Optimization I

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Notification

- During this lecture we will need Excel and its solver add-in

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Resource Allocation

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Red Brand Canners

- Canner and distributor of fruits and vegetables in West USA
- Starring executives

- Meeting

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Supply & Demand

- Supply (Tucker)
- Demand forecasts (Myers)

Product	Selling Price per Case	Demand Forecast (Cases)
whole tomatoes	$ 12.00	800,000
tomato juice	$ 13.50	50,000
tomato paste	$ 11.40	80,000

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Management Accounting

- Profit contributions (Cooper)

Product	Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$ 12.00	$ 13.50	$ 11.40
Direct labour	$ 3.54	$ 3.96	$ 1.62
Variable overhead	$ 0.72	$ 1.08	$ 0.78
Variable selling	$ 1.20	$ 2.55	$ 1.14
Packaging material	$ 2.10	$ 1.95	$ 2.31
Fruit	$ 3.24	$ 3.60	$ 4.50
Total Variable Costs	$ 10.80	$ 13.14	$ 10.35
Contribution	$ 1.20	$ 0.36	$ 1.05
Allocated Overhead	$ 0.84	$ 0.63	$ 0.69
Net Profit	**$ 0.36**	**-$ 0.27**	**$ 0.36**

Product	Pounds per Case
whole tomatoes	18
tomato juice	20
tomato paste	25

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Quality

- Quality requirements limit production (Tucker)
- RBC quality scale: 0 (low quality) – 10 (high quality)
- Tomato grades
- Product quality requirements
- Conclusions: “Whole tomato production is limited to 800,000 pounds” (see “Maximum Whole tomatoes.xls”)
- Extra supply (Gordon)

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Tomato Cost

- Myers does not agree with Coopers
- Tomato Cost (see “Myers Tomato Cost.xlsx”)
- Production Cost

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Revised Tomato Cost

Product	Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$ 12.00	$ 13.50	$ 11.40
Direct labour	$ 3.54	$ 3.96	$ 1.62
Variable overhead	$ 0.72	$ 1.08	$ 0.78
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Myers' Recommendation

Product	Selling Price per Case	Demand Forecast (Cases)	Pounds per Case
whole tomatoes	$ 12.00	800,000	18
tomato juice	$ 13.50	50,000	20
tomato paste	$ 11.40	80,000	25

Evaluation of Myer's Analysis

1. Using 400,000 pounds of grade B and all grade A (600,000 pounds) for tomato juice implies that the average quality of the tomatoes used in the juice is

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Revised Profit Contributions

Product	Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$ 12.00	$ 13.50	$ 11.40
Direct labour	$ 3.54	$ 3.96	$ 1.62
Variable overhead	$ 0.72	$ 1.08	$ 0.78
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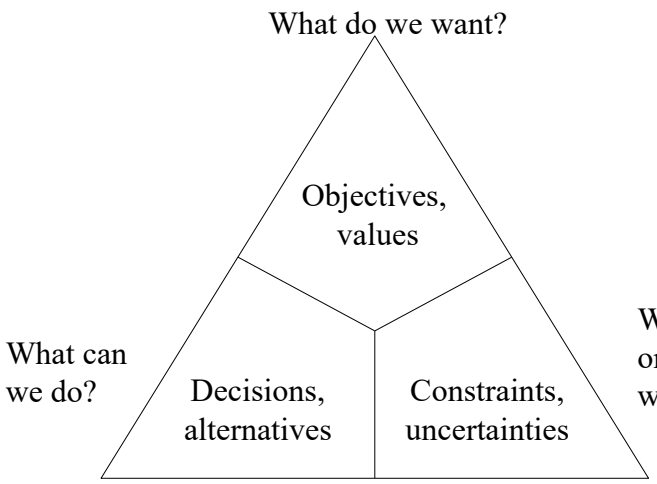
Decision Ingredients in Optimization Problems



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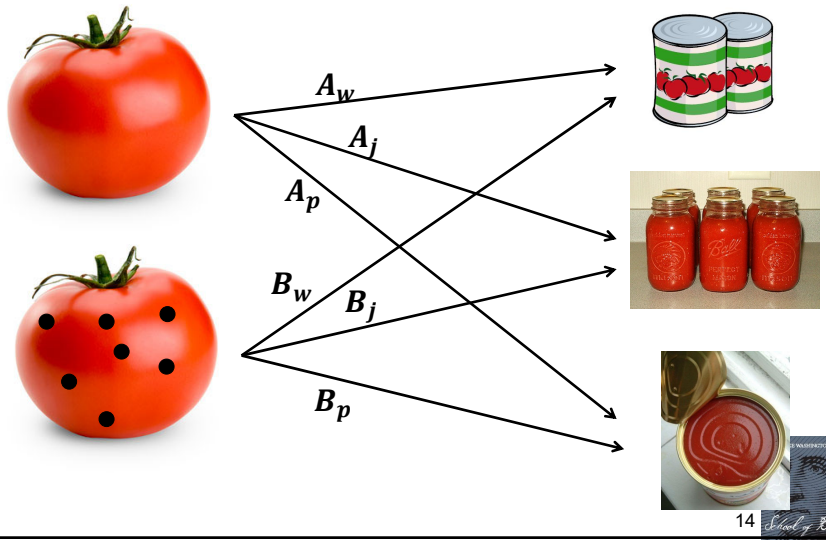
Decisions?



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Algebraic Model

- Maximize profit contribution
  Subject to
  Demand constraints
  Supply constraints
  Quality constraints
  Non-negativity constraints
- Maximize
  Subject to

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Excel Model (with Myers' suggestion RBC.xls)

RED BRAND CANNERS

MIX DECISION	Whole	Juice	Paste	Total Required	Available
Grade A	0	600	0	600	600
Grade B	0	400	2,000	2,400	2,400
Total Production	0	1,000	2,000		
Demand	14,400	1,000	2,000		

QUALITY	Whole	Juice	Paste	Quality
Grade A	0	5,400	0	9
Grade B	0	2,000	10,000	5
Total Quality	0	7,400	10,000	
Required Total Quality	0	6,000	10,000	
Average Quality	#DIV/0!	7.4	5.0	
Required Average Quality	8.0	6.0	5.0	

PROFIT	Whole	Juice	Paste	Total Contribution	Total Profit
Contribution Margin	$246.67	$198	$222	$642,000	$102,000

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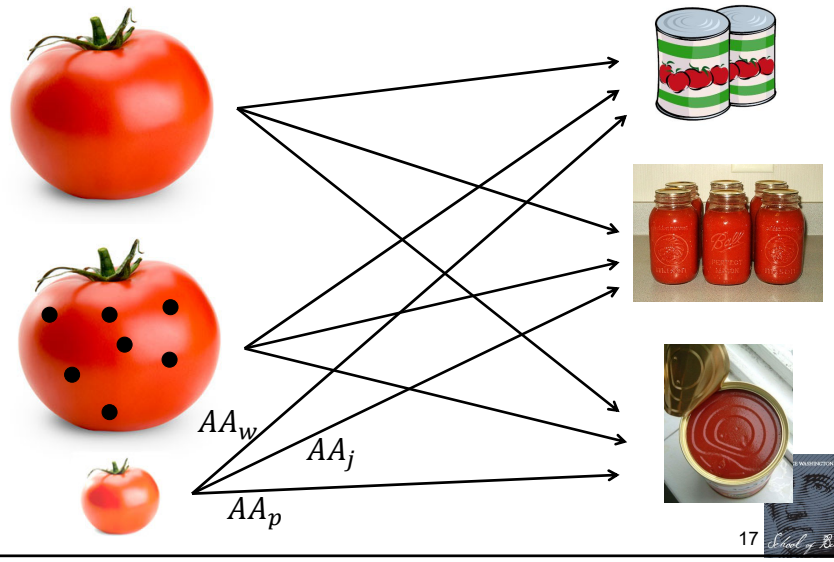
Additional Grade A Tomatoes?



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Objective with AA Tomatoes

- Maximize

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Optimal Solution with AA tomatoes

RED BRAND CANNERS

MIX DECISION	Whole	Juice	Paste	Total Required	Available
Grade A	600	0	0	600	600
Grade AA	15	65	0	80	80
Grade B	205	195	2,000	2,400	2,400
Total Production	820	260	2,000		
Demand	14,400	1,000	2,000		

QUALITY	Whole	Juice	Paste	Quality
Grade A	5,400	0	0	9
Grade AA	135	585	0	9
Grade B	1,025	975	10,000	5
Total Quality	6,560	1,560	10,000	
Required Total Quality	6,560	1,560	10,000	
Average Quality	8.0	6.0	5.0	
Required Average Quality	8.0	6.0	5.0	

PROFIT	Whole	Juice	Paste	Total Contribution	Total Profit
Contribution Margin	$246.67	$198	$222	$681,177	
Contr. Margin AA	-$8.33	-$57.00	-$33.00	-$3,830	
				$677,347	$137,347

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Sensitivity Analysis (Original Model without Extra A Tomatoes)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Grade A Whole	525	0	246.6666667	463.1111111	64.88888889
C4	Grade A Juice	75	0	198	64.88888889	463.1111111
D4	Grade A Paste	0	0	222	97.33333333	1E+30
B5	Grade B Whole	175	0	246.6666667	1389.333333	64.88888889
C5	Grade B Juice	225	0	198	42.96296296	154.3703704
D5	Grade B Paste	2000	0	222	1E+30	48.33333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B12	Total Quality Whole	5600	-24.33	0	466.6666667	600
C12	Total Quality Juice	1800	-24.33	0	1400	200
D12	Total Quality Paste	10000	-24.33	0	1400	0
E4	Grade A Total Required	600	271.00	600	600	466.6666667
E5	Grade B Total Required	2400	173.67	2400	466.6666667	200
B6	Total Production Whole	700	0.00	14400	1E+30	13700
C6	Total Production Juice	300	0.00	1000	1E+30	700
D6	Total Production Paste	2000	48.33	2000	200	466.6666667

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Shadow Price = additional profit generated by 1(000) pounds extra A tomatoes, exceeds cost

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D5	Grade B Paste	2000	0	222	1E+30	48.33333333

If additional grade A tomatoes are purchased (80,000 pounds). How would the profit contribution change?

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B12	Total Quality Whole	5600	-24.33	0	466.6666667	600
C12	Total Quality Juice	1800	-24.33	0	1400	200
D12	Total Quality Paste	10000	-24.33	0	1400	0
E4	Grade A Total Required	600	271.00	600	600	466.6666667
E5	Grade B Total Required	2400	173.67	2400	466.6666667	200
B6	Total Production Whole	700	0.00	14400	1E+30	13700
C6	Total Production Juice	300	0.00	1000	1E+30	700
D6	Total Production Paste	2000	48.33	2000	200	466.6666667

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Key Takeaways

- Overhead and sunk costs are irrelevant for decision making
- Formulate model
  - What do we want? -> objective (goal, values)
  - What can we do? -> decisions (decision variables)
  - What limits our decision making? -> constraints
- Do not make decision solely on intuition (e.g., Cooper, Myers' accounting data, extra A tomatoes with negative contributions)
- Use optimization (Excel's solver in simple problems, Python with Gurobi)
  - For linear optimization problems, optimality of the solution can be guaranteed
  - If negative values do not make sense, add constraint >=0
  - Use sensitivity analysis to price additional resources

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