
EE 204 - Analog Circuits

Lecture 8

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1 Transistor Review

So far, we have seen transistors being used for a varied number of applications. We have used transistor to build a switch (ON/OFF switch), as an Amplifier, as a Cascade/Summing Amplifier, also we have used Transistors to create OpAmp (Operational Amplifier).

To summarise -

1. Switch

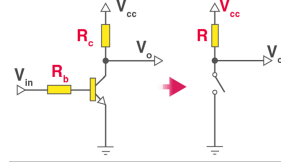


Figure 1: ON/OFF Switch

2. Amplifier

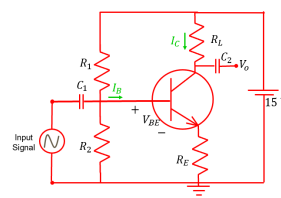


Figure 2: Amplifier

3. Summing Amplifier

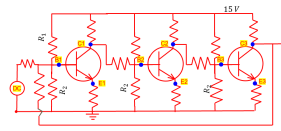


Figure 3: Summing Amplifier

4. OpAmp

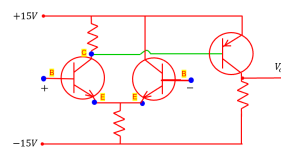


Figure 4: OpAmp Circuit

So, now let's begin with the Hybrid Parameter Model.

2 Hybrid Parameter Model

A transistor can be thought of as a two-port network. Any two-port network's terminal behaviour can be determined by the terminal voltages V_1 and V_2 , and the respective currents, I_1 and I_2 at the ports 1 and 2 respectively.

Always, we can represent two out of the four variables in terms of the remaining two. Therefore, it suffices to choose i_1 and v_2 as Independent variables, and thus represent v_1 and i_2 in terms of these two.

Therefore, we get the following two relations -

$$v_1 = h_{11}i_1 + h_{12}v_2$$



Figure 5: Hybrid Parameter Model

$$i_2 = h_{21}i_1 + h_{22}v_2$$

where, h_{11} , h_{12} , h_{21} , and h_{22} are the hybrid parameters

Let's quickly go over how the hybrid parameters are defined.

2.1 Hybrid Parameters

$$h_{11} = \frac{v_1}{i_1} \text{ at } v_2 = 0$$

Input impedance with output port short circuited for ac

$$h_{12} = \frac{v_1}{v_2} \text{ at } i_1 = 0$$

Reverse voltage amplification factor with output port open circuited for ac

$$h_{21} = \frac{i_2}{i_1} \text{ at } v_2 = 0$$

Forward Current gain with output port short circuited for ac

$$h_{22} = \frac{i_2}{v_2} \text{ at } i_1 = 0$$

Output admittance with output port open circuited for ac

As can be clearly seen, h_{12} and h_{21} are dimensionless, whereas h_{11} and h_{22} have dimensions of Impedance and Admittance ($= 1/\text{Impedance}$) respectively.

2.2 h-equivalent circuit model

Now, that we know the parameters and the relevant equations; for a two-port network, we get the following equivalent circuit which is called as the 'h-equivalent' circuit.

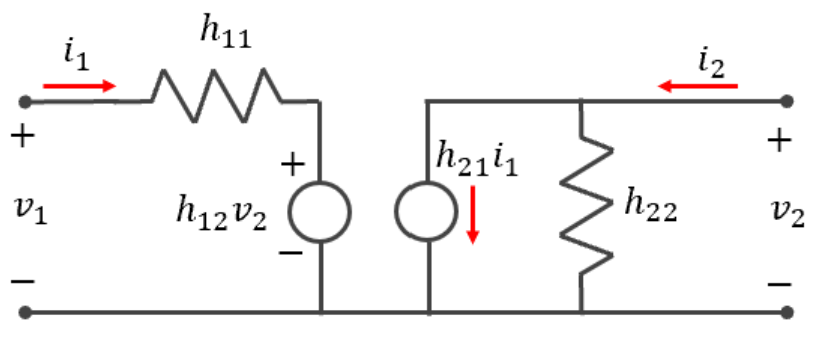


Figure 6: A two-port network h-equivalent circuit model

3 Transistor Amplifier as a two-port network

For transistors, to have better clarity about the coefficients we will use the following notation -

- $h_{11} = h_i$
- $h_{12} = h_r$
- $h_{21} = h_f$
- $h_{22} = h_o$

Here, i, r, f, o stands for input, reverse, forward and output. We will also use a second subscript b, e, c in order to designate the circuit configuration of the transistor.

Also, Input port 1 is the base B and the Output port 2 is Collector C. Therefore, the port 1 variables

are now B variables and port 2 variables are now C variables.

Using Taylor Series expansion on the two linear equations that we got above, we get

$$h_{ie} = \frac{\delta v_B}{\delta i_B} \quad h_{re} = \frac{\delta v_B}{\delta v_C}$$

$$h_{fe} = \frac{\delta i_C}{\delta i_B} \quad h_{oe} = \frac{\delta i_C}{\delta v_C}$$

The values of the hybrid parameters are still evaluated at the initial conditions as stated earlier. For the transistor those conditions correspond to the Q-point on the I_C vs V_{CE} graph.

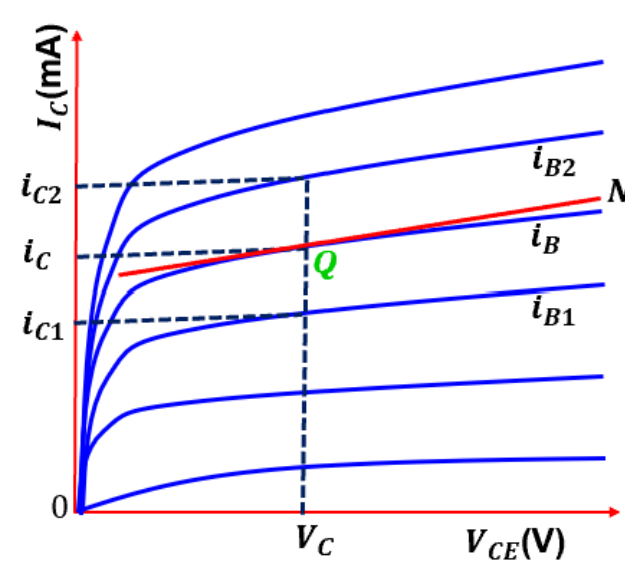


Figure 7: I_C vs V_{CE} graph for measuring the h-parameters and the transconductance

A new parameter transconductance is defined as

$$g_m = \frac{\delta i_C}{\delta v_B} = \frac{h_{fe}}{h_{ie}}$$

3.1 Hybrid Parameter Relationships

Now, since we can apply KVL, KCL equations for the transistor configurations, we can land up at a relationship between the h-parameters for the b, c and e configurations (b is common base, c is common collector and e is common emitter configuration).

What we get on simplifying the equations are the following:

$$I_C = -\frac{h_{fe}}{1 + h_{fe}} I_E + \frac{h_{oe}}{1 + h_{fe}} V_{cb}$$

$$I_C = -h_{fb} I_E + h_{ob} V_{cb}$$

Therefore, by comparing the two equations we get -

$$h_{fb} = \frac{h_{fe}}{1 + h_{fe}}$$

$$h_{ob} = \frac{h_{oe}}{1 + h_{fe}}$$

Similarly, we arrive at a relation for b and e parameters.

The following chart gives a comparison of the values of the hybrid parameters for the different configurations.

Parameters	CE	CC	CB
h_i	$2 \times 10^3 \Omega$	$2 \times 10^3 \Omega$	40Ω
h_r	2×10^{-3}	-1	4×10^{-4}
h_f	50	-51	-0.98
h_o	$40 \mu A/V$	$40 \mu A/V$	$0.8 \mu A/V$

Figure 8: Values of Hybrid-Parameters for Transistor Configurations

3.2 h-equivalent parameter model

So, the equivalent representation of the transistor as we know it in the form a two-port network using the h-parameter values is shown below -

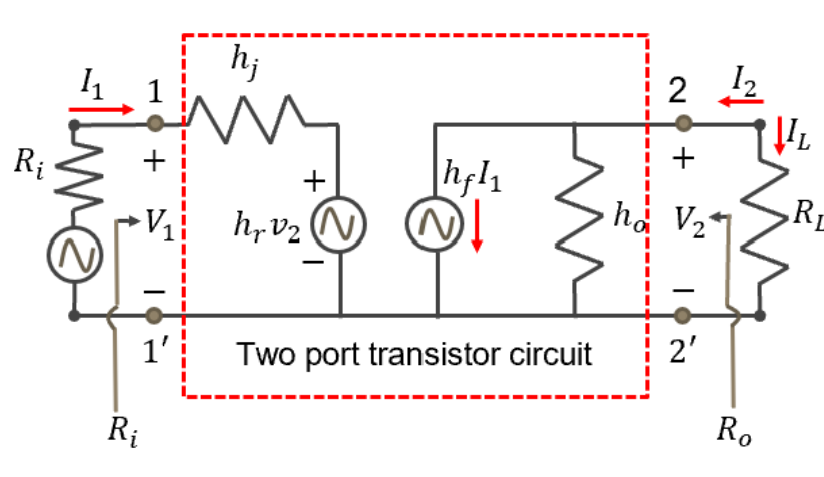


Figure 9: The h-parameter model of a transistor amplifier