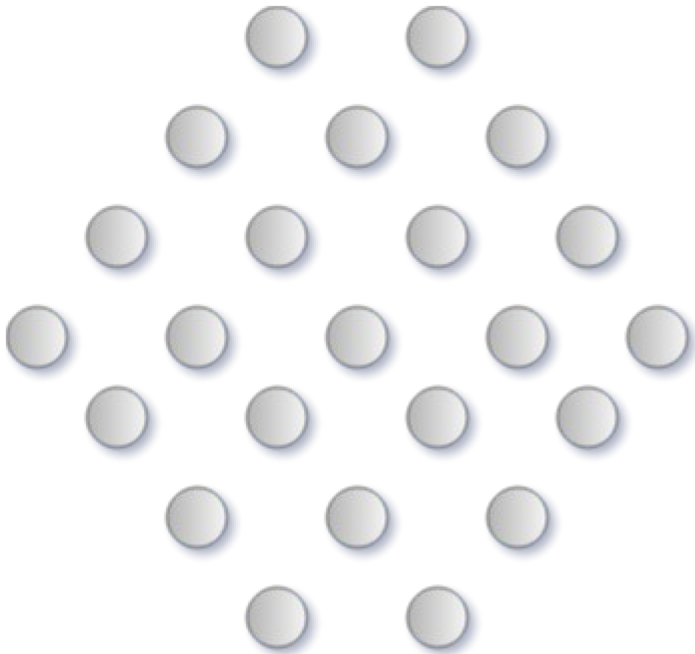
Análise de Redes Sociais e Econômicas

Alguns pontos do Paper

Erdös e Rényi (1960)

Henrique Solér Kalinovski (2023)

Algumas definições



$$G = V, E$$

$$G' = V', E' : (el \in G' \rightarrow el \in G) \wedge (e_{v1,v2} \in E' \rightarrow v_1, v_2 \in V')$$

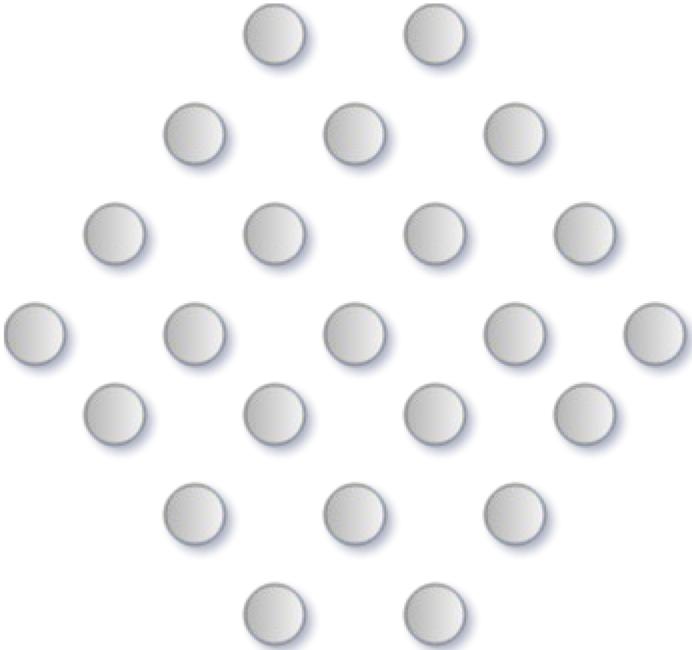
$$G_{balanceado} = G : \nexists G' : \frac{2N'}{n'} > \frac{2N}{n}$$

$$G_{conectado}$$

$$G_{árvore}$$

$$G_{ciclo}$$

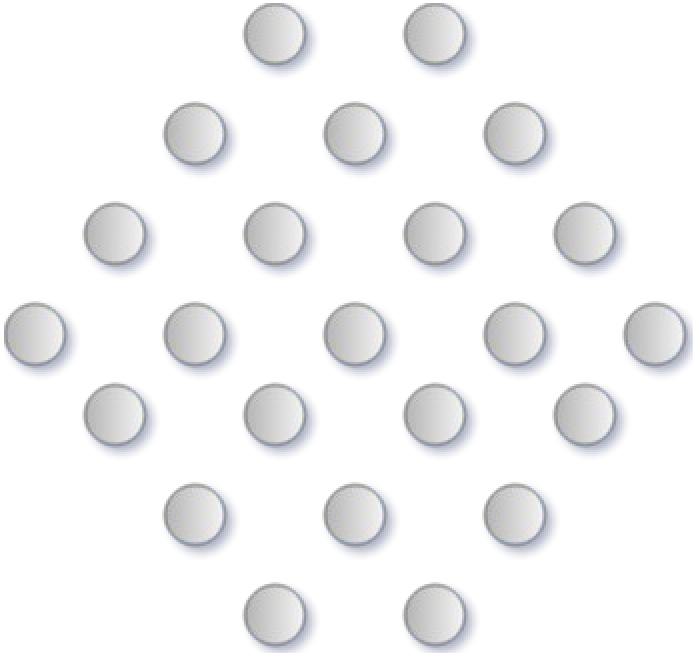
Função limiar



$$\lim_{n\to\infty} P_{n,N(n)}(A)=\begin{cases} 0\,,se\lim\limits_{n\to\infty}\left(\dfrac{N(n)}{A(n)}\right)=0 \\ 1\,,se\lim\limits_{n\to\infty}\left(\dfrac{N(n)}{A(n)}\right)=\infty \end{cases}$$

$$\lim_{n\to\infty} P_{n,N(n)}(A)=F(x)\,se\lim_{n\to\infty}\left(\frac{N(n)}{A(n)}\right)=x$$

Teorema 1



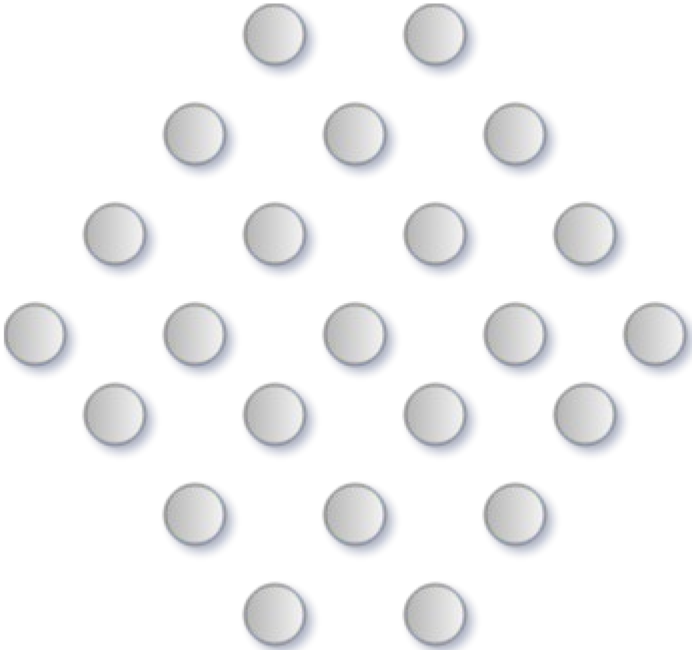
$$Sejam\ k \geq 2\ e\ k-1 \leq l \leq \binom{k}{2}\ inteiros\ positivos$$

$$A\left(B_{k,l}\right) = n^{\left(2-\frac{k}{l}\right)}$$

Onde B é a classe de grafos balanceados e conectados com k vértices e l arestas arbitrariamente escolhidos, por exemplo:

Árvores de ordem k
Ciclos de ordem k

Prova do teorema 1
(parte 1)

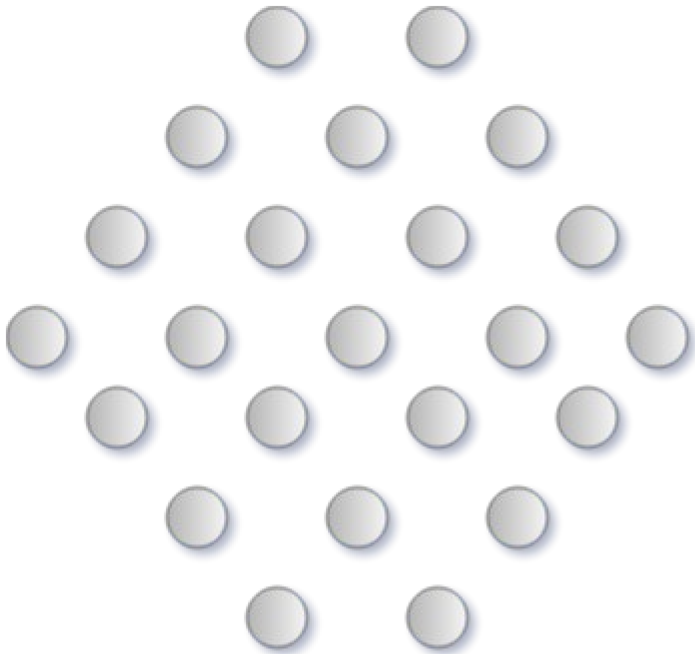


$$P_{n,N}(B_{k,l}) \le \frac{\binom{n}{k} * B^c_{k,l} * \binom{\binom{n}{2}-l}{N-l}}{\binom{\binom{n}{2}}{N}}$$

$$\Rightarrow \frac{B^c_{k,l}}{k!} * \frac{n!}{(n-k)!} * \frac{\left(\binom{n}{2}-l\right)!}{\binom{n}{2}!} * \frac{N!}{(N-l)!}$$

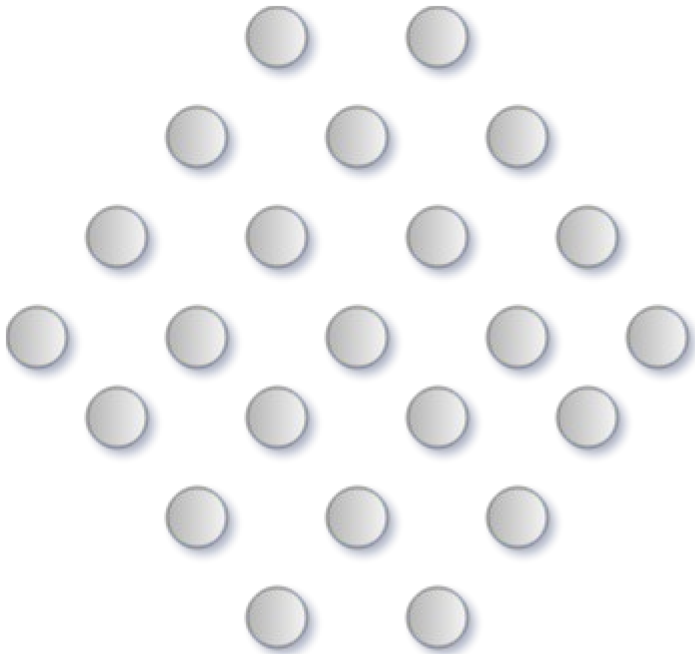
$$.\sim \frac{B^c_{k,l}}{k!} * n^k * \frac{2^l}{n^{2l}} * N^l = \frac{B^c_{k,l}}{k!} * \frac{(2N)^l}{n^{2l-k}} = O\left(\frac{N^l}{n^{2l-k}}\right)$$

Prova do teorema 1
(parte 2)



Muito grande

Teorema 1
(alguns corolários)

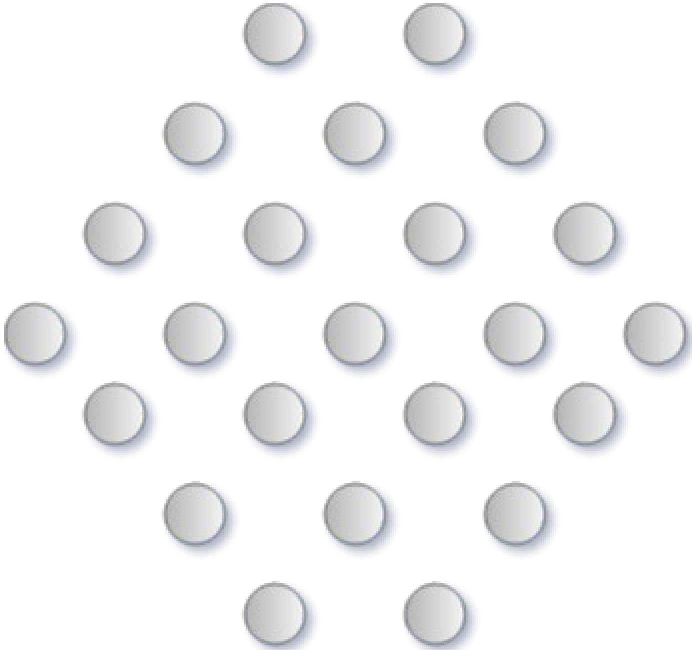


$$A_{árvore_k}(n) = n^{\frac{k-2}{k-1}}$$

$$A_{Ciclo}(n) = n$$

$$A_{Completo}(n) = n^{2\left(1-\frac{1}{k-1}\right)}$$

Exemplo de teorema provado a partir disso



§ 4. The total number of points belonging to trees

We begin by proving

Theorem 4a. *If $N = o(n)$ the graph $\Gamma_{n,N}$ is, with probability tending to 1 for $n \to +\infty$, the union of disjoint trees.*

Proof of Theorem 4a. A graph consists of disjoint trees if and only if there are no cycles in the graph. The number of graphs $G_{n,N}$ which contain at least one cycle can be enumerated as was shown in § 1 for each value k of the length of this cycle. In this way, denoting by T the property that the graph is a union of disjoint trees, and by $\overline{T}$ the opposite of this property, i. e. that the graph contains at least one cycle, we have

$$(4.1) \qquad \mathbf{P}_{n,N}(\overline{T}) \leq \sum_{k=3}^{n} \binom{n}{k} (k-1)! \frac{\binom{\binom{n}{2}-k}{N-k}}{\binom{\binom{n}{2}}{N}} = O\left(\frac{N}{n}\right).$$

It follows that if $N = o(n)$ we have $\lim\limits_{n \to +\infty} \mathbf{P}_{n,N}(T) = 1$ which proves Theorem 4a.

Discussão final

Muito obrigado