Self-driving Car Assignment 2 學號：0851924 姓名：許朝鈞

#2

(a)

$P(\text{Day } 2 = \text{cloudy} \mid \text{Day } 1 = \text{sunny}) = 0.2$

$P(\text{Day } 3 = \text{cloudy} \mid \text{Day } 2 = \text{cloudy}) = 0.4$

$P(\text{Day } 4 = \text{rainy} \mid \text{Day } 3 = \text{cloudy}) = 0.2$

$\because$ the weather transition function is a Markov chain

$\therefore P(\text{Day } 2 = \text{cloudy}, \text{Day3} = \text{cloudy}, \text{Day4} = \text{rainy} \mid \text{Day } 1 = \text{sunny})$

$= P(\text{Day } 2 = \text{cloudy} \mid \text{Day } 1 = \text{sunny}) * P(\text{Day3} = \text{cloudy} \mid \text{Day2} = \text{cloudy}) * P(\text{Day4} = \text{rainy} \mid \text{Day3} = \text{cloudy})$

$= 0.2 * 0.4 * 0.2$

$= 0.016$ ※

(b)(c)

```cpp
# include <iostream>
# include <stdio.h>
# include <stdlib.h>
# include <time.h>
using namespace std;

enum weather{
    sunny = 0, cloudy, rainy
};

weather nextWeather(weather);
void printWeather(weather);

int main(void){
    srand(time(NULL));

// 2_b
    int todayWeather;
    int length;

    do{
        cout << "Please enter today's weather (sunny: 0, cloudy: 1, rainy: 2) -> ";
        cin >> todayWeather;
    }while(todayWeather != (int)todayWeather || todayWeather > 2 || todayWeather < 0);

    do{
        cout << "Enter the length of the weather sequence (0-100) -> ";
        cin >> length;
    }while(length != (int)length || length > 100 || length < 0);

    weather w = (weather)todayWeather;
    for(int i = 0; i < length; i++){
        printWeather((weather)w);
        w = nextWeather(w);
    }
    cout << endl;
// end of 2_b
```

```cpp
// 2_c
    int seq_duration = 10000;
    int seq_length = 10000;
    int sunny_count = 0;
    int cloudy_count = 0;
    int rainy_count = 0;

    for(int i = 0; i < seq_duration; i++){
        weather w = (weather)(rand() % 3);
        for(int j = 0; j < seq_length; j++){
            if((weather)w == sunny) sunny_count++;
            else if((weather)w == cloudy) cloudy_count++;
            else rainy_count++;

            w = nextWeather(w);
        }
    }

    cout << "Prob. of sunny days:\t" << ((double)sunny_count / (seq_duration * seq_length)) << endl;
    cout << "Prob. of cloudy days:\t" << ((double)cloudy_count / (seq_duration * seq_length)) << endl;
    cout << "Prob. of rainy days:\t" << ((double)rainy_count / (seq_duration * seq_length)) << endl;
// end of 2_c

    return 0;
}
```

```cpp
weather nextWeather(weather todayWeather){
    double trans[3][3] = {
        {0.8, 0.2, 0},
        {0.4, 0.4, 0.2},
        {0.2, 0.6, 0.2}
    };

    double num = (double)(rand() % 100) / 100;
    if(num < trans[todayWeather][0]) return sunny;
    else if(num < trans[todayWeather][0] + trans[todayWeather][1]) return cloudy;
    else return rainy;
}

void printWeather(weather w){
    if(w == sunny) cout << "sunny\t";
    else if(w == cloudy) cout << "cloudy\t";
    else cout << "rainy\t";
}
```

(b)result

```
parallels@hsuchaochundeubuntu:~/SelfDriving_Car$ ./2
Please enter today's weather (sunny: 0, cloudy: 1, rainy: 2) -> 0
Enter the length of the weather sequence (0-100) -> 10
sunny   sunny   sunny   sunny   sunny   sunny   sunny   sunny   sunny   sunny
```

```
parallels@hsuchaochundeubuntu:~/SelfDriving_Car$ ./2
Please enter today's weather (sunny: 0, cloudy: 1, rainy: 2) -> 1
Enter the length of the weather sequence (0-100) -> 10
cloudy  rainy   sunny   sunny   sunny   sunny   cloudy  sunny   sunny   cloudy
```

(c)result

```
parallels@hsuchaochundeubuntu:~/SelfDriving_Car$ ./2
Prob. of sunny days:    0.642687
Prob. of cloudy days:   0.285809
Prob. of rainy days:    0.071504
```

(d) let m = the stationary distribution based on the state transition matrix

$(m_1\ m_2\ m_3)$

$$m\begin{pmatrix} .8 & .2 & 0 \\ .4 & .4 & .2 \\ .2 & .6 & .2 \end{pmatrix} = m$$

$$\begin{cases} 0.8m_1 + 0.4m_2 + 0.2m_3 = m_1 \quad \cdots ① \\ 0.2m_1 + 0.4m_2 + 0.6m_3 = m_2 \quad \cdots ② \\ 0\,m_1 + 0.2m_2 + 0.2m_3 = m_3 \quad \cdots ③ \\ m_1 + m_2 + m_3 = 1 \quad \cdots ④ \end{cases}$$

$① - ② \Rightarrow 0.6m_1 - 0.4m_3 = m_1 - m_2 \Rightarrow m_2 - 0.4m_3 = 0.4m_1$ ⑤

$① - ③ \Rightarrow 0.8m_1 + 0.2m_2 = m_1 - m_3 \Rightarrow 0.2m_2 + m_3 = 0.2m_1$ ⑥

$\Rightarrow m_2 - 0.4m_3 = 0.4m_2 + 2m_3$

$\Rightarrow 0.6m_2 = 2.4m_3$

$\Rightarrow m_2 = 4m_3 \quad \cdots ⑦$

$\Rightarrow m_2 : m_3 = 4:1$

⑦ 代入 ⑤ $\Rightarrow 4m_3 - 0.4m_3 = 0.4m_1$

$\Rightarrow 3.6m_3 = 0.4m_1$

$\Rightarrow 9m_3 = m_1 \Rightarrow m_1 : m_3 = 9:1$

$\therefore m_1 : m_2 : m_3 = 9:4:1$ 代入 ④ $\Rightarrow m_1 = \frac{9}{14}\ m_2 = \frac{4}{14} = \frac{2}{7}\ m_3 = \frac{1}{14}$ ※

(e) $H_p(x) = E[-\log_2 p(x)]$

$\because$ days are discrete

$\therefore H_p(x) = -\sum_x p(x)\log_2 p(x)$

$$= -\left(\frac{9}{14}\log_2\left(\frac{9}{14}\right) + \frac{2}{7}\log_2\left(\frac{2}{7}\right) + \frac{1}{14}\log_2\left(\frac{1}{14}\right)\right)$$

$$= -\left(\frac{9}{14}(\log_2 9 - \log_2 14) + \frac{2}{7}(\log_2 2 - \log_2 7)\right) + \frac{1}{14}(\log_2 1 - \log_2 14)\Big)$$

$$= -\left(\frac{9}{7}\log_2 3 - \frac{9}{14} - \frac{9}{14}\log_2 7 + \frac{2}{7} - \frac{2}{7}\log_2 7 - \frac{1}{14} - \frac{1}{14}\log_2 7\right)$$

$$= -\frac{9}{7}\log_2 3 + \log_2 7 + \frac{3}{7} \quad ※$$

$(= 1.19811742.)$

(f) Bayes rule: $P(x|y) = \frac{P(y|x)P(x)}{P(y)} = \frac{P(y|x)P(x)}{\sum_{x'} P(y|x')P(x')}$

let $\begin{cases} x \text{ present yesterday's weather.} \\ y \text{ present today's weather.} \end{cases}$

$\Rightarrow$

		tomorrow sunny	cloudy	rainy
today	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2

$\rightarrow$

		yesterday sunny	cloudy	rainy
today	sunny	$\frac{0.8 * \frac{9}{14}}{\frac{9}{14}}$	$\frac{0.4 * \frac{2}{7}}{\frac{9}{14}}$	$\frac{0.2 * \frac{1}{14}}{\frac{9}{14}}$
	cloudy	$\frac{0.2 * \frac{9}{14}}{\frac{2}{7}}$	$\frac{0.4 * \frac{2}{7}}{\frac{2}{7}}$	$\frac{0.6 * \frac{1}{14}}{\frac{2}{7}}$
	rainy	$\frac{0 * \frac{9}{14}}{\frac{1}{14}}$	$\frac{0.2 * \frac{2}{7}}{\frac{1}{14}}$	$\frac{0.2 * \frac{1}{14}}{\frac{1}{14}}$

$\rightarrow$

		yesterday sunny	cloudy	rainy
today	sunny	0.8	0.178	0.022
	cloudy	0.45	0.4	0.15
	rainy	0	0.8	0.2 ✱

(g) No, it wouldn't violate the Markov property. Due to the value of the next day's weather still only depends on today's weather. We could add the "season" as another variable, then we get another transition table which dimension is 12*12, that can transmit the state from $\{x_t, S_t\}$ to $\{x_{t+1}, S_{t+1}\}$ ✱

#3

#3 (a)

$\because$ Day 4 is rainy, the measurement doesn't make mistake and by Markov assumption

$\therefore$ $P(\text{Day 5 actual} = \text{sunny} \mid \text{Day 5 measure} = \text{sunny}) = \frac{P(\text{Day 5 measure} = \text{sunny} \mid \text{Day 5 actual} = \text{sunny})\, P(\text{Day 5 actual} = \text{sunny})}{P(\text{Day 5 measure} = \text{sunny})}$

$= \frac{0.6 * 0.2}{0.6 * 0.2 + 0.3 * 0.6 + 0 * 0.2}$

$= \frac{0.12}{0.12 + 0.18} = 0.4$ ✱

(c) $P(x_{2\sim4} \mid x_1, z_{2\sim4}) = \eta\, \underbrace{P(z_{2\sim4} \mid x_1, x_{2\sim4})}_{P(z_4|x_4)P(z_3|x_3)P(z_2|x_2)}\, \underbrace{P(x_{2\sim4} \mid x_1)}_{P(x_4|x_3)P(x_3|x_2)P(x_2|x_1)}$

$\therefore$ the most likely sequence of weather is "sunny, cloudy, rainy" $\left(\frac{0.00576}{0.00576 + 0.00144} = 80\%\right)$ ✱