CS663: Digital Image Processing - Homework 3

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1 Homework 3 - Question 4

The Fourier transform of such an image can be computed as follows:

$$F(u,v) = \frac{1}{201} \sum_{x=1}^{201} \sum_{y=1}^{201} \delta(x-101) e^{-j2\pi\left(\frac{ux}{201}+\frac{vy}{201}\right)}$$

Since the image is zero everywhere except in the 101st column, we can simplify the summation by fixing $x = 101$:

$$F(u,v) = \frac{1}{201} \sum_{y=1}^{201} 255 \cdot e^{-j2\pi\left(\frac{101u}{201}+\frac{vy}{201}\right)}$$

Factoring out constants, we get:

$$F(u,v) = \frac{255}{201} e^{-j2\pi\frac{101u}{201}} \sum_{y=1}^{201} e^{-j2\pi\frac{vy}{201}}$$

The summation is a geometric series, and it evaluates to:

$$F(u,v) = \frac{255}{201} e^{-j2\pi\frac{101u}{201}} \cdot \frac{1-e^{-j2\pi v}}{1-e^{-j\frac{2\pi v}{201}}}$$

This evaluates to:

$$F(u,v) = \begin{cases} Ke^{-j2\pi\frac{101u}{201}} \cdot 201, & v = 201n, \quad n \in \mathbb{Z}, \\ Ke^{-j2\pi\frac{101u}{201}} \cdot \frac{1-e^{-j2\pi v}}{1-e^{-j\frac{2\pi v}{201}}}, & \text{otherwise} \end{cases}$$

where K is a constant.
This is zero when v is an integer not divisible by 201.

MATLAB Code

The following MATLAB code computes the Fourier transform using `fft2`, shifts the zero frequency component to the center using `fftshift`, and plots the logarithm of the Fourier magnitude:

```
% MATLAB code to compute the Fourier transform
I = zeros(201, 201); % Create a 201x201 black image
I(:, 101) = 255; % Set the central column to 255

% Compute the 2D Fourier transform and shift zero-frequency to center
F = fft2(I);
F_shifted = fftshift(F);

% Compute the logarithm of the magnitude
log_magnitude = log(abs(F_shifted) + 1);

% Plot the result
figure;
imagesc(log_magnitude);
colorbar;
title('Logarithm of Fourier Magnitude');
```

Output Image

The output image is shown in Figure 1.

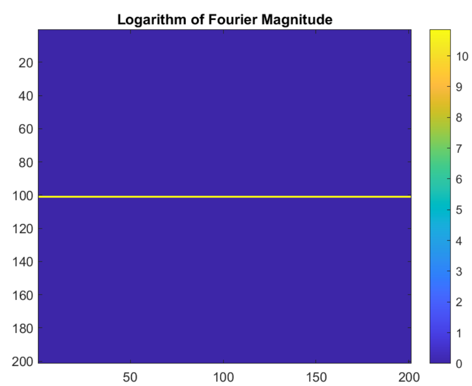

Figure 1: Logarithm of Fourier Magnitude

Conclusion

The Fourier transform of the given image has been derived analytically and computed using MATLAB. The image contains a strong response along the vertical frequency components due to the constant value in the central column of the image.