Optimization Theory for Communications
Final Project

Due: 2022/06/19

1. Nutrition Problem

The following table shows the amount of nutrition that contains in each fruit:

	A(i.u./100g)	**B1**(mg/100g)	**C**(mg/100g)	price($/100g)
litchi	0	0.19	63	14
banana	200	0.03	8	7
pineapple	110	0.1	29	6
Nutrition per day	**2000**	**1.2**	**100**	

The required nutrition per day is given in the last row.

(a) According to the above table, please give the food plan with the less price.

(b) Assume that the price of the fruits has the uncertainty, and the litchi, banana, pineapple have the average price of 14, 7, and 6, respectively. Please consider the robust linear programming with the deterministic model for the uncertainty of the price, where $\mathbf{P} = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 1.4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Please give the food plan with the less price under the worst case consideration.

2. Worst-case Beamforming Design Problem:

(a) Design beamforming weights for the worst-case sidelobe energy, and plot the angle spectrum.

$$\min_{\mathbf{w}} \max_{\theta\in\mathbf{\Theta}} \mathbf{w}^H \mathbf{a}(\theta)$$

$$\text{s.t.} \quad \mathbf{w}^H \mathbf{a}(\theta_d) = 1$$

where $\mathbf{P} = \int_{\Theta} \mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta$, $\theta \in \Theta = [-90^\circ, \theta_l] \cup [\theta_u, 90^\circ]$, and $\theta_l = 0^\circ, \theta_u = 15^\circ$. Compare the results with the number of antennas $M = 10$ and $M = 30$ with $\theta_d = 10°$.

3. FIR Design Problem:

Give a specified Gaussian filter with frequency coefficients $\left|H_{spec}(\omega)\right| = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\omega-\theta)^2}{2\sigma^2}\right)$, where

$\theta = \frac{\pi}{2}$; $\sigma^2 = 0.05$; $0 \le \omega \le \pi$, with group delay $\tau = 10$ (i.e., the filter has linear phase with $e^{-j10\omega}$).

Please use FIR filter:

Impulse response: $h(n) = \sum_{k=0}^{L-1} h_k x[n-k]$

Frequency response: $H(\omega) = \sum_{k=0}^{L-1} h_k e^{-j\omega k}$ where L: the number of FIR taps

to find the FIR filter coefficients which satisfy the specification. Here, please adopt the worst-case design criterion to minimize the absolute error between the desired and the actual channel frequency response:

$$\min_{\{h_k\}} \max_{\omega\in[0,\pi]} \left|H(\omega) - H_{spec}(\omega)\right|$$

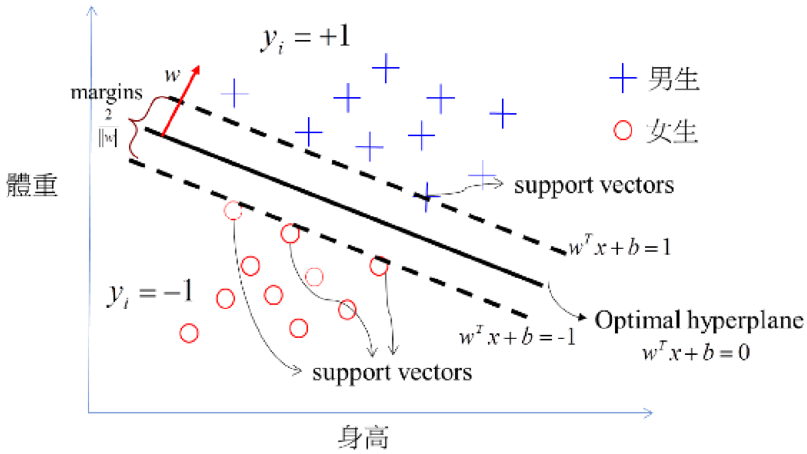
(a) When $L = 20$, please show the impulse, magnitude, phase response of the designed FIR filter and compare with the specification.

(b) As L increases, what do you observe ?

4. Support Vector Machine Design Problem

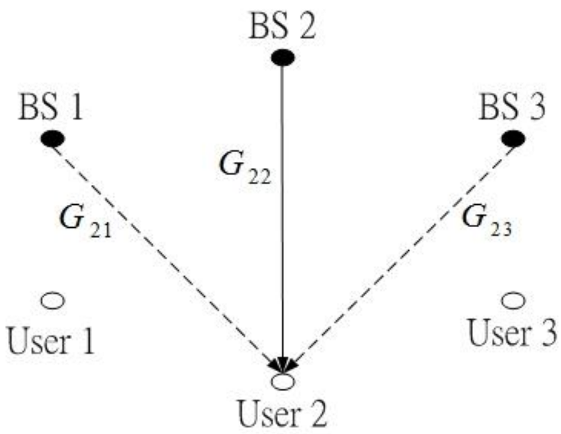
We have 20 male and female students. Give 20 groups of height and weight information to judge whether it is a boy or a girl. We want to classify data to either one of the two classes under different information with the support vector machine. Please find and plot the line to classify the information.

$$\max_{\mathbf{w}} \frac{2}{\|\mathbf{w}\|_2^2}$$
$$\text{s.t. } y_i\left(\mathbf{w}^H\mathbf{x}_i + b\right) \geq 1,\ \forall i = 1,\ldots,N$$



5. Transmit Power Control Problem

Consider an optimal power allocation problem with three users, as shown in the following figure.



where the base stations "BS 1~3" intend to communicate with the users "User 1~3", respectively. Define p_i as the transmit power of the ith BS for the ith user, G_{ij} as the channel gain from the jth BS to the ith user, and σ_i^2 as the noise power at the ith user. The signal-to-interference plus noise ratio (SINR) at the ith user is defined by

$$\gamma_i = \frac{G_{ii} p_i}{\sum_{j \neq i} G_{ij} p_j + \sigma_i^2}$$

Assume that $\mathbf{G} = \begin{bmatrix} 1 & 0.02 & 0.1 \\ 0.05 & 1 & 0.06 \\ 0.002 & 0.003 & 1 \end{bmatrix}$, $\sigma_i^2 = 0.01,\ P_{\max,i} = 10,\ i = 1 \cdots 3$.

(a) Consider the worst case design, and apply geometric programming (GP) to find the total required transmit power and the SINR at each user. (Please refer to lecture 4, page 10.)
(Note that please rewrite the associated GP problem to a convex form.)

(b) Minimize the total transmit power, subject to all the users' SINRs $\gamma_0 \geq 10\ dB$. Please find the total required transmit power and the SINR at each user. (Please refer to lecture 3, page 22.)

(c) The worst design problem in (a) is actually a quasi-convex problem. Please apply the bisection method to find the optimal total required transmit power and the SINR at each user, and give comments on the results obtained from the GP and the bisection method.

<General Information> deliverables for this project include:

- Simulation results
- a paragraph summarizing your observations and any conclusions you can draw from this
- source code
- bonus points for any extra plots with meaningful explanations and conclusions.

<Notice>

- The report and simulation codes are uploaded to ee-class
- File: ID_name.rar (e.g., 965403001_王小明.rar)