CS-385 Hw: Recurrence Relations

Name: Harris Spahic Date: 09/29/21

Pledge: "I pledge my honor that I've abided by the Steven's Honor System." - Harris Spahic

---

Pg 67

#4. Mystery(n) computes the sum of squares upto n.

b. $i \cdot i$; i multiplication

c. $n-1$ times

d. $\text{mystery}(n) \in \Theta(n)$

e. Notice mystery(n) returns

$1^2 + 2^2 + 3^2 + \dots + n^2$

$= \boxed{\frac{1}{6} n(n+1)(2n+1)} = \text{mystery}'(n)$

This solution takes constant time to compute thus $\text{mystery}'(n) \in \Theta(1)$

Pg 76

1. $x(n) = x(n-1) + 5$ $\quad n>1$, $x(1)=0$

$\rightarrow x(n-1) = x(n-2) + 5$

$\rightarrow x(n) = x(n-2) + 10$

$\vdots$

$\rightarrow x(n) = 5(n-1)$

$\rightarrow x(n) \in \Theta(n)$

---

b. $x(n) = 3(x(n-1))$ $\quad x>1$ $\quad x(1)=4$

$x(n-1) = 3(x(n-2))$

$\Rightarrow x(n) = 3^2(x(n-2))$

$\Rightarrow x(n) = 3^{n-1} \cdot 4 \rightarrow x(n) \in \Theta(3^n)$

c. $x(n) = x(n-1) + n$ $\quad n>0$ $\quad x(0)=0$

$x(n-1) = x(n-2) + (n-1)$

$\Rightarrow x(n) = x(n-2) + n + (n-1)$

$\Rightarrow x(n) = n + (n-1) + (n-2) + \dots 1$

$= \frac{n(n+1)}{2} \Rightarrow x(n) \in \Theta(n^2)$

d.* $x(n) = x(n/2) + n \quad n > 1 \quad x(1) = 1 \quad n = 2^k$

$x(2^k) = x(2^{k-1}) + 2^k$

$x(2^{k-1}) = x(2^{k-2}) + 2^{k-1}$

$\rightarrow x(2^k) = x(2^{k-2}) + 2^k + 2^{k-1}$

$x(2^k) = x(2^{k-i}) + 2^k + 2^{k-1} \dots + 2^{k-i+1}$

$\rightarrow X(1) = 1$

$\rightarrow 1 = 2^{k-i}$

$\rightarrow i = k$

$x(2^k) = x(2^0) + 2^k + 2^{k-1} + \dots + 2^1$

$= 2^k + 2^{k-1} + \dots + 2 + 1$

$= 2^{k+1} - 1$

Since $n = 2^k \rightarrow 2n = 2^{k+1}$

$\rightarrow \boxed{X(n) = 2n - 1} \rightarrow x(n) \in \Theta(n)$

e.* $x(n) = x(n/3) + 1 \quad n > 1$

$x(1) = 1, \quad n = 3^k$

$\rightarrow x(3^k) = x(3^{k-1}) + 1$

$x(3^{k-1}) = x(3^{k-2}) + 1$

$\rightarrow x(3^k) = x(3^{k-2}) + 2$

$= x(3^{k-i}) + i$

$\rightarrow x(1) = 1$

$\rightarrow 3^{k-i} = 1$

$\rightarrow i = k$

$x(3^k) = x(3^0) + k$

$x(3^k) = k + 1$

Since $n = 3^k$

$\rightarrow \boxed{x(n) = \log_3 n + 1}$

$\rightarrow x(n) \in \Theta(\log_3 n)$

Pg 76-77

#3)* $S(n) = S(n-1)+2$ $\quad n>1,\ S(1)=0$

$\rightarrow S(n-1) = S(n-2)+2$

$\rightarrow S(n) = S(n-2)+4$

$= S(1)+2(n-(n-1))$

$= 2n-2$

b) $\left(n\left(\frac{n+1}{2}\right)\right)^2 = 1+2^3+3^3+ \dots + n^3$

$\rightarrow$ Notice $\left(n\left(\frac{n+1}{2}\right)\right)^2$ is a constant time operation, but the recursive function is linear time.

More specifically $(n_1 \cdot ((n +_2 1) /_3 2)^2_4$ is 4 seperate operations, where $S(n)$ is $2n-2$. Obviously the constant one is better.