1 MEASUREMENTS AND EXPERIMENTATION

Syllabus :

(i) *International system of units (the required S.I. units with correct symbols are given at the end of this syllabus). Other commonly used system of units – FPS and CGS.*

(ii) *Measurements using common instruments, Vernier callipers and micrometre screw gauge for length and simple pendulum for time.*

Scope – Measurement of length using vernier callipers and micrometre screw gauge. Decreasing least count leads to an increase in accuracy; least count (L.C.) of vernier callipers and screw gauge, zero error (basic idea) (no numerical problems on callipers and screw gauge). Simple pendulum; time period, frequency, graph of length l vs. T^2 only; slope of the graph. Formula $T = 2\pi\sqrt{l/g}$ (No derivation). Only simple numerical problems.

(A) SYSTEMS OF UNIT AND UNITS IN S.I. SYSTEM

1.1 NEED OF UNIT FOR MEASUREMENT

Physics, like other branches of science require experimental studies which involve measurements. For the measurement of a physical quantity, we consider a constant quantity of same nature as a standard and then we compare the given quantity with the standard quantity *i.e.* we find the number which expresses, how many times the *standard* quantity is contained in the given physical quantity. Thus

Measurement is the process of comparison of the given physical quantity with the known standard quantity of the same nature.

The standard quantity used to measure the given physical quantity is called the ***unit***. For quantities of different nature, we use different units.

Unit is the quantity of a constant magnitude which is used to measure the magnitudes of other quantities of the same nature.

The result of measurement of a physical quantity is expressed in terms of the following *two* parameters :

(i) The ***unit*** in which the quantity is being measured, and

(ii) The ***numerical value*** which expresses, how many times the above selected unit is contained in the given quantity.

Thus the magnitude of a physical quantity is expressed as :

Physical quantity = (numerical value) × (unit)

Examples : (i) If the length of a piece of cloth is 10 metre, it means that the length is measured in the unit metre and this unit is contained 10 times in the length of that piece of cloth.

(ii) If the mass of a given quantity of sugar is 5 kilogram, it means that the mass is measured in the unit kilogram and this unit is contained 5 times in the given quantity of sugar.

Choice of unit

To measure a physical quantity, the unit chosen should have the following properties :

(i) The unit should be of *convenient size.*

(ii) It should be possible to define the unit *without ambiguity,*

(iii) The unit should be *reproducible.*

(iv) The value of unit should *not change with space and time.* (*i.e.*, it must always remain same everywhere).

The last three conditions (ii), (iii) and (iv) are essential for the unit to be accepted internationally.

Kinds of unit

Units are of *two* kinds : (i) Fundamental or basic units, and (ii) Derived units.

(i) Fundamental or basic units

A fundamental (or basic) unit is that which is independent of any other unit or which can neither be changed nor can be related to any other fundamental unit.

Examples: The units of mass, length, time, temperature, current and amount of substance are independent of each other. They can not be obtained from any other unit. These are the fundamental units.

(ii) Derived units

The units of quantities other than those measured in fundamental units, can be expressed in terms of the fundamental units and they are called the derived units. Thus

Derived units are those which depend on the fundamental units or which can be expressed in terms of the fundamental units.

Examples : (i) For the measurement of area, we need to measure length and breadth in the unit of length and then express area in a unit which is length × length or $(\text{length})^2$.

(ii) The volume is expressed in a unit which is length × length × length or $(\text{length})^3$.

(iii) The unit of speed of a moving body is obtained by dividing the unit of distance (*i.e.*, length) by the unit of time *i.e.*, it can be expressed in terms of the units of length and time.

Thus the units used to measure area, volume, speed, *etc*. are the derived units. More examples of derived units are given ahead in article 1.6.

1.2 SYSTEMS OF UNIT

In mechanics, ***length, mass*** and ***time*** are the *three* fundamental quantities. For the units of these three *basic quantities*, following systems have been used :

(i) **C.G.S. system (or French system) :** In this system, the unit of length is centimetre (cm), of mass is gram (g) and of time is second (s).

(ii) **F.P.S. system (or British system) :** In this system, the unit of length is foot (ft), of mass is pound (lb) and of time is second (s).

(iii) **M.K.S. system (or metric system) :** In this system, the unit of length is metre (m), of mass is kilogram (kg) and of time is second (s).

The above mentioned systems are now no longer in use and are only of historical importance. Now we use the S.I. system of units which is an enlarged and modified version of the metric system.

Systeme Internationale d'Unites (or S.I. system)

In 1960, the General Conference of Weights and Measures recommended that in addition to the units of length, mass and time, the units of temperature, luminous intensity, current and the amount of substance also be taken as fundamental units and the units of angle and solid angle as the complementary fundamental units. Thus in all, now there are *seven* fundamental units and *two* complementary fundamental units.

For **S.I. system,** the following table gives the fundamental quantities, their units and their standard symbols.

Fundamental quantities, units and symbols in S.I. system

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Luminous intensity	candela	cd
Electric current	ampere	A
Amount of substance	mole	mol*
Angle	radian	rd
Solid angle	steradian	st-rd

Use of prefix with a unit

For expressing large measurements, we use deca, hecto, kilo *etc*., as *prefixes* with the units.

** Nowadays 1 mol means 1 kg mol equal to $6{\cdot}02 \times 10^{26}$ entities (i.e., atoms or molecules or ions).*

The symbol and meaning of each prefix are given below.

Some prefixes used for big measurements

Prefix	Symbol	Meaning
deca	da	10^1
hecto	h	10^2
kilo	k	10^3
mega	M	10^6
giga	G	10^9
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}
zetta	Z	10^{21}
yotta	Y	10^{24}

The various small measurements are expressed by using the *prefixes* deci, centi, milli, micro, *etc.*, with the units. The symbol and meaning of each such prefix are given below.

Some prefixes used for small measurements

Prefix	Symbol	Meaning
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p (or μμ)	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yoeto	y	10^{-24}

Example : 2·5 GHz will mean $2{\cdot}5 \times 10^9$ Hz, 5·0 pF will mean $5{\cdot}0 \times 10^{-12}$ F, 5·0 MΩ will mean $5{\cdot}0 \times 10^6\ \Omega$, 2·0 ms will mean $2{\cdot}0 \times 10^{-3}$ s and so on.

1.3 UNITS OF LENGTH

S.I. unit of length

The S.I. unit of the length is metre (m).

> ***A metre was originally defined in 1889 as the distance between two marks drawn on a platinum-iridium (an alloy with 90% platinum and 10% iridium) rod kept at 0°C in the International Bureau of Weights and Measures at Sevres near Paris.***

Later, in 1960, the metre was re-defined as 1,650,763·73 times the wavelength of a specified orange-red spectral line in the emission spectrum of Krypton-86. It is also defined as : 'one metre is 1,553,164·1 times the wavelength of the red line in the emission spectrum of cadmium'.

> *In 1983, the metre was re-defined in terms of speed of light according to which one metre is the distance travelled by the light in* $\frac{1}{299{,}792{,}458}$ *of a second in air (or vacuum).*

Sub units of metre

For the measurement of small lengths, the metre is considered too big a unit. The most commonly used sub units of metre are **(i) centimetre** (cm), **(ii) millimetre** (mm), **(iii) micron** (μ) and **(iv) nanometre** (nm).

(i) centimetre (cm) : *One centimetre is one-hundredth part of a metre. i.e.,*

$$1 \text{ cm} = \frac{1}{100} \text{ m} = 10^{-2} \text{ m}$$

(ii) millimetre (mm) : *One millimetre is one-thousandth part of a metre. i.e.,*

$$1 \text{ mm} = \frac{1}{1000} \text{ m} = 10^{-3} \text{ m} = \frac{1}{10} \text{ cm}$$

(iii) micrometre or micron : *It is one-millionth* (10^{-6}) *part of a metre.* It is expressed by the symbol μ. It is also called micrometre (symbol μm).

$$1 \text{ micron } (\mu) = 10^{-6} \text{ metre} = 10^{-4} \text{ cm} = 10^{-3} \text{ mm.}$$

(iv) nanometer (nm) : *It is one billionth* (10^{-9} *th) part of a metre. i.e.,* $1 \text{ nm} = 10^{-9}$ m.

Multiple units of metre

For the measurement of large lengths (or distances), the metre is considered as too small a unit. The most commonly used multiple unit of metre is kilometre.

kilometre (km) : *One kilometre is the one-thousand multiple of a metre. i.e.,*

$$1 \text{ km} = 1000 \text{ m (or } 10^3 \text{ m).}$$

Non-metric units of length

Bigger units : For the measurement of distance between two heavenly bodies, the kilometre is considered a too small unit. The commonly used units for this purpose are : **(i) astronomical unit** (A.U.), **(ii) light year** (ly) and **(iii) parsec.**

(i) Astronomical unit (A.U.) : *One astronomical unit is equal to the mean distance between the earth and the sun. i.e.,*

$$\boxed{1 \text{ A.U.} = 1{\cdot}496 \times 10^{11} \text{ metre}}$$

(ii) Light year (ly) : A *light year is the distance travelled by light in vacuum, in one year. i.e.,*

$$1 \text{ light year} = \text{speed of light} \times \text{time 1 year} = 3 \times 10^8 \text{ m s}^{-1} \times (365 \times 24 \times 60 \times 60 \text{ s}) = 9{\cdot}46 \times 10^{15} \text{ m} = 9{\cdot}46 \times 10^{12} \text{ km}$$

The distance of stars from earth is generally expressed in light years. However, light minute and light second are its smaller units.

$$1 \text{ light minute} = 3 \times 10^8 \text{ m s}^{-1} \times 60 \text{ s} = 1{\cdot}8 \times 10^{10} \text{ m}$$

and $1 \text{ light second} = 3 \times 10^8 \text{ m s}^{-1} \times 1 \text{ s} = 3 \times 10^8 \text{ m}$

(iii) Parsec : *One parsec* is the distance from where the semi major axis of orbit of earth* (1 A.U.) *subtends an angle of one second.*

i.e., Parasec × 1 = 1 A.U.

or $$1 \text{ Parasec} = \frac{1{\cdot}496 \times 10^{11} \text{ m}}{(1/3600) \times (\pi/180)} = 3{\cdot}08 \times 10^{16} \text{ m}$$

$$= \frac{3{\cdot}08 \times 10^{16}}{9{\cdot}46 \times 10^{15}} \text{ ly} = 3{\cdot}26 \text{ ly}$$

Smaller units : To express the wavelength of light, size and separation between two molecules (or atoms), radius of orbit of electron, etc. a small size unit called the **Angstrom** (Å) is used, while the size of the nucleus is expressed by a still smaller unit called **fermi** (f).

(i) Angstrom (Å) : *It is 10^{-10}th part of a metre.* It is expressed by the symbol Å. *i.e.*,

$$1 \text{ Angstrom (Å)} = 10^{-10} \text{ metre} = 10^{-8} \text{ cm} = 10^{-1} \text{ nm}$$

∴ 1 micron = 10,000 Å

and 1 nm = 10 Å

Nowadays, Å is outdated and nm is preferred over the Å. The wavelength of light, inter-atomic or inter-molecular separation, *etc*. are now commonly expressed in nm.

(ii) fermi (f) : *It is 10^{-15} th part of a metre. i.e.,*

$$1 \text{ fermi (f)} = 10^{-15} \text{ m}$$

The commonly used smaller and bigger units of length are summarized in the following table.

Smaller and bigger units of length

Smaller units	Value in metre	Bigger units	Value in metre
cm	10^{-2} m	km	10^3 m
		A.U.	$1{\cdot}496 \times 10^{11}$ m
mm	10^{-3} m	ly	$9{\cdot}46 \times 10^{15}$ m
μ (or μm)	10^{-6} m	parsec	$3{\cdot}08 \times 10^{16}$ m
nm	10^{-9} m		
Å	10^{-10} m		
f	10^{-15} m		

** Parsec is constituted from the combination of two words, parallax (par) and arc-second (sec).*

1.4 UNITS OF MASS

S.I. unit of mass

The S.I. unit of mass is **kilogram** (kg).

In 1889, one kilogram was defined as the mass of a cylindrical piece of platinum-iridium alloy kept at International Bureau of Weights and Measures at Sevres near Paris.

However, the mass of 1 litre (= 1000 ml) of water at 4°C is also taken as 1 kilogram.

Sub units of kilogram

For measurement of small masses, kilogram (kg) is a bigger unit of mass. The smaller units of mass in common use are **(i) gram** (g) and **(ii) milligram** (mg).

(i) gram (g) : *One gram is the one-thousandth part of a kilogram i.e.,*

$$1 \text{ g} = \frac{1}{1000} \text{ kg} = 10^{-3} \text{ kg}$$

or $1 \text{ kg} = 1000 \text{ g}$

(ii) milligram (mg) : *One milligram is one-millionth (10^{-6}) part of a kilogram* or it is one-thousandth (10^{-3}) part of a gram. *i.e.*,

$$1 \text{ mg} = 10^{-6} \text{ kg} \quad \text{or} \quad 1 \text{ mg} = 10^{-3} \text{ g}$$

Multiple units of kilogram

The bigger common units of mass used in daily life are **(i) quintal** and **(ii) metric tonne**.

(i) quintal : It is one hundred times a kilogram,

i.e., 1 quintal = 100 kg

(ii) metric tonne : It is one-thousand times a kilogram. *i.e*,

1 metric tonne = 1000 kg = 10 quintal.

Non-metric unit of mass

The mass of atomic particles such as proton, neutron and electron is expressed in a unit called the **atomic mass unit** (symbol a.m.u) or the **unified atomic mass unit** (symbol u). It is defined as below :

1 a.m.u. (or u) is $\frac{1}{12}$ th the mass of one carbon-12 atom.

The mass of $6{\cdot}02 \times 10^{26}$ atoms of carbon –12 is 12 kg*.

** Avogadro number $N = 6{\cdot}02 \times 10^{26}$ per kg atom.*

$\therefore$ 1 a.m.u (or u) $= \frac{1}{12} \times \frac{12}{6{\cdot}02\times10^{26}}$ kg

$= 1{\cdot}66 \times 10^{-27}$ kg

The mass of large heavenly bodies is measured in terms of **solar mass** where 1 solar mass is the mass of the sun, *i.e.*,

1 solar mass $= 2 \times 10^{30}$ kg

The commonly used smaller and bigger units of mass are summarized in the following table.

Smaller and bigger units of mass

Smaller units	Value in kg	Bigger units	Value in kg
g	10^{-3} kg	quintal	100 kg
mg	10^{-6} kg	metric tonne	1000 kg
u (or a.m.u.)	$1{\cdot}66 \times 10^{-27}$ kg	solar mass	2×10^{30} kg

1.5 UNITS OF TIME

S.I. unit of time

The S.I. unit of time is second (s).

A second is defined as* 1/86400 th *part of a mean solar day. i.e.,

$$1 \text{ s} = \frac{1}{86400} \times \textit{one mean solar day}$$

One solar day is the time taken by the earth to complete one rotation on its own axis.

For many years, the above definition of second remained in use. But mean solar day varies over the years, therefore in 1956, scientists agreed to consider one year 1900 and 12 hours as the ephemeris time and one year 1900 to be equal to 365·2422 days. Thus,

$$\begin{aligned} 1 \text{ year } 1900 &= 365{\cdot}2422 \text{ days} \\ &= 365{\cdot}2422 \times 86400 \text{ s} \\ &= 31556925{\cdot}9747 \text{ s} \end{aligned}$$

Hence one second is defined as $\frac{1}{31556925{\cdot}9747}$ th part of the year 1900. *i.e.*,

$$1 \text{ s} = \frac{1}{31{,}556{,}925{\cdot}9747} \text{ th part of the year 1900.}$$

In 1964, a second was defined in terms of energy change in cesium atom as follows :

One second is the time interval of 9,192,631,770 vibrations of radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium - 133 atom.

Smaller units of time

The common smaller units of time are millisecond (ms), microsecond (μs), shake and nanosecond (ns) where

1 ms $= 10^{-3}$ s; 1 μs $= 10^{-6}$ s;

1 shake $= 10^{-8}$ s and 1 ns $= 10^{-9}$ s.

Bigger units of time

Sometimes second is a smaller unit of time and so we use other units of time such as **(i) minute, (ii) hour, (iii) day, (iv) month, (v) lunar month, (vi) year, (vii) leap year, (viii) decade, (ix) century and (x) millennium**. They are defined as below.

(i) minute (min) : *One minute is the duration of 60 second. i.e.,* 1 min = 60 s

(ii) hour (h) : *One hour is the duration of 60 minutes. i.e.,*

$$\begin{aligned} 1 \text{ h} &= 60 \text{ min} \\ &= 60 \times 60 \text{ s} = 3600 \text{ s} \end{aligned}$$

(iii) day : *The time taken by the earth to rotate once on its own axis is called a day*. One day is divided in 24 hours. Thus,

$$\begin{aligned} 1 \text{ day} &= 24 \text{ h} \\ &= 24 \times 60 \text{ min} = 1440 \text{ min} \\ &= 24 \times 60 \times 60 \text{ s} = 86400 \text{ s} \end{aligned}$$

(iv) month : The western or Gregorian Calendar has January, March, May, July, August, October and December each of 31 days; April, June, September and November each of 30 days and Feburary of 28 days (or 29 days in a leap year). To an approximation, a month is considered to be of 30 days and a year of 12 months to be of 365 days.

(v) lunar month : The western or Gregorian Calendar is based on the period of revolution of earth around the sun, but our Hindu (Vikram and Shak) and Muslim (Hizri) Calendars are based on the phases of moon as seen from our earth. In these calendars, one month is the time of one lunar cycle which is nearly 29.5 days. The period of 12 lunar months is 354·37 days.

(vi) year (yr) : *One year is defined as the time in which the earth completes one revolution around the sun.* The period of revolution of earth around the sun is nearly 365 days. Thus,

$$\begin{aligned} 1 \text{ yr} &= 365 \text{ days} \\ &= 365 \times 86400 \text{ s} = 3{\cdot}1536 \times 10^{7} \text{ s} \end{aligned}$$

(vii) Leap year : *A leap year is the year in which the month of February is of 29 days, i.e.,*

$$1 \text{ Leap year} = 366 \text{ days}$$

Since the exact period of revolution of the earth around the sun is 365·2422 days, therefore to compensate for the excess of 0·2422 days in a year, the English calendar has been modified as follows :

Every fourth year (*i.e.*, the year divisible by 4) has one day extra in the month of February (*i.e.*, the February then has 29 days) and so it is the leap year. For example, the years like 1904, 1908, ..., 2000, 2004, 2008, 2012, were the leap years and the years 2016, 2020, ... will also be the leap years. The exception is :

The century years (*i.e.*, 1800, 1900 etc.) though divisible by 4, are not the leap years *e.g.* the year 2100 will not be a leap year. But the years which are divisible by 400, will be the leap year. e.g. the year 2000 was a leap year and the year 2400 will also be a leap year.

(viii) Decade : *A decade is of 10 years.* Thus,

$$1 \text{ Decade} = 10 \text{ years} = 3{\cdot}1536 \times 10^8 \text{ s}$$

(ix) Century : *A century is of 100 years.* In a century, there will be 24 years each of 366 days and 76 years each of 365 days. Thus,

$$\begin{aligned} 1 \text{ Century} &= 100 \text{ years} \\ &= (24 \times 366 + 76 \times 365) \text{ days} \\ &= 36524 \text{ days} = 3{\cdot}16 \times 10^9 \text{ s.} \end{aligned}$$

(x) Millennium : *A millennium is of 1000 years, i.e.* $1 \text{ Millennium} = 3{\cdot}16 \times 10^{10} \text{ s}$

The commonly used bigger units of time are summarized in the following table.

Bigger units of time

Bigger units	Value in second	Bigger units	Value in second
min	60 s	year	$3{\cdot}1536 \times 10^7$ s
h	3600 s	Decade	$3{\cdot}1536 \times 10^8$ s
day	86400 s	Century	$3{\cdot}16 \times 10^9$ s
month	$2{\cdot}592 \times 10^6$ s	Millennium	$3{\cdot}16 \times 10^{10}$ s

1.6 SOME EXAMPLES OF DERIVED UNITS

We have read that apart from the seven fundamental quantities used in S.I. system such as length, mass, time, temperature, luminous intensity, current and the amount of substance, the units of all other physical quantities are obtained in terms of the fundamental units. The units so obtained are called the derived units. Some examples of derived units are listed below.

Derived units of some physical quantities

Quantity	Definition	Derived unit	Abbreviation/symbol
1. Area	length × breadth	metre × metre	m^2
2. Volume	length × breadth × height	metre × metre × metre	m^3
3. Density	$\frac{\text{mass}}{\text{volume}}$	$\frac{\text{kilogram}}{(\text{metre})^3}$	kg m^{-3}
4. Speed or velocity	$\frac{\text{distance}}{\text{time}}$	$\frac{\text{metre}}{\text{second}}$	m s^{-1}
5. Acceleration	$\frac{\text{velocity}}{\text{time}}$	$\frac{\text{metre/second}}{\text{second}}$	m s^{-2}
6. Force	mass × acceleration	kilogram × $\frac{\text{metre}}{(\text{second})^2}$ or newton	kg m s^{-2} or N
7. Work or energy	force × displacement	kilogram × $\frac{\text{metre}}{(\text{second})^2}$ × metre or joule	kg m^2 s^{-2} or J
8. Momentum	mass × velocity	kilogram × $\frac{\text{metre}}{\text{second}}$ or newton × second	kg m s^{-1} or N s

9. Moment of force or torque	force × distance	$\text{kilogram} \times \frac{\text{metre}}{(\text{second})^2} \times \text{metre}$ or newton-metre	$kg\ m^2\ s^{-2}$ or N m
10. Power	$\frac{\text{work}}{\text{time}}$	$\frac{\text{kilogram}\,(\text{metre})^2}{(\text{second})^2} \Big/ \text{second}$ or joule/second or watt	$kg\ m^2\ s^{-3}$ or $J\ s^{-1}$ or W
11. Pressure	$\frac{\text{force}}{\text{area}}$	$\text{kilogram} \times \frac{\text{metre}}{(\text{second})^2} \Big/ (\text{metre})^2$ or newton/(metre)^2 or pascal	$kg\ m^{-1}\ s^{-2}$ or $N\ m^{-2}$ or Pa
12. Frequency	$\frac{1}{\text{time period}}$	$\frac{1}{\text{second}}$ or second^{-1} or hertz	s^{-1} or Hz
13. Electric charge	current × time	ampere × second or coulomb	A s or C
14. Electric potential or electromotive force (e.m.f.)	$\frac{\text{work}}{\text{charge}}$	$\frac{\text{kilogram} \times \text{metre}^2}{\text{second}^2} \Big/ \text{ampere} \times \text{second}$ or joule/coulomb or volt	$kg\ m^2\ A^{-1}\ s^{-3}$ or $J\ C^{-1}$ or V
15. Electrical resistance	$\frac{\text{potential}}{\text{current}}$	$\frac{\text{kilogram} \times \text{metre}^2}{\text{ampere} \times \text{second}^3} \Big/ \text{ampere}$ or $\frac{\text{volt}}{\text{ampere}}$ or ohm	$kg\ m^2\ A^{-2}\ s^{-3}$ or $V\ A^{-1}$ or Ω
16. Electrical power	potential × current	volt × ampere or watt	V A or W

From the above list, it can be noted that some derived units are complex when they are expressed in terms of fundamental units. Such derived units have been given special names after the name of the scientist who has contributed in that field. For example, newton for force, joule for work (or energy), watt for power, pascal for pressure, hertz for frequency, coulomb for electric charge, volt for electric potential (or e.m.f.) and ohm for electrical resistance. Full name of such a unit is always written with small initial letter, while its symbol is written with the first capital letter.

1.7 GUIDELINES FOR WRITING THE UNITS

Conventionally following rules are observed while writing the unit of a physical quantity :

(i) The symbol for a unit, which is not named after a scientist, is written in small letter. *For example*, symbol for metre is m, for second is s, for kilogram is kg and so on.

(ii) The symbol for a unit, which is named after a scientist, is written with first letter of his name in capital. *For example*, N for newton, J for joule, W for watt, Pa for pascal, Hz for hertz, C for coulomb and V for volt.

(iii) The full name of the unit, irrespective of the fact whether it is named after a scientist or not, is always written with a lower initial letter e.g., unit for mass is written as kilogram, not as Kilogram; unit of length is written as metre, not as Metre; unit of force is written as newton and not as Newton ; unit of energy is written as joule and not as Joule; unit of power is written as watt and not as Watt.

(iv) A compound unit formed by multiplication of two or more units is written after putting a dot, cross or leaving a space between the two symbols. For example, the unit of torque is written as N.m or N × m or N m.

(v) Negative power is used for compound units, which are formed by dividing one unit by the other.

Examples :

(1) The unit of velocity is $\frac{\text{metre}}{\text{second}}$. It is expressed as m s^{-1}.

(2) The unit of power is $\frac{\text{joule}}{\text{second}}$. It is expressed as J s^{-1}.

(vi) A unit in its short form is never written in plural. *For example*, 10 metres can not be written as 10 ms, because ms would mean millisecond.

(vii) To avoid powers of ten in the magnitude of a quantity, prefix can be used with its unit. But a unit must not be written with more than one prefixes. *For example*, instead of kMW, we must write GW.

(viii) When prefix is used with the symbol of unit, the prefix and symbol combined becomes the new symbol of the unit. *For example*, km^3 means $(10^3 \text{ m})^3 = 10^9 \text{ m}^3$; it does not mean 10^3 m^3.

EXAMPLES

1. (i) The mass of an atom of oxygen is 16·0 u. Express it in kg.

(ii) The mass of a molecule of hydrogen is $3{\cdot}332 \times 10^{-27}$ kg. Find the mass of 1 kg mol of hydrogen gas.

(i) Given : mass of one atom of oxygen = 16·0 u

$1 \text{ u} = 1{\cdot}66 \times 10^{-27}$ kg

∴ The mass of an atom of oxygen

$= 16{\cdot}0 \times 1{\cdot}66 \times 10^{-27}$ kg

$= \mathbf{2656 \times 10^{-26}}$ **kg**

(ii) Given : mass of a molecule of hydrogen

$= 3{\cdot}332 \times 10^{-27}$ kg

1 kg mol of hydrogen gas will contain $6{\cdot}02 \times 10^{26}$ molecules of hydrogen

∴ The mass of 1 kg mol of hydrogen gas

$= (6{\cdot}02 \times 10^{26}) \times 3{\cdot}332 \times 10^{-27}$ kg

= **2·0 kg**

2. The size of a particle is 4·6 μ. Express it in metre.

Given : size of a particle = 4·6 μ

Since $1 \mu = 10^{-6}$ m

∴ Size of a particle = $\mathbf{4{\cdot}6 \times 10^{-6}}$ **m**

3. It takes 5 years for light to reach the earth from a star. Express the the distance of star from the earth in (i) light year, (ii) km. Take speed of light = 3×10^8 m s^{-1} and 1 year = $3{\cdot}1 \times 10^7$ s.

(i) The distance travelled by light in 1 year is called 1 light year. Thus the distance travelled by light in 5 years = 5 light year.

∴ Distance of star from the earth = **5 light year.**

(ii) 1 light year = speed of light × time 1 year

$= (3 \times 10^8 \text{ m s}^{-1}) \times (3{\cdot}1 \times 10^7 \text{ s})$

$= 9{\cdot}3 \times 10^{15} \text{ m} = 9{\cdot}3 \times 10^{12}$ km

∴ Distance of star from the earth = $5 \times 9{\cdot}3 \times 10^{12}$ km

$= \mathbf{4{\cdot}65 \times 10^{13}}$ **km**

4. The average mass of an atom of uranium is $3{\cdot}9 \times 10^{-25}$ kg. Find the number of atoms in 1 g of uranium .

Given : mass of one atom of uranium

$= 3{\cdot}9 \times 10^{-25} \text{ kg} = 3{\cdot}9 \times 10^{-22}$ g

(since 1 kg = 10^3 g)

∴ Number of atoms in 1 g uranium

$= \frac{1}{3{\cdot}9 \times 10^{-22}} = \mathbf{2{\cdot}6 \times 10^{21}}$

EXERCISE 1(A)

1. What is meant by measurement ?
2. What do you understand by the term unit ?
3. What are the *three* requirements for selecting a unit of a physical quantity ?
4. Name the *three* fundamental quantities.
5. Name the *three* systems of unit and state the various fundamental units in them.
6. Define a fundamental unit.
7. What are the fundamental units in S.I. system ? Name them along with their symbols.
8. Explain the meaning of derived unit with the help of *one* example.
9. Define standard metre.
10. Name *two* units of length which are bigger than a metre. How are they related to the metre ?
11. Write the names of *two* units of length smaller than a metre. Express their relationship with metre.

12. How is nanometre related to Angstrom ?

13. Name the *three* convenient units used to measure length ranging from very short to very long value. How are they related to the S.I. unit ?

14. Name the S.I. unit of mass and define it.

15. Complete the following :

(a) 1 light year = ...m

(b) 1 m = ... Å

(c) 1 m = ...μ

(d) 1 micron = ... Å

(e) 1 fermi = ... m

Ans. (a) $9{\cdot}46 \times 10^{15}$ (b) 10^{10} (c) 10^{6} (d) 10^{4} (e) 10^{-15}

16. State *two* units of mass smaller than a kilogram. How are they related to the kilogram ?

17. State *two* units of mass bigger than a kilogram. Give their relationship with the kilogram.

18. Complete the following :

(a) 1 g = ... kg

(b) 1 mg = ... kg

(c) 1 quintal = ... kg

(d) 1 a.m.u (or u) = ... kg

Ans. (a) 10^{-3} (b) 10^{-6} (c) 100 (d) $1{\cdot}66 \times 10^{-27}$

19. Name the S.I. unit of time and define it.

20. Name *two* units of time bigger than a second. How are they related to the second ?

21. What is a leap year ?

22. 'The year 2016 will have February of 29 days'. Is this statement true ? **Ans.** Yes

23. What is a lunar month ?

24. Complete the following :

(a) 1 nano second = ...s.

(b) 1 μs = ...s.

(c) 1 mean solar day = ...s.

(d) 1 year = ...s.

Ans. (a) 10^{-9} (b) 10^{-6} (c) 86400 (d) $3{\cdot}15 \times 10^{7}$

25. Name the physical quantities which are measured in the following units :

(a) u (b) ly (c) ns (d) nm

Ans. (a) mass (b) distance (or length) (c) time (d) length

26. Write the derived units of the following :

(a) speed (b) force

(c) work (d) pressure.

Ans. (a) $m\ s^{-1}$ (b) $kg\ m\ s^{-2}$ (c) $kg\ m^2\ s^{-2}$ (d) $kg\ m^{-1}\ s^{-2}$

27. How are the following derived units related to the fundamental units ?

(a) newton (b) watt

(c) joule (d) pascal.

Ans. (a) $kg\ m\ s^{-2}$ (b) $kg\ m^2\ s^{-3}$ (c) $kg\ m^2\ s^{-2}$ (d) $kg\ m^{-1}\ s^{-2}$

28. Name the physical quantities related to the following units :

(a) km^2 (b) newton (c) joule

(d) pascal (e) watt

Ans. (a) area (b) force (c) energy (d) pressure (e) power

Multiple choice type :

1. The fundamental unit is :

(a) newton (b) pascal

(c) hertz (d) second. **Ans.** (d) second

2. Which of the following unit is not a fundamental unit :

(a) metre (b) litre

(c) second (d) kilogram.

Ans. (b) litre

3. The unit of time is :

(a) light year (b) parsec

(c) leap year (d) angstrom.

Ans. (c) leap year

4. 1 Å is equal to :

(a) 0·1 nm (b) 10^{-10} cm

(c) 10^{-8} m (d) 10^{4} μ.

Ans. (a) 0·1 nm

5. ly is the unit of :

(a) time (b) length

(c) mass (d) none of these.

Ans. (b) length

Numericals :

1. The wavelength of light of a particular colour is 5800 Å. Express it in (a) nanometre and (b) metre.

Ans. (a) 580 nm, (b) $5{\cdot}8 \times 10^{-7}$ m

2. The size of a bacteria is 1 μ. Find the number of bacteria in 1 m length. **Ans.** 10^{6}

3. The distance of a galaxy is $5{\cdot}6 \times 10^{25}$ m. Assuming the speed of light to be $3 \times 10^{8}\ m\ s^{-1}$ find the time taken by light to travel this distance.

[**Hint :** Time taken = $\dfrac{\text{Distance travelled}}{\text{speed}}$]

Ans. $1{\cdot}87 \times 10^{17}$ s

4. The wavelength of light is 589 nm. What is its wavelength in Å ?

Ans. 5890 Å

5. The mass of an oxygen atom is 16·00 u. Find its mass in kg. **Ans.** $2{\cdot}656 \times 10^{-26}$ kg

6. It takes time 8 min for light to reach from the sun to the earth surface. If speed of light is taken to be 3×10^8 m s^{-1}, find the distance from the sun to the earth in km. **Ans.** $1{\cdot}44 \times 10^8$ km

7. 'The distance of a star from the earth is 8·33 light minutes.' What do you mean by this statement ? Express the distance in metre. **Ans.** $1{\cdot}5 \times 10^{11}$ m

(B) MEASUREMENT OF LENGTH

1.8 LEAST COUNT OF A MEASURING INSTRUMENT

The measurement of a physical quantity (such as length, mass, time, current, etc.), requires an instrument. For example, a metre rule (or vernier callipers) is used for length, a balance for mass, a watch for time, a thermometer for temperature and an ammeter for current. Each instrument has a definite limit for accuracy of measurement which is expressed in terms of its least count.

The least count of an instrument is the smallest measurement that can be taken accurately with it.

A measuring instrument is provided with a graduated scale for measurement and *the least count is the value of one smallest division on its scale. For example*, the least count of a metre rule is the value of its one division which is one-tenth of a centimetre (or 1 mm). The least count of a stop watch is 0·5 second if there are 10 divisions between 0 and 5 s mark. The least count of an ammeter having 5 divisions between the marks 0 and 1 A, is 0·2 A.

Smaller the least count of an instrument, more precise is the measurement made by using it.

1.9 MEASUREMENT OF LENGTH

Generally a metre rule having its zero mark at one end and 100 cm mark at the other end is used to measure the length of an object. It has 10 subdivisions in each one centimetre length, so the value of its one small division is 1 mm (or 0·1 cm). Thus a metre rule can be used to measure length correct only up to 1 mm *i.e.*, one decimal place of a centimetre. It cannot measure length with still more accuracy *i.e.,* up to second decimal place of a centimetre. The reason is that if one end of object lies between two small divisions on metre rule, the mark nearer the end of the object is read and thus its length correct up to the second decimal point can not be measured. However it becomes possible with the help of the *vernier callipers and screw gauge*. They are more accurate since they have least count smaller than 0·1 cm.

1.10 PRINCIPLE OF VERNIER

Pierre Vernier devised a method by which length up to 2nd decimal place of a cm *i.e.,*correct up to 0·1 mm (or 0·01 cm) can be measured. In this technique, *two* scales are used. One scale, called the ***main scale***, is fixed, while the other scale, called the ***vernier scale***, slides along the main scale.

The main scale is graduated with value of one division on it equal to 1 mm. The graduations on the vernier scale are such that the length of n divisions on vernier scale is equal to the length of $(n - 1)$ divisions of the main scale. Generally, a vernier scale has 10 divisions and the total length of these 10 divisions is equal to the length of $10 - 1 = 9$ divisions of the main scale *i.e.,* equal to 9 mm. Thus each division of the vernier scale is of length 0·9 mm (*i.e.*, smaller in size by $\frac{1}{10}$ mm than a division on the main scale). This difference is utilised as least count for the measurement.

Least count of vernier or vernier constant

The least count of vernier is equal to the difference between the values of one main scale division and one vernier scale division. It is also called the ***vernier constant***. Thus,

Vernier constant or least count of vernier,
L.C. = value of 1 main scale division
– value of 1 vernier scale division.

...(1.1)

Let n divisions on vernier be of length equal to that of $(n - 1)$ divisions on main scale and the value of 1 main scale division be x. Then

Value of n divisions on vernier $= (n - 1)x$

$\therefore$ Value of 1 division on vernier $= \frac{(n-1)x}{n}$

Hence from eqn. (1.1),

$$\text{L.C.} = x - \frac{(n-1)x}{n} = \frac{x}{n}$$

i.e., $$\boxed{\text{L.C.} = \frac{\text{Value of one main scale division } (x)}{\text{Total number of divisions on vernier } (n)}}$$ ...(1.2)

Thus, the least count of a vernier is obtained simply by dividing the value of one division of main scale by the total number of divisions on vernier scale.

Example : Fig. 1.1 shows a main scale graduated to read up to 1 mm and a vernier scale on which the length of 10 divisions is equal to the length of 9 divisions on main scale. We are to find its least count.

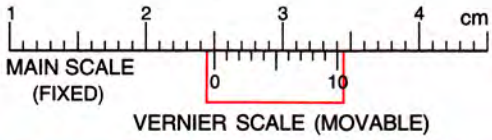

Fig 1.1 To find the least count of the vernier

In Fig. 1.1,

Value of 1 division of main scale (x) = 1 mm.

Total number of divisions on vernier (n) = 10

∴ From eqn. (1.2),

$$\text{L.C.} = \frac{x}{n} = \frac{1\text{ mm}}{10} = 0{\cdot}1 \text{ mm} = 0{\cdot}01 \text{ cm.}$$

Use of vernier scale

Fig. 1.2 illustrates the use of a vernier scale. The two scales (main scale and vernier scale) are so made that *when the movable vernier scale touches the fixed end, its zero mark coincides with the zero mark of the main scale*. The rod whose length is to be measured, is placed along the main scale and vernier scale is moved so as to hold the rod between the fixed end and the movable vernier scale. In this position, the zero mark of the vernier scale is ahead of 1·2 cm mark on main scale. Thus the actual length of the rod = 1·2 cm + the length *ab* (*i.e.*, the length between the 1·2 cm mark on main scale and 0 mark on vernier scale). The length *ab* cannot be measured by the main scale as its value is less than one division marked on the main scale.

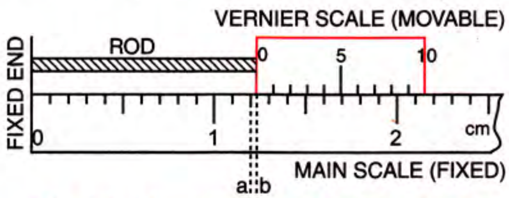

Fig. 1.2 Measurement with vernier scale

To measure the length *ab,* first we find the least count of the vernier scale. It is $\frac{0{\cdot}1}{10}$ cm = 0·01 cm. Then we note that p^{th} division of vernier scale which coincides (or which is in line) with any division of the main scale. The product of this number of vernier division *p* with the least count gives the length *ab.** This is called the *vernier reading*. Thus

$$\boxed{\begin{aligned}&\text{length } ab = \text{vernier reading} = p \times \text{least count,}\\ &\text{and Total reading} = \text{main scale reading}\\ &\qquad\qquad + \text{vernier reading}\end{aligned}}$$ ..(1.3)

In Fig. 1.2, the main scale reading is 1·2 cm and 4th division of vernier scale coincides with a main scale division and so the length *ab* (or vernier reading) is 4 × 0·01 cm = 0·04 cm.

Hence the length of rod is 1·2 + 0·04 = 1·24 cm.

1.11 VERNIER CALLIPERS

A vernier callipers is also called the *slide callipers*. It is used to measure the length of a rod, the diameter of a sphere, the internal and external diameters of a hollow cylinder, the depth of a small beaker (or bottle), etc.

(a) *Description*

A vernier callipers is shown in Fig. 1.3. It consists of a long and thin steel strip provided with a jaw J_1 at one end. On the strip, a scale is graduated with the value of one division equal to 1 mm. This is the *main scale*. Another small steel strip provided with a jaw J_2 at its end, can slide over the main scale strip. This strip also has a scale graduated with 10 divisions on it, the length of which is equal to 9 mm. It is called the *vernier scale*. For more precise measurement, the vernier scale can have 20, 25 or 50 divisions marked on it and the total length of vernier divisions will be equal to the length of one division less (*i.e.*, 19, 24 or 49 divisions respectively) on the main scale. The vernier scale which slides over the main scale, can

* If p^{th} division of vernier scale coincides with any division of main scale, then
ab + length of *p* divisions on vernier scale
= length of *p* divsions on main scale
length *ab* = length of *p* divisions on main scale
− length of *p* divisions on vernier scale
= *p* × (length of 1 division on main scale
− length of 1 division on vernier scale)
= *p* × L.C.

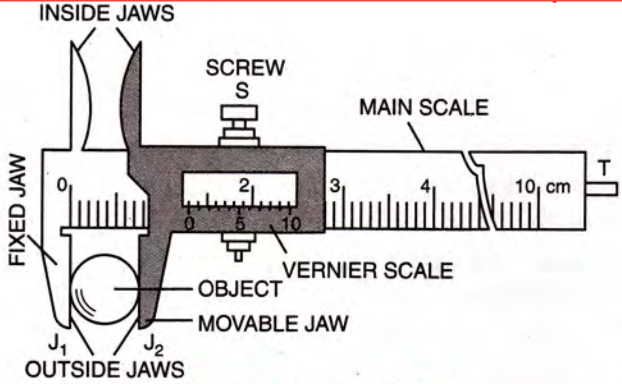

Fig. 1.3 Vernier callipers

be fixed at any position on the main scale with the help of a screw S. Both jaws are parallel to each other and are projected on either side of the main scale to hold the given object. The lower jaws are called the *outside jaws* and they are used to measure the length of a rod, diameter of a sphere or external diameter of a cylinder. The upper jaws are called the *inside jaws* which are used to measure the internal diameter of a hollow cylinder (or pipe).

A vernier callipers is also provided with a thin and long strip T attached to the vernier scale strip, at the back of the main scale strip. It slides with the vernier scale. When jaws J_1 and J_2 are in contact, the end of the strip T touches the end at the back of the main scale strip. The strip T is used to measure the depth of a small beaker (or bottle).

The table below gives the main parts of a vernier callipers with their functions.

Vernier callipers — main parts and their functions

Part	Function
1. Outside jaws	To measure the length of a rod, diameter of a sphere, external diameter of a hollow cylinder.
2. Inside jaws	To measure the internal diameter of a hollow cylinder or pipe.
3. Strip	To measure the depth of a beaker or a bottle.
4. Main scale	To measure length correct up to 1 mm.
5. Vernier scale	Helps to measure length correct up to 0·1 mm.

(b) *Least count of vernier callipers*

The least count of vernier callipers is equal to the difference between the values of one main scale division and one vernier scale division. It is calculated by using the eqn. (1.2), *i.e.*,

$$\text{L.C.} = \frac{\text{Value of one main scale division } (x)}{\text{Total number of divisions on vernier } (n)}$$

The least count of a vernier callipers can be decreased by (i) increasing the number of divisions on the vernier scale and (ii) decreasing the value of one division on main scale.

(c) *Zero error in vernier callipers*

On bringing the movable jaw J_2 in contact with the fixed jaw J_1, the zero mark of the vernier scale should coincide with the zero mark of the main scale. If it is so, the vernier is said to be *free from zero error.* In this condition, the end of strip T also touches the end of the main scale strip.

But sometimes there is a mechanical error in the vernier callipers due to which the zero mark of the vernier scale does not coincide with the zero mark of the main scale when the two jaws J_1 and J_2 are in contact. It is then said to have *zero error*. In such a case, *the zero error is equal to the length between the zero mark of the main scale and the zero mark of the vernier scale.* It is necessary to account for this error for a correct (or true) measurement from this instrument.

Kinds of zero error : The zero error is of the following *two* kinds :

(i) Positive zero error, and

(ii) Negative zero error.

(i) Positive zero error : On bringing the two jaws together, if zero mark of the vernier scale is on the ***right of zero mark*** of the main scale, the zero error is said to be ***positive.*** Fig. 1.4 shows the two scales of a vernier callipers with positive zero error. The positive zero error is equal to the length between the zero mark of the vernier scale from the zero mark of the main scale.

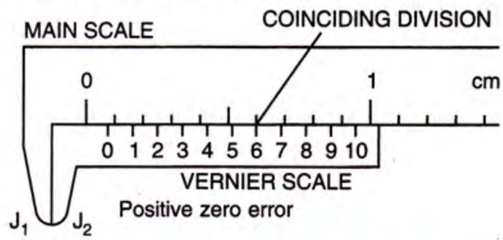

Fig. 1.4 Positive zero error

To find this error, we note that division of the vernier scale which coincides with any division of the main scale. The number of this vernier division when multiplied by the least count of the vernier, gives the zero error.

For example, for the scales shown in Fig. 1.4, the least count is 0·01 cm and the 6th division of vernier scale coincides with a main scale division.

$\therefore$ Zero error $= + 6 \times$ L.C. $= + 6 \times 0{\cdot}01$ cm

$= + 0{\cdot}06$ cm.

(ii) Negative zero error : On bringing the two jaws together, if zero mark of the vernier scale is to the ***left of zero mark*** of the main scale, the zero error is said to be ***negative***. Fig. 1.5 shows the two scales of a vernier callipers with negative zero error. The negative zero error is equal to the length between the zero mark of the main scale from the zero mark of the vernier scale.

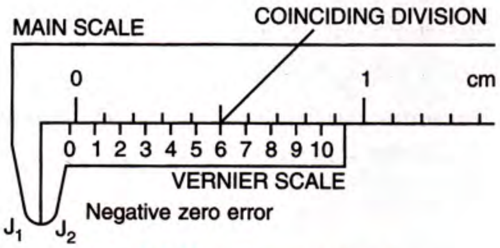

Fig. 1.5 Negative zero error

To find this error, we note that division of the vernier scale which coincides with any division of the main scale. The number of this vernier division is subtracted from the total number of divisions on the vernier scale and then the difference is multiplied by the least count.

In Fig. 1.5, the least count is 0·01 cm and the sixth division of the vernier scale coincides with a certain division of the main scale. The total number of divisions on vernier are 10.

$\therefore$ Zero error $= -(10 - 6) \times$ L.C.

$= -4 \times 0{\cdot}01$ cm $= -0{\cdot}04$ cm.

(d) ***Correction due to zero error i.e., correct measurement with a vernier callipers having a zero error***

To get the correct reading, *the zero error with its proper sign is always subtracted from the observed reading. i.e.,*

Correct reading = Observed reading – Zero error (with sign) ...(1.4)

Thus the positive zero error gets subtracted from the observed reading, while the negative zero error gets added to the observed reading.

(e) ***Measurement of length of an object with a vernier callipers***

Procedure :

(i) Find the least count and zero error of the vernier callipers.

(ii) Move the jaw J_2 away from the jaw J_1 and place the object to be measured, between the jaws J_1 and J_2. Move the jaw J_2 towards the jaw J_1 till it touches the object. Tighten the screw S to fix the vernier scale in its position.

(iii) Note the main scale reading.

(iv) Note that division p on vernier scale which coincides or is in line with any division of the main scale. Multiply this vernier division p with the least count. This is the vernier scale reading *i.e.*, Vernier scale reading $= p \times$ L.C.

(v) Add the vernier scale reading to the main scale reading. This gives the observed length.

(vi) Repeat it *two* times and record the observations as below.

Observations :

Total number of divisions on vernier scale $n =$ ....

Value of one division on main scale $x =$ .... cm

Least count (L.C.) $= \dfrac{x}{n} =$ .... cm

Zero error = ... cm

S. No.	*Main scale reading a (in cm)*	*Vernier divsion coinciding p*	*Vernier scale reading b = p × L.C. (in cm)*	*Observed length = a + b (in cm)*
1.				
2.				
3.				

Mean observed length = ... cm

From the mean observed length, subtract the zero error, it any, with its proper sign to obtain the true measurement of the length of the given object. Thus,

Observed length = main scale reading + (vernier division p coinciding with any division on the main scale) × least count.

True length = observed length – zero error (with sign)

Example : Fig. 1.6 illustrates how to read a vernier callipers.

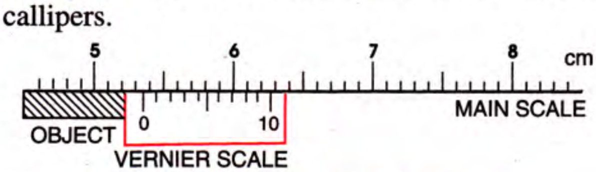

Fig. 1.6 To read a vernier callipers

In Fig. 1.6,

The least count of vernier callipers = 0·01 cm

Main scale reading = 5·3 cm

6th division of vernier scale coincides with a division on main scale *i.e.*, $p = 6$

$\therefore$ Vernier scale reading $= 6 \times 0{\cdot}01 = 0{\cdot}06$ cm

$$\begin{aligned}\text{Hence observed reading} &= \text{main scale reading} + \text{vernier scale reading}\\ &= 5{\cdot}3 \text{ cm} + 0{\cdot}06 \text{ cm}\\ &= 5{\cdot}36 \text{ cm}\end{aligned}$$

If the vernier callipers is free from zero error, then true length = 5·36 cm.

1.12 PRINCIPLE OF A SCREW

An ordinary screw has threads on it at an equal distance along its length. On rotating the head of the screw, it moves forward or backward linearly along its axis. The linear distance which the screw moves in one round, is equal to the distance between the two consecutive threads on it. This distance is called the ***pitch*** of the screw. Thus,

> ***The pitch of a screw is the distance moved along its axis by the screw in one complete rotation of its head.***

Generally, the pitch of a screw is 1 mm or 0·5 mm.

To use the linear movement of a screw for measuring small lengths, the head of the screw is made large and it is graduated along its circumference. Normally it has 50 or 100 equal divisions on it. This is called the ***circular*** or ***head scale***.

Least count of a screw

If pitch of a screw is 1 mm, and it has 100 divisions on its head, then on rotation of 100 divisions of its circular scale, the pointed end of the screw moves by a distance equal to 1 mm. Hence the distance moved by the screw along its axis, on rotation of 1 division of the circular scale will be $\frac{1\text{ mm}}{100} = 0{\cdot}01$ mm $= 0{\cdot}001$ cm. This is the least distance which can be measured by the movement of screw and is therefore called its ***least count***. Thus,

> ***The least count of a screw is the distance moved along the axis by it in rotating the circular scale by one division.***

The least count of a screw can be obtained by dividing the pitch of the screw by the total number of divisions on its circular scale. *i.e.*,

$$L.C. = \frac{\textit{Pitch of screw}}{\textit{Total number of divisions on circular scale}} \qquad ..(1.5)$$

Examples :

(1) If a screw moves by 1 mm in one rotation and it has 100 divisions on its circular scale, then pitch of the screw = 1 mm and least count of the screw $= \frac{1\text{mm}}{100} = 0{\cdot}01$ mm $= 0{\cdot}001$ cm.

(2) If a screw moves by 1 mm in two rotations and its circular scale has 50 divisions, then pitch of the screw $= \frac{1}{2}$ mm (or 0·5 mm) and the least count of the screw $= \frac{0{\cdot}5\text{ mm}}{50}$ $= 0{\cdot}01$ mm $= 0{\cdot}001$ cm.

(3) If a screw moves by 1 mm in two rotations and its circular scale has 500 divisions, then pitch of the screw is $\frac{1}{2}$ mm (or 0·5 mm) and the least count of the screw is equal to $\frac{0{\cdot}5\text{ mm}}{500}$ $= 0{\cdot}001$ mm (or 1 μ).

1.13 SCREW GAUGE

A screw gauge works on the principle of a screw. It is used to measure the diameter of a wire or thickness of a paper, etc., usually up to an accuracy of third decimal place of a cm (*i.e.*, correct up to 0·001 cm). However, the least count of a micrometer screw gauge is 1 μ (or 0·0001 cm).

(a) *Description*

A screw gauge is shown in Fig. 1.7. It has a U-shaped frame with a flat end *A* called the *stud* at one end and a *nut N* with a cylindrical sleeve at the other end. *Both the nut and sleeve are threaded from inside*. A screw with its one flat end *B* can be moved inside the nut *N* by rotating its head which is in form of a hollow cylinder (or *thimble*) provided at

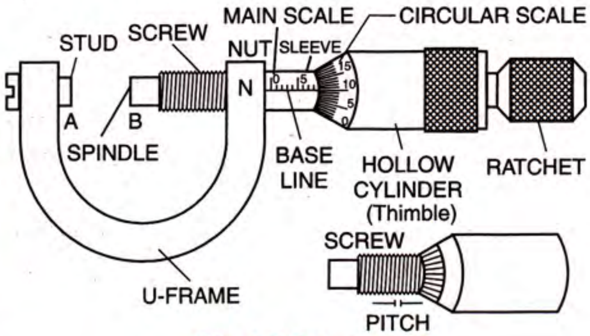

Fig. 1.7 Screw gauge

the other end of the screw. A reference line or *base line* graduated in mm is drawn on the cylindrical sleeve, parallel to the axis of the screw. This is the *main scale*. The hollow cylinder (or thimble) is also graduated and it is divided into 100 equal parts. This is the *circular scale* (or *head scale*). The thimble is attached with a *ratchet* by a spring. *The screw is always rotated by turning the ratchet.* As soon as the flat end *B* of the screw comes in contact either with the stud *A* or with the object in between *A* and *B*, any further rotation of the ratchet does not move the screw linearly to press *B* against *A* (or the object). Thus a ratchet helps in holding the given object *gently* between the stud *A* and the end *B* of the screw.

The table below gives the main parts of a screw gauge and their functions.

Screw gauge — main parts and their functions

Part	Function
1. Ratchet	To advance the screw by turning it till the object is gently held between the stud and spindle of the screw.
2. Sleeve	To mark main scale and base line.
3. Thimble	To mark circular scale.
4. Main scale	To read length correct up to 1 mm.
5. Circular scale	Helps to read length correct up to 0·01 mm.

(b) *Pitch and least count of a screw gauge*

The pitch of a screw gauge is the linear distance moved by its screw on the main scale when the circular scale is given one complete rotation.

However, the least count of a screw gauge is the linear distance moved by its screw along the main scale when the circular scale is rotated by one division on it. Thus,

Least count of screw gauge,

$$\text{L.C.} = \frac{\text{Pitch of the screw gauge}}{\text{Total number of divisions on its circular scale}} \quad ...(1.6)$$

Way to decrease the least count of a screw gauge : The least count of a screw gauge can be decreased by (i) decreasing the pitch and (ii) increasing the total number of divisions on the circular scale. In Fig. 1.8, a screw gauge is designed to have 20 divisions in 1 cm on main scale and 500 divisions on its circular scale. Thus the value of one division on main scale is $\frac{1}{20}$ cm = 0·05 cm or 0·5 mm. The screw moves 1 division along the main scale in one complete rotation of circular scale so its pitch is 0·5 mm and the least count is $\frac{0{\cdot}5\text{ mm}}{500}$ = 0·001 mm or 10^{-6} m. Therefore it is called the micrometer screw gauge. Such a screw gauge is used where high degree of accuracy is required *e.g.* in optical measurements.

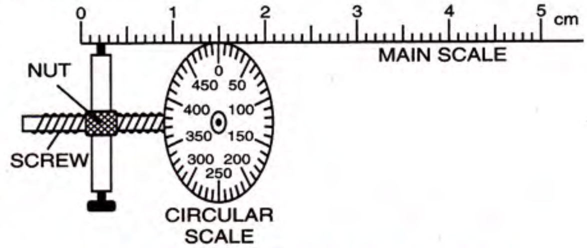

Fig. 1.8 Micrometer screw gauge

(c) *Zero error in a screw gauge*

On bringing the flat end *B* of the screw in contact with the stud *A*, if the zero mark of circular scale coincides with the base line of main scale, the screw gauge is said to be *free from zero error.*

But sometimes, due to mechanical error, on bringing the stud *A* in contact with the stud *B*, the zero mark of the circular scale is either below or above the base line of main scale, then the screw gauge is said to have a zero error.

Kinds of zero error : The zero error is of the following *two* kinds :

(i) Positive zero error, and (ii) Negative zero error.

(i) Positive zero error : If on bringing the flat end *B* of the screw in contact with the stud *A*, the zero mark on the circular scale is ***below the base line*** of main scale, the zero error is said to be ***positive.*** Fig. 1.9 shows the position of two scales of a screw gauge with positive zero error. To find it, we note that division of the circular scale which coincides with the base line. This number when multiplied with the least count of the screw gauge, gives the value of positive zero error.

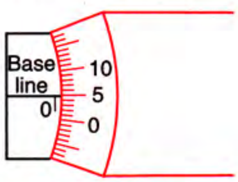

Fig. 1.9 Positive zero error

In Fig. 1.9, the 5th division of circular scale coincides with the base line. If the least count of the screw gauge is 0·001 cm, then zero error = + 5 × 0·001 cm = + 0·005 cm.

(ii) Negative zero error : If on bringing the flat end *B* of the screw in contact with the stud *A*, the zero mark on the circular scale is ***above the base line*** of main scale, the zero error is said to be ***negative.*** Fig. 1.10 shows the position of two scales of a screw gauge with negative zero error. To find it, we note the division of the circular scale coinciding with the base line. This number is subtracted from the total number of divisions on the circular scale and is then multiplied with the least count of the screw gauge. This gives the value of *negative zero error.*

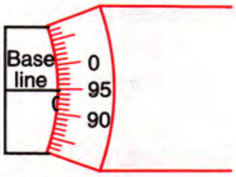

Fig. 1.10 Negative zero error

In Fig. 1.10, 95th division of circular scale coincides with the base line and total number of divisions on the circular scale are 100. If least count of the screw gauge is 0·001 cm, then zero error = – (100 – 95) × 0·001 cm = – 0·005 cm.

(d) *Correction due to zero error i.e., correct reading with a screw gauge having a zero error*

To find the correct reading, the zero error *with its sign* is subtracted from the observed reading. Thus,

Correct reading = observed reading – zero error (with sign) ....(1.7)

(e) *Measurement of diameter of a wire with a screw gauge*

Procedure :

(i) Find the least count and the zero error of the screw gauge.

(ii) Turn the ratchet anticlockwise so as to obtain a gap between the stud *A* and the flat end *B*. Place the wire in the gap between the stud *A* and the flat end *B*. Then turn the ratchet clockwise so as to hold the given wire gently between the stud *A* and the flat end *B* of the screw.

(iii) Note the main scale reading.

(iv) Note that division *p* of the circular scale which coincides with the base line of the main scale. This circular scale division *p* when multiplied by the least count, gives the circular scale reading *i.e.*,

Circular scale reading = p × L.C.

(v) Add the circular scale reading to the main scale reading to obtain the total reading (*i.e.*, the observed diameter of the wire).

(vi) Repeat it by keeping the wire in perpendicular direction. Take *two* more observations at different places of wire and record them in the table below.

Observation : Pitch of the screw = ... cm

Total number of divisions on circular scale = ...

Least counting screw gauge (L.C.)

$$= \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}} = \text{...........}\ \text{cm}$$

Zero error = ...

S.No.	Main scale reading *a* (in cm)	Circular reading *b* = number of division of circular scale in line of bass line *p* × L.C. (in cm)	Observed diameter = *a* + *b* (in cm)
1. (i) in one direction (ii) in ⊥ direction			
2. (i) (ii)			
3. (i) (ii)			

Mean observed diameter = ... cm

From the observed mean diameter, subtract the zero error (if any) with its sign to get the true diameter. Thus,

Observed diameter = main scale reading + (circular scale division p coinciding the base line of main scale × least count).
True diameter = observed diameter − zero error (with sign). ...... (1.8)

Example : Fig. 1.11 illustrates how to read a screw gauge.

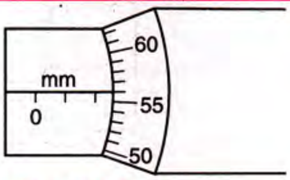

Fig. 1.11 Reading a screw gauge

In Fig. 1.11,

The pitch of the screw = 0·1 cm

The least count of the screw gauge = 0·001 cm.

Main scale reading = 2 mm = 0·2 cm.

56th division of circular scale coincides with the base line *i.e.*, $p = 56$

∴ Circular scale reading = 56 × 0·001 cm
= 0·056 cm.

Hence, observed reading = 0·2 cm + 0·056 cm.
= 0·256 cm.

If the screw gauge is free from zero error, then true reading = 0·256 cm.

(f) *Backlash error*

Sometimes due to wear and tear of threads of screw, it is observed that on reversing the direction of rotation of the thimble, the tip of the screw does not start moving in the opposite direction *at once*, but it remains stationary for a part of rotation. This causes error in the observation which is called the *backlash error*.

To avoid the backlash error, while taking the measurements, the screw should be rotated in one direction only. If it is required to change the direction of rotation of the screw, do not change the direction of rotation at once. Move the screw still further, stop there for a while and then rotate it in the reverse direction.

Note : From above discussion, we find that a metre rule (L.C. = 1 mm), vernier callipers (L.C. = 0·1 mm) and screw gauge (L.C. = 0·01 mm) have decreasing least count which leads to the increase in accuracy.

EXAMPLES

1. In an instrument, there are 25 divisions on the vernier scale which have length of 24 divisions of the main scale. 1 cm on main scale is divided in 20 equal parts. Find the least count.

The value of one main scale division $x = \frac{1}{20}$ cm

The number of divisions on vernier scale $n = 25$

∴ L.C. of vernier

$$= \frac{\text{Value of one main scale division } (x)}{\text{Number of divisions on vernier scale } (n)}$$

$$= \frac{(1/20)\text{ cm}}{25} = \frac{1}{500}\text{ cm} = \mathbf{0{\cdot}002\ cm.}$$

2. Fig. 1.12 given below shows the two scales of a vernier callipers. Find : (i) the least count of the vernier, and (ii) the reading shown in the diagram.

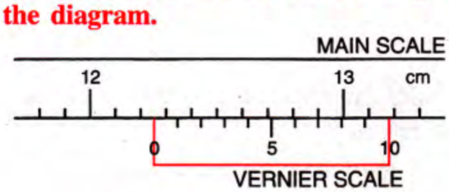

Fig. 1.12

(i) There are 10 divisions in 1 cm on the main scale.

∴ The value of one main scale division $x = \frac{1}{10}$ cm
= 0·1cm.

The number of divisions on vernier scale $n = 10$.

∴ The least count of vernier $= \frac{x}{n} = \frac{0{\cdot}1\text{cm}}{10} = \mathbf{0{\cdot}01\ cm.}$

(ii) Main scale reading = 12·2 cm.

Since 7th division of vernier scale coincides with the main scale division, so, $p = 7$

∴ Vernier scale reading = 7 × 0·01 cm = 0·07 cm.

Total reading = main scale reading + vernier scale reading
= 12·2 + 0·07 = **12·27 cm**

3. The least count of a vernier callipers is 0·01 cm and its zero error is + 0·02 cm. While measuring the length of a rod, the main scale reading is 4·8 cm and sixth division on vernier scale is in line with a marking on the main scale. Calculate the length of the rod.

Given, L.C. = 0·01cm, zero error = + 0·02 cm, main scale reading = 4·8 cm, and number of vernier division coinciding with the main scale division is 6, so, $p = 6$

Length of the rod = observed reading − zero error

= [main scale reading + (coinciding vernier division × L.C.)] − zero error.

= [(4·8 cm) + (6 × 0·01 cm)] + (− 0·02 cm)

= (4·8 cm + 0·06 cm) − (+ 0·02 cm)

= 4·86 cm − 0·02 cm = **4·84 cm.**

4. The circular head of a screw gauge is divided into 50 divisions and the screw moves 1 mm ahead in two revolutions of the circular head. Find its (a) pitch and (b) least count.

Given, number of divisions on circular head = 50, and distance moved in two revolutions = 1 mm

(a) Pitch = distance moved ahead in 1 revolution

$$= \frac{1 \text{ mm}}{2} = 0{\cdot}5 \text{ mm} = \mathbf{0{\cdot}05 \text{ cm}}$$

$$\text{(b) Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular head}}$$

$$\therefore \quad \text{L.C.} = \frac{0{\cdot}05 \text{ cm}}{50} = \mathbf{0{\cdot}001 \text{ cm.}}$$

5. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular head. While measuring the diameter of a wire, the main scale reads 3 mm and the 55th division is in line with the base line. Find the diameter of the wire. Assume that the screw gauge is free from zero error.

Given, pitch = 1 mm, number of divisions on circular head = 100, main scale reading = 3 mm, number of division of circular head in line with base line is 55, so, $p = 55$.

$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular head}}$$

$$= \frac{1 \text{ mm}}{100} = 0{\cdot}01 \text{ mm}$$

Circular scale reading $= p \times \text{L.C.} = 55 \times 0{\cdot}01$ mm

Diameter of the wire = main scale reading + circular scale reading

= 3 mm + (55 × 0·01 mm)

= 3 mm + 0·55 mm

= **3·55 mm** or **0·355 cm.**

6. In Fig. 1.13, the pitch of the screw is 1 mm. Find : (i) the least count of screw gauge and (ii) the reading represented in the diagram.

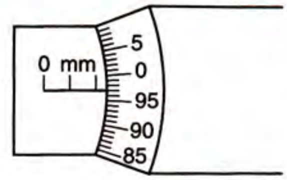

Fig. 1.13

(i) Given, pitch of the screw = 1 mm

Number of divisions on the circular scale = 100

∴ L.C. of the screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \frac{1 \text{ mm}}{100}$$

= **0·01 mm** or **0·001 cm.**

(ii) Main scale reading = 2 mm = 0·2 cm

Since 97th division of head scale coincides with base line, *i.e.*, $p = 97$

∴ Circular scale reading $= p \times$ L.C.

$= 97 \times 0{\cdot}001$cm $= 0{\cdot}097$ cm

Total reading = main scale reading + circular scale reading

= 0·2 cm + 0·097 cm = **0·297 cm.**

7. A boy measures the length of a piece of pencil by metre rule, vernier callipers and screw gauge to be 1·2 cm, 1·24 cm and 1·243 cm respectively. State (a) the least count of each measuring instrument, (b) the accuracy in each measurement.

(a) Least count of metre rule = **0·1 cm**

Least count of vernier callipers = **0·01 cm**

Least count of screw gauge = **0·001 cm**

(b) Accuracy in 1·2 cm = **0·1 cm**

Accuracy in 1-24 cm = **0·01 cm**

Accuracy in 1·243 cm = **0·001 cm**

EXERCISE 1 (B)

1. Explain the meaning of the term 'least count of an instrument' by taking a suitable example.
2. A boy makes a ruler with graduations in cm on it (*i.e.*, 100 divisions in 1 m). To what accuracy this ruler can measure ? How can this accuracy be increased ?
3. A boy measures the length of a pencil and expresses it to be 2·6 cm. What is the accuracy

of his measurement ? Can he write it as 2·60 cm ?

4. Define least count of a vernier callipers. How do you determine it ?
5. Define the term 'Vernier constant'.
6. When is a vernier callipers said to be free from zero error ?
7. What is meant by zero error of a vernier callipers ? How is it determined ? Draw neat diagrams to explain it. How is it taken in account to get the correct measurement ?
8. A vernier callipers has a zero error + 0·06 cm. Draw a neat labelled diagram to represent it.
9. Draw a neat labelled diagram of a vernier callipers. Name its main parts and state their functions.
10. State *three* uses of a vernier callipers.
11. Name the *two* scales of a vernier callipers and explain, how is it used to measure a length correct up to 0·01 cm.
12. Describe in steps, how would you use a vernier callipers to measure the length of a small rod ?
13. Name the part of the vernier callipers which is used to measure the following :
    (a) external diameter of a tube,
    (b) internal diameter of a mug,
    (c) depth of a small bottle,
    (d) thickness of a pencil.
14. Explain the terms (i) pitch, and (ii) least count of a screw gauge. How are they determined ?
15. How can the least count of a screw gauge be decreased ?
16. Draw a neat and labelled diagram of a screw gauge. Name its main parts and state their functions.
17. State *one* use of a screw gauge.
18. State the purpose of ratchet in a screw gauge.
19. What do you mean by zero error of a screw gauge ? How is it accounted for ?
20. A screw gauge has a least count 0·001 cm and zero error + 0·007 cm. Draw a neat diagram to represent it.
21. What is backlash error ? Why is it caused ? How is it avoided ?
22. Describe the procedure to measure the diameter of a wire with the help of a screw gauge.
23. Name the instrument which can measure accurately the following :
    (a) the diameter of a needle,
    (b) the thickness of a paper,
    (c) the internal diameter of the neck of a water bottle,
    (d) the diameter of a pencil.

    Ans. (a) screw gauge (b) screw gauge (c) vernier callipers (d) screw gauge.
24. Which of the following measures a small length to a high accuracy : metre rule, vernier callipers, screw gauge ? **Ans.** screw gauge
25. Name the instrument which has the least count: (a) 0·1 mm (b) 1 mm (c) 0·01 mm.

    Ans. (a) vernier callipers (b) metre rule (c) screw gauge.

Multiple choice type :

1. The least count of a vernier callipers is :
   (a) 1 cm (b) 0·001 cm
   (c) 0·1 cm (d) 0·01 cm

   Ans. (c) 0·01 cm
2. A microscope has its main scale with 20 divisions in 1 cm and vernier scale with 25 divisions, the length of which is equal to the length of 24 divisions of main scale. The least count of microscope is :
   (a) 0·002 cm (b) 0·001 cm
   (c) 0·02 cm (d) 0·01 cm

   Ans. (a) 0·002 cm
3. The diameter of a thin wire can be measured by :
   (a) a vernier callipers (b) a metre rule
   (c) a screw gauge (d) none of these.

   Ans. (c) a screw gauge

Numericals :

1. A stop watch has 10 divisions graduated between the 0 and 5 s marks. What is its least count ? **Ans.** 0·5 s.
2. A vernier has 10 divisions and they are equal to 9 divisions of main scale in length. If the main scale is calibrated in mm, what is its least count ? **Ans.** 0·01 cm.
3. A microsocpe is provided with a main scale graduated with 20 divisions in 1 cm and a vernier scale with 50 divisions on it of length same as of 49 divisions of main scale. Find the least count of the microscope. **Ans.** 0·001 cm
4. A boy uses a vernier callipers to measure the thickness of his pencil. He measures it to be 1·4 mm. If the zero error of vernier callipers is + 0·02 cm, what is the correct thickness of pencil ? **Ans.** 1·2 mm

5. A vernier callipers has its main scale graduated in mm and 10 divisions on its vernier scale are equal in length to 9 mm. When the two jaws are in contact, the zero of vernier scale is ahead of zero of main scale and 3rd division of vernier scale coincides with a main scale division. Find : (i) the least count and (ii) the zero error of the vernier callipers.

Ans. (i) 0·01 cm (ii) + 0·03 cm

6. The main scale of a vernier callipers is calibrated in mm and 19 divisions of main scale are equal in length to 20 divisions of vernier scale. In measuring the diameter of a cylinder by this instrument, the main scale reads 35 divisions and 4th division of vernier scale coincides with a main scale division. Find : (i) least count and (ii) radius of cylinder.

Ans. (i) 0·005 cm, (ii) 1·76 cm

7. In a vernier callipers, there are 10 divisions on the vernier scale and 1 cm on the main scale is divided in 10 parts. While measuring a length, the zero of the vernier lies just ahead of 1·8 cm mark and 4th division of vernier coincides with a main scale division.

(a) Find the length.

(b) If zero error of vernier callipers is – 0·02 cm, what is the correct length ?

Ans. (a) 1·84 cm, (b) 1·86 cm

8. While measuring the length of a rod with a vernier callipers, Fig. 1.14 below shows the position of its scales. What is the length of the rod ?

Fig. 1.14 **Ans.** 3·36 cm

9. The pitch of a screw gauge is 0·5 mm and the head scale is divided in 100 parts. What is the least count of screw gauge ?

Ans. 0·005 mm or 0·0005 cm.

10. The thimble of a screw gauge has 50 divisions. The spindle advances 1 mm when the screw is turned through two revolutions.

(i) What is the pitch of screw gauge ?

(ii) What is the least count of the screw gauge?

Ans. (i) 0·5 mm (ii) 0·01 mm.

11. The pitch of a screw gauge is 1 mm and its circular scale has 100 divisions. In measurement of the diameter of a wire, the main scale reads 2 mm and 45th mark on circular scale coincides with the base line. Find :

(i) the least count, and

(ii) the diameter of the wire.

Ans. (i) 0·001cm (ii) 0·245 cm.

12. When a screw gauge of least count 0·01 mm is used to measure the diameter of a wire, the reading on the sleeve is found to be 1 mm and the reading on the thimble is found to be 27 divisions. (i) What is the diameter of the wire in cm ? (ii) If the zero error is + 0·005 cm, what is the correct diameter ?

Ans. (i) 0·127 cm (ii) 0·122 cm.

13. A screw gauge has 50 divisions on its circular scale and its screw moves by 1 mm on turning it by two rotations. When the flat end of the screw is in contact with the stud, the zero of circular scale lies below the base line and 4th division of circular scale is in line with the base line. Find : (i) the pitch, (ii) the least count and (iii) the zero error, of the screw gauge.

Ans. (i) 0·5 mm (ii) 0·01 mm (iii) + 0·04 mm

14. Fig. 1.15 below shows the reading obtained while measuring the diameter of a wire with a screw gauge. The screw advances by 1 division on main scale when circular head is rotated once.

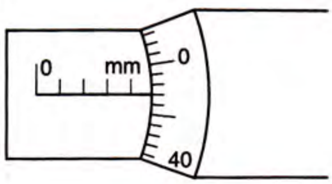

Fig 1.15

Find : (i) pitch of the screw gauge,

(ii) least count of the screw gauge, and

(iii) the diameter of the wire.

Ans. (i) 1 mm (ii) 0·02 mm (iii) 4·94 mm.

15. A screw has a pitch equal to 0·5 mm. What should be the number of divisions on its head so as to read correct up to 0·001 mm with its help ? **Ans.** 500

1.14 MEASUREMENT OF TIME

In offices and home, we commonly use a *pendulum clock* to note time which is based on the periodic oscillations of a pendulum. Here we shall study the principle of a simple pendulum.

Simple pendulum

A simple pendulum is a heavy point mass (known as bob) suspended from a rigid support by a massless and inextensible string. This is an ideal case because we cannot have a heavy mass having the size of a point and a string which has no mass. Fig. 1.16 shows a simple pendulum. Here a heavy solid (iron or brass) ball is suspended by a light, but strong thread from a rigid support. The ball is called the bob. In Fig. 1.16, the rest (or mean) position of bob is O.

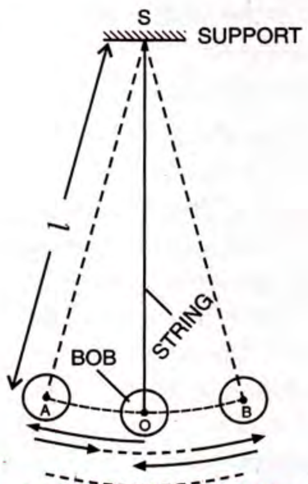

Fig. 1.16 A simple pendulum

When the bob from its mean position O is pulled to one side and then released, the pendulum is set in motion and the bob moves alternately on either side of its mean position.

Note : The pendulum used in a clock is not a simple pendulum, but it is a ***compound pendulum*** (*i.e.*, a body capable of oscillating about a *horizontal axis passing through it).*

(i) *Some terms related to simple pendulum*

Oscillation : *One complete to and fro motion of the bob of pendulum is called one oscillation.* For example in a simple pendulum shown in Fig. 1.16, the rest (or mean) position of bob is O, while its extreme positions on left and right sides are A and B respectively. One oscillation is the motion of the bob from O to A, A to B and then back from B to O as shown by arrows in Fig. 1.16. The motion of bob from A to B and then back from B to A also represent one oscillation.

Period of oscillation or time period : *This is the time taken to complete one oscillation.* It is denoted by the symbol T. Its unit is second (s).

Frequency of oscillation : *It is the number of oscillations made in one second.* It is denoted by the letter f or n. Its unit is per second (s^{-1}) or hertz (Hz).

Relationship between time period and frequency: If T is the time period of a simple pendulum, then

In time T second, the number of oscillation is 1.

∴ In time 1 second, the number of oscillations will be $\frac{1}{T}$ which is the frequency f.

i.e., $$\boxed{f = \frac{1}{T} \text{ or } T = \frac{1}{f}} \quad .....(1.18)$$

Amplitude : *The maximum displacement of the bob from its mean position on either side, is called the amplitude of oscillation*. In Fig. 1.16, the amplitude is OA or OB. It is denoted by the letter a or A and is measured in metre (m).

Effective length of a pendulum : *It is the distance of the point of oscillation O (i.e., the centre of gravity of the bob) from the point of suspension S.* In Fig. 1.16 it is shown by l.

(ii) *Measurement of time period of a simple pendulum*

To measure the time period of a simple pendulum, the bob is slightly displaced from its rest (mean) position O and is then released. It begins to move to and fro about its mean position O in a vertical plane along with the string. The time t for 20 complete oscillations is measured with the help of a stop watch and then dividing t by 20, its time period T is calculated*.

* To find time period, the time for number of oscillations more than 1 is noted because the least count of stop watch is either 1 s or 0·5 s. It can not record the time period in fraction such as 1·2 s or 1·4 s and so on. It is made possible by noting the time t for 20 oscillations or more than it and then dividing t by the number of oscillations.

The experiment is then repeated for different lengths of the pendulum. The observations are recorded in the table given below.

Time period for pendulum of different lengths

S.No	Length l (in cm)	Time for 20 oscillations t (in s)	Time period $T = \frac{t}{20}$ (in s)	$\frac{l}{T^2}$ (in cm s^{-2})
1	25	20	1·0	25
2	36	24	1·2	25
3	49	28	1·4	25
4	64	32	1·6	25
5	81	36	1·8	25
6	100	40	2·0	25

From the above observations 1 and 6, it can be noted that if the length of a pendulum is made four times, the period of oscillation gets doubled *i.e.*, now it takes twice the time for one complete to and fro motion. Thus time period T is directly proportional to the square root of length l of the pendulum ($T \propto \sqrt{l}$) or the square of time period T^2 is directly proportional to the length l of the pendulum (*i.e.*, $T^2 \propto l$) or l/T^2 is a constant.

(iii) *Graph showing the variation of square of time period (T^2) with the length (l) of a pendulum*

If a graph is plotted for the square of time period (T^2) taken on Y-axis against the length l taken on X-axis, it comes out to be a straight line inclined to the l-axix as shown in Fig. 1.17. This shows that T^2 is directly proportional to l.

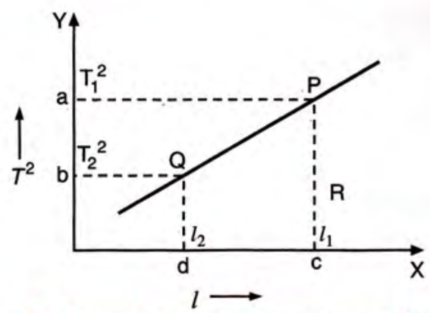

Fig. 1.17 Graph showing variation of T^2 with l

(iv) *Slope of T^2 vs l graph*

The slope of the straight line obtained in the T^2 vs l graph as shown in Fig. 1.17 can be obtained by taking two points P and Q on the straight line and drawing normals from these points on the X and Y axes. Then note the value of T^2 say T_1^2 and T_2^2 at a and b respectively, and also the value of l say l_1 and l_2 respectively at c and d. Then

$$Slope = \frac{PR}{QR} = \frac{ab}{cd} = \frac{T_1^2 - T_2^2}{l_1 - l_2}$$

This slope is found to be a constant at a place and is equal to $\frac{4\pi^2}{g}$ where g is the acceleration due to gravity at that place. Thus g can be determined at a place from these measurements by using the following relation :

$$\boxed{g = \frac{4\pi^2}{\text{Slope of } T^2 \text{ vs } l \text{ graph}}} \quad ....(1.9)$$

(v) *Factors affecting the time period of a simple pendulum*

From experiments on simple pendulum, it is observed that

(i) *The time period of oscillation is directly proportional to the square root of its effective length* *i.e.*, $T \propto \sqrt{l}$ or in other words, the square of time period of oscillation (T^2) is directly proportional to its effective length (l) *i.e.*, $T^2 \propto l$.

A pendulum clock has a compound pendulum made up of a metal like brass (or steel). Due to seasonal change of temperature, the effective length of pendulum changes, due to which the clock goes fast in winter and slow in summer. In winter due to contraction, the effective length of the pendulum gets shortened, and so its time period is decreased and the pendulum completes more oscillations in a given time *i.e.*, the clock goes fast. But in summer because of expansion *i.e.*, increase in the effective length of pendulum, its time period is increased and it completes less number of oscillations in a given time *i.e.*, the clock goes slow.

Similarly, while swinging, if we *stand* on the swing, the time period of swing decreases (*i.e.*, the swing moves faster). This is because when we stand, the centre of gravity rises, so the effective length of the swing decreases due to which its time period decreases.

(ii) *The time period of oscillation is inversely proportional to the square root of acceleration due to gravity i.e., $T \propto \frac{1}{\sqrt{g}}$*

For this reason, a pendulum clock goes slow (*i.e.*, the time period of oscillation increases) when it is taken to mountains or to mines due to decrease in the value of g.*

(iii) *The time period of oscillation does not depend on the mass or material of the body suspended (i.e., bob).* If we take two pendulums of equal lengths, but with bobs of different masses or different materials, their time periods will remain same.

(iv) *The time period of oscillation does not depend on the extent of swing on either side (i.e., on amplitude)* provided the swing is not too large.

(vi) *Expression for the time period of simple pendulum*

The time period of oscillation of the pendulum is given by the following relation :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or
$$\boxed{T^2 = 4\pi^2\frac{l}{g}} \quad .....(1.10)$$

where T = time period,

l = effective length of pendulum,

and g = acceleration due to gravity.

At a given place, since g is constant, the time period of a pendulum of given length is constant. This is why a pendulum can be used to measure the time.

(vii) *Seconds' pendulum*

The pendulum of a clock which we use to note time in our house, is a seconds' pendulum. It takes time 1 s in moving from one extreme to the other extreme, so its time period is 2 s. Thus, *a pendulum with a time period of oscillation equal to two seconds, is known as a seconds pendulum.* The effective length of the seconds' pendulum, at a place where $g = 9{\cdot}8$ m s^{-2} (the average value), is nearly 1 metre.

* The value of acceleration due to gravity g decreases with altitude as well as with depth from the earth surface.

EXAMPLES

1. Calculate the length of a seconds' pendulum at a place where $g = 9{\cdot}8$ m s^{-2}.

For seconds' pendulum $T = 2{\cdot}0$ s, $g = 9{\cdot}8$ m s^{-2}.

From the relation $T = 2\pi\sqrt{\frac{l}{g}}$,

length of pendulum, $l = \frac{gT^2}{4\pi^2}$

$\therefore \quad l = \frac{9{\cdot}8 \times (2{\cdot}0)^2}{4 \times (3{\cdot}14)^2} = \mathbf{0{\cdot}994\ m}$

2. Compare the time periods of a simple pendulum at places where g is $9{\cdot}8$ m s^{-2} and $4{\cdot}36$ m s^{-2} respectively.

Given : $g_1 = 9{\cdot}8$ m s^{-2}, $g_2 = 4{\cdot}36$ m s^{-2}

Since $\quad T \propto \frac{1}{\sqrt{g}} \quad \therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

or $\quad \frac{T_1}{T_2} = \sqrt{\frac{4{\cdot}36}{9{\cdot}8}} = \frac{1}{1{\cdot}5} = \frac{2}{3}$

i.e., $T_1 : T_2 = \mathbf{2 : 3}$ (or $\mathbf{0{\cdot}667 : 1}$)

3. Compare the time periods of two simple pendulums of length 1 m and 16 m at a place.

Given : $l_1 = 1$ m and $l_2 = 16$ m

Since $\quad T \propto \sqrt{l} \quad \therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{1}{16}} = \frac{1}{4} \quad$ *i.e.*, $T_1 : T_2 = \mathbf{1 : 4}$

4. (a) A simple pendulum is made by suspending a bob of mass 500 g by a string of length 1 m. Calculate its time period at a place where $g = 10$ m s^{-2}.

(b) How will the time period in part (a) be affected if bob of mass 100 g is used, keeping the length of string unchanged ?

(a) Given : $l = 1$ m, $g = 10$ m s^{-2}

Time period $T = 2\pi\sqrt{\frac{l}{g}} = 2 \times 3{\cdot}14 \times \sqrt{\frac{1}{10}}$

$= \mathbf{1{\cdot}99\ s}$

(b) On changing the bob by other bob of different mass, the time period will **remain unaffected** because it does not depend on the mass of bob.

EXERCISE 1 (C)

1. What is a simple pendulum ? Is the pendulum used in a pendulum clock simple pendulum ? Give reason to your answer.

2. Define the terms : (i) oscillation, (ii) amplitude, (iii) frequency, and (iv) time period as related to a simple pendulum.

3. Draw a neat diagram of a simple pendulum. Show on it the effective length of the pendulum and its one oscillation.

4. Name *two* factors on which the time period of a simple pendulum depends. Write the relation for the time period in terms of the above named factors.

5. Name *two* factors on which the time period of a simple pendulum does not depend.

6. How is the time period of a simple pendulum affected, if at all, in the following situations :
   (a) the length is made four times,
   (b) the acceleration due to gravity is reduced to one-fourth,

   Ans. (a) doubled, (b) doubled,

7. How are the time period T and frequency f of an oscillation of a simple pendulum related ?

   Ans. $f = \frac{1}{T}$

8. How do you measure the time period of a given pendulum ? Why do you note the time for more than one oscillation ?

9. How does the time period (T) of a simple pendulum depends on its length (l) ? Draw a graph showing the variation of T^2 with l. How will you use this graph to determine the value of g (acceleration due to gravity) ?

10. Two simple pendulums A and B have equal length, but their bobs weigh 50 gf and 100 gf respectively. What would be the ratio of their time periods ? Give reason for your answer.

    Ans. 1 : 1 ; **Reason :** Time period does not depend on the weight of the bob.

11. Two simple pendulums A and B have length 1·0 m and 4·0 m respectively at a certain place. Which pendulum will make more oscillations in 1 minute ? Explain your answer.

    Ans. A. The reason is that $T \propto \sqrt{l}$ $\therefore$ time period of B is more (twice) than that of A. Hence A will make more oscillations (twice) in a given time, than B.

12. State how does the time period of a simple pendulum depend on (a) length of pendulum, (b) mass of bob, (c) amplitude of oscillation and (d) acceleration due to gravity.

13. What is a seconds' pendulum ?

14. State the numerical value of the frequency of oscillation of a seconds' pendulum. Does it depend on the amplitude of oscillation ?

    Ans. 0·5 s^{-1}, No.

Multiple choice type :

1. The length of a simple pendulum is made one-fourth. Its time period becomes :
   (a) four times (b) one-fourth
   (c) double (d) half.

   Ans. (d) half

2. The time period of a pendulum clock is :
   (a) 1 s (b) 2 s
   (c) 1 min (d) 12 h

   Ans. (b) 2 s

3. The length of a seconds' pendulum is nearly :
   (a) 0·5 m (b) 9·8 m
   (c) 1·0 m (d) 2·0 m.

   Ans. (c) 1·0 m

Numericals :

1. A simple pendulum completes 40 oscillations in one minute. Find its (a) frequency, (b) time period.

   Ans. (a) 0·67 s^{-1} (b) 1·5 s

2. The time period of a simple pendulum is 2 s. What is its frequency ? What name is given to such a pendulum ?

   Ans. 0·5 s^{-1}, seconds' pendulum

3. A seconds' pendulum is taken to a place where acceleration due to gravity falls to one-fourth. How is the time period of the pendulum affected, if at all ? Give reason. What will be its new time period ?

   Ans. The time period increases (it is doubled) because $T \propto \frac{1}{\sqrt{g}}$. Its new time period will be 4 s.

4. Find the length of a seconds' pendulum at a place where $g = 10 \text{ m s}^{-2}$ (Take $\pi = 3{\cdot}14$).

Ans. 1·0142 m

5. Compare the time periods of two pendulums of length 1 m and 9 m. **Ans.** 1 : 3

6. A pendulum completes 2 oscillations in 5 s. (a) What is its time period ? (b) If $g = 9{\cdot}8 \text{ m s}^{-2}$, find its length. **Ans.** (a) 2·5 s, (b) 1·55 m

7. The time periods of two simple pendulums at a place are in ratio 2 : 1. What will be the ratio of their length ?

Ans. 4 : 1

8. It takes 0·2 s for a pendulum bob to move from mean position to one end. What is the time period of pendulum ? **Ans.** 0·8 s

9. How much time does the bob of a seconds' pendulum take to move from one extreme of its oscillation to the other extreme ? **Ans.** 1 s