CS663: Digital Image Processing - Homework 2

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1 Homework 2 - Question 5

Part a (Gaussian):

A Gaussian kernel $g(k)$ with mean 0 and standard deviation σ is symmetric and normalized such that:

$$\sum_{k=-\infty}^{+\infty} g(k) = 1 \quad \text{and} \quad \sum_{k=-\infty}^{+\infty} kg(k) = 0$$

The Gaussian filtered image is given by the convolution:

$$J(x) = (g * I)(x) = \sum_{k=-\infty}^{+\infty} I(x-k)g(k)$$

Substitute $I(x-k) = c(x-k) + d$ into the convolution:

$$J(x) = \sum_{k=-\infty}^{+\infty} [c(x-k) + d]g(k)$$

$$J(x) = c \sum_{k=-\infty}^{+\infty} (x-k)g(k) + d \sum_{k=-\infty}^{+\infty} g(k)$$

Using the properties of the Gaussian kernel:

$$J(x) = c(x-0) + d = cx + d$$

Thus, the Gaussian filter preserves the ramp image:

$$J(x) = cx + d = I(x)$$

Part b (Bilateral):

The bilateral filter is defined as:

$$J(x) = \frac{1}{L} \sum_{k=-\infty}^{+\infty} I(x-k) \cdot e^{-\frac{k^2}{2\sigma_s^2}} \cdot e^{-\frac{(I(x-k)-I(x))^2}{2\sigma_r^2}}$$

where L is the normalization factor:

$$L = \sum_{k=-\infty}^{+\infty} e^{-\frac{k^2}{2\sigma_s^2}} \cdot e^{-\frac{(I(x-k)-I(x))^2}{2\sigma_r^2}}$$

Substituting $I(x-k) = c(x-k) + d$ into the bilateral filter:

$$J(x) = \frac{1}{L} \sum_{k=-\infty}^{+\infty} [c(x-k)+d] e^{-\frac{k^2}{2\sigma_s^2}} \cdot e^{-\frac{(ck)^2}{2\sigma_r^2}}$$

Factor out the constants:

$$J(x) = cx \frac{1}{L} \sum_{k=-\infty}^{+\infty} e^{-\frac{k^2}{2\sigma_s^2}} e^{-\frac{c^2k^2}{2\sigma_r^2}} + d \frac{1}{L} \sum_{k=-\infty}^{+\infty} e^{-\frac{k^2}{2\sigma_s^2}} e^{-\frac{c^2k^2}{2\sigma_r^2}}$$

The sums of the Gaussian kernels evaluate to 1, and the terms involving k vanish:

$$J(x) = cx + d = I(x)$$

Thus, the bilateral filter also preserves the ramp image