The Network Dynamic of Conformity

—How does the non-linear pattern of conformity adjust the risk of Groupthink in Poisson

Random Network?

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Abstract

It is a long tradition for social scientists to both find reasons for groupthink and use assumption to model conformity behavior. In this work, we consider the different risk of groupthink from the linear assumption and non-linear conformity behavior. We use network simulation strategy to model the process from the individual conformity behavior to the groupthink. We use the data collected from behavior experiment and then build a regression model for the conformity behavior. We then generate a Poison random network to for the emerging of groupthink. We demonstrate (i) that contrary to linear conformity, people use non-linear pattern of conformity and thus result in higher risk of groupthink, but (ii) that people with different personalities could counterbalance it.

Key words: Social Network, Conformity, Groupthink, Poison random network

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1. Introduction:

It is inherent for western philosophic tradition to concept of "radical evil". As for Immanuel Kant, (1998) this originates from a kind of human propensity to evil or the so-called original perversion. This means that people are always and by nature characterized by a tendency to subordinate truth to the pathological order of inclination freely. Hana Ardent (1965) in her book Eichmann in Jerusalem: A Report on the Banality of Evil described the destructive obedience of the forming of wrong consensus. She developed "radical evil" to "thoughtless" for individuals. Not like the inherited evil notion, social scientists are always willing to owe this problem to the outside influential factors which results in the misbehavior of a group. "Groupthink" (Janis, 1972) is a concept pointing to the situation that group members with homogeneity of ideology under high stress from external threats have tendency of groupthink. Tetlock (1992) conducted a LISREL analysis, assessing the casual relationships in the groupthink model, and confirmed the importance of structural and procedural faults of the organization as antecedents of groupthink. Researchers adopted different methods of analysis also argue on whether other factors such as group cohesiveness (Janis, 1982; Tetlock, 1992; McCauley, 1989; Turner, 1992) and provocative situational context (Janis, 1982, Tetlock, 1994) could be antecedents for the groupthink.

However, despite the social context that could influence the emerging of groupthink, none of the research has extracted the influence from Kant and Arendt's consideration about human nature. Newly developing social network study provides me with method to think back about the innate factors of human which could probably result in high risk of groupthink. Alongside with the laboratorial efforts, this work presents a cohesive way of modeling the personal behavior to emerge as a group-level result.

In the traditional modeling process, researchers tend to simplify the human

behavior pattern in a linear way under the condition of no social influence, especially in studies from the field of behavior economics and the social network, though they would complicate the model to involve other social factors. Diminishing marginal utility is another commonly used assumption for micro-behavior analysis in economics, but it is probably not true in many situations such as the consumption of drug and also the conformity behavior in this study. Even though, not many researchers use the true behavior pattern to model the social fact. Some studies focus on the non-linear way of conforming to other people's opinion, but they do not include the issue of conformity and network modeling technique into their study. In this work, we collect the real data from laboratorial experiment as micro parameter for individuals, which is quite different from other theoretical studies. What's more, simulation process needs agents' behavioral mode to be relatively stable, thus we enrolled a small number of participants but each of them conducted a same series of questions for hundreds of times.

Since Watts and Strogatz's (1998) benchmark work, there has been a large body of literatures focusing on the true network pattern in the human society, especially to testify the existence of small world phenomenon in real world. However, other researchers found that small world network does not have the property of fatter tailed distribution of degree, which is found in many real networks (Matthew, 2009). Preferential attachment (Barabasi et al, 1999) and richer sequential link formation model (Vasquez, 2003) developed later to fix the inadequacy of the Small World Model. However, as the antecedent of all these models, the Poisson Random networks (Erdos & Renyi,1959) could have most compatibility as exponential network to fit various kinds of network style. Any real network could show some property of a random network. As we do not specify the network to represent a specific group in this work, we adopt this old setting of network with some adjustment to represent the largely defined true situations.

Conformity is the concept as the micro basis for the groupthink in this work. According to the classic definition of conformity, it is a kind of social influence resulting from exposure to the opinions of a majority, or the majority of one's group (P228). This concept also interpreted by some social psychologists as majority influence. The

extension of this concept should actually be different in this work since that even exposed to the results from minorities, the subjects are still possible to change their choice.

In the data collection part, this work combines two classic paradigms of social conformity study to collect behavioral data. The social psychologist Sherif (1936) was one of the few who studied the conformity behavior in early time. He initiate the autokinetic paradigm in order to determine the social influence by operating the confederates. Asch paradigm, another benchmark has been popular since 1950s, testified that individual could change their opinion under the influence from other members (Asch, 1950). The Asch paradigm asks participants to judge the length of lines with exposure to other people's judgment. Our experiment combines these two paradigms that, the participants make judgment about objective tasks—similar as Asch's content, but neither answer in the task is correct, like Sherif's experiment. The advantage of this design is that the influence of degree of difficulty could be balanced within one participants answer and thus could be eliminated in our analysis. There is also no problem of compliance in our design since the participants are isolated in a lab environment to communicate with confederates via computer.

There are two folders of questions in this research: (i) is the true behavior pattern different from linear assumption and if so, how the result of the simulation in the Poisson random network different from that under linear assumption? (ii) How to understand the human conformity nature and the emerging groupthink risk? To answer the first question, it is necessary to incorporate the experimental study and the network simulation strategy. To answer the second question, we must analysis the varieties of results under different network constructions.

The challenge of incorporating the experimental results to the network simulation is that the statistical results of behaviors are in inconsistent way, while network connection is not always suitable for the data collected in that way. Thus, the regression strategy is important in this work to convert the inconsistent result to the consistent function thus could make it be fitted to the requirement of simulation tasks. The innate character of this operation is to detach the probability calculating process and the decision

process.

We generate the Poisson random network in this work by two steps. First we initiate with a set of nodes { n_i }, 1 <= i <= n in sequence and use the least links in this graph to form an interconnected network: connect n_1 and n_2 , and then add n_3 to this graph with the same possibility 0.5 to connect n_1 or n_2 . We add n_4 with possibility of 0.33 to each of the existing nodes. This process ends till the last node n_n is added in this graph. The possibility of a new node n_k to join each of the already connected nodes is 1/(k-1), k=2,3...n. The second step is to generate the p as the most important parameter in a Poisson Network. We denote the variable q to be the new links that needed to be randomly added to the connected network. The outcome p is the proportion of the links in a complete graph:

$$P=(n-1+q)/C(\frac{2}{n})$$

The network generated in way above is an interconnected Poisson network with the possibility of connectivity between each two nodes equals to p, if we consider the generation of this graph in a non-sequential way.

The model is set up as follows. Each agent in a social network has a true or false judgment about a specific question at first. And each of them tends to communicate with their neighbors, who weigh the same to the judgment in the second round. Each agent could probably change their initial judgment and this possibility is set according to the result in the experiment. This process will continue until all of the agents in this network have gained the same judgment, whether true or false. The most important result, denoted as "N", is the average number of periods of this process lasting, which aims to measure the speed of becoming convergence in network. Although, it is not reasonable to assume that the network is inflexible during the evolution, and the judgment is made at the same time during every period, this design could relatively reflect the influence of individual behavior pattern towards the risk of macro-level groupthink.

The key simulation results of the paper are summarized as follows. First, we present the network antecedents of groupthink. We use several statistical items, including the relative advantages, the clustering coefficient and the average distance between the

nodes in the graph to elaborate the network factors for the final results. And second, we make comparison between the linear and non-linear patterns of conforming on group level. It is contrary to our conjecture that the risk of groupthink will always be higher under the non-linear pattern than the linear pattern. In some situations, especially when the network is of smaller p but larger n, the result could probably be opposite. In the end, we use the conformity pattern of two different personalities to resimulate the same process.

The organization of the paper goes in the following sequence. In section 2, we introduce the behavior experiment and the operation of its result. In section 3, we elaborate the network generating process and the algorithm for simulation as well as the definitions of several important statistical items. Section 4 mainly presents the simulation results. Conclusion remarks follow in Section 5.

2. Experiment Design and Data Collection

2.1Participants

Twenty-four undergraduate and graduate students (13 females; mean age 22.5 years, SD = 1.93) participated in the experiment. We also enrolled four students, who were strangers to the participants as confederates.

All the participants were right-handed and had normal or corrected-to-normal vision. They had no history of neurological or psychiatric disorders. Informed consent was obtained from each participant before the test. We performed this experiment in accordance with the Declaration of Helsinki and the Ethics Committee of the Department of Psychology also approved our experiment. We paid each participant 60 Chinese Yuan (about \$ 9.5) as basic payment and informed them that additional monetary reward would be paid according to their performance in the task, although in the end all the participants were paid the same on top of the basic payment.

2.2 Design and procedures

The experiment has five consistency levels. For highly inconsistency condition, four group members choose the different line with participant whereas for moderately consistency condition, two group members choose the different line with the participant.

For consistency condition, one or none group member chooses the different line as the participant does.

We assemble one participant and four confederates and tell them that they would sit in separate rooms to finish a line judgment task together through the computer network. The participant sits comfortable about 1.5 m in front of a computer screen in a dimly light room. In fact, we generate this procedure by computer program with a Del 22-in. CRT display. We administer the experiment on a computer, using Presentation software (Neurobehavioral System Inc.) to control the presentation and timing of the stimuli, which is out of participant's knowledge. The detail of the process during the experiment is in Figure 1. Neither of the two vertical lines is coordinated with the horizontal one and the difference between them is very small-- only 0.48 degree visual angle. And most people feel difficult to accomplish the tasks in this experiment. The mean difficulty is 4.21, scored (1-7 the Likert Scale, SD=1.13) by the participants after the experiment in a self-report questionnaire.

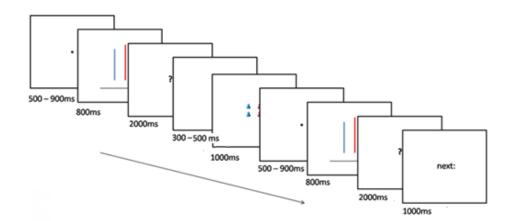


Fig1. The flow diagram of the experiment: each trial begins with a white dot against black background jittered between 500 – 900 ms. Then two vertical lines, one in red and the other in blue on either the right or left side of the screen (counterbalanced within participant), are presented together with a horizontal line (in black) for 800 ms. The participant need to judge which one of the vertical lines has the same length with the horizontal one in 2000 ms. After presentation of blank screen for 300 – 500 ms, group opinion screen was presented for 1000 ms. On this screen, four head portraits in either red or blue color indicated the group members' choices, with specific color corresponding to the group members' choice of line. A white dot was

then presented for 500 - 900 ms, and the line stimulus were presented for another 800 ms. The participant had to indicate his/her final choice in 2000 ms. The next trial began after 1000 ms after the button press.

For highly incongruent condition, four group members' choices were different from the participant's in 120 trials and three were different in 60 trials. For moderately incongruent condition, two group members' choices were different from the participant's in 140 trials. For less incongruent condition, one group member's choice is different from the participant's in 60 trials and none group member's choice is different in 120 trials. We categorize the group opinion in such a way to make the trial numbers roughly the same across conditions. We mix 500 trials randomly and divide them in equal numbers into 5 blocks with the restriction that no more than five trials are on the same incongruent level. A practice block of 30 trials is administered before the formal test to familiarize the participants with the task. Participants are debriefed, paid and thanked at the end of the experiment.

2.3 Data analysis and result

Trials in which participants did not respond following the initial or second presentation of the line stimulus were excluded from data analysis, amounting to 1.18% of the total data. What's more, among the twenty-four participants, four of them in the interview after the test stated that they disbelieved the setup of the experiment that they were communicating with true people. We also find that one participant changed initial choice in less than 5 trials in either highly or lowly incongruent conditions and another two participants react randomly after exposed to others' judgment. We treat participants in these circumstances above as invalid samples, although they possibly behave like that in real life. Anyhow, they were not suitable for the following simulation design since their behaviors are structure-independent—they did not vary according to the choice of their neighbors. These participants were excluded from data analysis, leaving seventeen participants for the following analysis.

We define the consistency as the follows: the proportion of the neighbors of the participant who have the same choice with me (including the participant). That means if all of the rest participants' choices are different from that of the participant in our

experiment, the consistency is 0.2; and if all are the same, the consistency is 1. Trials in which the participants changed the initial choice during the second presentation of line stimulus are marked as changed trials. We then calculated the change rate as the percent of change trials in each condition. The result could also be separated to two groups according to the participants' scores of Self-construal Scale (SCS) after the experiment. We minus the score of independent items from that of dependent items in the SCS to get the final score. The lower score group contains 7 samples while the higher score group contains 10 samples; the cut point score for this classification is -5. The relation of consistency and change rate is illustrated by Figure 2.

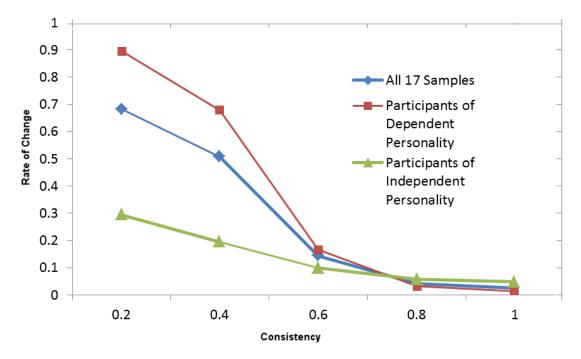


Fig 2. Result of change rate under different consistency.

The finding in the experiment session is that the model fits the human conformity behavior in different level of consistency is a logistic curve, rather than a linear curve. All of the sample show the trend to decrease sharply between the consistency level of 0.4 and 0.6. We compare the model-data fit between both the linear and logistic model and find that the logistic model (R Square=0.6636) have higher goodness of fit than the linear model(R Square =0.5952), though both of them are very high. Thus, we choose logistic regression for the following simulation model. This non-linear trend is also seen in almost every participant's change rate distribution. If change rate is denoted as y and, the

consistency is donated as x. The expression of this model is as follows:

$$Log (y/(1-y)) = 1.050-2.856x, x \in [0,1]$$

The regression models for the two personalities--dependent personality and independent personality are as the following:

$$Log (y_1/(1-y_1)) = 1.881-4.028x, x \in [0,1]$$

$$Log (y_2/(1-y_2)) = -0.138-1.421x, x \in [0,1]$$

3. Simulation in a random Network

3.1 The evolution rule

Many strategy evolution and social network study set some similar simulation model for the emerging of micro-basis. As a benchmark, we describe the learning updating in network setting—the DeGroot model (1974). In his model, agents observe signals just once and then repeatedly communicate with each other and update their beliefs after every round of communication. The social network is described by a weighted and possibly directed "trust" matrix $\mathbf{T} \in [0,1]^{n*n}$, with Σ_j $\mathbf{T}_{ij}=1$. When the agent gives equal weight of his or her neighbor, the T_{ij} is like this:

$$\mathbf{T}_{ij}=\mathbf{g}_{ij}/\mathbf{d}_{i}\left(\mathbf{g}\right)$$

In this equation, the society is an undirected social network g, with $g_{ij}=1$ indicating that i and j are linked or $g_{ij}=0$ indicating that i and j are not linked, and $d_i(g)$ is n_i 's degree. In the DeGroot model, agents begin with some initial opinions described by a belief $b_i(0) \in [0,1]$ and the update those over time. The updating rule is just:

$$b_i(t) = \sum \mathbf{T}_{ij} b_j (t-1)$$

Thus,
$$b(t)=T b(t-1)$$

$$b(t)=T^{t}b(0)$$

It could also be more complicated to involve the different weights on different friends based on frequencies of interaction, envisioned reliability, affinity, or other reasons. We just not focus on these influences but narrow down our scope to the non-linear conformity nature. Even without these influences, the agent would still not weigh others' opinions equally according to our experiment's result. The DeGroot model also denotes b_i to be a consistent variable, but in our model the b_i is a 0-1 variable which is not consistent.

Thus, the final equilibrium is also not a convergent number between zero to one, but the system is at its first time ended with all 0 or 1. Second, the most important difference from the DeGroot Model is that the T_{ij} is objective result of the opinion and a function f is used to connect the objective result to the subjective judgment. This operation also implies that the essence of this model is the detachment of the distribution of opinions and the final decisions, which is obtained from the regression of the inconsistent data in the experiment.

$$U=f(T)$$

$$B_t = U^t b(0)$$

3.2 The statistical items for network

The experiment adopted classic Poison Network with the parameter that the connectivity between each of the two nodes is denoted as p. And we have already introduced the algorithm in the introduction part. At each attempts of simulation, the graph is regenerated randomly according to the algorithm. Several other classic statistical items for a network are also introduced in this work. We define the extension of all these items here.

3.2.1 Probability of link:

The network structure of G is represented by G_{ij} , $1 \le i \le n-1, j > I$, where $G_{ij} = p$ denotes as i and j are linked with probability p

3.2.2 Relative Advantages:

Another measurement designed for this simulation experiment is the relative advantage of the initial opinion:

Radv =
$$\sum d_1(i) / \sum d_0(i)$$

This item is used to consider the initial distribution of the choice and further check whether the strong iniquity of the two patterns will influence the speed of convergence.

3.2.3 Clustering Coefficient

The clustering coefficient is calculated as follows. The degree of agent n_i is k, and the number of links existing among these k neighbors' is r. Then the clustering

coefficient for the agent n_i is:

$$CC_i = r/C(_k^r)$$

And thus, the Clustering Coefficient for the whole graph is $CC = \sum CC_i / n$.

3.2.4 Average Distance of the Graph

This item is defined as the follows. In an interconnected and undirected graph with n nodes, the shortest distance between two nodes n_i and n_j $1 \le i \le j \le n$ is d_{ij} . The average distance of the graph is $d = \sum d_{ij} / C \binom{n}{n}$.

3.2.5 Average Periods

This item is a core item to measure the convergent speed of the opinions on network. Given the fix n and p, then the first time for the network system to converge to a same opinion lasts Ni period. Then the average periods is defined as the average number of N_i in m attempts: $N=N_i/m$.

4. Result of Simulation

We run the simulation program with different initial setting and each for 1000 times. The random network in simulation task consists of four conditions--10, 20, 50 and 100 nodes, and we also try several p from the lowest (n-1)/ $C(_n^2)$ to 1 to depict the rough shape of the graph. We find that all results of average number of periods for converging conforms to normal distribution under a given p and n.

4.1 Network Influence to N.

We first summarize the influence from the network itself. Three items are measured to illustrate the property of the network every time. Since in all of the four conditions the result are almost the same, we use the logistic model at n=20 as an example. The N has shown some positive relation with the clustering coefficient. That means in a condensed network, meeting a consensus needs a longer time than when the network is lower condensed. The average length of a graph is also an influential factor for the speed on convergent; the negative relation between it and N also means that a condensed network is harder for meeting a consensus quickly. The third item is Radv, measuring the relative advantage between the two opinions has shown a peak the graph. The more

inequity between the two opinions at the initial stage will enhance the speed of convergent. When Ad is around 1, which means the relative potential is similar between the two opinions, the convergent speed is slower than the more imbalanced situation. This easily matches the true situation, people with more degrees in a network have more influence to this network rather than those with less ones.

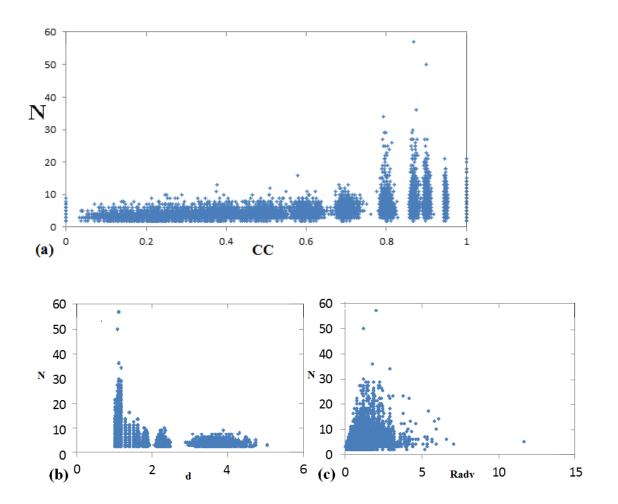


Fig 3. (a) Clustering Coefficient vs N, (b) d vs N, (c) Radv vs N.

4.2 Linear and Non-linear Conformity.

In this section, we summarize the influences of different conformity patterns between the linear conformity and the non-linear conformity.

4.2.1The N for linear conformity follows a power-law (Figure 4. (a)).

The distribution of N is in a Poisson pattern in the linear conformity. The N increases slowly as p increases and when p is beyond 0.8, the N increase in an exponential or even a super exponential way. We use exponential regression to fit the data from

observation. For all of the four conditions, the model fitness is very high (Anova, P<0.006) and the super exponential modeling strategy could find an even higher goodness of fit (Anova, P<0.002).

If people conform to other people's opinions according to a linear rule, they may probably very hard to reach a consensus when p is at a high level, which in other words, means that members within a group that are familiar with each other are hard to reach consensus. This may be controversial to our common sense about group behavior—low risk of groupthink. (a) (b)

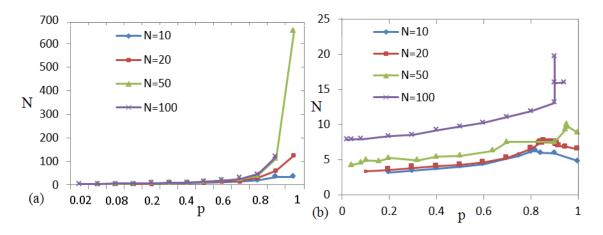
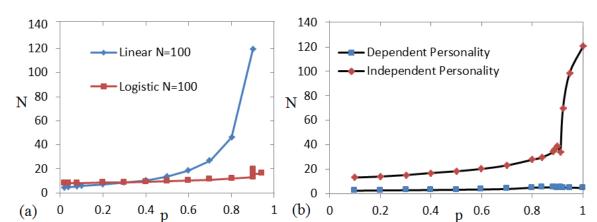


Fig. 4 (a) Linear Conformity: p vs N, (b) Non-Linear Conformity: p vs N

4.2.2 Non-linear conformity results in relative stable N and have peaks as p increases

The non-linear conformity pattern is quite different from linear conformity pattern in the convergent speed, though the regression line is only slightly different (Fig. 4(b)). All of the simulations have a turning point when p is at about 0.9. In a smaller network, its N increase slower and the complete graph has a lower convergent speed than the situation when p is at some number within [0.8, 0.9]. However, the N under a provided n is relative stable compare to the linear model. What' also different is that logistic model does not follow a power law.

In small networks with n less than 50, the N of linear conformity is always higher than that of non-linear conformity. But this is not true when the network is larger. We use n=100 as an example. In a larger social network like n=100, the non-linear and linear conformity transect with each other at P=0.3. When P is at low level, N of non-linear



conformity is higher than that on linear condition; while p is at high level, that reverses.

Fig. 5 (a) N=100: p vs N; (b) the conformity of two personalities, N=20: p vs N

4.3 Conformity of People with different personalities.

We discuss the different macro order of the two people with different personalities. In the last section, we introduce high risk of groupthink in the non-linear pattern of conformity. After distinguishing the two different pattern between the two personalities, we use the two regression models corresponding to the two personalities respectively to simulate the emerging process again on the condition of n=20. The finding showing in the following graph is that the risk of groupthink differs a lot within a group containing people with only one of the personalities. The speed of convergent decrease a lot after the crisis of p=0.9105 for people with independent personalities, while the people with dependent personality always convergent to a consensus very quickly—the slowest process lasts only about 5.205 periods in average when p=0.8895. These two patterns of conformity down turn the risk of groupthink compared to the integrative operation of data in an overall estimation of conformity pattern.

5 Conclusion.

The linear conformity assumption is quite different from the actual distribution of conformity, as is shown in the experiment session. The linear and non-linear will lead to totally different order in a Poison random network. Although, the speed of convergent is sensitive to the structural of the network itself, our simulation has shown that the non-linear conformity has relatively stable speed of convergent compared with the linear

conformity. In a small network, non-linear conformity converges to a mutual opinion quicker than the linear conformity in average; while in a big network, linear conformity converge to an opinion quicker when P is very small but could be very slow when P is at a higher level. For example, in a network with 100 nodes, the linear conformity goes beyond the non-linear conformity from the crisis of about p=0.3. Non-linear conformity makes it possible that people converge to a mutual opinion quickly in a relative big network with concentrated links.

Linear modeling strategy, commonly used by pervious researchers when making assumptions of individual behaviors, is not suitable for modeling the conformity behavior according to our results. The promotion of this work could be conducted in following methods: enlarge the n of the network or use advanced network algorithm which needs higher-speed computer; the introduction of the Bayesian network algorithm (Stuart, 1998) that could add structure among the agents in this network.

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