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Better to give than to receive: Predictive directional measurement of volatility spillovers

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ABSTRACT

Using a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to the variable ordering, we propose measures of both the total and directional volatility spillovers. We use our methods to characterize daily volatility spillovers across US stock, bond, foreign exchange and commodities markets, from January 1999 to January 2010. We show that despite significant volatility fluctuations in all four markets during the sample, cross-market volatility spillovers were quite limited until the global financial crisis, which began in 2007. As the crisis intensified, so too did the volatility spillovers, with particularly important spillovers from the stock market to other markets taking place after the collapse of the Lehman Brothers in September 2008.

1. Introduction

Financial crises occur with notable regularity; moreover, they display notable similarities (e.g., Reinhart & Rogoff, 2008). During crises, for example, the financial market volatility generally increases sharply and spills over across markets. Naturally, one would like to be able to measure and monitor such spillovers, both to provide "early warning systems" for emergent crises, and to track the progress of extant crises.

Motivated by such considerations, Diebold and Yilmaz (2009) introduce a volatility spillover measure based on forecast error variance decompositions from vector autoregressions (VARs).[1] It can be used to measure the spillovers in returns or return volatilities (or, for that matter, any return characteristic of interest) across individual assets, asset portfolios, asset markets, etc., both within and across countries, revealing spillover trends, cycles, bursts, etc. In addition, although it conveys useful information, it nevertheless sidesteps the contentious issues associated with the definition and existence of episodes of "contagion" or "herd behavior".[2]

However, the Diebold and Yilmaz (DY) framework, as currently developed and implemented, has several limitations, both methodological and substantive. Consider the methodological side. First, DY relies on the Cholesky-factor identification of VARs, and thus the resulting variance decompositions can be dependent on variable ordering. One would prefer a spillover measure which was invariant to ordering. Second, and crucially, DY only addresses the *total* spillovers (from/to each market i, to/from all other markets, added across i). One would also like to examine *directional* spillovers (from/to a particular market).

Now consider the substantive side. DY consider only the measurement of spillovers across identical assets (equities) in different countries, but various other possibilities are also of interest, including individual-asset spillovers

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[1] VAR variance decompositions, introduced by Sims (1980), record how much of the H-step-ahead forecast error variance of some variable i is due to innovations in *another* variable j.

[2] On contagion (or a lack thereof), see for example Forbes and Rigobon (2002).

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within countries (e.g., among the thirty Dow Jones Industrials in the US), across asset classes (e.g., between stock and bond markets in the US), and of course various blends. Spillovers across asset classes, in particular, are of special interest, given the recent global financial crisis (which appears to have started in credit markets but spilled over into equities), but they have not yet been investigated in the DY framework.

In this paper we fill these methodological and substantive gaps. We use a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to the variable ordering, and we explicitly include directional volatility spillovers. We then use our methods in a substantive empirical analysis of daily volatility spillovers across US stock, bond, foreign exchange and commodities markets over a ten year period, including the recent global financial crisis.

We proceed as follows. In Section 2 we discuss our methodological approach, with a particular emphasis on our new use of generalized variance decompositions and directional spillovers. In Section 3 we describe our data and present our substantive results. We conclude in Section 4.

2. Methods: generalized spillover definition and measurement

Here we extend the DY spillover index, which follows directly from the familiar notion of a variance decomposition associated with an N-variable vector autoregression. Whereas DY focus on the *total* spillovers in a *simple* VAR framework (i.e., with potentially order-dependent results driven by Cholesky factor orthogonalization), we progress by measuring the *directional* spillovers in a *generalized* VAR framework that eliminates the possible dependence of the results on ordering.

Consider a covariance stationary N-variable VAR(p), $x_t = \sum_{i=1}^{p} \Phi_i x_{t-i} + \varepsilon_t$, where $\varepsilon \sim (0, \Sigma)$ is a vector of independently and identically distributed disturbances. The moving average representation is $x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}$, where the $N \times N$ coefficient matrices A_i obey the recursion $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \cdots + \Phi_p A_{i-p}$, with A_0 being an $N \times N$ identity matrix and with $A_i = 0$ for $i < 0$. The moving average coefficients (or transformations such as impulse-response functions or variance decompositions) are the key to understanding the dynamics of the system. We rely on variance decompositions, which allow us to parse the forecast error variances of each variable into parts which are attributable to the various system shocks. The variance decompositions allow us to assess the fraction of the H-step-ahead error variance in forecasting x_i that is due to shocks to x_j, $\forall j \neq i$, for each i.

The calculation of variance decompositions requires orthogonal innovations, whereas our VAR innovations are generally contemporaneously correlated. Identification schemes such as that based on Cholesky factorization achieve orthogonality, but the variance decompositions then depend on the ordering of the variables. We circumvent this problem by exploiting the generalized VAR framework of Koop, Pesaran, and Potter (1996) and Pesaran and Shin (1998), hereafter KPPS, which produces variance decompositions which are invariant to the ordering. Instead of attempting to orthogonalize shocks, the generalized approach allows correlated shocks but accounts for them appropriately using the historically observed distribution of the errors. As the shocks to each variable are not orthogonalized, the sum of the contributions to the variance of the forecast error (that is, the row sum of the elements of the variance decomposition table) is not necessarily equal to one.

2.1. Variance shares

Let us define *own variance shares* as the fractions of the H-step-ahead error variances in forecasting x_i that are due to shocks to x_i, for $i = 1, 2, \ldots, N$, and *cross variance shares*, or *spillovers*, as the fractions of the H-step-ahead error variances in forecasting x_i that are due to shocks to x_j, for $i, j = 1, 2, \ldots, N$, such that $i \neq j$.

Denoting the KPPS H-step-ahead forecast error variance decompositions by $\theta_{ij}^{g}(H)$, for $H = 1, 2, \ldots$, we have

$$\theta_{ij}^{g}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \Sigma A_h' e_i)}, \tag{1}$$

where Σ is the variance matrix for the error vector ε, σ_{jj} is the standard deviation of the error term for the jth equation, and e_i is the selection vector, with one as the ith element and zeros otherwise. As was explained above, the sum of the elements in each row of the variance decomposition table is not equal to 1: $\sum_{j=1}^{N} \theta_{ij}^{g}(H) \neq 1$. In order to use the information available in the variance decomposition matrix in the calculation of the spillover index, we normalize each entry of the variance decomposition matrix by the row sum as:[3]

$$\tilde{\theta}_{ij}^{g}(H) = \frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{N} \theta_{ij}^{g}(H)}. \tag{2}$$

Note that, by construction, $\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = 1$ and $\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = N$.

2.2. Total spillovers

Using the volatility contributions from the KPPS variance decomposition, we can construct the total volatility spillover index:

$$S^{g}(H) = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}_{ij}^{g}(H)}{\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H)} \cdot 100 = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}_{ij}^{g}(H)}{N} \cdot 100. \tag{3}$$

[3] Alternatively, we can normalize the elements of the variance decomposition matrix with the column sum of these elements and compare the resulting total spillover index with the one obtained from the normalization with the row sum.

This is the KPPS analog of the Cholesky factor based measure used by Diebold and Yilmaz (2009). The total spillover index measures the contribution of spillovers of volatility shocks across four asset classes to the total forecast error variance.

2.3. Directional spillovers

Although it is sufficient to study the total volatility spillover index is sufficient to enable us to understand how much of shocks to the volatility spill over across major asset classes, the generalized VAR approach enables us to learn about the direction of volatility spillovers across major asset classes. As the generalized impulse responses and variance decompositions are invariant to the ordering of variables, we calculate the directional spillovers using the normalized elements of the generalized variance decomposition matrix. We measure the directional volatility spillovers received by market i from all other markets j as:

$$S_{i\cdot}^{g}(H)=\frac{\sum\limits_{\substack{j=1\\ j\neq i}}^{N}\tilde{\theta}_{ij}^{g}(H)}{\sum\limits_{i,j=1}^{N}\tilde{\theta}_{ij}^{g}(H)}\cdot 100=\frac{\sum\limits_{\substack{j=1\\ j\neq i}}^{N}\tilde{\theta}_{ij}^{g}(H)}{N}\cdot 100. \tag{4}$$

In a similar fashion, we measure the directional volatility spillovers transmitted by market i to all other markets j as:

$$S_{\cdot i}^{g}(H)=\frac{\sum\limits_{\substack{j=1\\ j\neq i}}^{N}\tilde{\theta}_{ji}^{g}(H)}{\sum\limits_{i,j=1}^{N}\tilde{\theta}_{ji}^{g}(H)}\cdot 100=\frac{\sum\limits_{\substack{j=1\\ j\neq i}}^{N}\tilde{\theta}_{ji}^{g}(H)}{N}\cdot 100. \tag{5}$$

One can think of the set of directional spillovers as providing a decomposition of the total spillovers to those coming from (or to) a particular source.

2.4. Net spillovers

We obtain the net volatility spillover from market i to all other markets j as

$$S_{i}^{g}(H)=S_{\cdot i}^{g}(H)-S_{i\cdot}^{g}(H). \tag{6}$$

The net volatility spillover is simply the difference between the gross volatility shocks transmitted to and those received from all other markets.

2.5. Net pairwise spillovers

The net volatility spillover in Eq. (6) provides summary information about how much each market contributes to the volatility in other markets, in net terms. It is also of interest to examine the net pairwise volatility spillovers, which we define as:

$$S_{ij}^{g}(H)=\left(\frac{\tilde{\theta}_{ji}^{g}(H)}{\sum\limits_{i,k=1}^{N}\tilde{\theta}_{ik}^{g}(H)}-\frac{\tilde{\theta}_{ij}^{g}(H)}{\sum\limits_{j,k=1}^{N}\tilde{\theta}_{jk}^{g}(H)}\right)\cdot 100$$

$$=\left(\frac{\tilde{\theta}_{ji}^{g}(H)-\tilde{\theta}_{ij}^{g}(H)}{N}\right)\cdot 100. \tag{7}$$

The net pairwise volatility spillover between markets i and j is simply the difference between the gross volatility shocks transmitted from market i to market j and those transmitted from j to i.

3. Empirics: estimates of volatility spillovers across US asset markets

Here, we use our framework to measure the volatility spillovers among four key US asset classes: stocks, bonds, foreign exchange and commodities. This is of particular interest, because spillovers across asset classes may be an important aspect of the global financial crisis that began in 2007.

The remainder of this section proceeds as follows. We begin by describing our data in Section 3.1, then calculate the average (i.e., total) spillovers in Section 3.2. We then quantify spillover dynamics, examining rolling-sample total spillovers, rolling-sample directional spillovers, rolling-sample net directional spillovers and rolling-sample net pairwise spillovers in Sections 3.3–3.5.

3.1. Stock, bond, exchange rate, and commodity market volatility data

We examine the daily volatilities of returns on the US stock, bond, foreign exchange, and commodity markets. In particular, we examine the S&P 500 index, the 10-year Treasury bond yield, the New York Board of Trade US dollar index futures, and the Dow-Jones/UBS commodity index.[4] The data span the period January 25, 1999, to January 29, 2010, with a total of 2771 daily observations.

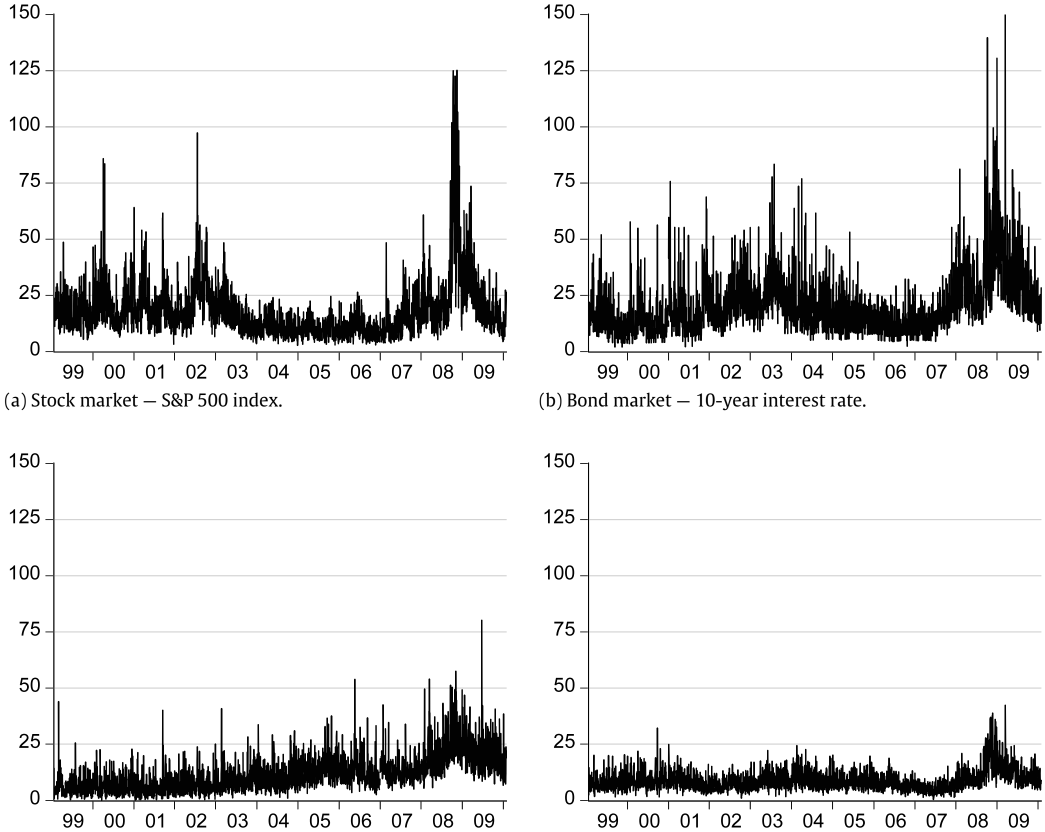
In the tradition of a large body of literature dating back at least to Parkinson (1980), we estimate the daily variance using daily high and low prices.[5] For market i on day t we have

$$\tilde{\sigma}_{it}^{2}=0.361[\ln(P_{it}^{\max})-\ln(P_{it}^{\min})]^{2},$$

where $P_{it}^{\max}$ is the maximum (high) price in market i on day t, and $P_{it}^{\min}$ is the daily minimum (low) price. Because $\tilde{\sigma}_{it}^{2}$ is an estimator of the daily variance, the corresponding estimate of the annualized daily percent standard deviation (volatility) is $\hat{\sigma}_{it}=100\sqrt{365\bullet\tilde{\sigma}_{it}^{2}}$. We plot the four markets' volatilities in Fig. 1, and provide summary statistics of the log volatility in Table 1. Several interesting facts emerge, including: (1) the bond and stock markets have been the most volatile (roughly equally so), with the commodity and FX markets being comparatively less volatile; (2) the volatility dynamics appear to be highly persistent, in keeping with the large body of literature summarized by Andersen, Bollerslev, Christoffersen, and Diebold (2006), for example; and (3) all volatilities are high during the recent crisis, with the stock and bond market volatilities, in particular, displaying huge jumps.

[4] The DJ/AIG commodity index was re-branded as the DJ/UBS commodity index following the acquisition of the AIG Financial Products Corp. by UBS Securities LLC on May 6, 2009.

[5] For background, see Alizadeh, Brandt, and Diebold (2002) and the references therein.



(a) Stock market — S&P 500 index.

(b) Bond market — 10-year interest rate.

(c) Commodity market — DJ/UBS index.

(d) FX market — US dollar index futures.

Fig. 1. Daily US financial market volatilities (annualized standard deviations, percentages).

Table 1
Log volatility summary statistics, four asset classes.

	Stocks	Bonds	Commodities	FX
Mean	−9.70	−9.44	−10.69	−11.00
Median	−9.74	−9.44	−10.50	−10.99
Maximum	−5.45	−4.23	−6.34	−7.62
Minimum	−13.09	−13.79	−18.33	−16.86
Std. deviation	1.19	1.19	1.54	0.98
Skewness	0.21	0.019	−0.73	−0.21
Kurtosis	3.18	3.16	4.21	3.87

Over the sample, the stock market went through two major periods of volatility. In 1999, the daily stock market volatility was close to 25%, but it then increased significantly to above 25% and fluctuated around that level until mid-2003, occasionally moving above 50%. After mid-2003, it declined to less than 25% and stayed there until August 2007. Since August 2007, the stock market volatility has reflected the dynamics of the sub-prime crisis quite well.

In the first half of our sample, the interest rate volatility, as measured by the annualized standard deviation, was comparable to the stock market volatility. While it was below the 25% mark for most of 2000, it then increased and fluctuated between 25% and 50% in the first and last few months of 2001. The bond market volatility remained high until mid-2005, then fell below 25% between late 2005 and the first half of 2007. Since August 2007, the volatility in bond markets has also increased significantly.

The commodity market volatility used to be very low compared to the stock and bond markets, but it has increased slightly over time, especially over the period 2005–2006, and more recently in 2008. The FX market volatility has been the lowest among the four markets. It increased in 2008 and moved to the 25%–50% band following the collapse of the Lehman Brothers in September 2008. The FX market volatility has declined since then, but it is still above its average for the last decade.

3.2. *Unconditional patterns: the full-sample volatility spillover table*

We call Table 2 the volatility spillover table. Its *ij*th entry is the estimated contribution *to* the forecast error variance of market *i* coming *from* innovations to market *j*.[6] Hence,

[6] All of the results are based on vector autoregressions of order 4 and generalized variance decompositions of 10-day-ahead volatility forecast errors. To check for the sensitivity of the results to the choice of the order of the VAR, we calculate the spillover index for orders 2 to 6, and plot the minimum, the maximum and the median values obtained in Fig. A.1 of the Appendix. Similarly, we calculated the spillover index for forecast horizons varying from 4 to 10 days. Both Figs. A.1 and A.2 of the Appendix show that the total spillover plot is not sensitive to the choice of the order of the VAR or the choice of the forecast horizon.

Table 2
Volatility spillover table, four asset classes.

	Stocks	Bonds	Commodities	FX	Directional FROM others
Stocks	88.76	7.28	0.34	3.62	11.24
Bonds	10.17	81.49	2.69	5.65	18.51
Commodities	0.46	3.69	93.71	2.14	6.29
FX	5.66	6.99	1.59	85.76	14.24
Directional TO others	16.29	17.95	4.63	11.41	
Directional including own	105.0	99.4	98.3	97.2	Total spillover index (50.3/400): 12.6%

the off-diagonal column sums (labeled contributions TO others) and row sums (labeled contributions FROM others) are the "to" and "from" directional spillovers, and the "from minus to" differences are the net volatility spillovers. In addition, the total volatility spillover index appears in the lower right corner of the spillover table. It is approximately the grand off-diagonal column sum (or row sum) relative to the grand column sum including diagonals (or row sum including diagonals), expressed as a percentage.[7] The volatility spillover table provides an approximate "input–output" decomposition of the total volatility spillover index.

Consider first what we learn from the table about directional spillovers (gross and net). From the "directional to others" row, we can see that gross directional volatility spillovers to others from each of the four markets are not very different. We can also see from the "directional from others" column that the gross directional volatility spillovers from others to the bond market is relatively large, at 18.5%, followed by the FX market, with the spillovers from others explaining 14.2% of the forecast error variance. As for the net directional volatility spillovers, the largest are from the stock market to others ($16.29 - 11.24 = 5.05\%$) and from others to the FX market ($11.41 - 14.24 = -2.8\%$).

Now consider the total (non-directional) volatility spillover, which is effectively a distillation of the various directional volatility spillovers into a single index. The total volatility spillover appears in the lower right corner of Table 2, which indicates that, on average, across our entire sample, 12.6% of the volatility forecast error variance in all four markets comes from spillovers. The summary of Table 2 is simple: both the total and directional spillovers over the full sample period were quite low.

3.3. Conditioning and dynamics I: the rolling-sample total volatility spillover plot

Clearly, many changes took place during the years in our sample, January 1999–January 2010. Some are well-described as a more-or-less continuous evolution, such as increased linkages among global financial markets and

Fig. 2. Total volatility spillovers, four asset classes.

an increased mobility of capital, due to globalization, the move to electronic trading, and the rise of hedge funds. Others, however, may be better described as bursts that subsequently subside.

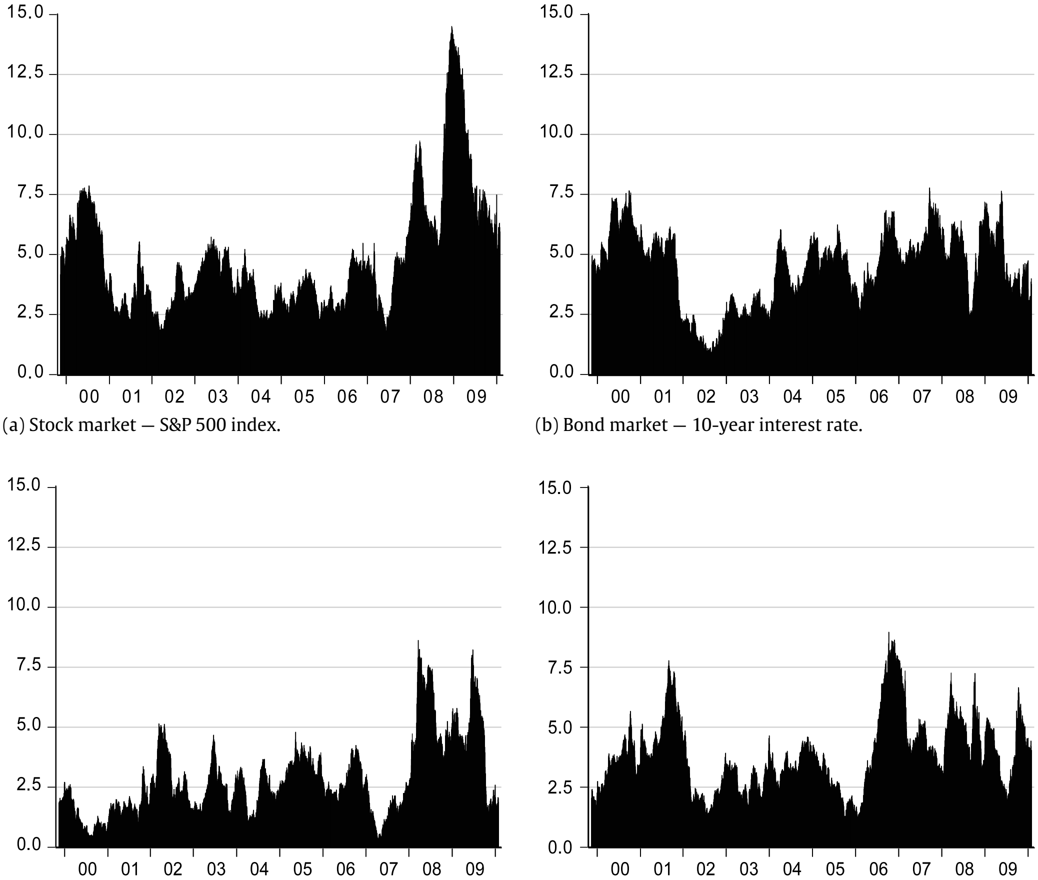
Given this background of financial market evolution and turbulence, it seems unlikely that any single fixed-parameter model would apply over the entire sample. Hence, the full-sample spillover table and spillover index constructed earlier, while providing a useful summary of the "average" volatility spillover behavior, probably miss potentially important secular and cyclical movements in spillovers. To address this issue, we now estimate volatility spillovers using 200-day rolling samples, and assess the extent and nature of the spillover variation over time via the corresponding time series of spillover indices, which we examine graphically in the so-called total spillover plot in Fig. 2.

Starting at a value slightly below 15% in the first window, the total volatility spillover plot mostly fluctuates between ten and twenty percent. However, there are important exceptions: the spillovers exceed the twenty percent mark in mid-2006, and, most importantly, far exceed the thirty percent level during the global financial crisis of 2007–2009.

We can identify several cycles in the total spillover plot. The first cycle started with the burst of the tech bubble in 2000, when the index climbed from 13% to 20%. In the second half of 2001, the index increased to 20% again, before dropping back to 10% at the end of January 2002. After hitting the bottom in mid-2002, the index went through three relatively small cycles until the end of 2005. The first cycle started in mid-2002 and lasted until the last quarter of 2003. The second cycle was shorter, starting in the first quarter of 2004 and ending in the third quarter. The third cycle during this period started in the middle of 2004 and lasted until the end of 2005. All three cycles involve movements of the index of between 10% and 15%.

After the rather calm era from 2003 to 2005, the spillover index recorded a significant upward movement from May to the end of 2006. On May 9, 2006, the Federal Open Market Committee of the Federal Reserve decided to increase the federal funds target rate from 4.75% to 5.00%, and signaled the likelihood of another increase in

[7] As we have already discussed in detail in Section 2, the approximate nature of the claim stems from the properties of the generalized variance decomposition. With Cholesky factor identification, the claim is exact rather than approximate; see also Diebold and Yilmaz (2009).



(a) Stock market — S&P 500 index.

(b) Bond market — 10-year interest rate.

(c) Commodity market — DJ/UBS index.

(d) FX market — US dollar index futures.

Fig. 3. Directional volatility spillovers, *FROM* four asset classes.

its June meeting.[8] After this decision, the total spillover index increased from 12% at the end of April 2006 to 24% by November 2006. The fact that FED was continuing to tighten the monetary policy led to an increase in volatility in the bond and FX markets which spilled over to other markets.

Finally, the most interesting part of the total spillover plot concerns the recent financial crisis. One can see four volatility waves during the recent crisis: July–August 2007 (credit crunch), January–March 2008 (panic in stock and foreign exchange markets, followed by an unscheduled rate cut of three-quarters of a percentage point by the Federal Reserve and Bear Stearns' takeover by JP Morgan in March), September–December 2008 (following the collapse of the Lehman Brothers), and the first half of 2009 (as the financial crisis started to have its full effects all round the world). During the January–March 2008 episode, and even more following the collapse of the Lehman Brothers in mid-September, and consistent with an unprecedented evaporation of liquidity world-wide, the spillover index surges above thirty percent.

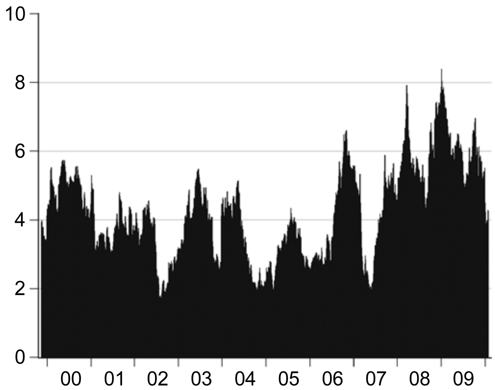
[8] Indeed, the FOMC increased the federal funds target rate to 5.25% in its June meeting and kept it at that level for more than a year, until its September 2007 meeting.

3.4. Conditioning and dynamics II: rolling-sample gross directional volatility spillover plots

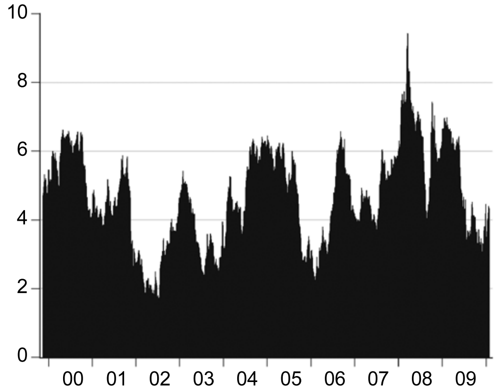
Thus far, we have only discussed the *total* spillover plot, which is of interest but which discards directional information. That information is contained in the "Directional *TO* others" row (the sum of which is given by $S^g_{i\cdot}(H)$ in Eq. (4)) and the "Directional *FROM* others" column (the sum of which is given by $S^g_{\cdot i}(H)$ in Eq. (5)).

We now estimate the above-mentioned row and column of Table 2 dynamically, in a fashion precisely parallel to the earlier-discussed total spillover plot. We call these *directional* spillover plots. In Fig. 3, we present the directional volatility spillovers from each of the four asset classes *to* others (corresponding to the "directional *to* others" row in Table 2). They vary greatly over time. During tranquil times, the spillovers from each market are below five percent, but at volatile times, the directional spillovers increase to close to 10%. Among the four markets, the gross volatility spillovers from the commodity markets to the others are generally smaller than the spillovers from the other three markets.

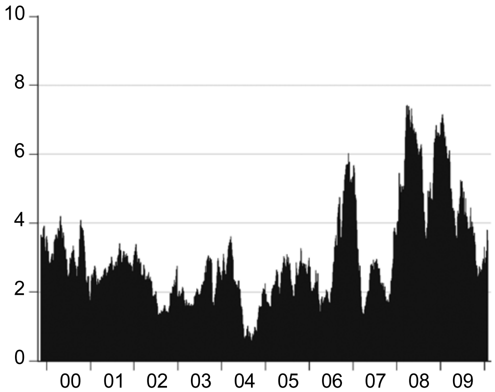
In Fig. 4, we present the directional volatility spillovers *from* the others to each of the four asset classes (corresponding to the "directional *from* others" column in Table 2). As with the directional spillovers *to* others, the



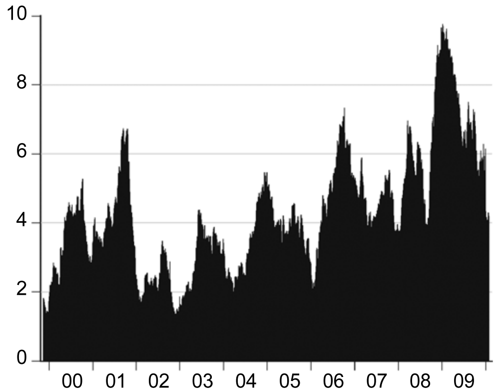
(a) Stock market — S&P 500 index.



(b) Bond market — 10-year interest rate.



(c) Commodity market — DJ/UBS index.



(d) FX market — US dollar index futures.

Fig. 4. Directional volatility spillovers, *TO* four asset classes.

spillovers *from* others vary noticeably over time. However, the relative variation pattern is reversed, with directional volatility spillovers *to* commodities and FX increasing relatively more in turbulent times.

3.5. Conditioning and dynamics III: rolling-sample net directional volatility spillover plots

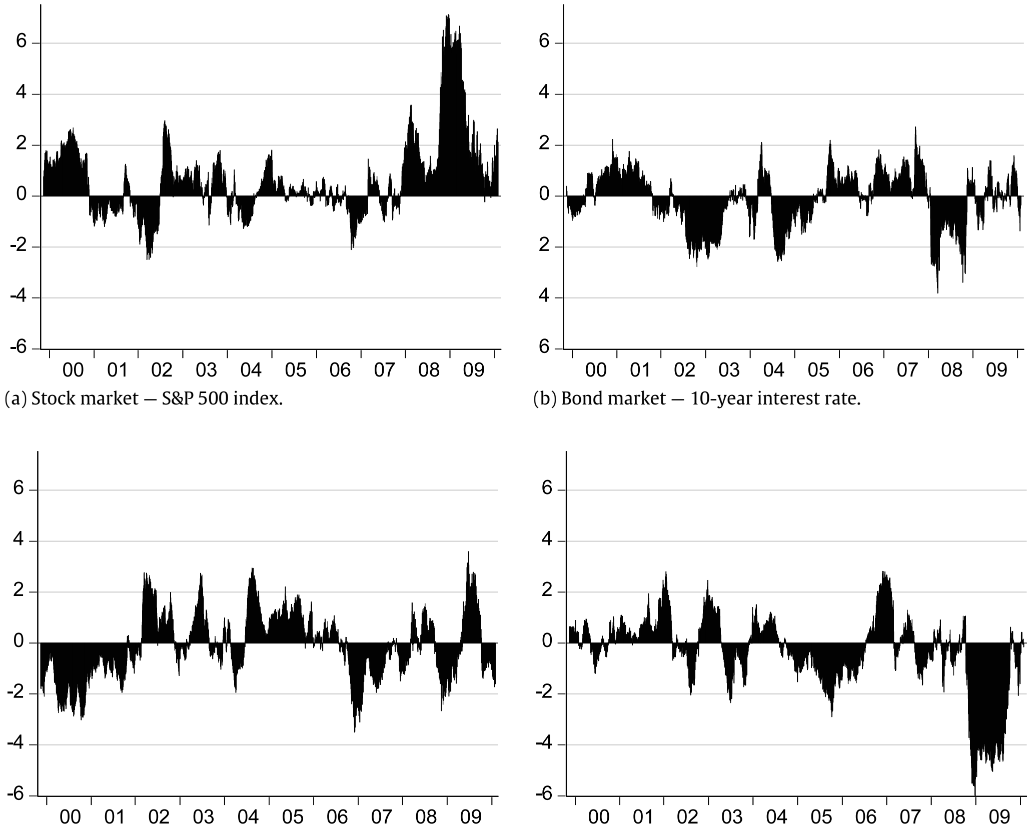
Above, we discussed the gross spillover plots briefly, because our main focus point is the net directional spillover plot presented in Fig. 5. Each point in Fig. 5(a) to 5(d) corresponds to $S_i^g(H)$ (Eq. (6)), and is the difference between the "Contribution from" column sum and the "Contribution to" row sum. In addition, as we mentioned briefly at the end of Section 2, we also calculate the net pairwise spillovers between two markets (Eq. (7)) and present these plots in Fig. 6.

Until the recent global financial crisis, the net volatility spillovers from/to each of the four markets never exceeded the three percent mark (Fig. 5). Furthermore, until 2007 all four markets were at both the giving and receiving ends of the net volatility transmissions, with almost equal magnitudes. However, things changed dramatically after January 2008. The net volatility spillovers from the stock market stayed positive throughout the several stages of the crisis, reaching as high as six percent after the collapse of the Lehman Brothers in September 2008.

As we have already introduced the net spillover and net pairwise spillover plots, we now provide a detailed analysis of the spillovers from each market to the others using Figs. 5 and 6. From 1999 to 2009, there were three major episodes of net volatility spillovers taking place from the stock market to other markets (Fig. 5(a)): during 2000, in 2002–2003, and after January 2008. In our sample period, the first round of volatility spillovers from the stock market took place with the burst of the technology bubble in 2000. As the troubles of the technology stocks intensified after March 2000, the spillover index reached close to 20% in the second to the last quarters of 2000 (Fig. 2). At the time, the bulk of the volatility spillovers from the stock market were transmitted first to the bond, then to the commodity markets (Fig. 6(a) and (b)).

The second period when the stock market was a net transmitter of volatility to other markets was from the second half of 2002 to the third quarter of 2003. The technology stocks continued to be under pressure until October 2002, as the Nasdaq Composite Index hit its lowest level since 1997. In addition, the Iraqi crisis and the prospects of a war in the region increased the volatility in the US stock markets.[9] During this episode, the total

[9] Leigh, Wolfers, and Zitzewitz (2003) showed that a 10 percentage point rise in the probability of a war on Iraq lowered the S&P500 by about one and a half percentage points.



(a) Stock market — S&P 500 index.

(b) Bond market — 10-year interest rate.

(c) Commodity market — DJ/UBS index.

(d) FX market — US dollar index futures.

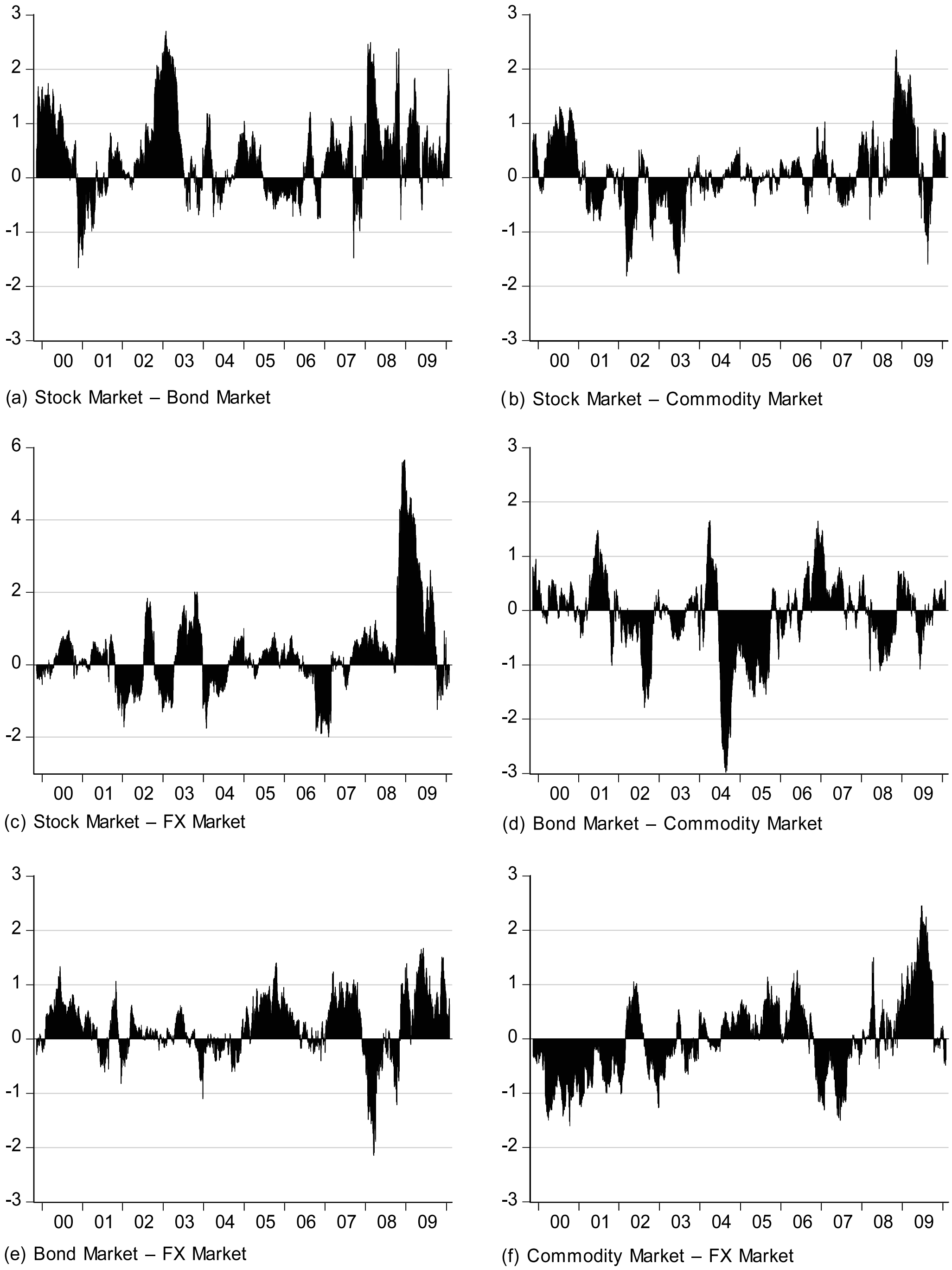
Fig. 5. Net volatility spillovers, four asset classes.

spillover index increased from 7.5% in June 2002 to 15% in June 2003. The net volatility spillovers from the stock market reached close to 3% (Fig. 5(a)), and affected mostly the bond market (Fig. 6(a)). The fact that the stock market was at the same time a net receiver of volatility from the commodity market (Fig. 6(b)) shows the link between the increased volatility in stock markets and the impending Iraqi War.

While the first two episodes of net volatility spillovers from the stock market were important, the third took place during the worst financial crisis yet to hit the global financial markets. Since January 2008, the total spillovers have jumped to above 30% twice, in the first and fourth quarters of 2008. During these two bouts of hefty volatility spillovers across financial markets, the net spillovers from the stock market jumped to more than 3% and 7%, respectively (Fig. 5(a)). The volatility from the stock market was transmitted to all three markets, but especially to the FX market (close to 5%), following the collapse of the Lehman Brothers (Fig. 6(c)). Actually, during the global financial crisis the FX market also received sizeable net volatility spillovers from both the bond market (Fig. 6(e)) and the commodity market (Fig. 6(f)).

Net volatility spillovers from the bond market tend to be smaller than net spillovers from the other markets. We identify three episodes of net volatility spillovers from the bond market: from the second half of 2000 to 2001; from the end of 2005 to 2006; and throughout 2007 (Fig. 5(b)). In 2000, the spillovers went in the direction of the FX market (Fig. 6(e)), whereas in 2001, on the other hand, the majority of volatility spillovers from the bond market were transmitted to the stock market (Fig. 6(a)) and the commodity market (Fig. 6(d)). In the second half of 2005 and the first half of 2006, spillovers from the bond market were transmitted mostly to the FX market. In 2007, on the other hand, the bond market spillovers mostly affected the FX market, followed by the commodity market.

We identify four episodes of net volatility spillovers from the commodity markets: throughout 2002, in the first five months of 2003 (leading up to and immediately following the invasion of Iraq in March 2003), in late 2004 and through 2005, and in the second half of 2009 (Fig. 5(c)). During the period 2002–2003, the commodity market was a net transmitter of the volatility (Fig. 5(c)). The oil prices started to increase from less than \$20 at the end of 2001 to close to \$40 by February 2003, before falling again to almost \$25 by the end of April 2003. Volatility spillovers from commodity markets increased in 2003 just before and during the invasion of Iraq by US forces. Volatility spillovers from the commodity markets also increased, at the end of 2004 and early 2005, when the surge in the Chinese demand for oil and



Note: The left axis scale ranges from -3 to 3 percent in all panels except for panel c), where it ranges from -3 to 6 percent.

Fig. 6. Net pairwise volatility spillovers.

metals surprised investors, sending commodity prices higher (these shocks were mostly transmitted to the bond and FX markets, see Fig. 6(d) and (e)), and especially from March to September 2009 (the shocks were mostly transmitted to the FX market). The volatility shocks in the commodity market in 2002 and during the initial phases of the Iraqi invasion spilled over mostly to the stock market (Fig. 6(b)), but also to the bond market (Fig. 6(d)). During the periods from late 2004 to early 2005 and the first half of 2008, the volatility shocks in the commodity market spilled over mostly to the bond market (Fig. 6(d)), but also to the FX market (Fig. 6(f)). While the commodity market was a net recipient of modest levels of volatility shocks from the stock and bond markets, it was a net transmitter to the FX market during 2009.

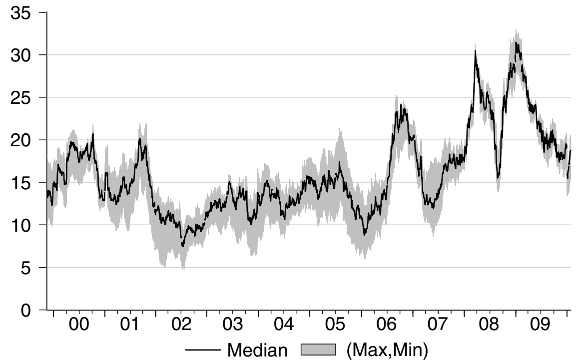


Fig. A.1. Sensitivity of the index to the VAR lag structure (max, min and median values of the index for VAR orders of 2–6).

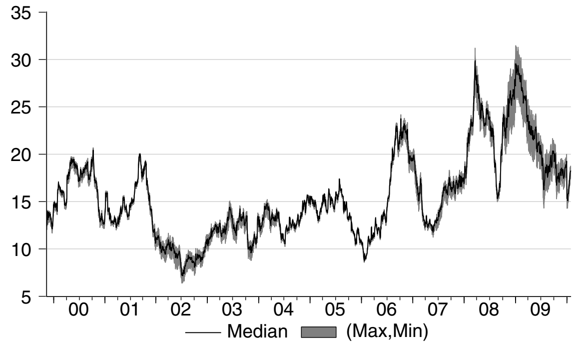


Fig. A.2. Sensitivity of the index to the forecast horizon (min, max and median values over 5- to 10-day horizons).

In the case of FX markets, there were three major episodes of positive net spillovers. The net volatility spillovers from FX markets had little impact on the volatility in the other markets, perhaps with the exception of the modest spillovers at the end of 2001 and early 2002, from the end of 2002 to the first half of 2003, and finally in the second half of 2006 (Fig. 5(d)). The net volatility spillovers from the FX market increased at the end of 2001 and in early 2002. They also increased in May 2006, following the FED's decision to tighten the monetary policy further (Fig. 5(d)). In both episodes, the volatility shocks in the FX market spilled over to the stock and commodity markets (Fig. 6(c) and (f)).

4. Concluding remarks

We have provided both gross and net *directional* spillover measures that are independent of the ordering used for the volatility forecast error variance decompositions. When applied to US financial markets, our measures shed new light on the nature of cross-market volatility transmission, pinpointing the importance of volatility spillovers from the stock market to other markets during the recent crisis.

Of course, we are not the first to consider issues related to volatility spillovers (e.g., Edwards & Susmel, 2001; Engle, Ito, & Lin, 1990; King, Sentana, & Wadhwani, 1994), but our approach is very different. It produces continuously-varying indexes (unlike, for example, the "high state/low state" indicator of Edwards & Susmel), and is econometrically tractable even for very large numbers of assets. Although it is beyond the scope of this paper, it would be interesting in future work to further investigate the relationship between our spillover measure and a variety of others, based on measures ranging from traditional (albeit time-varying) correlations (e.g., Engle, 2002, 2009) to the recently-introduced CoVaR of Adrian and Brunnermeier (2008).

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Appendix

See Figs. A.1 and A.2.

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