Question 7:

Exercise 3.1.1:

a. True.
b. False.
c. True.
d. False.
e. True.
f. False.
g. False.

Exercise 3.1.2:

a. False.
b. True.
c. False.
d. True.
e. False.

Exercise 3.1.5:

b. $S = \{3, 6, 9, 12, ...\}$

$S = \{x \in N: x \text{ is an integer multiple of } 3\}$
Set S is an infinite set.

---

d. $S = \{0, 10, 20, 30, ..., 1000\}$

$S = \{x \in N: 0 \leq x \leq 1000 \text{ and an integer multiple of } 10\}$
Set S is a finite set.
$Cardinality = |S| = 101$

Exercise 3.2.1:

a. True.
b. True.
c. False.
d. False.
e. True.
f. True.
g. True.
h. False.
i. False.
j. False.
k. False.

Question 8:

Exercise 3.2.4:

b. Let A = {1, 2, 3}. What is {x ∈ P(A): 2 ∈ x}?

Explanation:

$P(A) = \{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

2 is not an element of $P(A)$.

Therefore the roster notation for the expression {x ∈ P(A): 2 ∈ x} would be $\emptyset$.

Question 9:

Exercise 3.3.1:

c. $A \cap C = \{-3, 1, 17\}$

---

d. $A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

---

e. $A \cap B \cap C = \{1\}$

Exercise 3.3.3:

a. $A_i = \left\{i^0, i^1, i^2\right\}$

$\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5 = \left\{x\colon x \in A_i \; for\ all\ i\ such\ that\ 2 \le x \le 5\right\}$

---

b.

$\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5 = \left\{x\colon x \in A_i \; for\ some\ i\ such\ that\ 2 \le x \le 5\right\}$

---

e.

$\bigcap_{i=1}^{100} C_i = C_1 \cap C_2 \cap C_3 \cap \ldots \cap C_{100} = \left\{x\colon x \in C_i \; for\ all\ i\ such\ that\ \frac{-1}{i} \le x \le \frac{1}{i}\right\}$

---

f.

$$\bigcup_{i=1}^{100} C_i = C_1 \cup C_2 \cup C_3 \cup \ \dots \ \cup C_{100} = \left\{x:\ x \in C_i \ for\ some\ i\ such\ that\ \frac{-1}{i} \leq x \leq \frac{1}{i}\right\}$$

<u>Exercise 3.3.4</u>:

b. $P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

---

d. $P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

<u>Question 10</u>:

<u>Exercise 3.5.1</u>:

b. $\in B\ X\ A\ X\ C\ = \{foam,\ tall,\ non - fat\}$

---

c.
$B\ X\ C\ = \{(foam,\ non - fat),\ (foam,\ whole), (no - foam, non - fat), (no - foam,\ whole)\}$

<u>Exercise 3.5.3</u>:

b. True.

$Z^2 \subseteq R^2$

C. False.

$Z^2 \cap Z^3 = \varnothing$

<u>Explanation</u>:

$8^2 = 4^3$

Therefore, $Z^2 \cap Z^3 \neq \varnothing$

---

e. True.

For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.

Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ and $C = \{1, 2, 3, 4\}$.

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)

Exercise 3.5.6:

d. $S = \{xy\colon\ where\ x \in \{0\} \cup \{0\}^2\ and\ y \in \{1\} \cup \{1\}^2\}$

Solution:

Let $x = \{0, 00\}$ and $y = \{1, 11\}$

	1	11
0	(0,1)	(0,11)
00	(00,1)	(00,11)

$S = \{0, 1, 00, 11\}$

---

e. $\{xy\colon\ x \in \{aa,\ ab\}\ and\ y \in \{a\} \cup \{a\}^2\}$

Solution:

Let $x = \{aa,\ ab\}$ and $y = \{a,\ aa\}$

	a	aa
aa	aaa	aaaa
ab	aba	abaa

$S = \{aaa, aba, abaa, aaaaa\}$

Exercise 3.5.7:

c. $(A \times B) \cup (A \times C) = \{\{aa\}, \{ab\}, \{ac\}, \{ad\}\}$

---

f. $P(A \times B) = \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$

---

g.
$P(A) \times P(B) = \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

<u>Question 11</u>:

<u>Exercise 3.6.2</u>:

b. $(B \cup A) \cap (\overline{B} \cup A) \;=\; A$

Line Number	Symbols	Law Applied
1	$(B \cup A) \cap (\overline{B} \cup A)$	Hypothesis
2	$(A \cup B) \cap (\overline{B} \cup A)$	Commutative Law, 1
3	$A \cup (B \cap \overline{B}) \cup A$	Associative Law, 2
4	$A \cup \emptyset \cup A$	Complement Law, 3
5	$A \cup A$	Identity Law, 4
6	A	Idempotent Law, 5

---

c. $\overline{A \cap \overline{B}} = \overline{A} \cup B$

Line Number	Symbols	Law Applied
1	$\overline{A \cap \overline{B}}$	Hypothesis
2	$\overline{A} \cup \overline{\overline{B}}$	DeMorgan’s Law,1
3	$\overline{A} \cup B$	Double Complement Law, 2

<u>Exercise 3.6.3</u>:

b. $A \;-\; (B \cap A) \;=\; A$

<u>Solution</u>:

Let $A = \{1, 2\}$ and $B = \{1\}$

$(B \cap A) = \{1\}$

$A - \{1\} = \{1, 2\} - \{1\} = \{2\}$

$A - (B \cap A) = \{2\} \neq \{1, 2\}$

In conclusion, $\{2\} \neq A$.

---

d. $(B - A) \cup A = A$

<u>Solution</u>:

Let $A = \{2\}$ and $B = \{2, 3, 4\}$

$(B - A) = (\{2, 3, 4\} - \{2\}) = \{3, 4\}$

$\{3, 4\} \cup A = \{3, 4\} \cup \{2\}$

$(B - A) \cup A = \{2, 3, 4\} \neq \{2\}$

In conclusion, $\{2, 3, 4\} \neq A$.

<u>Exercise 3.6.4</u>:

b. $A \cap (B - A) = \emptyset$

Line Number	Symbols	Law Applied
1	$A \cap (B - A)$	Hypothesis
2	$A \cap (B \cap \overline{A})$	Subtraction Law, 1
3	$A \cap (\overline{A} \cap B)$	Commutative Law, 2

4	$(A \cap \overline{A}) \cap B)$	Associate Law, 3
5	$\varnothing \cap B$	Complement Law, 4
6	$\varnothing$	Domination Law, 5

---

c. $A \cup (B - A) = A \cup B$

Line Number	Symbols	Law Applied
1	$A \cup (B - A)$	Hypothesis
2	$A \cup (B \cap \overline{A})$	Subtraction Law, 1
3	$(A \cup B) \cap (A \cup \overline{A})$	Distributive Law, 2
4	$(A \cup B) \cap U$	Complement Law, 3
5	$A \cup B$	Identity Law, 4
