Question 1:

A. Convert the following numbers to their decimal representation. Show your work:

1. 100110111_2 = 154_{10}

Explanation:

To reach the conclusion, 100110111_2 = 154_{10}, I took each character of the binary number 10011011 and found the index of each character displayed in the "Index" row of the following table.

Character	1	0	0	1	1	0	1	1
Index	7	6	5	4	3	2	1	0
Power	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Finally I constructed the following formula (if the original character was a 1 then it is included in the formula, else it is omitted):

$$(1 * 2^0) + (1 * 2^1) + (1 * 2^3) + (1 * 2^4) + (1 * 2^7) = 154$$

---

2. 456_7 = 237_{10}

Explanation:

Following similar logic to previous questions, I assigned an index to each character in the 456_7 string seen in the table below.

For this scenario, we use a base of 7:

Character	4	5	6
Index	2	1	0
Power	7^2	7^1	7^0

Finally I constructed the following formula:

$$(6 \ * 7^{0}) \ + \ (5 \ * 7^{1}) \ + \ (4 \ * 7^{3}) \ = 237$$

---

3. $38A_{16}$ = 906_{10}

<u>Explanation</u>:

Following similar logic to previous questions, I assigned an index to each character in the $38A_{16}$ string seen in the table below.

For this scenario, we use a base of 16. In the hexadecimal system the letter “A” carries a value of 10, in this scenario A = 10.

Character	3	8	A
Index	2	1	0
Power	16^{2}	16^{1}	16^{0}

Finally I constructed the following formula:

$$(10 \ * 16^{0}) \ + \ (8 \ * 16^{1}) \ + \ (3 \ * 16^{2}) \ = 906$$

---

4. 2214_{5} = 310_{10}

<u>Explanation</u>:

Following similar logic to previous questions, I assigned an index to each character in the 2214_{5} string seen in the table below.

For this scenario, we use a base of 5.

Character	2	2	1	4
Index	3	2	1	0
Power	5^3	5^2	5^1	5^0

Finally I constructed the following formula:

$$(4 * 5^0) + (1 * 5^1) + (2 * 5^2) + (2 * 5^3) = 310$$

B. Convert the following numbers to their binary representation.

To find the results for following question(s), I used the method below:

(1) Take the intended number to convert and divide it by 2 (i.e $\frac{69}{2}$).

(2) If the value has a remainder (i.e $\frac{69}{2}$ = 34.5) this would result in a 1 (seen in the table below).

(3) If the value had NO remainder (i.e $\frac{34}{2}$ = 17) that would result in a 0.

(4) Repeat steps (2) and (3) until $\frac{1}{2}$.

(5) Concatenate the binary values from bottom to top (or reverse the final column).

---

1. 69_{10} = 01000101_2

<u>Explanation</u>:

Equation	Quotient	Result
$\frac{69}{2}$	34.5	1

$\frac{34}{2}$	17	0
$\frac{17}{2}$	8.5	1
$\frac{8}{2}$	4	0
$\frac{4}{2}$	2	0
$\frac{2}{2}$	1	0
$\frac{1}{2}$	0.5	1

The result is the last column from bottom to top: 01000101

---

2. 485_{10} = 1010101101_2

<u>Explanation</u>:

Equation	Quotient	Result
$\frac{485}{2}$	242.5	1
$\frac{242}{2}$	171	0
$\frac{171}{2}$	85.5	1
$\frac{85}{2}$	42.5	1
$\frac{42}{2}$	21	0
$\frac{21}{2}$	10.5	1
$\frac{10}{2}$	5	0

$\frac{5}{2}$	2.5	1
$\frac{2}{2}$	1	0
$\frac{1}{2}$	0	1

The result is the last column from bottom to top: 1010101101

---

3. $6D1A_{16}$ = 0110110100011010_2

<u>Explanation</u>:

To find the binary representation of $6D1A_{16}$ I wrote out the following table that corresponds to all possible hexadecimal values in binary:

Hexadecimal Value	Binary Value
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010

B	1011
C	1100
D	1101
E	1110
F	1111

Based on the table above, I found the values for 6, D, 1 and A respectively. These being:

Hexadecimal Value	Binary Value
6	0110
D	1101
1	0001
A	1010

Taking the first 4 digits of each value and concatenating them I found the result to be 0110110100011010.

C. Convert the following numbers to their hexadecimal representation.

1. 1101011_2 = $6B_{16}$

Explanation:

Using the table from above I found the binary values for the hexadecimal characters 6 and B seen below:

Hexadecimal Value	Binary Value
6	0110

A	1010

Taking each value and concatenating them together I found the result to be 6A.

---

2. 895_{10} = $E84_{16}$

Explanation:

To arrive at 895_{10} = $E84_{16}$, I first converted 895_{10} into its binary representation displayed in the table below:

Equation	Quotient	Result
$\frac{895}{2}$	447.5	1
$\frac{447}{2}$	223.5	1
$\frac{223}{2}$	111.5	1
$\frac{111}{2}$	55.5	1
$\frac{55}{2}$	27.5	1
$\frac{27}{2}$	13.5	1
$\frac{13}{2}$	6.5	1
$\frac{6}{2}$	3	0
$\frac{3}{2}$	1.5	1
$\frac{1}{2}$	0.5	1

Taking each value and concatenating them together I found the binary representation of 895_{10} = 1101111111_2

Firstly, I broken 1101111111_2 into bits of 4. They are presented below:

- 0011 (Prepended two zeros to make the value 4 bits)
- 0111
- 1111

Then I converted 1101111111_2 to hexadecimal using the table above:

Hexadecimal Value	Binary Value
3	0011
8	0111
F	1111

The final result is 38F.

Question 2:

A. Solve the following, do all calculations in the given base. Show your work:

1. $7566_8 + 4515_8 =$

Explanation:

$$\begin{array}{r} 111 \\ 7566_8 \\ +\ 4515_8 \\ 14303_8 \end{array}$$

---

2. $10110011_2 + 1101_2 =$

Explanation:

$$\begin{array}{r} 111111 \\ 10110011_2 \\ +\quad 1101_2 \\ 11000000_2 \end{array}$$

---

3. $7A66_{16} + 45C5_{16} =$

Explanation:

$$\begin{array}{r} 11 \\ 7A66_{16} \\ +\ 45C5_{16} \\ C02B_{16} \end{array}$$

---

4. $3022_5 + 2433_5 =$

<u>Explanation</u>:

$$\begin{array}{r} 111 \\ 3022_5 \\ +\ 2433_5 \\ 11010_5 \end{array}$$

<u>Question 3</u>:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work:

To find the results for following question(s), I used the method below:

(1) Take the intended number to convert and divide it by 2 (i.e $\frac{69}{2}$).

(2) If the value has a remainder (i.e $\frac{69}{2}$ = 34.5) this would result in a 1 (seen in the table below).

(3) If the value had NO remainder (i.e $\frac{34}{2}$ = 17) that would result in a 0.

(4) Repeat steps (2) and (3) until $\frac{1}{2}$.

(5) Concatenate the binary values from bottom to top (or reverse the final column).

---

1. 124_{10} = 01111100_{8}

<u>Explanation</u>:

Equation	Quotient	Result
$\frac{124}{2}$	62	0
$\frac{62}{2}$	31	0
$\frac{31}{2}$	15.5	1
$\frac{15}{2}$	7.5	1
$\frac{7}{2}$	3.5	1
$\frac{3}{2}$	1.5	1

$\frac{1}{2}$	0.5	1

Finally the result is the last column from bottom to top, pre-pending a 0: 01111100

---

2. $-\ 124_{10}$ = 10000100_{8}

<u>Explanation</u>:

Equation	Quotient	Result
$\frac{124}{2}$	62	0
$\frac{62}{2}$	31	0
$\frac{31}{2}$	15.5	1
$\frac{15}{2}$	7.5	1
$\frac{7}{2}$	3.5	1
$\frac{3}{2}$	1.5	1
$\frac{1}{2}$	0.5	1

The result is the last column from bottom to top, pre-pending a 0: 01111100
We see the the initial value ($-\ 124_{10}$) is negative and follow the steps below:

(1) Flip the bits
(2) Add 1 to the result from Step (1)

Original Bits	0	1	1	1	1	1	0	0
Flipped Bits	1	0	0	0	0	0	1	1

Add 1	1	0	0	0	0	1	0	0

The result is: 10000100

---

3. $109_{10} = 01101101_{8}$

Explanation:

Equation	Quotient	Result
$\frac{109}{2}$	54.5	1
$\frac{54}{2}$	27	0
$\frac{27}{2}$	13.5	1
$\frac{13}{2}$	6.5	1
$\frac{6}{2}$	3	0
$\frac{3}{2}$	1.5	1
$\frac{1}{2}$	0.5	1

Finally the result is the last column from bottom to top, pre-pending a 0: 01101101

---

4. $-79_{10} = 10110011_{8}$

Explanation:

Equation	Quotient	Result
$\frac{79}{2}$	38.5	1
$\frac{38}{2}$	19	0
$\frac{19}{2}$	9.5	1
$\frac{9}{2}$	4.5	1
$\frac{4}{2}$	2	0
$\frac{2}{2}$	1	0
$\frac{1}{2}$	0.5	1

The result is the last column from bottom to top, pre-pending a 0: 01001101
We see the the initial value ($-\ 124_{10}$) is negative and follow the steps below:

(1) Flip the bits
(2) Add 1 to the result from Step (1)

Original Bits	0	1	0	0	1	1	0	1
Flipped Bits	1	0	1	1	0	0	1	0
Add 1	1	0	1	1	0	0	1	1

The result is: 10110011

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work:

1. $00011110_{8\ bit\ 2's\ comp}$ = 30_{10}

Explanation:

Looking at the MSB (Most Significant Bit) to the far left (0), we know that the decimal representation will be a positive number. Therefore, we use the following table to find 00011110's decimal representation:

Digit	0	0	0	1	1	1	1	0
Index	7	6	5	4	3	2	1	0
Power	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Finally I constructed the following formula:

$$(1 * 2^4) + (1 * 2^3) + (1 * 2^2) + (1 * 2^1) + (1 * 2^0) = 30$$

---

2. $11100110_{8\ bit\ 2's\ comp} = -26_{10}$

Explanation:

Looking at the MSB (Most Significant Bit) to the far left (1), we know the value is negative and follow the procedure below:

(1) Flip the bits
(2) Add 1 to the result from Step (1)
(3) Take the result from Step (2) and convert it's value to a decimal value (base 10)
(4) Multiply the result from Step (3) by (-1)

Original Bits	1	1	1	0	0	1	1	1
Flipped Bits	0	0	0	1	1	0	0	0
Add 1	0	0	0	1	1	0	1	0

Digit	0	0	0	1	1	0	1	0
Index	7	6	5	4	3	2	1	0
Power	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Finally I constructed the following formula:

$$(1 * 2^4) + (1 * 2^3) + (1 * 2^1) = 26 * (-1) = -26$$

---

3. $00101101_{8\ bit\ 2's\ comp}$ = 45_{10}

<u>Explanation</u>:

Looking at the MSB (Most Significant Bit) to the far left (0), we know that the decimal representation will be a positive number. Therefore, we use the following table to find 00101101 decimal representation:

Digit	0	0	1	0	1	1	0	1
Index	7	6	5	4	3	2	1	0
Power	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Finally I constructed the following formula:

$$(1 * 2^5) + (1 * 2^3) + (1 * 2^2) + (1 * 2^0) = 45$$

---

4. $10011110_{8\ bit\ 2's\ comp}$ = -98_{10}

<u>Explanation</u>:

Looking at the MSB (Most Significant Bit) to the far left (1), we know the value is negative and follow the procedure below:

(1) Flip the bits
(2) Add 1 to the result from Step (1)
(3) Take the result from Step (2) and convert it's value to a decimal value (base 10)
(4) Multiply the result from Step (3) by (-1)

Original Bits	1	0	0	1	1	1	1	0
Flipped Bits	0	1	1	0	0	0	0	1
Add 1	0	1	1	0	0	0	1	0

Digit	0	1	1	0	0	0	1	0
Index	7	6	5	4	3	2	1	0
Power	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Finally I constructed the following formula:

$$(1 * 2^6) + (1 * 2^5) + (1 * 2^1) = 98 * (-1) = -98$$

Question 4:

(1) Exercise 1.2.4: Questions b,c:

Write a truth table for each expression:

b. $\neg (p \lor q)$

Explanation:

p	q	$(p \lor q)$	$\neg (p \lor q)$
T	T	T	F
T	T	T	F
T	F	T	F
T	F	T	F
F	T	T	F
F	T	T	F
F	F	F	T
F	F	F	T

Write a truth table for each expression:

c. $r \lor (p \land \neg q)$

Explanation:

p	q	r	$\neg q$	$(p \land \neg q)$	$r \lor (p \land \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T

T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

---

(2) Exercise 1.3.4: Questions b,d:

Give a truth table for each expression.

b. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

Truth Table:

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	T	T	T	T
T	F	F	T	T
T	F	F	T	T
F	T	T	F	F
F	T	T	F	F
F	F	T	T	T
F	F	T	T	T

Give a truth table for each expression.

d. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

<u>Truth Table</u>:

p	q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	T	T	F	T
T	F	F	T	T
T	F	F	T	T
F	T	F	T	T
F	T	F	T	T
F	F	T	F	T
F	F	T	F	T

Question 5:

Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions:

(1) Exercise 1.2.7: Questions b,c:

b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

Result: $(b \land d) \lor (b \land m) \lor (d \land m)$

---

c. Applicant must present either a birth certificate or both a driver's license and a marriage license.

Result: $b \lor (d \land m)$

---

(2) Exercise 1.3.7: Questions b-d:

Define the following propositions:

s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot

Express each of the following English sentences with a logical expression:

b. A person can park in the school parking lot if they are a senior or at least seventeen years of age.

Result: $p \rightarrow (s \lor y)$

---

c. Being 17 years of age is a necessary condition for being able to park in the school parking lot.

Result: $y \leftrightarrow p$

---

d. A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

Result: $p \leftrightarrow (s \wedge y)$

---

e. Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

Result: $p \leftrightarrow (s \vee y)$

---

(1) Exercise 1.3.9: Questions c,d:

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

- y: the applicant is at least eighteen years old
- p: the applicant has parental permission
- c: the applicant can enroll in the course

c. The applicant can enroll in the course only if the applicant has parental permission.

Result: $c \leftrightarrow p$

---

d. Having parental permission is a necessary condition for enrolling in the course.

Result: $p \leftrightarrow c$

<u>Question 6</u>:

(1) <u>Exercise 1.3.6</u>: <u>Questions b-d</u>:

b. Maintaining a B average is necessary for Joe to be eligible for the honors program.

<u>Result</u>:

If Joe maintains a B average, Joe will be eligible for the honors program.

---

c. Rajiv can go on the roller coaster only if he is at least four feet tall.

<u>Result</u>:

If Rajiv is at least four feet tall, then he can go on the roller coaster.

---

d. Rajiv can go on the roller coaster if he is at least four feet tall.

<u>Result</u>:

If Rajiv is at least four feet tall, then he can go on the roller coaster.

---

(2) <u>Exercise 1.3.10</u>: <u>Questions c-d</u>:

c. $(p \lor r) \leftrightarrow (q \land r) = False$

<u>Explanation & Result</u>:

Let p = True and q = False.

$$(p \lor r) \leftrightarrow (q \land r) =$$
$$(T \lor r) \leftrightarrow (F \land r)$$

With a biconditional if the values on both sides of the expression are not equal then the expression evaluates to false.

Therefore, regardless of the value of “r” in the expression $(F \wedge r)$, the entire expression will always evaluate to False.

---

d. $(p \wedge r) \leftrightarrow (q \wedge r) = False$

<u>Explanation & Result</u>:

Let p = True and q = False.

$$(p \wedge r) \leftrightarrow (q \wedge r) =$$
$$(T \wedge r) \leftrightarrow (F \wedge r)$$

With a biconditional if the values on both sides of the expression are not equal then the expression evaluates to false.

Therefore, regardless of the value of “r” in the expressions $(T \wedge r)$ and $(F \wedge r)$, the entire expression will always evaluate to False.

---

e. $p \rightarrow (r \vee q) = Unknown$

<u>Explanation & Result</u>:

Let p = True and q = False.

$$p \rightarrow (r \vee q) =$$
$$T \rightarrow (r \vee F)$$

In this scenario, the only way in which the full expression evaluates to False is if r = False. If r = True then the full expression evaluates to True

Therefore, we would need to know the value of “r” to evaluate the expression. The truth value of the expression is Unknown.

---

f. $(p \wedge q) \rightarrow r = True$

<u>Explanation & Result</u>:

Let p = True and q = False.

$$(p \ \wedge \ q) \ \rightarrow r \ =$$
$$(T \ \wedge \ F) \ \rightarrow r \ =$$
$$F \rightarrow r \ =$$

In this scenario, the expression $(T \ \wedge \ F)$ always evaluates to false.

Therefore, regardless of the value of “r” the full expression will always evaluate to True.

<u>Question 7</u>:

Define the following propositions:

- j: Sally got the job.
- l: Sally was late for her interview
- r: Sally updated her resume.

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

(1) Exercise 1.4.5: Questions b-d:

b. If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

Solution: Not logically equivalent.

Explanation:

If Sally did not get the job, then she was late for her interview or did not update her resume =

$$\neg j \rightarrow (l \vee \neg r)$$

If Sally updated her resume and was not late for her interview, then she got the job =

$$(r \wedge l) \rightarrow j$$

j	l	r	$\neg j \rightarrow (l \vee \neg r)$	$(r \wedge l) \rightarrow j$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T	T	F
F	T	F		
F	F	T		

F	F	F		

Looking at the row colored in red, you can see that there is a scenario in which the expressions $\neg j \rightarrow (l \vee \neg r)$ and $(r \wedge l) \rightarrow j$ are not logically equivalent.

---

c. If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

Solution: Not logically equivalent.

Explanation:

If Sally got the job then she was not late for her interview.

$$j \rightarrow \neg l$$

If Sally did not get the job, then she was late for her interview.

$$\neg j \rightarrow l$$

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
T	T	F	T
T	F		
F	T		
F	F		

Looking at the row colored in red, you can see that there is a scenario in which the expressions $j \rightarrow \neg l$ and $\neg j \rightarrow l$ are not logically equivalent.

---

d. If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

Solution: Not logically equivalent.

Explanation:

If Sally updated her resume or she was not late for her interview, then she got the job.

$$(r \lor l) \rightarrow j$$

If Sally got the job, then she updated her resume and was not late for her interview.

$$j \rightarrow (r \land \neg l)$$

j	l	r	$(r \lor l) \rightarrow j$	$j \rightarrow (r \land \neg l)$
T	T	T		
T	T	F	T	F
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Looking at the row colored in red, you can see that there is a scenario in which the expressions $(r \lor l) \rightarrow j$ and $j \rightarrow (r \land \neg l)$ are not logically equivalent.

Question 8:

(1) Exercise 1.5.2: Questions c,f,i:

c. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

Explanation & Result:

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r) \quad =$$
$$(\neg p \ \vee \ q) \ \wedge \ (\neg p \ \vee \ r) \equiv \neg p \ \vee \ (q \ \wedge \ r) \quad Conditional\ Law\ (both\ sides)$$
$$\neg p \ \vee \ (q \ \wedge \ r) \equiv \neg p \ \vee \ (q \ \wedge \ r) \quad Distributive\ Law\ (left\ side)$$

---

f. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Explanation & Result:

$$\neg(p \vee (\neg p \wedge q)) \ \equiv \ \neg p \wedge \neg q \quad =$$
$$\neg p \ \vee \ \neg(\neg p \ \wedge \ q) \quad DeMorgan's\ Law$$
$$\neg p \ \vee \ (\neg\neg p \ \vee \ \neg q) \quad DeMorgan's\ Law$$
$$\neg p \ \vee \ (p \ \vee \ \neg q) \quad Double\ Negation\ Law$$
$$(\neg p \ \vee \ p) \ \vee \ \neg q \quad Associate\ Law$$
$$(p \ \wedge \ \neg p) \ \vee \ \neg q \quad Commutative\ Law$$
$$\neg p \ \vee \ \neg q \quad Complement\ Law$$

$$\neg p \ \vee \ \neg q \ \equiv \ \neg p \wedge \neg q$$

---

i. $(p \wedge q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow \neg q$

Explanation & Result:

$$(p \wedge q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow \neg q \quad =$$
$$(p \wedge q) \rightarrow r \quad = \ (Right\ Side)$$
$$(\neg p \wedge \neg q) \ \vee \ r \quad Conditional\ Law$$
$$\neg p \ \vee \ (\ \neg q \ \wedge \ r) \quad Associative\ Law$$
$$\neg p \ \vee \ (\ r \ \wedge \ \neg q) \quad Communicative\ Law$$
$$(\neg p \wedge \ r) \ \vee \ \neg q) \quad Associative\ Law$$

$$(p \wedge \neg r) \rightarrow \neg q \quad = \quad (Left\ Side)$$
$$(\neg p \wedge \neg\neg r) \ \vee \ \neg q \quad Conditional\ Law$$
$$(\neg p \wedge r) \ \vee \ \neg q \quad Double\ Negation\ Law$$

$$(\neg p \wedge r) \ \vee \ \neg q \equiv (\neg p \wedge r) \ \vee \ \neg q$$

---

(2) Exercise 1.5.3: Questions c,d:

c. ¬r ∨ (¬r → p)

Explanation & Result:

p	r	$\neg r$	$(\neg r \rightarrow p)$	$\neg r \vee (\neg r \rightarrow p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T

Looking at the last column colored in green you will see that the expression always evaluates to True (a Tautology).

---

d. ¬(p → q) → ¬q

Explanation & Result:

p	q	$\neg q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

Looking at the last column colored in green you will see that the expression always evaluates to True (a Tautology).

Question 9:

(1) Exercise 1.6.3: Questions c,d:

c. There is a number that is equal to its square.

Result: $\exists x\,(x = x^2)$

---

d. Every number is less than or equal to its square plus 1.

Result: $\forall x(x \leq x^2 + 1)$

---

(2) Exercise 1.7.4: Questions b-d:

In the following question, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

S(x): x was sick yesterday
W(x): x went to work yesterday
V(x): x was on vacation yesterday

Translate the following English statements into a logical expression with the same meaning.

b. Everyone was well and went to work yesterday.

Result: $\forall x(\neg S(x) \land w(x))$

---

c. Everyone who was sick yesterday did not go to work.

Result: $\forall x(S(x) \rightarrow \neg w(x))$

---

d. Yesterday someone was sick and went to work.

<u>Result</u>: $\exists(S(x) \land w(x))$

Question 10:

(1) Exercise 1.7.9: Questions c-i:

c. $\exists x((x = c) \rightarrow P(x))$ = Evaluates to True

Explanation & Result:

Let P(a) = True and c = False

$$\exists x((x = c) \rightarrow P(x)) =$$
$$\exists x((T = F) \rightarrow T) =$$
$$\exists x(F \rightarrow T) =$$
$$T$$

---

d. $\exists x(Q(x) \land R(x))$ = Evaluates to True

Explanation & Result:

Let Q(e) = True and R(e) = True

$$\exists x(Q(e) \land R(e)) =$$
$$\exists x(T \land T) =$$
$$T$$

---

e. $Q(a) \land P(d)$ = Evaluates to True

Explanation & Result:

Let Q(a) = True and P(d) = True

$$Q(a) \land P(d) =$$
$$T \land T =$$
$$T$$

---

f. $\forall x\ ((x \neq b) \rightarrow Q(x))$ = Evaluates to True

<u>Explanation & Result</u>:

Let Q(b) = False

$$\forall x\ ((b \neq b) \rightarrow Q(b)) =$$
$$\forall x\ ((F \neq F) \rightarrow F) =$$
$$\forall x\ (F \rightarrow F) =$$
$$T$$

---

g. $\forall x\ (P(x) \lor R(x))$ = Evaluates to False

<u>Explanation & Result</u>:

Let P(c) = False and R(c) = False

$$\forall x\ (P(c) \lor R(c)) =$$
$$\forall x\ (F \lor F) =$$
$$F$$

---

h. $\forall x\ (R(x) \rightarrow P(x))$ = Evaluates to True

<u>Explanation & Result</u>:

Let R(e) = True and P(e) = True

$$\forall x\ (R(e) \rightarrow P(e)) =$$
$$\forall x\ (T \rightarrow T) =$$
$$T$$

There is never a scenario in the table in which R(x) = T and P(x) = F resulting in the expression $\forall x\ (R(x) \rightarrow P(x))$ always equally True.

---

i. $\exists x(Q(x) \lor R(x))$

<u>Explanation & Result</u>:

Let Q(a) = True and R(a) = True

$$\exists x(Q(a) \lor R(a)) =$$
$$\exists x(T \lor T) =$$
$$T$$

---

(2) Exercise 1.9.2: Questions b-i:

b. $\exists x \ \forall y \ Q(x, y)$ = True

Explanation & Result:

Let x = 2

When x = 2, the only values “y” can be equal to are the following:

- 1 = T
- 2 = T
- 3 = T

Therefore, when x = 2, $\exists x \ \forall y \ Q(x, y)$ is True.

---

c. $\exists y \ \forall x \ P(x, y)$ = True

Explanation & Result:

Let y = 1

When y = 1, the only values “x” can be equal to are the following:

- 1 = T
- 2 = T
- 3 = T

Therefore, when y = 1, $\exists y \ \forall x \ P(x, y)$ is True.

---

d. ∃x ∃y S(x, y) = False

Explanation & Result:

The entire “S” table only has False values.

Therefore, any combination of values for x and y in the expression, ∃x ∃y S(x, y), is False.

---

e. ∀x ∃y Q(x, y) = False

Explanation & Result:

Let x = 1

When x = 1, the only values “y” can be equal to are the following:

- 1 = F
- 2 = F
- 3 = F

Therefore, when x = 1, ∀x ∃y Q(x, y) is False.

---

f. ∀x ∃y P(x, y) = True

Explanation & Result:

For the “P” table “x” can only be a True value.

Therefore, “∃y” will choose the value that makes the expression, P(x, y), True (i.e. P(1,1)). The expression ∀x ∃y P(x, y) is True.

---

g. ∀x ∀y P(x, y) = False

Explanation & Result:

Let x = 1, y = 2

The universal quantifier “$\forall$” is actively trying to make the expression $\forall x\ \forall y\ P(x, y)$ evaluate to False.

Therefore, $P(1, 2)$ is False causing the expression $\forall x\ \forall y\ P(x, y)$ to be False.

---

h. $\exists x\ \exists y\ Q(x, y)$ = True

Explanation & Result:

Let $x = 2$, $y = 2$

The existential quantifier “$\exists$” is actively trying to make the expression $\exists x\ \exists y\ Q(x, y)$ evaluate to True.

Therefore, $Q(2, 2)$ is True causing the expression $\exists x\ \exists y\ Q(x, y)$ to be True.

---

i. $\forall x\ \forall y\ \neg S(x, y)$ = True

Explanation & Result:

The entire “S” table holds only False values. Any combination of x and y will initially evaluate to False and is then negated in the expression $\forall x\ \forall y\ \neg S(x, y)$.

Therefore, the expression $\forall x\ \forall y\ \neg S(x, y)$ will always evaluate to True.

Question 11:

(1) Exercise 1.10.4: Questions c-g:

Translate each of the following English statements into logical expressions. The domain is the set of all real numbers.

---

c. There are two numbers whose sum is equal to their product.

Result :$\exists x \, \exists y \, (x + y = x * y)$

---

d. The ratio of every two positive numbers is also positive.

Explanation & Result:

Let k equal some positive integer.

$$\forall x \, \forall y \, (\frac{x}{y} = 2k) \ \land \ (x = 2k \ \land \ y = 2k)$$

---

e. The reciprocal of every positive number less than one is greater than one.

Explanation & Result:

Let k equal some positive integer.

$$\forall x (\frac{x}{1} < 1 > 1) \ \land \ (x = 2k)$$

---

f. There is no smallest number.

Result: $\neg \exists x \, \forall y \, (y \geq x)$

---

g. Every number other than 0 has a multiplicative inverse.

Result: $\forall((x \neq 0) \rightarrow \frac{1}{x} > 0)$

---

(2) Exercise 1.10.7: Questions c-f:

The domain is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.

- P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)
- D(x): x missed the deadline.
- N(x): x is a new employee.

Give a logical expression for each of the following sentences.

---

c. There is at least one new employee who missed the deadline.

Explanation & Result:

$\exists x\, (N(x) \ \wedge \ D(x))$

There is at least one new employee = $\exists x\, (N(x)$

And they missed the deadline = $\wedge \ D(x)$

---

d. Sam knows the phone number of everyone who missed the deadline.

Explanation & Result:

Let S = Sam

$\forall y\, (P(S, y) \ \rightarrow \ D(y))$

If Sam knows the phone number = $P(S, y)$

Then it is the phone number of everyone who missed the deadline = $\forall y\, D(y)$

---

e. There is a new employee who knows everyone's phone number.

Explanation & Result:

$$\exists x \, \forall y \, (N(x) \ \land \ P(x, y))$$

There is a new employee = $\exists x \, N(x)$

And they know everyone's phone number = $\land \ \ \forall y \, P(x, y)$

---

f. Exactly one new employee missed the deadline.

Explanation & Result:

$$\forall x(\, N(x) \ \land \ D(x))$$

There is exactly one employee = $\forall x \, N(x)$

And they missed the deadline = $\land \ \ D(x)$

---

(3) Exercise 1.10.10: Questions c-f:

The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate T(x, y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

---

c. Every student has taken at least one class other than Math 101.

Explanation & Result:

Let z = Math 101

$$\forall x \, \exists y \, (T(x, y) \ \land \ (y \ \neq \ z))$$

Every student has taken at least one class = $\forall x \, \exists y \, (T(x, y))$

And z is not one of them = $\land \ \ (y \ \neq \ z)$

---

d. There is a student who has taken every math class other than Math 101.

Explanation & Result:

Let z = Math 101

$$\exists x\, \forall y\, (T(x, y) \ \wedge \ (y \ \neq \ z))$$
There is a student that has taken every math class = $\exists x\, \forall y\, (T(x, y))$
And z is not one of them = $\wedge \ (y \ \neq \ z)$

---

e. Everyone other than Sam has taken at least two different math classes.

Explanation & Result:

Let S = Sam

$$\forall x\, \forall y\, (T(x, y) \ \wedge \ (x \ \neq \ S) \ \wedge (y \ \neq \ y))$$
Everyone other than Sam = $\forall x \ T(x, y) \ \wedge \ (x \ \neq \ S)$
And they have taken at least two math classes = ∀y$(y \ \neq \ y)$

---

f. Sam has taken exactly two math classes.

Explanation & Result:

Let S = Sam

$$\exists y\, (S = y + 2)$$
Sam has taken some class = $\exists y\, (S = y)$
The number of classes being = $y + 2$

<u>Question 12</u>:

(1) <u>Exercise 1.8.2</u>: <u>Questions b-e</u>:

In the following question, the domain is a set of male patients in a clinical study. Define the following predicates:

P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

---

b. Every patient was given the medication or the placebo or both.

<u>Explanation & Result</u>:

$$\neg\exists x(\neg D(x) \land \neg P(x))$$

Some patients were not given the medication and the placebo

---

c. There is a patient who took the medication and had migraines.

<u>Explanation & Result</u>:

$$\neg\forall x(\neg D(x) \lor \neg M(x))$$

Every patient did not take the medication or did not have migraines

---

d. Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \rightarrow q \equiv \neg p \lor q$.)

<u>Explanation & Result</u>:

$$\neg\exists x(P(x) \land \neg Q(x))$$

Some patient took the placebo and did not have migraines

---

e. There is a patient who had migraines and was given the placebo.

Explanation & Result:

$$\neg\forall x(\neg M(x) \lor \neg(P(x))$$

Every patient did not have migraines or did not take the placebo

---

(2) Exercise 1.9.4: Questions c-e:

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

---

c. ∃ x ∀ y (P(x, y) → Q(x, y))

Result: $\neg\forall x\ \neg\exists y(P(x,y)\ \land\ \neg Q(x,y))\ Conditional\ Law$

---

d. ∃ x ∀ y (P(x, y) ↔ P(y, x))

Result: $\neg\exists x\ \neg\forall y(\neg P(x,y)\ \lor\ P(y,x)\ \ \land\ \ \neg P(y,x)\ \ \lor\ \ P(x,y))$

$$\exists x\ \forall y\ (P(x,\ y) \leftrightarrow P(y,\ x))\ \ =$$
$$P(x,y)\ \rightarrow\ P(y,x)\ \ \land\ \ P(y,x)\ \rightarrow\ P(x,y)\quad Conditional\ Law$$
$$\neg P(x,y)\ \ \lor\ \ P(y,x)\ \ \land\ \ \neg P(y,x)\ \ \lor\ \ P(x,y)\quad Conditional\ Law$$

---

e. ∃ x ∃ y P(x, y) ∧ ∀ x ∀ y Q(x, y)

Result: $\neg\forall x\ \neg\forall y\ \neg P(x,y)\ \lor\ \neg\exists x\ \neg\exists y Q(x,y)$
