Quiz on Infinity and Basic Counting

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Please show the definition of C(n,k) or P(n,k) somewhere if you use them unexpanded. You need not evaluate these expressions to a final numeric answer.

1. Classify the following sets as finite, countably infinite, or uncountably infinite and explain how you know. (All credit is for the explanation, not the answer.)

a. Suppose we have a function $f(x, y)$ that takes two natural number inputs and outputs a natural number greater than x. Classify the least set S such that $0 \in S$ and whenever $i \in S$ and $j \in S$, the number $f(i, j)$ is also in S.

This set is the sum of all natural numbers, therefore it's the set of all natural numbers. clearly the set of all natural number is countably infinity, e.g. 0, 1, 2, 3.... i where i is ith element in the set.

b. Classify the set T containing all possible sets of even natural numbers.

Uncountably infinite set, since we know there are infinity amount of even natural number, but each set of even natural number have different size, eg. {2} and {2.4} have different size, as the size increase, it's not countable.

2. Count the following sets and explain your counts.

a. How many sets of exactly four 2-digit even numbers are there (leading zeroes not allowed)? Explain. (example set: $\{12, 24, 36, 78\}$)

~~There are 5×9=45 2-digit even umbers amount numbers~~ (from 10 – 98, there are 5 2-digit even umber in 10~18, 5 in 20~28 ..., so totally 5×9=45

Since we can reuse the numbers order does no matter, so we are choosing 4 number out of 45 possible number, by definition, it's $C(45,5) = \frac{45!}{5!(40)!}$

Since $C(n,k) = \frac{n!}{k!(n-k)!}$

b. How many different ways can 160 distinct students take seats in a 250-seat room, with everyone seated and no seat sharing? Explain.

Since no seat is sharing, order matter as one student take that seat and the other student cannot use it anymore. So we want to choose 160 from 250 with order matters which is $P(250, 160) = \frac{250!}{90!}$

by definiton, ~~its~~ $P(n.k) = \frac{n!}{(n-k)!}$

c. How many different ways can 160 distinct students be assigned grades from the eleven choices A$^+$/A/A$^-$/B$^+$/B/B$^-$/C$^+$/C/C$^-$/D/F? Explain.

we know different student can have the same grades. so ~~the order~~ does not matters. we want to choose 11 grades from 160 students which by the definiton is $C(160, 11) = \frac{160!}{11!\,149!}$ since $C(n,K) = \frac{n!}{K!(n-K)!}$