CS663: Digital Image Processing - Homework 2

Harsh | Pranav | Swayam

September 6, 2024

1 Homework 2 - Question 1

Image Intensity:

The clean image $I(x, y)$ has continuous-valued intensities at every pixel position (x, y). Let's assume that a noisy image $I_N(x, y)$ is formed by adding Gaussian noise $N(x, y)$ to each pixel value of the clean image. Mathematically, this can be written as:

$$I_N(x, y) = I(x, y) + N(x, y)$$

PDF of the Gaussian Noise:

$$p(N) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{(-\frac{N}{2\sigma^2})}$$

PDF of the Noisy Image:

Since:

$$I_N(x, y) - I(x, y) = N(x, y)$$

Image PDF will be:

$$p(I_N) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{(-\frac{I_N(x,y) - I(x,y)}{2\sigma^2})}$$

Resemblance:

This PDF resembles the Gaussian blur operation in image processing. A Gaussian blur smooths an image by averaging neighboring pixel values, with the weights of the average determined by a Gaussian distribution. In a similar manner, the noisy image's pixel intensities are perturbed by values drawn from a Gaussian distribution, thus resulting in a similar smoothing or blurring effect.

Modified PDF:

So for uniform PDF we have to make the probability of every value in the range between $-r$ to r same. Which is:

$$p(N) = \begin{cases} \frac{1}{2r}, & \text{if } -r \leq N \leq r \\ 0, & \text{otherwise} \end{cases}$$

For the noisy image $I_N(x, y) = I(x, y) + N(x, y)$, the PDF of the noisy image will also be uniform, shifted by the clean pixel value $I(x, y)$. The PDF for $I_N(x, y)$ becomes:

$$p(I_N) = \begin{cases} \frac{1}{2r}, & \text{if } -r \leq I_N(x, y) + I(x, y) \leq r \\ 0, & \text{otherwise} \end{cases}$$

In other words, the noisy pixel value $I_N(x, y)$ will be uniformly distributed within the range $[I(x, y) - r, I(x, y) + r]$, which means the noise will deviate the clean pixel values uniformly within this range.