CS 334 ~ Hw #3 :

Partners: Harris Spahic
Hiya Bhavsar

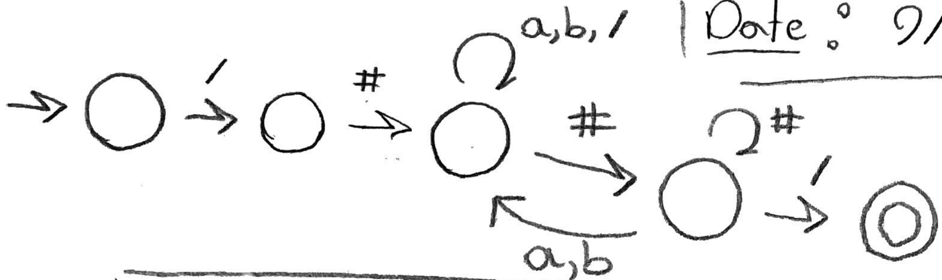
Pledge: "I pledge my honor I have abided by the Steven's Honor System."

Date: 9/24/21

1. α: $0^*(10^*1)^*0^*$

β: $1^*(01^*0)^*1^*$

$\rightarrow$ $\boxed{\alpha \cdot \beta = (0^*(10^*1)^*0^*)^*(1^*(01^*0)^*1^*)^*}$

1b. 

Thus, $\boxed{/\#\ (a \cup b \cup ((a \cup b)/) \cup \#)^* \#//}$

? /#/////#/

~~2*. Let R be any regular expression. We show R^R is also regular.~~

~~Let the magnitude of R denote the # of external concatenated expressions in R.~~

~~For example: $0^*(10^*1)0^*1^*(01^*0)1^* = R$~~

~~$\rightarrow \|R\| = 10$~~

~~And example: $/\#(a \cup b \cup / \cup (\#^*(a \cup b)))^*\#/ = R$~~

~~$\rightarrow \|R\| = 5$~~

~~We prove by induction:~~

~~Base Case: $\|R\| = 0$, obviously $R = R^R = \emptyset$ thus R^R is a regular expression.~~

2. Let R be any regular expression. We show R^R is also regular.

Notice for every operational expression except concatonation

1. a
2. ε
3. $\emptyset$
4. R_1^*

$\Bigg\}$ the reverse of the regular expression (for their base case (one element)) are the same.

Thus we work with concatonation, $R_1 \circ R_2$ & show the concatonenation of two sub expressions R_1 & R_2, s.t $R_1, R_2 \in R$ reversed still result in a regular expression.

Let the magnitude of R denote the # of <u>external</u> concatenated sub expressions in R. (IMPORTANT: Each sub expression can be represented as a single term in the regular expression R.)

<u>Ex:</u> $\underbrace{0^*}_{r_1}\underbrace{1}_{r_2}\underbrace{0^*}_{r_3}\underbrace{1}_{r_4}\underbrace{0^*}_{r_5} - R$ where r_i is a sub expression of R.

$\rightarrow \|R\| = 5$

<u>Ex 2:</u> $\underbrace{/}_{r_1}\underbrace{\#}_{r_2}\underbrace{(a \cup b \cup / \cup ((a \cup b)/) \cup \#)^*}_{r_3}\underbrace{\#}_{r_4}\underbrace{/}_{r_5}$

$\rightarrow \|R\| = 5$

By induction on # of external concatenations in R,

<u>Base Case:</u> $\|R\| = 1$

$\rightarrow R^R = R$, thus R^R is a regular expression.

Assume if $||R|| = k$, R^R is regular expression.
Show $||R|| = k+1$ implies R^R is regular.

~~Two cases:~~

Remove the $k+1^{th}$ sub expression from R.
Then, the remaining sub expression R' has $||R'|| = k$.
→ R'^R is regular by inductive hypothesis.

We add the $k+1^{th}$ expression to the front of R', and concatenate the two. Since the $k+1^{th}$ expression is regular & R'^R is regular & the concatenation of regular expressions is regular.

R^R is a regular expression.

We still need to show the reverse of the union of l regular expressions is also regular.

This is trivial, note for any union of regular expressions, the resulting accepted language is a single Regular expression.

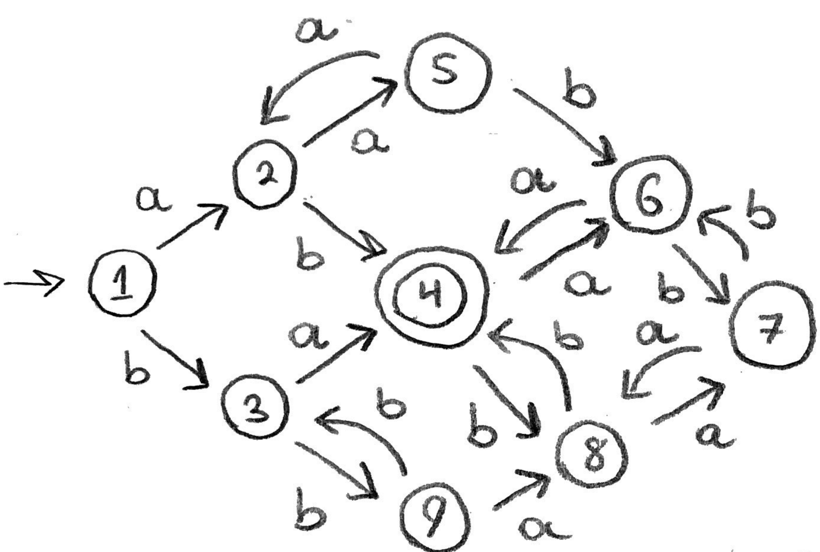
For Ex: $R_1 = (a)$
$R_2 = (b)$
$R_3 = (a^*bc)$
→ $R_1 \cup R_2 \cup R_3$ only returns 1 expression of the three.

Since we showed the reverse of any regular expression not containing a union is regular, & we can simplify the union of the expressions reversal to a single expression,

→ The reverse of the union is regular. ∎

~~Assume if ||R|| = k then R is regular.~~
~~Show if ||R+1|| = k+1 then R is regular.~~

3.



	1	2	3	4	5	6	7	8	9
1		✓	✓	✓	X	✓	X	✓	X
2			✓	✓	✓	✓	✓	X	✓
3				✓	✓	X	✓	✓	✓
4					✓	✓	✓	✓	✓
5						✓	X	✓	X
6							✓	✓	✓
7								✓	X
8									✓
9									

Where X is equivalent

$\{1,2\} \xrightarrow{a} \{2,5\}$
$\xrightarrow{b} \{3,4\}$ ✓

$\{1,3\} \xrightarrow{a} \{2,4\}$ ✓
$\xrightarrow{b}$

$\{1,5\} \xrightarrow{a} \{2,2\}$
$\xrightarrow{b} \{3,6\}$

$\{2,3\} \xrightarrow{a} \{5,4\}$

but

$\{3,2\} \xrightarrow{a} \{4,5\}$ ✓

$\{2,5\} \xrightarrow{a} \{5,2\}$
$\xrightarrow{b} \{4,6\}$

$\{2,6\} \xrightarrow{b} \{4,7\}$

$\{2,7\} \xrightarrow{b} \{4,6\}$

$\{2,8\} \xrightarrow{b} \{4,4\}$

$\{2,9\} \xrightarrow{b} \{4,3\}$

⋮

IS node has path to accept, then distinguishable <u>unless</u> both have same transition

4

3. cont'd:

Based on our table we see the following pairs are equivalent.

{1,5}, {2,8}, {5,9}
{1,7}, {3,6}, {7,9}
{1,9}, {5,7}

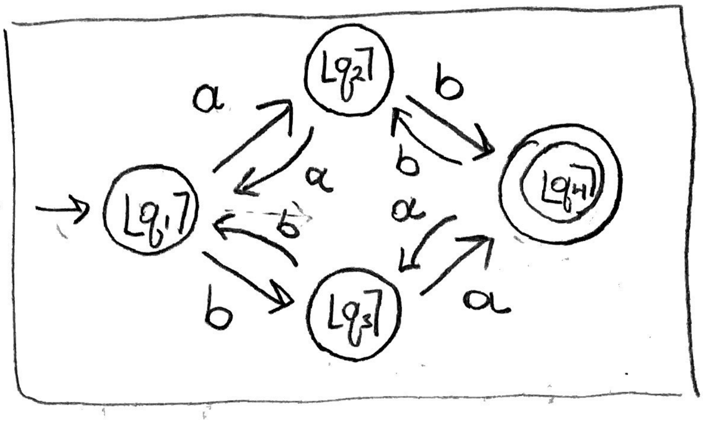
That is to say we have the following equivalence classes:

$[q_1] = \{q_1, q_5, q_7, q_9\}$

$[q_2] = \{q_2, q_8\}$

$[q_3] = \{q_3, q_6\}$

$[q_4] = \{q_4\}$

$\Rightarrow$ 

Reduced DFA.