Stokes equations in two-dimensional space

Problem: The unsteady Stokes equations

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} - \nu \nabla^2 \boldsymbol{u} + \nabla p = \boldsymbol{f}, \\ \nabla \cdot \boldsymbol{u} = 0. \end{cases}$$

defined on the domain $\Omega = [-1, 1]^2$ and time interval $[0, T] = [0, 0.1]$ with $\boldsymbol{f} = \mathbf{0}$ and the analytical solution as

$$\begin{cases} \boldsymbol{u}(\boldsymbol{x}, t) = \begin{bmatrix} \sin(x_1)(a \sin(a x_2) - \cos(a) \sinh(x_2)) \\ \cos(x_1)(\cos(a x_2) + \cos(a) \cosh(x_2)) \end{bmatrix} \exp(-\lambda t), \\ p(\boldsymbol{x}, t) = \lambda \cos(a) \cos(x_1) \sinh(x_2) \exp(-\lambda t). \end{cases}$$

where $\lambda = \nu(1 + a^2)$, $\nu = 1$ and $a = 2.883356$. The initial conditions and the Dirichlet boundary condition are all deduced from the analytical solution.

Discretizations:

The temporal discretization follows the dual splitting scheme described in [1] and [2]. The spatial Discontinuous Galerkin discretization follows that presented in [3].

To run this example, go to the `~/examples/stokes_2d` folder, run `make`, and then execute `./stokes_2d`. To configure the running mode (CPU or GPU), edit the `~/src/config.h` file to enable or disable USE_CPU_ONLY before running `make`.

Meshes: Three versions of the mesh are included for this example: coarse, fine, and super fine.

Run times of dg-on-cuda: Comparison of the CPU (serial) execution on Nvidia Jetson Xavier NX (Carmel ARMv8.2 64-bit 6MB L2 + 4MB L3) with the GPU execution (Volta GPU with 384 CUDA cores) on the fine mesh (1,064 elements) is shown below. The GPU executions are all timed with block size 64.

	CPU						GPU					
Approx. order	1	2	3	4	5	6	1	2	3	4	5	6
Time (ms)	-	-	-	-	-	-	-	-	-	-	-	-

References

[1] G.E. Karniadakis, M. Israeli, and S.A. Orszag, *High-order splitting methods for the incompressible Navier-Stokes equations*, Journal of Computational Physicals, vol. 97, 414-443, 1991.

[2] J.S. Hesthaven and T. Warburton, *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications*, Springer, 2007.

[3] N. Fehn, W.A. Wall, and M. Kronbichler, *On the stability of projection methods for the incompressible Navier–Stokes equations based on high-order discontinuous Galerkin discretizations*, Journal of Computational Physicals, vol. 351, 392-421, 2017.