Ec2610 (IO) – Problem Set I

Differentiated Product Demand

Ariel Pakes, Robin Lee, TF David Hao Zhang *

Due: October 10, 2018

September 24, 2018

Preliminaries

This problem set consists of one empirical exercise. You are going to estimate different models of differentiated product demand on market level data.

The due date for submission of this problem set is as indicated above. All submissions need to be in electronic form in a zip file on Canvas, and include both your documented (!) programming code and your answers to the questions in the problem set. When you are asked for results, please present them in separate and clearly marked tables in your write-up. Please package your code in a single folder with a single file that I can run (ie. there should be a master file that calls other programs which I can run to get all the results). Code should be written in Matlab, Python, R, or Julia, although Matlab is recommended unless you strongly prefer the other languages.

I would also like you to place your code in the appendix of your problem set[1] as well as include the actual program files in your submission. The filename of your write-up needs to be "PS[#]_[last name]_[first name].pdf".

On non-programming questions, please answer them in such a way that an intelligent research assistant who knows Matlab could implement your suggestion, without being unnecessarily verbose.

* dhzhang@g.harvard.edu

[1] You can use the package mcode; include, on separate lines in your preamble, the options "\usepackage[numbered,framed]{mcode}" and "\lstset{breaklines=true}". Use, e.g., "\lstinputlisting{zhang_ps1.m}" to automatically insert your code.

There are several bonus questions on this problem set. One of them asks you to use your estimates to evaluate the effects of a hypothetical merger. The others are typically a high-level review or critical assessment of some of the theoretical issues in demand estimation. View them as entirely optional, a complimentary learning opportunity.

Please submit your problem set write-up and code in a zip file on Canvas (preferred).

Collaboration is encouraged. However, every student must submit their own write-up and code, with write-up and code written independently. In addition, if you work in a group I would like you to indicate your group members.

The problem set largely derives from previous problem sets by Daniel Pollmann, Tom Wollmann, Michael Sinkinson, and Ashvin Gandhi.

1 Background on demand estimation

Berry, Levinsohn, and Pakes (1995) ("BLP") has become very influential in demand estimation. I suggest you have a look at Aviv Nevo's "Practitioner's Guide to Estimation of Random-Coefficients Logit Models of Demand" (Nevo, 2000), which provides some context in its introduction and discusses many issues related to the estimation of differentiated product demand; the appendix goes over some of the practical aspects (slightly dated, but useful). An excellent overview is also presented in the first part of the Handbook of Econometrics chapter on market outcomes (Ackerberg et al., 2007).

Nevo describes random-coefficient discrete-choice models of demand as follows: "The new method maintains the advantage of the logit model in handling a large number of products. It is superior to prior methods because (1) the model can be estimated using only market-level price and quantity data, (2) it deals with the endogeneity of prices, and (3) it produces demand elasticities that are more realistic—for example, cross-price elasticities are larger for products that are closer together in terms of their characteristics."

When I was first presented with this material, I found it hard it to put the different papers that followed into context, so I will try to do my best to summarize a subset of them in one paragraph here. Berry, Levinsohn, and Pakes (1999) apply a demand model to the evaluation of trade policy using a more advanced approximation to the optimal instruments than the original BLP paper.[2] Berry,

[2]To approximate the optimal instruments, they use the implicit function theorem to replace $\mathbb{E}\left[\frac{\partial \xi_j}{\partial \theta}|Z_j\right]$ by $\frac{\partial \xi_j}{\partial \theta}|Z_j$ evaluated at $\mathbb{E}(\xi_j) = 0$. The original BLP paper instead used only the first-order basis functions of the optimal instruments under symmetric NE (the different sums of characteristics) as an approximation. See also Reynaert and Verboven (2012)

Levinsohn, and Pakes (2004) ("Micro BLP") incorporate marketing survey data and match additional "micro" moments given by the covariance of observed consumer attributes and product characteristics and the covariance of first- and second-choice characteristics. Petrin (2002) analyzes the welfare effect of the introduction of the minivan and adds as micro information the average characteristics of buyers (income, family size, age) from the automobile supplement of the Consumer Expenditure Survey. Berry, Linton, and Pakes (2004) provide limit theory for estimators in BLP-type models as the number of products grows large, while there are more recent papers that instead consider the many-markets setting (Freyberger, 2012; Gandhi, Lu, and Shi, 2012). Berry, Gandi, and Haile (2012) prove that given very primitive conditions,[3] the inversion $\xi = s^{-1}(\mathfrak{s})$ (the contraction mapping in BLP; differenced log shares in pure logit) is a one-to-one mapping, generalizing the result in Berry (1994) that under mild assumptions on the joint individual error distribution, there exists a one-to-one mapping between the observed market share vector $\mathfrak{s}$ and the vector of mean utilities δ. Berry and Haile (2010) apply this condition to the identification of demand using only "macro", i.e., market share, data.

The original BLP paper proposed a nested fixed-point (NFP) approach, which has become the standard in empirical implementations. At every value of the parameter vector θ, the market shares are inverted using a contraction mapping to obtain a vector holding the mean utility level for each product. Recently, Dubé, Fox, and Su (2012) proposed an approach based on MPEC ("Mathematical Programming with Equilibrium Constraints"), suggesting to rewrite the problem as a constrained optimization problem, for which researchers have developed very efficient methods. MPEC can be much faster than NFP methods because the equilibrium constraints only need to hold at the optimum, while NFP methods impose them at every value of the parameter vector that the optimization algorithm searches over. We will use the traditional NFP method here, but feel free to implement MPEC as an additional exercise, and let us know about your experience with it.

2 Estimation exercise

2.1 Setting

A number of national producers sell substitute products in regional markets. The government intends to bail out a struggling firm and and allow it to merge with one of its healthy competitors. What do

for discussion and a two-step approach to the implementation of optimal instruments.

[3]The main condition, which they term "connected substitutes", requires, roughly speaking, that the set of products cannot be split into subsets such that any two products from different subsets exhibit no substitutability.

you expect the welfare consequences to be?

2.2 Data description

For the empirical exercise, we are giving you data on $T = 10$ markets. In these markets, 11 different firms sell a total of $J = 247$ products. All of the products are unique, so none of them are offered in multiple markets. The dataset is simulated, but you can still think of a product as a passenger vehicle with a set of characteristics if you like, although the units do not have an interpretation.

The dataset contains the following pieces of data, where products are ordered by market (1-10):

- "prodsMarket": T-vector of the number of products in each market
- "share": J-vector of market shares
- "f": J-vector denoting the firm that sells the product
- "ch": $J \times 4$-matrix of constant and three product characteristics
- "pr": J-vector of prices
- "costShifters": $J \times 2$-matrix of cost shifters

2.3 Basic summary statistics

1. Prepare a table with the following pieces of information for each market: How many firms are active? How many products do they market in total? What fraction of agents bought one of the goods in the sample period?

2. Prepare a table with summary statistics for market share, characteristics, price, and cost shifters. Please include mean, median, minimum, maximum, and standard deviation. You can inspect these statistics separately for each market, but in what you report, you may pool all markets.

2.4 Pure logit model

1. Suppose agents have the following utility function, where i denotes the agent, and j denotes the product:

$$u_{ij} = \underbrace{\delta_j}_{x_j'\beta - \alpha p_j + \xi_j} + \epsilon_{ij},$$

where ϵ_{ij} is an iid error following a standard Type-I Extreme Value distribution with $F(\epsilon) = e^{-e^{-\epsilon}}$ ("logit" errors). Suppose further that the firms know ξ when setting prices but did not knowξ when setting characteristics.

(a) What statistical assumptions can you make based on this? Which of your conditions, based on data provided to you, identify the parameter vector of interest, $\theta = (\alpha, \beta)$? In other words, what are valid (and relevant) instruments? Is the model over-identified?

(b) Show how you can invert market shares to obtain the mean utility level δ_j for each product.

(c) Estimate $\theta = (\alpha, \beta)$ and provide standard errors for your estimate. You can try different combinations of instruments, but please use all the different types of instruments that are included or can be constructed from the data (i.e., "BLP instruments").

2. Estimate and present the matrix of cross- and own-price elasticities for market 10 based on your model and parameter estimates.[4]

3. In the next question, we are going to free up the substitution pattern by introducing random coefficients as in BLP. Alternatively, we could think about implementing nested logit, the pure characteristics model, or multinomial probit. Would they be appealing in this setting? Why or why not?

2.5 Random-coefficient logit model

1. Suppose agents have the following utility function, where i denotes the agent, and j denotes the product:

$$u_{ij} = \underbrace{\delta_j}_{x_j'\beta - \alpha p_j + \xi_j} + \sum_{k \in \{1,2\}} \sigma_k \nu_{i,k} x_{j,k} - \sigma_p \nu_{i,p} p_j + \epsilon_{ij},$$

where ϵ_{ij} is an iid error following a standard Type-I Extreme Value distribution, and $\nu_{i,\cdot} \overset{iid}{\sim} \mathcal{N}(0,1)$ is an iid standard normal error. To summarize: the model is as before, but with random coefficients on the constant, the first characteristic, and price. The orthogonality/exogeneity assumptions remain the same as before.

[4]The symmetry of the matrix of share derivatives with respect to price (though not of elasticities) may have reminded you of the symmetry of the Slutsky matrix of the derivative of uncompensated demand with respect to price (Mas-Colell, Whinston, and Green, 1995, Proposition 3.G.2). Anderson, de Palma, and Thisse (1992, p. 67) (available on Hollis) show that it also holds in discrete choice settings with constant marginal utility of income in the region of interest, e.g., with quasilinear utility. Goolsbee and Petrin (2004) take advantage of symmetry to identify a cross-price derivative which would not be identified otherwise due to lack of variation in one of the prices.

(a) What is the contraction mapping used here for the inner loop? Is there a way to reduce the computational burden from the contraction mapping? (Hint: take a look at page 4 of the appendix to Nevo (2000).) In the following, make sure to set the "inner tolerance" level for the contraction mapping very tight, in your final run ideally on the order of 10^{-14}.

(b) Write the parameter vector of interest as $\theta = (\theta_1, \theta_2)$, where θ_1 are the "linear" parameters, and θ_2 are the "nonlinear" parameters. Which parameters are in θ_1 and which are in θ_2? What does this imply for estimation?

(c) *Bonus question*: Explain how the variance terms σ are identified from variation in the choice set and prices.

(d) Estimate the model using 2-step optimal GMM. In addition to your point estimates, please provide standard errors. (Hint: take a look at page 6 of the appendix to Nevo (2000) for analytic standard errors, and/or use finite differences for a numerical approximation.) If you try different starting values, do your estimates change?

(e) Provide an explicit expression for the variance-covariance matrix of your estimates, and discuss how simulation error affects it.

2. Compare the cross- and own-price elasticities for market 10 for the RC logit and pure logit model.

3. We are assuming here that demand in all markets is identical. With data on the distribution of income within each market, how could you let the distribution of α_i (the random variable coefficient on price) vary systematically across markets?

4. Let's assume, only for this question, that you had "micro moments" as in Berry, Levinsohn, and Pakes (2004): the covariance of consumer attributes and product characteristics as well as the covariance between first- and second-choice characteristics. How could you integrate them into your estimation procedure to improve the precision of your estimates? Which of the coefficients would each of the different sets of moments be particularly useful in "pinning down"?

5. Suppose in addition that each product has the following marginal cost structure:

$$mc_j = \begin{bmatrix} x_j & cs_j \end{bmatrix}' \gamma + \omega_j,$$

where cs is the $J \times 2$ matrix of cost shifters.

(a) Explain how you can estimate γ based on your estimates for the demand side and different assumptions on the supply side (product-level profit maximization, firm-level profit maximization, collusion to maximize total profit). How could you obtain valid standard errors?

(b) Can you also estimate the demand side and supply side jointly? How would you do so? What are the "linear" and "nonlinear" parameters now? Can joint estimation improve the precision of your estimates for the demand side parameters? Please explain how. Is there a caveat?

(c) Estimate the demand side and supply side parameters jointly under the assumption that firms maximize total firm profits. You don't need to provide standard errors. (Hint: since we have two sets of moment conditions, we can use multiple-equation GMM for the linear parameters.[5])

(d) *Bonus question*: suppose you had to select among the different pricing assumptions; how could you do so? Hint: have a look at Nevo (2001).

2.6 Counterfactual (bonus exercise)

Firms 1 and 10 have announced their intention to merge.[6] As the regulator in market 9, you are concerned that this will lead to an increase in prices and a decrease in product variety. Are these concerns justified? Please explain carefully and show your estimates. What concerns do you have about this counterfactual?

3 Feedback

Which parts of the problem set did you find more interesting or useful, and which less? Suppose you were the TF, and you had to give this problem set again next year. Which additional exercises or questions would you pose to students (or which would you scrap), and what would you add to the empirical model to make it more realistic or interesting? Can you think of other interesting counterfactuals?

[5] See, e.g., Eric Zivot's notes on multiple-equation linear GMM – notice that he flips the notation for X and Z relative to their typical use.

[6] Mergers will be discussed in lecture later on in the course.

References

Ackerberg, Daniel, C. Lanier Benkard, Steven Berry, and Ariel Pakes. 2007. "Econometric Tools for Analyzing Market Outcomes." In *Handbook of Econometrics*, vol. 6, edited by J.J. Heckman and E.E. Leamer, chap. 63. Elsevier. URL `http://ideas.repec.org/h/eee/ecochp/6a-63.html`.

Anderson, Simon P., André de Palma, and Jacques-Francois Thisse. 1992. *Discrete choice theory of product differentiation*. Cambridge, Mass.: MIT Press.

Berry, Steve, Oliver B. Linton, and Ariel Pakes. 2004. "Limit Theorems for Estimating the Parameters of Differentiated Product Demand Systems." *Review of Economic Studies* 71:613–54. URL `http://ideas.repec.org/a/bla/restud/v71y2004ip613-654.html`.

Berry, Steven, Amit Gandi, and Philip Haile. 2012. "Connected Substitutes and Invertibility of Demand." Cowles Foundation Discussion Papers 1806R, Cowles Foundation for Research in Economics, Yale University. URL `http://EconPapers.repec.org/RePEc:cwl:cwldpp:1806r`.

Berry, Steven, James Levinsohn, and Ariel Pakes. 1995. "Automobile Prices in Market Equilibrium." *Econometrica* 63 (4):841–90. URL `http://ideas.repec.org/a/ecm/emetrp/v63y1995i4p841-90.html`.

———. 1999. "Voluntary Export Restraints on Automobiles: Evaluating a Trade Policy." *American Economic Review* 89 (3):400–30. URL `http://ideas.repec.org/a/aea/aecrev/v89y1999i3p400-430.html`.

———. 2004. "Differentiated Products Demand Systems from a Combination of Micro and Macro Data: The New Car Market." *Journal of Political Economy* 112 (1):68–105. URL `http://ideas.repec.org/a/ucp/jpolec/v112y2004i1p68-105.html`.

Berry, Steven T. 1994. "Estimating Discrete-Choice Models of Product Differentiation." *RAND Journal of Economics* 25 (2):242–62. URL `http://ideas.repec.org/a/rje/randje/v25y1994isummerp242-262.html`.

Berry, Steven T. and Philip A. Haile. 2010. "Identification in Differentiated Products Markets Using Market Level Data." NBER Working Papers 15641, National Bureau of Economic Research, Inc. URL `http://ideas.repec.org/p/nbr/nberwo/15641.html`.

Dubé, Jean-Pierre, Jeremy T. Fox, and Che-Lin Su. 2012. “Improving the Numerical Performance of Static and Dynamic Aggregate Discrete Choice Random Coefficients Demand Estimation.” *Econometrica* 80 (5):2231–67. URL `http://ideas.repec.org/a/ecm/emetrp/v80y2012i5p2231-2267.html`.

Freyberger, Joachim. 2012. “Asymptotic theory for differentiated products demand models with many markets.” CeMMAP working papers CWP19/12, Centre for Microdata Methods and Practice, Institute for Fiscal Studies. URL `http://EconPapers.repec.org/RePEc:ifs:cemmap:19/12`.

Gandhi, Amit, Zhentong Lu, and Xiaoxia Shi. 2012. “Demand Estimation for Differentiated Products: The Many Markets Case.” Tech. rep., University of Wisconsin-Madison.

Goolsbee, Austan and Amil Petrin. 2004. “The Consumer Gains from Direct Broadcast Satellites and the Competition with Cable TV.” *Econometrica* 72 (2):351–81. URL `http://ideas.repec.org/a/ecm/emetrp/v72y2004i2p351-381.html`.

Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green. 1995. *Microeconomic Theory*. Oxford University Press.

Nevo, Aviv. 2000. “A Practitioner’s Guide to Estimation of Random-Coefficients Logit Models of Demand.” *Journal of Economics & Management Strategy* 9 (4):513–48. URL `http://ideas.repec.org/a/bla/jemstr/v9y2000i4p513-548.html`.

———. 2001. “Measuring Market Power in the Ready-to-Eat Cereal Industry.” *Econometrica* 69 (2):307–42. URL `http://dx.doi.org/10.1111/1468-0262.00194`.

Petrin, Amil. 2002. “Quantifying the Benefits of New Products: The Case of the Minivan.” *Journal of Political Economy* 110 (4):705–29. URL `http://www.jstor.org/stable/10.1086/340779`.

Reynaert, Mathias and Frank Verboven. 2012. “Improving the performance of random coefficients demand models: the role of optimal instruments.” Center for Economic Studies - Discussion papers ces12.07, Katholieke Universiteit Leuven, Centrum voor Economische Studiën. URL `http://ideas.repec.org/p/ete/ceswps/ces12.07.html`.