Practice Exam 1

Tables Provided on Exam 1

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	p ∨ p ≡ p	p ∧ p ≡ p
Associative laws:	(p ∨ q) ∨ r ≡ p ∨ (q ∨ r)	(p ∧ q) ∧ r ≡ p ∧ (q ∧ r)
Commutative laws:	p ∨ q ≡ q ∨ p	p ∧ q ≡ q ∧ p
Distributive laws:	p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)	p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)
Identity laws:	p ∨ F ≡ p	p ∧ T ≡ p
Domination laws:	p ∧ F ≡ F	p ∨ T ≡ T
Double negation law:	¬¬p ≡ p	
Complement laws:	p ∧ ¬p ≡ F ¬T ≡ F	p ∨ ¬p ≡ T ¬F ≡ T
De Morgan's laws:	¬(p ∨ q) ≡ ¬p ∧ ¬q	¬(p ∧ q) ≡ ¬p ∨ ¬q
Absorption laws:	p ∨ (p ∧ q) ≡ p	p ∧ (p ∨ q) ≡ p
Conditional identities:	p → q ≡ ¬p ∨ q	p ↔ q ≡ (p → q) ∧ (q → p)

Table 1.12.1: Rules of inference known to be valid arguments.

Rule of inference	Name
p p → q ∴ q	Modus ponens
¬q p → q ∴ ¬p	Modus tollens
p ∴ p ∨ q	Addition
p ∧ q ∴ p	Simplification

Rule of inference	Name
p q ∴ p ∧ q	Conjunction
p → q q → r ∴ p → r	Hypothetical syllogism
p ∨ q ¬p ∴ q	Disjunctive syllogism
p ∨ q ¬p ∨ r ∴ q ∨ r	Resolution

Table 1.13.1: Rules of inference for quantified statements

Rule of Inference	Name
c is an element (arbitrary or particular) $\forall x\ P(x)$ ∴ P(c)	Universal instantiation
c is an arbitrary element P(c) ∴ $\forall x\ P(x)$	Universal generalization
$\exists x\ P(x)$ ∴ (c is a particular element) ∧ P(c)	Existential instantiation*
c is an element (arbitrary or particular) P(c) ∴ $\exists x\ P(x)$	Existential generalization

Table 3.6.1: Set identities.

Name	Identities	
Idempotent laws	A ∪ A = A	A ∩ A = A
Associative laws	(A ∪ B) ∪ C = A ∪ (B ∪ C)	(A ∩ B) ∩ C = A ∩ (B ∩ C)
Commutative laws	A ∪ B = B ∪ A	A ∩ B = B ∩ A
Distributive laws	A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)	A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
Identity laws	A ∪ ∅ = A	A ∩ U = A
Domination laws	A ∩ ∅ = ∅	A ∪ U = U
Double Complement law	$\overline{\overline{A}} = A$	
Complement laws	A ∩ $\overline{A}$ = ∅ $\overline{U}$ = ∅	A ∪ $\overline{A}$ = U $\overline{\varnothing}$ = U
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	A ∪ (A ∩ B) = A	A ∩ (A ∪ B) = A

Set Theory

Given:

A = { 1, {2}, {{3, 4}} }
For each of the following statements, state whether they are **true** or **false.**

      a. $1 \in A$
      b. $1 \subseteq A$
      c. $\{2\} \in A$
      d. $\{2\} \subseteq A$
      e. $\{3, 4\} \in A$
      f. $\{3, 4\} \subseteq A$
      g. $\{\{3,4\}\} \in A$
      h. $\{\{3,4\}\} \subseteq A$
      i. $\emptyset \in A$
      j. $\emptyset \subseteq A$

1.2 Let A = {1, 2, 3, 4}. Select the statement that is **false**.

    a. $\emptyset \in P(A)$
    b. $\emptyset \subseteq P(A)$
    c. $\{2, 3\} \in A$
    d. $\{2, 3\} \subseteq A$

Functions

Choose the property for which the function satisfies if well defined.

        a. Neither one-to-one, nor onto

        b. One-to-one, but not onto

        c. Onto, but not one-to-one

        d. one-to-one and onto

        e. not well defined

Given a function whose domain is the **set of all integers and whose target is the set of all positive integers:**

a)

$$f(x) \;=\; 2x + 1$$

b)

$$f(x) \;=\; |x| + 1$$

c)

$$f(x) \;=\; x^\wedge 2 + 1$$

d)

$$f(x) \;=\; \{(x > 0: 2x + 1) \wedge (x <= 0: -2x)\}$$

e)

$$f(x) \;=\; \{(x >= 0: 2x + 1) \wedge (x < 0: -2x + 2)\}$$

Proofs

4.1 Direct Proof
Prove that the product of two odd integers is an odd integer.

4.2 Proof by Contrapositive
Prove that if n^2 is even, then n is even.

Consider the contraposition of the proposition which is $\sim q \rightarrow \sim p$. Show that if n is odd, then n^2 is odd.

4.2 Proof by Contradiction
Prove by contradiction that if 3n+5 is odd, then n is even.
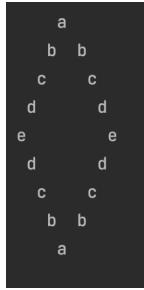
Number Systems Conversion

5.1 Decimal to 8-bit Two's Complement

$(-43)_{10} =$

5.2 Binary to Hexadecimal

$(110011100)_2 =$

Coding

```
    a
   b   b
  c     c
 d       d
e         e
 d       d
  c     c
   b   b
    a
```

Make a hollowed-out diamond made up of in-order alphabet letters. An example is below when n=5.

Challenge Question

For this question, you have to create the following Barn Door shape with `n=10`:

```
##########
#$        $#
# $      $ #
#  $    $  #
#   $  $   #
#   $$    #
#   $$    #
#  $  $   #
# $      $ #
#$        $#
##########
```