# CS663: Digital Image Processing - Homework 2

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## 1 Homework 2 - Question 2

### Given Convolution:

$$g(x) = \sum_{k=-3}^{3} f(x-k)w(k)$$

where $f(x)$ is the input image, $w(k)$ is the convolution kernel, and $g(x)$ is the resultant image. We express this operation as matrix multiplication after zero-padding the image.

Taking the image $f(x)$ is zero-padded to handle boundary effects. If the original image has $n$ pixels, the zero-padded image has $n+6$ pixels. The zero-padded image vector is:

$$f = \begin{bmatrix} f(-3) & f(-2) & f(-1) & f(0) & \dots & f(n+3) \end{bmatrix}^T$$

Similarly, the output image $g$ is:

$$g = \begin{bmatrix} g(-3) & g(-2) & g(-1) & g(0) & \dots & g(n+3) \end{bmatrix}^T$$

### Convolution Matrix:

The convolution matrix $W$ is constructed using the flipped kernel $w'_i = w(-i)$. That is:

$$w = \begin{bmatrix} w_0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{bmatrix}$$

And

$$w' = \begin{bmatrix} w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 \end{bmatrix}$$

It is a $(n+6) \times (n+6)$ matrix where each row is a shifted version of the flipped kernel:

$$W = \begin{bmatrix} w'_0 & w'_1 & w'_2 & w'_3 & w'_4 & w'_5 & w'_6 & \dots & 0 & 0 \\ 0 & w'_0 & w'_1 & w'_2 & w'_3 & w'_4 & w'_5 & w'_6 & \dots & 0 \\ \vdots & & \ddots & & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w'_0 & w'_1 & w'_2 & w'_3 & w'_4 & w'_5 & w'_6 \end{bmatrix}$$

## Properties of this Matrix:

1. The matrix $W$ may or may not be invertible depending on the properties of the convolution kernel. If $W$ is invertible, it implies that we can recover the original image $f$ from the output $g$ and the convolution kernel $w$.

2.$W$ is a Toeplitz matrix, meaning each diagonal (and sub-diagonal) of the matrix contains constant values, which correspond to the flipped kernel values $w'$

## Application:

Convolution as a Linear Operation: The matrix formulation reinforces the idea that convolution is a linear operation.
This allows us to use linear algebra tools to analyze and manipulate convolutions.This can help us in more efficient computation and enables us to use linear algebra algorithms.