CS 334 Fall 2021: Problem Set 6.

Problem 1. (30 points) A 2-stack PDA (2PDA for short) is an automaton with a non-deterministic finite state control and two stacks that operate independently. Just like a PDA, in one step the automaton can either read an input symbol or choose not to; it can choose to pop either one or both or neither of the two stacks. Depending on the symbols read and popped, the automaton changes state and can choose to push on one, both or neither stack.

a. (5 points) Describe informally a 2PDA to recognize the non-CFL $\{w\#w: w \in \{a,b\}^*\}$
b. (5 points) Now describe informally a 2PDA for $\{ww: w \in \{0,1\}^*\}$ (Hint: magic!)
c. (10 points) Describe informally a 2PDA for $\{a^{i^2}: i \geq 0\}$
d. (10 points) Show that a 2PDA can recognize the language of any Turing Machine. Hint: simulate each step of a TM by steps of a 2PDA, keeping track of the configuration of the TM (state, tape contents, and position of tape head).

Problem 2. (15 points) Prove that the intersection of a CFL and a Regular language is always context free.

Problem 3. (20 points)

a. (10 points) Show that the class of TM-decidable languages is closed under the following operations: union, concatenation, star, intersection, and complement.
b. (10 points) Show that the class of TM-recognizable languages is closed under the following operations: union, concatenation, star, and intersection. Is it closed under complement?

Optional Problem 4. (20 points) A ***queue automaton*** is like a push-down automaton except that the stack is replaced by a queue. A queue is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we'll call it a push) adds a symbol to the left-hand end of the queue and each read operation (we'll call it a pull) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at any time. Show that a language can be recognized by a deterministic queue automaton iff the language is Turing-recognizable.