CS 334 Fall 2021: Problem Set 1.

Problem 1. (10 points) Consider the following language D, defined over the alphabet $\{a, b\}$ as:
$\{w: w\ contains\ an\ even\ number\ of\ a's\ and\ an\ odd\ number\ of\ b's\ and\ does\ not\ contain\ the\ substring\ ab\}$.
Construct a 5-state DFA that recognizes D. Give an explanation for why your construction is correct.

Problem 2. (10 points)
(a) Prove that the state diagram of every DFA contains a directed cycle.
(b) State a necessary and sufficient condition a DFA must satisfy for its language to be infinite. Prove the correctness of your statement.

Problem 3. (20 points) Construct deterministic FSAs for each of the following languages over the alphabet $\{a, b\}$:

1. $A = \{w: w\ starts\ with\ an\ a\ and\ has\ at\ most\ one\ b.\}$

2. $L_4 = \{w: the\ 4^{th}\ symbol\ from\ the\ end\ of\ w\ is\ a\}$.
   So, for example, $abbabaaba \in L_4\ \ but\ \ abababababa \notin L_4$. To get started see how trivial it would be if the FSA had to check the last symbol of the input. Then try to design an FSA that must check only the 2nd last symbol. What does each state in this FSA represent? Now extend this idea to design an FSA that accepts L_3 before proceeding to the DFA for L_4.

Extra Credit Problem (10 points) Modify the proof of Theorem 1.25 in the textbook to cover the case when the machines M_1, M_2 have *different* input alphabets Σ_1, Σ_2. Hint: the machine M that recognizes the union of the languages of M_1, M_2 will have input alphabet $\Sigma = \Sigma_1 \cup \Sigma_2$. Take care when defining $\delta((r_1, r_2), a)$ as the symbol a could belong to one alphabet but not the other!

Optional Exercise. (Present your ideas anytime this term during Sandeep's office hours.) You have been asked to design an FSA to operate a building elevator. How would you go about this design process? What would a state of the FSA correspond to? What should be the alphabet? What constraints would be reasonable to impose on the movement of the elevator? This is an open-ended problem – we don't expect a full design, but rather things you considered and what constraints you found difficult to design for. To get started, imagine a 2-storey building, then a 3-storey building, etc. How do your ideas scale for a very tall building?