CS663: Digital Image Processing - Homework 3

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1 Homework 3 - Question 6

First we prove $F(F(f(t))) = f(-t)$:

$$\begin{aligned}
G(f) = F(f(t)) &= \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}dt \\
H(\tau) = F(G(f)) = F(F(f(t))) &= \int_{-\infty}^{\infty} G(f)e^{-j2\pi f\tau}df \\
&= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}dt\right) e^{-j2\pi f\tau}df \\
&= \int_{-\infty}^{\infty} f(t) \left(\int_{-\infty}^{\infty} e^{-j2\pi f(t+\tau)}df\right) dt \\
&(\because \text{ f and t are independent}) \\
&= \int_{-\infty}^{\infty} f(t)\delta(t+\tau)dt \quad \text{(Shifting)} \\
&= f(-\tau) \quad \text{(Sifting)}
\end{aligned}$$

$\therefore H(\tau) = f(-\tau) \implies H(t) = f(-t) \implies F(F(f(t))) = f(-t)$
Now, if $F(F(f(t))) = f(-t)$, then
$F(F(F(F(f(t))))) = F(F(f(-t))) = f(-(-t)) = f(t)$
One very important application of this is that it offers a reasoning for the construction of the inverse fourier transform. Operations in the spatial domain affect the frequency domain and vice versa. For instance, applying a low-pass or high-pass filter in the frequency domain affects the smoothness or edge characteristics of an image in the spatial domain. The reverse holds due to duality.