MA 232 HW #4

Hyman Sabin

– "I pledge my Honor I have abided by the Stevens Honor Code"

#1)

$$\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \quad A^T \cdot A\vec{x} = A^T b$$

$$\begin{bmatrix} 5 & 0 \\ 0 & -10 \end{bmatrix} \vec{x} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\rightarrow e = \vec{b} - A\vec{x} \qquad \Rightarrow \boxed{\begin{matrix} C = 1 \\ D = -1 \end{matrix}}$$

Actual – predicted

$$= \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ -1 \end{bmatrix} \leftarrow \sum_{i=1}^{5} x_i = 0 \ \checkmark$$

#2)

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \rightarrow A = \hat{a} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ +1 \\ -1 \\ 0 \end{bmatrix} \rightarrow B = (\vec{b} - \mathrm{proj}_{\hat{a}} \vec{b}) = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - (\vec{b} \circ \hat{a})\hat{a}$$

$$= \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{(-1)}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \hat{b} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \rightarrow C = \vec{c} - \mathrm{proj}_{\hat{a}} \vec{b} - \mathrm{proj}_{\hat{b}} \vec{c}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 0 \end{bmatrix} + \frac{2}{3}(+1) \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/2 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1/3 \\ 1/3 \\ -2/3 \\ 0 \end{bmatrix} =$$

#3) Q_1 & Q_2 are orthanormal.

$$\Rightarrow Q_1^T Q_1 = I_n$$

$$Q_2^T Q_2 = I_n$$

Notice this implies

$$Q_2^T = Q_2^{-1} \Rightarrow Q_2^T \cdot Q_2 = Q_2 \cdot Q_2^T$$

Thus,

$$Q_1^T Q_1 Q_2 Q_2^T = I_n$$

$$Q_1 Q_2 Q_2^T = Q_1$$

$$Q_1 Q_2 Q_2^T Q_1^T = I_n$$

$$Q_1 Q_2 (Q_1 Q_2)^T = I_n$$

Thus $(Q_1 Q_2)^T = (Q_1 Q_2)^{-1} \Rightarrow Q_1 Q_2$ is orthanormal.

#4) Counter example: $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ← All non invertible matrices have det = 0

$B = O_{2\times2}$

$C = O_{2\times2}$

$D = I_2$

$$\Rightarrow \det \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \det\left(1\left(I_3\right) + (I_3)\right) = (1+1) = 2$$

But $\det(A) = 0 \Rightarrow \det(A)\det(D) - \det(B)\det(C)$

$$= 0 \neq 2$$

Thus Contradiction.

#5)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 - R_1 \\ R_3 - R_1 \end{array} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 - R_2 \end{array} = U$$

$$\rightarrow \boxed{\det(A) = 1 = 1 \cdot 1 \cdot 1} = 1$$