# CS 334 Fall 2021: Problem Set 7.

**Problem 1.** (15 points) Prove that every infinite TM-recognizable language has an infinite decidable subset.

**Problem 2.** (15 points) Prove that the following language is decidable by giving a high-level description of a TM that decides the language.

$$\{< G >: G\ is\ a\ CFG\ in\ CNF\ \ and\ L(G)\ is\ an\ infinite\ language\ \}$$

You may assume that a string of terminal symbols can be derived from every non-terminal in the CFG, and that every non-terminal appears in some string derivable from the start symbol.

**Problem 3.** (15 points) Let $G$ be a context-free grammar that generates strings over the alphabet $\Sigma = \{a, b\}$. Show that the problem of determining if $G$ generates a string in $a^*$ is decidable. In other words, show that the following language is decidable:

$$\{< G >: G\ is\ a\ CFG\ over\ \{a, b\}\ and\ a^* \cap L(G) \neq \phi\ \}$$