CS663: Digital Image Processing - Homework 3

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1 Homework 3 - Question 5

We have,

$$F(u,v) = \sum_{x,y} f(x,y) \exp\left(-j2\pi \frac{(ux+vy)}{N}\right)$$

$$F(-u,-v) = \sum_{x,y} f(x,y) \exp\left(j2\pi \frac{(ux+vy)}{N}\right).$$

$$\begin{aligned}\therefore F^*(u,v) &= \left(\sum_{x,y} f(x,y) \exp\left(-j2\pi \frac{(ux+vy)}{N}\right)\right)^* \\ &= F(-u,-v)\end{aligned} \tag{1}$$

`The conjugation step is valid because` $f(x,y)$ `is real-valued`.

If f is both real and even, i.e. $f(x,y) = f(-x,-y)$. Then we have:

$$\begin{aligned}F^*(u,v) &= \left(\sum_{x=0}^{N}\sum_{y=0}^{N} f(x,y) \exp\left(-j2\pi \frac{(ux+vy)}{N}\right)\right)^* \\ &= \sum_{x=0}^{N}\sum_{y=0}^{N} f(x,y) \exp\left(j2\pi \frac{(ux+vy)}{N}\right) \\ &= \sum_{x=0}^{N}\sum_{y=0}^{N} f(-x,-y) \exp\left(j2\pi \frac{(-ux-vy)}{N}\right)\end{aligned} \tag{2}$$

By substituting $x' = -x$ and $y' = -y$, we get -

$$= \sum_{x'=0}^{N}\sum_{y'=0}^{N} f(x',y') \exp\left(-j2\pi \frac{(ux'+vy')}{N}\right)$$

$$= F(u,v) \tag{3}$$

Hence, $F(u,v)$ is proven to be real-valued.

We already proved $F^*(u,v) = F(-u,-v)$ which in this case shows that $F(u,v) = F(-u,-v)$ and hence $F(u,v)$ is even.
Therefore, if f is real and even, then $F(u,v)$ is also real and even.