TMD phenomenology and nonperturbative structures

Ted Rogers
Jefferson Lab & Old Dominion University

Based on:

- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- J.O. Gonzalez, T. Rainaldi, TCR (2023), Phys.Rev.D 107 (2023) 9, 094029
- F. Aslan, M. Boglione, J.O. Gonzalez, T. Rainaldi, TCR, A. Simonelli, 2401.14266 [hep-ph]

March 29, 2024

"Structure"

https://www.bnl.gov/eic/

Precision 3D imaging of protons and nuclei

The Electron-Ion Collider will take three-dimensional precision snapshots of the internal structure of protons and atomic nuclei. As they pierce through the larger particles, the high-energy electrons will interact with the internal microcosm to reveal unprecedented details—zooming in beyond the simplistic structure of three valence quarks bound by a mysterious force. Recent experiments indicate that the gluons—which carry the strong force—multiply and appear to linger within particles accelerated close to the speed of light, and play a significant role in establishing key properties of protons and nuclear matter. By taking images at a range of energies, an EIC will reveal features of this "ocean" of gluons and the "sea" of quark-antiquark pairs that form when gluons interact—allowing scientists to map out the particles' distribution and movement within protons and nuclei, similar to the way medical imaging technologies construct 3D dynamic images of the brain. These studies may help reveal how the energy of the massless gluons is transformed through $E=mc^2$ to generate most of the mass of the visible universe.

Typical HEP colloquium

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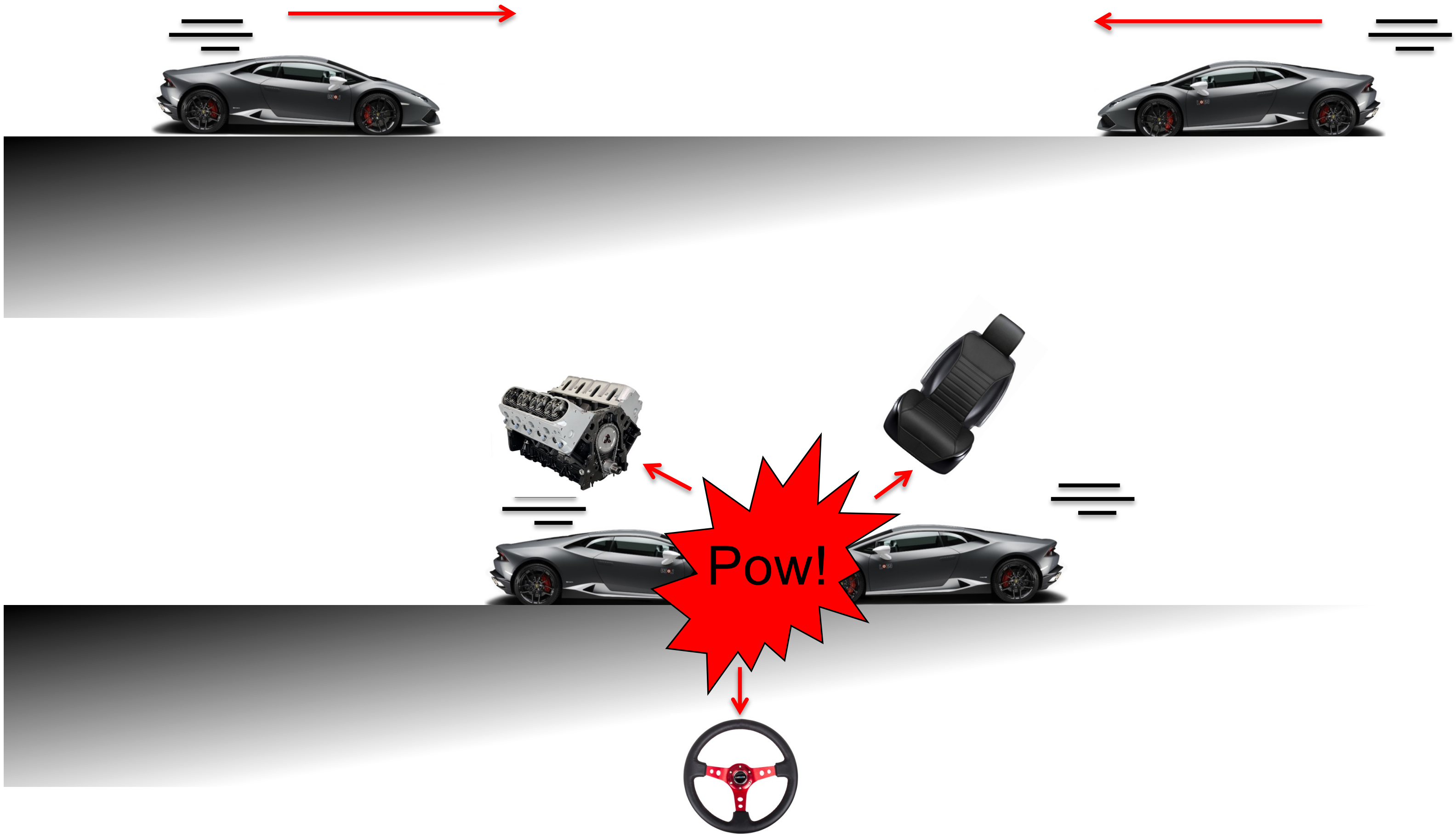
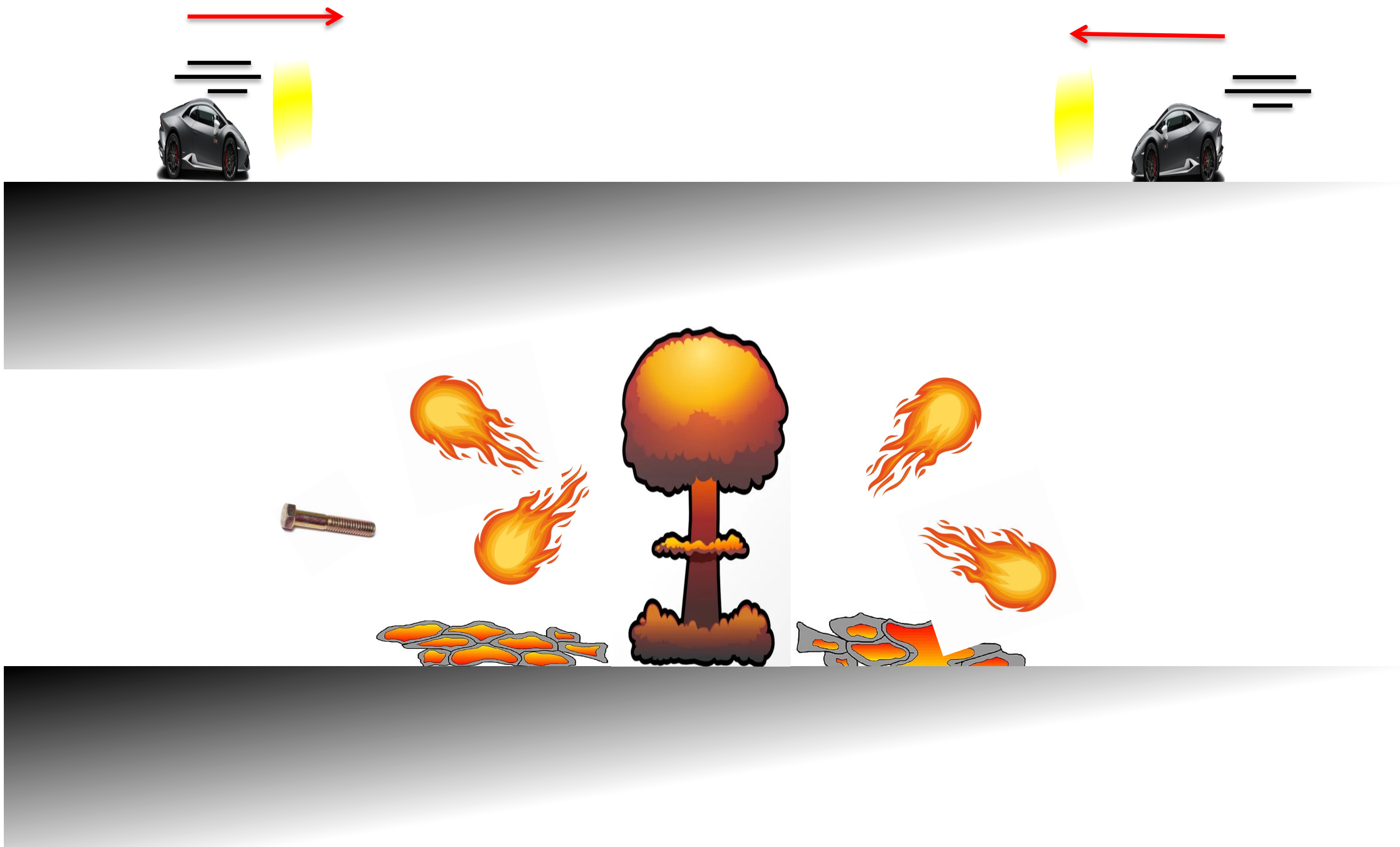


Better

Better



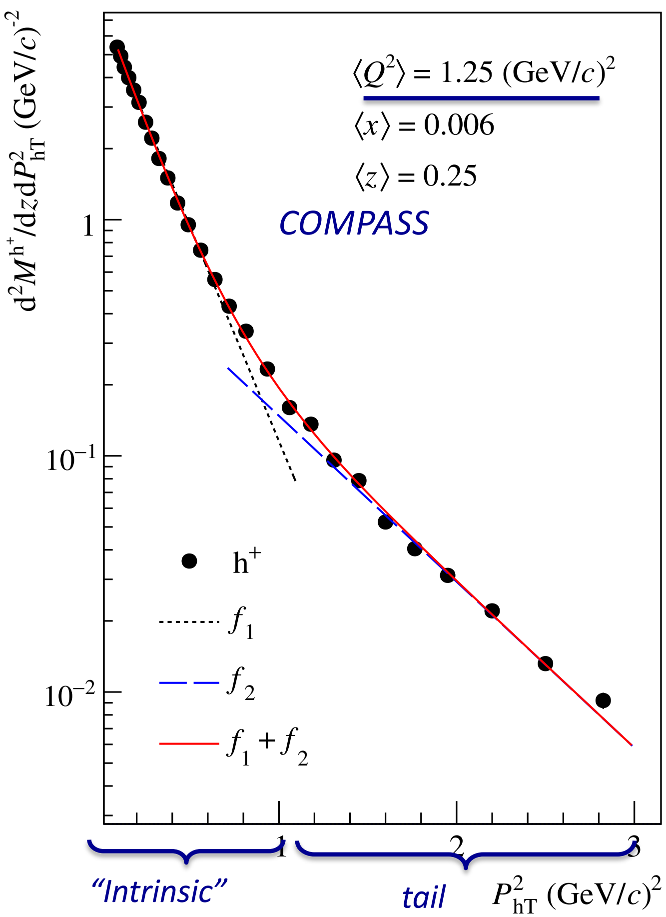
Factorization

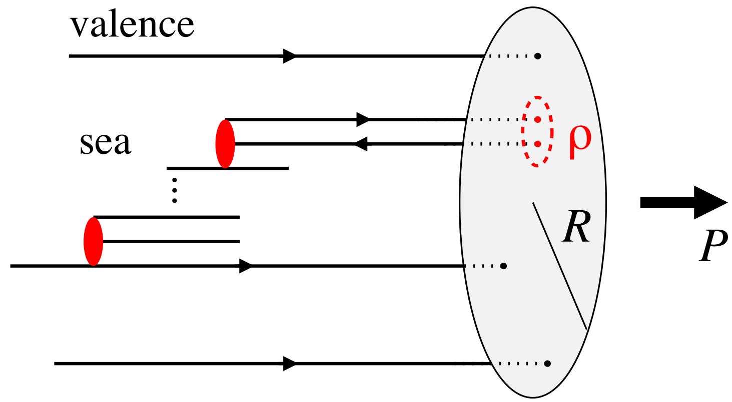
- How do (can?) we separate fundamental interactions into:
  - Stuff that belongs inside the separate particles
  - Stuff associated with ***specific*** high energy interactions and processes
- EIC: Learn about nonperturbative QCD structures

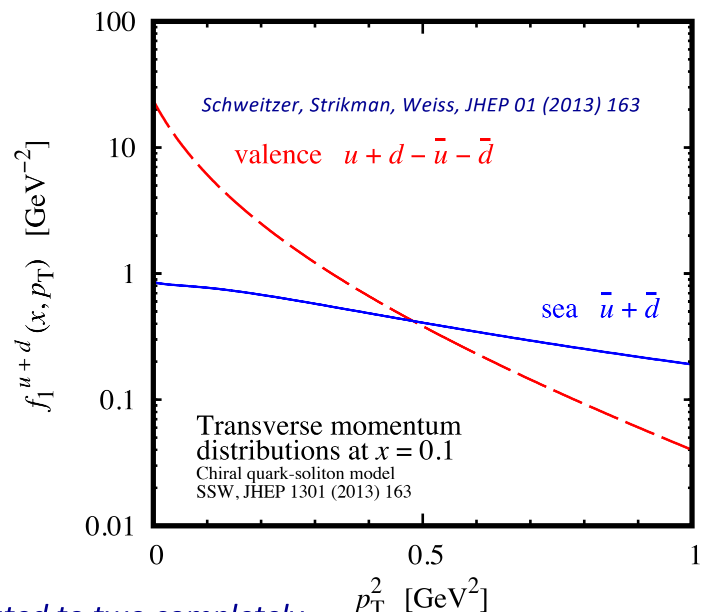
- “There are some interesting issues in statistics and the philosophy of science here, which do not appear to arise in such a strong form in other areas of science.”

- J. Collins, ***Foundations of Perturbative QCD,*** 2011

Nonperturbative structures in pheno





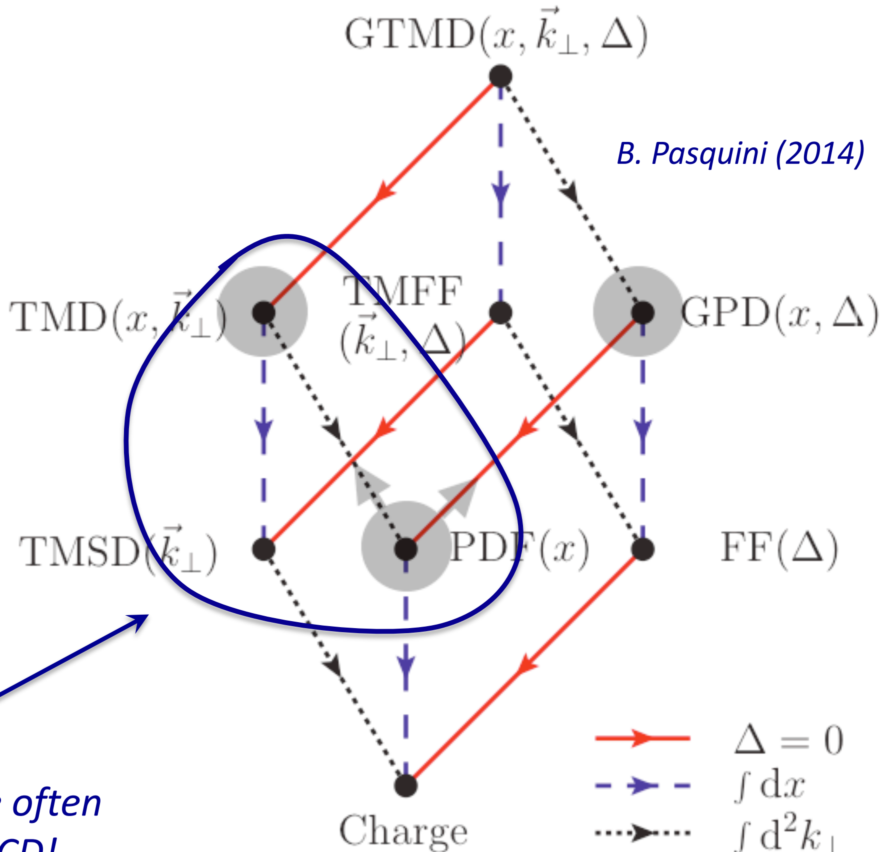


"… the two exponential functions in our parametrizations F_1 can be attributed to two completely different underlying physics mechanisms that overlap in the region $P_{hT} \simeq 1\ \mathrm{GeV}^2$."

Nonperturbative structures in pheno

Interpretations

B. Pasquini (2014)



These integrals are often divergent in real QCD!

A “hadron structure oriented” approach

- TMD factorization is well understood at a formal level

- How to construct phenomenological parametrizations that allow us to learn the most from using TMD factorization?

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$$\underbrace{f(x, k_T; \mu_Q, Q)}_{\textit{Abstract theory object}} \quad \Longrightarrow \quad \underbrace{f_{\text{param}}(x, k_T; \mu_Q, Q)}_{\textit{Function to be compared with / fitted to data}}$$

Where is the hadron structure!?

$$f(x, k_T; Q) = f(x; Q) \underbrace{\frac{e^{\frac{-k_T^2}{4B}}}{4\pi B}}_{??}$$

Where is the hadron structure!?

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} e_j^2 \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$
$$\times \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\ \tilde{C}^{\mathrm{CSS1,\ DY}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)$$
$$\times \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\ \tilde{C}^{\mathrm{CSS1,\ DY}}_{\bar{\jmath}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)$$
$$\times \exp\left\{-\int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left[A_{\mathrm{CSS1}}(a_s(\mu');C_1)\ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\mathrm{CSS1,\ DY}}(a_s(\mu');C_1,C_2)\right]\right\}$$
$$\times \exp\left[-g^{\mathrm{CSS1}}_{j/A}(x_A, b_{\mathrm{T}}; b_{\max}) - g^{\mathrm{CSS1}}_{\bar{\jmath}/B}(x_B, b_{\mathrm{T}}; b_{\max}) - g^{\mathrm{CSS1}}_{K}(b_{\mathrm{T}}; b_{\max})\ln(Q^2/Q_0^2)\right]$$
$$+\ \text{suppressed corrections.}$$

CSS1 = Collins-Soper-Sterman (≈1985)

A “hadron structure oriented” approach

- Adhere strictly to logic of TMD factorization
- Preserve a parton model interpretation (as much as possible)
- Preserve relationships between TMD and collinear factorization
- Allow to compare/contrast different models and descriptions of nonperturbative parts

How to test consistency

- What does $q_T \approx Q$ mean?
  - No sensitivity to parameters related nonperturbative transverse momentum
  - $\Lambda_{QCD} \ll \; q_T \ll Q$? Look for matching between fixed order x-section and asymptotic term
    - Is there a region where both

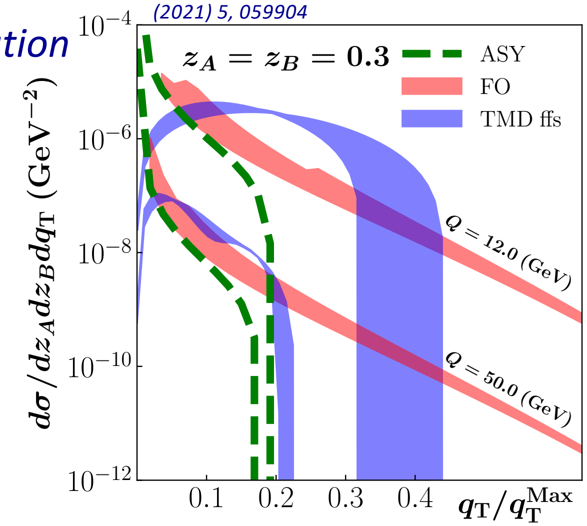
$$\frac{\Lambda_{QCD}}{q_T} \;\&\; \frac{q_T}{Q}$$

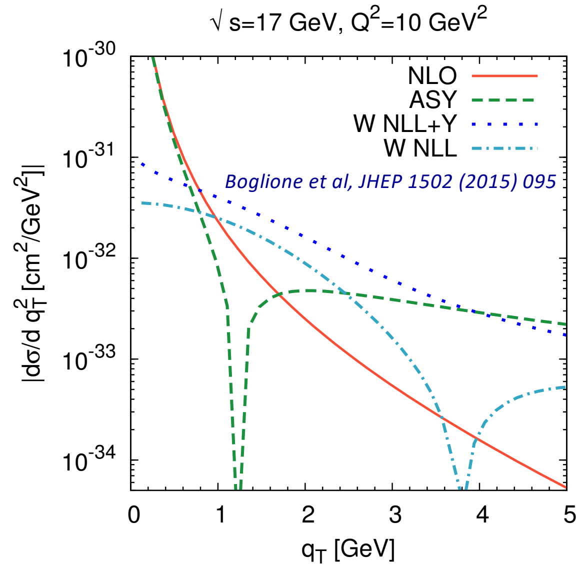
powers are simultaneously negligible?

  - No large logarithms: Look for node in asymptotic term

E. Moffat, T. Rogers, N. Sato, A. Signori Phys.Rev.D 104 (2021) 5, 059904

e^+e^- annihilation





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$$\tilde{f}(x, b_T; \mu_{Q_0}, Q_0)$$

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Example: $$\boldsymbol{b}_* = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2/b_\mathrm{max}^2}}$$

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$$\tilde{f}(x, b^*(b_T); \mu_{Q_0}, Q_0) \left(\frac{\tilde{f}(x, b_T; \mu_{Q_0}, Q_0)}{\tilde{f}(x, b^*(b_T); \mu_{Q_0}, Q_0)} \right)$$

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$$\frac{d}{db^*} \left[\tilde{f}(x, b^*(b_T); \mu_{Q_0}, Q_0) e^{-g(b_T)} \right] = 0$$

The b* method

- Preserve identities integrals?
  - Parton model / superrenormalizable

$$\int d^2\mathbf{k}_T f(x,\mathbf{k}_T) = f(x)$$

  - QCD /renormalizable

$$\int^{Q^2} d^2\mathbf{k}_T f(x,\mathbf{k}_T;\mu_Q,Q) = f(x;Q) + \alpha_s(Q)\Delta\left(f(x),\alpha_s(Q)\right)$$

- Behavior at intermediate transverse momentum?

$$\Lambda_{\mathrm{QCD}} \ll k_T \lesssim Q_0$$

Where is the hadron structure!?

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = {}& \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
& \times \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\ \tilde{C}_{j/j_A}^{\mathrm{CSS1,\ DY}}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\ \tilde{C}_{\bar{\jmath}/j_B}^{\mathrm{CSS1,\ DY}}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \exp\left\{ -\int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \left[A_{\mathrm{CSS1}}(a_s(\mu'); C_1) \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\mathrm{CSS1,\ DY}}(a_s(\mu'); C_1, C_2) \right] \right\} \\
& \times \exp\Big[-g_{j/A}^{\mathrm{CSS1}}(x_A, b_{\mathrm{T}}; b_{\max}) - g_{\bar{\jmath}/B}^{\mathrm{CSS1}}(x_B, b_{\mathrm{T}}; b_{\max}) - g_K^{\mathrm{CSS1}}(b_{\mathrm{T}}; b_{\max}) \ln(Q^2/Q_0^2) \Big] \\
& + \text{suppressed corrections.}
\end{aligned}
$$

Impose continuity of derivatives to preserve predictive power
(Grewal, Kang, Qiu 2020)

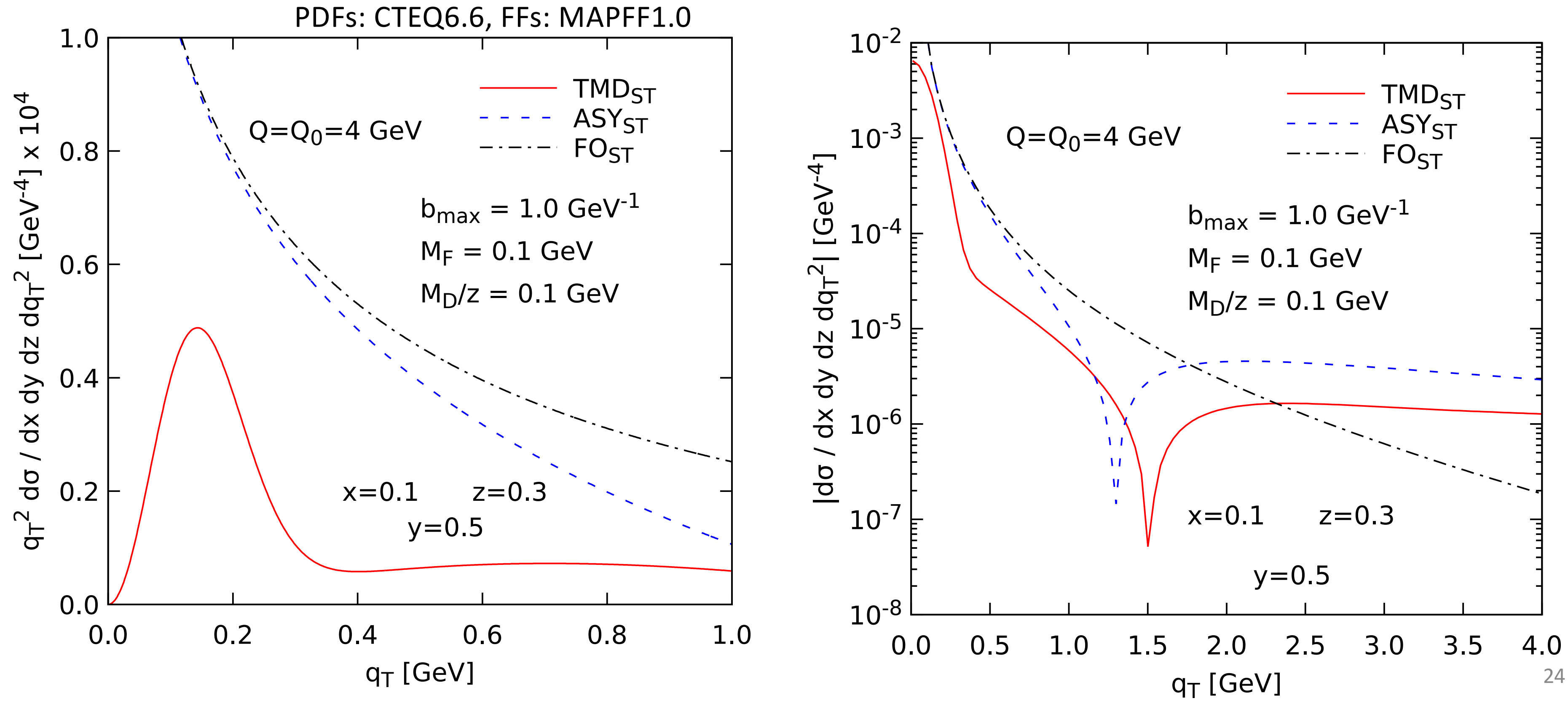
CSS1 = Collins-Soper-Sterman (≈1985)

Conventional organization & its complications

(An example typical of conventional approach)

$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4} M_F^2 \, b_{\mathrm{T}}^2$$

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\,z^2} M_D^2 \, b_{\mathrm{T}}^2$$



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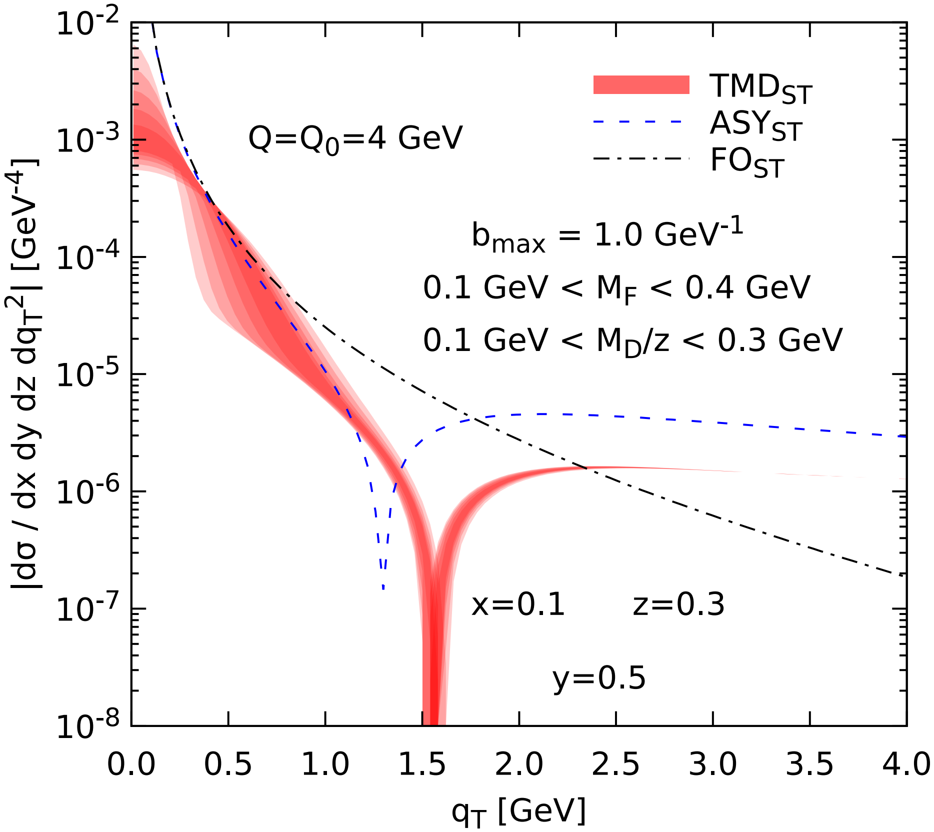
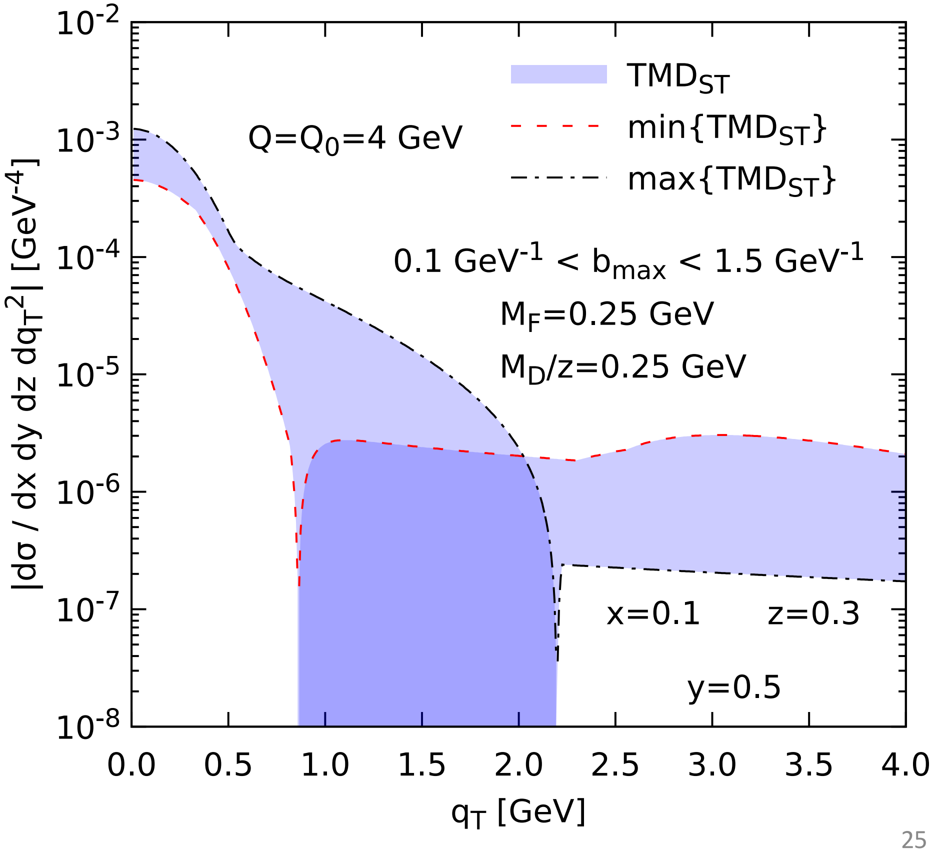
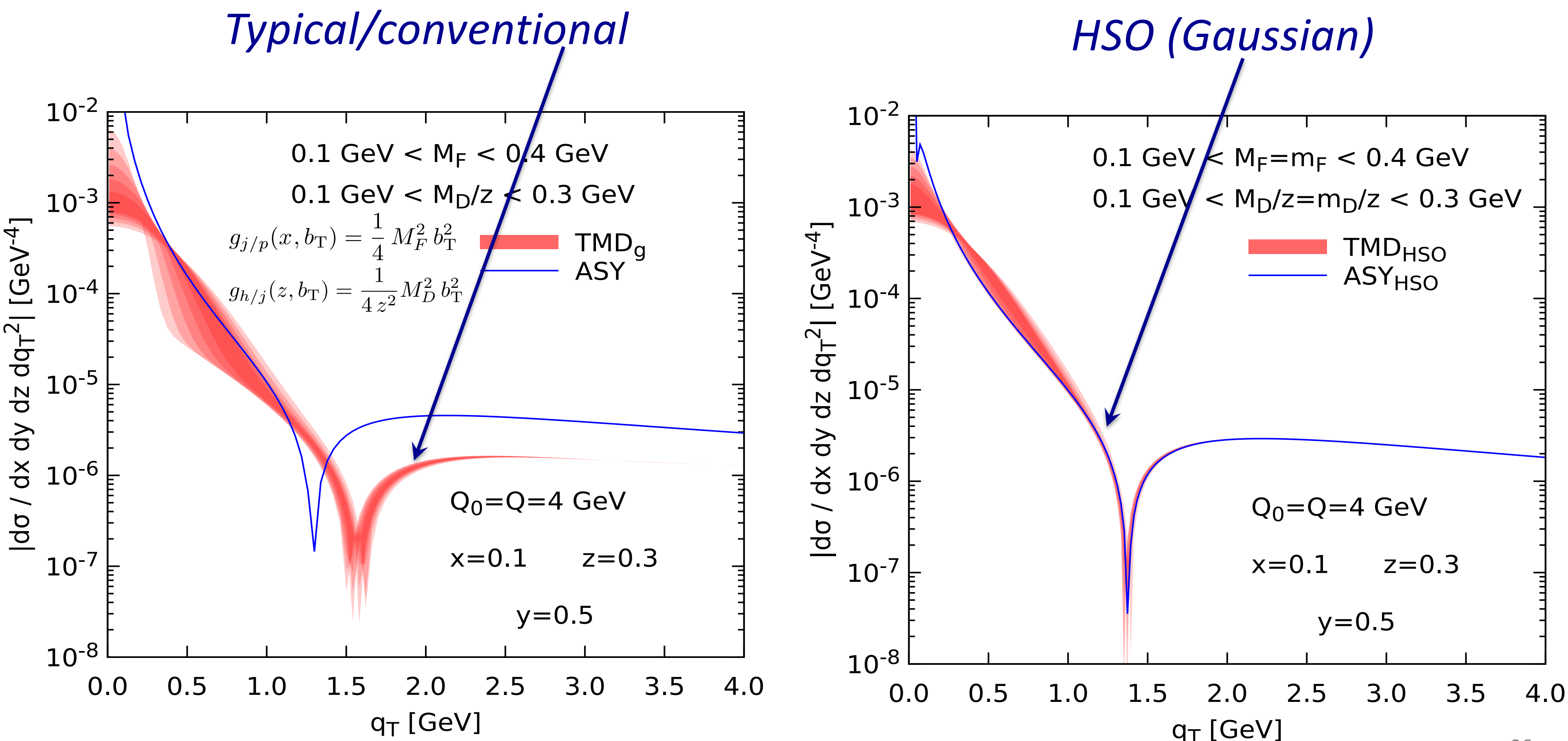




Compare standard/unconstrained with HSO ($O(\alpha_s)$)



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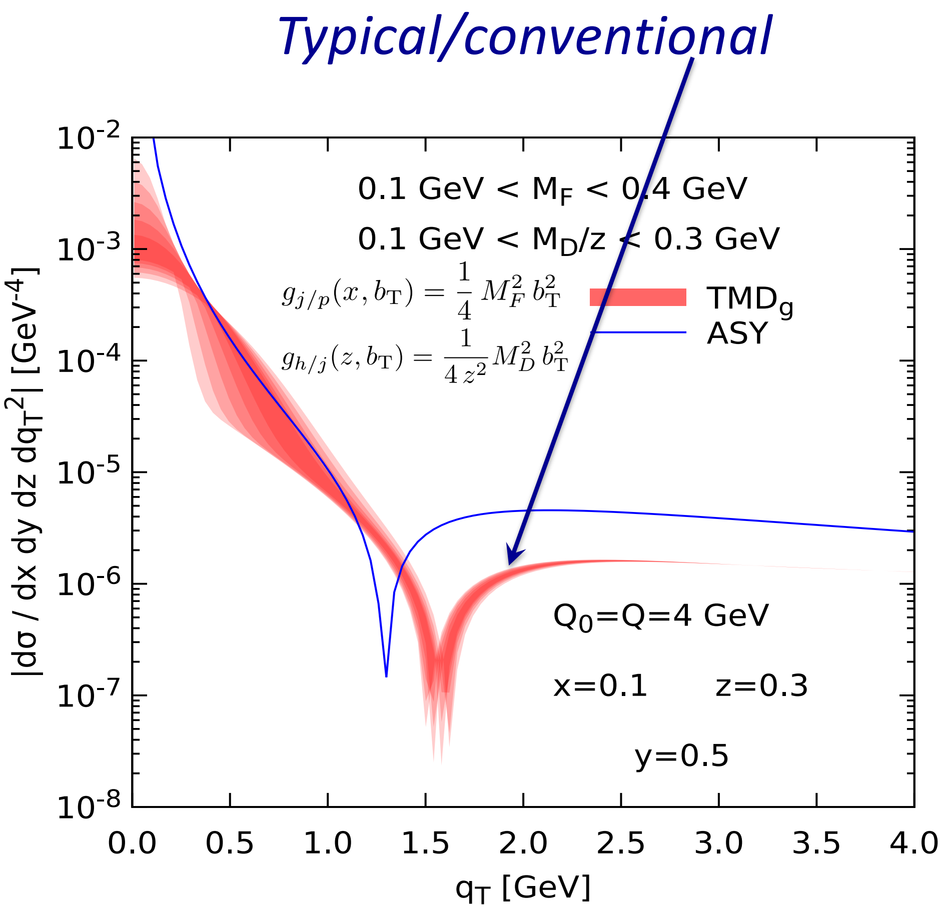
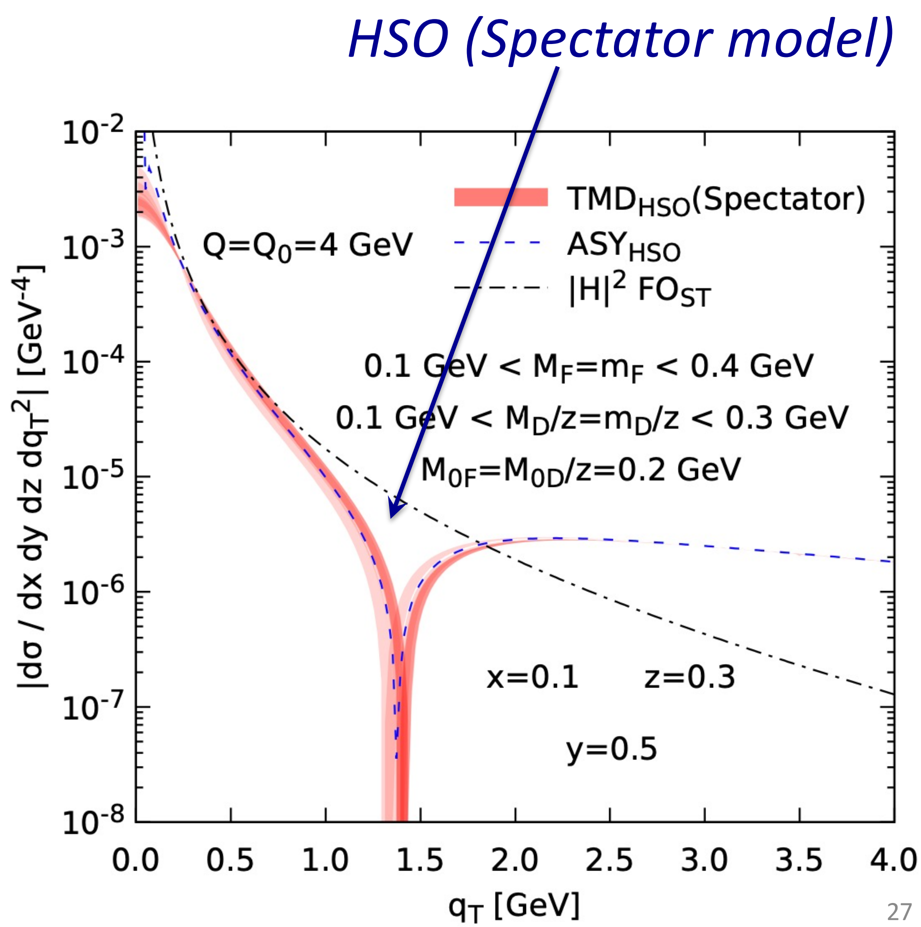




An $O(\alpha_s)$ example with $\overline{\mathrm{MS}}$ pdfs and ffs

- Parametrizing the very small transverse momentum

A. Gaussian model (very commonly used)

$$f^{\mathrm{Gauss}}_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2) = \frac{e^{-k_{\mathrm{T}}^2/M_{\mathrm{F}}^2}}{\pi M_{\mathrm{F}}^2}\,, \qquad D^{\mathrm{Gauss}}_{\mathrm{core},h/j}(z,z\boldsymbol{k}_{\mathrm{T}};Q_0^2) = \frac{e^{-z^2\,k_{\mathrm{T}}^2/M_{\mathrm{D}}^2}}{\pi M_{\mathrm{D}}^2}$$

B. Spectator model

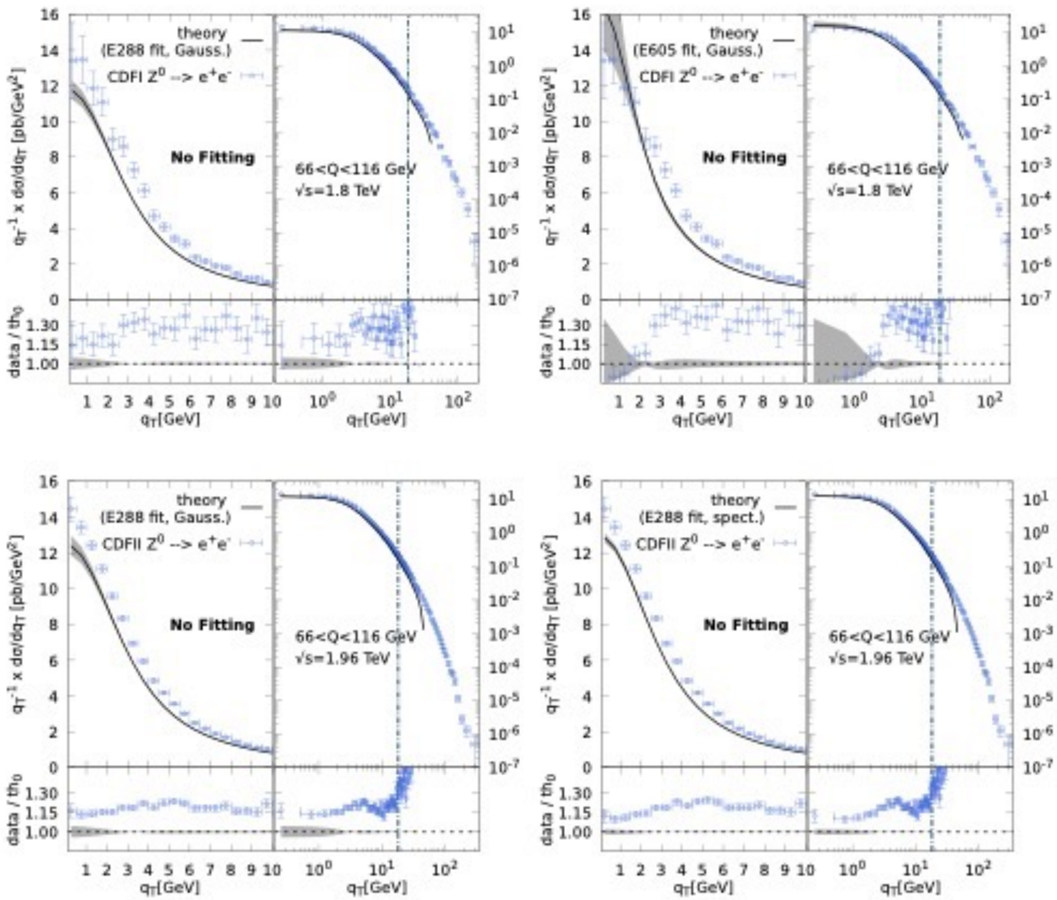
$$f^{\mathrm{Spect}}_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2) = \frac{6M_{0\mathrm{F}}^6}{\pi\left(2M_{\mathrm{F}}^2+M_{0\mathrm{F}}^2\right)}\frac{M_{\mathrm{F}}^2+k_{\mathrm{T}}^2}{\left(M_{0\mathrm{F}}^2+k_{\mathrm{T}}^2\right)^4}\,, \quad D^{\mathrm{Spect}}_{\mathrm{core},h/j}(z,z\boldsymbol{k}_{\mathrm{T}};Q_0^2) = \frac{2M_{0\mathrm{D}}^4}{\pi\left(M_{\mathrm{D}}^2+M_{0\mathrm{D}}^2\right)}\frac{M_{\mathrm{D}}^2+k_{\mathrm{T}}^2z^2}{\left(M_{0\mathrm{D}}^2+k_{\mathrm{T}}^2z^2\right)^3}$$

Updates

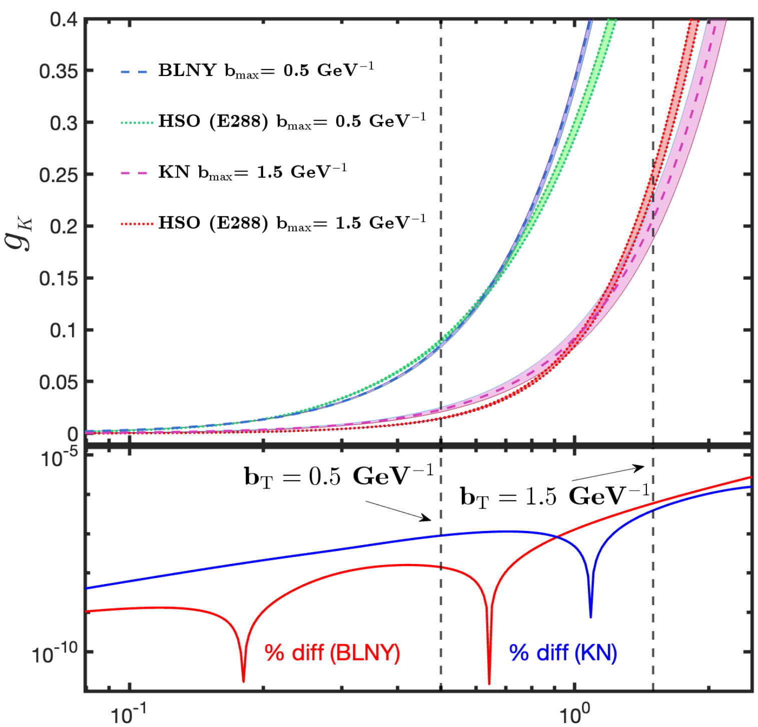
Examples of fitting to Drell-Yan

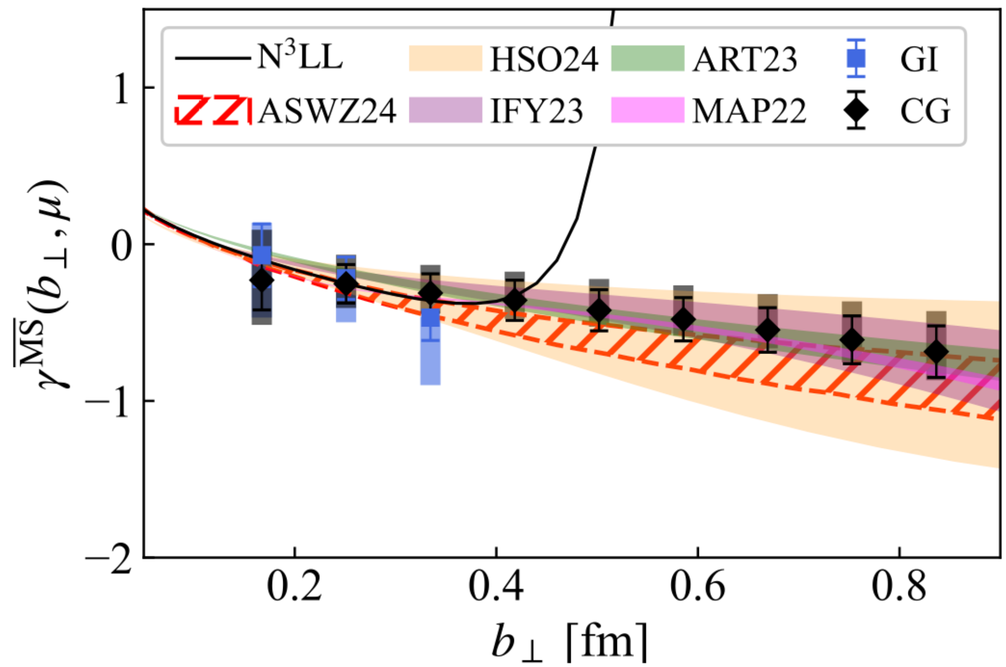
F. Aslan, M. Boglione, J.O. Gonzalez, T. Rainaldi, TCR, A. Simonelli, 2401.14266 [hep-ph]

Fit lower Q to predict high Q



Consistency checks





Lattice calculation from

Bollweg, Gao, Mukherjee, Zhao, (2024), 2403.00664 [hep-lat]

Summary

- Switching to a hadron-structure-oriented approach to pheno with TMD factorization improves consistency in the large transverse behavior of TMD correlation functions

- Necessary for understanding the shapes of nonperturbative distributions, separating perturbative and nonpeturbative parts, etc

- Not a new formalism, just a way of organizing parametrizations; **HSO = "standard CSS"!**

See video:
https://youtu.be/7Wqx9yhBXuI