How to turn on the D-DABIC inversion option

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Outline

1. Motivations

- During the use of FEMTIC, my collaborators and I identified that the L-curve method (originally used in the code to select the final model) suffers from subjective bias:
  - *The point of maximum curvature on the L-curve is strongly influenced by human-defined settings such as the range and discrete values of regularization parameters(α).*

  This issue compromises the objectivity of model selection and the reproducibility of results to a certain extent (different scholars may obtain distinct results using the L-curve method with various parameter settings). For a more comprehensive discussion of this problem, refer to Constable et al. (2015) and Song et al. (2025).

- To address this issue, we tested various strategies and ultimately established an α selection and model selection approach based on Akaike's Bayesian Information Criterion (ABIC). Building on this, we proposed a 3-D data-space inversion method based on a data-space variant of ABIC (D-DABIC) and implemented it into the FEMTIC code.

References
Constable, S., A. Orange, and K. Key, 2015, And the geophysicist replied: "Which model do you want?": GEOPHYSICS, 80, no. 3, E197–E212, https://doi.org/10.1190/geo2014-0381.1.
Song,H., Peng, Yu., Usui, Y., et al. Three-dimensional Magnetotelluric Inversion based on a Data Space variant of Akaike's Bayesian Information Criterion. Geophysics 2025, https://doi.org/10.1190/geo-2025-0233.

2. Theoretical Basis

2.1 Why ABIC?

2.2 The Data-space inversion based on DABIC (D-DABIC)

2.3 The flowchart of D-DABIC inversion

2.1 Why ABIC?

$$U(\mathbf{m}) = \|\mathbf{W_d}\,(\mathbf{d} - \mathbf{F}(\mathbf{m})\|^2 + \alpha^2 \|\mathbf{Cm})\|^2$$

The role of ABIC

- Akaike's Bayesian Information Criterion (ABIC) : **selecting the statistical optimal hyperparameter.**. A smaller ABIC represents a better hyperparameter (Akaike 1980).
- In MT inversion, **it can help us determine the optimal regularization parameter** in statistical view (e.g., Uchida 1993).

The merits of ABIC

ABIC: The negative logarithmic form of the marginal likelihood function

$$\mathrm{ABIC} = (-2)\log(L(\mathbf{Wd}\,|\,v_0^2,\alpha^2)) \;+\; 2\dim(\text{hyperparameters})$$

Marginal likelihood function

$$L(\mathbf{Wd}\,|\,v_0^2,\alpha^2) = \int f(\mathbf{Wd}\,|\,\mathbf{m},v_0^2)\,p(\mathbf{m}\,|\,v_0^2,\alpha^2)\,d\mathbf{m}$$

$$= \int \left(2\pi v_0^2\right)^{-(N+M)/2} \left|\alpha^2\mathbf{C}^T\mathbf{C}\right|^{\frac{1}{2}} \times \exp\left(-\frac{1}{2v_0^2}\left(\|\mathbf{Wd} - \mathbf{WF(m)}\|^2 + \alpha^2\|\mathbf{Cm}\|^2\right)\right) d\mathbf{m}$$

Substitute into the linearized objective function：

$$\mathrm{ABIC}_k(\alpha) = N\log\left(2\pi\frac{U(\mathbf{m}_{k+1})}{N}\right) - \log\left|\alpha^2\mathbf{C}^T\mathbf{C}\right| + \log\left|\left(\mathbf{WA}_k\right)^T\mathbf{WA}_k + \alpha^2\mathbf{C}^T\mathbf{C}\right| + N + 2$$

- The value of ABIC incorporates a comprehensive consideration of the **data misfit** term, **model constraint** term, and **inversion properties** (i.e., the Hessian matrix), **it is a univariate function of the regularization parameter**. So the advantage of this method is that it accounts for all aspects of the regularized inversion and is not influenced by any other subjective empirical parameters (e.g., Uchida 1993).

References
Akaike, H., 1980, Likelihood and the Bayes procedure: Trabajos de Estadística y de Investigación Operativa, 31, 143–166, https://doi.org/10.1007/BF02888350.
Uchida, T., 1993, Smooth 2-D Inversion for Magnetotelluric Data Based on Statistical Criterion ABIC: Journal of Geomagnetism and Geoelectricity, 45, 841–858, https://doi.org/10.5636-/jgg.45.841.

2.2 The Data-space inversion based on DABIC (D-DABIC)

Derivation of DABIC

- ➢ Owing to its merits of objectivity and effectiveness, ABIC has gained attention in EM inversion studies (e.g., Uchida, 1993; Ogawa & Uchida, 1996; Mitsuhata et al., 2002).

- ➢ **The high computational cost of the ABIC has limited its practical application in 3-D scenarios :**

ABIC（Akaike, 1980）：

$$\mathrm{ABIC}_k(\alpha) = N\log\left(2\pi\frac{U(\mathbf{m}_{k+1})}{N}\right) - \log\left|\alpha^2\mathbf{C}^T\mathbf{C}\right| + \boxed{\log\left|(\mathbf{WA}_k)^T\mathbf{WA}_k + \alpha^2\mathbf{C}^T\mathbf{C}\right|} + N + 2$$

***M* dimensional**

Low-rank update (Rasmussen & Williams, 2006)

DABIC（this work）：

***N* dimensional**

$$\mathrm{DABIC}_k(\alpha) = N\log\left(2\pi\frac{U(\mathbf{m}_{k+1})}{N}\right) + \log\left|\mathbf{I} + \mathbf{WA}_k\left(\alpha^2\mathbf{C}^T\mathbf{C}\right)^{-1}(\mathbf{WA}_k)^T\right| + N + 2$$

The computational complexity of determinant calculation from O(M^3) to O(N^3)

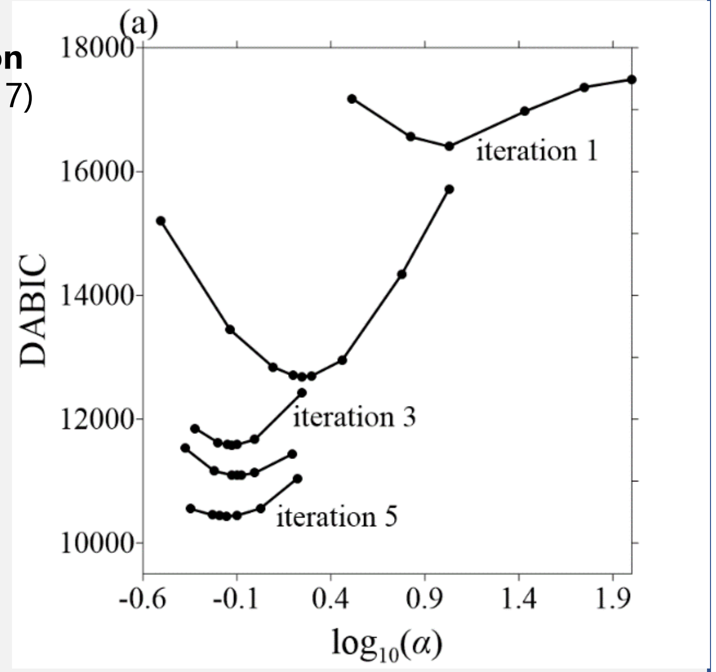
M :number of model parameter;
N: number of data points

D-DABIC inversion method

- ➢ Based on FEMTIC code (Usui, 2015; Usui et al., 2017, 2021)

- **Brent line search (Brent, 2018):**
finding α^2 that minimize DABIC in each iteration

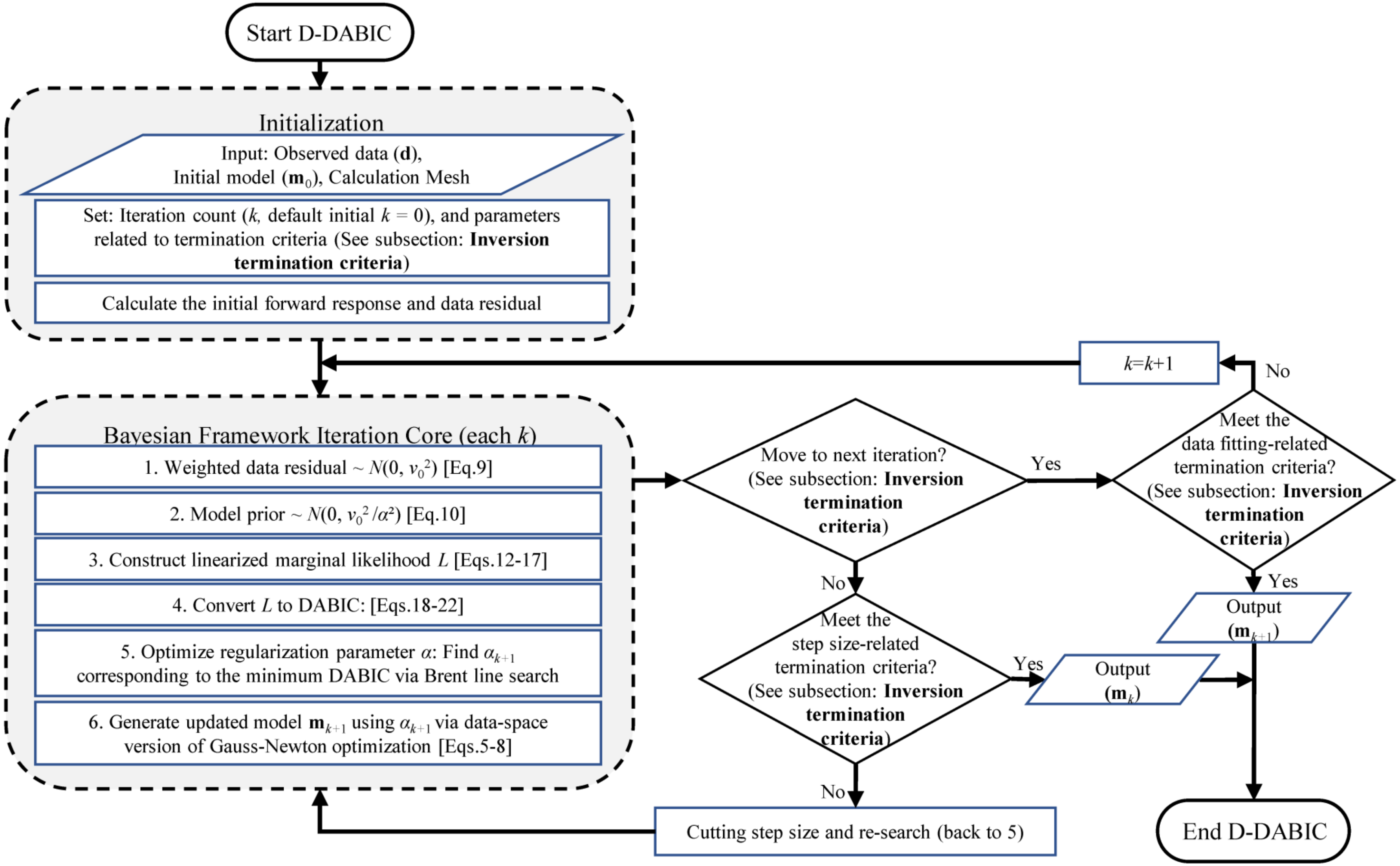
- **Data-space Gauss-Newton Optimization**(Usui et al., 2017)



- We adopt the condition that neither ABIC nor RMS can decrease further as the iterative convergence criterion.

References
For more details, please refer to our recent GEOPHYSICS paper:
Song,H., Peng, Yu., Usui, Y., et al. Three-dimensional Magnetotelluric Inversion based on a Data Space variant of Akaike's Bayesian Information Criterion. Geophysics 2025, https://doi.org/10.1190/geo-2025-0233.

2.3 The flowchart of D-DABIC inversion



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3. Turn on the D-DABIC inversion

- Step 1: Compile the source code;
- Step 2: Prepare the basic files (control.dat, observe.dat, mesh.dat, resistivity_block_iter#.dat, etc.) following the user manual of FEMTIC (https://github.com/yoshiya-usui/femtic/tree/main/doc)
- Step 3: Modify control.dat:

  (1) Change:

  INV_METHOD,

  TRADE_OFF_PARAM.

  (2) add three new keywords:

  PARAM_DISTORTION,

  TYPEOF_TO

Example:

```
…
INV_METHOD
2
DISTORTION
0
PARAM_DISTORTION
TYPEOF_TO
1
TRADE_OFF_PARAM
100.0 1.0
BOTTOM_RESISTIVITY
200.0
…
END
```

A detailed explanation of this example can be accessed in the next slide

- Step 4: Run D-DABIC Inversions.

3. Turn on the D-DABIC inversion

Keyword	Content	Data type	Option	Example
INV_METHOD *(modified)*	Type of inversion method	Integer (0, 1 or 2)	0: Model space method 1: Data space method 2: D-DABIC inversion	**INV_METHOD** 2
TYPEOF_TO *(newly added)*	Type of trade-off parameter selection	Integer (0 or 1)	The number of option depends on the option of the keyword 'INV_METHOD'. If the option of 'INV_METHOD' is '0' or '1' then: 0: Fixed value If the option of 'INV_METHOD' is '2' then: 1: updated by DABIC minimization	**TYPEOF_TO** 1
TRADE_OFF_PARAM *(modified)*	Initial Trade-off parameter and target RMS*1)	Two real positive values		**TRADE_OFF_PARAM** 100.0 1.0
PARAM_DISTORTION *(newly added)*	Trade-off parameters for distortion estimation	Real positive value(s)	The number of option(s) depends on the option of the keyword 'DISTORTION'. If the option of 'DISTORTION' is '0', you should not write anything. If the option of 'DISTORTION' is '1' or '3', you should write trade-off parameters β. If the option of 'DISTORTION' is '2', you should write trade-off parameters β_1 and β_2.	**PARAM_DISTORTION**

*1) When data uncertainty is accurately estimated, the target RMS can be set to 1.0; when it is not precisely known, a small value (e.g., 0.01) should be specified to allow the D-DABIC inversion to converge to the termination criteria described in Song et al. (2025).

References

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