

## Maxwell's equation in two-dimensional space

**Problem:** Find the magnetic fields  $\mathbf{H} = (H^x, H^y)$  and the electric field  $E^z$  governed by the equation:

$$\begin{cases} \frac{\partial H^x}{\partial t} = -\frac{\partial E^z}{\partial y}, \\ \frac{\partial H^y}{\partial t} = \frac{\partial E^z}{\partial x}, \\ \frac{\partial E^z}{\partial t} = \frac{\partial H^y}{\partial x} - \frac{\partial H^x}{\partial y}. \end{cases} \quad x, y \in [-1, 1],$$

with the boundary conditions  $\mathbf{H} = \mathbf{H}^{interior}$  and  $E^z = 0$  at the boundary, and the initial condition:

$$\begin{cases} H^x(x, y, 0) = 0, \\ H^y(x, y, 0) = 0, \\ E^z(x, y, 0) = \sin(\pi x) \cos(\pi y). \end{cases}$$

**Exact solution:**

$$\begin{cases} H^x(x, y, t) = -\frac{1}{\sqrt{2}} \sin(\pi x) \cos(\pi y) \sin(\sqrt{2}\pi t), \\ H^y(x, y, t) = \frac{1}{\sqrt{2}} \cos(\pi x) \sin(\pi y) \sin(\sqrt{2}\pi t), \\ E^z(x, y, t) = \sin(\pi x) \sin(\pi y) \cos(\sqrt{2}\pi t). \end{cases}$$

To run this example, go to the `~/examples/maxwell_2d` folder, run `make`, and then execute `./maxwell_2d`. To configure the running mode (CPU or GPU), edit the `~/src/config.h` file to enable or disable `USE_CPU_ONLY` before running `make`.

**Meshes:** Three versions of the mesh are included for this example: coarse, fine, and super fine.

**Run times of dg-on-cuda:** Comparison of the CPU (serial) execution on Nvidia Jetson Xavier NX (Carmel ARMv8.2 64-bit 6MB L2 + 4MB L3) with the GPU execution (Volta GPU with 384 CUDA cores) on the coarse mesh (16 elements) is shown below. The GPU executions are all timed with block size 64.

	CPU							GPU					
Approx. order	1	2	3	4	5	6		1	2	3	4	5	6
Time (ms)	4	13	38	68	100	162		20	61	140	215	431	783

On the fine mesh (1,064 elements), the comparison is:

	CPU							GPU					
Approx. order	1	2	3	4	5	6		1	2	3	4	5	6
Time (s)	5.5	20.0	52.2	125.3	380.9	487.8		1.6	4.2	12.3	27.8	63.9	112.4

On the super fine mesh (105,678 elements), the comparison is only done on the first order approximation — it took too long to run higher orders on CPU:

	CPU	GPU
Approx. order	1	1
Time (s)	98506.5	4231.6

As expected, on the coarse mesh CPU runs much faster than GPU. When the work load increases, however, GPU executions surpass CPU — it runs up to 23 times faster than CPU.