CS663: Digital Image Processing - Homework 1

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1 Homework 1 - Question 4

To perform motion estimation using control points for the given motion model, we need to estimate the unknown constants $a, b, c, d, e, f, A, B, C, D, E, F$.

Given

The motion models:

$$x_2 = ax_1^2 + by_1^2 + cx_1y_1 + dx_1 + ey_1 + f$$
$$y_2 = Ax_1^2 + By_1^2 + Cx_1y_1 + Dx_1 + Ey_1 + F$$

Here, (x_1, y_1) are the coordinates of a point in the first image, and (x_2, y_2) are the coordinates of the corresponding point in the second image. The goal is to estimate the coefficients $a, b, c, d, e, f, A, B, C, D, E, F$ using a set of known control points.

System of Equations

Suppose we have N control points, where each control point in Image 1 is denoted as (x_{1i}, y_{1i}) and the corresponding point in Image 2 as (x_{2i}, y_{2i}) for $i = 1, 2, \ldots, N$.

For each control point, we have two equations:

$$x_{2i} = ax_{1i}^2 + by_{1i}^2 + cx_{1i}y_{1i} + dx_{1i} + ey_{1i} + f$$
$$y_{2i} = Ax_{1i}^2 + By_{1i}^2 + Cx_{1i}y_{1i} + Dx_{1i} + Ey_{1i} + F$$

Matrix Form

For the x_2 coordinate:

$$\begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2N} \end{bmatrix} = \begin{bmatrix} x_{11}^2 & y_{11}^2 & x_{11}y_{11} & x_{11} & y_{11} & 1 \\ x_{12}^2 & y_{12}^2 & x_{12}y_{12} & x_{12} & y_{12} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{1N}^2 & y_{1N}^2 & x_{1N}y_{1N} & x_{1N} & y_{1N} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

This can be simplified as:

$$\mathbf{x}_2 = \mathbf{X}_1 \cdot \mathbf{p}$$

where

$$\mathbf{x}_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2N} \end{bmatrix}, \quad \mathbf{X}_1 = \begin{bmatrix} x_{11}^2 & y_{11}^2 & x_{11}y_{11} & x_{11} & y_{11} & 1 \\ x_{12}^2 & y_{12}^2 & x_{12}y_{12} & x_{12} & y_{12} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{1N}^2 & y_{1N}^2 & x_{1N}y_{1N} & x_{1N} & y_{1N} & 1 \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

Similarly, for the y_2 coordinate:

$$\mathbf{y}_2 = \mathbf{X}_1 \cdot \mathbf{P}$$

where

$$\mathbf{y}_2 = \begin{bmatrix} y_{21} \\ y_{22} \\ \vdots \\ y_{2N} \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix}$$

Solving the System of Equations

To estimate the parameters $\mathbf{p} = [a, b, c, d, e, f]^T$ and $\mathbf{P} = [A, B, C, D, E, F]^T$, we solve the linear systems for $\mathbf{p}$ and $\mathbf{P}$:

$$\mathbf{p} = \mathbf{X}_1^{\dagger}\mathbf{x}_2$$

$$\mathbf{P} = \mathbf{X}_1^{\dagger}\mathbf{y}_2$$

where $\mathbf{X}^{\dagger} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ of $\mathbf{X}$. This is used here because $\mathbf{X}_1$ might not be a square matrix.

Finally

After solving these systems, the vectors $\mathbf{p}$ and $\mathbf{P}$ contain the estimated coefficients that describe the motion between the two images. These coefficients can then be used to map any point (x_1, y_1) in Image 1 to its corresponding location (x_2, y_2) in Image 2 according to the given motion model.