Optimization I

Janne Kettunen

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Mask Mandate

- “The mask mandate implies that everyone needs to be masked. Faculty can remove their mask, if they are at least 6 feet away from everyone at all times and if every student in the class is masked. Further students are not to remove masks.” - Vice Deans Rodney Lake and Shivraj Kanungo Aug 2021
- Also, as a reminder eating and drinking in class is not allowed!

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Instructor: Janne Kettunen

- Associate Professor of Decision Sciences
  MS in Electrical Engineering, MBA (Finance), D.Sc. in Operations and Systems Research
- Work experience
  Business development analyst: Compagnie Financiere Tradition and Nokia Japan
- Current teaching
  Undergraduate: Decision Models
  Graduate: Optimization I and Optimization II
  PhD: Decision Analysis
- Research
  Decision and risk analysis in (i) project management, especially in innovation management and new product development, and (ii) operations management in energy and environmental context
- Contact
  jkettune@gwu.edu, 202 994 3029, Funger hall 409, virtual office hours: Wed 2pm – 4pm, please reserve your appointment by e-mail beforehand

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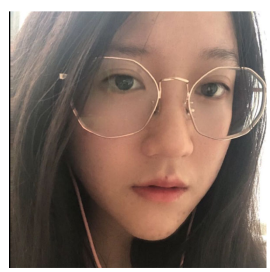
Teaching Assistant



- Gaoyu Xie (PhD student)
- E-mail: gaoyux@gwmail.gwu.edu
- Virtual office hours: Fridays 3-4pm
- Please reserve your appointment by e-mail beforehand

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Introduce Yourself

- Background
- Experience
- Expectations for the course

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Course Description

- The course focuses on optimization problems in business context, such as, resource allocation, work shift planning, or selecting portfolio of investments.
- The emphasis is on (i) problem formulation and (ii) interpretation of the results in an intuitive and practical manner.
- The core concepts behind the solution approaches are covered to the extent that are required to understand and interpret the results and their sensitivities to the model parameters.
- The optimization problems are formulated and solved using Excel's solver and Python programming language and Gurobi optimization engine.
- The covered models include linear, multiobjective, and network optimization models.

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Learning Objectives

1. To competently formulate practical business problems as optimization models (linear, multiobjective, and network).
2. To acquire skills to program the optimization models using Excel and Python.
3. To be able to efficiently represent, analyze, and interpret the optimization model results.
4. To obtain fundamental understanding how the different optimization algorithms work.

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Study Approaches

1. “Go for good grade” = focus only on the material that is needed to do well in the course (i.e., lecture and workshop material)
   - Not a recommended strategy as you miss learning additional skills that are covered in book and you are unlikely to have a comprehensive picture of the topics
2. “Go for big picture” = read through all material (lecture slides and book) without getting a good handle of details
   - Not a recommended strategy as you fail to prepare well for the exam, which can result in lower grade
3. “Go for comprehensive learning” = read through all material (lecture slides and book) and also spend additional time for understanding thoroughly details in lecture and workshop material
   - Recommended strategy resulting in good grade and thorough understanding of all topics

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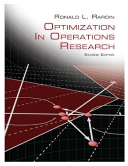
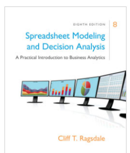
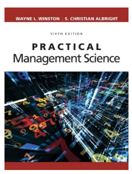
Recommended Books



- Rardin, R. L., Optimization in Operations Research, 2nd Edition, Pearson 2017.
  - Holistic overview of optimization, most comprehensive and rigorous. Includes content that is beyond this course.



- Ragsdale, C. T., Spreadsheet Modeling and Decision Analysis, 8th Edition, Cengage, 2018.
  - Good coverage of course topics. Easy to understand. Not as rigorous or comprehensive as Rardin's book.



- Winston *W*. L. and Albright S. C., Practical Management Science, Cengage, 2019.
  - Good coverage of course topics. Easiest but also least rigorous.

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Required Software

- MS Excel's Solver,
  - Free from IT support?
- Python programming language
  - Free
  - Download Anaconda (instructions on Blackboard)
- Gurobi optimization engine
  - Free academic licence
  - Install Gurobi into Anaconda (instructions on Blackboard)

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Course Expectations

- Read syllabus carefully through:
  - includes explanation about all assignments
  - all due dates
  - explains how to prepare for each lecture and recommends after-class reading material
- Students are responsible for studying and understanding all assigned reading materials (not including recommended material)
- Students will use MS Excel's solver (1st workshop) and python with Gurobi thereafter
- Use Blackboard as a course management system

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Grading Schemes

Assignment	Points	Effort	Due
Participation (best 2 classes)	2x50	Individual	Every lecture
List of team members	Required	Team	Start of 2nd lecture
Team charter	Required	Team	Start of 3rd lecture
LP workshop report	400	Team	Start of 5th lecture
Homework	150	Individual	Start of 7th lecture
Final exam	350	Individual	As scheduled by registrar's office
EXTRA CREDIT			
Feedback survey	15 (if > 70% of students respond)	Individual	End of feedback period

The grades will be assigned based on the total sum of points at the end of the semester. The average grade will be B+. I will apply a curve, with the following approximate proportions of grades:

A:	5-10% of students
A-:	10-25% of students
B+:	25-50% of students
B:	10-25% of students
B- or lower:	0-10% of students

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Class Participation

- Best 2 lecture sessions count
- The following rubric is applied for participation:
  - 50 points: student is engaged for the entire class or blackboard collaborate chat session and offers good insights (or one excellent insight), asks helpful questions, builds on other students' or the instructor's comments, e.g., by elaborating or offering counterpoints, or gives good examples.
  - 25 points: student is engaged during the class/chat session but does not participate verbally beyond simple questions.
  - 0 points: student is absent.

Please bring and use name tags!

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Recordings and COVID-19 Accommodation

- Lectures are recorded (breaks including) and the recodings are available for all students enrolled on the course. These recordings can be accessed from blackboard via link:
  - "Lecture recordings / Synchronous participation"
- COVID-19 accommodation:
  - In case you contract COVID-19 and need to quarantine, catch another illness, or you have been approved later arrival to campus, you can attend the class remotely and synchronously via the same balckboard link "Lecture recordings / Synchronous participation". This will open up a zoom meeting.
  - Students in the classroom cannot log into the zoom web conferencing platform as otherwise the voice starts circulating!

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Group Formation for the Workshop

- Group formation:
  - Teams of 3-4
  - Submit the names of the team members via Blackboard
  - Develop a team charter (about 1 page) and submit it via Blackboard
  - In the event of team dysfunctions, we will follow the charter for proper procedures and actions
- Workshop and report
  - The purpose of the workshop is to allow you to practice hands-on the learned concepts and develop confidence in applying them
  - Deliverables about 3 pages report plus an appendix of any length

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WS and HW Resource Usage

Resource	Permitted	Permitted with Citation	Prohibited
Chegg, Course Hero, Quizlet, and similar sites focused on academic assessments.			X
Classmates in your assigned group. **ONLY IN WORKSHOP REPORT. NOT IN INDIVIDUAL HOMEHORK!**	X		
Classmates, including via GroupMe or other shared conversations.			X
Classmates in other groups, not your own.			X
Course materials on Blackboard.	X		
Course materials not on Blackboard.		X	
Gelman Library Research Services.		X	
Google translate, other translation services and tools, or other tools of "artificial intelligence".		X	
GW Writing Center	X		
Material from outside of this course (e.g., library books, notes from other courses, online material, Wikipedia, YouTube).		X	
Material from students formerly enrolled in the course (academic integrity violation for all students involved).			X
Notes taken in course meetings (including office hours).	X		
Other people (not classmates as noted above).			X
Recorded lectures (from this class, if recording was done or permitted by instructor).	X		
Recorded lectures, talks, podcasts, videos (from a source other than this class).	X		
A tutor from GW's Academic Commons or elsewhere at GW.		X	
A tutor not affiliated with a GW service.			X
All other resources not specified, unless you receive direction otherwise from the course instructors.			X
Copying (including and pasting) text or answers from a resource without citation or if that resource is prohibited.			X

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The Team Charter

- Must be an **operational document** (guidelines for co-operation)
  - probably even more important for virtual teams (distance)
- Should encourage good behavior and discourage poor behavior
  - Must have some teeth in it!
  - **In creating it, you must think like lawyers**
- Joining the team implies its acceptance
  - so in the long run, it will save time
- Set:
  - roles and responsibilities of team members
  - broad performance objectives (team and team members) e.g., quality of work
  - expectations for the success of the project[s]
  - ground rules for the administrative operation of the project team related to: software, document control, communication management, points of contact, status reporting, feedback from higher level meetings, escalation procedures (both emotional and project)
  - expectations for team members' behavior and punishments for poor behavior related to: timeliness (meetings and correspondence), mutual respect, openness, commitment (e.g., work hours, including overtime)
  - "rules of engagement": meeting protocols, discussion protocols, conflict management, support of agreements (e.g., schedules!), **decision making** (e.g., consensus)

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Final Exam

- The final exam includes all covered material. It is administered in-class at the scheduled time
- Individual effort, about 2 hours. You can bring to the exam a one A4 size notes page
- The exam consists of multiple-choice questions, short answer questions, and calculation questions
- To best prepare for the exam, make sure you understand all aspects of the covered material

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Course Schedule

- Session 1: Introduction to linear programming (LP)
- Session 2: Simplex algorithm, duality, and solving LP problems

Theory focused

- Session 3: RBC case
- Session 4: LP workshop

Problem focused

- Session 5: LP using Python and Gurobi

Software focused

- Session 6: Multiobjective optimization
- Session 7: Optimization in networks

Modeling focused

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Introduction to Optimization

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What Is "Optimization" (Mathematical Programming)?

Mathematical optimization (alternatively spelled *optimisation*) or **mathematical programming** is the selection of a best element, with regard to some criterion, from some set of available alternatives.[1] Optimization problems of sorts arise in all quantitative disciplines from computer science and engineering to operations research and economics, and the development of solution methods has been of interest in mathematics for centuries.[2]

In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations constitutes a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain (or input), including a variety of different types of objective functions and different types of domains.

-- Wikipedia, 2021

- Solving problems about how to best (optimally) use limited resources (people, machines, money, time, land)
- Elements of an optimization model:
  - Decisions: Variables whose values the decision maker can choose
  - Objective: Value that is optimized (maximized or minimized)
  - Constraints: Requirements that must be met
- Widely applicable for both organizations and individuals:
  - Organizations
    - Optimal mix of products or services, location of facilities, resource allocation, scheduling, hiring best workers based on multiple criteria
    - Minimizing costs (production, transportation, inventory management)
    - Maximizing profits, value of advertising
  - Individuals
    - Career choices, time usage
    - Investment decisions, purchasing decisions

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Optimization Is Utilized in

- Operations: e.g., factory layout, scheduling, logistics, supply-chain management, product development, resource allocation, sourcing, risk management and mitigation, investment decisions
- Marketing: e.g., product pricing, product configuration, revenue management
- Economics: e.g., supply-demand models, managerial economics
- Finance: e.g., portfolio selection, risk management
- Statistics: e.g., parameter estimation in curve fitting, optimal sample sizes, pattern recognition
- Airspace: e.g., cost models, satellite component selection, spacecraft design
- Engineering: e.g., chemical equilibriums, product designs
- Government: e.g., military operations, fund allocations

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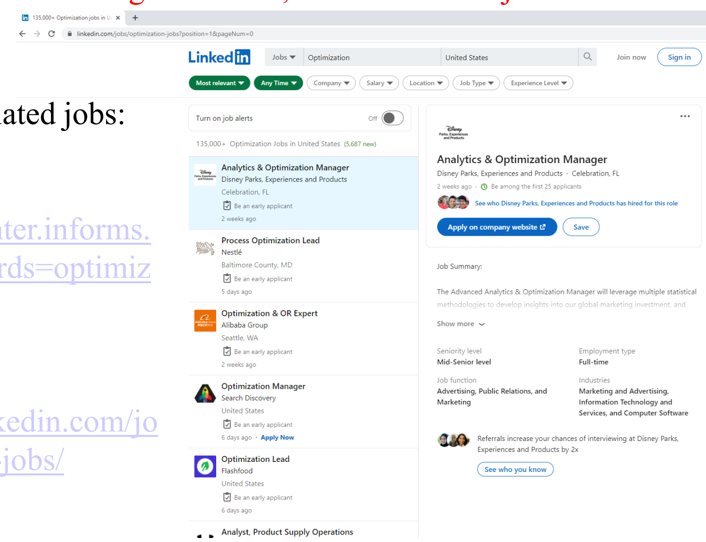
Job Opportunities in Optimization

In Aug 2021: 135,000+ advertised jobs in linked-in

Optimization related jobs:

INFORMS:
https://careercenter.informs.org/jobs?keywords=optimization&sort=

Linked-in:
https://www.linkedin.com/jobs/optimization-jobs/



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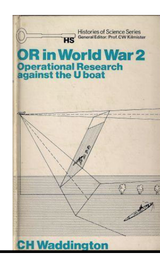
The History of Optimization

- World War II: Scientists and engineers analyzed military problems that were too complex to address using intuition/experience
  - Deployment of radars
  - Organizing convoy, bombing, antisubmarine, mining operations
- The overarching area is often called as operations research (engineering) or management science or decision sciences (business)
  - The goal is to make better decisions employing advanced analytics tools, such as optimization



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Benefits of Optimization

- Make sense of data (particularly important in the era of "big data" and business analytics)
- Allows to deal with complex otherwise intractable problems (involving billions of combinations)
- Can incorporate uncertainty in outcomes
- Help to build insights to the problem and develop a plan how to utilize the available resources

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Major Types of Mathematical Programming

MP	**Characteristics**
Linear Programs (**LP**)	Continuous variables and linear objective and constraints
Nonlinear Programs (**NLP**)	Continuous variables with nonlinear terms in either the objective or the constraints
Integer (or Mixed Integer) Linear Programs (**ILP** or **MILP**)	Integer values required for all (or some) of the variables with linear objective and constraints
Integer (or Mixed Integer) Nonlinear Programs (**INLP** or **MINLP**)	Integer values required for the variables with nonlinear objective or constraints
Multiple Objective Programs	Multiple objective functions are considered simultaneously
Stochastic Programs (**SP**)	Uncertainty in the data parameters explicitly modeled in the formulation

More on Optimization II DNSC 6308!

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Mathematical Programming: General Form

$x_1, x_2, \ldots, x_n$ are the n decision variables

max (or min): $f_0(x_1, x_2, \ldots, x_n)$ Objective function

∀ denotes "for all"

Left-hand-side (LHS) functions for the constraints

M_1 is the index set of constraints of the ≤ type.

subject to: $f_i(x_1, x_2, \ldots, x_n) \le b_i, \ \forall\, i \in M_1$

$f_i(x_1, x_2, \ldots, x_n) \ge b_i, \ \forall\, i \in M_2$

$f_i(x_1, x_2, \ldots, x_n) = b_i, \ \forall\, i \in M_3$

Right-hand-side (RHS) values for the constraints

M_2 is the index set of constraints of the ≥ type.

M_3 is the index set of constraints of the = type.

$x_j \ge 0, \ \forall\, j \in N_1$

$x_j \le 0, \ \forall\, j \in N_2$

x_j unrestricted, $\forall\, j \in N_3$

N_1, N_2 and N_3 and are the index sets of nonnegative, nonpositive and unrestricted variables, respectively.

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Linear Programming

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Linear Programming (LP) Founders and the Term

- Founders:
  - Kantorovich developed LP problems (1939)
  - Danzig developed simplex solution method (1947)
  - Von Neumann developed the theory of the duality (1947)
- Danzig (2002) explains that the linear programming term comes from:
  - "The military refer to their various plans or proposed schedules of training, logistical supply and deployment of combat units as a *program*. When I first analyzed the Air Force planning problem and saw that it could be formulated as a system of linear inequalities, I called my paper *Programming in a Linear Structure*. Note that the term 'program' was used for linear programs long before it was used as the set of instructions used by a computer. In the early days, these instructions were called *codes*. In the summer of 1948, Koopmans and I visited the Rand Corporation. One day we took a stroll along the Santa Monica beach. Koopmans said: "Why not shorten 'Programming in a Linear Structure' to 'Linear Programming'?" I replied: "That's it! From now on that will be its name."

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LP in General Form

Objective function cost coefficients

$$\text{min (or max)} \quad c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$\begin{aligned} \text{subject to} \quad & a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \le b_i, \forall i \in M_1 \\ & a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \ge b_i, \forall i \in M_2 \\ & a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n = b_i, \forall i \in M_3 \\ & x_j \ge 0, \forall j \in N_1 \\ & x_j \le 0, \forall j \in N_2 \\ & x_j \text{ unrestricted}, \forall j \in N_3 \end{aligned}$$

Constraints' coefficients

- Linear objective function and linear constraints

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LP General Form: Using Summation Notation

$$\text{min (or max)} \sum_{j \in N_1 \cup N_2 \cup N_3} c_j x_j$$

$$\text{subject to} \sum_{j \in N_1 \cup N_2 \cup N_3} a_{ij} x_j \leq b_i, \forall i \in M_1$$

$$\sum_{j \in N_1 \cup N_2 \cup N_3} a_{ij} x_j \geq b_i, \forall i \in M_2$$

$$\sum_{j \in N_1 \cup N_2 \cup N_3} a_{ij} x_j = b_i, \forall i \in M_3$$

$$x_j \geq 0, \forall j \in N_1$$

$$x_j \leq 0, \forall j \in N_2$$

$$x_j \text{ unrestricted}, \forall j \in N_3$$

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Terminology

- A feasible solution
  - is any such vector ***x*** of decision variables that satisfies all the constraints
- A feasible region
  - is the set of all feasible solutions
- An infeasible solution
  - is a solution where at least one constraint is not satisfied
- The optimal solution
  - ***x**** is the feasible solution which depending on the problem type either maximizes or minimizes the objective function
- The optimal objective value
  - is the value of the objective function corresponding to the optimal solution ***x****

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Solving LP Problems Graphically

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A Two Variable Example Problem

- Blue Ridge Spa company produces two types of hot tubs: (i) standard tubs and (ii) luxury tubs

	Standard	Luxury	
Pumps	1	1	Resource requirements
Labor	9 hrs	6 hrs	
Tubing	12 ft	16 ft	
Profit / unit	$350	$300	

- For the upcoming production period, they have the following resources available:
  - 200 pumps, 1,566 hrs of labor, and 2,880 ft of tubing
- The demand for both tubs is high, non-constraining
- How many standard and luxury hot tubs the company should produce to maximize its profit?

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Formulating Problem

- Decision variables:

  x_s=number of standard hot tubs to produce
  x_l=number of luxury hot tubs to produce

- Objective function:

  $$Max\ 350x_s + 300x_l$$

- Constraints:

  pumps: $x_s + x_l \leq 200$
  labor: $9x_s + 6x_l \leq 1566$
  tubing: $12x_s + 16x_l \leq 2880$

  non-negativity: $x_s \geq 0$
  $x_l \geq 0$

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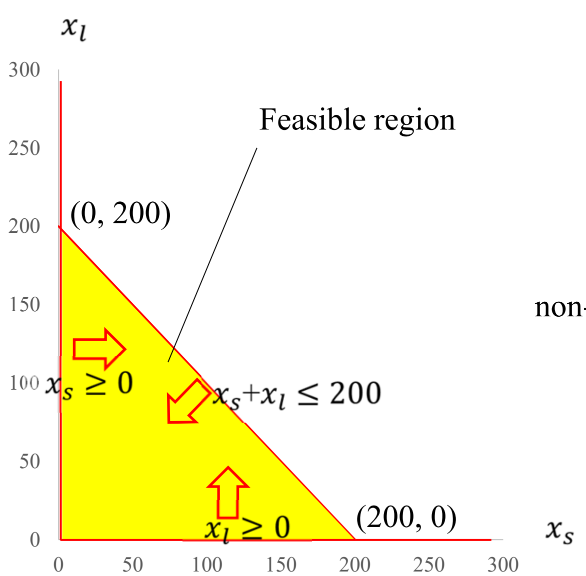
Constraints Graphically 1/3



Constraints:

pumps: $x_s + x_l \leq 200$

non-negativity: $x_s \geq 0$
$x_l \geq 0$

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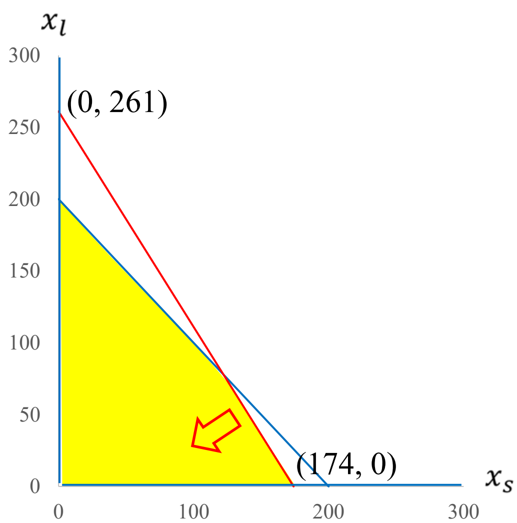
Constraints Graphically 2/3



Adding constraint:

labor: $9x_s+6x_l \leq 1566$

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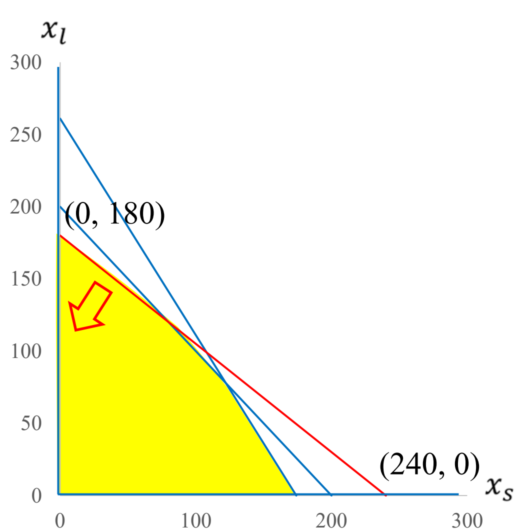
Constraints Graphically 3/3



Adding constraint:

tubing: $12x_s+16x_l \leq 2880$

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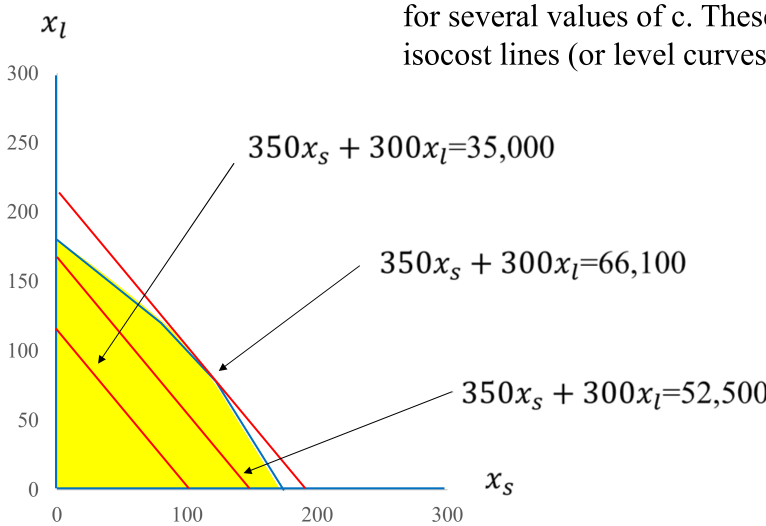
Objective Function Graphically

Draw lines such that $350x_s + 300x_l$=c for several values of c. These are called isocost lines (or level curves or contours).



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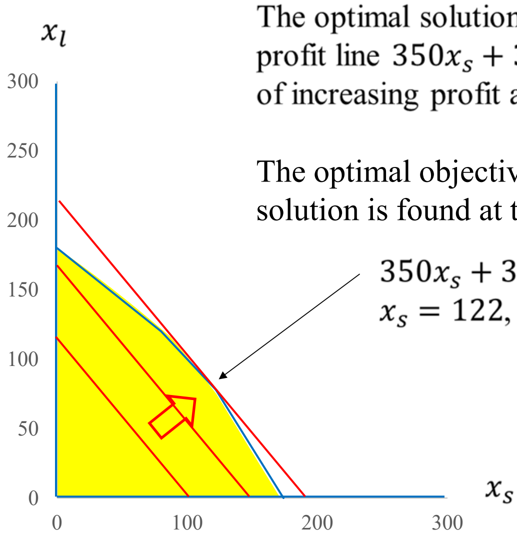
Objective Function Graphically

The optimal solution can be found by moving the profit line $350x_s + 300x_l$ parallel to the direction of increasing profit as much as possible.

The optimal objective value and solution is found at the corner point:

$350x_s + 300x_l$=66,100
$x_s = 122,\ x_l = 78$



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Limitations of the Graphical Approach

- Becomes very difficult to apply for 3-variable models
- Cannot be applied if there are more than 3 variables due to humans limitations to visualize beyond three dimensions
- Also, the approach is cumbersome and slow for problems with many constraints
- Consequently, better techniques are required to solve optimization problems (more about those during the second session)

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Summary

- Linear programming (LP) problems can be solved graphically up to three decision variables
- This approach relies on
  - drawing the feasible region as defined by the constraints and
  - finding the optimal objective value and decisions by moving along the isocost lines of the objective function to the direction of increasing (maximization problem) objective function
- Solving LP problems graphically is slow and cumbersome and more powerful solution algorithms have been developed and will be covered during the second session

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