Advection in two-dimensional space

Problem: Find the scalar field $u = u(x, y, t)$ governed by the equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 0, & x, y \in [-1, 1], \\ u(x, y, 0) = \sin((x+1)\pi)\sin((y+1)\pi), \\ u(-1, -1, t) = \sin(\pi t)\sin(\pi t). \end{cases}$$

Exact solution: $u(x, y, t) = \sin((x + 1 - t)\pi)\sin((y + 1 - t)\pi)$.

To run this example, go to the $\sim$/`examples/advection_2d` folder, run `make`, and then execute `./advection_2d`. To configure the running mode (CPU or GPU), edit the $\sim$/`src/config.h` file to enable or disable the macro USE_CPU_ONLY before running `make`.

Meshes: Three versions of the mesh are included for this example: coarse, fine, and super fine.

Run times of dg-on-cuda: Comparison of the CPU (serial) exection on Nvidia Jetson Xavier NX (Carmel ARMv8.2 64-bit 6MB L2 + 4MB L3) with the GPU execution (Volta GPU with 384 CUDA cores) on the coarse mesh (16 elements) is shown below. The GPU executions are all timed with block size 64.

	CPU						GPU					
Approx. order	1	2	3	4	5	6	1	2	3	4	5	6
Time (s)	0.7	1.4	2.1	3.9	5.6	8.2	4.0	5.6	8.7	14.8	25.2	39.5

On the fine mesh (1,064 elements), the comparison is:

	CPU						GPU					
Approx. order	1	2	3	4	5	6	1	2	3	4	5	6
Time (s)	32.6	51.2	79.5	145.9	235.7	334.8	4.2	7.4	15.4	27.3	46.3	73.2

On the super fine mesh (105,678 elements), the comparison is only done up to the approximation order of 4 — it took too long to run higher orders on CPU:

	CPU				GPU			
Approx. order	1	2	3	4	1	2	3	4
Time (s)	4043.7	5961.5	8937.2	15642.9	156.4	418.0	807.5	1608.5

As expected, on the coarse mesh CPU runs much faster than GPU. When the work load increases, however, GPU executions surpass CPU — it runs up to 25 times faster than CPU.