Optimization I

Janne Kettunen

1

1

Course Evaluation

- Please take time to provide course evaluation
- Evaluation available 12 Oct – 17 Oct
- I value and use your responses
- If you have suggestions how to improve the course or even if you are happy how things have been taught, I would appreciate to hear that!
- Incentive: if at least 70% of students fill up the course evaluation, I will grant everybody 15 additional points

Thanks!

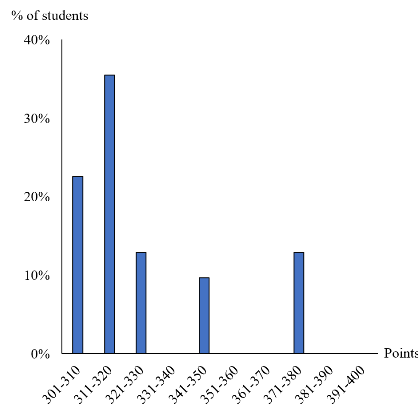
2

2

Workshop Results

	Points	% of total
max	374	94%
min	302	76%
average	326	82%
st dev	23	6%
range	72	18%

Well done!



If you have grading related questions, please contact the TA Gaoyu Xie, who graded the reports. If something is still unclear after contacting her, you can escalate the issue to me and then both Gaoyu and I will look the report again.

3

3

Multiobjective Optimization

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4

4

What Is Multiobjective Optimization?

- Occurs when we have to make tradeoffs among conflicting objectives
- Understanding the tradeoffs that are made is essential for making the right decision
- Topics covered in multiobjective optimization are:
  - Pareto optimality and efficient frontiers
  - Preemptive optimization
  - Goal programming (standard or preemptive)
  - Weighted objective functions

5

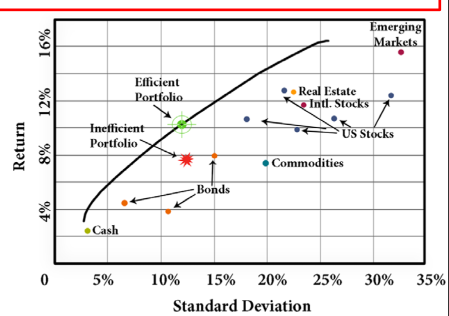
5

Efficient point and Frontier

A feasible solution to a multiobjective optimization model is an efficient point if the solution scores (i) at least as well in all objective functions as other feasible solutions and (ii) strictly better in at least one objective. Such a solution is also called a Pareto optimal one.

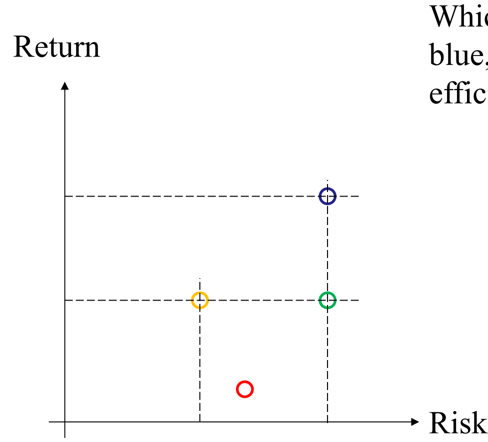
The efficient frontier of a multiobjective optimization model is the set of all efficient points for the model.

- When there are two objectives the efficient frontier is often computed and showed graphically



6

Example



Which of the (yellow, red, blue, green) points are efficient?

7

7

Preemptive Optimization

Preemptive or lexicographic optimization performs multiobjective optimization by considering one objective at a time, in order of importance

- Once the most important objective is optimized then the second most important is optimized, subject to a constraint that the first objective achieves its optimal value; and so on

- Since each optimization run optimizes the value for a single objective, in sequence, the final solution is Pareto-optimal (efficient) for the multiobjective model, i.e., no further improvement is possible for any of the objectives without detoriating some other objectives.

- Limitation: Once the first optimization model (for the most important objective) is completed, then all solutions that are obtained in subsequent optimization must be alternate optima for the first optimization model, which is quite restrictive.

8

8

Goal Programming

Standard: The problem is constructed in terms of goals to be achieved (or target levels to be met) rather than objectives to be optimized.

- Rationale: the importance of any objective diminishes once a target level is reached
- Implementation relatively easy using constraints and deficiency variables.

Preemptive: Goals are considered one at a time, in order of importance

- Deficiency for the most important goal is first minimized
- Next, deficiency for the second most important goal is minimized while ensuring that the first goal deficiency achieves its minimum value; and so on

- A caveat: a solution in which goals are achieved may not be efficient, i.e., it might be possible to improve
- To fix this problem and obtain an efficient solution, add small positive (negative) multiplier of each of the original minimization (maximization) objective functions

9

9

Weighted-Sum of Objectives Approach

Multiple functions are combined into a single objective function by applying weights to the individual objectives:

- If the combined single objective is to be minimized (maximized), then positive (negative) weights should be applied to all minimization (maximization) objectives and negative (positive) weights to all maximization ones

- Optimal solutions obtained using this approach are efficient
  - Reason: Any solution that can improve upon the optimal solution for an objective would have scored better in the weighted objective function, thereby, only an efficient point can be optimal
- Assessing weights (more on Decision Analytics course):
  - The swing method
  - The analytic hierarchy process (AHP)
  - Utility functions

10

10

An Example of Constructing the Efficient Frontier

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11

11

Example Problem

- "Bank Three" has:
  - $20 million own capital, $150 million on demand deposits (customers' checking accounts), and $80 million on time deposits (customers' savings accounts). The bank can invest its funds ($250 million in total) as follows

Investment Category	Return Rate (%)	Liquid Part (%)	Required Capital* (%)	Risk Asset?
Cash	0.0	100.0	0.0	No
Short term	4.0	99.5	0.5	No
Government: 1 – 5 years	4.5	96.0	4.0	No
Government: 5 – 10 years	5.5	90.0	5.0	No
Government: over 10 years	7.0	85.0	7.5	No
Installment loans	10.5	0.0	10.0	Yes
Mortgage loans	8.5	0.0	10.0	Yes
Commercial loans	9.2	0.0	10.0	Yes

* These are approximate requirements by the U.S. Government for calculating the bank's capital-adequacy ratio (CAR), which is an indicator of solvency.

- Constraints:
  - Cash reserves must be at least 14% of demand deposits plus 4% of time deposits.
  - Liquid investments should be at least 47% of demand deposits plus 36% of time deposits.
  - At least 5% of funds should be invested in each of the eight categories for diversity.
  - At least 30% of funds should be invested in commercial loans to maintain community status.
- Objectives:
  - *maximize* profit.
  - *minimize* "capital-adequacy ratio". Low required capital / own capital implies lower risk.
  - *minimize* "illiquid risk asset ratio". Low illiquid risk asset/own capital implies financially secure institution.

12

Problem Formulation 1/2

Decision Variables

Let,
x_j = amount invested in investment category j ($ million), j =1 , 2, ... , 8

Objectives

(1) Profit maximization:

$$\max\ 0.040x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8$$

(2) Capital-adequacy ratio minimization:

$$\min\ \frac{1}{20}(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8)$$

(3) Illiquid risk assets/capital ratio minimization:

$$\min\ \frac{1}{20}(x_6 + x_7 + x_8)$$

13

13

Problem Formulation 2/2

Constraints:

Investments should equal the available capital plus deposits:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 20 + 150 + 80$$

Cash reserves must be at least 14% of demand deposits plus 4% of time deposits:

$$x_1 \geq 0.14 \times 150 + 0.04 \times 80$$

Liquid investments should be at least 47% of demand deposits plus 36% of time deposits:

$$x_1 + 0.995x_2 + 0.96x_3 + 0.9x_4 + 0.85x_5 \geq 0.47 \times 150 + 0.36 \times 80$$

At least 5% of funds should be invested in each of the eight categories for diversity:

$$x_j \geq 0.05 \times (20 + 150 + 80), \quad \forall j = 1, \ldots, 8$$

At least 30% of funds should be invested in commercial loans to maintain community status:

$$x_8 \geq 0.3 \times (20 + 150 + 80)$$

Finally, all the variables should be nonnegative:

$$x_j \geq 0, \quad \forall j = 1, \ldots, 8$$

See "Multiobjective Optimization.xlsx". "3-Objective Model" tab

14

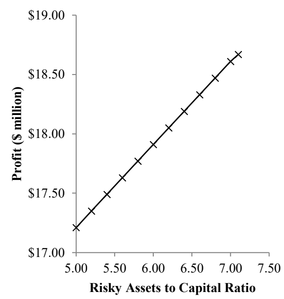
14

Constructing Efficient Frontier

Up to three objectives it is possible to graphically construct the efficient frontier:

1. Optimize objective 1 and note the corresponding objective function values for the two objectives: $(V_1^*, V_2^*$
2. Optimize objective 2 and note the corresponding objective function values for the two objectives: (V_1^{**}, V_2^{**})
3. Construct the efficient frontier by repeatedly optimizing the problem to determine points $(V_1^{*\prime}, V_2^{*\prime})$ for the plot by adding an ever tighter constraint for the objective 2 starting from V_2^* and ending at V_2^{**}.

Example: Efficient frontier for “profit” – “risky assets to capital ratio”



- The higher the fraction of risky assets allowed, the higher the possible profit .
- For each profit amount, there is a *minimum* fraction of risky assets; *any fraction higher than that is inefficient*.
- For each fraction of risky assets, there is a *maximum* profit that can be achieved; *any profit less than that is inefficient*.

See “Multiobjective Optimization.xlsx”, “Efficient Frontier – Model” and “Efficient Frontier – Plot” tabs.

15

15

An Example of Preemptive Optimization

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16

16

Example Problem

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Government: over 10 years	7.0	85.0	7.5	No
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Mortgage loans	8.5	0.0	10.0	Yes
Commercial loans	9.2	0.0	10.0	Yes

* These are approximate requirements by the U.S. Government for calculating the bank's capital-adequacy ratio (CAR), which is an indicator of solvency.

- Constraints:
  - Cash reserves must be at least 14% of demand deposits plus 4% of time deposits.
  - Liquid investments should be at least 47% of demand deposits plus 36% of time deposits.
  - At least 5% of funds should be invested in each of the eight categories for diversity.
  - At least 30% of funds should be invested in commercial loans to maintain community status.

- Objectives in order of importance:
  - *minimize* illiquid risk asset. A low risk/capital ratio indicates a financially secure institution.
  - *maximize* profit.
  - *minimize* "capital-adequacy ratio", i.e., the required capital for bank solvency to own capital. Note that the *lower the ratio the less risky the bank is*.

17

Preemptive Optimization: First Optimization Problem

1st objective: minimize illiquid risk asset:

$$\min \frac{1}{20}(x_6 + x_7 + x_8)$$

Subject to:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 20 + 150 + 80$$
$$x_1 \ge 0.14(150) + 0.04(80)$$
$$1.00x_1 + 0.995x_2 + 0.960x_3 + 0.900x_4 + 0.850x_5 \ge 0.47(150) + 0.36(80)$$
$$x_j \ge 0.05(20 + 150 + 80) \text{ for all } j = 1, \dots, 8$$
$$x_8 \ge 0.30(20 + 150 + 80)$$
$$x_1, \dots, x_8 \ge 0$$

The optimal solution is:

$$x_1^* = 100.00, x_2^* = 12.50, x_3^* = 12.50, x_4^* = 12.50$$
$$x_5^* = 12.50, x_6^* = 12.50, x_7^* = 12.50, x_8^* = 75.00$$

The corresponding values for the three objectives:
Objective 1 (Profit) = \$11.90 million
Objective 2 (Capital Adequacy Ratio) = 0.61
Objective 3 (Risk-Asset Ratio) = 5.00

18

18

Preemptive Optimization: Second Optimization Problem

2^{nd} objective: maximize profit:

$$\max\ 0.040x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8$$

Subject to:

$$\frac{1}{20}(x_6 + x_7 + x_8) \le 5.00$$ Added constraint to retain risk-asset ratio at 5.00

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 20 + 150 + 80$$

$$x_1 \ge 0.14(150) + 0.04(80)$$

$$1.00x_1 + 0.995x_2 + 0.960x_3 + 0.900x_4 + 0.850x_5 \ge 0.47(150) + 0.36(80)$$

$$x_j \ge 0.05(20 + 150 + 80) \text{ for all } j = 1, \dots, 8$$

$$x_8 \ge 0.30(20 + 150 + 80)$$

$$x_1, \dots, x_8 \ge 0$$

The optimal solution is:

$$x_1^* = 24.20, x_2^* = 12.50, x_3^* = 12.50, x_4^* = 12.50$$

$$x_5^* = 88.30, x_6^* = 12.50, x_7^* = 12.50, x_8^* = 75.00$$

The corresponding values for the three objectives:
Objective 1 (Profit) = $17.21 million
Objective 2 (Capital Adequacy Ratio) = 0.89
Objective 3 (Risk-Asset Ratio) = 5.00

19

19

Preemptive Optimization: Third Optimization Problem

3^{rd} objective: minimize capital adequacy:

$$\min\ \frac{1}{20}(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8)$$

Subject to:

$$\frac{1}{20}(x_6 + x_7 + x_8) \le 5.00$$

Added constraint to retain profit at $17.2 million

$$\boxed{0.040x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8 \ge 17.20}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 20 + 150 + 80$$

$$x_1 \ge 0.14(150) + 0.04(80)$$

$$1.00x_1 + 0.995x_2 + 0.960x_3 + 0.900x_4 + 0.850x_5 \ge 0.47(150) + 0.36(80)$$

$$x_j \ge 0.05(20 + 150 + 80) \text{ for all } j = 1, \dots, 8$$

$$x_8 \ge 0.30(20 + 150 + 80)$$

$$x_1, \dots, x_8 \ge 0$$

The optimal solution does <u>not</u> change:

$$x_1^* = 24.20, x_2^* = 12.50, x_3^* = 12.50, x_4^* = 12.50$$

$$x_5^* = 88.30, x_6^* = 12.50, x_7^* = 12.50, x_8^* = 75.00$$

The corresponding values for the three objectives:
Objective 1 (Profit) = $17.21 million
Objective 2 (Capital Adequacy Ratio) = 0.89
Objective 3 (Risk-Asset Ratio) = 5.00

See "Multiobjective Optimization.xlsx" and "Preemptive Optimization" tab.

20

20

An Example of Goal Programming

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21

21

Example Problem

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Government: over 10 years	7.0	85.0	7.5	No
Installment loans	10.5	0.0	10.0	Yes
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* These are approximate requirements by the U.S. Government for calculating the bank's capital-adequacy ratio (CAR), which is an indicator of solvency.

- Constraints:
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  - Liquid investments should be at least 47% of demand deposits plus 36% of time deposits.
  - At least 5% of funds should be invested in each of the eight categories for diversity.
  - At least 30% of funds should be invested in commercial loans to maintain community status.
- Objectives, goals:
  - profit$\geq$18.5
  - "capital-adequacy ratio" $\leq$0.8 (the required capital / own capital)
  - "risk-asset ratio $\leq$ 7 (low "risk asset/own capital" ratio indicates a financially secure institution)
  - Since it may not be possible to find a solution that satisfies the goals, they are treated as soft rather than hard constraints

22

22

Goal Programming Formulation 1/3

Deficiency Variables

Let,

$d_1^- =$ amount profit ***falls short of*** its goal

$d_2^+ =$ amount capital-adequacy ratio ***exceeds*** its goal

$d_3^+ =$ amount risk-asset ratio ***exceeds*** its goal

The three goals can then be written as:

(1) Profit goal:

$$0.040x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8 + d_1^- \geq 18.5$$

(2) Capital-adequacy ratio goal:

$$\frac{1}{20}(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8) - d_2^+ \leq 0.8$$

(3) Illiquid risk assets/capital ratio goal:

$$\frac{1}{20}(x_6 + x_7 + x_8) - d_3^+ \leq 7.0$$

NOTE :
An objective (i) that requires ***exactly*** meeting a goal will need two ***deficiency*** variables: d_i^+ and d_i^-.

23

23

Goal Programming Formulation 2/3

Objective

$\min\ d_1^- + d_2^+ + d_3^+ \longrightarrow$ Minimize the sum of the deficiency variables to try to meet all three goals

If weights can be assessed for the relative importance of not meeting the three goals, then a weighted objective function can be used:

$$\min\ w_1^- d_1^- + w_2^+ d_2^+ + w_3^+ d_3^+$$

where,

$w_1^- =$ penalty per unit violation of goal 1

$w_2^+ =$ penalty per unit violation of goal 2

$w_3^+ =$ penalty per unit violation of goal 3

24

24

Goal Programming Formulation 3/3

Complete Formulation

$$\min d_1^- + d_2^+ + d_3^+$$

Subject to:

$$0.040x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8 + d_1^- \geq 18.5$$

$$\frac{1}{20}\left(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8\right) - d_2^+ \leq 0.8$$

$$\frac{1}{20}\left(x_6 + x_7 + x_8\right) - d_3^+ \leq 7.0$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 20 + 150 + 80$$

$$x_1 \geq 0.14(150) + 0.04(80)$$

$$1.00x_1 + 0.995x_2 + 0.960x_3 + 0.900x_4 + 0.850x_5 \geq 0.47(150) + 0.36(80)$$

$$x_j \geq 0.05(20 + 150 + 80) \text{ for all } j = 1, \ldots, 8$$

$$x_8 \geq 0.30(20 + 150 + 80)$$

$$x_1, \ldots, x_8, d_1^-, d_2^+, d_3^+ \geq 0$$

25

Goal Programming Solution

Solution

The optimal solution will now be:

$$x_1^* = 24.20, x_2^* = 16.03, x_3^* = 12.50, x_4^* = 12.50$$

$$x_5^* = 44.77, x_6^* = 52.50, x_7^* = 12.50, x_8^* = 75.00$$

$d_1^- = 0 \longrightarrow$ First goal is exactly met.

$d_2^+ = 0.13 \longrightarrow$ Second goal is violated by 0.13.

$d_3^+ = 0 \longrightarrow$ Third goal is exactly met.

The corresponding values for the three goals are:
Goal 1 (Profit) = $18.5 million
Goal 2 (Capital Adequacy Ratio) = 0.93
Goal 3 (Risk-Asset Ratio) = 7.00

See "Multiobjective Optimization.xlsx" and "Goal Programming" tab.

26

Preemptive Goal Programming and Example Problem

- First goal (profit ≥ 18.5):
  - Solve for: min d_1^-
  - The solution is: $d_1^- = 0$
  - This implies that the profit goal can be achieved
- Second goal "capital-adequacy ratio" ≤ 0.8:
  - Solve for: min d_2^+
  - Add constraint on 1st goal's deficiency variable: $d_1^- \leq 0$
  - The solution is: $d_2^+ = 0.12$
  - This implies that the capital-adequacy ratio is violated by 0.12
- Third goal "risk-asset ratio ≤ 7:
  - Solve for: min d_3^+
  - Add constraints on 1st and 2nd goals' deficiency variables: $d_1^- \leq 0$, $d_2^+ \leq 0.12$
  - The solution is: $d_3^+ = 0.14$
  - This implies that the risk-asset ratio is violated by 0.14

Solution

The optimal solution will now be:

$$x_1^* = 24.20, x_2^* = 19.37, x_3^* = 12.50, x_4^* = 12.53$$

$$x_5^* = 38.52, x_6^* = 55.38, x_7^* = 12.50, x_8^* = 75.00$$

The corresponding values for the three goals are:
Goal 1 (Profit) = $18.5 million
Goal 2 (Capital Adequacy Ratio) = 0.92
Goal 3 (Risk-Asset Ratio) = 7.14

See "Multiobjective Optimization.xlsx" and "Preemptive Goal Programming" tab.

27

27

An Example of Weighted-Sum of Objectives Approach

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28

28

Reminder: Weighted-Sum of Objectives Approach

Multiple functions are combined into a single objective function by applying weights to the individual objectives:

- If the combined single objective is to be minimized (maximized), then positive (negative) weights should be applied to all minimization (maximization) objectives and negative (positive) weights to all maximization ones

- Optimal solutions obtained using this approach are efficient
  - Reason: Any solution that can improve upon the optimal solution for an objective would have scored better in the weighted objective function, thereby, only an efficient point can be optimal
- Assessing weights (more on Decision Analytics course):
  - The swing method
  - The analytic hierarchy process (AHP)
  - Utility functions

29

29

Example Problem

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  - Liquid investments should be at least 47% of demand deposits plus 36% of time deposits.
  - At least 5% of funds should be invested in each of the eight categories for diversity.
  - At least 30% of funds should be invested in commercial loans to maintain community status.
- Objectives with weights:
  - profit, weighted at 0.5
  - "capital-adequacy ratio", weighted at 0.3 (the required capital for bank solvency to own capital)
  - "risk-asset ratio, weighted at 0.2 (low risk/capital ratio indicates a financially secure institution)

30

30

Weighted-Sum of Objectives Formulation

Weighted-sum objective function is:

$$\min \; -w_1\left(0.040x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8\right)$$
$$+ w_2\left(\frac{1}{20}\left(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8\right)\right)$$
$$+ w_3\left(\frac{1}{20}\left(x_6 + x_7 + x_8\right)\right)$$

where $w_1 = 0.5$, $w_2 = 0.3$, and $w_3 = 0.2$

The optimal solution is:
$x_1^* = 24.2, x_2^* = 12.5, x_3^* = 12.5, x_4^* = 12.5$
$x_5^* = 46.37, x_6^* = 54.43, x_7^* = 12.5, x_8^* = 75.0$
The corresponding values for the three objectives:
Objective 1 (Profit) = $18.67 million
Objective 2 (Capital Adequacy ratio) = 0.94
Objective 3 (Risk-Asset Ratio) = 7.10

See "Multiobjective Optimization.xlsx" and "Weighted-sum" tab.

31

31

Multiobjective Optimization Using Python and Gurobi

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32

32

Multiobjective Optimization Problem

A university library must cut annual subscription expenses s_j to some scientific journals j=1,…,40 to absorb a \$5000 - \$6000 per year budget cut. One consideration will be the impact factor c_j of journal j, which is a measure of how seminal a journal is to research. Another is the usefulness rating of a journal r_j (1=low to 10=high) solicited from university faculty. Finally, the library wants to consider the ratings a_j of the relative availability (1=low to 8=high) in nearby libraries, believing that journals readily available elsewhere need not be retained. Journal data is given in file "library_data.csv".

(a) Formulate a weighted-sum multiobjective integer linear programming problem to choose which journals to drop. The weights are as follows 0.2 for the impact factor, 0.3 for the usefulness, and 0.5 for the availability.

(b) Code the weighted-sum multiobjective integer linear programming problem using Python and Gurobi and solve your model. Which journals will be cut and how much costs can be saved?

(c) Formulate and solve the same problem using preemptive (lexicographic) programming. Priority order is as follows availability, usefulness, impacts factor. Compare results to those obtained in part (b).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Subscription expense	1358	860	372	602	318	269	1860	1084	1018	1111	1741	443	605	825	314	1637	733	1993	676	796
Average citations / article	5.2	4.5	2.1	3	3.7	0.9	3.4	2.5	1.8	2.8	4.5	0.6	5.2	4.4	0.1	5.6	0.1	3.1	2	3.2
Usefulness	6	10	1	5	4	9	8	8	3	5	5	2	4	5	7	4	8	4	7	5
Availability	7	1	8	3	3	3	3	4	7	2	3	3	4	6	6	7	8	6	8	4

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Subscription expense	66	660	1351	686	1943	1313	1036	160	689	661	300	1138	998	1398	1158	176	576	1025	1816	938
Average citations / article	1.5	0.1	4.8	4.5	1.2	2.8	5.7	3.8	1.7	2.1	2.4	0.9	1.5	1.3	0.1	4.4	0.5	2.1	1.4	0.8
Usefulness	10	4	2	9	3	3	4	10	9	4	7	6	1	2	3	8	8	10	3	9
Availability	8	1	8	6	7	2	5	4	2	4	4	6	1	6	3	8	5	8	4	3

33

33

Problem Formulation

Decision variables: $x_j \triangleq 1$, if a journal j is dropped and 0 otherwise

$$Min - 0.5\sum_{j=1}^{40} a_j x_j + 0.3\sum_{j=1}^{40} r_j x_j + 0.2\sum_{j=1}^{40} c_j x_j$$ objective function

Subject to

$$\left.\begin{aligned}\sum_{j=1}^{40} s_j x_j &\geq 5000\\ \sum_{j=1}^{40} s_j x_j &\leq 6000\end{aligned}\right\}$$ constraints on expenses

$$\left. x_j \in \{0,1\}\ \forall\, j = 1, \dots, 40 \right\}$$ integrality constraints

34

34

Weighted-Sum Optimization: Results

- Optimal to drop journals: 3, 9, 12, 15, 17, 21, 23, 34, 36 (note that the numbering for decision variables starts from 0 in "Library (weighted-sum).ipynb")
- Cost savings: $5871

35

35

Discussion and Comparison

- Solve the same problem using preemptive (lexicographic) programming. Priority order is as follows availability, usefulness, impacts factor.
  - Edit "Library_preemptive_template.ipynb" and solve the following 3 optimization problems consecutively
    - 1st maximize availability of the dropped journals
    - 2nd minimize usefulness with a constraint on availability (1st optimization result)
    - 3rd minimize impact factor with constraints on availability (1st opt) and usefulness (2nd opt)
- Compare results to those obtained in part (b), i.e., weighted-sum optimization
- Why the dropped journals differ?
- Which approach you would prefer if you were librarian? How about if you were an administrator?
- Can you argue that both approaches are valid?

36

36

Preemptive Optimization: Results

- After 1st optimization (max availability):
  - Drop journals: 3, 6, 13, 15, 17, 19, 21, 24, 28, 31, 36, 37, 38
  - Cost savings: $5958
- After 2nd optimization (min usefulness):
  - Drop journals: 3, 5, 14, 15, 17, 19, 21, 24, 28, 31, 36, 38
  - Cost savings: $5920
- After 3rd optimization (min citations):
  - Drop journals: 3, 5, 6, 12, 14, 15, 17, 19, 21, 24, 28, 31, 36, 37
  - Cost savings: $5914

37

37

Discussion and Comparison

- Weighted-sum optimization:
  - Drop journals: 3, 9, 12, 15, 17, 21, 23, 34, 36
  - Cost savings: $5871
- Preemptive optimization:
  - Drop journals: 3, 5, 6, 12, 14, 15, 17, 19, 21, 24, 28, 31, 36, 37
  - Cost savings: $5914
- Why the dropped journals differ?
- Which approach you would prefer if you were librarian? How about if you were an administrator?
- Can you argue that both approaches are valid?

38

38

Summary

- An efficient point is such the solution scores (i) at least as well in all objective functions as other feasible solutions and (ii) strictly better in at least one objective

- The efficient frontier is the set of all efficient points

- Multiobjective optimization problems are solved using:
  - preemptive optimization
  - goal programming
  - weighted-sum of objectives approach