Harris Spahic
"I pledge my honor I have abided by the Steven's honor system."
3/7/21

Problem #1)
Part 1:

Problem #1) Notice $U(0,\theta) = 1/\theta$ for $\theta \in (0,\theta)$ by def.

$\rightarrow E[x] = \int_{-\infty}^{\infty} f(x) \cdot x\, dx = \int_0^\theta f(\theta) \cdot x\, dx = \int_0^\theta 1/\theta \, x\, dx = \frac{\theta^2}{2\theta} = \frac{\theta}{2}$

By symmetry, makes sense. ✓

$\rightarrow E[x^2] = \int_0^\theta f(\theta) x^2 dx = \frac{\theta^3}{3\theta} = \frac{\theta^2}{3}$

$\Rightarrow E[X] = \bar{X} \Rightarrow \frac{\theta}{2} = 1/n\,(x_1 + \dots + x_n) \Rightarrow \boxed{\hat{\theta}_m = \frac{2(\sum_{i=1}^n x_i)}{n}}$

Part 2:

1ii) $L(x,\theta) = \prod_{i=1}^n f(x_i, n) = \prod_{i=1}^n 1/\theta = \theta^{-n}$

$\uparrow$ constant uniform func.

$\frac{d(\ln(L(x,\theta)))}{d\theta} = \frac{d(\ln(\theta^{-n}))}{d\theta}$

$= \frac{d(-n\ln\theta)}{d\theta} = \frac{-n}{\theta} < 0 \rightarrow L(x,\theta)$ is decreasing for $\theta \geq x_n$.

Thus $\max(\theta_L) = \max(x_i \mid i \in [1,n])$

$\rightarrow \boxed{\hat{\theta}_L = \max(x_1, x_2, \dots, x_n)}$

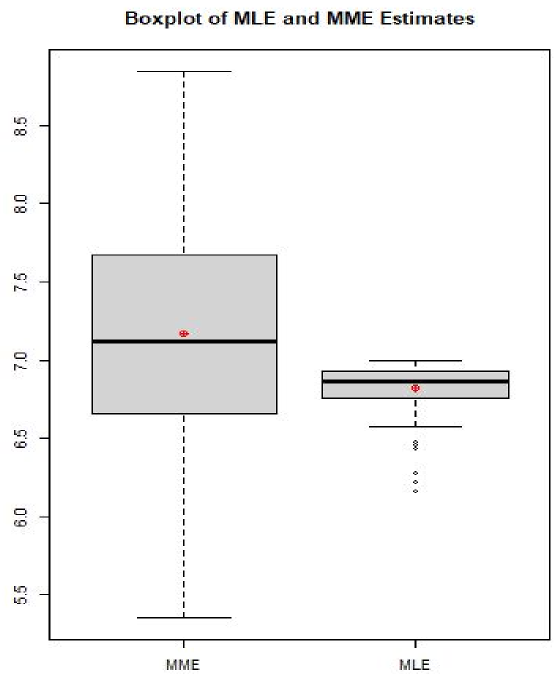
Part 3:

1 iii)* $\hat{\theta}_M = 2/7\,(1.0+2.4+3.2+1.2+0.5+3.1+6.8) = \boxed{5.2}$

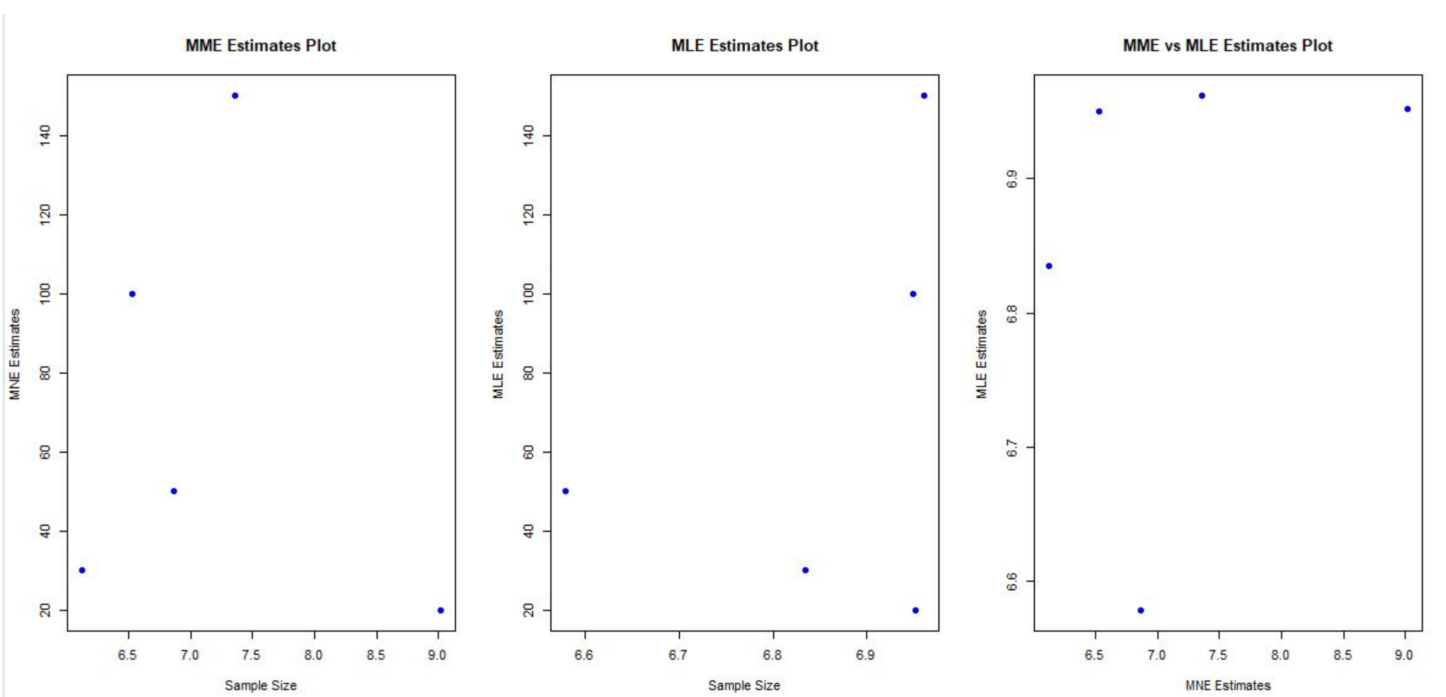
$\hat{\theta}_L = \max(\;\downarrow\;) = \boxed{6.8}$

The moment estimator is the best estimator as it has the smallest value.

Part 4:


Boxplot of MLE and MME Estimates
8.5
8.0
7.5
7.0
6.5
6.0
5.5
MME
MLE

Part 5:


MME Estimates Plot
MNE Estimates
Sample Size
MLE Estimates Plot
MLE Estimates
Sample Size
MME vs MLE Estimates Plot
MLE Estimates
MNE Estimates

Problem #2)

Part 1:

Problem #2) Given SRS $X_1, X_2, \dots, X_n$ with $X \sim N(\mu, \sigma^2)$

$$E[X] = \mu$$

$$E[X^2] = Var[X] + E[X]^2 = \sigma^2 + \mu^2$$

$$\rightarrow E[X] = \bar{X} = \hat{\mu} \rightarrow \boxed{\hat{\mu}_m = 1/n \sum_{i=1} X_i = \bar{X}}$$

$$\rightarrow Var[X] = E[X^2] - E[X]^2 = 1/n \sum_{i=1}^{n} X_i^2 - \mu^2 = \boxed{1/n \sum_{i=1} X_n^2 - \bar{X}^2 = \hat{\sigma}_n^2}$$

Part 2:

2ii) We know $f_X(x_i) = (2\pi\sigma^2)^{-1/2} e^{\left(-\frac{(X_i - \mu)^2}{2\sigma^2}\right)}$

$$\rightarrow L\underbrace{((\mu, \sigma^2), (x_1, x_2, \dots, x_n))} = (2\pi\sigma^2)^{-n/2} \cdot e^{\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2\right)}$$

$$\ln(L(\,\text{"}\,)) = -n/2 \ln(2\pi\sigma^2) - \left(1/2\sigma^2 \sum_{i=1}^{n} (x_i - \mu)^2\right)$$

$$= -n/2 \ln(2\pi) - n/2 \ln(\sigma^2) - 1/2\sigma^2 \left(\sum_{i=1}^{n} (x_i - \mu)^2\right)$$

$$\rightarrow \frac{d(\ln(L(\dots)))}{d\mu} = 0 - 0 - 1/2\sigma^2 \left(2 \cdot \sum_{i=1}^{n} (\mu - x_i)\right)$$

$$= 1/\sigma^2 \sum_{i=1}^{n} (x_i - \mu)$$

$$= 1/\sigma^2 \left(\sum_{i=1}^{n} x_i - \mu n\right) = 0 \leftarrow \text{set} = 0 \text{ \& solve}$$

$$\rightarrow \sum_{i=1}^{n} x_i - \mu n = 0 \rightarrow \boxed{\tilde{\mu} = 1/n \sum_{i=1}^{n} x_i = \tilde{X}}$$

2ii contd)

$$\frac{\partial \left(\ln \left(L((\mu, \sigma^2), (x_1, x_2, \ldots, x_n)\right)\right)}{\partial \sigma^2} = 0 - \frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2} \sum_{i=1}^{n} (x_i - \mu)^2$$

$$= \frac{1}{2\sigma^2} \left(\frac{1}{\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 - n \right) = 0 \leftarrow \text{Set} = 0 \text{ \& solve}$$

$$\Rightarrow \sigma^2 = 0 \quad \text{or} \quad \frac{1}{\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 - n = 0$$

Note σ is impossible so ignored.

$$\boxed{\frac{1}{n} \sum_{i=1}^{n} (x_i - \tilde{\mu}_n)^2 = \tilde{\sigma}_n^2}$$

Problem #3)

Problem 6.17:
Part 1:
Margin of Error: 0.2444761
95% Confidence Interval: (5.155524 , 5.644476)

Part 2:
Margin of Error: 0.3212961
99% Confidence Interval: (5.078704 , 5.721296)

Margin of error for 99% CI > 95% CI. Confidence interval is also larger for higher % of confidence
This makes sense, because the larger the interval the more likely we can say the event will occur

Problem 6.27:
Part 1:
95% Confidence Interval: (11.03039 , 11.96961)

Part 2:
Obviously not since 17% of students said they didn't even listen to the radio, yet the interval is from
(11.03, 11.97) > 0 --> its impossible 95% of students fall into this interval

Part 3:
Since the sample size is sufficiently large (1200 samples), even if 204 of the students don't listen to the radio the confidence interval of the normal distribution still provides a good apx. of how long university students listen to the radio weekly (Especially given 95% confidence, i.e large interval)

Problem 6.28:
Part 1:
95% Confidence Interval: (11.03039 , 11.96961)

Part 2:
Obviously not since 17% of students said they didn't even listen to the radio, yet the interval is from
(11.03, 11.97) > 0 --> its impossible 95% of students fall into this interval

Part 3:
Since the sample size is sufficiently large (1200 samples), even if 204 of the students don't listen to the radio the confidence interval of the normal distribution still provides a good apx. of how long university students listen to the radio weekly (Especially given 95% confidence, i.e large interval)