



D.

$$E[D] \stackrel{\Delta}{=} \mu$$

$$X_1, \dots, X_n \sim D.$$

$$X \sim D \quad \text{positive}$$

Markov's.

$$P_r(X > t \cdot \mu) < \frac{1}{t}$$

$$t = 100$$

w.p.

$$\approx 99.$$

$$X \leq 100 \cdot \mu.$$

$\Leftrightarrow$

$$\mu \geq \frac{X}{100}$$

$\alpha = \text{Var}(D)$  is given

$$\Pr(|X - \mu| > t) \leq \frac{\text{Var}(X)}{t^2}$$

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$$X = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\text{Var}[X] = \frac{1}{n} \cdot \text{Var}[X] = \frac{\alpha}{n}$$

$$\Pr(|X - \mu| > t) \leq \frac{\alpha/n}{t^2}$$

↓

$$\underline{t = \epsilon}$$

$$|X - \mu| > \epsilon$$

$$\frac{\alpha/n}{\epsilon^2}$$

w.p.

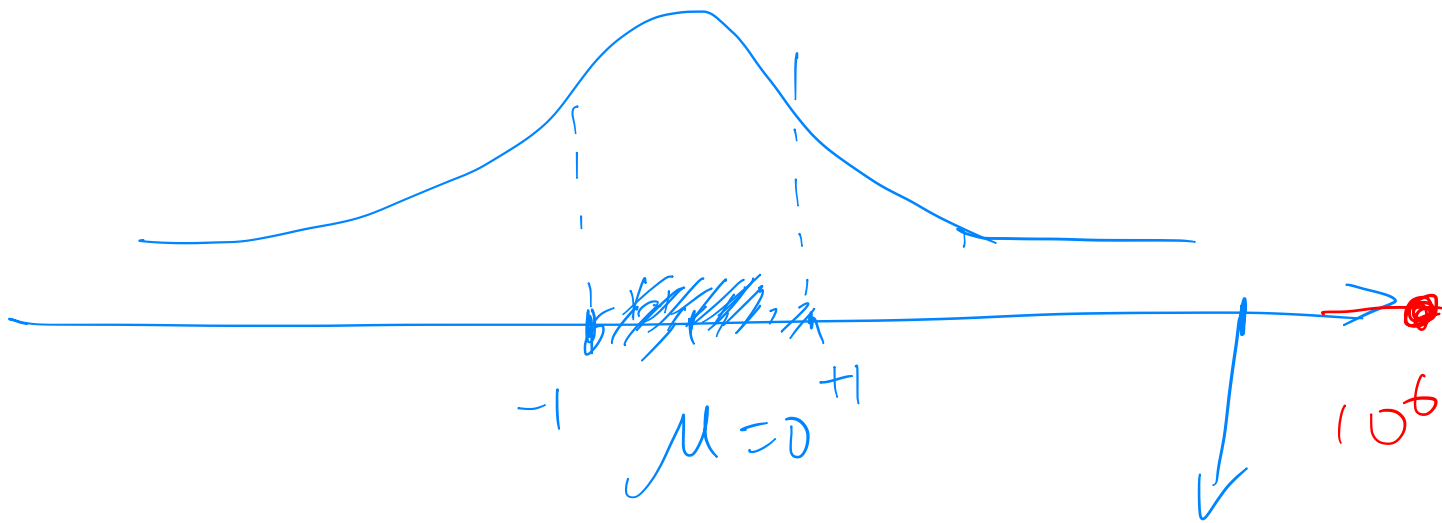
$$1 - \frac{\alpha/n}{\epsilon^2}$$

$$|X - \mu| \leq \epsilon$$

$$n = \frac{100\alpha}{\epsilon^2}$$

$$\approx 0.99$$

$$\frac{\alpha/n}{\epsilon^2} = 0.01$$

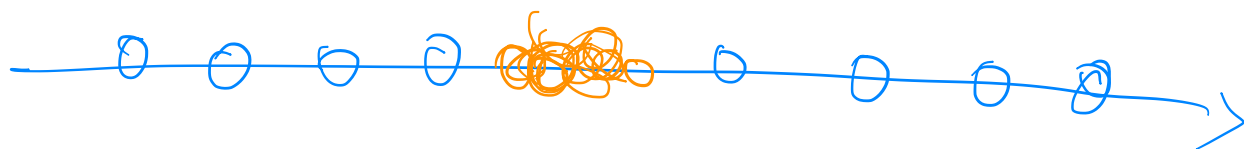
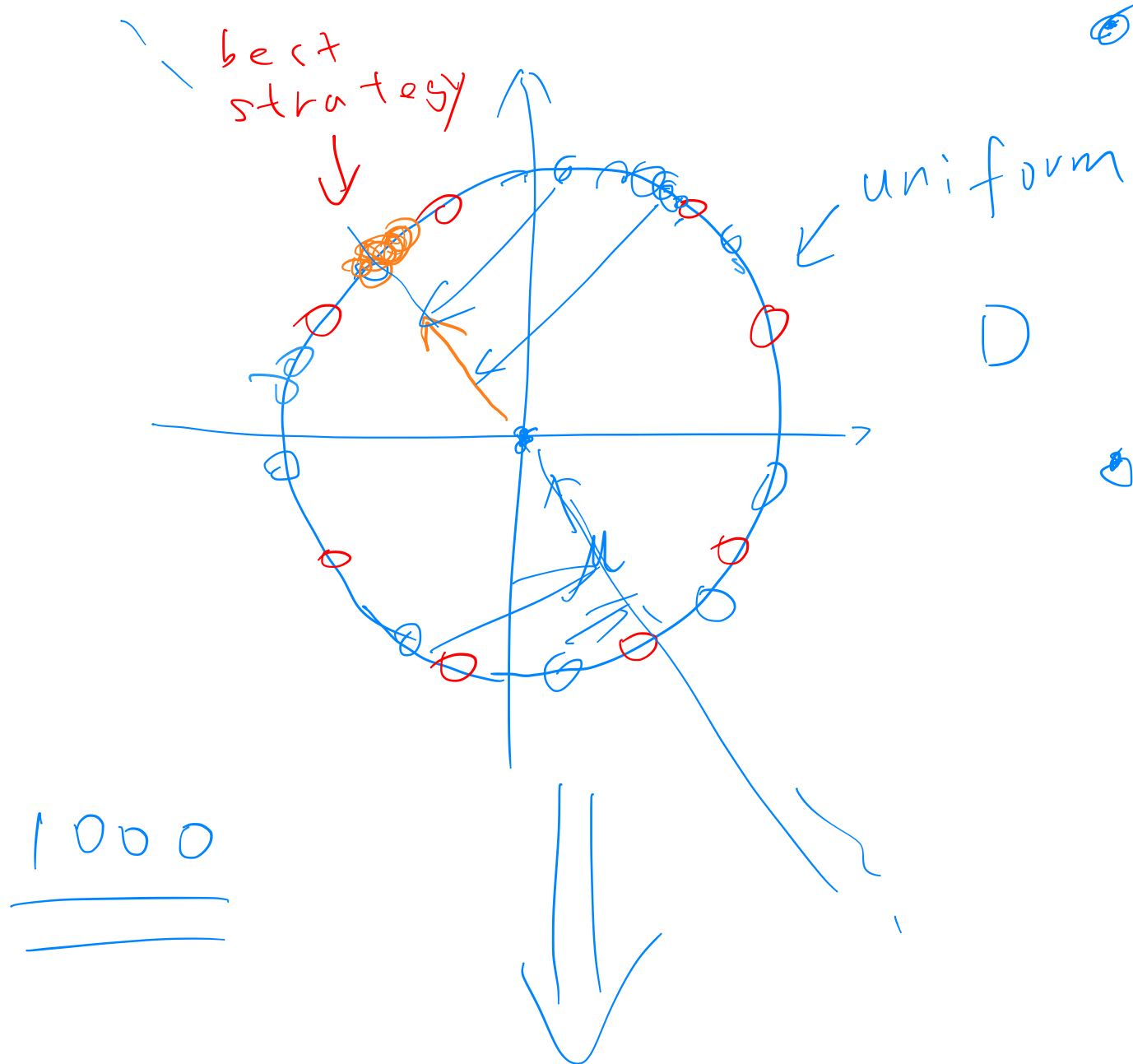


$$\frac{1}{n} \sum x_i$$

$$R_{\frac{d}{n}}$$

$$\frac{E}{d}$$

$$E$$



large variance

comp. w/ variance if  
all data are clean.

$x_1 \quad \dots$  $x_n$  $\downarrow$  $\dots$  $\downarrow$  $q_1$  $q_n$ 

$$0 \leq q_i \leq 1$$

Goal: if  $\underline{x_i}$  is clean.

$$\underline{q_i} = 1$$

if  $x_i$  is corrupted.

$$\underline{q_i} = 0$$

$$\Rightarrow \frac{1}{(1-\eta)n} \sum_{i=1}^n q_i x_i \geq x_i$$

 $\Rightarrow$ 

$\eta$  = frac. of corrupted data.

$$(1) \quad 0 \leq q_i \leq 1$$

$$(2) \quad \frac{1}{(1-\eta)n} \cdot \sum q_i (x_i - \hat{\mu})(x_i - \hat{\mu})^T$$

$$\hat{\mu} = \frac{1}{(1-\eta)n} \cdot \sum q_i x_i$$

↓ ≤  
cor. data  
have small  
weight.

⌈ → I  
↓  
given as  
far as we  
know clean  
data dist.

LHS = cov. of D.

I

$$D = N(\mu, I)$$

$q_i = 1$ ,  $x_i$  corrupted.

$$(3) \quad \sum q_i \geq (1-\eta)n$$

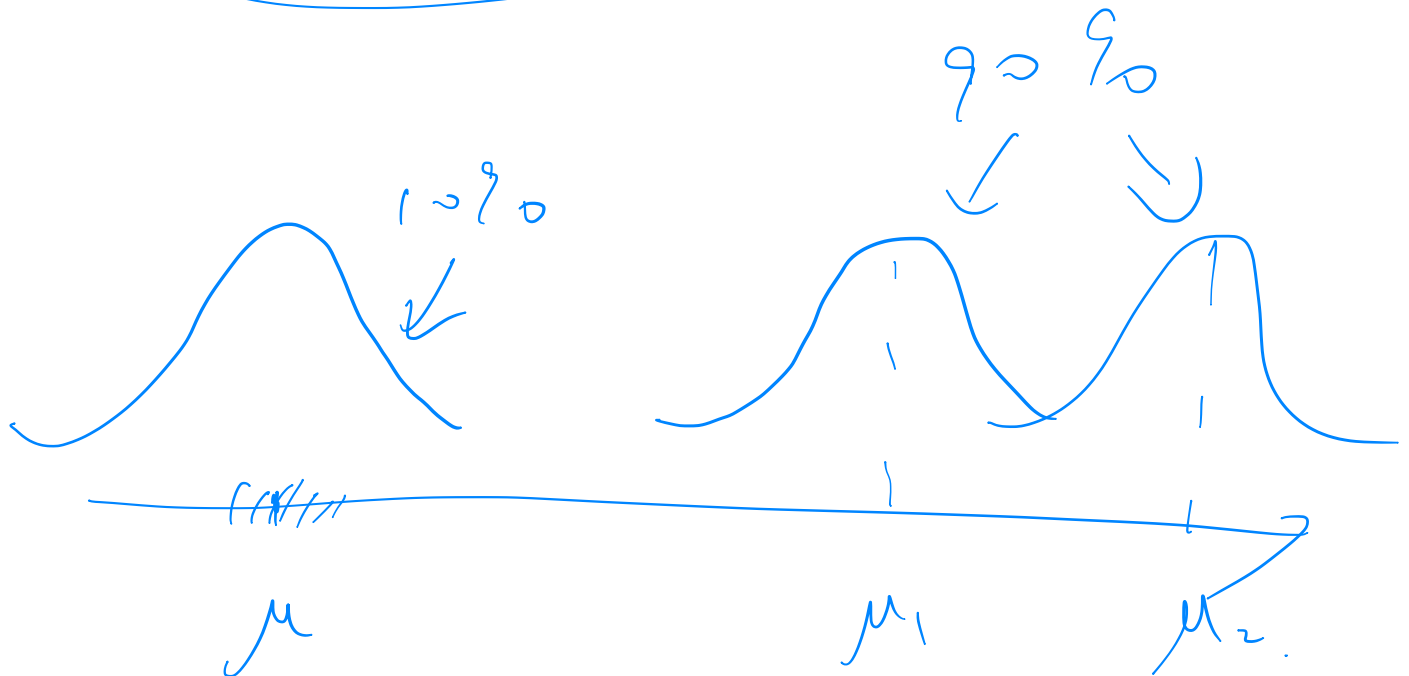
clean data have large weight

$$\eta < 1/2$$

99% false

← 4.9 (100 people)

← Warning: - -



→  $\{\hat{\mu}_1, \dots, \hat{\mu}_k\}$



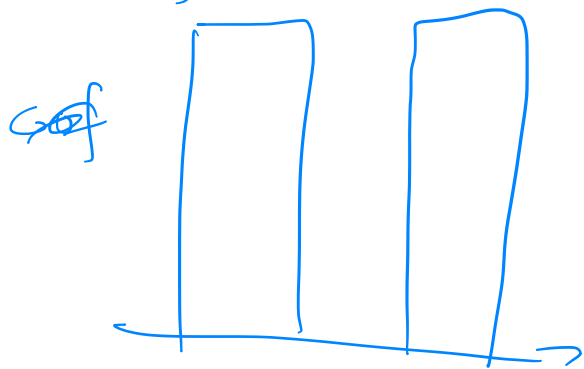
$$\min_{1 \leq i \leq k} \|\hat{\mu}_i - \mu\| \leq \epsilon$$

$\uparrow$   
 true mean

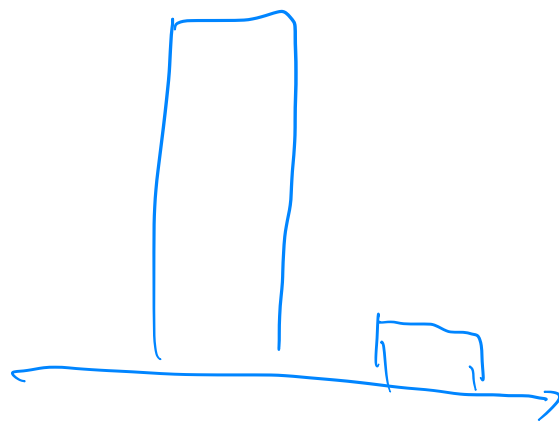
$\rightarrow 95\%$   
 $\rightarrow pl.$

$\rightarrow$  Harrisburg  
 $\rightarrow \leq 10\%$

conf.



conf



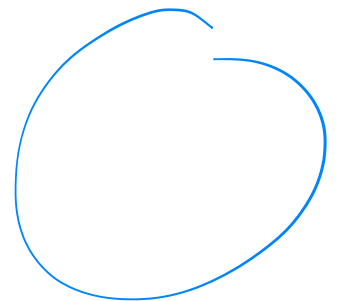
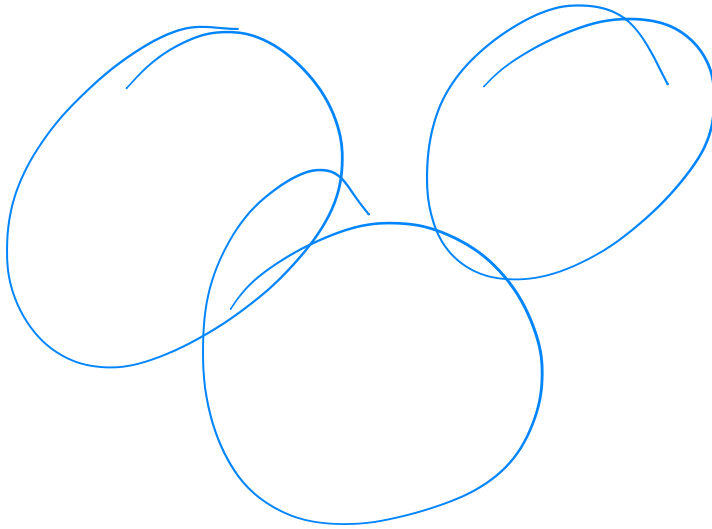
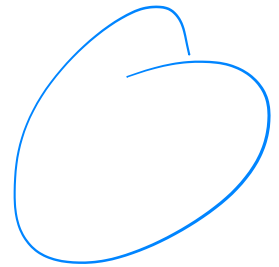
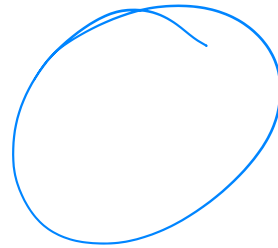
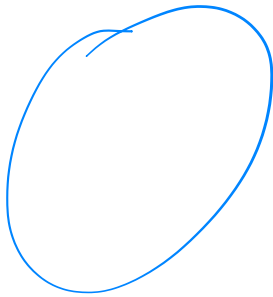
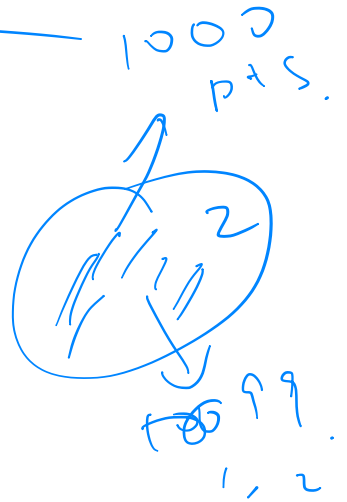
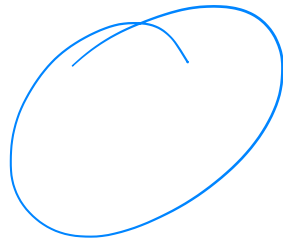
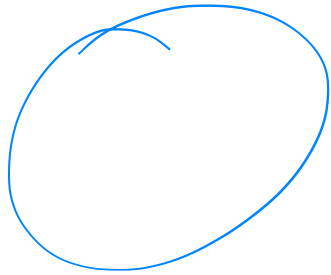
$n$   $X_{train}$

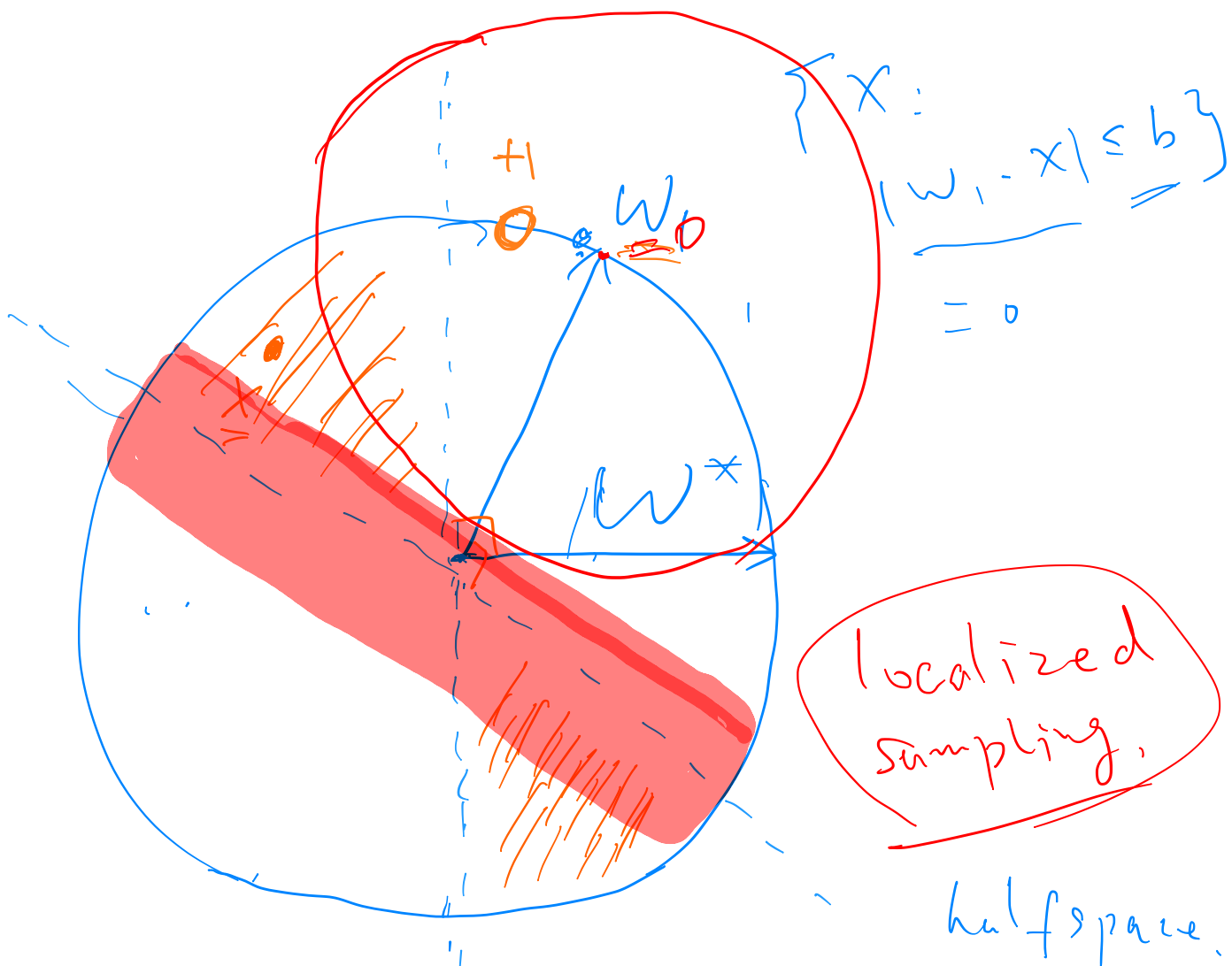
$m$   $X_{test}$

$M$   $n \leq m$

# Active Learning

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eg

$x,$

$$y = \text{sign}(w^* \cdot x)$$

$$w^* \cdot x > 0$$

$$y = 1$$

$$w^* \cdot x < 0$$

$$y = -1$$

$$\underline{w}_0 = (1, 0, 0, \dots, 0)$$

$$k = 1, \dots, T$$

$$X_k = \{x : |w_{k-1} \cdot x| \leq b_k\}$$

• draw  $n_k$  pts from  $X_k$ .  
 • label them  
 • "strategy" for labeling.

→ SVM.

$$\min_w \sum_{t=1}^{n_k} \max\{0, 1 - y_t \cdot w \cdot x_t\}$$

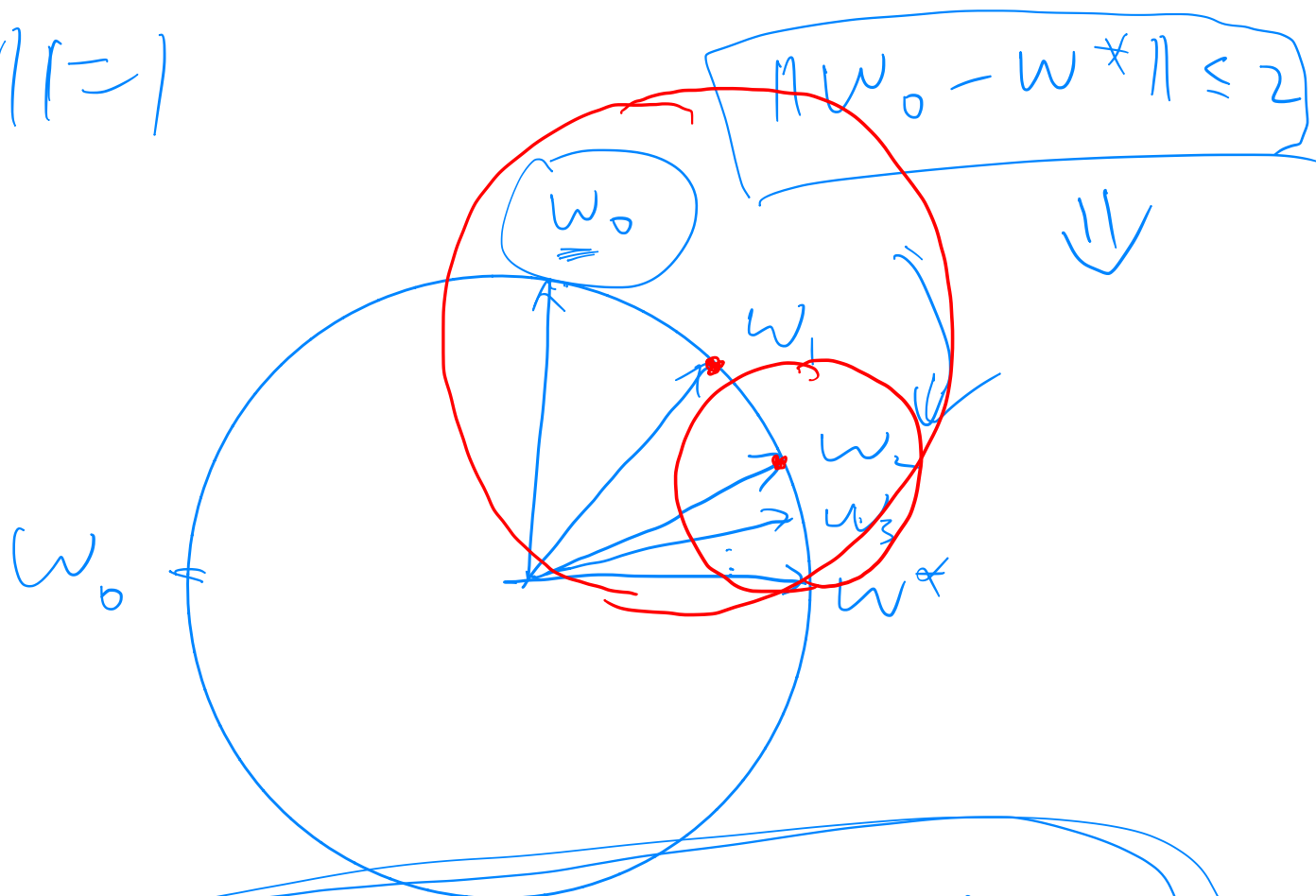
$w_1$  ←

s.t.

~~$$\|w\|_2 \leq r_k$$~~

$$\|w - w_{k-1}\|_2 \leq r_k$$

$$\|w\| = 1$$



$$\# \text{ labels/iter} = d.$$

$$\text{init angle} = O(1).$$

$$\# \text{ iter} = \log \frac{1}{\epsilon}$$

$$\boxed{d \cdot \log \frac{1}{\epsilon}} \leftarrow$$

$$\textcircled{1} \approx (d \cdot \log \frac{1}{\epsilon}) \quad \textcircled{2} d \cdot \frac{1}{\epsilon}$$

$$\|w_0 - w^*\| \leq 2$$

$\Downarrow$

$$\underline{V_0 = \{w : \|w - w_0\| \leq 2\}}$$

$$w^* \in V_0$$

$V_0$  : trust region for  $w^*$

$\Downarrow$

$$\underline{\|w_1 - w^*\| \leq 1}$$

$\Downarrow$

$$\underline{V_1 = \{w : \|w - w_1\| \leq 1\}}$$

$\Downarrow$

$\vdots$

# Margin-based Active Learning.



Condition on  $D$ .

2007

2014  $\rightarrow y$   
            
          ↓  
          :

2020  
            
          ↓  
           $(x, y)$

$d - \log \frac{1}{\epsilon}$



$s \cdot \log d - \log \frac{1}{\epsilon}.$

$$d/\epsilon.$$

$$\# \text{ label} = d \cdot \log \frac{1}{\epsilon}$$

$$\# \text{ comp} = d \cdot \frac{1}{\epsilon}$$

