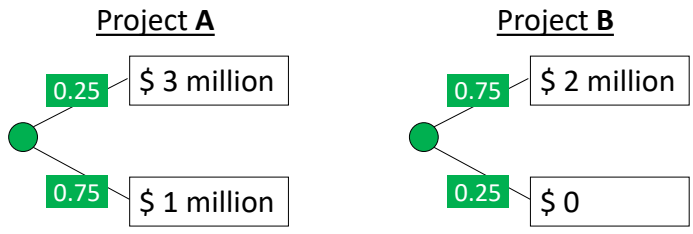
1

Risk and Risk Tolerance in Decision Making

GW Business

2

Expected Value does not reflect Risk Preferences



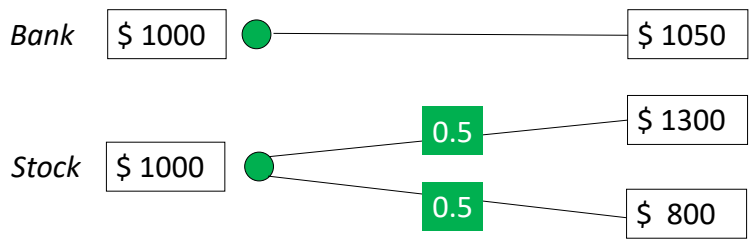
- Expected Values of **A** and **B** are the same.
- Variance of **A** and **B** is the same.
- Still, we are not indifferent between **A** and **B**…
- How much would you pay/ask for switching from **B** to **A**?

4

4

Expected Value vs. Risk

Two investments of $1000:
– Savings account with annual yield of 5%
– Stock with a 50:50 chance of $1300 (+30%) or $800 *(–20%) in a year



- Expected returns are identical:
  – Savings account = 5%
  – Stock = 0.5*30% + 0.5*(–20%) = 5%
- Which would you prefer?
- In general, for the same expected return, investors prefer more reliable, less uncertain → they will require a **higher return** for a **riskier** asset

5

5

Risk Tolerance in Decisions involving Risk

- What is Risk Tolerance (or Risk Aversion)?
- How do we use Risk Tolerance to prescribe decisions?
- How do we measure a person's Risk Tolerance?

6

What is Risk Tolerance (or Risk Aversion)?

- A coefficient that measures an individual's willingness to accept a quantifiable risk
- If Person 1 has higher risk tolerance than Person 2, then Person 1 will accept all the risks that Person 2 accepts
- A person's Risk-Tolerance coefficient is derived from the individual's **utility function** for wealth, an abstract measure of economic well-being
- Risk-Tolerance and Risk-Aversion: simply reciprocal of each other
  - α = Risk-Aversion coefficient
  - $\tau = 1/\alpha$ = Risk-Tolerance coefficient

Formal definition: $\alpha(x) = -\frac{u''(x)}{u'(x)}$ where u is the utility function, x is the payoff (e.g., $)

7

“ Mathematicians evaluate money in proportion to its quantity, while people with common sense in proportion to the usage that they can make of it.”

Gabriel Cramer (1704-1752)

Swiss mathematician

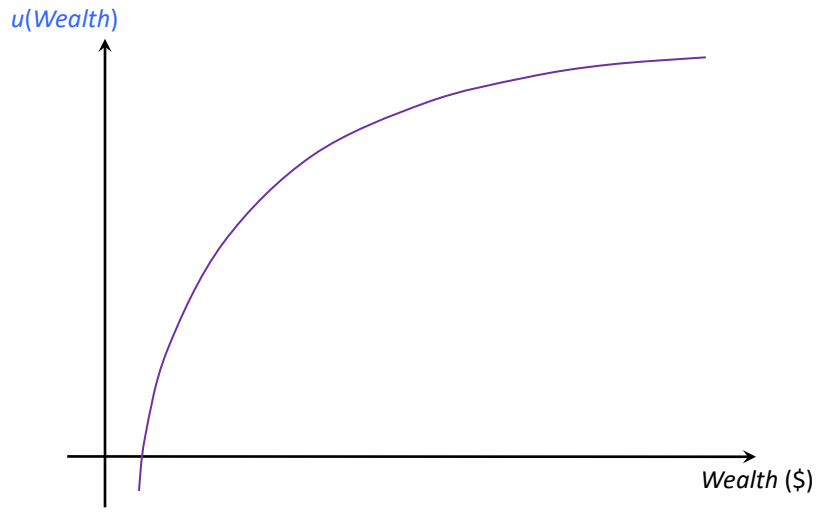
Correspondence with Nicolas Bernoulli, 1728

8

8

9

Utility of Wealth: typically, concave (diminishing marginal utility)



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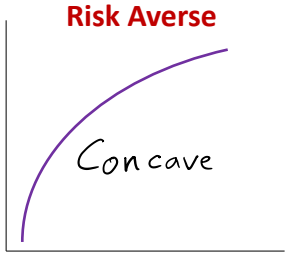
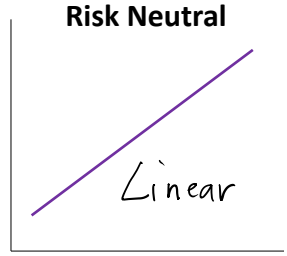
10

The shape of the utility function determines Risk Attitudes

Risk Averse	Risk Neutral	Risk Seeking
		
A **downside** change in payoff carries **more weight** than the same-size upside	Downside and upside changes in payoff have the **same weight**	An **upside** change in payoff has **more weight** than the same-size downside
For any gamble:	*For any gamble:*	*For any gamble:*
CE < EV	**CE = EV**	**CE > EV**

12

12

Utility Functions: Some Common Mathematical Forms

- **Exponential**

  $u(x) = 1 - e^{-x/\tau}$ τ is the **Risk Tolerance** coefficient
  $1/\tau$ is the **Risk Aversion** coefficient

- **Power**

  $u(x) = \frac{(x+w)^{\alpha}}{\alpha}$ for $x \geq -w$

- **Logarithm**

  $u(x) = \mathrm{Ln}(x+w)$ for $x > -w$

- **Others…**

  Quadratic, Linear + Exponential, etc…

Inspecting Excel graphs…

13

13

Risk Tolerance of the Exponential Utility Function

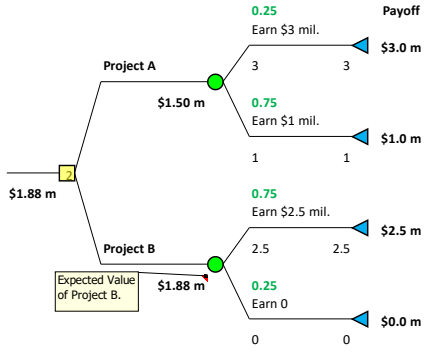
- The **exponential utility function:** $u(x) = 1 - e^{-x/\tau}$
- Has **desirable mathematical properties** for prescriptive analysis
- We can ignore background wealth, reason on gains & losses
- The **Risk Tolerance coefficient** is the parameter τ, a constant
- τ is in the same units as x, the payoff
- Lower value of τ means less risk tolerant, or more risk averse
- Built into *TreePlan* software: ready-made formulas to calculate EU and CE

14

14

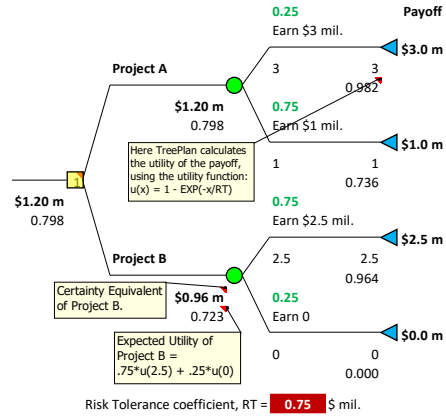
How do we use Risk Tolerance to prescribe decisions?

Decision tree with *Expected Values*



*Under **EV**, Project B is the best choice*

Decision tree with *Risk Tolerance*



*Under **EU**, Project A is the best choice*

See separate instructions for how to incorporate Risk Tolerance in TreePlan.

15

Certainty Equivalent: a sure thing equal to a gamble

Definition CE = the risk-free amount, C, that is worth exactly as much as the Gamble X

$$u(C) = \mathrm{E}[u(X)] \Rightarrow \boxed{C = u^{-1}(\mathrm{E}[u(X)])}$$

The math formulas for this are built into decision tree software

Interpretations

- CE measures the value of a gamble, in the same units as the payoffs, a risk-free cash equivalent of the gamble.
- A higher CE is preferred!
- The individual is indifferent between having CE for sure or taking the Gamble
- CE is the minimum selling price of the Gamble (lowest payment the decider would accept in exchange for giving up the gamble)

16

16

Direct Formula for CE of Bell-shaped Distributions

If a **gamble** has a **bell-shaped** distribution and the **utility** function is **exponential**,

then the **CE of the gamble** can be approximated as:

$$CE = EV - \sigma^2/(2\tau)$$

where: $EV =$ Expected Value of the gamble (the Mean)
$\sigma^2 =$ Variance of gamble
$\tau =$ Decision maker's Risk Tolerance coefficient

- ✓ Formula is exact if the gamble has a Normal distribution
- ✓ Approximates well if distribution is bell shaped
- ✓ Approximates well if risk is small relative to risk tolerance ($\sigma \ll \tau$)

Very practical, because the Mean and Variance of a random variable can be easily calculated or estimated by MC simulation.

17

17

How do we measure a person's Risk Tolerance?

Risk Tolerance/Expected Utility analysis: solidly grounded on *Rational Choice Theory*, the **gold-standard** for decision-making under risk.

Yet…

Measuring Risk Tolerance is probably the thorniest part of all this:

- ❑ Financial institutions and fiduciaries use questionnaires to assess risk tolerance. Those produce a rough score with *limited predictive and prescriptive validity*.
- ❑ A person's choices may not all be consistent with one stable risk tolerance, making it *elusive to measure*.
- ❑ People's responses to risk situations are a mix of economic and psychological factors, *fraught with bias and error*.
- ❑ People find it *difficult to answer* questions that probe their risk tolerance.

18

18

Corporate Risk Tolerance

- Risk aversion is a concern in few, **high-stake** decisions
- Some **rules of thumb** for setting the Risk-Tolerance coefficient: [*]
  - For companies taking moderate risks, set: $\tau \approx$ Net Income [1]
  - For large, diversified firms, set: $\tau \approx 1/6 \times$Firm's Equity [2]
  - For oil exploration units, set: $\tau \approx 1/4 \times$Unit's Annual Budget [3]
- The **same Risk Tolerance** should be used throughout the company
- Because the Certainty Equivalent of a **risky NPV** accounts for the idiosyncratic risk of the project, the discount rate should <u>not</u> be risk-adjusted for calculating NPV (otherwise you will double-count risk: once with discounting, then again with CE)

[*] based on empirical studies with senior management and business unit managers

[1] McNamee, P. & J. Celona 1990. *Decision Analysis with Supertree* (2nd ed), The Scientific Press: San Francisco, p. 122.
[2] Howard, R. A. 1988. Decision Analysis: Practice and Promise. *Management Science*, **34**, 679-695.
[3] Walls, M. R., T. Morahan & J. S. Dyer 1995. Decision Analysis of Exploration Opportunities in the Onshore US at Phillips Petroleum Company. *Interfaces*, **25**, 39-56.

19

19

So, what's <u>your</u> Risk Tolerance?

Some ways to assess your risk-tolerance coefficient (τ):

- ❑ Subjectively estimate your certainty equivalent(s) for some gamble(s) that are representative of the risky decisions you face. Then solve for the value of τ that best fits your certainty equivalents. (Can use Solver for this! ☺)
- ❑ You have a one-time opportunity to invest in a start-up that has 2/3 chance to make you a multi-billionaire, and 1/3 chance to lose all the money invested. Ask yourself: What is the <u>largest amount</u>, M, of my own money that I would be willing to put in it? Then, your $\tau \approx M$.
- ❑ What is your <u>discretionary wealth</u>, W_d? You could take your $\tau \approx W_d$.

20

20

Risk and Risk Attitude - Summary

- Expected Value does **not account for risk** in decisions, because it weighs upside (gains) and downside (losses) equally, linearly.
- To reflect the risk and risk preferences, we evaluate gambles using the decision maker's **risk tolerance**.
- The Risk Tolerance coefficient is derived from the decision maker's utility for money. The **utility function** measures relative preferences for payoffs; if concave, it will weigh downside variations in payoffs more heavily than upside variations of the same magnitude.
- A gamble is evaluated by its **Expected Utility** (EU), the expected value of *utility* scores. From EU, we can calculate the **Certainty Equivalent** (CE) of the gamble: CE is the **risk-adjusted** value of the gamble.
- Decision rule: select alternatives with **highest CE** (same as **highest EU**).
- Risk Aversion has far-reaching implications for economic decisions, notably in insurance, investment, financial risk management, diversification, …