<u>Question 5</u>:

a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$

<u>Solution</u>:

Let $f(n) = 5n^3 + 2n^2 + 3n$.

To find Θ we must bound $f(n)$. To do this we need to determine $O(n)$ and $\Omega(n)$.

In order to find $O(n)$, we take the following expression, $5n^3 + 2n^2 + 3n$, and raise all the terms to the same exponent. Finally, we add all the terms together to derive $O(n)$ and take the sum of those terms to get c_1:

$$5n^3 + 2n^3 + 3n^3 \le 10n^3$$

$$O(n) = 10n^3 = O(n^3)$$

$$c_1 = 10$$

In order to find $\Omega(n)$, we take the following expression, $5n^3 + 2n^2 + 3n$, and remove all the lower order terms to leave us with the following:

$$5n^3 + 2n^2 + 3n \ge 5n^3$$

$$\Omega(n) = 5n^3 = \Omega(n^3)$$

$$c_2 = 5$$

Using $n_0 = 1$, we can conclude that $5n^3 + 2n^2 + 3n = \Theta(n^3)$. ■

---

b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

<u>Solution</u>:

Let $f(n) = \sqrt{7n^2 + 2n - 8}$.

To find Θ we must bound $f(n)$. To do this we need to determine $O(n)$ and $\Omega(n)$.

In order to find $O(n)$, we take the following expression, $\sqrt{7n^2 + 2n - 8}$, and raise all the non-negative terms to the same exponent. Finally, we add all the terms together and find the square root of the result to derive $O(n)$ and take the sum of those terms to get c_1

$$\sqrt{7n^2 + 2n^2} \leq \sqrt{9n^2}$$

$$\sqrt{9n^2} = 3n \;= O(n)$$

$$c_1 = 3$$

In order to find $\Omega(n)$, we take the following expression, $\sqrt{7n^2 + 2n - 8}$, we ignore the lower order terms and conduct the following:

$$\sqrt{7n^2}$$

$$= \sqrt{7} * n$$

$$c_2 = \frac{\sqrt{7}}{2}$$

$$\frac{\sqrt{7}}{2} * n \;= \Omega(n)$$

Using $n_0 = 1$, we can conclude that $\sqrt{7n^2 + 2n - 8} = \Theta(n)$. ■