Practice Exercises
Risk and Risk Aversion in Expected Utility Analysis

1. Certainty Equivalent and Risk Premium

Consider the following two prospects:

- Prospect A: 25% chance of a $3 million payoff, and 75% chance of a $1 million payoff.
- Prospect B: 75% chance of a $2 million payoff, and 25% chance of a $0 payoff.

a) Calculate the expected values and variances of A and B to show that they are equal. Which prospect do you personally judge to be riskier?

Suppose an individual has utility for wealth given by $u(x) = 1 - e^{-x/0.75}$ (the parameter 0.75 is the risk-tolerance coefficient of the individual).

b) Calculate the Expected Utility, Certainty Equivalent and Risk Premium of each prospect. Which prospect would the individual chose?

2. Use of Risk Tolerance in a Decision Tree

Revisit Decision Tree Exercise #1 (Insurance Coverage). Suppose the business owner's risk tolerance coefficient is $200,000. Evaluate the decision tree model previously developed for this problem using the expected utility criterion (see separate instructions for using Risk Tolerance in TreePlan).

a) What is the best alternative?

b) What is the associated certainty equivalent (CE)? How do you interpret the CE?

3. Impact of Risk Tolerance on Decisions

Revisit Decision Tree Exercise #2 (Contract Bid). We will now evaluate the decision tree model developed for this problem using risk tolerance.

a) What is the best strategy if the corporate risk tolerance is RT = 600 (in £-thousand)?

b) What is the certainty equivalent (CE) associated with the optimal strategy? How does it compare to the Expected Value?

c) How would the best strategy change if the risk tolerance was RT = 300? If RT = 100?

4. Diversification with Expected Utility

An investor has \$1000 to invest in two types of share. If he invests \$$x$ in Share A, he will invest the remainder \$(1000 – x) in Share B. Any amount, x, invested in Share A has a 70% chance of doubling in value (that is, ending up with +x net gain) and a 30% chance of total loss (end up with –x). An investment in Share B has a 60% chance of doubling in value and a 40% chance of a total loss. The variations of Share A and Share B are independent.

a) Determine the optimal investment if the investor's utility function for a gain or loss of \$$y$ is $u(y) = \text{Ln}(y+3000)$. *Hint*: construct the 4-outcome combined gamble A + B, and calculate the expected utility of the combined gamble as a function of x, the amount invested in A.

b) What would be the optimal investment with the utility function $u(y) = 3y + 2000$?