Optimization I

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Notification

- During this lecture we will need Excel and its solver add-in

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Resource Allocation

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Red Brand Canners

- Canner and distributor of fruits and vegetables in West USA
- Starring executives
  - Mitchell Gordon, VP of operations
  - William Cooper, Controller
  - Charles Myers, Sales Manager
  - Dan Tucker, Production Manager
- Meeting
  - Tomato crop has already been bought and was beginning to arrive
  - Amount of tomato products to pack for the coming season?
  - Packing operations start Monday

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Supply & Demand

- Supply (Tucker)
  - Quantity: 3,000,000 pounds (lb)
  - Quality: 20% grade A, 80% grade B
  - 18 cents / pound
- Demand forecasts (Myers)
  - "Selling prices are set in light of the long-term marketing strategy of the company, potential sales are forecast at these prices"
  - "We can sell all the whole tomatoes we can produce"
  - "Demand for tomato juice and paste is limited"

Product	Selling Price per Case	Demand Forecast (Cases)
whole tomatoes	$ 12.00	800,000
tomato juice	$ 13.50	50,000
tomato paste	$ 11.40	80,000

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Management Accounting

- Profit contributions (Cooper)
  - "Incremental profit is greatest for whole tomatoes"

Product	Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$ 12.00	$ 13.50	$ 11.40
Direct labour	$ 3.54	$ 3.96	$ 1.62
Variable overhead	$ 0.72	$ 1.08	$ 0.78
Variable selling	$ 1.20	$ 2.55	$ 1.14
Packaging material	$ 2.10	$ 1.95	$ 2.31
Fruit	$ 3.24	$ 3.60	$ 4.50
Total Variable Costs	$ 10.80	$ 13.14	$ 10.35
Contribution	$ 1.20	$ 0.36	$ 1.05
Allocated Overhead	$ 0.84	$ 0.63	$ 0.69
Net Profit	**$ 0.36**	**-$ 0.27**	**$ 0.36**

Product	Pounds per Case
whole tomatoes	18
tomato juice	20
tomato paste	25

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Quality

- Quality requirements limit production (Tucker)
  - "Impossible to produce all whole tomatoes because too small portion of the crop is grade A"
- RBC quality scale: 0 (low quality) – 10 (high quality)
- Tomato grades
  - Grade A 9 points
  - Grade B 5 points
- Product quality requirements
  - Whole canned tomatoes 8 points
  - tomato juice 6 points
  - Tomato paste 5 points
- Conclusions: "Whole tomato production is limited to 800,000 pounds" (see "Maximum Whole tomatoes.xls")
- Extra supply (Gordon)
  - "Additional 80,000 pounds grade A tomatoes available at 25.50 cents per pound"

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Tomato Cost

- Myers does not agree with Coopers
  - Whole canned tomatoes look attractive, but require more grade A tomatoes (higher quality, fewer available) compared to other products
  - "Tomato cost has to be allocated on the basis of quantity and quality" (different tomato cost for different products)
  - Computes cost of A and B tomatoes based on their quality (quality 10 tomatoes should cost twice as much as quality 5 tomatoes)
- Tomato Cost (see "Myers Tomato Cost.xlsx")
  - A tomatoes (9 points) = 27.93 cents/pound
  - B tomatoes (5 points) = 15.52 cents/pound
- Production Cost
  - whole (8 points) = 8/9*27.93 = 24.83 cents/pound
  - juice (6 points) = 6/9*27.93 = 18.62 cents/pound
  - paste (5 points) = 5/9*27.93 = 15.52 cents/pound

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Revised Tomato Cost

Production Cost

- whole = 24.83 cents/pound = 24.83*18 cents/case = $4.47/case
- juice = 18.62 cents/pound = $3.72/case
- paste = 15.52 cents/pound = $3.88/case

Product	Pounds per Case
whole tomatoes	18
tomato juice	20
tomato paste	25

Product	Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$ 12.00	$ 13.50	$ 11.40
Direct labour	$ 3.54	$ 3.96	$ 1.62
Variable overhead	$ 0.72	$ 1.08	$ 0.78
Variable selling	$ 1.20	$ 2.55	$ 1.14
Packaging material	$ 2.10	$ 1.95	$ 2.31
Fruit	$ 3.24 $4.47	$ 3.60 $3.72	$ 4.50 $3.88
Total Variable Costs	$ 10.80	$ 13.14	$ 10.35
Contribution	$ 1.20 -$0.03	$ 0.36 $0.24	$ 1.05 $1.67
Allocated Overhead	$ 0.84	$ 0.63	$ 0.69
Net Profit	**$ 0.36** -$0.87	**-$ 0.27** -$0.39	**$ 0.36** $0.98

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Myers' Recommendation

- "Make all the tomato paste we can!"
- "Use 2,000,000 pounds of grade B tomatoes to make 80,000 cases of tomato paste (satisfy demand)."
- "Use the remaining 400,000 pounds of B tomatoes and the 600,000 grade A tomatoes to make 50,000 cases of tomato juice (satisfy demand),"
- Total profit contribution of paste = $133,600 (80,000*$1.67)

  Total profit contribution of juice = $12,000 (50,000*$0.24)

  Total profit contribution = $145,600!

Product	Selling Price per Case	Demand Forecast (Cases)	Pounds per Case
whole tomatoes	$ 12.00	800,000	18
tomato juice	$ 13.50	50,000	20
tomato paste	$ 11.40	80,000	25

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Evaluation of Myer's Analysis

1. Using 400,000 pounds of grade B and all grade A (600,000 pounds) for tomato juice implies that the average quality of the tomatoes used in the juice is

$$\frac{9(600{,}000)+5(400{,}000)}{1{,}000{,}000}=7.4\ \geq 6$$

This implies a tomato cost of 7.4/9 * 27.93=22.96 cents/pound ($4.59 per case), whereas he used a cost of 18.62 cents/pound ($3.72 per case) for computing the contribution of juice! Therefore, the actual contribution of juice (per case) will not be $0.24 but $0.24+($3.72-$4.59)=-$0.63!
Therefore, the predicted profit contribution will not be $145,600, but only $102,000!

2. TOMATO COST IS SUNK COST
$540,000 are spent on tomatoes, and no decision will affect this cost outlay (if we do not produce anything, the tomatoes will still have to be paid for). Disregard the tomato costs for decision making!

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Revised Profit Contributions

Product	Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$ 12.00	$ 13.50	$ 11.40
Direct labour	$ 3.54	$ 3.96	$ 1.62
Variable overhead	$ 0.72	$ 1.08	$ 0.78
Variable selling	$ 1.20	$ 2.55	$ 1.14
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Contribution	$ 1.20	$ 0.36	$ 1.05
Allocated Overhead	$ 0.84	$ 0.63	$ 0.69
Net Profit	**$ 0.36**	**-$ 0.27**	**$ 0.36**

We need to add fruit cost back:

Product	Pounds per Case
whole tomatoes	18
tomato juice	20
tomato paste	25

$4.44/case → **$0.247 per lbs.** → **$246.67** per 1000 lbs.

$3.96/case → **$0.198 per lbs.** → **$198** per 1000 lbs.

$5.55/case → **$0.222 per lbs.** → **$222** per 1000 lbs.

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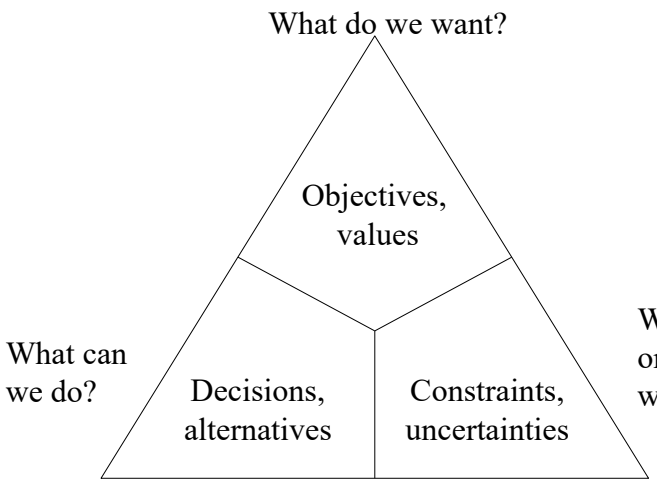
Decision Ingredients in Optimization Problems



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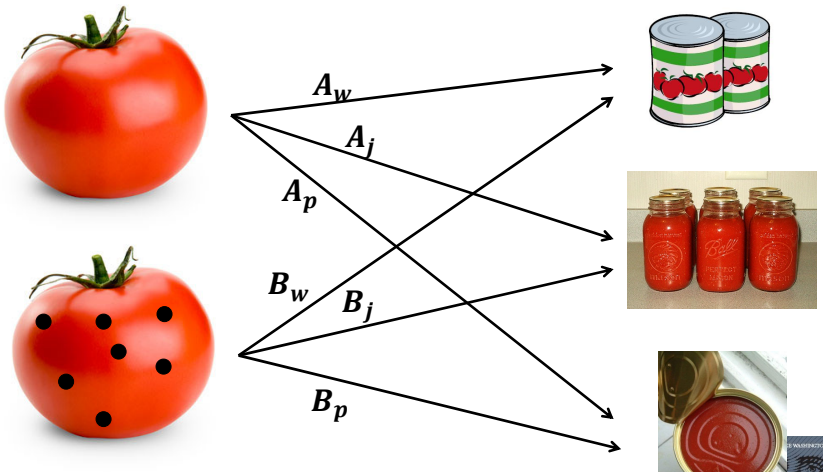
Decisions?



per 1000 lbs. tomatoes utilized

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Algebraic Model

- Maximize profit contribution
  Subject to
  Demand constraints
  Supply constraints
  Quality constraints
  Non-negativity constraints

- Maximize $246.67A_w + 198A_j + 222A_p + 246.67B_w + 198B_j + 222B_p$
  Subject to

$$A_w + B_w \leq 14{,}400$$
$$A_j + B_j \leq 1{,}000$$
$$A_p + B_p \leq 2{,}000$$
$$A_w + A_j + A_p \leq 600$$
$$B_w + B_j + B_p \leq 2{,}400$$
$$9A_w + 5B_W \geq 8(A_w + B_w)$$
$$9A_j + 5B_j \geq 6(A_j + B_j)$$

$$A_w, B_w, A_j, B_j, A_p, B_p \geq 0$$

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Excel Model (with Myers' suggestion RBC.xls)

RED BRAND CANNERS

MIX DECISION	Whole	Juice	Paste	Total Required	Available
Grade A	0	600	0	600	600
Grade B	0	400	2,000	2,400	2,400
Total Production	0	1,000	2,000		
Demand	14,400	1,000	2,000		

QUALITY	Whole	Juice	Paste	Quality
Grade A	0	5,400	0	9
Grade B	0	2,000	10,000	5
Total Quality	0	7,400	10,000	
Required Total Quality	0	6,000	10,000	
Average Quality	#DIV/0!	7.4	5.0	
Required Average Quality	8.0	6.0	5.0	

PROFIT	Whole	Juice	Paste	Total Contribution	Total Profit
Contribution Margin	$246.67	$198	$222	$642,000	$102,000

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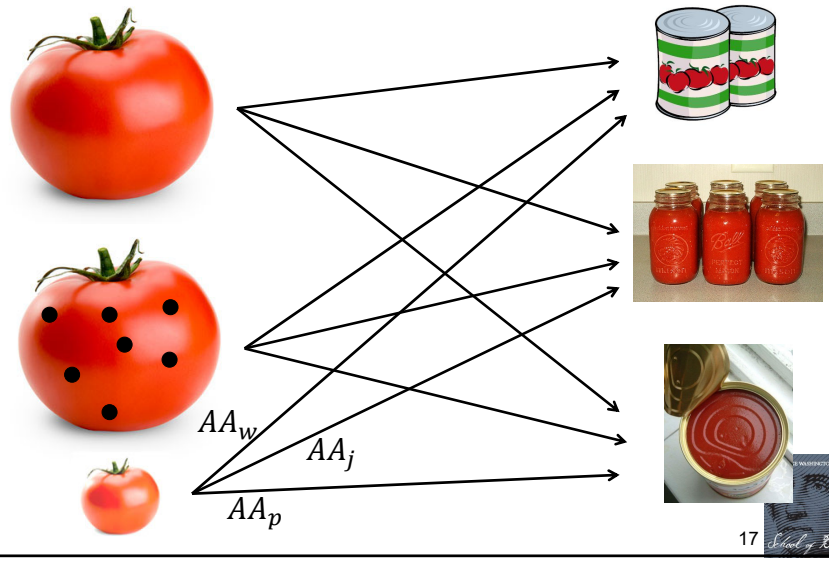
Additional Grade A Tomatoes?



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Objective with AA Tomatoes

- Maximize $246.67A_w+198A_j+222A_p + 246.67B_w+198B_j+222B_p$

- If we decide to buy the additional AA tomatoes, we incur an additional cost ($255 per 1000 lbs). Therefore, the additional tomato cost is now variable! Therefore, subtract from contribution
  - whole tomatoes: $246.67 - $255 = -$8.33
  - tomato juice: $198 - $255 = -$57
  - tomato paste: $222 - $255 = -$33

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Model for AA Tomatoes

- Maximize $246.67A_w+198A_j+222A_p + 246.67B_w+198B_j+222B_p$
  $-8\ AA_w\ -57\ AA_j\ -33\ AA_p$

Subject to

$$A_w + AA_w + B_w \le 14{,}400$$
$$A_j + AA_j + B_j \le 1{,}000$$
$$A_p + AA_p + B_p \le 2{,}000$$
$$A_w + A_j + A_p \le 600$$
$$B_w + B_j + B_p \le 2{,}400$$

$$9A_w + 9AA_w + 5B_W \ge 8(A_w + AA_w + B_w)$$
$$9A_j + 9AA_j + 5B_j \ge 6(A_j + AA_j + B_j)$$

$$A_w, AA_w, B_w, A_j, AA_j, B_j, A_p, AA_p, B_p \ge 0$$

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Optimal Solution with AA tomatoes

RED BRAND CANNERS

MIX DECISION	Whole	Juice	Paste	Total Required	Available
Grade A	600	0	0	600	600
Grade AA	15	65	0	80	80
Grade B	205	195	2,000	2,400	2,400
Total Production	820	260	2,000		
Demand	14,400	1,000	2,000		

QUALITY	Whole	Juice	Paste	Quality
Grade A	5,400	0	0	9
Grade AA	135	585	0	9
Grade B	1,025	975	10,000	5
Total Quality	6,560	1,560	10,000	
Required Total Quality	6,560	1,560	10,000	
Average Quality	8.0	6.0	5.0	
Required Average Quality	8.0	6.0	5.0	

PROFIT	Whole	Juice	Paste	Total Contribution	Total Profit
Contribution Margin	$246.67	$198	$222	$681,177	
Contr. Margin AA	-$8.33	-$57.00	-$33.00	-$3,830	
				$677,347	$137,347

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Sensitivity Analysis (Original Model without Extra A Tomatoes)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Grade A Whole	525	0	246.6666667	463.1111111	64.88888889
C4	Grade A Juice	75	0	198	64.88888889	463.1111111
D4	Grade A Paste	0	0	222	97.33333333	1E+30
B5	Grade B Whole	175	0	246.6666667	1389.333333	64.88888889
C5	Grade B Juice	225	0	198	42.96296296	154.3703704
D5	Grade B Paste	2000	0	222	1E+30	48.33333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B12	Total Quality Whole	5600	-24.33	0	466.6666667	600
C12	Total Quality Juice	1800	-24.33	0	1400	200
D12	Total Quality Paste	10000	-24.33	0	1400	0
E4	Grade A Total Required	600	271.00	600	600	466.6666667
E5	Grade B Total Required	2400	173.67	2400	466.6666667	200
B6	Total Production Whole	700	0.00	14400	1E+30	13700
C6	Total Production Juice	300	0.00	1000	1E+30	700
D6	Total Production Paste	2000	48.33	2000	200	466.6666667

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Shadow Price = additional profit generated by 1(000) pounds extra A tomatoes, exceeds cost

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D5	Grade B Paste	2000	0	222	1E+30	48.33333333

If additional grade A tomatoes are purchased (80,000 pounds). How would the profit contribution change?

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B12	Total Quality Whole	5600	-24.33	0	466.6666667	600
C12	Total Quality Juice	1800	-24.33	0	1400	200
D12	Total Quality Paste	10000	-24.33	0	1400	0
E4	Grade A Total Required	600	271.00	600	600	466.6666667
E5	Grade B Total Required	2400	173.67	2400	466.6666667	200
B6	Total Production Whole	700	0.00	14400	1E+30	13700
C6	Total Production Juice	300	0.00	1000	1E+30	700
D6	Total Production Paste	2000	48.33	2000	200	466.6666667

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Sensitivity Analysis

- The shadow price is the increase in the objective if we increase the right hand side of the corresponding constraint by 1 unit.
- Shadow price of "Grade A Total Required" (grade A supply constraint): $271
- We should be willing to pay up to $271 to obtain an additional 1000 pounds of grade A tomatoes (if we had 1000 lbs. of extra grade A tomatoes, we could generate $271 extra profit)
- This is true for up to additional 600,000 grade A tomatoes ("Allowable Increase"), beyond that the value of extra A tomatoes may be less
- We should buy the 80,000 extra pounds at a price of $255/1000 pounds: net benefit =$271-$255=$16/1000 pounds, $1,280 in total

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Key Takeaways

- Overhead and sunk costs are irrelevant for decision making
- Formulate model
  - What do we want? -> objective (goal, values)
  - What can we do? -> decisions (decision variables)
  - What limits our decision making? -> constraints
- Do not make decision solely on intuition (e.g., Cooper, Myers' accounting data, extra A tomatoes with negative contributions)
- Use optimization (Excel's solver in simple problems, Python with Gurobi)
  - For linear optimization problems, optimality of the solution can be guaranteed
  - If negative values do not make sense, add constraint >=0
  - Use sensitivity analysis to price additional resources

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