1.

$$\begin{bmatrix} 10^{-6} & 1 \\ 1 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 10^{-6} & 1 \\ 0 & 1-10^6 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2-10^6 \end{bmatrix}$$

$$x_2 = \frac{999,998}{999,999}$$

$$\Rightarrow 10^{-6}x_1 + \frac{999,998}{999,999} = 1$$

$$\Rightarrow 10^{-6}x = \frac{1}{999,999}$$

$$\Rightarrow x_1 = \frac{1,000,000}{999,999}$$

1b.

$$\begin{bmatrix} 1 & 1 \\ 10^{-6} & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1-10^{-6} \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 1-2\times 10^{-6} \end{bmatrix}$$

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$$\left[\begin{array}{cc} 0 & 1.0 \end{array} \mid \right] \quad \left[\begin{array}{cc} 1 & 2 \cdot 0 \end{array} \mid \right]$$

$$1 - 10^{-6} x_2 = 1 - 2 \times 10^{-6}$$

$$\Rightarrow x_2 = \frac{1 - 2 \times 10^{-6}}{1 - 10^{-6}} \times \frac{10^6}{10^6}$$

$$= \frac{999,998}{999,999}$$

$$x_1 + x_2 = 2$$

$$x_1 + \frac{999,998}{999,999} = 2$$

$$x_1 = \frac{999,999 + 1}{999999} = \frac{1,000,000}{999999}$$

c) Notice 1a & 1b have the same solutions for x_1 & x_2.

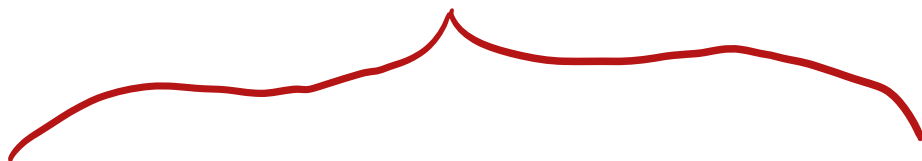This is because the order of equations is irrelevant to computing its solution.

2.

$$\left[\begin{array}{ccc|ccc} 2 & -3 & 2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 1 & 0 \\ 2 & -3 & 2 & 1 & 0 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 8 & -1 & 0 & -3 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & -1 & -8 & 13 & 1 \end{array}\right]$$

1)
$$\begin{vmatrix} 1 & -2 & 1 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & -1 & \vdots & -8 \end{vmatrix}$$

$b_{31} = 8$

$b_{21} = 1$

$b_{11} - 2b_{21} + b_{31} = 0$

$\rightarrow b_{11} - 2 + 8 = 0$

$\qquad b_{11} = -6$

2)
$$\begin{vmatrix} 1 & -2 & 1 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -2 \\ 0 & 0 & -1 & \vdots & 13 \end{vmatrix}$$

$b_{32} = -13$

$b_{22} = -2$

$b_{12} - 2b_{22} + b_{32} = 1$

$\rightarrow b_{32} + 4 - 13 = 1$

$\qquad b_{32} = 10$

3) $\begin{vmatrix} 1 & -2 & 1 & \vdots & 0 \end{vmatrix}$

$$\begin{vmatrix} 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & -1 & \vdots & 1 \end{vmatrix}$$

$b_{33} = -1$

$b_{23} = 0$

$b_{13} - 2b_{23} + b_{33} = 0$

$\Rightarrow b_{13} - 0 - 1 = 0$

$\quad b_{13} = 1$

$$A^{-1} = \begin{bmatrix} -6 & 10 & 1 \\ 1 & -2 & 0 \\ 8 & -13 & -1 \end{bmatrix}$$

Notice $\begin{bmatrix} 2 & -3 & 2 \\ 1 & -2 & 1 \\ 3 & 2 & 2 \end{bmatrix}\begin{bmatrix} -6 & 10 & 1 \\ 1 & -2 & 0 \\ 8 & -13 & -1 \end{bmatrix} = I_3 \checkmark$

$A \times A^{-1}$

b) $Ax = b$

$\rightarrow x = A^{-1}b$

$\rightarrow x = \begin{bmatrix} -6 & 10 & 1 \\ 1 & -2 & 0 \\ 8 & -13 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$