CS 334: Problem Set 1

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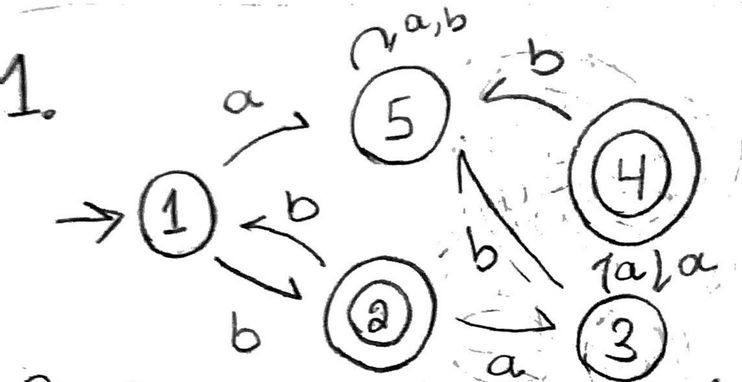
1.



PLEDGE: "I pledge my honor I have abided by the Stevens Honor System."

States:

1 ~ even b's, even a's

2 ~ odd b's, even a's

3 ~ even b's, odd a's } after b's

4 ~ even b's, even a's } after b's

5 ~ reject (ab state)

$Q = \{1, 2, 3, 4, 5\}$

$\Sigma = \{\text{"a"}, \text{"b"}\}$

$\delta =$

	a	b
1	5	2
2	3	1
3	4	5
4	3	5
5	5	5

$q_0 = 1$

$F = \{2, 4\}$

Explanation: we know that our DFA only accepts the language with the following properties.

1) odd # b's
2) even # a's
3) no "ab" substrings

→ Notice, property 1 implies the accepted language must also have at least 1 b.

→ That means if we start with a, then eventually a b must follow.

1) $\{1, \text{"a"}\} \rightarrow 5$ (reject state)

once @ state 5, no matter the input (a or b) the string should be rejected.

2) $\{5, \text{"a"}\} \rightarrow 5$

$\{5, \text{"b"}\} \rightarrow 5$

We continue from the initial state to a new state 2, which has odd b's & even a's & no "ab"s.

3) 2 is an accept state.

From 2, if we give "b" we want to return to our initial state, so that a subsequent "b" keeps the # of b's even & an "a" results in an eventual "ab".

4) $\{2, "b"\} \rightarrow 1$

Otherwise we give an "a", & have an odd # of b's & an odd # of a's. Call this state 3.

5) $\{2, "a"\} \rightarrow 3$

Once we give an "a" to our string, we no longer want any "b"s.

6) $\{3, "b"\} \rightarrow 5$

We also need keep alternating string's parity of "a" until it is even.

7) $\{3, "a"\} \rightarrow 4$

Once "a" is given, we once again have even "a"s & odd "b"s. Call state 4.

8) 4 is an accept state.

We no longer want to accept an subsequent "b" at 4, & return to failure 3 if "a" (odd a's) given.

9) $\{4, "a"\} \rightarrow 3$ & $\{4, "b"\} \rightarrow 5$

2. Prove that the state diagram of every DFA must contain a directed cycle.

<u>Def:</u> <u>Directed Cycle</u> ~ a sequence of directed edges $(v_1, v_2) \ldots (v_n, v_1)$ S.T. when v_i is a vertex in the sequence & $1 \leq i < n$, then there is an edge $i (v_i, v_{i+1})$. → We can follow 1-way edges to return to same point.

Notice every DFA can be generalized as a directed graph with $\|Q\|$ vertices, whom all have an outdegree of $\|\Sigma\|$. Since $\|\Sigma\|$ is always ≥ 1 then every DFA must have a directed cycle.

2 cont'd:

2b) Any DFA which contains a path from the initial state to an accept state, and also contains a directed cycle will have an infinite language.

Condition

Proof: If there exists a path that satisfies the conditions above in the DFA, then there exists a string which takes transitions upto the directed cycle in our path, call that string A. At A, we can construct a new string B which follows the directed cycle any # of times from start to to finish, which always ends at the same state. Since we can repeat this subpath, B can be infinitely long. Finally, we construct a new string C which takes transitions from the endstate of B to an accept state.

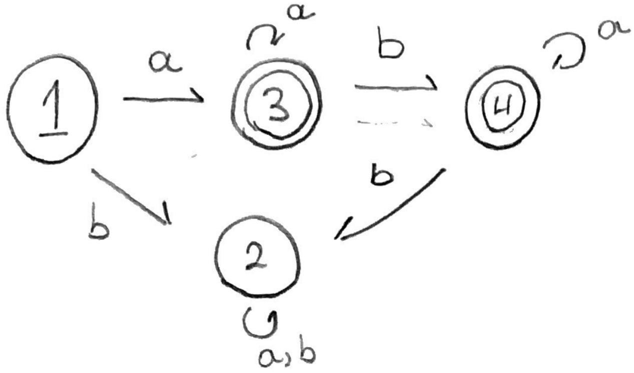
→ The infinite language of our NFA is the concatonation of the strings

$$A + B + C$$

where $B = \underbrace{bbbb\ldots}_{\text{any \# of times}}$ & b = string sequence traversing directed cycle from start to finish

Since B can be any length of multiple "b", our DFA's language is infinite.

3)



$Q = \{1, 2, 3, 4\}$

$\Sigma = \{\text{"a"}, \text{"b"}\}$

$\delta =$

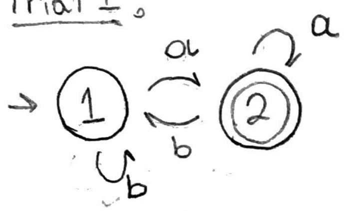
	"a"	"b"
1	3	2
2	2	2
3	3	4
4	4	2

$q_0 = 1$

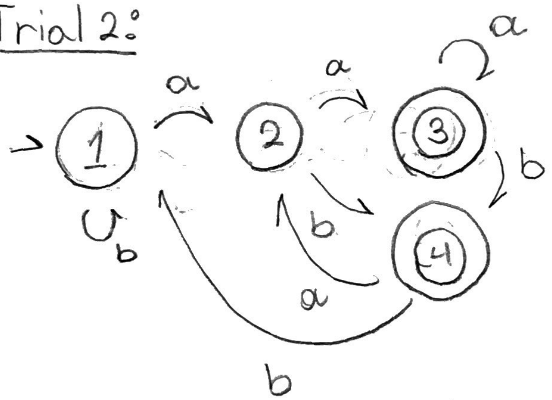
$F = \{3, 4\}$

3b)

Trial 1:



Trial 2:



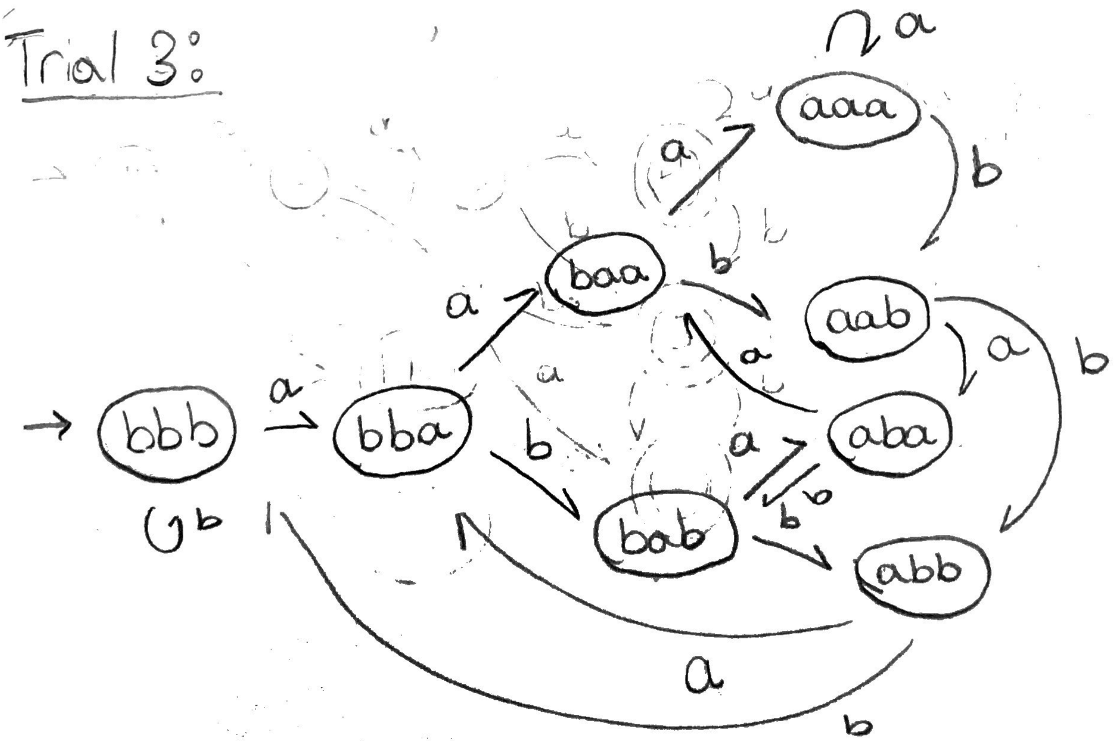
~1: 1st char

~2: 2nd char

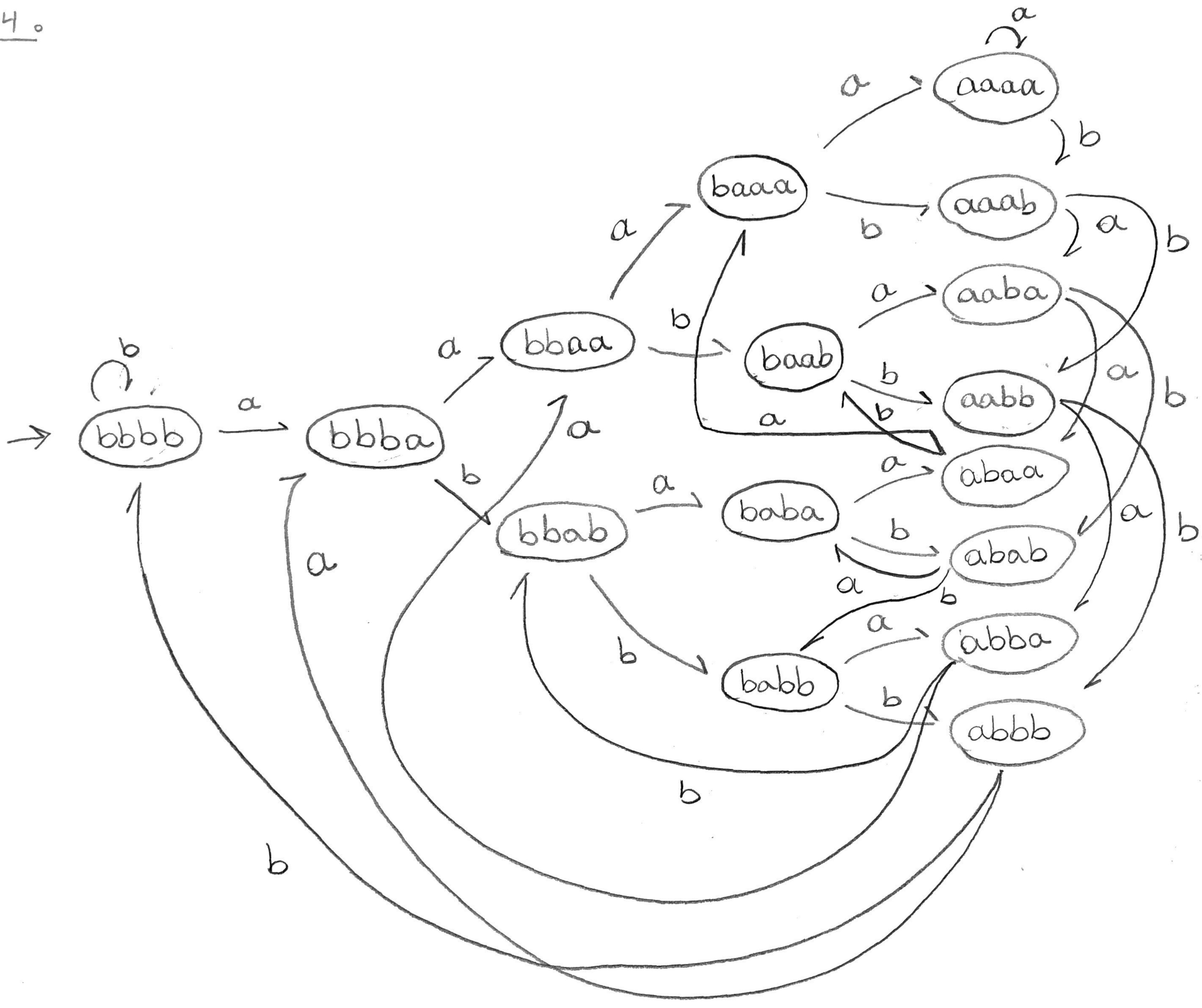
~3: Accept if

~4: Accept if

Trial 3:



L4.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4) Show $A \cup B = \{x \mid x \in A \vee x \in B\}$ S.T $\Sigma_A \neq \Sigma_B$

Proof:

Let M_1 recognize A_1, where $M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$ and M_2 recognize A_2, where $M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$.

Construct M to recognize $A_1 \cup A_2$, where $M = (Q, \Sigma, \delta, q_0, F)$.

1. $Q = \{(r_1, r_2) \mid r_1 \in Q_1 \wedge r_2 \in Q_2\}$

2. $\Sigma = \Sigma_1 \cup \Sigma_2$

3. δ is given as the union of 3 transition functions.

   i) For $a \in (\Sigma_1 \cap \Sigma_2)$

   $$\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$$

   ii) For $a \in \Sigma_1 \wedge a \notin \Sigma_2$

   $$\delta((r_1, r_2), a) = (\delta_1(r_1, a), r_2)$$

   iii) For $a \notin \Sigma_1 \wedge a \in \Sigma_2$

   $$\delta((r_1, r_2), a) = (r_1, \delta(r_2, a))$$

   iv) For $(a \notin \Sigma_1 \wedge a \notin \Sigma) \vee a = \phi$

   $$\delta((r_1, r_2), a) = (r_1, r_2)$$

4. $q_0 = (q_1, q_2)$

5. $F = \{(r_1, r_2) \mid r_1 \in F_1 \vee r_2 \in F_2\}$ ∎