CS 334 Fall 2021: Problem Set 4.

Problem 1. (15 points)

a) (10 points) Prove that the language of palindromes $\{w: w = w^R, w \in \{0,1\}^*\}$[1] is not regular. Make sure your argument is precise and complete.
b) (5 points) Is the language $\{w: w = xyx^R\ ,\ x, y \in \{0,1\}^*\}$ regular? Give your reasoning.

Problem 2. (25 points) For any language L over an alphabet Σ we say that string $u \in \Sigma^*$ is *compatible* with string $v \in \Sigma^*$ if for every $x \in \Sigma^*$, $ux \in L$ *if and only if* $vx \in L$.

a) (5 points) Prove that compatibility defines an equivalence relation over Σ^*. It follows that the relation partitions Σ^* into disjoint subsets of strings; strings within any one partition are all pairwise compatible.
b) (10 points) Show that if L is the regular language recognized by a DFA with n states, then the number of equivalence classes of L under the compatibility relation is no greater than n. (Hint: from the start state if two strings end up in the same state, are they compatible?)
c) (5 points) Give a high-level description of an algorithm which, given an n-state DFA for L computes the exact number of equivalence classes of L under the compatibility relation. You may invoke any algorithm presented in class without getting into its details.
d) (5 points) There is a theorem which states: "A language L is regular if and only if the compatibility relation partitions L into a finite number of equivalence classes." Use this theorem to prove that the language $\{0^n 1^n : n \geq 0\}$ is not regular, without using the pumping lemma.
e) Extra Credit (10 points) Show that if the number of equivalence classes of L under the compatibility relation is finite then L is regular. To get started, suppose there are n equivalence classes $C_1, \ldots, C_n$ and let $s_i \in C_i, 1 \leq i \leq n$. Construct a DFA with state q_i for equivalence class C_i. If, for $a \in \Sigma$ $s_i a$ and s_j are compatible then how would you use that to define δ?

Problem 3. (20 points)

a) (7 points) Show that the language

$$L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } i = 1 \Rightarrow j = k\}$$

satisfies the three conditions of the pumping lemma. Hint: set the pumping threshold to 2 and argue that every string in L can be divided into three parts to satisfy the conditions of the pumping lemma.
b) (8 points) Prove that L is not regular. Note that $L = b^* c^* \cup aaa^* b^* c^* \cup \{ab^i c^i : i \geq 0\}$, and use the fact that regular languages are closed under complement and difference.
c) (5 points) Explain why parts (c) and (d) do not contradict the pumping lemma.

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[1] w^R is the string w in reverse order.