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Homework for

MA 346 Numerical Methods

Spring 2022 — Homework 2

Submit via Canvas before February 6, 2022, 11:59 p.m..

Exercise 1 (Gaussian Elimination)

a) Use Gaussian Elimination without Pivoting to calculate the solution of

$$\begin{pmatrix} 10^{-6} & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

b) Then, apply Gaussian Elimination without Pivoting to calculate the solution of the following linear system of equations

$$\begin{pmatrix} 1 & 1 \\ 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

c) Compare the two results and reflect on these results.

Exercise 2 (Gaussian Elimination and Inverse of Matrix)

a) Use Gaussian Elimination without Pivoting to calculate the inverse of the matrix

$$A = \begin{pmatrix} 2 & -3 & 2 \\ 1 & -2 & 1 \\ 3 & 2 & 2 \end{pmatrix}.$$

Write down all intermediate steps. Do not forget to check after you have determined A^{-1} whether A^{-1} and A actually satisfy $AA^{-1} = A^{-1}A = I$, where $I \in \mathbb{R}^{3\times 3}$ denotes the identity matrix.

b) Use the result from a) to compute the solution of $Ax = b$ for $b = (2, 1, 2)^T$.

Exercise 3 (Gaussian Elimination (Deadline: February 20, 11:59pm))
Write a Matlab function `gauss_elim` which can be used as `[x] = gauss_elim(A,b)` for a vector $b \in \mathbb{R}^{n\times 1}$ and matrices $A \in \mathbb{R}^{n\times n}$ that do *not* require pivoting. The program does *not* need to check whether the matrix requires pivoting.

- The m-file has to start with a comment stating what the program is doing and what are the requirements for the matrix considered.
- The algorithm has to check whether the matrix is square and return an error otherwise. Hint: Use the Matlab function `size` to that end.
- The Matlab functions `lu`, `pcg`, `inv` or any Matlab function that solves a linear system of equations, inverts a matrix, or computes a decomposition of a matrix must not be used.
- Use comments in your code explaining what you are doing in the respective step.

Develop a testing strategy to convince us why we should trust that your code correctly implements the algorithm. Include a detailed explanation of this testing strategy and its results in the report; including screen shots or pictures of a Gaussian elimination written by hand is fine. Some suggestions:

- Think of various test matrices (one is not sufficient) including matrices that do not require pivoting and for which you can do Gauss elimination either by hand in an acceptable time frame (use e.g. the other exercises from this Exercise set) or where the result x is obvious (e.g. choose A as the identity matrix, first define A and x then use b as $b = Ax$). Give the test matrices in the report and explain why you chose those matrices.
- Compare the output of the algorithm in each iteration with the result you obtain in each iteration if you do Gauss elimination by hand.

How to submit your solution: Write a report following the instructions above (pdf) and submit both the report and the Matlab function as single files via Canvas.