MA331 Homework02

1 Simulation

Problem 1. Assume that $N \sim \mathcal{B}(n, p)$, a Binomial distribution with number of trials n and probability of success p. Set $p = 0.4$.

(i) For $n = 20, 30, 50, 75, 100$, accurately compute $P(N \leq 8.25)$ by using R function.

(ii) For $n = 20, 30, 50, 75, 100$, approximate $P(N \leq 8.25)$ by using Laplace theorem.

(iii) Evaluate and scatter plot errors of all approximations from (ii), i.e., the absolute difference between the accurate computation and the Laplace approximation.

(iv) Scatter plot of all errors of (iii). What do you perceive based on the plot.

Problem 2. Check the instruction of R commands '`density`(x)' and '`plot`(`density`(x))'. Generate a SRS $X_1, \cdots, X_n$ for the population $X \sim \mathcal{N}(2, 3^2)$ by using '`rnorm`(n,2,3)', and collect the observed samples of

$$\frac{\bar{X} - 2}{\sqrt{3^2/n}}, \qquad \frac{(n-1)S^2}{3^2}, \qquad \left(\frac{\bar{X} - 2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2} \right),$$

respectively. Then, based on the corresponding samples, plot estimated density curves of $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$, respectively, and also make the scatter plot of $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2} \right)$.

(i) For $n = 20$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(ii) For $n = 30$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(iii) For $n = 50$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(iv) For $n = 75$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(v) Based on the plots of $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and those of $\frac{(n-1)S^2}{3^2}$ in (i) - (iv) describe your findings on probability distributions of $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$, respectively.

(vi) Based on the scatter plots of $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2} \right)$ in (i) - (iv) describe your findings on the statistical association between $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$.

2 Computation

Problem 3. Assume a SRS $(x_1, \cdots, x_n) = (0.5, 0.9, -0.7, 1.5, -1, 2.5, 3.75, -1.6, 0.2, 3.15)$ from $X \sim \mathcal{N}(\mu, \sigma^2)$.

(i) For $\mu = 1$ and $\sigma^2 = 4$, compute $a = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$, determine the distribution of the statistic $\sum_{i=1}^{n} \frac{(X_i-\mu)^2}{\sigma^2}$, and then evaluate $\mathrm{P}\left(\sum_{i=1}^{n} \frac{(X_i-\mu)^2}{\sigma^2} \leq a\right)$.

(ii) For $\sigma^2 = 4$, compute $b = \sum_{i=1}^{n} \frac{(x_i-\bar{x})^2}{\sigma^2}$, determine the distribution of the statistic $\sum_{i=1}^{n} \frac{(X_i-\bar{X})^2}{\sigma^2}$, and then evaluate $\mathrm{P}\left(\sum_{i=1}^{n} \frac{(X_i-\bar{X})^2}{\sigma^2} \leq b\right)$.

(iii) For $\mu = 1$, compute $c = \frac{\bar{x}-\mu}{\sqrt{s^2/n}}$, determine the distribution of the statistic $\frac{\bar{X}-\mu}{\sqrt{S^2/n}}$, and then evaluate $\mathrm{P}\left(\frac{\bar{X}-\mu}{\sqrt{S^2/n}} \leq c\right)$.

Problem 4. Assume $X \sim \mathcal{N}(-1, 9)$, $Y \sim \chi^2_{12}$, $T \sim \mathcal{T}_{10}$ and $F \sim \mathcal{F}_{8,9}$.

(i) Calculate $\mathrm{P}(X \in (0, 1))$, $\mathrm{P}(Y \in (3, 14))$, $\mathrm{P}(T \in (0, 1))$, and $\mathrm{P}(F \in (0, 1))$.

(ii) For $\alpha = 0.05$, calculate $\alpha/2$ and $1 - \alpha/2$ quantiles of X, Y, T and F, respectively.

3 Verification

Problem 5. verify that (i) $\mathrm{E}[N] = np$ for $N \sim \mathcal{B}(n, p)$, (ii) $\mathrm{E}[T] = 0$ for $T \sim \mathcal{T}_n$.