EE324: Control Systems Lab
Experiment 2: Inverted Pendulum
Group 1 - Thursday

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1 Objective

To design and implement control action for maintaining a pendulum in the upright position (even when subjected to external disturbances) through LQR technique in an Arduino Mega.

The specific objectives were:

- To restrict the pendulum arm vibration (α) within $\pm 3°$.
- To restrict the base angle oscillation (θ) within $\pm 30°$.

2 Control Algorithm

The control algorithm used in this experiment is Linear Quadratic Regulator (LQR). Given the plant model

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t)$ is the state vector, $u(t)$ is the control input, A is the state matrix and B is the input matrix. The objective of the Linear Quadratic Regulator (LQR) problem is to find the control input $u(t)$ that minimizes the cost function J given by -

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))\mathrm{d}t \tag{2}$$

where Q and R are positive definite matrices. The control input $u(t)$ is given by the feedback law -

$$u(t) = -Kx(t) \tag{3}$$

where K is the state feedback gain matrix given by -

$$K = R^{-1}B^TP \tag{4}$$

and P is the solution to the equation -

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{5}$$

Some important code snippets showing the implementation of the LQR controller in Arduino.

```
/*
  Experiment 2 : Inverted Pendulum

  Group 1:
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*/

/*
Q = [410, 0, 0, 0;
    0,4, 0, 0;
    0, 0, 50, 0;
    0, 0, 0, 100];
R = 14;
*/

#include <SPI.h>

/* Serial rates for UART */
#define BAUDRATE        115200

/* SPI commands */
#define AMT22_NOP       0x00
#define AMT22_RESET     0x60
#define AMT22_ZERO      0x70

/* Define special ascii characters */
#define NEWLINE         0x0A
#define TAB             0x09

/* We will use these define macros so we can write code once compatible
   ↪ with 12 or 14 bit encoders */
#define RES12           12
#define RES14           14

/* SPI pins */
#define ENC_0            2
#define ENC_1            3
#define SPI_MOSI        51
#define SPI_MISO        50
#define SPI_SCLK        52

void setup()
{
  ⋮
}

void loop()
{
  uint16_t encoderPosition0, encoderPosition1;
  uint8_t attempts;
  float theta, alpha;
  float start_pos_arm = (float)getPositionSPI(ENC_0, RES14)*360/16383;
```

```
  float error_pendulum_cur,error_arm_cur, error_pendulum_prev, error_arm_prev, velocity_arm, velocity_pendulum, Vm_out;
  int fbsignal;
  float k[4] = {-5.41162769282160,  96.7114158315394,  -3.49111501865230, 13.0618707845219};

  encoderPosition1 = getPositionSPI(ENC_1, RES14);
  encoderPosition0 = getPositionSPI(ENC_0, RES14);

  theta = (float)encoderPosition0*360/16383;
  alpha = (float)encoderPosition1*360/16383;

  error_pendulum_prev = alpha - 180;
  error_arm_prev = theta - start_pos_arm;
  while(1){

    encoderPosition0 = getPositionSPI(ENC_0, RES14);
    encoderPosition1 = getPositionSPI(ENC_1, RES14);

    theta = (float)encoderPosition0*360/16383;
    alpha = (float)encoderPosition1*360/16383;

    error_pendulum_cur = alpha - 180;
    error_arm_cur = theta - start_pos_arm;
    velocity_pendulum = (error_pendulum_cur - error_pendulum_prev)/0.025;
    velocity_arm = (error_arm_cur - error_arm_prev)/0.025;
    //LQR CODE
    Vm_out = (k[0]*error_arm_cur + k[1]*error_pendulum_cur + k[2]* velocity_arm + k[3]*velocity_pendulum)*3.1415926535/180;

    fbsignal =map(abs(Vm_out),0,12,0,255);

    Serial.print("arm_error: ");
    Serial.print(error_arm_cur);
    Serial.print("pendulum_error: ");
    Serial.println(error_pendulum_cur);

    if(Vm_out>0){
      analogWrite(12, constrain(fbsignal,0,255));  // Set motor direction
      analogWrite(13, 0);
    }
    else{
      analogWrite(13, constrain(fbsignal,0,255));  // Set motor direction
      analogWrite(12, 0);
    }

    error_pendulum_prev = error_pendulum_cur;
    error_arm_prev=error_arm_cur;
    delay(25);
  }
}
```

3 Challenges Faced and Solutions

- **Tuning of the LQR Controller:** Finding the appropriate values for the LQR controllables (Q, R) to meet the design specifications (K) was challenging.
  <u>Solution:</u> Started with smaller values of Q and R and gradually increased them to meet the design specifications. Monitored the system's response closely using **MATLAB** during tuning to prevent instability.

- **Attaining Controller Objectives:** Achieving a balance between a fast response (stabilization) and restricting pendulum vibration (α) within $\pm 3°$ and base angle oscillation (θ) within $\pm 30°$ was challenging. Over-tuning for speed lead to excessive overshoot or instability.
  <u>Solution:</u> Adjusted the values of Q and R to satisfy the constraints. Monitored the system's response closely using Serial Plotter and its resilience to external disturbance during tuning. The step response plot obtained from **MATLAB** is shown in Figure 1.

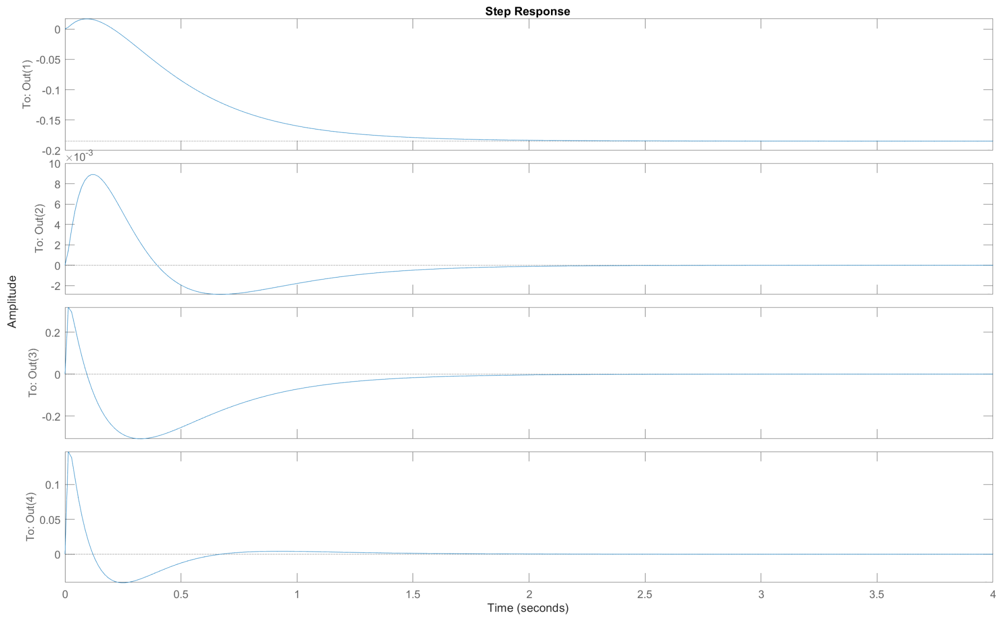

Figure 1: Step Response of the System

4 Results

- The final values of the LQR controllables were found to be

$$Q = \begin{bmatrix} 410 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$R = 14$$

- The feedback gain matrix K was found to be

$$K = \begin{bmatrix} -5.41 & 96.71 & -3.49 & 13.06 \end{bmatrix}$$

- The relative arm and pendulum position (error) vs time plot is shown in Figure 2

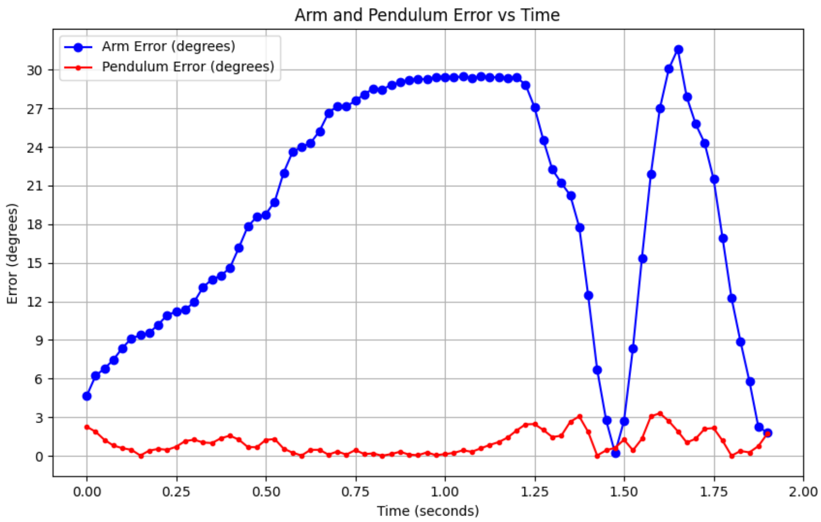

Figure 2: Relative Position vs Time

5 Observations and Inference

- The LQR controller was able to stabilize the inverted pendulum system within desired constraints.
- The system was able to maintain the pendulum arm within $\pm 3°$ and the base angle within $\pm 30°$ as can be seen in the plot 2.
- The system recovered from external disturbances and maintain stability when displaced from the equilibrium position.