MA 232 - Linear Algebra

Homework 3 (due March 12 at 5pm)

Problem 1 [15 pts]

Construct a matrix whose nullspace consists of all combinations of $\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$.

Problem 2 [15 pts]

Construct a matrix whose column space contains $\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ and whose nullspace contains $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

Problem 3 [10 pts]

Let $u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $u_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Show that u_1, u_2, u_3 are independent but u_1, u_2, u_3, u_4 are dependent.

Problem 4 [15 pts]

For which numbers c, d does the following matrix have rank 2?

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & \mathrm{c} & 2 & 2 \\ 0 & 0 & 0 & \mathrm{d} & 2 \end{bmatrix}$$

Problem 5 [15 pts]

Find a basis for each of the four fundamental subspaces (column, null, row, left null) associated with the following matrix:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Problem 6 [15 pts]

Suppose that S is spanned by $s_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$, $s_2 = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$. Find two vectors that span the orthogonal complement $S^{\perp}$. (Hint: this is the same as solving $Ax = 0$ for some A)

Problem 7 [15 pts]

Suppose P is the subspace of $\mathbb{R}^4$ that consists of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ that satisfy $x_1 + x_2 + x_3 + x_4 = 0$. Find a basis for the perpendicular complement $P^{\perp}$ of P.