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Appendix A

Example 1

Documentation of a PBL problem design: utilizing the 9-step process
Target learners: 5th grade students
Subject: Mathematics

Step 1 – Set goals and objectives

Instructional goal: Fifth grade students will apply knowledge of algebra in solving real life problems. (Note: This instructional goal reflects Arizona State Mathematics Standards, Strand 3, Concept 3.)

Objectives: The 5th grade students will demonstrate and perform the following knowledge and tasks at a level of 90% accuracy.

Domain knowledge objectives

Terminal objective: analyze, represent, and solve mathematical situations and structures using algebraic representations.
Enabling objective 1: solve expressions involving decimals and the four basic operations.
Enabling objective 2: solve one-step equations with one variable represented by a letter or symbol (e.g. 15 = 45/*N*).
Enabling objective 3: use variables in contextual situations.

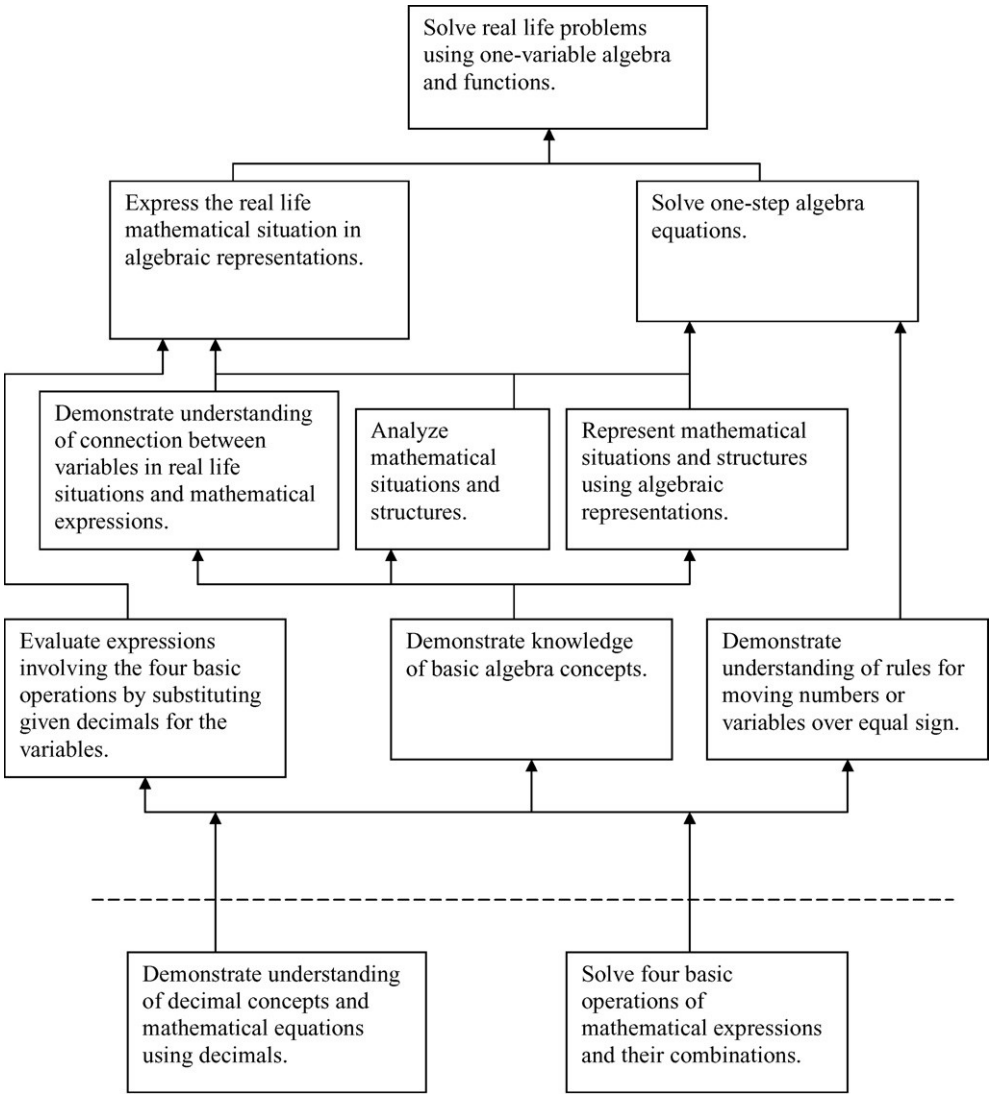
Problem solving skills objectives

1. With moderate-to-aggressive assistance, identify and gather all necessary information
2. With moderate-to-aggressive assistance, conduct simple hypothesis generation and testing
3. With moderate-to-aggressive assistance, select most viable solution

Self-directed learning skills objectives

1. With aggressive assistance, generate learning issues
2. With moderate-to-aggressive assistance, reflect on learning process

Step 2 – Conduct content/task analysis



Step 3 Analyze context specification

Projected context	Factors that influence Researching and Reasoning processes
General	Since no specific projected context can be identified, this section is not applicable

Step 4 – Select/generate PBL problem

List of potential problems:

1. Running a bakery: Student(s) have inherited a building from a deceased aunt who left specific instructions to turn the building into a family bakery, and must make a 5% profit at the end of one month.
2. Have a fund-raiser to pay for a field trip the Desert Museum. The students will have to help with this fund raising activity.

Decision: Fund raising problem is selected because this is something that the students are familiar with, it is relevant to their needs (going on a field trip), and appealing to them.

Step 5 – Conduct problem affordance analysis

Problem: Mrs. Holliday, your biology teacher, and I have decided to take this class to the Desert Museum for a field trip four weeks from now. After discussing this with Mrs. Holliday, we thought that having you ask your parents for money to go on the trip might be a burden for them, so we decided to have a fund-raiser to pay for our trip. Mrs. Holliday and I will pitch in $50 to help start the fund. You, as a group, need to earn the rest of the money for the trip.

In order to solve this problem, the following is the most common problem solving process as to this particular problem (with alternatives when applicable).

Full description:

1. Understanding the problem:
   a. Problem state (current state): a group of 23 fifth grade students lack funding for a day trip to Tucson to visit the Desert Museum. The students need to raise sufficient funds to support their trip.
   b. Goal state: raising sufficient funds to support a field trip for a group of 23 fifth grade students to the Desert Museum in Tucson.
   c. Known variables: duration of the trip (hours), a school bus can be used for the trip, however, there is a fee for ordering the bus. The school will provide financial support for 30% of the cost of transportation.
   d. Unknown variables: need to identify how much the total cost of the trip is, what constitutes the total cost of the trip (the admission fee, transportation, lunch, more?), what items might be possible to sell, what items will produce the most profit in a short period of time, who will be the target customers, what is a reasonable price for the items, what organizations or venders might supply the items, how much do the potential items identified cost, etc.
2. Problem solving processes:
   a. To figure out how much money needs to be raised, we need to first figure out how much the total cost of the trip is.
   b. The costs for the trip could include entry fees, transportation, and lunch. To save on costs, the students will bring their own lunch.
   c. To find out the total cost of the trip, we need to add up all the costs, which include the entry fees (entry fee per student times the number of students, *two prices depending on the season*), transportation ((1–30%)*the cost of one day use of school bus).
      i. Alternative: discount = transportation*30%, then final transportation cost = transportation − discount.
      ii. Alternative: convert 30% to 0.3 may be easier for calculation.
   d. Research information for unknown variables – need to find out how much the entry fee per student is, how much the transportation costs are.
   e. Calculate an estimated total cost for the trip (the total entry fees + transportation cost).
   f. Figure out how much money needs to be raised.
   g. With seed money of $50 that the teachers pitch in, there are two ways to figure out total cost:
      i. We can calculate the amount of money needed by devising an equation that represents
         1. Money needed to be raised [M] + $50 = $ total cost [result from 2e])
         2. M + $50 = $total cost
            a. Alternative: Money needed to be raised (instead of using a symbol) = $ total cost − $50 [teacher may want to encourage students not to use this method, so they will have more practice with the basic operations of algebraic representation.]
         3. Solve the equation: we first have to *isolate* the variable M by moving 50 from the left to the right. To do so, we must *add the opposite* (or add −50 to the right side), then we see that M + 50 − 50 = $total cost − 50. Next we do *Combining like terms* on each side, the + 50 and −50 *cancel*, and the equation becomes M = $total cost − 50.

h. To decide what fund raising activities to do, we need to identify what makes the most profit in a short period of time.
i. First of all, we need to identify fund raising options/activities.
j. Eliminate improper options/activities.
k. Inquire/ask organizations or venders about cost to purchase their items.
l. Analyze the profits for each candidate option.
m. Compare and identify the option with the best profits.
n. The profits from fund raising have to be equal to or greater than *M*.
o. A hypothesized scenario: the students decide to sell candy bars which are 25 cents a piece. We need to decide on a reasonable selling price for the candy bars so we can obtain the best profit. (Alternative: use $0.25 dollar (or whatever the amount is), instead of 25 cents to require students to work with decimals.)
   i. Use T chart to list all the different prices versus how many candy bars need to be sold to reach *M*
   ii. To calculate how many candy bars that need to be sold for each hypothesized price in the T chart, solve (number of candy bars [*N*] * price = *M*) (apply the solving principles/procedure in 2.g.i.3)
p. When deciding the sales price, also consider how, although a high price may reach *M* fast, fewer people may buy items priced too high.
q. Decide the selling price.
r. Now we need to figure out how many candy bars each student is responsible for selling. To do so,
   i. get the *N* of the price decided from the T chart
   ii. calculate (the number of candy bars each student needs to sell = *N*/23)

3. After sales are over, we need to calculate how much the actual profit is.
   a. Calculate the actual cost for purchasing candy bars
      i. Count the number of candy bars purchased from the vender
      ii. The actual cost = (number of candy bars purchased) * (the unit price)
   b. Calculate the actual sale
      i. Count the number of candy bars sold
         1. get the number of candy bars each student sold
         2. add the number of candy bars sold for all students
      ii. The actual sale = (number of candy bars sold) * (sale price)
   c. Calculate the actual profit
      i. The profit = (actual sale) − (actual cost)
   d. Determine whether or not we raised enough funds for the trip
      i. If the profit ≥ *M*, then yes (conditional statement)
      ii. If the profit < *M*, then no
4. If we have reached the fund raising goal, then Mrs. Holliday will arrange the trip. If not, Mrs. Holliday will seek help from the school to make up any shortage of funds, and then arrange the trip.

Domain knowledge needed for solving the problem:

- Concepts used in solving the problem:
  - basic algebra concepts
  - basic algebraic representations
  - profits
  - greater than, less than, equal to
  - concept of percentage
  - concept of decimals (peripheral)
- Principles used in solving the problem:
  - Solving
    - Isolate
    - Add the opposite
    - Combining like terms
    - Cancel
- Procedures used in solving the problem:
  - basic mathematical operations
  - solving one-variable algebraic equations
  - procedure for basic operations involving decimals (peripheral)
- Factual information needed for solving the problem:
  - Entry fee per student (season)
  - Cost of transportation (bus)
  - Possible activities for fund raising
  - Venders

Problem solving skills analysis:

Identify what known variables are: see above.
Identify what unknown variables are: see above.
Do research to find the unknown variables identified.
Figure out the total funds needed
Research possible fund raising activities.
Figure out the best way to reach the goal based on the information gathered (hypotheses generation process) and generate the most viable solution.
Execute the solution
Examine the outcome of solution.

Step 6 – Conduct correspondence analysis

Table 1. Content correspondence analysis chart.

Domain knowledge (researching)	*Overall goal*: analyze and represent mathematical situations and structures using algebraic representations								Prerequisites	Beyond scope of objectives
	Concept				Principle		Procedure			
	Variables (symbols)	Basic algebra	Mathematical situations and structures	Concept of decimals	Basic algebra representation	Solving	Solving one-step with one variable algebraic equation	Four operations involving decimals		
Core										
Concept (c)										
–Concepts of basic algebra	√	√	√		√	√	√			
–Basic algebraic representation	√		√		√	√	√			
–Isolate		√				√	√			
–Add the opposite		√				√	√			
–Combining like terms		√				√	√			
–Cancel		√				√	√			
–Concept of percentage									√	
–Convert real life situations into mathematical expression			√							
–Profits			√						√	
–Cost			√						√	
–Profitable price			√							√
–Sale			√						√	
–Greater than, less than, equal to									√	
Principle (PL)										
–Best profit			√						√	
–Solving			√						√	
–Conditional statement							√		√	
–Order of operations							√		√	
Procedure (PD)										
–Basic operations			√				√		√	
–Basics of algebra									√ (may not master)	
–Percentage operation										
Factual information (F)									√	
–Entry fee									√	
–Transportation										
Peripheral										
C				√						
–Decimals									√	
–Arithmetic										
P								√		
–Basic operations involving decimals										

Table 2. Correspondence analysis between problem-solving cognitive processes and learning objectives.

Cognitive processes	Description	PSO-1 Identify and gather all necessary information need to research with assistance	PSO-2 Conduct simple hypothesis generation and testing with assistance	PSO-3 Select most viable solution
Researching and reasoning processes				
Symptoms recognition/problem identification	–Raising sufficient funds for a field trip	√		
Known variables identification	–Total cost of the trip (Entry fees + Transportation (school supports 30% of the cost)	√		
	–Seed money of $50	√		
Unknown variables identification	–How much is the total needed for covering the cost in addition to the seed money and the 30% discount of the transportation cost (M + $50 = total cost)	√		
	the options of fund raising activities	√		√
	–Number of items need to sell	√		
Information/data searching	–Cost for entry fees	√		
	–Cost for transportation	√	√	
	the options of fund raising activities (venders or organizations)	√		
	–Inquire the cost of the options/activities	√		
Information/data analysis	–Find out appropriate entry fee rate (there are two rates depending on the season)		√	
	–Eliminate infeasible fund raising options/activities	√		√
	–Analyze the gross profit of each candidate fund raising option	√		√
	–Calculate the estimated total cost of the trip	√		√
Reasoning path	1. Need to know the total cost for the trip 2. Need to know the actual cost after subtracting the seed money and 30% discount for transportation			
Hypotheses generation and testing	–Use T-chart to analyze the number of items needed to be sold to reach the goal of fund needed		√	
Reasoning path and solutions generation	1. Once the item to be sold is decided, need to know the total number of the items needed to be sold 2. Need to know the number of items to be sold for each student			√

Problem context analysis: The context of this problem is general and applies to the students' everyday lives. Also, a field trip is generally appealing to students, therefore, the context can satisfy the motivational aspect of this PBL problem design.

Conclusion of correspondence analysis:

1. Domain knowledge:
   a. The degree of correspondence between the intended objectives and the scope of the PBL problem reaches a desirable level.
   b. Non-corresponding portion:
      i. The core domain knowledge in the PBL problem may not necessarily cover the objective "Evaluate expressions involving the four basic operations by substituting given decimals for the variables." Decimal concept may be used if alternative path is taken. The presentation of the problem should guide students to take this reasoning and solution path.
      ii. Several concepts (profits, profitable price, best profits) may be beyond the scope of the objectives.

2. Contextual information: The contextual information of the problem properly supports the projected context.
3. Problem solving and self-directed learning skills:
   a. Researching component: There are several pieces of information that the students need to research, compare (e.g. entry fee, transportation, fund raising options, etc.), and make a decision about what item to sell. To achieve the problem solving skill objectives, these pieces of information should be researched by the students, however, with sufficient guiding information to help the students initiate the researching process. The problem should give initial hints to guide the students to seek out this information.
   b. Reasoning component: the students need to reason: how to get (calculate) an estimated total cost for the trip, how to calculate how much profit is needed to cover the cost of the trip and the cost of obtaining the sale items, what mathematical concepts, principles, and procedures need to be used to obtain the numbers mentioned above, what items will produce the best profits (high price will produce better profit but fewer number of items may be sold, and vise versa), generate hypotheses about the possible profits for each candidate item, and select the best items to sell.

Results—the following are the elements that need to be calibrated in the problem presentation:

1. Domain knowledge:
   a. Convert the percentage (30%) component to decimals.
   b. Give brief explanations of the concepts (profits, cost, profitable price, best profits) in the problem presentation.
   c. The presentation of the problem should guide students to take this reasoning and solution path by requiring students to use dollar as unit to perform the calculation so that decimal representations and operations will be part of the requirement for solving the problem.
2. Problem solving and Self-directed learning skills:
   a. Based on problem solving skills and self-directed learning skills objectives (note: the objectives are devised based on that the targeted audience is 5th graders and this is the first experience with PBL), the following information should be given to guide their problem solving and self-directed learning process:
      i. Researching component:
         1. what information do they need to research (e.g. entry fee, transportation, fund raising options,) [give one as an initiating hint—transportation, then ask questions to guide students to identify other information that needs to be researched.]
         2. what mathematical concepts, principles, and procedures will they need to know to solve the problem [interlink with 2.a.ii.3].
      ii. Reasoning component:
         1. how to get (calculate) an estimated total cost for the trip [prompt the students to this reasoning path with guiding questions]
         2. how to calculate how much profits are needed to cover the cost of the trip and the cost of obtaining the sale items [prompt the students to this reasoning path with guiding questions]
         3. what mathematical concepts, principles, and procedures need to be used to obtain the numbers mention above [ask these questions as guiding questions]
         4. what items will produce the best profits (high price will produce better profit but fewer number of items may be sold, and vise versa) [5th grader may not be aware of this concept, this information, therefore, should be included in the problem presentation),
         5. generate hypotheses about the possible profits for each candidate items, and select the best selling items [giving hints to guide students to engage in this process by asking explicit questions as guiding questions].

Step 7 – Conduct calibration process

Problem: Mrs. Holliday, your biology teacher, and I have decided to take this class to the Desert Museum in Tucson for a field trip four weeks from now. After discussing this with Mrs. Holliday, we thought that having your parents pay for the trip might be a burden on them. So, we decided to have a fund-raiser to pay for our trip. Mrs. Holliday and I will pitch in $50 to help start the fund. You, as a group, need to help with this fund-raiser. Our goal is to raise enough money to support our field trip to the Desert Museum in Tucson. We will be taking a school bus for the trip, however, there is a fee for ordering the bus. The school will provide 30% of the cost of transportation, which can also be expressed as 0.3. We also need to think about what other costs there may be for the trip to figure out the total cost of the trip. So, we need to figure out how much the total cost of the trip will be, so we know what to sell to raise funds. To figure out how much money needs to be raised, we need to figure out how much the total cost of the trip is. To save money, you will need to bring your own lunch. How can we figure out the total cost for the trip? How much money do we need to earn to cover the cost of the trip, in addition to obtaining the fund-raiser items to sell?

What can we sell that will raise enough funds? How can we find out this information? Also, since we only have four weeks to raise the money, we may want to sell something that will make a big profit in a short period of time. So, we'd better think about what can be sold at a high price that a lot of people will want to buy. If we sell an item that is too

expensive, then there will be only a few people who can afford it, and we won't make as much money as we wish. So, let's also think about what a reasonable price for the item would be. What is a good way to help us make this decision? And after we finish our sales, we need to figure out how much we made. Is it enough to cover our trip? So that all of us are on the same page, we will use the dollar (vs. cents) as the unit for our calculation. Let's work together so we can have a fun field trip!

Step 8 – Constructing reflection component

Problem: Mrs. Holliday, your biology teacher, and I have decided to take this class to the Desert Museum in Tucson for a field trip four weeks from now. After discussing this with Mrs. Holliday, we thought that having your parents pay for the trip might be a burden on them. So, we decided to have a fund-raiser to pay for our trip. Mrs. Holliday and I will pitch in $50 to help start the fund. You, as a group, need to help with this fund-raiser. Our goal is to raise enough money to support our field trip to the Desert Museum in Tucson. We will be taking a school bus for the trip, however, there is a fee for ordering the bus. The school will provide 30% of the cost of transportation, which can also be expressed as 0.3. We also need to think about what other costs there may be for the trip to figure out the total cost of the trip. So, we need to figure out how much the total cost of the trip will be, so we know what to sell to raise funds. To figure out how much money needs to be raised, we need to figure out how much the total cost of the trip is. To save money, you will need to bring your own lunch. How can we figure out the total cost for the trip? How much money do we need to earn to cover the cost of the trip, in addition to obtaining the fund-raiser items to sell?

What can we sell that will raise enough funds? How can we find out this information? Also, since we only have four weeks to raise the money, we may want to sell something that will make a big profit in a short period of time. So, we'd better think about what can be sold at a high price that a lot of people will want to buy. If we sell an item that is too expensive, then there will be only a few people who can afford it, and we won't make as much money as we wish. So, let's also think about what a reasonable price for the item would be. What is a good way to help us make this decision? And after we finish our sales, we need to figure out how much we made. Is it enough to cover our trip? So that all of us are on the same page, we will use the dollar (vs. cents) as the unit for our calculation. Let's work together so we can have a fun field trip!

I am going to divide you into five groups. Each group will need to work on every step of this fund raising project because we want to gather as many ideas as possible to help us select the best way to do this. You should work with your group during the week, and we will have meetings each Fridays to see how each group is doing. During the meeting, you and your group will need to report your progress and how you came up with your ideas and why you made your decisions, and think of any better ways to do what you are doing.

OK, enough talking, let's get to work!

Step 9 – Examine Inter-supporting relationships of 3C3R components

	Content	Context	Connectin
Researching	Sufficiently support content knowledge acquisition.	Sufficiently support directing researching	Sufficiently mutually support integrating knowledge acquired by conning to prior knowledge.
Reasoning	Sufficiently support content knowledge acquisition and application.	Sufficiently support directing reasoning.	Sufficiently mutually support integrating knowledge acquired by conning to prior knowledge.
Reflecting	Does not support evaluating acquisition/ processing of content knowledge. Need revision.	Does not support directing reflecting. However, since the context component is general in this problem, it does not need revision.	Sufficiently mutually support integrating knowledge acquired by requiring students to reflect on their problem solving process.

Note: ==> supports

Conclusion: Revise reflecting component to support content component.

Final draft of problem presentation

Problem: Mrs. Holliday, your biology teacher, and I have decided to take this class to the Desert Museum for a field trip four weeks from now. As Mrs. Holliday and I discussed our options, we found that having you ask your parents for money to go on the trip may be a burden for your parents. So, we decided to have a fund raiser to pay for our trip. Mrs. Holliday and I will pitch in $50 to help start the fund. You, as a group, need to help with this fund raising. So, our goal for this fund raiser is to raise sufficient funds to support our field trip to the Desert museum in Tucson. We will be taking a school bus for the trip, however, there is a fee for ordering the bus. The school will provide financial support for 30% of the cost of transportation, which can also be expressed as 0.3. We may also need to think about other possible costs for the trip to figure out the total cost of the trip. So, we need to figure out how much the total cost of the trip is, and what to sell to raise funds. To figure out how much money needs to be raised, we need to figure out how much the total cost of the trip is. To save on costs, you will need to bring your own lunch. How can we find out the total cost of the trip? How much do we need to make to cover the cost of the trip and obtaining the items to sell?

What should we sell to raise enough funds? How can we find out this information? Also, since we only have four weeks to raise the money, we may want to sell something that will make the most profit in a short period of time. So, we better think about what items can be sold at higher price but still a good number of people will buy it. If we sell an item that is too expensive, then only a few people can afford it, and we won't make as much money as we wish. So, let's also think about what item to sell and what is a reasonable price for the items. What will be a better way to help us make this decision? And after we finish the selling, we also need to figure out what much do we make. Is it enough for covering our trip? Let's work together and so we can have a fun field trip!

I am going to divide you into 5 groups. Each group will need to work on every step of this fund raising project because we want to have as many ideas as possible to help us select the best way to raise money for our field trip. You should work with your group during the week, and we will have meetings on Fridays to see how each group is doing. During the meetings, you and your group will need to report your progress, share with the other groups how you found your information and how you have used or plan to use the information to solve our problem. You will also share how you came up with your ideas and why you made your decisions. You will also need to think about whether or not there might be a better way of doing what you are doing.

OK, enough talking, let's get to work!