Practice

$f(x) = x^3 - 6.1x^2 + 3.2x + 1.5$

$\rightarrow x = 4.71$

"I pledge my honor I've abided by the Stevens Honor Code"
– Yarisa Spalding

Excercise 1

5d. Compute absolute & relative error for 3 digit rounding

$\lfloor (121 - 119) - 0.327 \rceil$

Exact $\rightarrow (2) - 0.327 = \underline{1.673}$

Rounded $\rightarrow (2) - 0.327 = 1.673 \sim \underline{1.67}$

Absolute error $\rightarrow |p - p^*| = |0.003| = \boxed{0.003}$

Relative error $\rightarrow \dfrac{|p - p^*|}{|p|} = \dfrac{|0.003|}{|1.673|} = \boxed{1.73 \times 10^{-3}}$

17a) $1.130x - 6.990y = 14.20$
$1.013x - 6.099y = 14.22$

$\rightarrow m = \dfrac{1.013}{1.130} = 0.8965$

$\rightarrow d_1 = -6.099 - 0.8965(-6.990) = d - mb$
$= 0.1675$

$\rightarrow f_1 = 14.22 - 0.8965(14.20) = f - me$

$= 1.490$

$\rightarrow \boxed{Y} = \frac{f_1}{d_1} = \frac{1.490}{0.1673} = \boxed{8.906}$

$$\rightarrow \boxed{X} = \frac{(14.20 + 6.990(8.906))}{1.130}$$

$$= \frac{(14.20 + 62.25)}{1.130} = \boxed{67.65}$$

Excercise 2

$$Gc. \lim_{n \to \infty} \left(\sin \tfrac{1}{n}\right)^2 = 0$$

$$|\alpha_n - \alpha| \leq K|\beta_n|$$

Sequence

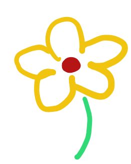
$\alpha_n = \left(\sin \tfrac{1}{n}\right)^2$

α_n converges to 0

Using taylor series ~ 4 terms

$$\rightarrow \sin(x) = \sin(0) + \frac{\cos(0)x}{1} - \frac{\sin(0)x^2}{2} - \frac{\cos(\xi)x^3}{6}$$

$$@\ x_0 = 0 \quad = x - \frac{x^3}{6}\cos\xi$$



$$\rightarrow \sin(1/n) = 1/n - \frac{1}{6n^3}\cos\xi$$

$$\rightarrow (\sin(1/n))^2 = 1/n^2 - \frac{1}{3n^4}\cos\xi + \frac{1}{36n^6}(\cos\xi)$$

$$\leq 1/n^2 - \frac{1}{3n^4} + \frac{1}{36n^6} = 1/n^2\left(1 - \frac{1}{3n^2} + \frac{1}{36n^4}\right)$$

$$= 1/n^2\left(1 - \frac{1}{6n^2}\right)^2 = 1/n^2\left(\frac{6n^2-1}{6n^2}\right)^2 \leq 1/n^2$$

$$\left|(\sin(1/n))^2 - 0\right| = \left|\sin(1/n)^2\right| = \left|1/n^2 \underbrace{\left(\frac{6n^2-1}{6n^2}\right)^2}_{\sim 1}\right| \leq \left|1/n^2\right|$$

$$\rightarrow \sin(1/n)^2 = 0 + O(1/n^2)$$

6d.* $\lim_{n\to\infty} [\ln(n+1) - \ln(n)] = 0$

$\ln(n+1) - \ln(n) = \ln\left(\frac{n+1}{n}\right)$

Since $n \geq 0$, we know $\ln(n+1) \leq n$

$$\rightarrow \ln(n) \leq n-1$$

Thus

$$\ln\left(\frac{n+1}{n}\right) \leq \frac{n+1}{n} - 1 = 1/n$$

$$\alpha_n = 0 + O(1/n)$$

7b. $\lim\limits_{h \to 0} \dfrac{1-\cos h}{h} = 0$

$$\cos x \underset{x_0=0}{\approx} \cos(0) - \frac{\sin(0)x}{1} - \frac{\cos(0)x^2}{2} + \frac{\sin(0)x^3}{6} + \frac{\cos(\xi)x^4}{24}$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24}\cos\xi$$

$$\rightarrow |\alpha_n| = \left|\frac{1-\left(1-\frac{h^2}{2}+\frac{h^4}{24}\cos\xi\right)}{h}\right| = \left|\frac{\frac{h^2}{2}-\frac{h^4}{24}\cos\xi}{h}\right|$$

$$= \left|\frac{h}{2} - \frac{h^3}{24}\cos\xi\right| \leq \left|\frac{h}{2} - \frac{h^3}{24}\right|$$

$$= \left|\frac{12h - h^3}{24}\right| \leq \left|\tfrac{1}{2}h\right|$$

$$\Rightarrow \alpha_n = 0 + O(h)$$

7c. $\lim\limits_{h \to 0} \dfrac{\sin h - h\cos h}{h} = 0$

$$\rightarrow \alpha_n = \frac{\sin x}{x} - \cos x = \frac{x - \frac{x^3}{6}\cos\xi}{x} - \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\cos\xi\right)$$

$$= 1 - \frac{x^2}{6}\cos\xi - 1 + \frac{x^2}{2} - \frac{x^4}{24}\cos\xi$$

$$= \left| \frac{x^2}{2} - \left(\frac{x^2}{6} - \frac{x^4}{24} \right) \cos \xi \right| \leq \left| x^2/2 \right|$$

$$\rightarrow \alpha_n = 0 + O(h^2)$$