

CS 334 Fall 2021: Problem Set 10.

Problem 1. (20 points) In this problem you will construct a linear-time algorithm to decide 2CNFSAT. Starting with an input formula in 2CNF with n variables x_1, \dots, x_n , we first construct a graph with $2n$ vertices, one for each literal. For every clause construct two edges as follows:

- If the clause is of the form $(x_i \vee x_j)$, add the directed edges $(\neg x_i, x_j)$ and $(\neg x_j, x_i)$.
- If the clause is of the form $(\neg x_i \vee x_j)$, add directed edges (x_i, x_j) and $(\neg x_j, \neg x_i)$.

Now, give a high level description of algorithm that navigates this graph and decides whether the input formula is satisfiable. (Hint: what does a directed edge from a to b imply? What about a directed path from a to b ? Finally, what would a directed path from x to $\neg x$ mean?)

Problem 2. (20 points) Behold, a genie appears before you! Given a formula $\phi(x_1, x_2, \dots, x_n)$ in conjunctive normal form with n boolean variables, the genie will correctly tell you (in one step) whether the formula is satisfiable. Unfortunately, the genie will not give you a truth assignment to the variables that makes the formula true.

Your problem is to figure out a satisfying truth assignment when the genie says the formula is satisfiable. You can present the genie with a polynomial (in the size of ϕ) number of queries.

- (5 points) Give a high-level description of your algorithm, with enough detail.
- (2 points) What is the maximum number of queries made by your algorithm?
- (3 points) Explain why your algorithm correctly finds a satisfying assignment for a satisfiable formula.
- (10 points) A second genie appears! Given an undirected graph, this genie will correctly tell you whether the graph has a Hamiltonian cycle. How will you use this genie to find a Hamiltonian cycle in any graph that has one in polynomial time?