Question 3:

Exercise 8.2.2:

b.

Solution:

Let $f(n) = n^3 + 3n^2 + 4$.

Taking the expression $n^3 + 3n^2 + 4$ we will determine $\Theta = (n^3)$ by finding $O(n^3)\ and\ \Omega(n^3)$.

To find $O(n^3)$ we use the following steps:

$$n^3 + 3n^2 + 4 \leq n^3 + 3n^3 + 4n^3$$
$$n^3 + 3n^2 + 4 \leq 8n^3$$

As a result $c_1 = 8$ (the leading coefficient on the right side of the inequality)and $O(n^3)$.

To find $\Omega(n^3)$ we use the following steps:

$$n^3 + 3n^2 + 4 \geq n^3$$

As a result $c_1 = 1$ (the leading coefficient on the right side of the inequality), $n_0 = 1$and $\Omega(n^3)$.

Therefore, $n^3 + 3n^2 + 4$ is $\Theta = (n^3)$. ■

Exercise 8.3.5:

a.

Solution:

In the initial while loop, the condition being checked is whether “i” is less than “j.”

```
i := 1
j := n
p := 0

While (i < j){
   …
}
```

Each of the two inner while loops are checking two conditions.

```
While (i < j and ai < p){
      i := i + 1
}

While (i < j and aj ≥ p){
      j := j - 1
}
```

The first inner while loop is checking whether “i” is less then “j” and if the i^{th} element in the collection is less than “p.” If **both** these conditions are true, “i” becomes its current value plus 1.

The second inner while loop is checking whether “i” is less then “j” and if the j^{th} element in the collection is greater than or equal to “p.” If **both** these conditions are true, “j” becomes it’s current value minus 1.

The conditional block in the outer while loop then checks the condition if “i” is less than “j” and swaps the current element of the collection at index “i” for the current element of the collection at index “j.”

```
If (i < j){
    swap ai and aj
}
```

Finally, the program returns the collection in its current state (altered or depending on the values in the collection un-altered).

```
Return( a1, a2,...,an )
```

---

b.

Solution:

```
i := i + 1
j := j - 1
```

For the operations above, the amount of times these operations are executed is contingent on both the length and contents of the collection.

For example:

Let i = 1, j = 2, p = 0 and a = {1, -2}.

The program will enter the outer while loop then the second while loop displayed below:

```
While (i < j and ai < p){
      i := i + 1
}
```

Substituting the relevant values:

```
While (1 < 2 and 1 < 0){
      i := i + 1
}
```

In this iteration the condition inside the while loop will not execute. If we were evaluating the second value in the collection (-2) then both the conditions would be met and the operations inside the while loop would be executed.

Same goes for the second inner while loop with a change in the second condition:

```
While (i < j and aj >= p){
      j := j - 1
}
```

Substituting the relevant values:

```
While (1 < 2 and 1 >= 0){
```

```
        j := j - 1
}
```

In this scenario for our first value in the collection (1), both conditions are met and the code within the while loop is executed.

Then we go to the outer while loop with the new values for “i” and “j”.

Since i = 1 and j = 1 the outer while loop breaks:

```
While (i < j){
    ...
}
```

This displays that the contents of the collection are significant.

If the length of the collection were say 3 (j = 3), we would have continued in the loop checking the conditions in both the while loops to see if either while loop is entered.

To maximize the number of executions, you would need to make the length of the collection as large as possible and all the elements be positive integers that are n, n+1, n+1 … ■

---

c.

Solution:

---

d.

Solution:

---

e.

Solution:

Question 4:

Exercise 5.1.2:

b.

Solution: $40^7 + 40^8 + 40^9$

---

c.

Solution: $(14 * 40^6) + (14 * 40^7) + (14 * 40^8)$

Exercise 5.3.2:

a.

Solution: $3 * 2^9 = 1536$

Explanation:

- Because the first character can be any character from the set $\{a, b, c\}$ and the letters cannot repeat, the next character in the set can only be the other two values that were not chosen to be the first character and so on.

Example Combination:

Character Index	Potential Character	Chosen Character	Num Possible Characters
0	$\{a, b, c\}$	$\{a\}$	3
1	$\{b, c\}$	$\{c\}$	2
2	$\{a, b\}$	$\{b\}$	2
3	$\{a, c\}$	$\{a\}$	2
4	$\{b, c\}$	$\{c\}$	2
5	$\{a, b\}$	$\{b\}$	2

6	$\{a, c\}$	$\{a\}$	2
7	$\{b, c\}$	$\{c\}$	2
8	$\{a, b\}$	$\{b\}$	2
9	$\{a, c\}$	$\{a\}$	2

Exercise 5.3.3:

b.

Solution: $10 * 26^4 * 9 * 8 = 329,022,720$

---

c.

Solution: $10 * 26 * 25 * 24 * 23 * 9 * 8 = 258,336,000$

Exercise 5.2.3:

a.

Solution:

---

b.

Solution: $2^9 = 512$

Question 5:

Exercise 5.4.2:

a.

Solution: $P(4, 3) * P(10, 4)$

Explanation:

$P(4, 3)$ = There are **"4"** different numbers that can be chosen $\{2, 4, 5, 8\}$ and can only be a length of **"3."**

$P(10, 4)$ = There are **"10"** different numbers that can be chosen $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and can only be a length of **"4."** (7 total digit, 4 digits remaining after the first 3 are defined)

---

b.

Solution: $2 * P(10, 4)$

$2 * P(10, 4)$ = There are **"2"** possible choices for the first three digits $\{824, 825\}$ and **"**$P(10, 4)$**"** possible choices for the remaining 4 digits.

Exercise 5.5.3:

a.

Solution: 2^{10}

---

b.

Solution: 2^{7}

---

c.

Solution: $2^{7} + 2^{8}$

---

d.

Solution: 2^8

---

e.

Solution: $C(10, 4)$

---

f.

Solution: $C(9, 3)$

---

g.

Solution: $C(5, 1) * C(5, 3)$

Exercise 5.5.5:

a.

Solution: $30\ chooses\ 10 + 35\ chooses\ 10$

Exercise 5.5.8:

c.

Solution: $C(26, 5)$

---

d.

Solution: $C(13, 1) * C(48, 1)$

---

e.

Solution: $C(13,1) * C(4,3) * C(12,1) * C(4,2)$

---

f.

Solution:

Exercise 5.6.6:

a.

Solution: $C(56,5) + C(44,5)$

---

b.

Solution: $P(56,2) * P(44,2)$

$Repudiators = P(56,2)$

$Demonstrators = P(44,2)$

Question 6:

Exercise 5.7.2:

a.

Solution: $C(52,5) - C(39,5)$

---

b.

Solution: $C(52,5) - C(13,5) * 4^5$

Exercise 5.8.4:

a.

Solution: 5^{20}

---

b.

Solution: $\frac{20!}{4!*4!*4!*4!*4!}$

Question 7:

How many one-to-one functions are there from a set with five elements to sets with the following number of elements?

a) 4 = 0, it is not possible to map a one-to-one function when 5 elements are mapping to 4 elements.

b) 5 = 5! = 120

c) 6 = $P(6, 5)$

d) 7 = $P(7, 5)$
