CS 334 Fall 2021: Problem Set 5.

Problem 1. (15 points)

- **a)** (5 points) If G is a CFG in Chomsky Normal Form, prove that $2n-1$ rule applications are necessary and sufficient to derive any string of length $n, n \geq 1$.
- **b)** (10 points) Convert the following CFG into an equivalent grammar in Chomsky Normal Form using the procedure described in the textbook. Show all steps of the conversion process.

$$S \rightarrow aAa \mid bBb \mid \epsilon$$
$$A \rightarrow C \mid a$$
$$B \rightarrow C \mid b$$
$$C \rightarrow CDA \mid \epsilon$$
$$D \rightarrow A \mid B \mid ab$$

Problem 2. (15 points) Let $G = (V, \Sigma, R, S)$ be a CFG where $V = \{S, T, U\}$, $\Sigma = \{0, \#\}$, and R is the set of rules:

$$S \rightarrow TT \mid U$$
$$T \rightarrow 0T \mid T0 \mid \#$$
$$U \rightarrow 0U00 \mid \#$$

Describe the language $L(G)$ in English and prove that it is not regular.

Problem 3. (15 points) Let $L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$ and $L_{mult} = \{a^i b^{ij} c^j : i, j \geq 0\}$. For each language, either give a CFG for it, or prove that it is not a CFL.