# *Practice Exercises*
# Linear Optimization Models

For each exercise below, create a *linear* optimization model in Excel, and use the Simplex LP method in Solver to find solutions. The data tables are available in the accompanying Excel file.

## 1. Project Funding

An organization is planning to finance a 3-year project by buying bonds now (in 2021). A management study has estimated the following cash requirements for the project:

| Year | Cash Required ($mil) |
|------|---------------------|
| 2022 | 20 |
| 2023 | 30 |
| 2024 | 40 |

The investment committee is considering four bonds for possible purchase. The unit price and cash flows of the bonds (in $) are given in the table below. For each bond, the cash flows include coupon payments, if any, and the principal payment at maturity:

| | Bond 1 | Bond 2 | Bond 3 | Bond 4 |
|---|--------|--------|--------|--------|
| | | *Bond Cash Flows* | | |
| 2021 | -1.04 | -1.00 | -0.98 | -0.92 |
| 2022 | 0.05 | 0.04 | 1.00 | 0.00 |
| 2023 | 0.05 | 1.04 | | 1.00 |
| 2024 | 1.05 | | | |

What is the least expensive portfolio of bonds whose cash flows meet the requirements for the project?

## 2. Diversifying Investments

An individual investor has $70,000 to divide among alternative investments: municipal bonds with a 4% annual return, certificates of deposit with a 2.5% return, treasury bills with a 3% return, and a growth stock fund with an expected annual return of 7%. The investments are all evaluated after one year.

The following diversification guidelines have been established for mitigating risk:

- No more than 20% of the total investment should be in Municipal Bonds.

- The amount invested in Growth Stocks should not exceed the amount invested in the other three alternatives.

- At least 30% of the investment should be in Treasury Bills and Certificates of Deposit.

How much should be invested in each alternative to maximize the return?

## 3. Advertising with decreasing impact

A company needs to allocate its $1.45 million advertising budget for a new product. The company is considering newspapers ads and television commercials as its primary channels for advertising. The following table summarizes the costs of advertising in these different media and the number of new customers reached by increasing amounts of advertising.

| Media and # of ads | New Customers reached per ad (000) | Cost per Ad |
|---|---|---|
| Newspaper 1-10 | 450 | $ 10,000 |
| Newspaper 11-20 | 350 | $ 9,000 |
| Newspaper 21-30 | 200 | $ 8,000 |
| Television 1-5 | 5000 | $ 120,000 |
| Television 6-10 | 3500 | $ 100,000 |
| Television 11-15 | 2500 | $ 80,000 |

For instance, each of the first ten ads placed in newspapers will cost $10,000 and is expected to reach 450,000 new customers. Each of the next 10 newspaper ads will cost $9,000 and is expected to reach 350,000 new customers. Note that the number of new customers reached by increasing amounts of advertising decreases as the advertising saturates the market. Assume the company will purchase no more than 30 newspaper ads and no more than 15 television ads.

a) Formulate a spreadsheet optimization model to maximize the number of new customers reached by advertising.

b) What is the optimal solution?

c) Suppose the number of new customers reached by 11-20 newspaper ads is 200,000 per ad and the number of new customers reached by 21-30 newspaper ads is 350,000 per ad. Make these changes in your spreadsheet and re-optimize the problem. What is the new optimal solution? What (if anything) is wrong with this solution and why?


## 4. Dealing with decreasing returns in a linear model

Revisit Exercise 1.1 (Product Mix). Suppose there are *decreasing returns*, such that the unit wholesale price of Smartphones is $300 for up to 60,000 unit, but $280 per unit thereafter. Modify the model, keeping a linear model formulation, to account for the decreasing selling price.