

A Declarative Semantics for SNOMED CT Expression Constraints

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Contents

1	Data Types	3
1.1	Atomic Data Types	3
1.2	Composite Data Types	4
2	The Substrate	5
2.1	Substrate Components	5
2.1.1	Quad	5
2.1.2	conceptReference	5
2.2	Substrate	6
2.2.1	Substrate Definition	6
2.2.2	Strict Substrate	9
2.2.3	Permissive Substrate	9
3	Interpretation of Expression Constraints	11
3.1	Interpretation Output	11
3.2	expressionConstraint	11
3.2.1	unrefinedExpressionConstraint	12
3.2.2	refinedExpressionConstraint	12
3.2.3	simpleExpressionConstraint	13
3.2.4	compoundExpressionConstraint	13
3.2.5	conjunctionExpressionConstraint	14
3.2.6	disjunctionExpressionConstraint	15
3.2.7	exclusionExpressionConstraint	15
3.2.8	subExpressionConstraint	16
3.3	refinement	17
3.3.1	conjunctionRefinementSet	17
3.3.2	disjunctionRefinementSet	18
3.3.3	subRefinement	18
3.4	attributeSet	19
3.4.1	conjunctionAttributeSet	20
3.4.2	disjunctionAttributeSet	20

3.4.3	subAttributeSet	21
3.5	attribute	21
3.5.1	attribute cardinality	23
3.5.2	expressionConstraintValue	25
3.6	attributeGroup	25
3.6.1	attribute group cardinality	26
3.6.2	attributeSet inside a group	27
3.6.3	conjunctionAttributeSet inside a group	27
3.6.4	disjunctionAttributeSet inside a group	28
3.6.5	subAttributeSet inside a group	28
3.6.6	attribute inside a group	28
3.6.7	cardinality inside a group	29
4	Substrate Interpretations	31
4.1	constraintOperator	31
4.2	attributeOperator	32
4.3	expressionComparisonOperator	32
4.4	numericComparisonOperator	33
4.5	stringComparisonOperator	34
4.6	focusConcept	35
4.6.1	memberOf	36
5	Types	37
5.1	Types	37
5.1.1	Quads_or_Error	37
5.1.2	setIdGroups	37
5.2	Result transformations	38
A	Optional elements	40
B	Generic cardinality evaluation	40
C	Generic sequence function	40

1 Data Types

1.1 Atomic Data Types

This section identifies the atomic data types that are assumed for the rest of this specification. These types are defined as lexical components in the specification.

```
sctId = digitNonZero 5*17( digit )
term = 1*nonwsNonPipe *( 1*SP 1*nonwsNonPipe )
numericValue = QM stringValue QM / “#” numericValue ( decimalValue / integerValue )
stringValue = QM 1*(anyNonEscapedChar / escapedChar) QM
integerValue = ( [“-”/“+”] digitNonZero *digit ) / zero
decimalValue = integerValue “.” 1*digit
nonNegativeIntegerValue = (digitNonZero *digit ) / zero
```

- **SCTID** – A *SNOMED CT* id is used to represent an attribute id or a concept id. The initial digit may not be zero. The smallest number of digits is six, and the maximum is 18.
- **TERM** – The term must be the term from a *SNOMED CT* description that is associated with the concept identified by the preceding concept identifier. For example, the term could be the preferred description, or the preferred description associated with a particular translation. The term may include valid UTF-8 characters except for the pipe “|” character. The term begins with the first non-whitespace character following the starting “|” character and ends with the last non-whitespace character preceding the next “|” character.
- **STRINGVALUE** – A string value includes one or more of any printable ASCII characters enclosed in quotation marks. Quotes and backslash characters within the string must be preceded by the escape character (“\”).
- **Z** – built-in Z equivalent of **INTEGERVALUE**. An integer may be positive, negative or zero. Positive integers optionally start with a plus sign (“+”), followed by a non-zero digit followed by zero to many additional digits. Negative integers begin with a minus sign (“-”) followed by a non-zero digit and zero to many additional digits.
- **DECIMALVALUE** – A decimal value starts with an integer. This is followed by a decimal point and one to many digits.
- **N** – built-in Z equivalent of **NONNEGATIVEINTEGERVALUE**
- **GROUPID** – a role group identifier. (Not explicated in the specification, but needed)

[*sctId*, *term*, *decimalValue*, *stringValue*, *groupId*]

We specifically define some well known identifiers: the *is_a* attribute, the *zero_group* and *attribute_concept*, and *refset_concept* the parents of all attributes and all refsets respectively.

	<i>is_a</i> : <i>sctId</i>
	<i>zero_group</i> : <i>groupId</i>
	<i>attribute_concept</i> : <i>sctId</i>
	<i>refset_concept</i> : <i>sctId</i>

1.2 Composite Data Types

concreteValue is in anticipation of the enhanced specification that includes target strings, integers or decimals.

- **concreteValue** – a string, integer or decimal literal
- **target** – the target of a relationship that is either an *sctId* or a *concreteValue*

concreteValue ::=
 cv_string⟨⟨*stringValue*⟩⟩ | *cv_integer*⟨⟨ \mathbb{Z} ⟩⟩ | *cv_decimal*⟨⟨*decimalValue*⟩⟩
target ::= *t_sctid*⟨⟨*sctId*⟩⟩ | *t_concrete*⟨⟨*concreteValue*⟩⟩

2 The Substrate

A substrate represents the context of an interpretation.

2.1 Substrate Components

2.1.1 Quad

Relationships in the substrate are represented a 4 element tuples or “quads” which consist of a source, attribute, target and role group identifier. The *is_a* attribute may only appear in the zero group, and the target of an *is_a* attribute must be a *sctId* (not a *concreteValue*)

<i>Quad</i>
<i>s</i> : <i>sctId</i>
<i>a</i> : <i>sctId</i>
<i>t</i> : <i>target</i>
<i>g</i> : <i>groupId</i>
$a = is_a \Rightarrow (g = zero_group \wedge t \in \text{ran } t_sctid)$

2.1.2 conceptReference

The root of the expression constraint language is concept references:

- **CONCEPTREFERENCE** – A *conceptReference* is represented by a *ConceptId* optionally followed by a term enclosed by a pair of “|” characters. Whitespace before or after the *ConceptId* is ignored as is any whitespace between the initial “|” characters and the first non-whitespace character in the term or between the last non-whitespace character and before second “|” character. **TERM** is ignored for the purposes of interpretation.
- **CONCEPTID** – The *ConceptId* must be a valid SNOMED CT identifier for a concept. The initial digit may not be zero. The smallest number of digits is six, and the maximum is 18.
- **ATTRIBUTEName** – The attribute name is the name of an attribute (or relationship type) to which a value is applied to refine the meaning of a containing expression. The attribute name is represented in the same way as other concept references. If the attribute name is not known then a wild card may be used to represent any attribute concept in the given substrate.

<pre>conceptReference = conceptId [ws “ ” ws term ws “ ”] conceptId = sctId attributeName = conceptReference / wildCard</pre>

$$\begin{aligned}
\text{conceptReference} &== \text{conceptId} \times \text{term}[0..1] \\
\text{conceptId} &== \text{sctId} \\
\text{attributeName} &::= \text{ancr}\langle\langle\text{conceptReference}\rangle\rangle \mid \text{anwc}
\end{aligned}$$

2.2 Substrate

2.2.1 Substrate Definition

A substrate consists of:

- **concepts** The set of *sctIds* (concepts) that are considered valid in the context of the substrate.
- **relationships** A set of relationship quads (source, attribute, target, group)
- **parentsOf** A function from an *sctId* to its asserted and inferred parents
- **equivalent_concepts** A function from an *sctId* to the set of *sctId*'s that have been determined to be equivalent to it.

The following functions can be computed from the basic set above

- **childrenOf** The inverse of the *parentsOf* function
- **descendants** The transitive closure of the *childrenOf* function
- **ancestors** The transitive closure of the *parentsOf* function

A substrate also implements three functions:

- **i_conceptReference** The interpretation of a concept reference. This function can return a (possibly empty) set of *sctId*'s or an error.
- **i_attributeName** The interpretation of an attribute name. This function can return a (possibly empty) set of *sctId*'s or an error.
- **i_refsetId** The interpretation of a refset identifier. This function can return a (possibly empty) set of *sctId*'s or an error.

The formal definition of substrate follows. The expressions below assert that:

1. All *relationship* sources, attributes and *sctId* targets are in *concepts*.
2. There is a *parentsOf* entry for every substrate *concept*.
3. Every *sctId* that can be returned by the *parentsOf* function is a *concept*.
4. Every *is_a* relationship entry is represented in the *parentsOf* function.
Note that there can be additional entries represented in the *parentsOf* function that aren't in the relationships table.
5. There is an *equivalent_concepts* assertion for every substrate *concept*.
6. The *equivalent_concepts* function is reflexive (i.e. every concept is equivalent to itself).
7. All equivalent concepts are in *concepts*.
8. If two concepts (*c1* and *c2*) are equivalent, then they:
 - Have the same parents
 - Appear the subject, attribute and object of the same set of relationships
 - If one is a concept *refset* the other is as well
9. Every concept *refset* is a substrate *concept*

10. Every concept *refset* is mappable via (i.e. is in the domain of) *i_refsetId*
11. *childrenOf* is the inverse of *parentsOf*, where any concept that doesn't appear a parent has no (the emptyset) children.
12. *descendants* is the transitive closure of the *childrenOf* function
13. *ancestors* is the transitive closure of the *parentsOf* function
14. No concept can be its own ancestor (or, by inference, descendant)
15. The *i_conceptReference*, *i_attributeName* and *i_refsetId* functions are defined for all possible *conceptReferences* and *attributeNames* (because they are complete functions).
16. All *sctId*'s that are produced by the The *i_conceptReference*, *i_attributeName* and *i_refsetId* functions are substrate *concepts*.

Substrate

$concepts : \mathbb{P} \text{ sctId}$
 $relationships : \mathbb{P} \text{ Quad}$
 $parentsOf : \text{ sctId} \rightarrow \mathbb{P} \text{ sctId}$
 $equivalent_concepts : \text{ sctId} \rightarrow \mathbb{P} \text{ sctId}$

 $childrenOf : \text{ sctId} \rightarrow \mathbb{P} \text{ sctId}$
 $descendants : \text{ sctId} \rightarrow \mathbb{P} \text{ sctId}$
 $ancestors : \text{ sctId} \rightarrow \mathbb{P} \text{ sctId}$

 $i_conceptReference : \text{ conceptReference} \rightarrow \text{ Sctids_or_Error}$
 $i_attributeName : \text{ attributeName} \rightarrow \text{ Sctids_or_Error}$
 $i_refsetId : \text{ sctId} \rightarrow \text{ Sctids_or_Error}$

$\forall rel : relationships \bullet rel.s \in concepts \wedge rel.a \in concepts \wedge$
 $(rel.t \in \text{ran } t_sctid \Rightarrow t_sctid \sim rel.t \in concepts)$

 $\text{dom } parentsOf = concepts$
 $\bigcup(\text{ran } parentsOf) \subseteq concepts$

 $\forall r : relationships \bullet r.a = is_a \Rightarrow (t_sctid \sim r.t) \in parentsOf \ r.s$

 $\text{dom } equivalent_concepts = concepts$
 $\bigcup(\text{ran } equivalent_concepts) \subseteq concepts$
 $\forall c : concepts \bullet c \in equivalent_concepts \ c$
 $\forall c1, c2 : concepts \mid c2 \in (equivalent_concepts \ c1) \bullet$
 $parentsOf \ c1 = parentsOf \ c2 \wedge$
 $\{r : relationships \mid r.s = c1\} = \{r : relationships \mid r.s = c2\} \wedge$
 $\{r : relationships \mid r.a = c1\} = \{r : relationships \mid r.a = c2\} \wedge$
 $\{r : relationships \mid t_sctid \sim r.t = c1\} = \{r : relationships \mid t_sctid \sim r.t = c2\}$

 $\text{dom } childrenOf = concepts$
 $\forall s, t : concepts \bullet t \in parentsOf \ s \Leftrightarrow s \in childrenOf \ t$
 $\forall c : concepts \mid c \notin \bigcup(\text{ran } childrenOf) \bullet childrenOf \ c = \emptyset$
 $\forall s : concepts \bullet$
 $descendants \ s = childrenOf \ s \cup \bigcup\{t : childrenOf \ s \bullet descendants \ t\}$
 $\forall t : concepts \bullet$
 $ancestors \ t = parentsOf \ t \cup \bigcup\{s : parentsOf \ t \bullet ancestors \ s\}$
 $\forall t : concepts \bullet t \notin ancestors \ t$

 $\forall cr_interp : \text{ran } i_conceptReference \mid cr_interp \in \text{ran } ok \bullet$
 $result_sctids \ cr_interp \subseteq concepts$
 $\forall att_interp : \text{ran } i_attributeName \mid att_interp \in \text{ran } ok \bullet$
 $result_sctids \ att_interp \subseteq concepts$
 $\forall refset_interp : \text{ran } i_refsetId \mid refset_interp \in \text{ran } ok \bullet$
 $result_sctids \ refset_interp \subseteq concepts$

Implementations may choose to implement “strict” substrates, where additional rules apply or “permissive” substrates where rules are relaxed.

2.2.2 Strict Substrate

A **strict_substrate** is a substrate where:

- If a `conceptReference` is not in the substrate concepts it returns an error, otherwise the set of equivalent concepts
- If an attribute name is a `conceptReference` and it is not in the substrate concepts or is not a descendant of the `attribute_concept` it returns an error, otherwise the set of equivalent attributes. If an attribute name is a wild card, it returns all descendants of `attribute_concept`
- If a concept reference that is a target of a `memberOf` function is not in the substrate concepts or is not a descendant of the `refset_concept` it returns an error, otherwise the set of equivalent refset identifiers

<i>strict_substrate</i>	<i>Substrate</i>
$\forall cr : \text{conceptReference} \bullet i_conceptReference\ cr =$	$\text{if } first\ cr \notin \text{concepts} \text{ then } error\ unknownConceptReference$ $\text{else } ok\ (equivalent_concepts\ (first\ cr))$
$\forall an : \text{attributeName} \bullet i_attributeName\ an =$	$\text{if } ancr \sim an \in \text{conceptReference} \text{ then}$ $\quad (\text{let } rslt == i_conceptReference\ (ancr \sim an) \bullet$ $\quad \quad \text{if } rslt \in \text{ran } error \text{ then } rslt$ $\quad \quad \text{else if } result_sctids\ rslt \subseteq (descendants\ attribute_concept)$ $\quad \quad \quad \text{then } rslt$ $\quad \quad \text{else}$ $\quad \quad \quad error\ unknownAttributeId)$
	else $\quad ok\ (descendants\ attribute_concept)$
$\forall rsid : \text{sctId} \bullet i_refsetId\ rsid =$	$\text{if } rsid \in \text{descendants } refset_concept$ $\quad \text{then } ok\ \{rsid\}$ $\quad \text{else } error\ unknownRefsetId$

2.2.3 Permissive Substrate

A **permissive_substrate** never raises an error condition. concept references that are not in the substrate are “discarded” (i.e. map to the empty set). Any valid `conceptReference` may be used in the position of an attribute or a refset id. The `attributeName` wild card returns the set of all possible concepts.

permissive_substrate

Substrate

```

 $\forall cr : \text{conceptReference} \bullet i\_conceptReference\ cr =$ 
  if first cr  $\notin$  concepts then ok  $\emptyset$ 
  else ok (equivalent_concepts (first cr))

 $\forall an : \text{attributeName} \bullet i\_attributeName\ an =$ 
if ancr $\sim an \in \text{conceptReference}$  then
  i_conceptReference (ancr $\sim an$ )
else
  ok concepts

 $\forall rsid : \text{sctId} \bullet i\_refsetId\ rsid =$ 
  if rsid  $\notin$  concepts then ok  $\emptyset$ 
  else ok (equivalent_concepts (rsid))
```

3 Interpretation of Expression Constraints

An `EXPRESSIONCONSTRAINT` is interpreted in the context of a *Substrate* and returns a set of *sctIds* or an error indicator.

3.1 Interpretation Output

The result of applying a query against a substrate is either a (possibly empty) set of *sctId*'s or an *ERROR*. An *ERROR* occurs when:

- The interpretation of a *conceptId* is not a substrate *concept*
- The interpretation of a relationship attribute is not a substrate *attributeId*
- The interpretation of a reset is not a substrate *refsetId*

$$\begin{aligned} \text{ERROR} &::= \text{unknownConceptReference} \mid \text{unknownAttributeId} \mid \text{unknownRefsetId} \\ \text{Sctids_or_Error} &::= \text{ok} \langle \mathbb{P} \text{ sctId} \rangle \mid \text{error} \langle \text{ERROR} \rangle \end{aligned}$$

Each interpretation that follows begins with a simplified version of the language construct in the specification. It then formally specifies the constructs that are used in the interpretation, followed by the interpretation itself. We start with the definition of *expressionConstraint*, which, once interpreted, returns either a set of *sctIds* or an error condition.

3.2 expressionConstraint

An expression constraint is either a refined expression constraint or an unrefined expression constraint.

$$\text{expressionConstraint} = \text{ws} \left(\text{refinedExpressionConstraint} \mid \text{unrefinedExpressionConstraint} \right)$$

The interpretation of an `EXPRESSIONCONSTRAINT` is the interpretation of either the `REFINEDEXPRESSIONCONSTRAINT` or the `UNREFINEDEXPRESSIONCONSTRAINT`.

$$\begin{aligned} \text{expressionConstraint} &::= \\ &\quad \text{expcons_refined} \langle \langle \text{refinedExpressionConstraint} \rangle \rangle \mid \\ &\quad \text{expcons_unrefined} \langle \langle \text{unrefinedExpressionConstraint} \rangle \rangle \end{aligned}$$

$$\text{i_expressionConstraint} :$$

$$\text{Substrate} \rightarrow \text{expressionConstraint} \rightarrow \text{Sctids_or_Error}$$

$$\forall ss : \text{Substrate}; \text{ec} : \text{expressionConstraint} \bullet \text{i_expressionConstraint} \text{ ss } \text{ec} =$$

$$\text{if } \text{ec} \in \text{ran expcons_refined}$$

$$\text{then}$$

$$\text{i_refinedExpressionConstraint} \text{ ss } (\text{expcons_refined} \sim \text{ec})$$

$$\text{else}$$

$$\text{i_unrefinedExpressionConstraint} \text{ ss } (\text{expcons_unrefined} \sim \text{ec})$$

3.2.1 unrefinedExpressionConstraint

An unrefined expression constraint is either compound or simple.

`unrefinedExpressionConstraint = compoundExpressionConstraint / simpleExpressionConstraint`

The interpretation of an `UNREFINEDEXPRESSIONCONSTRAINT` is either the interpretation of a `COMPOUNDEXPRESSIONCONSTRAINT` or a `SIMPLEEXPRESSIONCONSTRAINT`

unrefinedExpressionConstraint ::=
unrefined_compound⟨⟨*compoundExpressionConstraint*⟩⟩ |
unrefined_simple⟨⟨*simpleExpressionConstraint*⟩⟩

<p><i>i_unrefinedExpressionConstraint :</i> <i>Substrate</i> → <i>unrefinedExpressionConstraint</i> → <i>Sctids_or_Error</i></p> <hr/> <p>∀ <i>ss</i> : <i>Substrate</i>; <i>uec</i> : <i>unrefinedExpressionConstraint</i> • <i>i_unrefinedExpressionConstraint ss uec</i> = if <i>uec</i> ∈ ran <i>unrefined_compound</i> then <i>i_compoundExpressionConstraint ss (unrefined_compound~ uec)</i> else <i>i_simpleExpressionConstraint ss (unrefined_simple~ uec)</i></p>

3.2.2 refinedExpressionConstraint

A refined expression constraint starts with an unrefined expression constraint and adds a refinement. Refined expression constraints may optionally be placed in brackets.

`refinedExpressionConstraint = unrefinedExpressionConstraint ws “:” ws refinement / “(” ws refinedExpressionConstraint ws “)”`

The interpretation of `REFINEDEXPRESSIONCONSTRAINT` is the intersection of the interpretation of the `UNREFINEDEXPRESSIONCONSTRAINT` and the `REFINEMENT`, both of which return a set of `sctId`’s or an error. The second production defines `REFINEDEXPRESSIONCONSTRAINT` in terms of itself and has no effect on the results.

Note: The second declaration below (*refinedExpressionConstraint'*) is used to avoid circular definitions.

refinedExpressionConstraint ==
unrefinedExpressionConstraint × *refinement*
[refinedExpressionConstraint']

$i_refinedExpressionConstraint :$	$Substrate \rightarrow refinedExpressionConstraint \rightarrow Sctids_or_Error$
$\forall ss : Substrate; rec : refinedExpressionConstraint \bullet$	$(\mathbf{let} \text{ } unref_interp == (i_unrefinedExpressionConstraint \text{ } ss \text{ } (first \text{ } rec)) \bullet$
$i_refinedExpressionConstraint \text{ } ss \text{ } rec =$	$intersect \text{ } unref_interp(i_refinement \text{ } ss \text{ } (second \text{ } rec)))$

Duplicate signature for recursive definitions

$i_refinedExpressionConstraint' :$	$Substrate \rightarrow refinedExpressionConstraint' \rightarrow Sctids_or_Error$
-------------------------------------	--

3.2.3 simpleExpressionConstraint

A simple expression constraint includes exactly one focus concept, optionally preceded by a constraint operator.

simpleExpressionConstraint = [*constraintOperator* ws] *focusConcept*

The interpretation of **SIMPLEEXPRESSIONCONSTRAINT** is the application of an optional constraint operator to the interpretation of **FOCUSCONCEPT**, which returns a set of *sctId*'s or an error. The interpretation of an error is the error.

simpleExpressionConstraint == *constraintOperator*[0..1] × *focusConcept*

$i_simpleExpressionConstraint :$	$Substrate \rightarrow simpleExpressionConstraint \rightarrow Sctids_or_Error$
$\forall ss : Substrate; sec : simpleExpressionConstraint \bullet$	$i_simpleExpressionConstraint \text{ } ss \text{ } sec =$
$i_constraintOperator \text{ } ss \text{ } (first \text{ } sec) (i_focusConcept \text{ } ss \text{ } (second \text{ } sec))$	

3.2.4 compoundExpressionConstraint

A compound expression constraint contains two or more simple expression constraints joined by either a conjunction, disjunction or exclusion. Compound expression constraints may optionally be placed in brackets.

compoundExpressionConstraint = conjunctionExpressionConstraint / disjunctionExpressionConstraint / exclusionExpressionConstraint / "(" ws compoundExpressionConstraint ws ")"

The interpretation of a *compoundExpressionConstraint* is the interpretation the conjunction, disjunction or exclusion of nested compound constraint.

```
compoundExpressionConstraint ::=
    compound_conj⟨⟨conjunctionExpressionConstraint⟩⟩ |
    compound_disj⟨⟨disjunctionExpressionConstraint⟩⟩ |
    compound_excl⟨⟨exclusionExpressionConstraint⟩⟩ |
    compound_nested⟨⟨compoundExpressionConstraint⟩⟩
[compoundExpressionConstraint']
```

i_compoundExpressionConstraint :
Substrate → compoundExpressionConstraint → Sctids_or_Error

∀ ss : Substrate; cec : compoundExpressionConstraint •
i_compoundExpressionConstraint ss cec =
if cec ∈ ran compound_conj **then**
 i_conjunctionExpressionConstraint ss (compound_conj~cec)
else if cec ∈ ran compound_disj **then**
 i_disjunctionExpressionConstraint ss (compound_disj~cec)
else if cec ∈ ran compound_excl **then**
 i_exclusionExpressionConstraint ss (compound_excl~cec)
else
 i_compoundExpressionConstraint ss (compound_nested~cec)

i_compoundExpressionConstraint' :
Substrate → compoundExpressionConstraint' → Sctids_or_Error

3.2.5 conjunctionExpressionConstraint

A *conjunction* expression constraint combines two or more expression constraints with a conjunction (“and”) operator. More than one conjunction may be used without brackets. However any compound expression constraint (using a different binary operator) that appears within a conjunction expression constraint must be enclosed by brackets.

conjunctionExpressionConstraint = subExpressionConstraint 1*(ws conjunction ws subExpressionConstraint)

`CONJUNCTIONEXPRESSIONCONSTRAINT` is interpreted as the intersection of the interpretations of the `SUBEXPRESSIONCONSTRAINTS` .

$$\text{conjunctionExpressionConstraint} == \text{subExpressionConstraint} \times \text{seq}_1(\text{subExpressionConstraint})$$

Apply the intersection operator to the interpretation of each `subExpressionConstraint`

$ \begin{aligned} & i_conjunctionExpressionConstraint : \\ & \quad Substrate \rightarrow conjunctionExpressionConstraint \rightarrow Sctids_or_Error \\ & \hline & \forall ss : Substrate; cecr : conjunctionExpressionConstraint \bullet \\ & i_conjunctionExpressionConstraint ss cecr = \\ & \quad applyToSequence ss i_subExpressionConstraint intersect cecr \end{aligned} $

3.2.6 disjunctionExpressionConstraint

A *disjunction* expression constraint combines two or more expression constraints with a disjunction (“or”) operator. More than one disjunction may be used without brackets. However any compound expression constraint (using a different binary operator) that appears within a disjunction expression constraint must be enclosed by brackets.

$$\text{disjunctionExpressionConstraint} = \text{subExpressionConstraint} 1^*(\text{ws disjunction ws subExpressionConstraint})$$

$$\text{disjunctionExpressionConstraint} == \text{subExpressionConstraint} \times \text{seq}_1(\text{subExpressionConstraint})$$

`DISJUNCTIONEXPRESSIONCONSTRAINT` is interpreted as the union of the interpretations of the `SUBEXPRESSIONCONSTRAINTS` .

$ \begin{aligned} & i_disjunctionExpressionConstraint : \\ & \quad Substrate \rightarrow disjunctionExpressionConstraint \rightarrow Sctids_or_Error \\ & \hline & \forall ss : Substrate; decr : disjunctionExpressionConstraint \bullet \\ & i_disjunctionExpressionConstraint ss decr = \\ & \quad applyToSequence ss i_subExpressionConstraint union decr \end{aligned} $

3.2.7 exclusionExpressionConstraint

An *exclusion* expression constraint combines two expression constraints with an exclusion (“minus”) operator. A single exclusion operator may be used without brackets. However when the operands of the exclusion expression constraint are compound, these compound expression constraints must be enclosed by brackets.

exclusionExpressionConstraint = subExpressionConstraint ws exclusion ws subExpression-
Constraint

EXCLUSIONESPRESSIONCONSTRAINT is interpreted as the interpretation of the **SUBEXPRESSIONCONSTRAINT** minus the interpretation of the second.

$exclusionExpressionConstraint ==$
 $subExpressionConstraint \times subExpressionConstraint$

$i_exclusionExpressionConstraint :$ $Substrate \rightarrow exclusionExpressionConstraint \rightarrow Sctids_or_Error$
$\forall ss : Substrate; ecr : exclusionExpressionConstraint \bullet$ $(let\ first_sec == (i_subExpressionConstraint\ ss\ (first\ ecr));$ $second_sec == (i_subExpressionConstraint\ ss\ (second\ ecr)) \bullet$ $i_exclusionExpressionConstraint\ ss\ ecr = minus\ first_sec\ second_sec)$

3.2.8 subExpressionConstraint

A subexpression constraint, which appears within a compound expression constraint, must either be simple, or a bracketed compound or refined expression constraint.

subExpressionConstraint = simpleExpressionConstraint / "(" ws (compoundExpressionCon-
straint / refinedExpressionConstraint) ws ")"

The interpretation of **SUBEXPRESSIONCONSTRAINT** is the interpretation the **SIMPLEEXPRESSIONCONSTRAINT** **COMPOUNDEXPRESSIONCONSTRAINT** OR **REFINEDEXPRESSIONCONSTRAINT**

$subExpressionConstraint ::=$
 $subExpr_simple \langle \langle simpleExpressionConstraint \rangle \rangle \mid$
 $subExpr_compound \langle \langle compoundExpressionConstraint' \rangle \rangle \mid$
 $subExpr_refined \langle \langle refinedExpressionConstraint' \rangle \rangle$

$i_subExpressionConstraint :$ $Substrate \rightarrow subExpressionConstraint \rightarrow Sctids_or_Error$
$\forall ss : Substrate; sec : subExpressionConstraint \bullet$ $i_subExpressionConstraint\ ss\ sec =$ if $sec \in \text{ran } subExpr_simple$ then $i_simpleExpressionConstraint\ ss\ (subExpr_simple \sim sec)$ else if $sec \in \text{ran } subExpr_compound$ then $i_compoundExpressionConstraint'\ ss\ (subExpr_compound \sim sec)$ else $i_refinedExpressionConstraint'\ ss\ (subExpr_refined \sim sec)$

3.3 refinement

A *refinement* contains all the grouped and ungrouped attributes that refine the set of clinical meanings satisfied by the expression constraint. Refinements may represent the conjunction or disjunction of two smaller refinements, and may optionally be placed in brackets. Where both conjunction and disjunction are used, brackets are mandatory to disambiguate the intended meaning.

`refinement = subRefinement ws [conjunctionRefinementSet / disjunctionRefinementSet]`

The interpretation of `REFINEMENT` is one of:

- The intersection of the interpretation of the `SUBREFINEMENT` and the `CONJUNCTIONREFINEMENTSET`
- The union of the interpretation of the `SUBREFINEMENT` and the `DISJUNCTIONREFINEMENTSET`
- The interpretation of `SUBREFINEMENT` if neither conjunction nor disjunction set is present

$refinement == subRefinement \times refinementConjunctionOrDisjunction[0..1]$
 $[refinement']$

$refinementConjunctionOrDisjunction ::=$
 $refine_conjset \langle \langle conjunctionRefinementSet \rangle \rangle \mid$
 $refine_disjset \langle \langle disjunctionRefinementSet \rangle \rangle$

$i_refinement : Substrate \rightarrow refinement \rightarrow Sctids_or_Error$

$\forall ss : Substrate; rfnment : refinement \bullet$
 $i_refinement ss rfnment =$
 $(let lhs == i_subRefinement ss (first rfnment); rhs == second rfnment \bullet$
if $\#rhs = 0$ **then**
 lhs
else if $(head rhs) \in \text{ran } refine_conjset$ **then**
 $intersect lhs (i_conjunctionRefinementSet ss (refine_conjset \sim (head rhs)))$
else
 $union lhs (i_disjunctionRefinementSet ss (refine_disjset \sim (head rhs)))$

$i_refinement' : Substrate \rightarrow refinement' \rightarrow Sctids_or_Error$

3.3.1 conjunctionRefinementSet

A *conjunction refinement set* consists of one or more conjunction operators, each followed by a `subRefinement`.

$$\text{conjunctionRefinementSet} = 1^*(\text{ws conjunction ws subRefinement})$$

$$\text{conjunctionRefinementSet} == \text{seq}_1 \text{ subRefinement}$$

The interpretation of a `CONJUNCTIONREFINEMENTSET` is the intersection of the interpretation of the `SUBREFINEMENTS` .

$i_conjunctionRefinementSet :$ $Substrate \rightarrow conjunctionRefinementSet \rightarrow Sctids_or_Error$ <hr style="width: 100%;"/> $\forall ss : Substrate; conjset : conjunctionRefinementSet \bullet$ $i_conjunctionRefinementSet ss conjset =$ $\text{if } tail\ conjset = \langle \rangle \text{ then}$ $i_subRefinement ss (head\ conjset)$ else $intersect\ (i_subRefinement\ ss\ (head\ conjset))\ (i_conjunctionRefinementSet\ ss\ (tail\ conjset))$
--

3.3.2 disjunctionRefinementSet

$$\text{disjunctionRefinementSet} = 1^*(\text{ws disjunction ws subRefinement})$$

$$\text{disjunctionRefinementSet} == \text{seq}_1 \text{ subRefinement}$$

The interpretation of a `DISJUNCTIONREFINEMENTSET` is the union of the interpretation of the `SUBREFINEMENTS` .

$i_disjunctionRefinementSet :$ $Substrate \rightarrow disjunctionRefinementSet \rightarrow Sctids_or_Error$ <hr style="width: 100%;"/> $\forall ss : Substrate; disjset : disjunctionRefinementSet \bullet$ $i_disjunctionRefinementSet ss disjset =$ $\text{if } tail\ disjset = \langle \rangle \text{ then}$ $i_subRefinement ss (head\ disjset)$ else $union\ (i_subRefinement\ ss\ (head\ disjset))\ (i_disjunctionRefinementSet\ ss\ (tail\ disjset))$
--

3.3.3 subRefinement

A `subRefinement` is either an attribute set, an attribute group or a bracketed refinement.

$$\text{subRefinement} = \text{attributeSet} / \text{attributeGroup} / \text{"(" ws refinement ws "("}$$

The interpretation of a **SUBREFINEMENT** is the interpretation of the corresponding **ATTRIBUTESET** , **ATTRIBUTEGROUP** OR **REFINEMENT** .

```

subRefinement ::=
  subrefine_attset⟨⟨attributeSet⟩⟩ |
  subrefine_attgroup⟨⟨attributeGroup⟩⟩ |
  subrefine_refinement⟨⟨refinement'⟩⟩

```

<pre> i_subRefinement : Substrate → subRefinement → Sctids_or_Error ∀ ss : Substrate; subrefine : subRefinement • i_subRefinement ss subrefine = if subrefine ∈ ran subrefine_attset then i_attributeSet ss (subrefine_attset~subrefine) else if subrefine ∈ ran subrefine_attgroup then i_attributeGroup ss (subrefine_attgroup~subrefine) else i_refinement' ss (subrefine_refinement~subrefine) </pre>
--

3.4 attributeSet

An attribute set contains one or more attribute name-value pairs separated by a conjunction or disjunction operator. An attribute set may optionally be placed in brackets.

attributeSet = subAttributeSet ws [conjunctionAttributeSet / disjunctionAttributeSet]

The interpretation of an **ATTRIBUTESET** is one of:

- The intersection of the interpretation of the **SUBATTRIBUTESET** and the **CONJUNCTIONATTRIBUTESET**
- The union of the interpretation of the **SUBATTRIBUTESET** and the **DISJUNCTIONATTRIBUTESET**
- The interpretation of **SUBATTRIBUTESET** if neither conjunction nor disjunction set is present

```

attributeSet == subAttributeSet × conjunctionOrDisjunctionAttributeSet[0..1]
[attributeSet']

```

```

conjunctionOrDisjunctionAttributeSet ::=
  attset_conjattset⟨⟨conjunctionAttributeSet⟩⟩ |
  attset_disjattset⟨⟨disjunctionAttributeSet⟩⟩

```

$i_attributeSet :$ $Substrate \rightarrow attributeSet \rightarrow Sctids_or_Error$	
$\forall ss : Substrate; attset : attributeSet \bullet$ $i_attributeSet ss attset =$ $(let\ lhs == i_subAttributeSet ss (first\ attset); rhs == second\ attset \bullet$ if $lhs \in \text{ran } error \vee \#rhs = 0$ then lhs else if $head\ rhs \in \text{ran } attset_conjattset$ then $intersect\ lhs\ (i_conjunctionAttributeSet\ ss\ (attset_conjattset \sim (head\ rhs)))$ else $union\ lhs\ (i_disjunctionAttributeSet\ ss\ (attset_disjattset \sim (head\ rhs)))$	
$i_attributeSet' :$ $Substrate \rightarrow attributeSet' \rightarrow Sctids_or_Error$	

3.4.1 conjunctionAttributeSet

A conjunction attribute set consists of one or more conjunction operators, each followed by a subAttributeSet.

$conjunctionAttributeSet = 1^*(ws\ conjunction\ ws\ subAttributeSet)$

The interpretation of a `CONJUNCTIONATTRIBUTESET` is the intersection of the interpretations of its `SUBATTRIBUTESETS`.

$conjunctionAttributeSet == seq_1\ subAttributeSet$

$i_conjunctionAttributeSet :$ $Substrate \rightarrow conjunctionAttributeSet \rightarrow Sctids_or_Error$	
$\forall ss : Substrate; conjset : conjunctionAttributeSet \bullet$ $i_conjunctionAttributeSet ss conjset =$ if $tail\ conjset = \langle \rangle$ then $i_subAttributeSet ss (head\ conjset)$ else $intersect\ (i_subAttributeSet\ ss\ (head\ conjset))\ (i_conjunctionAttributeSet\ ss\ (tail\ conjset))$	

3.4.2 disjunctionAttributeSet

A disjunction attribute set consists of one or more disjunction operators, each followed by a subAttributeSet.

$$\text{disjunctionAttributeSet} = 1^*(\text{ws disjunction ws subAttributeSet})$$

The interpretation of a `DISJUNCTIONATTRIBUTESET` is the union of the interpretations of its `SUBATTRIBUTESETS` .

$$\text{disjunctionAttributeSet} == \text{seq}_1 \text{ subAttributeSet}$$

$i_disjunctionAttributeSet :$	$Substrate \rightarrow disjunctionAttributeSet \rightarrow Sctids_or_Error$
$\forall ss : Substrate; disjset : disjunctionAttributeSet \bullet$	$i_disjunctionAttributeSet ss disjset =$
$\text{if tail disjset} = \langle \rangle \text{ then}$	$i_subAttributeSet ss (\text{head disjset})$
else	$\text{union } (i_subAttributeSet ss (\text{head disjset})) (i_disjunctionAttributeSet ss (\text{tail disjset}))$

3.4.3 subAttributeSet

A `subAttributeSet` is either an attribute or a bracketed attribute set.

$$\text{subAttributeSet} = \text{attribute} / \text{"(" ws attributeSet ws "("}$$

The interpretation of a `SUBATTRIBUTESET` is either the interpretation of the `ATTRIBUTE` or the interpretation of the `ATTRIBUTESET` .

$$\begin{aligned} \text{subAttributeSet} ::= & \\ & \text{subaset_attribute} \langle \langle \text{attribute} \rangle \rangle \mid \\ & \text{subaset_attset} \langle \langle \text{attributeSet}' \rangle \rangle \end{aligned}$$

$i_subAttributeSet :$	$Substrate \rightarrow subAttributeSet \rightarrow Sctids_or_Error$
$\forall ss : Substrate; subaset : subAttributeSet \bullet$	$i_subAttributeSet ss subaset =$
$\text{if subaset} \in \text{ran subaset_attribute} \text{ then}$	$i_attribute ss (\text{subaset_attribute} \sim \text{subaset})$
else	$i_attributeSet' ss (\text{subaset_attset} \sim \text{subaset})$

3.5 attribute

An attribute is a name-value pair expressing a single refinement of the containing expression constraint. Either the attribute value must satisfy (or not) the given expression constraint, the attribute value is compared with a given numeric value (integer or decimal) using a numeric comparison operator, or the attribute value must be equal to (or not equal to) the given string value. The attribute may optionally be preceded by a cardinality constraint, a reverse flag and/or an attribute operator.

attribute = [cardinality ws] [reverseFlag ws] [attributeOperator ws] attributeName ws (expressionComparisonOperator ws expressionConstraintValue / numericComparisonOperator ws numericValue / stringComparisonOperator ws stringValue numericValue = “#” (decimalValue / integerValue)

Conceptually, the interpretation of `ATTRIBUTE` consists of the following steps:

1. Interpret `ATTRIBUTEName`
2. Interpret `ATTRIBUTEOperator` against the result of the previous step
3. Interpret one of:
 - The `EXPRESSIONCOMPARISONOperator` applied to the `REVERSEFLAG`, the interpretation of `ATTRIBUTEOperator` and `EXPRESSIONCONSTRAINTValue`.
 - The `NUMERICCOMPARISONOperator` applied to the interpretation of `ATTRIBUTEOperator` and `NUMERICValue`.
 - The `STRINGCOMPARISONOperator` applied to the interpretation of `ATTRIBUTEOperator` and `STRINGValue`.
4. Interpret `CARDINALITY` against the result of the previous step

Note that the `REVERSEFLAG` only applies in the case of `EXPRESSIONCOMPARISONOperator`.

. Not sure what to do if it is present in the other two cases...

The schema below provides names the various components of an attribute.

<i>attribute</i> <i>card</i> : <i>cardinality</i> [0 .. 1] <i>rf</i> : <i>reverseFlag</i> [0 .. 1] <i>attOper</i> : <i>constraintOperator</i> [0 .. 1] <i>name</i> : <i>attributeName</i> <i>opValue</i> : <i>attributeOperatorValue</i>

attributeOperatorValue ::=
attrib_expr⟨⟨*expressionComparisonOperator* × *expressionConstraintValue*⟩⟩ |
attrib_num⟨⟨*numericComparisonOperator* × *numericValue*⟩⟩ |
attrib_str⟨⟨*stringComparisonOperator* × *stringValue*⟩⟩
numericValue ::= *nv_decimal*⟨⟨*decimalValue*⟩⟩ | *nv_integer*⟨⟨ \mathbb{N} ⟩⟩
[*reverseFlag*]

```

i_attribute :
  Substrate → attribute → Sctids_or_Error
  -----
  ∀ ss : Substrate; att : attribute •
  i_attribute ss att =
  (let att_sctids == i_attributeOperator ss att.attOper (ss.i_attributeName att.name) •
  if att_sctids ∈ ran error then
    att_sctids

  else if att.opValue ∈ ran attrib_expr then
    (let expr_interp ==
    i_expressionComparisonOperator ss att.rf (result_sctids att_sctids) (attrib_expr ~ att.opValue) •
    i_att_cardinality ss att.card expr_interp)

  else if att.opValue ∈ ran attrib_num
  then
    (let num_interp ==
    i_numericComparisonOperator ss att.rf (result_sctids att_sctids) (attrib_num ~ att.opValue) •
    i_att_cardinality ss att.card num_interp)

  else
    (let str_interp ==
    i_stringComparisonOperator ss att.rf (result_sctids att_sctids) (attrib_str ~ att.opValue) •
    i_att_cardinality ss att.card str_interp)

```

3.5.1 attribute cardinality

The cardinality of an attribute represents a constraint on the minimum and maximum number of instances of the given attribute on each concept. The cardinality is enclosed in square brackets with the minimum cardinality appearing first, followed by two dots and then the maximum cardinality. The minimum cardinality must always be less than or equal to the maximum cardinality. A maximum cardinality of 'many' indicates that there is no limit on the number of times the attribute may appear on each concept.

cardinality = “[” nonNegativeIntegerValue to (nonNegativeIntegerValue / many) “]”

$unlimitedNat ::= num\langle\mathbb{N}\rangle \mid many$

$\frac{cardinality}{min : \mathbb{N}; max : unlimitedNat}$

The interpretation of *optional*[*cardinality*] in the context of an attribute is:

1. *error* if there is an error in the supplied list of quads

2. *i_required_cardinality* 1 many if the cardinality isn't supplied
3. *i_required_cardinality* cardinality if the minimum cardinality is > 0
4. *i_optional_cardinality* *ss* cardinality if the minimum cardinality = 0

$i_att_cardinality :$ $Substrate \rightarrow cardinality[0..1] \rightarrow Quads_or_Error \rightarrow Sctids_or_Error$	$\forall ss : Substrate; ocard : cardinality[0..1]; gore : Quads_or_Error \bullet$ $i_att_cardinality ss ocard gore =$ if $gore \in \text{ran } qerror$ then $\quad error(qerror \sim gore)$ else if $\#ocard = 0$ then $\quad ok(i_required_cardinality\ 1\ many\ gore)$ else (let $card == head\ ocard \bullet$ \quad if $card.min > 0$ then $\quad \quad ok(i_required_cardinality\ card.min\ card.max\ gore)$ \quad else $\quad \quad ok(i_optional_cardinality\ ss\ card.max\ gore))$
---	--

i_required_cardinality returns the set of subject or target sctids that meet the cardinality requirements

$i_required_cardinality :$ $\mathbb{N} \rightarrow unlimitedNat \rightarrow Quads_or_Error \rightarrow \mathbb{P}\ sctId$	$\forall min : \mathbb{N}; max : unlimitedNat; gore : Quads_or_Error \bullet$ $i_required_cardinality min max gore =$ if $quad_direction\ gore = source_direction$ then $\quad \{subj : \{q : quads_for\ gore \bullet q.s\} \mid$ $\quad \quad evalCardinality[Quad]\ min\ max\ \{q : quads_for\ gore \mid q.s = subj\} \neq \emptyset\}$ else $\quad \{targ : \{q : quads_for\ gore \bullet t_sctid \sim q.t\} \mid$ $\quad \quad evalCardinality[Quad]\ min\ max\ \{q : quads_for\ gore \mid (t_sctid \sim q.t) = targ\} \neq \emptyset\}$
--	--

i_optional_cardinality returns all of the concepts in the substrate that don't fail the cardinality test. The failing concepts are equivalent to the set of all subject (or target) concepts in the set of quads minus the set of concepts that pass.

$i_optional_cardinality :$ $Substrate \rightarrow unlimitedNat \rightarrow Quads_or_Error \rightarrow \mathbb{P}\ sctId$	$\forall ss : Substrate; max : unlimitedNat; gore : Quads_or_Error \bullet$ $i_optional_cardinality ss max gore =$ if $quad_direction\ gore = source_direction$ then $\quad ss.concepts \setminus (\{q : quads_for\ gore \bullet q.s\} \setminus (i_required_cardinality\ 0\ max\ gore))$ else $\quad ss.concepts \setminus (\{q : quads_for\ gore \bullet t_sctid \sim q.t\} \setminus (i_required_cardinality\ 0\ max\ gore))$
---	---

3.5.2 expressionConstraintValue

An expression constraint value is either a simple expression constraint, or is a refined or compound expression constraint enclosed in brackets.

expressionConstraintValue = simpleExpressionConstraint / "(" ws (refinedExpressionConstraint / compoundExpressionConstraint) ws ")"

The interpretation of `EXPRESSIONCONSTRAINTVALUE` is the interpretation of the corresponding `SIMPLEEXPRESSIONCONSTRAINT`, `REFINEDEXPRESSIONCONSTRAINT` or `COMPOUNDEXPRESSIONCONSTRAINT`

expressionConstraintValue ::=
expression_simple⟨⟨*simpleExpressionConstraint*⟩⟩ |
expression_refined⟨⟨*refinedExpressionConstraint'*⟩⟩ |
expression_compound⟨⟨*compoundExpressionConstraint*⟩⟩

i_expressionConstraintValue :
Substrate → *expressionConstraintValue* → *Sctids_or_Error*

∀ *ss* : *Substrate*; *ecv* : *expressionConstraintValue* •
i_expressionConstraintValue ss ecv =
if *ecv* ∈ ran *expression_simple* **then**
 i_simpleExpressionConstraint ss (expression_simple ~ ecv)
else if *ecv* ∈ ran *expression_refined* **then**
 i_refinedExpressionConstraint' ss (expression_refined ~ ecv)
else
 i_compoundExpressionConstraint ss (expression_compound ~ ecv)

3.6 attributeGroup

An attribute group contains a collection of attributes that operate together as part of the refinement of the containing expression constraint. An attribute group may optionally be preceded by a cardinality. An attribute group cardinality indicates the minimum and maximum number of attribute groups that must satisfy the given attributeSet constraint for the expression constraint to be satisfied.

attributeGroup = [cardinality ws] "{" ws attributeSet ws "}"

attributeGroup == *cardinality*[0 .. 1] × *attributeSet*

The difference between the interpretation of an `ATTRIBUTEGROUP` and an `ATTRIBUTESET` is that, within an `ATTRIBUTEGROUP` all of the qualifying `SCIDS` must belong to the same group ¹. Outside, a `SCID` would qualify an conjunction if one group passed the first match and a different one the second. *Within* a group, however, conjunctions and disjunctions only count if they both apply in the same group. This means that we have to re-interpret the meaning of `ATTRIBUTESET` on down in terms of *IDGroups* rather than *Quads*

The interpretation of an attribute group is the application of a cardinality constraint to all of the *sctIds* that pass the grouped attribute set.

$$\begin{array}{|l}
 i_attributeGroup : Substrate \rightarrow attributeGroup \rightarrow Sctids_or_Error \\
 \hline
 \forall ss : Substrate; ag : attributeGroup \bullet \\
 i_attributeGroup ss ag = \\
 \quad i_group_cardinality ss (first ag) (i_groupAttributeSet ss (second ag))
 \end{array}$$

3.6.1 attribute group cardinality

The interpretation of `CARDINALITY` within the context of an attribute group is as follows:

- If an error has been encountered anywhere in the process, it should be propagated.
- If the cardinality is omitted, it is interpreted as `[1 .. *]`
- If the minimum cardinality is `>0` then return the set of *sctids* with that appear at least *min* and at most *max* groups
- If the minimum cardinality is `0` then return every *sctid* that doesn't fail the max cardinality

$$\begin{array}{|l}
 i_group_cardinality : \\
 \quad Substrate \rightarrow cardinality[0 .. 1] \rightarrow sctIdGroups_or_Error \rightarrow Sctids_or_Error \\
 \hline
 \forall ss : Substrate; ocard : cardinality[0 .. 1]; idg : sctIdGroups_or_Error \bullet \\
 i_group_cardinality ss ocard idg = \\
 \quad \text{if } idg \in \text{ran } gerror \text{ then} \\
 \quad \quad error(gerror \sim idg) \\
 \quad \text{else (let } gvals == group_value \sim idg \bullet \\
 \quad \quad \text{if } \#ocard = 0 \text{ then} \\
 \quad \quad \quad ok(i_required_group_cardinality 1 many gvals) \\
 \quad \quad \text{else (let } card == head ocard \bullet \\
 \quad \quad \quad \text{if } card.min > 0 \text{ then} \\
 \quad \quad \quad \quad ok(i_required_group_cardinality card.min card.max gvals) \\
 \quad \quad \quad \text{else} \\
 \quad \quad \quad \quad ok(i_optional_group_cardinality ss card.max gvals)))
 \end{array}$$

¹The zero group is a special case, where each quad in a zero group is viewed as its own unique group

i_required_group_cardinality returns the set *sctids* that pass in at least *min* groups and at most *max* groups

$$\begin{array}{|l}
i_required_group_cardinality : \\
\mathbb{N} \rightarrow unlimitedNat \rightarrow \mathbb{P} sctIdGroup \rightarrow \mathbb{P} sctId \\
\hline
\forall min : \mathbb{N}; max : unlimitedNat; groups : \mathbb{P} sctIdGroup \bullet \\
i_required_group_cardinality min max groups = \\
\{sctid : \{s : groups \bullet first s\} \mid \\
evalCardinality[sctIdGroup] min max \{g : groups \mid first g = sctid\} \neq \emptyset\}
\end{array}$$

i_optional_group_cardinality returns the set of *sctids* that pass in at most *max* groups, which is equivalent to the set of all possible *sctIds* (i.e. *ss.concepts*) minus the set of *sctids* that pass in more than *max* groups.

$$\begin{array}{|l}
i_optional_group_cardinality : \\
Substrate \rightarrow unlimitedNat \rightarrow \mathbb{P} sctIdGroup \rightarrow \mathbb{P} sctId \\
\hline
\forall ss : Substrate; max : unlimitedNat; groups : \mathbb{P} sctIdGroup \bullet \\
i_optional_group_cardinality ss max groups = \\
ss.concepts \setminus \\
(\{s : groups \bullet first s\} \setminus (i_required_group_cardinality 0 max groups))
\end{array}$$

3.6.2 attributeSet inside a group

The interpretation of an **ATTRIBUTESET** a group is the interpretation of the **SUBATTRIBUTESET** optionally intersected or union with additional conjunction or disjunction attribute sets.

$$\begin{array}{|l}
i_groupAttributeSet : \\
Substrate \rightarrow attributeSet \rightarrow sctIdGroups_or_Error \\
\hline
\forall ss : Substrate; attset : attributeSet \bullet \\
i_groupAttributeSet ss attset = \\
(\text{let } lhs == i_groupSubAttributeSet ss (first attset); rhs == second attset \bullet \\
\text{if } \#rhs = 0 \text{ then} \\
\quad lhs \\
\text{else if } head\ rhs \in \text{ran } attset_conjattset \text{ then} \\
\quad gintersect\ lhs\ (i_groupConjunctionAttributeSet\ ss\ (attset_conjattset \sim (head\ rhs))) \\
\text{else} \\
\quad gunion\ lhs\ (i_groupDisjunctionAttributeSet\ ss\ (attset_disjattset \sim (head\ rhs)))) \\
\hline
i_groupAttributeSet' : \\
Substrate \rightarrow attributeSet' \rightarrow sctIdGroups_or_Error
\end{array}$$

3.6.3 conjunctionAttributeSet inside a group

Apply the intersect operator to the interpretation of each subAttributeSet in a group context. This is the same as the non-grouped *subAttributeSet* with the

exception that the inputs to and result of the function is *sctId* / *groupId* tuples rather than

$i_groupConjunctionAttributeSet :$	$Substrate \rightarrow conjunctionAttributeSet \rightarrow sctIdGroups_or_Error$
$\forall ss : Substrate; conjset : conjunctionAttributeSet \bullet$	$i_groupConjunctionAttributeSet ss conjset =$
$\text{if } tail\ conjset = \langle \rangle \text{ then}$	$i_groupSubAttributeSet ss (head\ conjset)$
else	$gintersect (i_groupSubAttributeSet ss (head\ conjset)) (i_groupConjunctionAttributeSet ss (tail\ conjset))$

3.6.4 disjunctionAttributeSet inside a group

Apply the union operator to the interpretation of each subAttributeSet in a group context. This is the same as the non-grouped *subAttributeSet* with the exception that the inputs to and result of the function is *sctId* / *groupId* tuples rather than just *sctIds*.

$i_groupDisjunctionAttributeSet :$	$Substrate \rightarrow disjunctionAttributeSet \rightarrow sctIdGroups_or_Error$
$\forall ss : Substrate; disjset : disjunctionAttributeSet \bullet$	$i_groupConjunctionAttributeSet ss disjset =$
$\text{if } tail\ disjset = \langle \rangle \text{ then}$	$i_groupSubAttributeSet ss (head\ disjset)$
else	$gunion (i_groupSubAttributeSet ss (head\ disjset)) (i_groupConjunctionAttributeSet ss (tail\ disjset))$

3.6.5 subAttributeSet inside a group

This is the same as the non-grouped *subAttributeSet* with the exception that the result of the function is *sctId* / *groupId* tuples rather than just *sctIds*.

$i_groupSubAttributeSet :$	$Substrate \rightarrow subAttributeSet \rightarrow sctIdGroups_or_Error$
$\forall ss : Substrate; subaset : subAttributeSet \bullet$	$i_groupSubAttributeSet ss subaset =$
$\text{if } subaset \in \text{ran } subaset_attribute$	$\text{then } i_groupAttribute ss (subaset_attribute \sim subaset)$
$\text{else } i_groupAttributeSet' ss (subaset_attset \sim subaset)$	

3.6.6 attribute inside a group

The interpretation of an attribute within the context of a group is the set of *sctIds* and the group or groups in which they were valid. It is possible for the same *sctId* to be valid in the interpretation of a given attribute for more than one

group. The union and intersection operators operate on the *sctId* / *groupId* tuples (ne. *sctIdGroup*).

$i_groupAttribute :$ $Substrate \rightarrow attribute \rightarrow sctIdGroups_or_Error$	$\forall ss : Substrate; att : attribute \bullet$ $i_groupAttribute\ ss\ att =$ $(\text{let } att_sctids == i_attributeOperator\ ss\ att.attOper\ (ss.i_attributeName\ att.name) \bullet$ $\text{if } att_sctids \in \text{ran } error$ $\text{then } gerror(error \sim att_sctids)$ $\text{else if } att.opValue \in \text{ran } attrib_expr \text{ then}$ $(\text{let } expr_interp ==$ $i_expressionComparisonOperator\ ss\ att.rf\ (result_sctids\ att_sctids)\ (attrib_expr \sim att.opValue) \bullet$ $i_att_group_cardinality\ ss\ att.card\ expr_interp)$ $\text{else if } att.opValue \in \text{ran } attrib_num \text{ then}$ $(\text{let } num_interp ==$ $i_numericComparisonOperator\ ss\ att.rf\ (result_sctids\ att_sctids)\ (attrib_num \sim att.opValue) \bullet$ $i_att_group_cardinality\ ss\ att.card\ num_interp)$ else $(\text{let } str_interp ==$ $i_stringComparisonOperator\ ss\ att.rf\ (result_sctids\ att_sctids)\ (attrib_str \sim att.opValue) \bullet$ $i_att_group_cardinality\ ss\ att.card\ str_interp))$
--	--

3.6.7 cardinality inside a group

Cardinality is interpreted differently within the context of a group. In particular:

- Zero groups are treated as “singletons” – every quad with a zero group is treated as a group unto itself.
- A sctid can pass in the context of more than one group. Intersections, unions, etc. apply to sctid’s in the *same* group.

i_att_group_cardinality takes an optional cardinality and a set of quads and returns a list sctids and the corresponding groups in which they passed the cardinality constraint. Note that, in the case of the zero group, the actual *Quad* that passed is associated with the sctid rather than the group identifier itself.

```

i_att_group_cardinality :
  Substrate → cardinality[0 .. 1] → Quads_or_Error → sctIdGroups_or_Error

∀ ss : Substrate; ocard : cardinality[0 .. 1]; qore : Quads_or_Error •
i_att_group_cardinality ss ocard qore =
if qore ∈ ran qerror then
  qerror(qerror~ qore)

else if #ocard = 0 then
  group_value (i_required_att_group_cardinality 1 many qore)

else (let card == head ocard •
  if card.min > 0 then
    group_value (i_required_att_group_cardinality card.min card.max qore)

  else
    group_value (i_optional_att_group_cardinality ss card.max qore))

```

i_required_att_group_cardinality returns the set of subject or target sctids combined with the group(s) (or *Quads*) in which they qualified

```

i_required_att_group_cardinality :
  ℕ → unlimitedNat → Quads_or_Error → ℙ sctIdGroup

∀ min : ℕ; max : unlimitedNat; qore : Quads_or_Error •
i_required_att_group_cardinality min max qore =
if quad_direction qore = source_direction then
  { subjgroup : { q : quads_for qore | q.g ≠ zero_group • (q.s, quadGroup q) } |
    evalCardinality[Quad] min max { q : quads_for qore |
      q.s = first subjgroup ∧ q.g = (ug~(second subjgroup)) } ≠ ∅ }
  ∪
  { q : quads_for qore | q.g = zero_group ∧
    evalCardinality[Quad] min max { q } ≠ ∅ • (q.s, quadGroup q) }

else
  { targgroup : { q : quads_for qore |
    q.g ≠ zero_group ∧ q.t ∈ ran t_sctid • (t_sctid~ q.t, quadGroup q) } |
    evalCardinality[Quad] min max { q : quads_for qore |
      t_sctid~ q.t = first targgroup ∧
      q.g = (ug~(second targgroup)) } ≠ ∅ }
  ∪
  { q : quads_for qore | q.g = zero_group ∧
    evalCardinality[Quad] min max { q } ≠ ∅ • (t_sctid~ q.t, quadGroup q) }

```

i_optional_att_group_cardinality returns the set of subject or target sctids along with all of the corresponding groups that *fail* the cardinality test. The failing list is equivalent to the set of all possible sctid / non-zero groups in the substrate minus the set of sctid / non-zero groups that fail the maximum cardinality. The set of failures is determined by removing the set of passing tuples from the total possible set in the supplied quads.

$ \begin{aligned} & i_optional_att_group_cardinality : \\ & \quad Substrate \rightarrow unlimitedNat \rightarrow Quads_or_Error \rightarrow \mathbb{P} \, sctIdGroup \\ & \forall ss : Substrate; max : unlimitedNat; gore : Quads_or_Error \bullet \\ & i_optional_att_group_cardinality \, ss \, max \, gore = \\ & \text{if } quad_direction \, gore = source_direction \text{ then} \\ & \quad (\{rel : ss.relationships \bullet (rel.s, quadGroup \, rel)\} \setminus \\ & \quad \{q : quads_for \, gore \bullet (q.s, quadGroup \, q)\}) \cup \\ & \quad \quad i_required_att_group_cardinality \, 0 \, max \, gore \\ & \text{else} \\ & \quad (\{rel : ss.relationships \mid rel.t \in \text{ran } t_sctid \bullet (t_sctid \sim rel.t, quadGroup \, rel)\} \setminus \\ & \quad \{q : quads_for \, gore \mid q.g \neq zero_group \bullet (t_sctid \sim q.t, quadGroup \, q)\}) \cup \\ & \quad \quad i_required_att_group_cardinality \, 0 \, max \, gore \end{aligned} $
--

4 Substrate Interpretations

This section defines the interpretations that are realized against the substrate.

4.1 constraintOperator

A constraint operator is either 'descendantOrSelfOf', 'descendantOf', 'ancestorOrSelfOf', or 'ancestorOf'.

constraintOperator = descendantOrSelfOf / descendantOf / ancestorOrSelfOf / ancestorOf

The interpretation of *constraintOperator*[0 .. 1] is one of:

- The input if an operator isn't supplied or if it contains an error
- The union of descendants of all members of the input, as provided by the substrate if the operator is *descendantOf*
- The union of descendants plus the input if the operator is *descendantOrSelfOf*
- The union of ancestors of all members of the input, as provided by the substrate if the operator is *ancestorOf*
- The union of ancestors plus the input if the operator is *ancestorsOrSelfOf*

Note that it is possible for some or all of the members of the input to be included in the *descendantOf* or *ancestorOf* operations as the descendants of one input sctid may include another input sctid.

The *completeFun* function assures that an operation on any identifier that isn't in the domain of the substrate *descendants* or *ancestors* function returns the empty set (\emptyset).

$$\begin{aligned}
constraintOperator ::= & \\
& descendantOrSelfOf \mid descendantOf \mid ancestorOrSelfOf \mid ancestorOf
\end{aligned}$$

```

i_constraintOperator :
  Substrate → constraintOperator[0 .. 1] → Sctids_or_Error → Sctids_or_Error

completeFun : (sctId →  $\mathbb{P}$  sctId) → sctId →  $\mathbb{P}$  sctId

 $\forall ss : \text{Substrate}; oco : \text{constraintOperator}[0 .. 1]; input : \text{Sctids\_or\_Error} \bullet$ 
i_constraintOperator ss oco input =
  if input ∈ ran error ∨ #oco = 0 then
    input
  else if head oco = descendantOrSelfOf then
    ok( $\bigcup \{id : \text{result\_sctids } input \bullet$ 
      completeFun ss.descendants id $\} \cup \text{result\_sctids } input$ )
  else if head oco = descendantOf then
    ok( $\bigcup \{id : \text{result\_sctids } input \bullet$ 
      completeFun ss.descendants id $\}$ )
  else if head oco = ancestorOrSelfOf then
    ok( $\bigcup \{id : \text{result\_sctids } input \bullet$ 
      completeFun ss.ancestors id $\} \cup \text{result\_sctids } input$ )
  else
    ok( $\bigcup \{id : \text{result\_sctids } input$ 
      • completeFun ss.ancestors id $\}$ )

 $\forall f : (sctId \rightarrow \mathbb{P} sctId); id : sctId \bullet \text{completeFun } f \text{ id} =$ 
  if id ∈ dom f then f id else  $\emptyset$ 

```

4.2 attributeOperator

An attribute operator indicates that instead of just matching the named attribute with the given attribute value, any descendants of or any descendants or self of the named attribute may match the given attribute value.

attributeOperator = descendantOrSelfOf / descendantOf

The interpretation of *attributeOperator* is the same as the interpretation of *constraintOperator*

attributeOperator == *constraintOperator*
i_attributeOperator == *i_constraintOperator*

4.3 expressionComparisonOperator

Attributes whose value is a concept may be compared to an expression constraint using either equals (“=”) or not equals (“!=”). In the full syntax “<>” and “not =” (case insensitive) are also valid ways to represent not equals.

expressionComparisonOperator = “=” / “!=”

The interpretation of **EXPRESSIONCOMPARISONOPERATOR** is the set of all quads in the substrate having a target (or subject if the direction is reverse) that are (or are not) in the interpretation of the **EXPRESSIONCONSTRAINTVALUE**

expressionComparisonOperator ::= eco_eq | eco_neq

QUESTION: does neq include attribute or not?

```

i_expressionComparisonOperator :
  Substrate → reverseFlag[0..1] →  $\mathbb{P}$  sctId →
  (expressionComparisonOperator × expressionConstraintValue) → Quads_or_Error

∀ ss : Substrate; rf : reverseFlag[0..1]; atts :  $\mathbb{P}$  sctId;
  ec : (expressionComparisonOperator × expressionConstraintValue) •
  i_expressionComparisonOperator ss rf atts ec =
  (let ecv == i_expressionConstraintValue ss (second ec) •
    if ecv ∈ ran error then
      qerror(error~ecv)
    else if #rf = 0 ∧ (first ec = eco_eq) then
      quad_value({q : ss.relationships |
        q.a ∈ atts ∧ q.t ∈ ran t_sctid ∧ (t_sctid~q.t) ∈ (result_sctids ecv)}, source_direction)
    else if #rf > 0 ∧ (first ec = eco_eq) then
      quad_value({q : ss.relationships |
        q.a ∈ atts ∧ q.s ∈ (result_sctids ecv)}, targets_direction)
    else if #rf = 0 ∧ (first ec = eco_neq) then
      quad_value({q : ss.relationships |
        q.a ∈ atts ∧ q.t ∈ ran t_sctid ∧ (t_sctid~q.t) ∉ (result_sctids ecv)}, source_direction)
    else
      quad_value({q : ss.relationships |
        q.a ∈ atts ∧ q.s ∉ (result_sctids ecv)}, targets_direction))

```

4.4 numericComparisonOperator

Attributes whose value is numeric (i.e. integer or decimal) may be compared to a specific concrete value using a variety of comparison operators, including equals (“=”), less than (“<”), less than or equals (“<=”), greater than (“>”), greater than or equals (“>=”) and not equals (“!=”). In the full syntax “<>” and “not =” (case insensitive) are also valid ways to represent not equals.

numericComparisonOperator = “=” / “!=” / “<=” / “<” / “>=” / “>”

The interpretation of `NUMERICCOMPARISONOPERATOR` is the set of all quads in the substrate having a *concreteValue* target of type *cv_integer* or *cv_decimal* that (or do not meet) the supplied criteria. As concrete values cannot occur as subjects, this function returns the empty set if the reverse flag is supplied

$numericComparisonOperator ::= nco_eq \mid nco_neq \mid nco_gt \mid nco_ge \mid nco_lt \mid nco_le$

$ \begin{aligned} &i_numericComparisonOperator : \\ &\quad Substrate \rightarrow reverseFlag[0..1] \rightarrow \mathbb{P} \text{ sctId} \rightarrow \\ &\quad (numericComparisonOperator \times numericValue) \rightarrow Quads_or_Error \\ \hline &\forall ss : Substrate; rf : reverseFlag[0..1]; atts : \mathbb{P} \text{ sctId}; \\ &\quad ncv : (numericComparisonOperator \times numericValue) \bullet \\ &i_numericComparisonOperator ss rf atts ncv = \\ &\text{if } \#rf > 0 \text{ then} \\ &\quad quad_value(\emptyset, targets_direction) \\ &\text{else} \\ &\quad quad_value(\{q : ss.relationships \mid q.a \in atts \wedge q.t \in \text{ran } t_concrete \wedge \\ &\quad numericComparison(t_concrete \sim q.t) (first\ ncv) = (second\ ncv)\}, source_direction) \end{aligned} $
--

Numeric comparison function. Compares a *concreteValue* with a *numericValue*.

TODO: Need to specify the rules for comparing integers and decimal numbers as well as decimal/decimal. As an example, does “5.00” = “5.0”

$ \begin{aligned} &numericComparison : \\ &\quad concreteValue \rightarrow numericComparisonOperator \rightarrow numericValue \end{aligned} $
--

4.5 stringComparisonOperator

Attributes whose value is numeric may be compared to an expression constraint using either equals (“=”) or not equals (“!=”). In the full syntax “ \neq ” and “not =” (case insensitive) are also valid ways to represent not equals.

$stringComparisonOperator = "=" \mid "!="$

$stringComparisonOperator ::= sco_eq \mid sco_neq$

$ \begin{aligned} & i_stringComparisonOperator : \\ & \quad Substrate \rightarrow reverseFlag[0..1] \rightarrow \mathbb{P} \text{ sctId} \rightarrow \\ & \quad (stringComparisonOperator \times stringValue) \rightarrow Quads_or_Error \\ & \forall ss : Substrate; rf : reverseFlag[0..1]; atts : \mathbb{P} \text{ sctId}; \\ & \quad scv : (stringComparisonOperator \times stringValue) \bullet \\ & i_stringComparisonOperator ss rf atts scv = \\ & \text{if } \#rf > 0 \text{ then} \\ & \quad quad_value(\emptyset, targets_direction) \\ & \text{else} \\ & \quad quad_value(\{q : ss.relationships \mid q.a \in atts \wedge q.t \in \text{ran } t_concrete \wedge \\ & \quad stringComparison(t_concrete \sim q.t) (first\ scv) = (second\ scv)\}, source_direction) \end{aligned} $	
--	--

String comparison function. Compares a *concreteValue* with a *stringValue*.

$ \begin{aligned} & stringComparison : \\ & \quad concreteValue \rightarrow stringComparisonOperator \rightarrow stringValue \end{aligned} $	
--	--

4.6 focusConcept

A *focus concept* is a *concept reference* or *wild card*, which is optionally preceded by a *member of function*. A *memberOf* function should be used only when the *conceptReference* refers to a *reference set concept*, or a *wild card* is used. A *wild card* represents any concept in the given substrate. In the brief syntax, a *wildcard* is represented using the “*” symbol. In the full syntax, the text “ANY” (case insensitive) is also allowed.

$$\text{focusConcept} = [\text{memberOf ws}] (\text{conceptReference} / \text{wildCard})$$

$$\begin{aligned}
crOrWildCard & ::= cr \langle\langle \text{conceptReference} \rangle\rangle \mid wc \\
focusConcept & ::= \\
& \quad focusConcept_m \langle\langle crOrWildCard \rangle\rangle \mid \\
& \quad focusConcept_c \langle\langle crOrWildCard \rangle\rangle
\end{aligned}$$

Interpretation: If `MEMBEROF` is present the interpretation of `FOCUSCONCEPT` is union the interpretation of `MEMBEROF` applied to each element in the interpretation of `CONCEPTREFERENCE`. Note that the interpretation of `WILDCARD` is different in the context of `MEMBEROF` than it is outside. `MEMBEROF WILDCARD` is interpreted as the interpretation of the domain of the *refset* function (i.e. all of the refsets in the substrate), where, without, it is interpreted as *all* of the concepts in the system.

If `MEMBEROF` isn’t specified, the interpretation is the substrate interpretations of `CONCEPTREFERENCE` itself

$$i_focusConcept : Substrate \rightarrow focusConcept \rightarrow Sctids_or_Error$$

```

∀  $ss : Substrate; fc : focusConcept \bullet$ 
 $i\_focusConcept\ ss\ fc =$ 
if  $focusConcept\_c \sim fc \in crOrWildCard$  then
  if  $cr \sim (focusConcept\_c \sim fc) \in conceptReference$  then
     $ss.i\_conceptReference\ (cr \sim (focusConcept\_c \sim fc))$ 
  else
     $ok\ ss.concepts$ 
else
   $i\_memberOf\ ss\ (focusConcept\_m \sim fc)$ 

```

4.6.1 memberOf

MEMBEROF returns the union of the application of the substrate *refset* function to a wild card or the the supplied reference set identifiers. An error is returned if (a) *refsetids* already has an error or (b) the substrate interpretation of a given *refsetId* returns an error.

$$i_memberOf : Substrate \rightarrow crOrWildCard \rightarrow Sctids_or_Error$$

```

∀  $ss : Substrate; crorc : crOrWildCard \bullet$ 
 $i\_memberOf\ ss\ crorc =$ 
(let  $refsetids ==$ 
  if  $cr \sim crorc \in conceptReference$ 
  then
     $ss.i\_conceptReference\ (cr \sim crorc)$ 
  else
     $ok\ (ss.descendants\ refset\_concept) \bullet$ 
if  $refsetids \in \text{ran error}$ 
  then
     $refsetids$ 
  else
     $bigunion\ \{sctid : result\_sctids\ refsetids \bullet ss.i\_refsetId\ sctid\}$ 
)

```

5 Types

This section carries various type transformations and error checking functions

5.1 Types

5.1.1 Quads_or_Error

Quads_or_Error is a collection of *Quads* or an error condition. If it is a collection of *Quads*, it also carries a direction indicator that determines whether it is the source or targets that carry the matching elements. Note:

- *source_direction* – matches were performed on attribute/target. Return the source *sctIds*.
- *targets_direction* – matches were performed on attribute/source (**REVERSEFLAG** was present). Return the target *sctIds* or, eventually, concrete values

$$\begin{aligned} direction &::= source_direction \mid targets_direction \\ Quads_or_Error &::= quad_value\langle\mathbb{P} Quad \times direction\rangle \mid qerror\langle ERROR \rangle \end{aligned}$$

5.1.2 sctIdGroups

The *groupId* identifies a set of one or more quads associated with the same subject. The zero group, however, is treated differently, where each individual quad is treated as a group unto itself. To implement this, we have to assign some sort of reproducible secondary "group" identifier to zero groups. While actual implementations will probably use a secondary identifier such as the RF2 relationship *sctid*, for the purposes of this specification we use the quad itself as the unique identity of the "group". This is represented by the *uniqueGroupId* construct below:

$$uniqueGroupId ::= ug\langle groupId \rangle \mid zg\langle Quad \rangle$$

The *sctidGroup* represents a subject or target *sctids* that meet the requirements of an attribute or attribute set within the context of a group. Each *sctidGroup* entry carries with the *sctid* that passed along with the group identifier of the group in which it passed, or the whole Quad in the case of zero groups of the specific group in which it passed. The *sctidGroup* construct is used for unions, intersections and differences within the context of a specific group.

sctIdGroups_or_Error represents a set of *sctIdGroups* or an error condition

$$\begin{aligned} sctIdGroup &== sctId \times uniqueGroupId \\ sctIdGroups_or_Error &::= group_value\langle\mathbb{P} sctIdGroup\rangle \mid \\ &\quad gerror\langle ERROR \rangle \end{aligned}$$

The *quadGroup* function converts a *Quad* to a corresponding *uniqueGroupId*, where the result is the *groupId* of the quad if the group is non-zero, and the quad itself when the group is zero.

$quadGroup : Quad \rightarrow uniqueGroupId$
$\forall q : Quad \bullet quadGroup\ q =$ $\quad \text{if } q.g \neq zero_group \text{ then}$ $\quad \quad ug\ q.g$ $\quad \text{else}$ $\quad \quad zg\ q$

5.2 Result transformations

- **result_sctids** – the set of *sctIds* in *Sctids_or_Error* or the empty set if there is an error
- **quads_for** – the set of quads in a *Quads_or_Error* or an empty set if there is an error
- **quad_direction** – the direction of a *Quads_or_Error* result. Undefined if error

$result_sctids : Sctids_or_Error \rightarrow \mathbb{P}\ sctId$
$quads_for : Quads_or_Error \rightarrow \mathbb{P}\ Quad$
$quad_direction : Quads_or_Error \rightarrow direction$
$\forall r : Sctids_or_Error \bullet result_sctids\ r =$ $\quad \text{if } r \in \text{ran error} \text{ then } \emptyset$ $\quad \text{else } ok \sim r$
$\forall q : Quads_or_Error \bullet quads_for\ q =$ $\quad \text{if } q \in \text{ran } qerror \text{ then } \emptyset$ $\quad \text{else } first\ (quad_value \sim q)$
$\forall q : Quads_or_Error \bullet quad_direction\ q =$ $\quad second\ (quad_value \sim q)$

Definition of the various functions that are performed on the result type.

- **firstError** – aggregate one or more *Sctids_or_Error* types, at least one of which carries and error and merge them into a single *Sctid_or_Error* instance propagating at least one of the errors (Not fully defined)
- **gfirstError** – convert two *sctIdGroups_or_Error* into a single *sctIdGroups_or_Error* propagating at least one of the errors.
- **union** – return the union of two *Sctids_or_Error* types, propagating errors if they exist, else returning the union of the sctId sets.
- **intersect** – return the intersection of two *Sctids_or_Error* types, propagating errors if they exist, else returning the intersection of the sctId sets.
- **minus** – return the difference of one *Sctids_or_Error* type and a second, propagating errors if they exist, else returning the set of sctId's in the first set that aren't in the second.

- **bigunion** – return the union of a set of *Sctids_or_Error* types, propagating errors if they exist, else returning the union of all of the *sctId* sets.
- **bigintersect** – return the intersection a set of *Sctids_or_Error* types, propagating errors if they exist, else returning the intersection of all of the *sctId* sets.

```

firstError :  $\mathbb{P}$  Sctids_or_Error  $\rightarrow$  Sctids_or_Error
gfirstError :  $\mathbb{P}$  sctIdGroups_or_Error  $\rightarrow$  sctIdGroups_or_Error

union, intersect, minus : Sctids_or_Error  $\rightarrow$  Sctids_or_Error  $\rightarrow$ 
    Sctids_or_Error
bigunion, bigintersect :  $\mathbb{P}$  Sctids_or_Error  $\rightarrow$  Sctids_or_Error

gunion, gintersect, gminus :
    sctIdGroups_or_Error  $\rightarrow$  sctIdGroups_or_Error  $\rightarrow$  sctIdGroups_or_Error

 $\forall x, y : \text{Sctids\_or\_Error} \bullet \text{union } x \ y =$ 
    if  $x \in \text{ran error} \vee y \in \text{ran error}$  then firstError  $\{x, y\}$ 
    else ok  $((\text{ok} \sim x) \cup (\text{ok} \sim y))$ 

 $\forall x, y : \text{Sctids\_or\_Error} \bullet \text{intersect } x \ y =$ 
    if  $x \in \text{ran error} \vee y \in \text{ran error}$  then firstError  $\{x, y\}$ 
    else ok  $((\text{ok} \sim x) \cap (\text{ok} \sim y))$ 

 $\forall x, y : \text{Sctids\_or\_Error} \bullet \text{minus } x \ y =$ 
    if  $x \in \text{ran error} \vee y \in \text{ran error}$  then firstError  $\{x, y\}$ 
    else ok  $((\text{ok} \sim x) \setminus (\text{ok} \sim y))$ 

 $\forall rs : \mathbb{P} \text{Sctids\_or\_Error} \bullet \text{bigunion } rs =$ 
    if  $\exists r : rs \bullet r \in \text{ran error}$  then firstError  $rs$ 
    else ok  $(\bigcup \{r : rs \bullet \text{result\_sctids } r\})$ 

 $\forall rs : \mathbb{P} \text{Sctids\_or\_Error} \bullet \text{bigintersect } rs =$ 
    if  $\exists r : rs \bullet r \in \text{ran error}$  then firstError  $rs$ 
    else ok  $(\bigcap \{r : rs \bullet \text{result\_sctids } r\})$ 

 $\forall x, y : \text{sctIdGroups\_or\_Error} \bullet \text{gintersect } x \ y =$ 
    if  $x \in \text{ran gerror} \vee y \in \text{ran gerror}$  then gfirstError  $\{x, y\}$ 
    else group_value  $((\text{group\_value} \sim x) \cap (\text{group\_value} \sim y))$ 

 $\forall x, y : \text{sctIdGroups\_or\_Error} \bullet \text{gunion } x \ y =$ 
    if  $x \in \text{ran gerror} \vee y \in \text{ran gerror}$  then gfirstError  $\{x, y\}$ 
    else group_value  $((\text{group\_value} \sim x) \cup (\text{group\_value} \sim y))$ 

 $\forall x, y : \text{sctIdGroups\_or\_Error} \bullet \text{gminus } x \ y =$ 
    if  $x \in \text{ran gerror} \vee y \in \text{ran gerror}$  then gfirstError  $\{x, y\}$ 
    else group_value  $((\text{group\_value} \sim x) \setminus (\text{group\_value} \sim y))$ 

```

A Optional elements

Representing optional elements of type T . Representing it as a sequence allows us to determine absence by $\#T = 0$ and the value by $\text{head } T$.

$$T[0 \dots 1] == \{s : \text{seq } T \mid \#s \leq 1\}$$

B Generic cardinality evaluation

evalCardinality Evaluate the cardinality of an arbitrary set of type T . If the number of elements in the set of T is greater than min and:

- max is a number and the number of elements in T is less than max
- max is *many*

Then return the the set. Otherwise return the empty set.

$[T]$	$\text{evalCardinality} : \mathbb{N} \rightarrow \text{unlimitedNat} \rightarrow \mathbb{P } T \rightarrow \mathbb{P } T$
$\forall \text{min} : \mathbb{N}; \text{max} : \text{unlimitedNat}; t : \mathbb{P } T \bullet$	$\text{evalCardinality } \text{min } \text{max } t =$
	if $(\#t \geq \text{min}) \wedge (\text{max} = \text{many} \vee \#t \leq (\text{num} \sim \text{max}))$
	then
	t
	else
	\emptyset

C Generic sequence function

A generic function that takes:

- A substrate
- A function that takes a substrate, a sequence of type T and returns *Sctids_or_Error* (example: *i_subExpressionConstraint*)
- An operator that takes two *Sctids_or_Error* and returns a combination (example: *union*)
- A tuple of the form “ $T \times \text{seq}_1 T$ ”

And returns *Sctids_or_Error*

In the formalization below, *first seq_e* refers to the left hand side of the $T \times \text{seq}_1 T$ and *second seq_e* to the right hand side. *head(second seq_e)* refers to the first element in the sequence and *tail(second seq_e)* refers to the remaining elements in the sequence, which may be empty ($\langle \rangle$).

$[T]$	<hr style="border: none; border-top: 1px solid black; margin-bottom: 5px;"/> <div style="margin-bottom: 10px;"> $applyToSequence : Substrate \rightarrow (Substrate \rightarrow T \rightarrow Sctids_or_Error) \rightarrow$ $(Sctids_or_Error \rightarrow Sctids_or_Error \rightarrow Sctids_or_Error) \rightarrow$ $(T \times seq_1 T) \rightarrow Sctids_or_Error$ </div> <hr style="border: none; border-top: 1px solid black; margin-top: 5px;"/> <div> $\forall ss : Substrate; f : (Substrate \rightarrow T \rightarrow Sctids_or_Error);$ $op : (Sctids_or_Error \rightarrow Sctids_or_Error \rightarrow Sctids_or_Error);$ $seq_e : (T \times seq_1 T) \bullet$ $applyToSequence\ ss\ f\ op\ seq_e =$ if $tail(second\ seq_e) = \langle \rangle$ then $op\ (f\ ss\ (first\ seq_e))(f\ ss\ (head\ (second\ seq_e)))$ else $op\ (f\ ss\ (first\ seq_e))(applyToSequence\ ss\ f\ op\ (head\ (second\ seq_e), tail\ (second\ seq_e)))$ </div> <hr style="border: none; border-top: 1px solid black; margin-top: 5px;"/>
-------	---