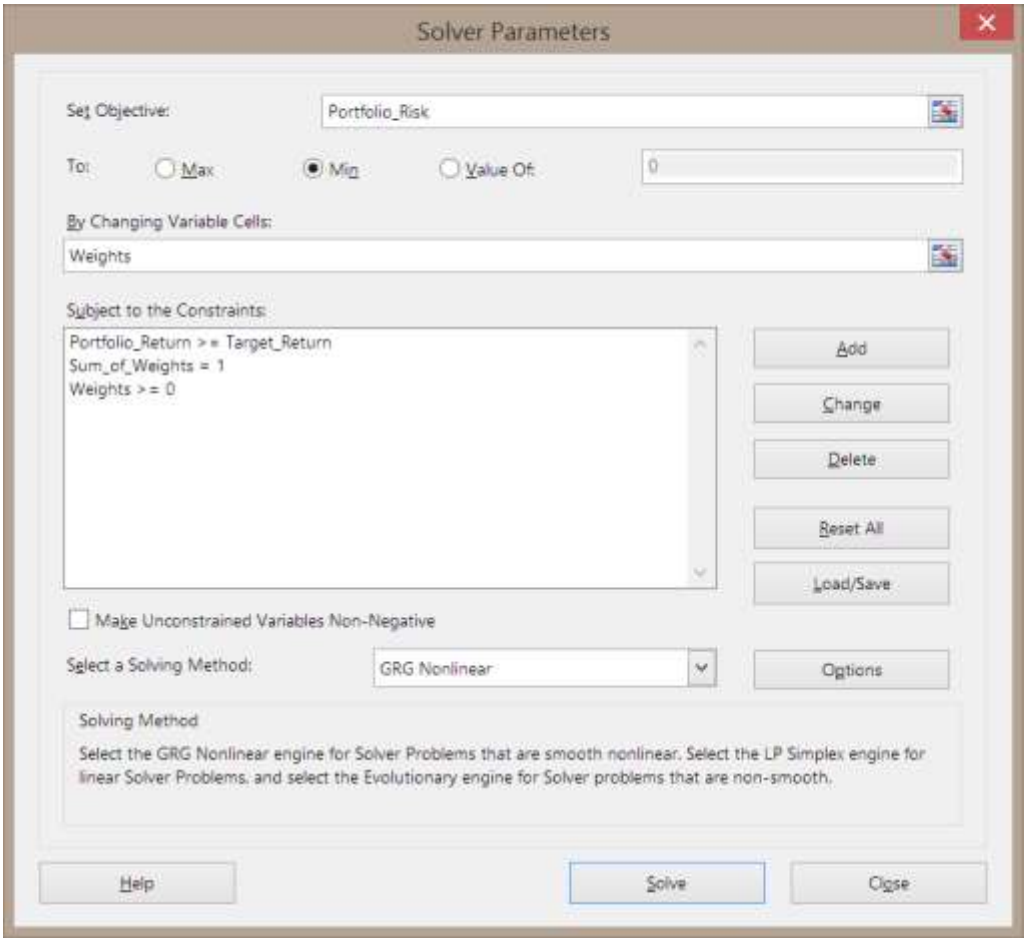
Using Excel's

Solver



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Answer Complex What-If Questions Using Microsoft Excel Solver

Microsoft Excel Solver is a powerful optimization and resource allocation tool. It can help you determine the best uses of scarce resources so that desired goals such as profit can be maximized or undesired goals such as cost can be minimized. Solver answers questions such as:

- What product price or promotion mix will maximize profit?
- How can I live within the budget?
- How fast can we grow without running out of cash?

Instead of guessing over and over, you can use Microsoft Excel Solver to find the best answers.

When to Use Solver

Use Solver when you need to find the optimum value for a particular cell by adjusting the values of several cells, or when you want to apply specific limitations to one or more of the values involved in the calculation.

If you want to find a specific value for a particular cell by adjusting the value of only one other cell, you can also use the Goal Seek command. For information about the Goal Seek command, see Microsoft Excel Help for “Goal Seek”.

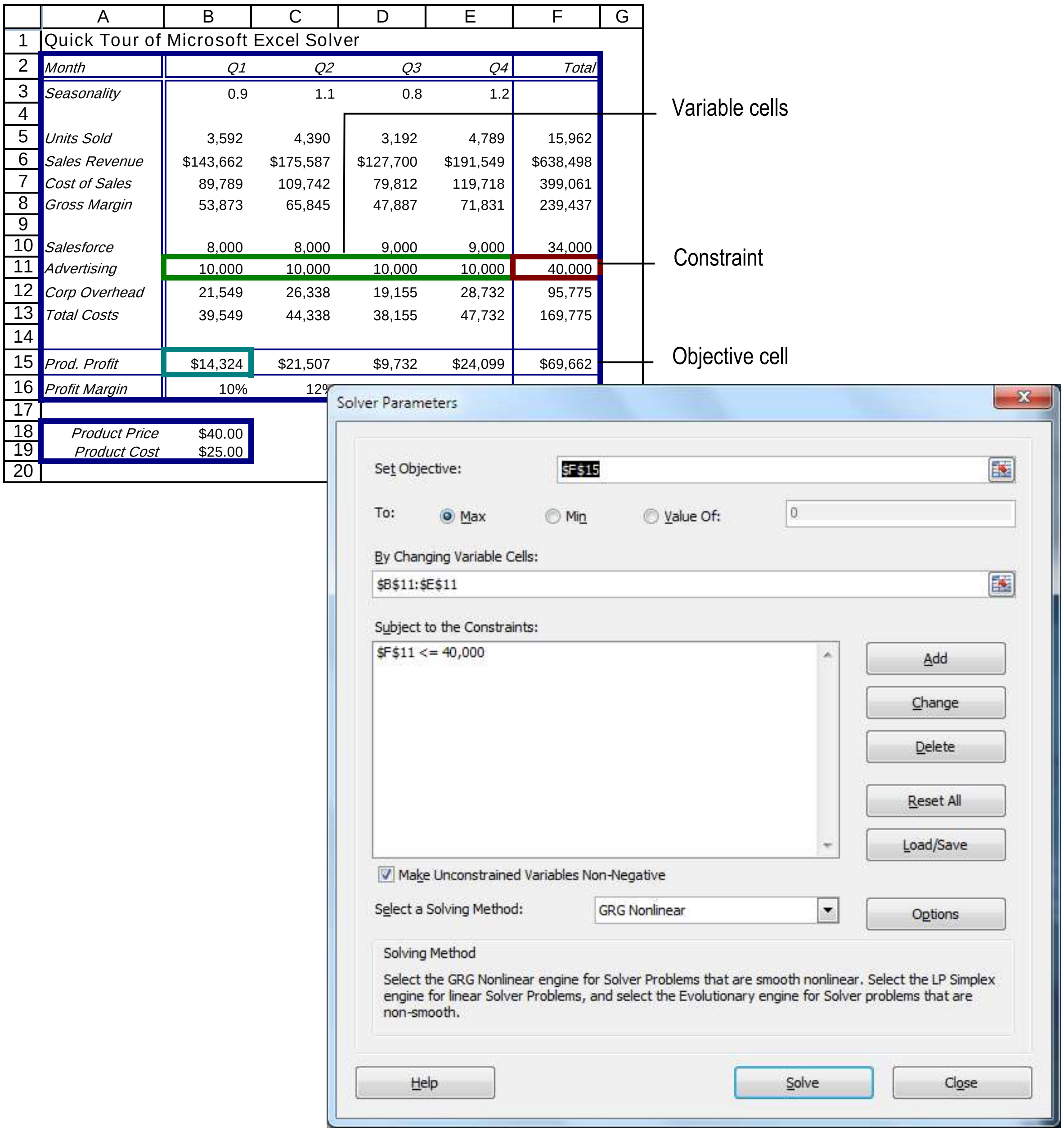
Identifying Key Cells in Your Worksheet

To use Microsoft Excel Solver with your worksheet model, you define a problem that needs to be solved by identifying an Objective, the Variable Cells, and the Constraints that you want used in the analysis.

After you define the problem and start the solution process. Solver finds values that satisfy the constraints and produce the desired value for the objective. Solver then displays the resulting values on your worksheet.

- The *objective* (also called the *objective function*) is the cell in your worksheet model that you want to minimize, maximize, or set to a certain value.
- The *variable cells* (also called *decision variables*) are cells that affect the value of the objective cell. Solver adjusts the values of the variable cells until a solution is found.
- A *constraint* is a cell value that must fall within certain limits or satisfy target values. Constraints may be applied to the objective cell and the variable cells.

Once these items are specified, you are ready to solve the problem. You can optionally adjust other parameters that control the reporting options, precision, and mathematical approach used to arrive at a solution.



Solver Settings Are Persistent

After you define a problem, Solver retains the settings you entered in the Solver Parameters and Solver Options dialog boxes. In a workbook, you can define separate problems on different sheets, and Solver retains the settings you made for each sheet individually. The next time you open the workbook and start Solver, the settings for each appear automatically. You can also try different settings for a problem on a sheet and save each different group of settings as a *model*.

Saving a Model or Scenario

After a solution is found, you can choose to save the current settings as a model using the Save Model button in the Solver Options dialog box.

Saving a model saves the objective, variable cells, constraints, and Solver settings in a range of cells on the worksheet. Each sheet in a workbook can contain a different Solver problem, with as many different models as you want for each problem. Once saved, you can easily load a model by specifying the cell range where it was saved in the Load Model dialog box.

If you use multiple models, you may want to use the Scenario Manager. The changing cells you specify with the Scenario Manager will be suggested automatically as the variable cells in Solver. You can save multiple scenarios for each worksheet using the Save Scenario button in the Solver Results dialog box.

Types of Problems Solver Can Analyze

The three types of optimization problems that the Solver can analyze are:

- Linear
- Nonlinear
- Integer

Linear and nonlinear optimization problems reflect the nature of the relationships between elements of the problem as expressed in formulas on your worksheet. If you know that the problem you are trying to solve is a linear optimization problem or a linear system of equations and inequalities, you can greatly enhance the solution process by selecting the ‘Simplex LP’ Solving Method in the Solver Parameters dialog box. This is particularly important if your problem is large and takes a long time to solve.

Integer problems are created by applying an integer constraint to any variable cell of the problem using the Solver. Use integer constraints when a variable cell used in the problem must take whole number values (not fractional or decimal), such as number of employees, for example. A binary constraint is a special case of integer constraint where a variable cell must be yes or no (1 or 0). Keep in mind that using the integer method can greatly increase the time necessary for Solver to reach a solution.

The Difference Between Linear and Nonlinear Problems

A majority of optimization problems involve *linear* relationship between variables. A linear relationship would be represented by a straight line on a graph. These include problems using simple arithmetic operations, such as:

- Addition and subtraction;
- Built-in functions such as SUM().

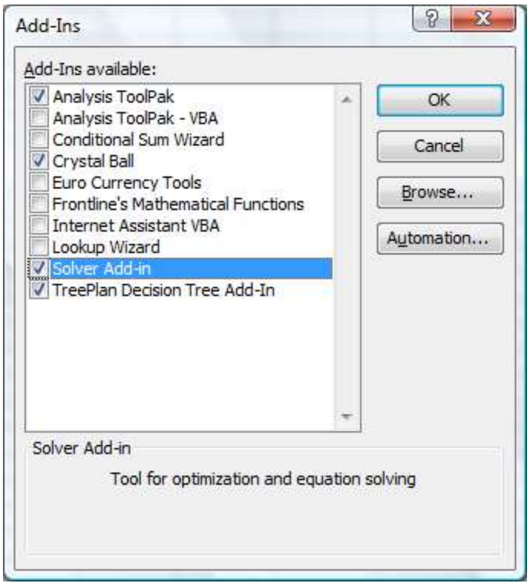
A problem becomes nonlinear when one or more elements share a disproportional relationship to one another. A nonlinear problem would be represented by a curved line on a graph. This can happen when:

- Pairs of variable cells are divided or multiplied by one another;
- Exponentiation is used in the problem;
- You use built-in functions such as GROWTH(), SQRT(), and all logarithmic functions.

Working with Microsoft Excel Solver

Solver Installation

The Solver macro is an add-in, a supplemental program that adds custom commands and features to Microsoft Excel. In Excel 2010 for Windows, the Solver command appears on the Data ribbon, within the Analysis subgroup. In Excel for Mac (and older versions of Windows Excel), Solver appears in the Tools menu.



If the Solver command is not present, go to the Add-Ins command (File > Options > Add-Ins > Go to Manage Excel Add-ins…) to see the list of add-ins currently available. If Solver appears there, make sure that its check box is selected. If Solver does not appear, you need to run the Microsoft Excel Setup program to install the Solver add-in.

Start Solver

Start by opening the worksheet model you want to use and then choose Solver from the Data tab (Tools menu in Excel 2003).

Specify the Objective

In the Set Objective box, enter the reference or name of the cell you want to minimize, maximize or set to a certain value.

- The Objective cell should contain a formula that depends, directly or indirectly, on the variable cells you specify in the By Changing Variable Cells box.
- If the objective cell does not contain a formula, it must also be a variable cell.
- If you don't specify an objective, Solver seeks a solution (adjusts values for the variable cells) that satisfies all of the constraints.

Specify the Variable Cells (Decision Variables)

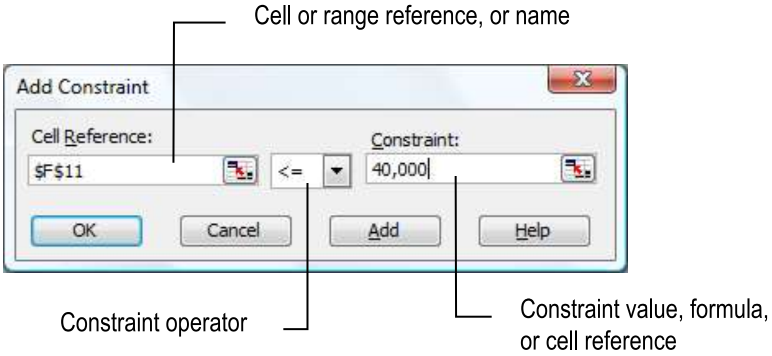
Variable Cells normally contain the key variables of your model whose values you are free to set, such as product prices, amounts to invest, or quantities of a good to produce. Enter in the By Changing Variable Cells box the references or names of the cells you want changed by Solver until the constraints in the problem are satisfied and the objective cell reaches its goal.

- You can specify up to 200 variable cells.
- Entries in the Variable Cells box consist of a reference to a range of cells, or references of several nonadjacent cells separated by commas.

Caution If your variable cells contain formulas, Solver will replace them with constant values if you choose to keep the solution.

Specify the Constraints

Using the Add, Change and Delete buttons in the Solver Parameters dialog box, build your list of constraints in the Subject to the Constraints box.



- Constraints can include upper and lower bounds for any cell in your model including the objective and the variable cells.
- The cell referred to in the Cell Reference box usually contains a formula that depends, directly or indirectly, on one or more of the cells you specify as variable cells.
- When you use the "int" operator, the constrained value is limited to whole numbers (integers) only. When using integer constraints, you can use the Tolerance setting in the Solver Options dialog box to adjust the allowable margin of error.
- When you use the "bin" operator, the constrained value is limited to binary values 0 and 1 only. When using binary constraints, you can use the Tolerance setting in the Solver Options dialog box to adjust the allowable margin of error.
- Only Variable Cells (decision variables) can be restricted to integer or binary values.
- For each problem, you can specify two constraints for each variable cell (one upper-limit and one lower-limit constraint), plus up to 200 additional constraints.

For more information about integer constraints and the Tolerance setting, see "Microsoft Excel Solver Tips and Troubleshooting" later in this document.

Solve the Problem

Pressing the Solve button initiates the problem-solving process.

Problem solution difficulty depends on:

- The size of the model, that is, the number of decision variables and constraints;
- The mathematical relationships between the objective and constraints, and the decision variables;
- The use of integer constraints.

Solver's solution process involves successive trials, or *iterations*. During each iteration, Solver uses a new set of variable cell values to recalculate the worksheet, and examines the constraints and optimum cell values. The process stops when a solution is found to acceptable precision, no further progress is possible, or the maximum time allowed or the maximum number of iterations is reached (see the Solver Options dialog box).

What to Do with the Solver Results

When the problem-solving process ends, a dialog box displays several choices. You can:

- Keep the solution Solver found, or restore the original values in your worksheet;
- Save the solution as a named scenario using the Scenario Manager;
- View any of Solver's built-in reports.

Save the model

You can save your model's settings (cell selections, constraints and options) by choosing the Save Model button in the Solver Options dialog box. When you save a model, Microsoft Excel stores its settings as a range of cells containing formulas. While the latest Solver settings are automatically displayed the next time you open the worksheet, you can save many different models using the Save Model button.

	A	B	C	D	E	F	G	H	I
1	Quick Tour of Microsoft Excel Solver								
2	*Month*	*Q1*	*Q2*	*Q3*	*Q4*	*Total*			
3	*Seasonality*	0.9	1.1	0.8	1.2				
4									
5	*Units Sold*	3,592	4,390	3,192	4,789	15,962			
6	*Sales Revenue*	$143,662	$175,587	$127,700	$191,549	$638,498			
7	*Cost of Sales*	89,789	109,742	79,812	119,718	399,061		$69,662	
8	*Gross Margin*	53,873	65,845	47,887	71,831	239,437		4	
9								TRUE	
10	*Salesforce*	8,000	8,000	9,000	9,000	34,000		100	
11	*Advertising*	10,000	10,000	10,000	10,000	40,000			
12	*Corp Overhead*	21,549	26,338	19,155	28,732	95,775			
13	*Total Costs*	39,549	44,338	38,155	47,732	169,775			
14									
15	*Prod. Profit*	$14,324	$21,507	$9,732	$24,099	$69,662			
16	*Profit Margin*	10%	12%	8%	13%	11%			
17									

Model saved on worksheet

Solver saves a model in a range starting with the cell you specify in the Save Model dialog box. The size of the range depends on the number of constraints you specify.

Tip Naming each model range that you save makes it easier to remember your models and load them later

Loading Solver Models

To replace the current Solver settings with a set of saved settings, choose the Load Model button in the Solver Options dialog box, and enter the name or reference of the range containing the settings you want to load.

Resetting the Solver

If you apply Solver to a problem previously undefined in Solver on a new sheet, the settings of the Solver Parameters dialog box appear in their default state. However, for the Solver Options dialog box, any settings you used during your current Microsoft Excel session remain in effect. You can easily reset them using the Reset All button in the Solver Parameters dialog box.

Solver Solutions and Special Reports

Using Microsoft Excel Solver, you can create three types of reports that summarize the results of the successful solution process.

Microsoft Excel creates each report on a separate sheet in the current workbook. To print the report, switch to the sheet containing the report, and choose the Print command from the File menu.

The Sensitivity Report

The Sensitivity report contains information demonstrating how sensitive a solution is to changes in the formulas used in the problem. There are two versions of this report, depending on whether you selected the Simplex LP solving method in the Solver Parameters dialog box.

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [Solvsamp.xls]Product Mix

Variable Cells

Cell	Name	Final Value	Reduced Gradient
$D$9	Number to Build-> TV set	159.8969096	0
$E$9	Number to Build-> Stereo	200	0
$F$9	Number to Build-> Speaker	80.20618074	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$C$11	Chassis No. Used	359.8969096	0
$C$12	Picture Tube No. Used	159.8969096	0
$C$13	Speaker Cone No. Used	800	6.173269272
$C$14	Power Supply No. Used	359.8969096	0
$C$15	Electronics No. Used	600	14.1451416

Sensitivity report for nonlinear problems

- **Reduced Gradient** Measures the increase in the objective per unit increase in the variable cell.
- **Lagrange Multiplier** Measures the increase in the objective per unit increase of the corresponding constraint.

A different version of the Sensitivity report is generated if you select the Simplex LP solving method in the Solver Parameters dialog box. For this the objective and all constraints must be linear functions of the variable cells.

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [Solvsamp.xls]Product Mix

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$9	Number to Build-> TV set	200	0	75	25	5
$E$9	Number to Build-> Stereo	200	0	50	25	12.5
$F$9	Number to Build-> Speaker	0	-2.5	35	2.5	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$11	Chassis No. Used	400	0	450	1E+30	50
$C$12	Picture Tube No. Used	200	0	250	1E+30	50
$C$13	Speaker Cone No. Used	800	12.5	800	100	100
$C$14	Power Supply No. Used	400	0	450	1E+30	50
$C$15	Electronics No. Used	600	25	600	50	200

Sensitivity report for linear problems

This version of the Sensitivity report adds the following information for each variable cell:

- **Reduced Cost** Replaces reduced Gradient, and measures the increase in the objective per unit increase in the variable cell.
- **Objective Coefficient** Measures the relative (linear) relationship between a variable cell and the objective.
- **Allowable Increase** Shows the increase in the objective coefficient before there would be a change in the optimal value of any of the variable cells.
- **Allowable Decrease** Shows the decrease in the objective coefficient before there would be a change in the optimal value of any of the variable cells.

The following information is added for each constraint cell:

- **Shadow Price** Replaces Lagrange Multiplier, and measures the increase in the objective per unit increase in the right side of the constraint equation.
- **Constraint RH Side (right-hand side)** Lists the constraint values you specified.
- **Allowable Increase** Shows the increase in the Constraint RH Side value before there would be a change in the Shadow Price.
- **Allowable Decrease** Shows the decrease in the Constraint RH Side value before there would be a change in the Shadow Price.

The Answer Report

This report shows:

- The objective (entered in the Set Objective box in the Solver Parameters dialog box);
- The variable cells, with their original and final values;
- The constraints and information about them.

Microsoft Excel 14.0 Answer Report
Worksheet: [Solvsamp.xls]Product Mix

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$18	Total Profits:	$10,095	$14,917

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$9	Number to Build-> TV set	100	160	Contin
$E$9	Number to Build-> Stereo	100	200	Contin
$F$9	Number to Build-> Speaker	100	80	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$11	Chassis No. Used	360	$C$11<=$B$11	Not Binding	90.10309037
$C$12	Picture Tube No. Used	160	$C$12<=$B$12	Not Binding	90.10309037
$C$13	Speaker Cone No. Used	800	$C$13<=$B$13	Binding	0
$C$14	Power Supply No. Used	360	$C$14<=$B$14	Not Binding	90.10309037
$C$15	Electronics No. Used	600	$C$15<=$B$15	Binding	0
$D$9	Number to Build-> TV set	160	$D$9>=0	Not Binding	160
$E$9	Number to Build-> Stereo	200	$E$9>=0	Not Binding	200
$F$9	Number to Build-> Speaker	80	$F$9>=0	Not Binding	80

Information about constraints appears in the Status and Slack columns. These columns tell you how each constraint was met.

In the Status column, one of the following values is displayed:

- **Binding** The final cell value equals the constraint value. For example, if the constraint is C11 <= 400 and the status is Binding, the final cell value would be 400.
- **Not Binding** The constraint is met but does not equal the constraint value. For example; if C11 contains 350 and the constraints is C11 <= 400, the status would be Not Binding.

The Slack column tells you the difference between the value displayed in the cell at solution time and the value of the constraint for that cell. For example, if the constraint is C11 <= 400 and cell C11 contains 350, the slack would be 400 – 350 = 50. When the constraint is binding, the slack value is zero.

The Limits Report

The Limits report lists the objective and the variable cells, with their values, lower and upper limits and objective results.

Microsoft Excel 14.0 Limits Report
Worksheet: [Solvsamp.xls]Product Mix

Cell	Objective Name	Value
$D$18	Total Profits:	$14,917

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$D$9	Number to Build-> TV set	160	0	7698	160	14917
$E$9	Number to Build-> Stereo	200	0	9030	200	14917
$F$9	Number to Build-> Speaker	80	0	13107	80	14917

- **Lower Limit** The smallest value that a variable cell can take while holding all the other variable cells fixed and still satisfying the constraints.
- **Upper Limit** The greatest value that a variable cell can take while holding all the other variable cells fixed and still satisfying the constraints.
- **Objective result** The value of the objective cell when the variable cell is at its lower or upper limit.

Microsoft Excel Solver Tips and Troubleshooting

When Variable Cells and the Objective Differ in Magnitude

To find a solution to a problem involving variable cells that differ from the objective cell by more than one order of magnitude, select the Use Automatic Scaling check box in the Solver Options dialog box. For example, you would select this option if you were planning to invest $10,000,000 in five different stocks and were trying to find the best return on the investment. The variable cells would be in the millions, and the objective cell would be a percentage value, eight or nine orders of magnitude smaller than the variable cells.

Mathematical Approaches Used by Solver

You can use the boxes and radio buttons in the Solver Options dialog box to choose among alternative technical approaches used by Solver at various points in the solution process. The default settings for these options are suitable for nearly all problems. Use of these options is primarily for those experienced in mathematical optimization methods: if you're having difficulty reaching the optimal solution you want, you can experiment with these options in an effort to obtain better results.

When Solver Finds a Solution

Solver will display one of the following messages when it has found a solution to your problem.

- **Solver found a solution** All constraints and optimality conditions are satisfied. All constraints are satisfied to within the precision and integer tolerance settings, and, if appropriate, a maximum, minimum, or target value has been found for the cell in the Set Objective box.

- **Solver has converged to the current solution** All constraints are satisfied. The value in the cell named in the Set Objective box is virtually unchanged for the last five trial solutions. A solution may have been found, but it is also possible that the iterative solution process is making very slow progress and is far from a solution, that the Precision setting (set with the Precision box in the Solver Options dialog box) is too low, or that the initial values for the variable cells were too far from the solution.

When Solver Stops Before a Solution Is Found

While solving a problem, Solver may stop before an optimal solution or even a feasible solution has been found. If this happens, a dialog box with one of the Solver completion messages will appear, and you'll have the choice of keeping the latest values of the variable cells or restoring their former contents.

Some of the reasons this may happen:

- You interrupted the solution process (e.g., by pressing the Esc key).
- You chose the Stop button while stepping through iterations.
- You chose the Stop button when the maximum time or maximum number of iterations was reached.
- The Set Objective value is increasing or decreasing without limit.
- The Simplex LP solving method is selected when the problem is nonlinear.
- You have a complex model containing integer constraints, and you need to adjust the tolerance setting to a higher percentage or increase the Max Time or Iterations setting.
- You need to select the Automatic Scaling option because some input values are several orders of magnitude apart, or input values are different from output values by several orders of magnitude.

Starting from Different Initial Solutions

The ultimate solution can depend on the initial values you supply for the variable cells. Setting the variable cells to values that you suspect are close to optimal can often reduce the solution time. This is especially important if you select the Use Automatic Scaling option or if you have applied integer constraints. If Solver finds a solution that is very different from what you expected, try rerunning Solver with different starting values for the variable cells.

A Case Study Using Microsoft Excel Solver

This section takes you on a quick tour of Microsoft Excel Solver's features. The sample worksheet used in this section is in a workbook called SOLVSAMP.XLS, available within a folder where Microsoft Excel is located, typically: C:\Program Files\Microsoft Office\Office14\Samples\. The file SOLVSAMP.XLS is also available in the electronic course material.

You can open the workbook and solve several problems by following the examples in this section. In this process, you will learn how to:

- Solve for one value or several values to maximize or minimize another value;
- Enter and change constraints;
- Save a problem model.

The following illustration shows the Solver sample worksheet.

	A	B	C	D	E	F	G
1	Quick Tour of Microsoft Excel Solver						
2	*Month*	*Q1*	*Q2*	*Q3*	*Q4*	*Total*	
3	*Seasonality*	0.9	1.1	0.8	1.2		
4							
5	*Units Sold*	3,592	4,390	3,192	4,789	15,962	
6	*Sales Revenue*	$143,662	$175,587	$127,700	$191,549	$638,498	
7	*Cost of Sales*	89,789	109,742	79,812	119,718	399,061	
8	*Gross Margin*	53,873	65,845	47,887	71,831	239,437	
9							
10	*Salesforce*	8,000	8,000	9,000	9,000	34,000	
11	*Advertising*	10,000	10,000	10,000	10,000	40,000	
12	*Corp Overhead*	21,549	26,338	19,155	28,732	95,775	
13	*Total Costs*	39,549	44,338	38,155	47,732	169,775	
14							
15	*Prod. Profit*	$14,324	$21,507	$9,732	$24,099	$69,662	
16	*Profit Margin*	10%	12%	8%	13%	11%	
17							
18	*Product Price*	$40.00					
19	*Product Cost*	$25.00					
20							

Row	**Contains**	**Explanation**
3	Fixed values	Seasonality factor: sales are higher in quarters 2 and 4, and lower in quarters 1 and 3
5	=35*B3*(B11+3000)^0.5	Forecast for units sold each quarter: row 3 contains the seasonality factor; row 11 contains the cost of advertising
6	=B5*$B$18	Sales revenue: forecast for units sold (row 5) times price (cell B18)
7	=B5*$B$19	Cost of sales: forecast for units sold (row 5) times product cost (cell B19)
8	=B6-B7	Gross margin: sales revenues (row 6) minus cost of sales (row 7)
10	Fixed values	Sales personnel expenses
11	Fixed values	Advertising budget
12	=0.15*B6	Corporate overhead expenses: sales revenues (row 6) times 15%
13	=SUM(B10:B12)	Total costs: sales personnel expenses (row 10) plus advertising (row 11) plus overhead (row 12)
15	=B8-B13	Product profit: gross margin (row 8) minus total costs (row 13)
16	=B15/B6	Profit margin: profit (row 15) divided by sales revenue (row 6)
18	Fixed values	Product price
19	Fixed values	Product cost

This is a typical marketing model that shows sales rising from a base figure (perhaps due to the sales personnel) along with increases in advertising, but with diminishing returns. For example, the first $5000 of advertising in Q1 yields about 1092 incremental units sold, but the next $5000 yields only about 775 units more.

You can use Solver to find out whether the advertising budget is too low, and whether advertising should be allocated differently over time to take advantage of the changing seasonality factor.

Solving for a Value to Maximize Another Value

One way you can use Solver is to determine the maximum value of a cell by changing another cell. The two cells must be related through the formulas on the worksheet. If they are not, changing the value in one cell will not change the value in the other cell.

For example, in the sample worksheet, you want to know how much you need to spend on advertising to generate the maximum profit for the first quarter. You are interested in maximizing profit by changing advertising expenditures.

- From the Data tab, choose Solver. In the Set Objective box, type **b15** or select cell B15 (first-quarter profits) on the worksheet. Select the Max option button. In the By Changing Variable Cells box, type **b11** or select B11 (first-quarter advertising) on the worksheet. Choose the Solve button.

You will see messages in the status bar as the problem is set up and Solver starts working. After a moment, you'll see a message that Solver has found a solution. Solver finds that Q1 advertising of $17,093 yields the maximum profit $15,093.

- After you examine the results, select the Restore Original values option button and choose the OK button to discard the results and return cell B11 to its former value.

Resetting the Solver Options

If you want to return the options in the Solver Parameters dialog box to the blank settings so that you can start a new problem, you can use the Reset All button.

Solving for a Value by Changing Several Values

You can also use Solver to solve for several values at once to maximize or minimize another value. For example, in the sample worksheet, you can solve for the advertising budget for each quarter that will result in the best profits for the entire year. Since the seasonality factor in row 3 enters into the calculation of unit sales in row 5 as a multiplier, it seems logical that you should spend more of your advertising budget in Q4 when the sales response is highest, and less in Q3 when the sales response is lowest. Use Solver to determine the best quarterly allocation.

- From the Data tab, choose Solver. In the Set Objective box, type **f15** or select cell F15 (total profits for the year) on the worksheet. Make sure the Max option button is selected. In the By Changing Variable Cells box, type **b11:e11** or select cells B11:E11 (the advertising budget for each of the four quarters) on the worksheet. Choose the Solve button.
- After you examine the results, select the Restore Original Values option button and choose the OK button to discard the results and return all cells to their former values.

You've just asked Solver to solve a moderately complex nonlinear optimization problem –that is, to find values for the four unknowns in cells B11 through E11 that will maximize profits. (This is a nonlinear problem because of the exponentiation that occurs in the formulas in row 5.) The results of this unconstrained optimization show that you can increase profits for the year to $79,706 if you spend $89,706 in advertising for the full year.

However, most realistic modeling problems have limiting factors that you will want to apply to certain values. These constraints may be applied to the objective, the variable cells or any other value that is related to the formulas in these cells.

Adding a Constraint

So far, the budget recovers the advertising cost and generates additional profit, but you're reaching a point of diminishing returns. Since you can never be sure that your model of sales response to advertising will be valid next year (especially at greatly increased spending levels), it doesn't seem prudent to allow unrestricted spending on advertising.

Suppose you want to maintain your original advertising budget of $40,000. Add the constraint to the problem that limits the sum of advertising during the four quarters to $40,000.

- From the Data tab, choose Solver. Choose the Add button. The Add Constraint dialog box appears. In the Cell Reference box, type **f11** or select cell F11 (advertising total) on the worksheet. Cell F11 must be less than or equal to $40,000. The relationship in the Constraint box is "<=" (less than or equal to) by default so you don't have to change it. In the box next to the relationship, type **40000**. Choose the OK button. Choose the Solve button.
- After you examine the results, select the Restore Original Values option button and choose the OK button to discard the results and return the cells to their former values.

The solution found by Solver allocates amounts ranging from $5117 in Q3 to $15,263 in Q4. Total Profit has increased from $69,662 in the original budget to $71,447, without any increase in the advertising budget.

Changing a Constraint

When you use Solver, you can experiment with slightly different parameters to decide the best solution to a problem. For example, you can change a constraint to see whether the results are better or worse than before. In the sample worksheet, try changing the constraint on advertising dollars to $50,000 to see what that does to total profits.

- From the Data tab, choose Solver. The constraint $F$11 <= 40000, should already be selected in the Subject to the Constraints box. Choose the Change button. In the Constraint box, change 40000 to 50000. Choose the OK button. Choose the Solve button. Select the Keep solver Solution option button and choose the OK button to keep the results that are displayed on the worksheet.

Solver finds an optimal solution that yields a total profit of $74,817. That's an improvement of $3,370 over the last figure of $71,447. In most firms, it's not too difficult to justify an incremental investment of $10,000 that yields an additional

$3,370 in profit or a 33.7% return on investment. This solution also results in profits of $4,889 less than the unconstrained result, but you spend $39,706 less to get there.

Saving a Problem Model

When you choose Save from the File menu, the last selections made in the Solver Parameters dialog box are attached to the worksheet and retained when you save the workbook. However, you can define more than one problem for a worksheet by saving them individually using the Load/Save Model button in the Solver Parameters dialog box. Each problem model consists of cells and constraints that you entered in the Solver Parameters dialog box.

When you choose the Save Model button, the Save Model dialog box appears with a default selection, based on the active cell, as the area for saving the model. The suggested range includes a cell for each constraint plus three additional cells. Make sure that this cell range is an empty range on the worksheet.

- From the Data tab, choose Solver. Choose the Options button. Choose the Save Model button. In the Select Model Area box, type **c17:c20** or select cells C17:C20 on the worksheet. Choose the OK button.

Note You can also enter a reference to a single cell in the Select Model Area box. Solver will use this reference as the upper-left corner of the range into which it will copy the problem specifications.

To load these problem specifications later, choose the Load Model button in the Solver Options dialog box, then type **c17:c20** in the Model Area box or select cells C17:C20 on the sample worksheet and choose the OK button. Solver displays a message asking if you want to reset the current Solver option settings with the settings for the model you are loading. Choose the OK button to proceed.

Microsoft Excel Solver Sample Worksheets

This section gives brief introductory information about the sample worksheets included for use with Solver.

The sample worksheets are available in the SOLVSAMP.XLS workbook typically located in a folder within the directory where Microsoft Excel is located. This file is also available among the electronic material supplied for the course.

When you open this workbook, and then switch to one of the worksheets and choose the Solver command, you will see that the objective, variable cells, and constraints are already specified for that worksheet.

The Most Profitable Product Mix

The sample worksheet named Product Mix provides data for several products using common parts, each with a different profit margin per unit. Parts are limited, so your problem is to determine the number of each product to build from the inventory on hand in order to maximize profits.

Problem Specifications

Objective	D18	Goal is to maximize profit
Variable cells	D9:F9	Units of each product to build
Constraints	C11:C15 <= B11:B15	Number of parts used must be less than or equal to the number of parts in inventory
	D9:F9 >= 0	Number to build value must be greater than or equal to 0

The formulas for profit per product in cells D17:F17 include the factor ^H15 to show that profit per unit diminishes with volume. H15 contains 0.9 which makes the problem nonlinear. If you change H15 to 1.0 to indicate that profit per unit remains constant with volume, and then choose the Solve button again, the optimal solution will change. This change also makes the problem linear.

The Least Costly Shipping Routes

The problem presented on the sample worksheet named Shipping Routes involves the shipment of goods from three plants to five regional warehouses. Goods can be shipped from any plant to any warehouse, but it obviously costs more to ship goods over long distances than over short distances. The problem is to determine the amounts to ship from each plant to each warehouse at minimum shipping cost in order to meet the regional demand, while not exceeding the plant supplies.

Problem Specifications

Objective	B20	Goal is to minimize total shipping cost
Variable cells	C8:G10	Amount to ship from each plant to each warehouse
Constraints	B8:B10 <= B16:B18	Total shipped must be less than or equal to supply at plant
	C12:G12 >= C14:G14	Totals shipped to warehouses must be greater than or equal to demand at warehouses
	C8:G10 >= 0	Number to ship must be greater than or equal to 0

You can solve this problem faster by selecting the Simplex LP solving method in the Solver Parameters dialog box before choosing the Solve button. A problem of this type will have an optimum solution in which amounts to ship are integers, if all of the supply and demand constraints are integers.

Staff Scheduling at Minimum Cost

The goal for the sample worksheet named Staff Scheduling is to schedule employees so that you have sufficient staff at the lowest cost. In this example, all employees are paid at the same rate, so by minimizing the number of employees working each day, you also minimize costs. Each employee works five consecutive days, followed by two days off.

Problem Specifications

Objective	D20	Goal is to minimize payroll cost
Variable cells	D7:D13	Employees on each schedule
Constraints	D7:D13 >= 0	Number of employees must be greater than or equal to 0
	D7:D13 = Integer	Number of employees must be an integer
	F15:L15 >= F17:L17	Employees working each day must be greater than or equal to the demand
Possible schedules	Rows 7-13	1 means employee on that schedule works that day

In this example, you use an integer constraint so that your solutions do not result in fractional numbers of employees on each schedule. Selecting the Simplex LP solving method in the Solver Parameters dialog box before you choose the Solve button will greatly speed up the solution process.

Maximizing Income from Working Capital

If you're a financial officer or a manager, one of your tasks is to manage cash and short-term investments in a way that maximizes interest income, while keeping funds available to meet expenditures. You must trade off the higher interest rates available from longer-term investments against the flexibility provided by keeping funds in short-term investments.

The sample worksheet named Maximizing Income calculates ending cash based on initial cash (from the previous month), inflows from maturing certificates of deposit (CDs), outflows for new CDs, and cash needed for company operations for each month.

You have a total of nine decisions to make: the amounts to invest in one-month CDs in months 1 through 6; the amounts to invest in three-month CDs in months 1 and 4; and the amount to invest in six-month CDs in month 1.

Problem Specifications

Objective	H8	Goal is to maximize interest earned
Variable cells	B14:G14 B15, E15, B16	Dollars invested in each type of CD
Constraints	B14:G14 >= 0 B15:B16 >= 0 E15 >= 0	Investment in each type of CD must be greater than or equal to 0
	B18:H18 >= 100000	Ending cash must be greater than or equal to $100,000
Amount to invest	B11	$400,000
Cash use aside from investments	Row 17	

The optimal solution determined by Solver earns a total interest income of $16,531 by investing as much as possible in six-month and three-month CDs and then turns to one-month CDs. This solution satisfies all of the constraints.

Suppose, however, that you want to guarantee that you have enough cash in month 5 for an equipment payment. Add a constraint that the average maturity of the investments held in month 1 should not be more than four months.

The formula in cell B20 computes a total of the amounts invested in month 1 (B14, B15 and B16) weighted by the maturities (1, 3 and 6 months), and then it subtracts from this amount the total investment, weighted by 4. If this quantity is zero or less, the average maturity will not exceed four months. To add this constraint, choose Solver from the Data tab, then choose the Add button. Type **b20** in the cell Reference box, type **0** in the Constraint box, and then choose the OK button. To solve the problem, choose the Solve button.

To satisfy the four-month maturity constraint, Solver shifts funds from six-month CDs to three month CDs. The shifted funds now mature in month 4 and, according to the present plan, are reinvested in new three-month CDs. If you need the funds, however, you can keep the cash instead of reinvesting. The $56,896 turning over in month 4 is more than sufficient for the equipment payment in month 5. You've traded about $460 in interest income to gain this flexibility.

An Efficient Portfolio of Securities

One of the basic principles of investment management is diversification. By holding a portfolio of several stocks, for example, you can earn a rate of return that represents the average of the returns from the individual stocks, while reducing your risk that any one stock will perform poorly.

Using the sample worksheet named Portfolio of Securities, you can use Solver to find the allocation of funds to stocks that minimizes the portfolio risk for a given rate of return, or that maximizes the rate of return for a given level of risk.

This worksheet contains figures for beta (market-related risk) and residual variance for four stocks. In addition, your portfolio includes investments in treasury bills (T-bills), assumed to have a risk-free rate of return and a variance of zero. Initially, equal amounts (20 percent of the portfolio) are invested in each security.

Use solver to try different allocations of funds to stocks and T-bills to either maximize the portfolio rate of return for a specified level of risk or minimize the risk for a given rate of return. With the initial allocation of 20 percent across the board, the portfolio return is 16.4 percent and the variance is 0.071.

Problem Specifications		
Objective	E18	Goal is to maximize Portfolio return
Variable cells	E10:E14	Weight of each stock
Constraints	E10:E14 >= 0	Weights must be greater than or equal to 0
	E16 = 1	Sum of weights must equal 1
	G18 <= 0.071	Variance must be less than or equal to 0.071
Beta for each stock	B10:B13	
Variance for each stock	C10:C13	

Cells D21:D29 contain the problem specifications to minimize risk for a required rate of return of 16.4 percent. To load these problem specifications into Solver, choose Solver from the Data tab, choose the Options button, choose the Load Model button, select cells D21:D29 on the worksheet, and then choose the OK button until the Solver Parameters dialog box is displayed. Choose the Solve button. As you can see, Solver finds portfolio allocations in both cases that surpass the rule of 20 percent across the board.

You can earn a higher rate of return (17.1 percent) for the same risk, or you can reduce your risk without giving up any return. These two allocations both represent efficient portfolios.

Cells A21:A29 contain the original problem model. To reload this problem, choose Solver from the Data tab, choose the Options button, choose the Load Model button, select cells A21:A29 on the worksheet, and then choose the OK button. Solver displays a message asking if you want to reset the current Solver option settings with the settings for the model you are loading. Choose the OK button to proceed.

Further References

Further information on Solver may be obtained from the following sources:

- On-line Solver tutorial: http://www.solver.com/tutorial.htm.

- Mac users: www.solver.com/welcome-mac-users-solver-now-included-excel-2011.

- Frontline Systems, Inc. developers of Excel Solver: www.solver.com.

- Article: "Design and Use of the Microsoft Excel Solver" by Daniel Fylstra, Leon Lasdon, John Watson, Allan Waren, *Interfaces* Vol. 28, 1998, pp. 29-55.