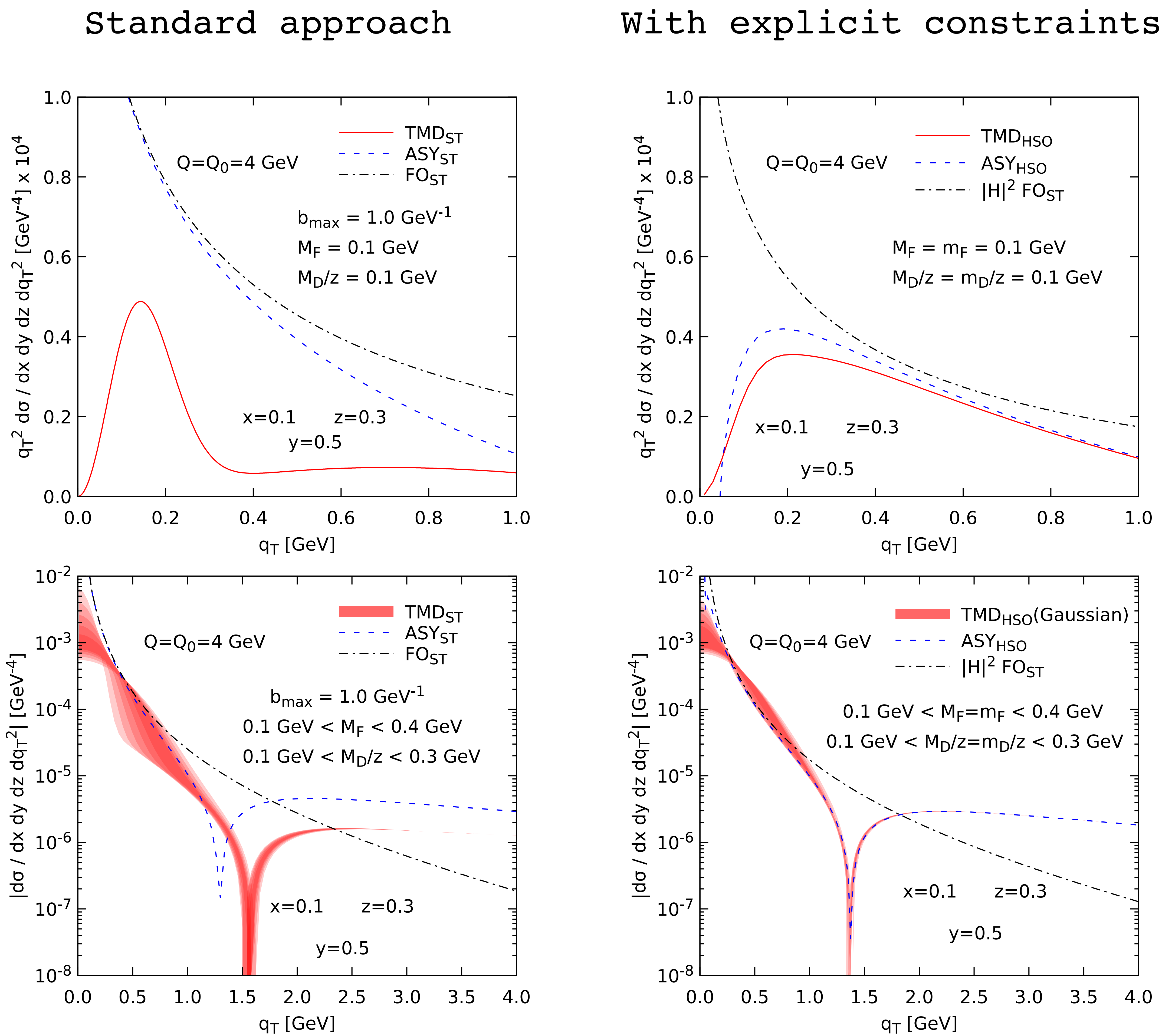

Standard approach
With explicit constraints
TMD$_{ST}$
ASY$_{ST}$
FO$_{ST}$
Q=Q$_0$=4 GeV
b$_{max}$ = 1.0 GeV^{-1}
M$_F$ = 0.1 GeV
M$_D$/z = 0.1 GeV
x=0.1 z=0.3
y=0.5
q_T^2 dσ / dx dy dz dq_T^2 [GeV^{-4}] x 10^4
q_T [GeV]
TMD$_{HSO}$
ASY$_{HSO}$
|H|^2 FO$_{ST}$
M$_F$ = m$_F$ = 0.1 GeV
M$_D$/z = m$_D$/z = 0.1 GeV
0.1 GeV < M$_F$ < 0.4 GeV
0.1 GeV < M$_D$/z < 0.3 GeV
|dσ / dx dy dz dq_T^2| [GeV^{-4}]
TMD$_{HSO}$(Gaussian)
0.1 GeV < M$_F$=m$_F$ < 0.4 GeV
0.1 GeV < M$_D$/z=m$_D$/z < 0.3 GeV

✳ Constraints on TMD models and HSO approach.

Model in the HSO approach

$$f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\left[A_{i/p}^{f}(x;\mu_{Q_0}) + B_{i/p}^{f}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\right]$$
$$+\frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0})$$
$$+C_{i/p}^{f}\,f_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2)\,,$$

Q_0 input scale: smallest scale where perturbation theory can be trusted

Model in the HSO approach

$$f_{\text{inpt},i/p}(x, \boldsymbol{k}_{\text{T}}; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi} \frac{1}{k_{\text{T}}^2 + m_{f_{i,p}}^2} \left[A_{i/p}^f(x; \mu_{Q_0}) + B_{i/p}^f(x; \mu_{Q_0}) \ln \frac{Q_0^2}{k_{\text{T}}^2 + m_{f_{i,p}}^2} \right]$$
$$+ \frac{1}{2\pi} \frac{1}{k_{\text{T}}^2 + m_{f_{g,p}}^2} A_{i/p}^{f,g}(x; \mu_{Q_0})$$
$$+ C_{i/p}^f \boxed{f_{\text{core},i/p}(x, \boldsymbol{k}_{\text{T}}; Q_0^2)\,,}$$

Any "core" model here

examples:

$$f_{\text{core},i/p}^{\text{Gauss}}(x, \boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{e^{-k_{\text{T}}^2/M_{\text{F}}^2}}{\pi M_{\text{F}}^2}$$

$$f_{\text{core},i/p}^{\text{Spect}}(x, \boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{6 M_{0\text{F}}^6}{\pi\,(2M_{\text{F}}^2 + M_{0\text{F}}^2)} \frac{M_{\text{F}}^2 + k_{\text{T}}^2}{(M_{0\text{F}}^2 + k_{\text{T}}^2)^{\,4}}$$

Model in the HSO approach

$$f_{\text{inpt},i/p}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2+\boxed{m_{f_{i,p}}^2}}\left[A_{i/p}^f(x;\mu_{Q_0}) + B_{i/p}^f(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\text{T}}^2+\boxed{m_{f_{i,p}}^2}}\right]$$
$$+\frac{1}{2\pi}\frac{1}{k_{\text{T}}^2+\boxed{m_{f_{g,p}}^2}}A_{i/p}^{f,g}(x;\mu_{Q_0})$$
$$+C_{i/p}^f\, f_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2)\,,$$

Transition between small and large k_T

Behaves as the pQCD tail, for large k_T

$$f_{\text{inpt},i/p}^{\text{pert}}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2}\left[A_{i/p}^f(x;\mu_{Q_0}) + B_{i/p}^f(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\text{T}}^2}\right] + \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0})\,,$$

Model in the HSO approach

$$
\begin{aligned}
f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) &= \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\left[A_{i/p}^{f}(x;\mu_{Q_0})+B_{i/p}^{f}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\right] \\
&+\frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0}) \\
&+\boxed{C_{i/p}^{f}}\,f_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2)\,,
\end{aligned}
$$

Determined by the integral relation

Integral relation

$$
f^{c}(x;\mu)\equiv\pi\int_0^{\mu^2}\mathrm{d}k_{\mathrm{T}}^2\,f_{i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu;\zeta)
$$

$$
\begin{aligned}
C_{i/p}^{f} \equiv \frac{1}{N_{i/p}^{f}}\Bigg[& f_{i/p}^{c}(x;\mu_{Q_0}) \\
& -A_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right)-B_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right)\ln\left(\frac{Q_0^2}{\mu_{Q_0}m_{f_{i,p}}}\right)\;-A_{i/p}^{f,g}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{g,p}}}\right)\Bigg]
\end{aligned}
$$

Model in the HSO approach

$$
\begin{aligned}
f_{\text{inpt},i/p}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = {} & \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2+m_{f_{i,p}}^2}\left[A_{i/p}^{f}(x;\mu_{Q_0}) + B_{i/p}^{f}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\text{T}}^2+m_{f_{i,p}}^2}\right] \\
& + \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0}) \\
& + \boxed{C_{i/p}^{f}}\, f_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2)\,,
\end{aligned}
$$

Determined by the integral relation

Integral relation (using $\overline{\text{MS}}$ functions)

$$
f^{c}(x;\mu) \equiv \pi\int_0^{\mu^2} \mathrm{d}k_{\text{T}}^2\, f_{i/p}(x,\boldsymbol{k}_{\text{T}};\mu;\zeta)
$$

$$
\begin{aligned}
C_{i/p}^{f} \equiv \frac{1}{N_{i/p}^{f}}\Bigg[& f_{i/p}^{\overline{\text{MS}}}(x;\mu_{Q_0}) + \frac{\alpha_s(\mu_{Q_0})}{2\pi}\left\{\sum_{jj'}\delta_{j'j}[\mathcal{C}_{\Delta}^{j'/j}\otimes d_{h/j'}](z;\mu_{Q_0}) + [\mathcal{C}_{\Delta}^{g/j}\otimes d_{h/g}](z;\mu_{Q_0})\right\}\Bigg] \\
& - A_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) - B_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right)\ln\left(\frac{Q_0^2}{\mu_{Q_0}m_{f_{i,p}}}\right) \; - A_{i/p}^{f,g}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{g,p}}}\right)\Bigg]
\end{aligned}
$$

Model in the HSO approach

$$
\begin{aligned}
f_{\text{inpt},i/p}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2+m_{f_{i,p}}^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\text{T}}^2+m_{f_{i,p}}^2}\right] \\
+ \frac{1}{2\pi}\frac{1}{k_{\text{T}}^2+m_{f_{g,p}}^2}A^{f,g}_{i/p}(x;\mu_{Q_0}) \\
+ C^f_{i/p}\, f_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2)\,,
\end{aligned}
$$

In b_{T} space

$$
\begin{aligned}
\tilde{f}_{\text{inpt},i/p}(x,\boldsymbol{b}_{\text{T}};\mu_{Q_0},Q_0^2) = K_0\left(b_{\text{T}}m_{f_{i,p}}\right)\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\left(\frac{b_{\text{T}}Q_0^2e^{\gamma_E}}{2m_{f_{i,p}}}\right)\right] \\
+ K_0\left(b_{\text{T}}m_{f_{g,p}}\right)A^f_{g/p}(x;\mu_{Q_0}) \\
+ C^f_{i/p}\,\tilde{f}_{\text{core},i/p}(x,\boldsymbol{b}_{\text{T}};Q_0^2)\,,
\end{aligned}
$$

from this expression one can recover the OPE

Model in the HSO approach

$$f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\left[A_{i/p}^{f}(x;\mu_{Q_0}) + B_{i/p}^{f}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\right]$$
$$+ \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0})$$
$$+ C_{i/p}^{f}\, f_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2)\,,$$

In b_{T} space

$$\tilde{f}_{\mathrm{inpt},i/p}(x,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = K_0\left(b_{\mathrm{T}}m_{f_{i,p}}\right)\left[A_{i/p}^{f}(x;\mu_{Q_0}) + B_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{b_{\mathrm{T}}Q_0^2e^{\gamma_E}}{2m_{f_{i,p}}}\right)\right]$$
$$+ K_0\left(b_{\mathrm{T}}m_{f_{g,p}}\right)A_{g/p}^{f}(x;\mu_{Q_0})$$
$$+ C_{i/p}^{f}\,\tilde{f}_{\mathrm{core},i/p}(x,\boldsymbol{b}_{\mathrm{T}};Q_0^2)\,,$$

Expressions useful for pheno at $Q \approx Q_0$

Model in the HSO approach: CS kernel

$$K^{(1)}_{\text{input}}(k_{\text{T}};\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_{\text{T}}^2+m_K^2} + C_K\delta^{(2)}(\boldsymbol{k}_{\text{T}})\,.$$

$$C_K = \frac{2\alpha_s(\mu_{Q_0})C_F}{\pi}\ln\left(\frac{m_K}{\mu_{Q_0}}\right)$$

Behaves as the pQCD tail, for large k_T

$$K^{(1)}(k_{\text{T}};\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_{\text{T}}^2}\,.$$

C_K ensures that

$$\frac{\mathrm{d}\tilde{K}^{(n)}_{\text{input}}(b_{\text{T}};\mu)}{\mathrm{d}\ln\mu} = -\gamma_K^{(n)}(\alpha_s(\mu)) + O\left(\alpha_s(\mu)^{n+1}\right)$$

Expressions useful for pheno at $Q \approx Q_0$

Model in the HSO approach

Need RG improvements for pheno at $Q \gg Q_0$

$$\sim \alpha_s(Q_0)^n \ln^m \left(\frac{q_{\rm T}}{Q_0} \right)$$

Wider range of qT available
upon evolution to large Q

Model in the HSO approach

Need RG improvements for pheno at $Q >> Q_0$

$$\sim \alpha_s(Q_0)^n \ln^m \left(\frac{q_{\mathrm{T}}}{Q_0} \right)$$

Wider range of qT available upon evolution to large Q

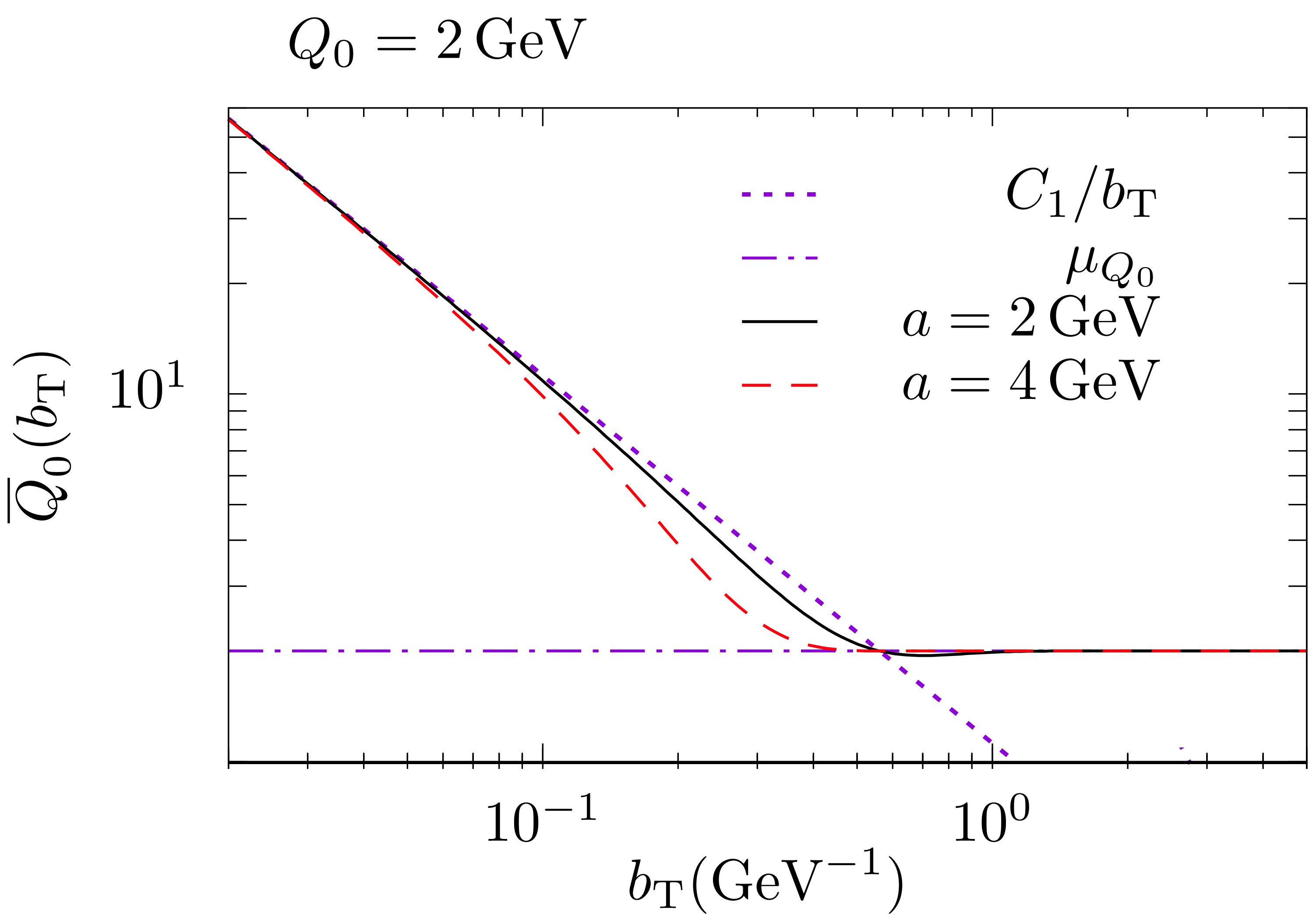
define scale transformation

$$\overline{Q}_0(b_{\mathrm{T}}) = Q_0 \ \mathrm{GeV} \left[1 - \left(1 - \frac{C_1}{Q_0 b_{\mathrm{T}}} \right) e^{-a^2 \, b_{\mathrm{T}}^2} \right]$$

- ✳ goes as $1/b_T$ for small b_T
- ✳ approaches input scale Q_0 at large b_T
- ✳ analogous to b_* in usual treatment

Scale setting for evolution to large Q

$$\overline{Q}_0(b_T) = Q_0 \text{ GeV}\left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$



Model in the HSO approach

Need RG improvements for pheno at Q >> Q_0

$$\sim \alpha_s(Q_0)^n \ln^m \left(\frac{q_\mathrm{T}}{Q_0} \right)$$

Wider range of qT available upon evolution to large Q

$$\tilde{f}_{i/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2) = \tilde{f}_{\mathrm{inpt}, i/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{\bar{Q}_0}, \bar{Q}_0^2) E(\bar{Q}_0/Q_0, b_\mathrm{T})$$

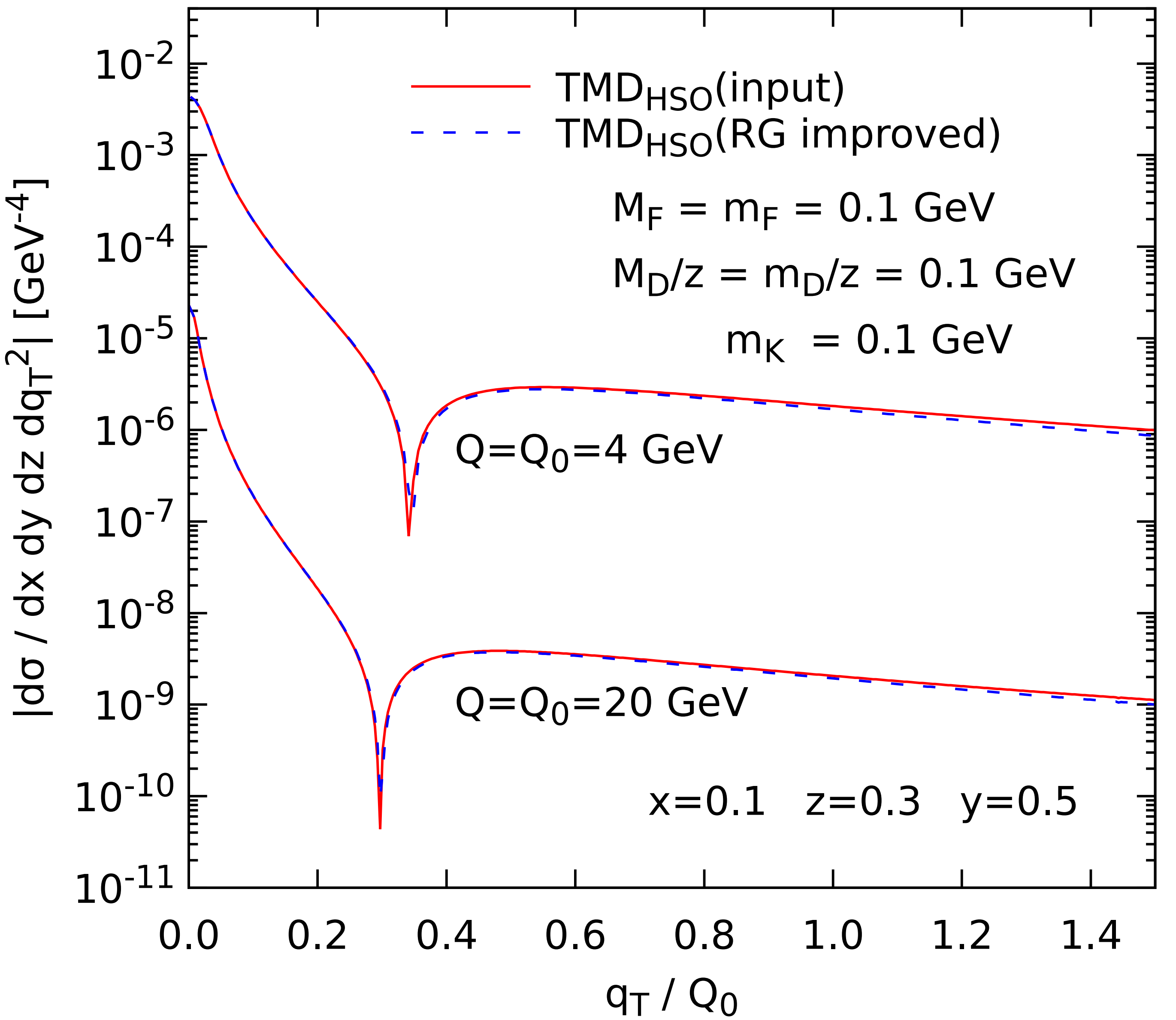
$$\overline{Q}_0(b_\mathrm{T}) = Q_0 \text{ GeV} \left[1 - \left(1 - \frac{C_1}{Q_0 b_\mathrm{T}} \right) e^{-a^2 b_\mathrm{T}^2} \right]$$

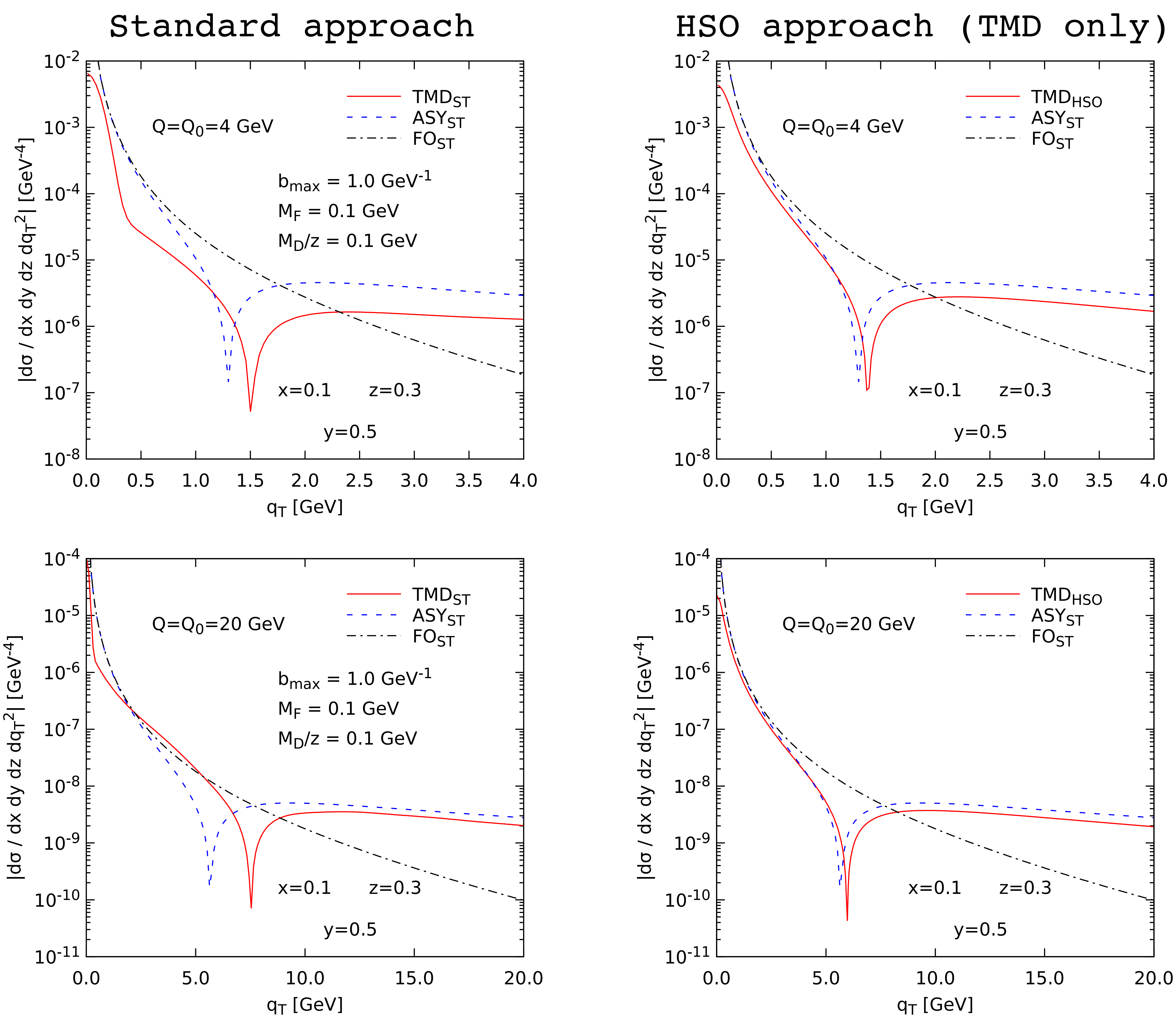
$$E(\bar{Q}_0/Q_0, b_\mathrm{T}) \equiv \exp\left\{ \int_{\mu_{\bar{Q}_0}}^{\mu_{Q_0}} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q_0}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln\frac{Q_0}{\bar{Q}_0} \tilde{K}_{\mathrm{inpt}}(b_\mathrm{T}; \mu_{\bar{Q}_0}) \right\}.$$

The usual evolution factor

Scale transformation not really needed for pheno at $Q \approx Q_0$

Work with $Q=Q_0$ for now





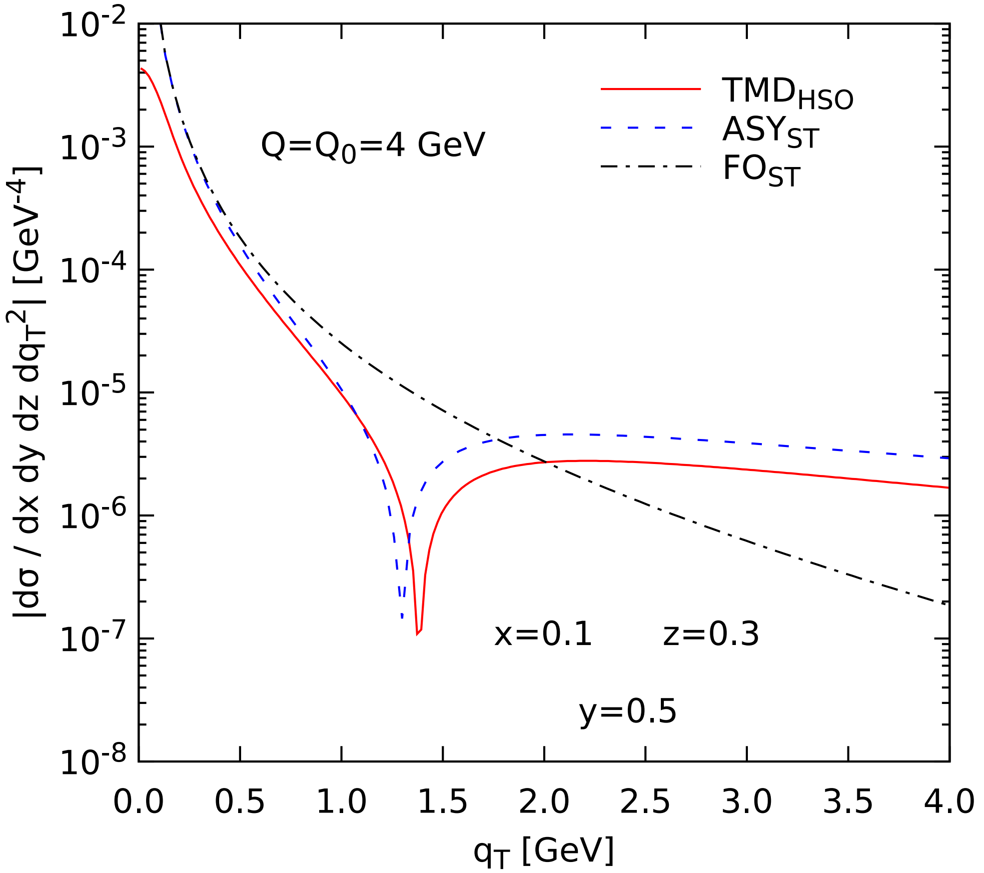
Standard approach
HSO approach (TMD only)
Q=Q$_0$=4 GeV
Q=Q$_0$=20 GeV
TMD$_{ST}$
TMD$_{HSO}$
ASY$_{ST}$
FO$_{ST}$
b_{max} = 1.0 GeV^{-1}
M_F = 0.1 GeV
M_D/z = 0.1 GeV
x=0.1 z=0.3
y=0.5
$|d\sigma / dx\, dy\, dz\, dq_T^2|$ [GeV^{-4}]
q_T [GeV]

Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{\rm FO}$$

Still not a good approximation to the TMD term at large q_T



Asymptotic term

The usual asymptotic term

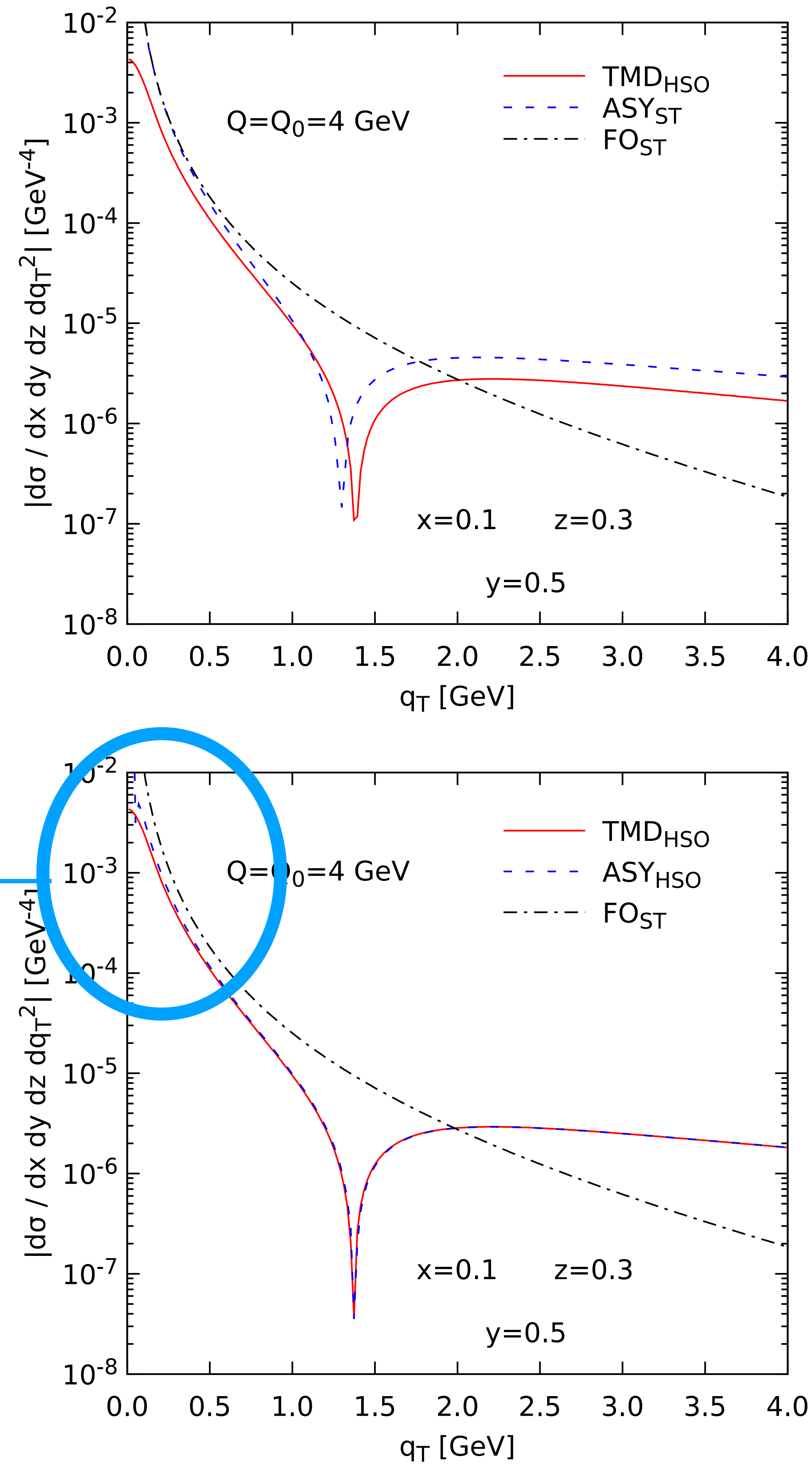
$$\lim_{q_{\rm T}/Q \to 0} F^{\rm FO}$$

Still not a good approximation to the TMD term at large q_T

We compute instead

$$\lim_{m/q_{\rm T} \to 0} F^{\rm TMD}$$

Stays a good approximation to the TMD term at large q_T, from around **this region**



Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{\rm FO}$$

We compute instead

$$\lim_{m/q_T \to 0} F^{\rm TMD}$$

$$\left[\lim_{q_T/Q \to 0} F^{\rm FO}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_T \to 0} F^{\rm TMD}\right]^{O(\alpha_s^n)} = O\left(\alpha_s^{n+1}, \boxed{m^2/Q^2}\right)$$

If using different schemes for collinear functions

Asymptotic term

The usual asymptotic term

$$\lim_{q_\mathrm{T}/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

$$\lim_{m/q_\mathrm{T} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_\mathrm{T}/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_\mathrm{T} \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2\right)$$

From two places (fixing the scheme for collinear functions)

Asymptotic term

The usual asymptotic term

We compute instead

$$\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}} \qquad\qquad \lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}} \right]^{O(\alpha_s^n)} - \left[\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}} \right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2 \right)$$

1) Additional terms in the bracket

$$[f, D] = \boxed{D^{\mathrm{pert}}(z, z\boldsymbol{q}_{\mathrm{T}}; \mu_Q; Q^2) f^c(x; \mu_Q) + \frac{1}{z^2} f^{\mathrm{pert}}(x, -\boldsymbol{q}_{\mathrm{T}}; \mu_Q; Q^2) d^c(z; \mu_Q)}$$

$$+ \boxed{\begin{aligned} \int \mathrm{d}^2\boldsymbol{k}_{\mathrm{T}} \, \Big\{ & f^{\mathrm{pert}}(x, \boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2; \mu_Q; Q^2) D^{\mathrm{pert}}(z, z\left(\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2\right); \mu_Q; Q^2) \\ & - D^{\mathrm{pert}}(z, z\boldsymbol{q}_{\mathrm{T}}; \mu_Q; Q^2) f^{\mathrm{pert}}(x, \boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2; \mu_Q; Q^2) \Theta(\mu_Q - |\boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2|) \\ & - D^{\mathrm{pert}}(z, z(\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2); \mu_Q; Q^2) f^{\mathrm{pert}}(x, -\boldsymbol{q}_{\mathrm{T}}; \mu_Q; Q^2) \Theta(\mu_Q - |\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2|) \Big\} \end{aligned}} + O\left(\frac{m^2}{q_{\mathrm{T}}^2} \right)$$

Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{FO}$$

We compute instead

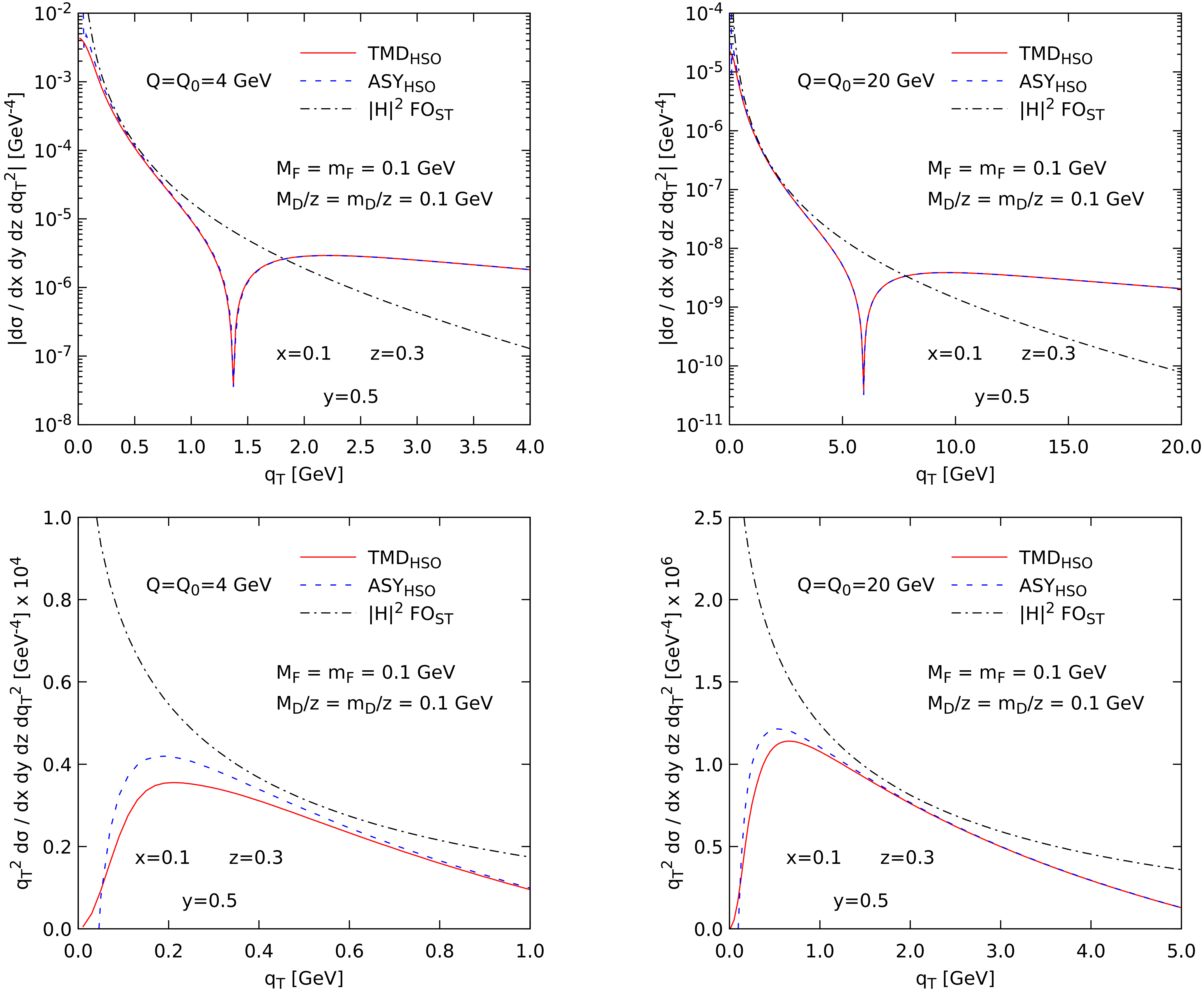
$$\lim_{m/q_T \to 0} F^{TMD}$$

$$\left[\lim_{q_T/Q \to 0} F^{FO}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_T \to 0} F^{TMD}\right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2\right)$$

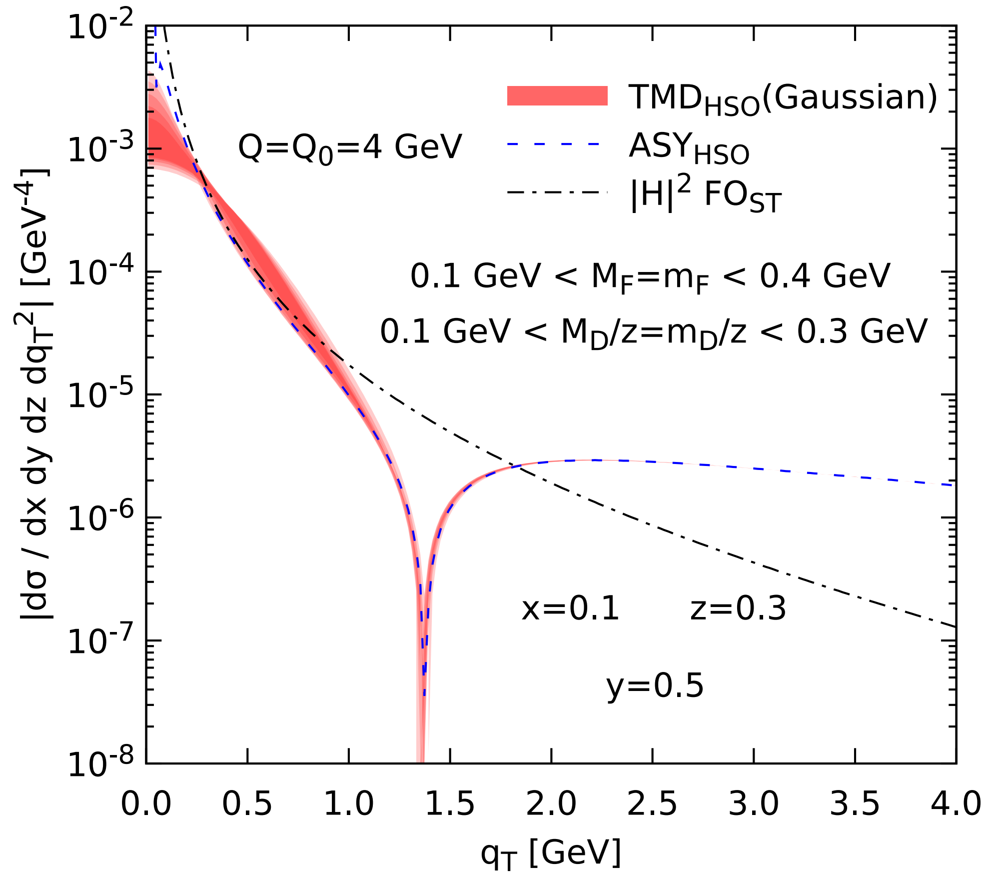
2) Hard coefficient in TMD term

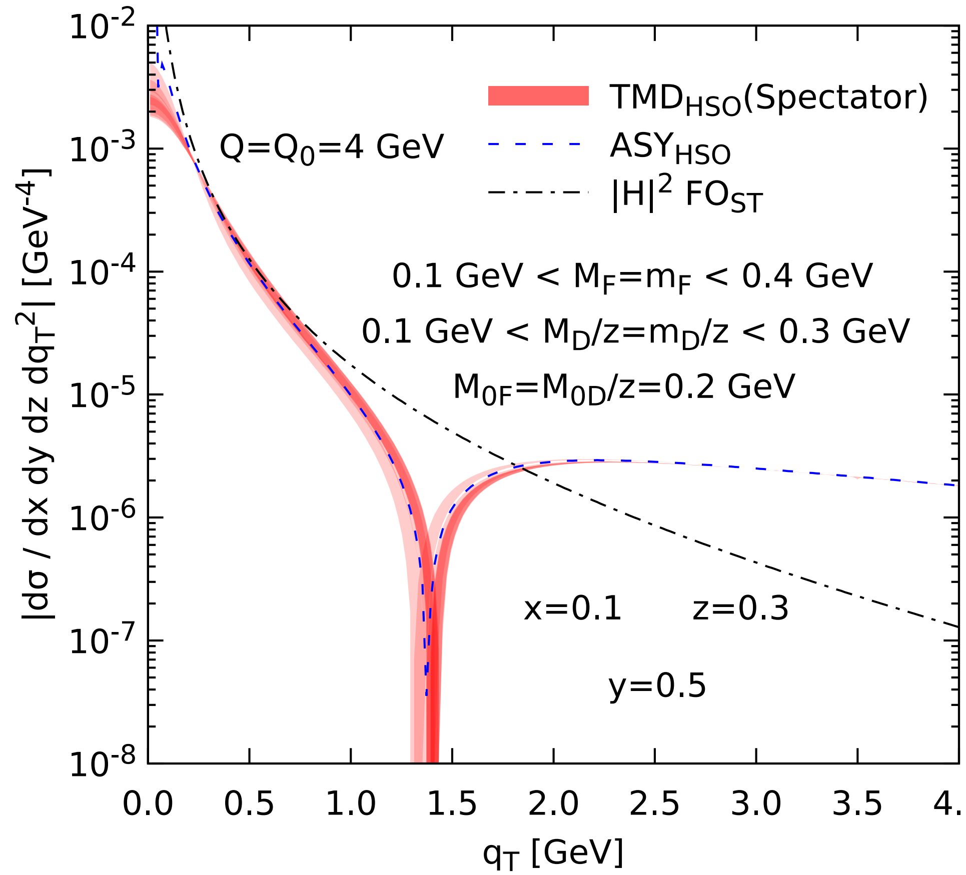
$$F_1^{TMD} \equiv 2\,z \sum_j \boxed{|H|_j^2} [f_{j/p}, D_{h/j}] \qquad F_2^{TMD} \equiv 4\,z\,x \sum_j \boxed{|H|_j^2} [f_{j/p}, D_{h/j}]$$

HSO approach



HSO approach

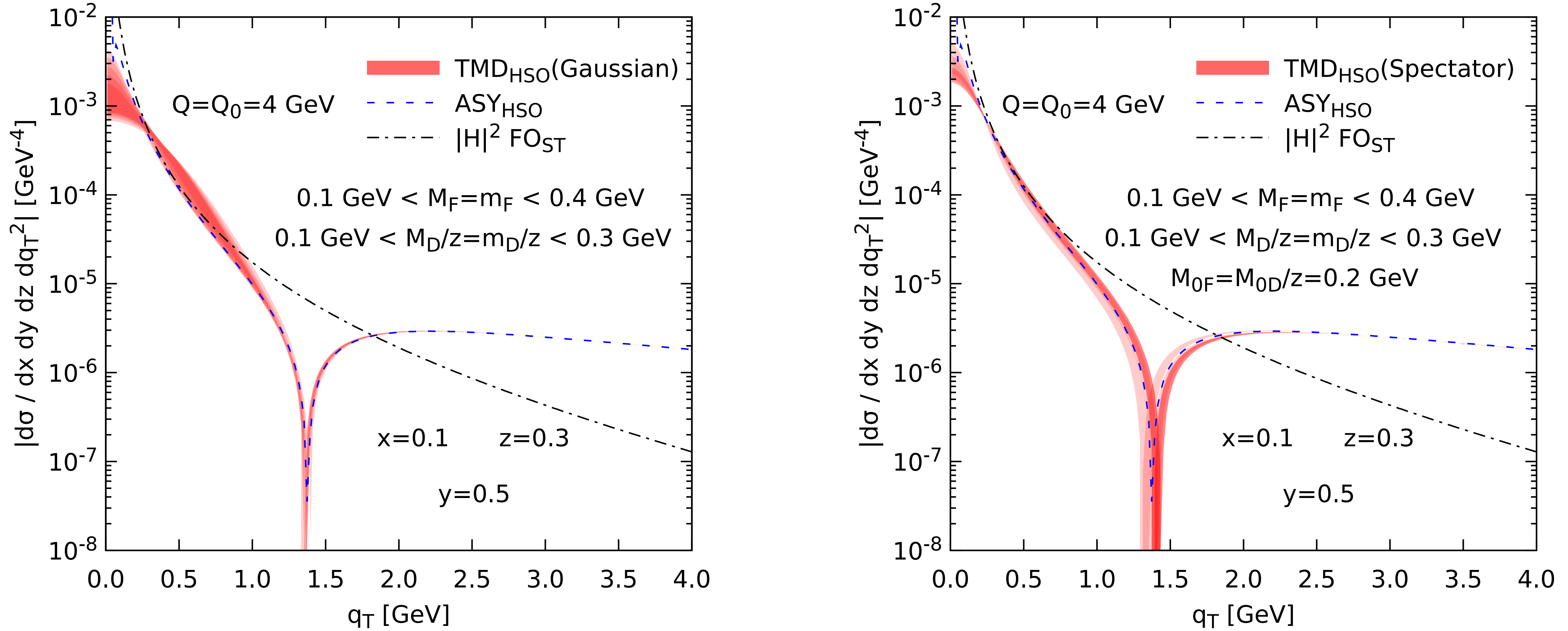




$$f^{\rm Gauss}_{{\rm core},i/p}(x,\boldsymbol{k}_{\rm T};Q_0^2) = \frac{e^{-k_{\rm T}^2/M_{\rm F}^2}}{\pi M_{\rm F}^2}$$

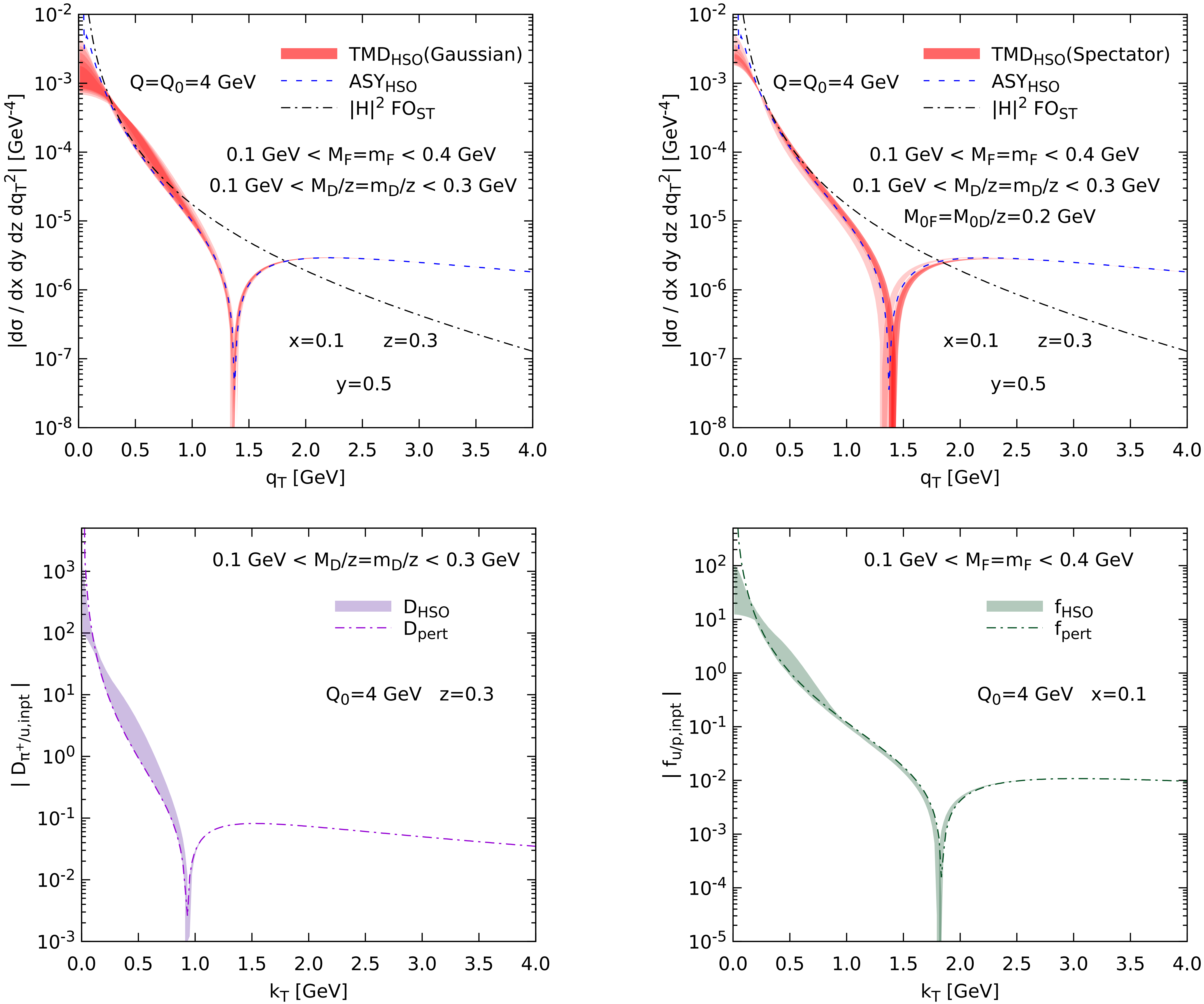
$$f^{\rm Spect}_{{\rm core},i/p}(x,\boldsymbol{k}_{\rm T};Q_0^2) = \frac{6M_{\rm 0F}^6}{\pi\,(2M_{\rm F}^2+M_{\rm 0F}^2)}\,\frac{M_{\rm F}^2+k_{\rm T}^2}{(M_{\rm 0F}^2+k_{\rm T}^2)^{\,4}}$$

HSO approach



Consistency of the band with the asymptotic term means the models for TMDs have been made consistent with collinear factorization. In the usual approach, this is the **aim** when embedding the OPE.

HSO approach



✳Standard treatment vs HSO approach.

b_{max} sensitivity

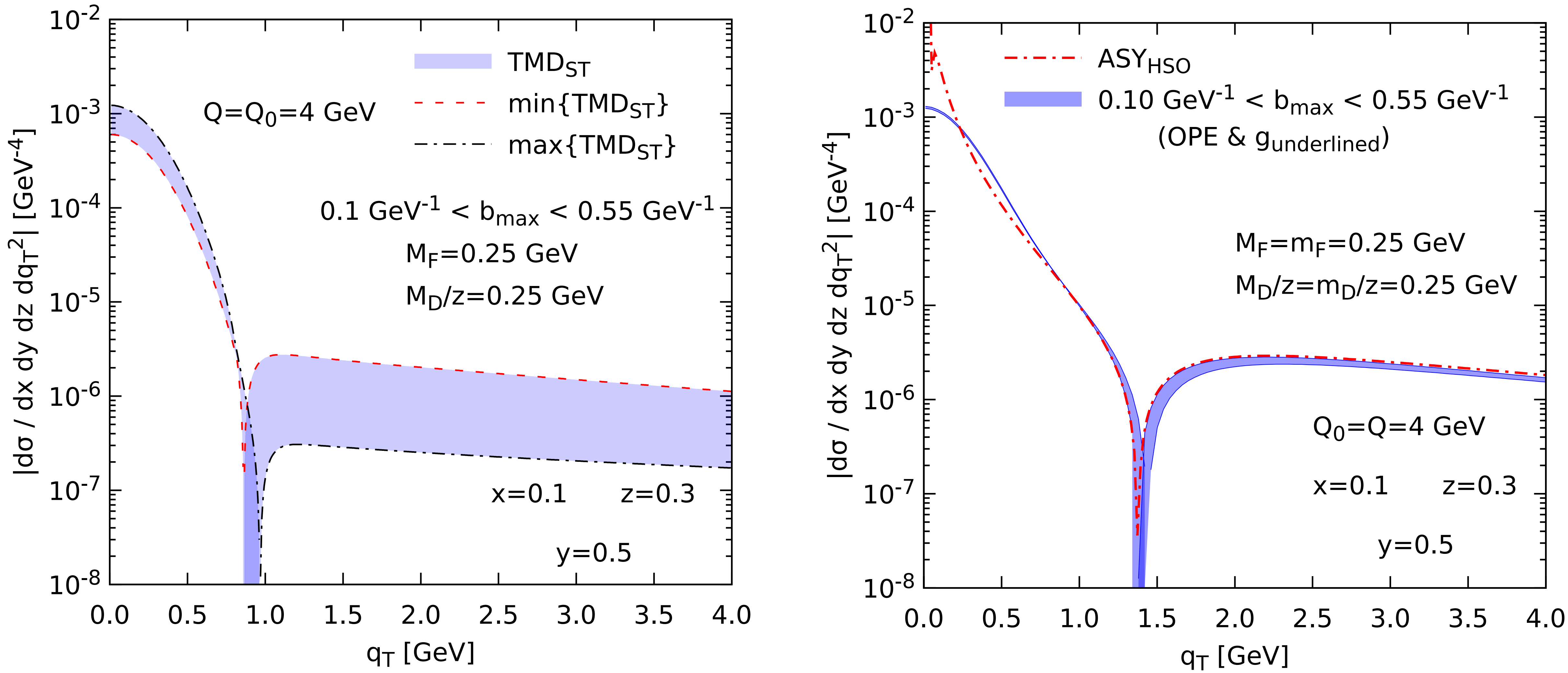
b_* prescription **not used** in HSO. It is instructive though to construct g-functions from HSO approach

$$-g_{j/p}(x, b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right), \qquad -g_{h/j}(z, b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right),$$

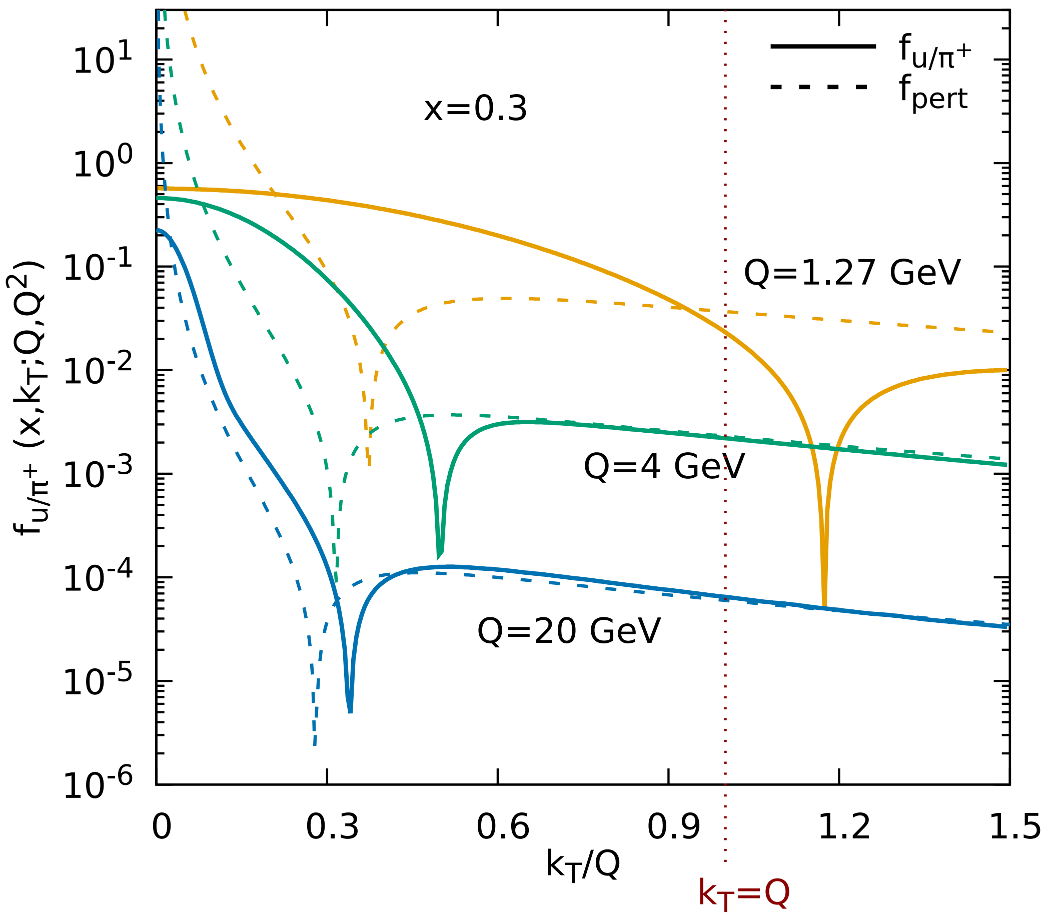
$$g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_\mathrm{T}; \mu).$$

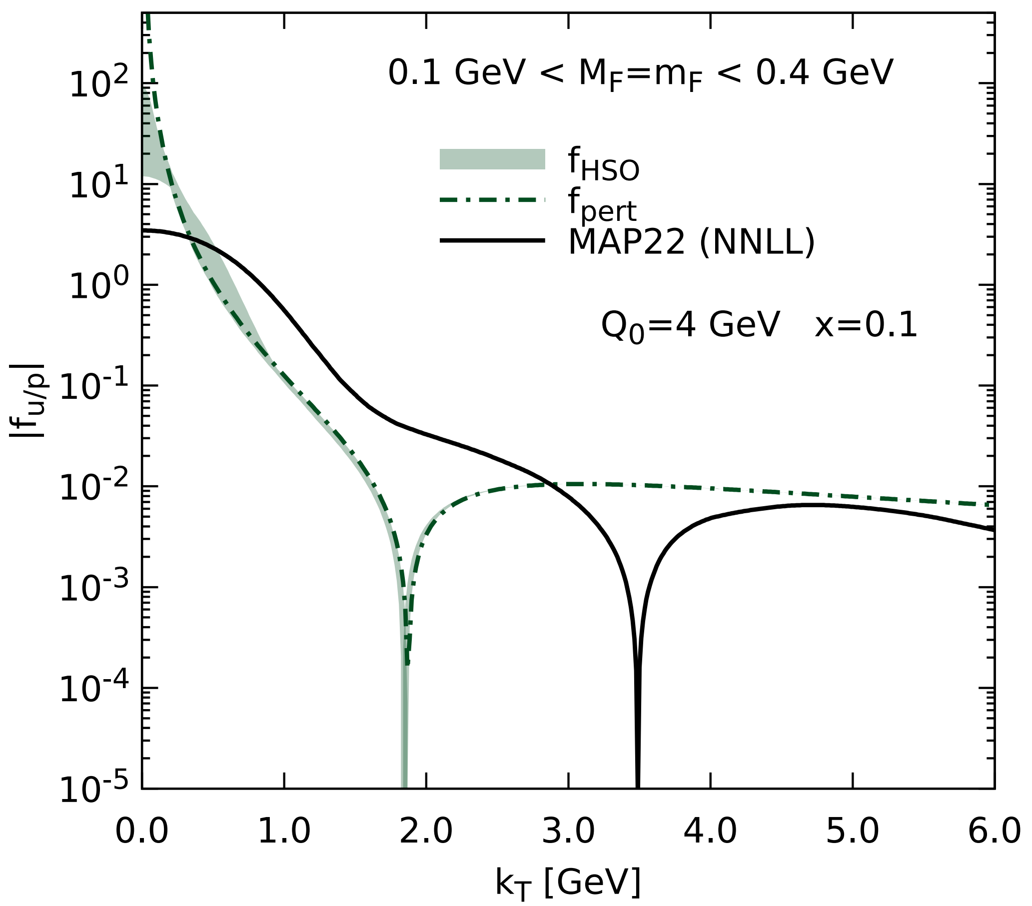
b_{max} sensitivity

b_* prescription **not used** in HSO. It is instructive though to construct g-functions from HSO approach



Some other comparisons





Final Remarks

Theoretical constraints are important to really assess/study hadronic structure

We propose an approach to treat TMDs in full consistency with collinear factorization.

We call it HSO “Hadron structure oriented” approach. A framework to embed models of nonperturbative behavior into the CSS formalism

No b_* prescription

Effectively, imposes constraints to models, like g-functions.

Pheno applications to come.

Thanks!