CS663: Digital Image Processing - Homework 3

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1 Homework 3 - Question 2

Part 1. Correlation of Two Continuous 2D Signals in the Continuous Domain

Definition of Correlation in the Continuous Domain

Given two continuous 2D signals $f(x, y)$ and $g(x, y)$, the correlation between these two signals is defined as:

$$h(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x' + x, y' + y) \, dx' \, dy'$$

This is an integral that shifts the signal $g(x, y)$ over $f(x, y)$ and computes their similarity at each point.

Fourier Transform of Correlation in the Continuous Domain

The Fourier transform $\mathcal{F}\{f(x, y)\}$ of a continuous signal $f(x, y)$ is defined as:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i2\pi(ux+vy)} \, dx \, dy$$

We can convert x to -x and we get h(x,y) as:

$$h(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x' - (-x), y' - (-y)) \, dx' \, dy'$$

Applying the Fourier Transform to both sides of the correlation equation for $h(x, y)$, we use the Convolution Theorem. Then we get:

$$\mathcal{F}\{h(x, y)\} = \mathcal{F}\{f(x, y)\} \cdot \mathcal{F}\{g(-x, -y)\}$$

The term $g(-x, -y)$ represents the flipped version of $g(x, y)$. Using the property that the Fourier transform of a flipped function is the complex conjugate of the original Fourier transform, we get:

$$H(u, v) = F(u, v) \cdot G^*(u, v)$$

Where $F(u, v)$ and $G(u, v)$ are the Fourier transforms of $f(x, y)$ and $g(x, y)$, and $G^*(u, v)$ is the complex conjugate of $G(u, v)$.

Part 2. Correlation of Two Discrete 2D Signals in the Discrete Domain

Definition of Correlation in the Discrete Domain

Consider two discrete 2D signals $f[m,n]$ and $g[m,n]$. The correlation in the discrete domain is defined as:

$$h[m,n] = \sum_{m'} \sum_{n'} f[m',n'] g[m'+m, n'+n]$$

This is a summation over all possible shifts of $g[m,n]$ relative to $f[m,n]$.

2D Discrete Fourier Transform (DFT)

The 2D Discrete Fourier Transform (DFT) of a discrete signal $f[m,n]$ is defined as:

$$F[k,l] = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f[m,n] e^{-i2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

Similarly, the DFT of $g[m,n]$ is:

$$G[k,l] = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g[m,n] e^{-i2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

Fourier Transform of Correlation in the Discrete Domain

h(m,n) can be written as:

$$h[m,n] = \sum_{m'} \sum_{n'} f[m',n'] g[m'-(-m), n'-(-n)]$$

By applying Fourier on both sides and using the Discrete Convolution Theorem, we get the following relationship between the correlation and Fourier transforms:

$$H[k,l] = F[k,l] \cdot G[-k,-l]$$

Which turns to the following due to the flipping property:

$$H[k,l] = F[k,l] \cdot G^*[k,l]$$

Where $H[k,l]$ is the 2D DFT of the correlation $h[m,n]$, and $F[k,l]$ and $G[k,l]$ are the 2D DFTs of $f[m,n]$ and $g[m,n]$, with $G^*[k,l]$ being the complex conjugate of $G[k,l]$.