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Homework for

MA 346 Numerical Methods

Spring 2022 — Homework 3

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Exercise 1 (LU decomposition)
Consider the linear system of equations $Ax = b$ with

$$A = \begin{pmatrix} 6 & 4 & 4 \\ 3 & 4 & 6 \\ 6 & 6 & 4 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}. \tag{1}$$

a) Compute the LU decomposition of A defined in (1) without pivoting.

b) Use the just determined LU decomposition of the matrix A defined in (1) to solve the linear system of equations $Ax = b$.

c) Use the just determined LU decomposition of the matrix A defined in (1) to calculate the determinant of A.

Exercise 2 (LU decomposition)
We have given the following matrix A:

$$A := \begin{pmatrix} \alpha & -1 & 0 \\ -1 & \alpha & -1 \\ 0 & -1 & \alpha \end{pmatrix} \qquad \text{with } \alpha \in \mathbb{R}. \tag{2}$$

a) Compute the LU decomposition of A defined in (2) without pivoting, where L has to be a unit lower triagonal matrix.

b) Give conditions for α that are sufficient for the existence of a unit lower triagonal matrix L and an upper triagonal matrix U such that $A = LU$ for A defined in (2).

c) Use the just determined LU decomposition of the matrix A defined in (2) to compute $\det(A)$.

Exercise 3 (LU decomposition with pivoting)
Consider the linear system of equations $Ax = b$ with

$$A = \begin{pmatrix} 0 & 1 & 5 \\ 6 & 1 & 3 \\ 5 & 1 & 2 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}.$$

a) Employing partial pivoting (exchanging rows), compute the decomposition $PA = LU$ with a permutation matrix P, a unit lower triagonal matrix L and an upper triagonal matrix U.

b) Use the just determined decomposition to solve the linear system of equations $Ax = b$.