MA 232 - Linear Algebra

Homework 2 (due February 26 at 5pm)

Problem 1 [20pts] Which matrices E_{21}, E_{31} produce zero in the $(2,1)$ respectively $(3,1)$ position of $E_{21} \cdot A$ respectively $E_{31} \cdot A$ for

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$$

Find the single matrix E that produces both zeros at once and calculate $E \cdot A$.

Problem 2 [20pts] Use the Gauss elimination method in order to find the inverses of the following matrices:

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix}$$

Problem 3 [20pts] Factor the symmetric matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ as $A = LDL^T$.

Problem 4 [20pts] Find the space of vectors to which $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ must belong, so the following system of equations has a solution $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$?

Problem 5 [20 pts] Reduce the following matrices to their row reduced echelon form: $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$