

CS 334 Fall 2021: Problem Set 8.

Problem 1. (10 points) Define the set of TMs that are deciders, i.e., TMs that halt on every input string and either accept or reject the input.

$$DECIDERS = \{ \langle M \rangle : M \text{ encodes a TM that halts on every input to it} \}$$

Prove that $DECIDERS$ is not a TM-recognizable language.

Problem 2. (15 points) Consider the problem of determining whether a TM M on input w ever attempts to move its head left when the head is on the leftmost tape cell.

$$LEFT_{TM} = \{ \langle M, w \rangle : M \text{ on } w \text{ moves left from the leftmost cell during the computation} \}$$

Prove that A_{TM} is undecidable by showing that $A_{TM} < LEFT_{TM}$.

Hint: construct a TM N that simulates M such that $\langle N, w \rangle \in LEFT_{TM} \iff \langle M, w \rangle \in A_{TM}$.

Problem 3. (15 points) Show that the language

$L = \{ \langle M, w, k \rangle : \text{TM } M \text{ accepts input } w \text{ and never moves its head beyond the first } k \text{ tape cells} \}$ is decidable.