Practice Exercises
Integer and Non-Linear Optimization Models

For each exercise below, you will need to create a model in Excel and use Solver to find solutions. The data tables are available in the accompanying Excel file.

1. Capital Budgeting - Project Selection

An organization has received thirty capital investment proposals and identified twenty projects as being consistent with its mission after some pre-screening. However, the organization does not have the funds to support all twenty projects, and it must determine which of them to select. The NPV that each project is expected to generate and the funding requirements for each project are summarized in the following table (all values are in £000s).

Table. Estimated NPV and annual cash requirements for 20 projects

Project	Estimated	Capital required in each year				
ID	NPV	Year 1	Year 2	Year 3	Year 4	Year 5
1	141	75	25	20	15	10
2	187	35	35	0	0	30
3	121	15	15	15	15	15
4	83	20	20	10	5	5
5	265	25	25	20	20	20
6	127	50	20	10	30	40
7	925	35	35	0	0	30
8	844	15	15	15	15	15
9	765	15	15	10	5	5
10	676	20	20	10	5	5
11	699	25	25	20	20	20
12	56	15	15	0	0	30
13	24	20	20	15	15	15
14	575	25	25	10	5	5
15	610	75	25	0	0	30
16	649	35	35	15	15	15
17	439	20	20	20	20	20
18	812	25	25	0	0	30
19	66	35	35	0	20	20
20	127	20	20	10	30	40

The organization currently has £250,000 available to invest in new projects. It has budgeted £80,000 for continued support for these projects in year 2 and £50,000 per year for years 3, 4 and 5.

a) Build an optimization model to help decide which projects to select.

b) Compare the optimal solution obtained by the model to a heuristic solution that selects projects by highest NPV contribution until the budget is depleted.

c) Suppose that any unused budget from a given year can be carried over to the following year. Modify the original model from (a) to reflect this and resolve the problem.

d) Project 9 is in fact a variant of Project 8. Therefore, one or the other can be done, not both. Add a constraint to your model to take this into account.

2. Dealing with increasing returns with a linear model

Revisit Exercise 1.1 (Product Mix). Suppose there are *increasing returns*, such that the marginal cost of Smartphones drops from $150 to $130 per unit if more than 65,000 units are produced. Modify the optimization model, *keeping a linear* formulation, to account for these economies of scale. *Hint*: this can be done by introducing an "artificial" binary decision variable and associated constraints that will force the production of up to 65,000 low-return smartphones before the number of higher-return smartphones can become non-zero.

3. Mean-Variance portfolio selection

The accompanying Excel file contains data on periodic returns for several stocks. Based on these data, your task is to formulate an optimization model to help select portfolios of these stocks that will minimize the portfolio's risk for a given level of portfolio expected return.

a) Using variance as a measure of risk, set up the optimization model to minimize the portfolio variance for a specified level of portfolio return. This is the classic "Markowitz" portfolio selection model.

b) Use the model repeatedly to compute several optimal portfolios for different levels of expected return. Plot the Mean vs. Standard Deviation of the optimal portfolios in a graph; this is called the Risk-Return Efficient Frontier. Also, plot the individual stocks on this graph.