CS663: Digital Image Processing - Homework 5

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Homework 5 - Question 2

Given the images g_1 and g_2 taken under different focus settings, we are expressing them mathematically as:

- $g_1 = f_1 + h_2 * f_2$, where:
  - f_1 is the scene outside (in focus for g_1),
  - h_2 is the blur kernel applied to the reflection f_2,
- $g_2 = h_1 * f_1 + f_2$, where:
  - h_1 is the blur kernel applied to the outside scene f_1 when the reflection f_2 is in focus.

Fourier Transform Approach

To solve for f_1 and f_2, let's take the Fourier Transform of both equations:

$$G_1(\mu) = F_1(\mu) + H_2(\mu)F_2(\mu)$$
$$G_2(\mu) = H_1(\mu)F_1(\mu) + F_2(\mu)$$

Now we can rearrange these equations to isolate $F_1(\mu)$ and $F_2(\mu)$:

1. Solving for $F_2(\mu)$:

$$F_2(\mu) = \frac{G_2(\mu) - H_1(\mu)G_1(\mu)}{1 - H_1(\mu)H_2(\mu)}$$

2. Solving for $F_1(\mu)$:

$$F_1(\mu) = \frac{G_1(\mu) - H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu)}$$

Observations on the Formula

This solution is defined as long as $1 - H_1(\mu)H_2(\mu) \neq 0$. However, due to the nature of h_1 and h_2 as low-pass blur kernels, each integrates to 1 over the entire domain (i.e., $H_1(0) = H_2(0) = 1$). This implies that at the frequency $\mu = 0$, we encounter:

$$1 - H_1(0)H_2(0) = 0$$

As a result, the formula becomes undefined at low frequencies, particularly at the DC component (zero frequency). This introduces an issue where we can recover high-frequency details in the images, but we cannot reliably reconstruct low-frequency components.

In practice, this lack of low-frequency information leads to an artificial-looking result because the image's lower frequencies, which contribute to smooth transitions and shading, are not accurately reconstructed. A common workaround is to add a small ϵ to the denominator to stabilize the solution:

$$F_2(\mu) = \frac{G_2(\mu) - H_1(\mu)G_1(\mu)}{1 - H_1(\mu)H_2(\mu) + \epsilon}$$

$$F_1(\mu) = \frac{G_1(\mu) - H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu) + \epsilon}$$

Effect of Noise

If the images are noisy, with $g_1 = f_1 + h_2 * f_2 + \eta_1$ and $g_2 = h_1 * f_1 + f_2 + \eta_2$, where η_1 and η_2 are noise terms, this affects our results:

For high frequencies, the denominator remains large (close to 1), so noise does not amplify significantly. However, for low frequencies, as $H_1(\mu)H_2(\mu) \to 1$, the denominator approaches zero, potentially amplifying noise. Since the signal strength in these frequencies is also high, the relative error won't become overwhelming, unlike with an inverse low-pass filter under noise.