

Harris Spahic

“I pledge my honor I have abided by the Steven’s honor system.”

2/21/21

Problem #1 :

Part 1:

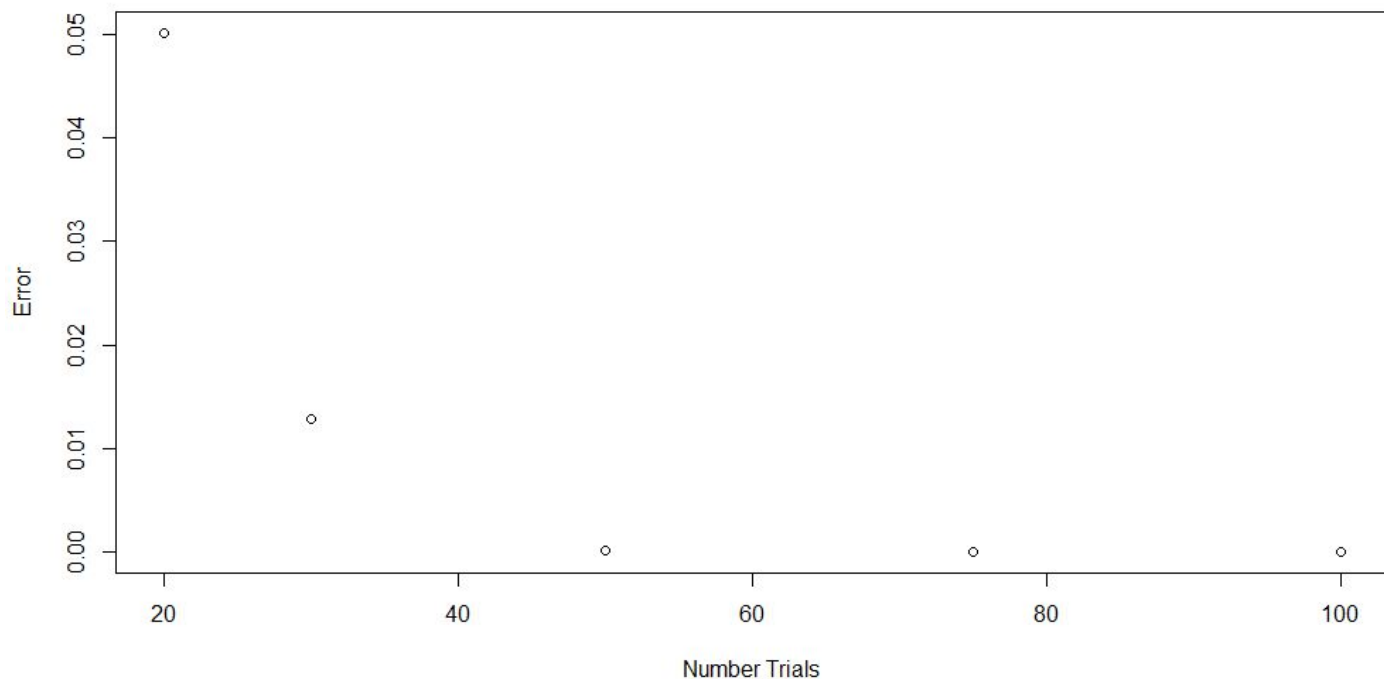
```
> pbinom(8.25, 20, 0.4)
[1] 0.5955987
> pbinom(8.25, 30, 0.4)
[1] 0.09401122
> pbinom(8.25, 50, 0.4)
[1] 0.0002305229
> pbinom(8.25, 75, 0.4)
[1] 1.826106e-08
> pbinom(8.25, 100, 0.4)
[1] 5.431127e-13
```

Part 2:

```
> pnorm(8.25, mean = 20 * 0.4, sd = sqrt(20 * .4 * .6))
[1] 0.5454243
> pnorm(8.25, mean = 30 * 0.4, sd = sqrt(30 * .4 * .6))
[1] 0.08112525
> pnorm(8.25, mean = 50 * 0.4, sd = sqrt(50 * .4 * .6))
[1] 0.0003470073
> pnorm(8.25, mean = 75 * 0.4, sd = sqrt(75 * .4 * .6))
[1] 1.475701e-07
> pnorm(8.25, mean = 100 * 0.4, sd = sqrt(100 * .4 * .6))
[1] 4.557597e-11
```

Part 3:

Approximation Error Scatter Plot



Part 4:

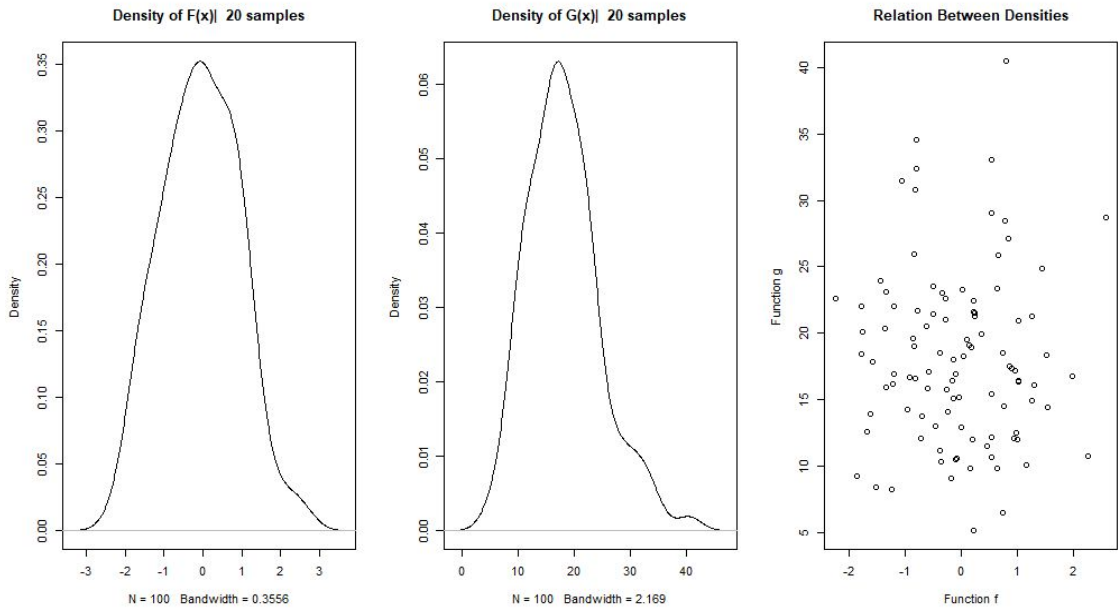
Based on the above scatter plot the apx. error appears to be in the form of a rational function, i.e the error bound is very large for small numbers of trials and likewise decreases at an exponential rate

as the number of trials tends towards infinity.

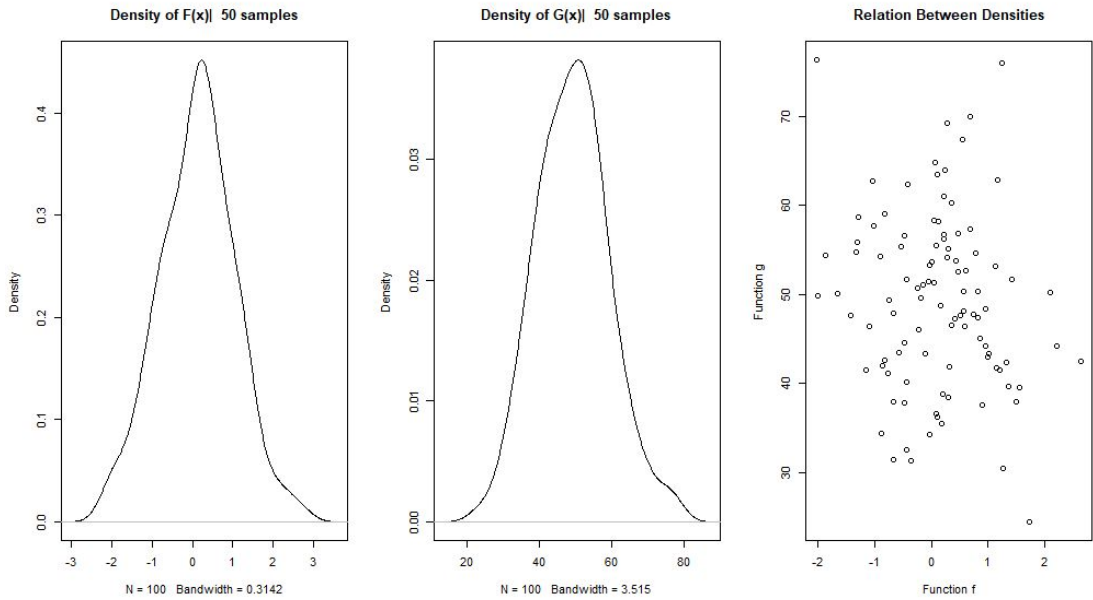
In other words, the Laplace function produces increasingly accurate results given larger sample size.

Problem #2: || NOTE: $F(X) = \frac{\bar{X} - 2}{\sqrt{3^2/n}}$, $G(X) = \frac{(n - 1)S^2}{3^2}$

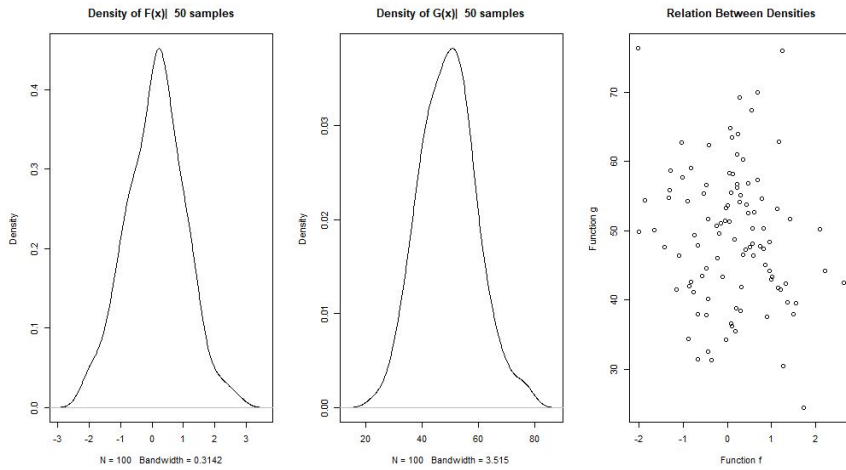
Part 1:



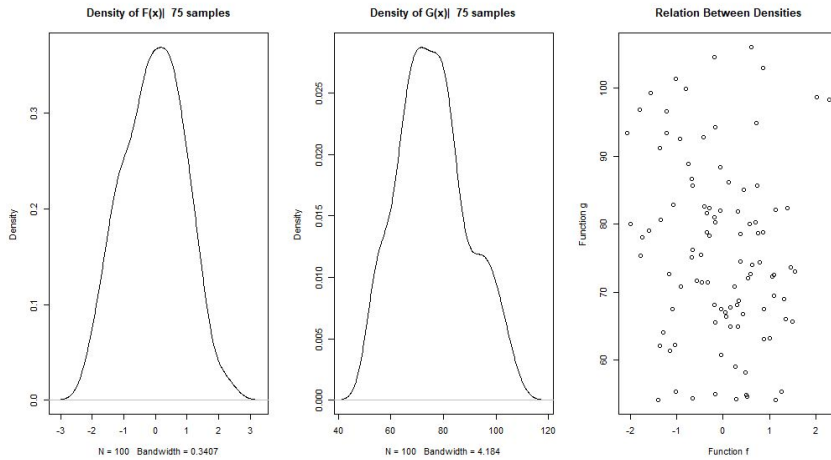
Part 2:



Part 3:



Part 4:



Part 5:

The probability distribution of $F(X)$ remains centered around 0 with an apx. normal distribution. Interestingly, despite having a factor proportional to the size n of the samples (\sqrt{n}) the range remains apx. constant with maxima & minima not surpassing $\text{abs}(4)$.

The probability distribution of $G(x)$ in contrast is always positive and appears to increase linearly (note the $(n-1)$ factor) as n increases. Its range moreover grows at a slower but still positive rate with n as well.

Part 6:

There appears to be no statistical correlation between the distributions $F(x)$ & $G(x)$, as observed from

their scatter plots."

Problem 3:

Part 1:

```
> a <- EQ1(samples, 1, 4)
> print(a)
[1] 7.30875
> #Chisq Distribution
> pchisq(a, 9)
[1] 0.3949919
```

Part 2:

```
> b <- EQ2(samples, 4)
> print(b)
[1] 7.29275
> # Chisq Distribution
> pchisq(b, 9)
[1] 0.3933312
```

Part 3 :

```
> c <- EQ3(samples, 1)
> print(c)
[1] -0.1405192
> #Student Distribution
> pt(c, 9)
[1] 0.4456723
```

Problem 4:

Part 1:

```
> pnorm(1, -1, 3) - pnorm(0, -1, 3)
[1] 0.1169488
> pchisq(14, 12) - pchisq(3, 12)
[1] 0.6948357
> pt(1, 10) - pt(0, 10)
[1] 0.3295534
> pf(1, 8, 9) - pf(0, 8, 9)
[1] 0.5054556
> |
```

Part 2:

```
> qnorm(.025, -1, 3)
[1] -6.879892
> qnorm(1- .025, -1, 3)
[1] 4.879892
>
> qchisq(.025, 12)
[1] 4.403789
> qchisq(1-.025, 12)
[1] 23.33666
>
> qt(.025, 10)
[1] -2.228139
> qt(1-.025, 10)
[1] 2.228139
>
> qf(.025, 8, 9)
[1] 0.2295034
> qf(1-.025, 8, 9)
[1] 4.101956
```

Problem 5: -----

#5) $E[N]$ for $N \sim B(N, p)$ *Shuen's Honor System*
Consider each trial N_i independently & compute $\sum_{i=1}^n E[N_i]$.
For any N_i , $E[N_i] = \underbrace{P(N_i=0)}_{\text{Failure}} \cdot 0 + \underbrace{P(N_i=1)}_{\text{Success}} \cdot 1 = 0 + p = p$.
 $\rightarrow \boxed{\sum_{i=1}^n E[N_i] = n \cdot p}$

#5b) Verify $E[T]=0$ for $T \sim T_n$.
Notice, $T = \frac{X}{\sqrt{Y/n}}$ where $X \sim N(0, 1)$ and $Y \sim \chi_n^2$.
 $E[T] = E\left[\frac{X}{\sqrt{Y/n}}\right] = \frac{E[X]}{E[\sqrt{Y/n}]}$ since X & Y are independent $= \boxed{\frac{0}{E[\sqrt{Y/n}]} = 0}$

