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Homework for

MA 346 Numerical Methods

Spring 2022 — Homework 7

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Exercise 1
Write codes[1] to approximate integrals of the form

$$I(f) = \int_a^b f(t)\,dt$$

using the trapezoidal and Simpson's rule on the sub-intervals $[x_{i-1}, x_i]$, $i = 1, \ldots, n$, where $x_i = a + ih$, $i = 0, \ldots, n$ with $h = (b - a)/n$.

(a) Hand in your Matlab files, and use them to approximate the integral

$$\int_{0.1}^{1} \sqrt{x}\,dx.$$

Compare the numerical errors E_n for both quadrature rules (the exact value of the integral is $\frac{2}{3} - \frac{1}{15\sqrt{10}}$). Try different n (e.g., $n = 10, 20, 40, 80, \ldots$) and plot the quadrature errors versus n in a double-logarithmic plot.[2]

(b) To numerically study how the errors E_n decrease with n, we assume that the errors behaves like Cn^{κ}, with to-be-determined $\kappa \in \mathbb{R}$. Applying the logarithm to $E_n = Cn^{\kappa}$ results in

$$\log(E_n) = \log(C) + \kappa \log(n). \tag{1}$$

Use the values for n and $\log(E_n)$ you computed in (a) to find a good estimate for κ. Compare your findings for κ with the theoretical estimates for the composite trapezoidal and Simpson's rules.

(c) Repeat steps (a) and (b) using $a = 0$ instead of $a = 0.1$ as lower integration bound. Can the theoretical estimates for the composite rules still be applied and why/why not?

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[1] Ideally, you write functions `trapez(f,a,b,n)` and `simpson(f,a,b,n)`, where f is a function handle (see `http://www.mathworks.com/help/matlab/matlab_prog/creating-a-function-handle.html` if you are not familiar with that concept) or f is the vector $(f(x_0), \ldots, f(x_n))$.

[2] You can use the Matlab function `loglog` for that.

Exercise 2 (Gaussian quadrature)
Recall that Gaussian quadrature is the unique quadrature rule with n points that is exact for polynomials of degree up to and including $2n-1$.

(a) Show that this statement is equivalent to saying that the quadrature rule is exact for the monomials x^0, x^1, $x^2, \ldots, x^{2n-1}$.

(b) Compute the formula for the 3-point Gaussian quadrature rule on the interval $(-1, 1)$. To do this, use symmetry and the fact 3 is an odd number to conclude that the nodes must be the points $\boldsymbol{x} = [-x_1, 0, x_1]$ where $x_1 \in (0, 1)$ is unknown, and the weights must be $\boldsymbol{w} = [w_1, w_2, w_1]$. Write down a system of 3 equations for the three unknowns $\{x_1, w_1, w_2\}$ and solve it.