Gaussian distributions

Logout

Samples from Gaussian distributions

in this chapter we will discuss how to deal with data that were drawn from a Gaussian distribution and we want to get an estimate of the mean and standard deviation of that distribution. To remind us:

$$P(y \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/2\sigma^2}$$

that leads us to the likelihood function to draw a set of $\{y_i\}$ from that Gaussian:

$$P(\{y_i\} \mid \mu, \sigma) \propto \prod_i \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i-\mu)^2}{2\sigma^2}\right)$$

the first situation that we want to consider is that we know σ. The posterior in this case is:

$$P(\mu \mid \{y_i\}, \sigma^2) \propto P(\mu \mid \sigma^2) P(\{y_i\} \mid \mu, \sigma^2)$$

if we assume that the prior for μ is also a Gaussian with standard deviation τ_0 then:

$$P(\mu \mid \{y_i\}, \sigma^2) \propto \exp\left(-\frac{(\mu-\mu_0)^2}{2\tau_0^2}\right) \exp\left(-\sum_i \frac{(y_i-\mu)^2}{2\sigma^2}\right)$$

By collecting the terms proportional to μ^2 and μ we can rearrange and write the posterior as:

$$P(\mu \mid \{y_i\}, \sigma^2) \propto \exp\left(-\frac{(\mu-\mu_N)^2}{2\tau_N^2}\right)$$

with

$$\frac{1}{\tau_N^2} = \frac{1}{\tau_0^2} + \frac{N}{\sigma^2}$$

and

$$\mu_N = \frac{\frac{1}{\tau_0^2}}{1 \qquad N} \mu_0 + \frac{\frac{N}{\sigma^2}}{1 \qquad N} \bar{y}$$

$$\frac{1}{\tau_0^2} + \frac{N}{\sigma^2} \qquad\qquad \frac{1}{\tau_0^2} + \frac{N}{\sigma^2}$$

using a non-informative prior of $\tau_0 = 0$ we get the expected results of $\mu_N = \bar{y}$ and $\tau_N^2 = \sigma^2/N$

Now lets consider the case that we do not know σ. For simplicity we assume that the prior $P(\mu, \sigma) = const$ for $\sigma \geq 0$. We then integrate out σ to get

$$P(\mu \mid \{y_i\}) \propto \int_0^\infty t^{N-2} \exp\Big(-\frac{t^2}{2} \sum_i (y_i - \mu)^2 \Big) dt$$

where we made the substitution $\sigma = 1/t$. A rescaling of t by substituting $\tau = t\sqrt{\sum_i (y_i - \mu)^2}$ reduces the posterior to:

$$P(\mu \mid \{y_i\}) \propto \Big[\sum_i (y_i - \mu)^2 \Big]^{-(N-1)/2}$$

This distribution turns out to be the Student t-distribution for degree of freedom $N - 2$. Using our trick from before we can estimate the position of the maximum and the width of the maximum, which can be summarized by

$$\mu = \bar{y} \pm \frac{S}{\sqrt{N}} \quad \text{where} \quad S^2 = \frac{1}{N-1} \sum_i (y_i - \bar{y})^2$$

the fact that we don't know σ forces us to estimate σ from the data.

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