CS 224S / LINGUIST 285
Spoken Language Processing

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Lecture 4: ASR: Word Error Rate, Training, Advanced Decoding

Original slides by Dan Jurafsky

Outline for Today

- Word error rate (WER) computation
- Training
  - Baum-Welch = EM = Forward Backward
    - Detailed example in slides appendix
  - How we train LVCSR systems in practice
- Advanced decoding

Administrative items

- Homework 1 due by 11:59pm tonight on Gradescope
- Homework 2 released tonight (due in 2 weeks)
- Project handout released tonight
  - We will compile and post to piazza project ideas over the next 1-2 weeks
  - Proposals due May 1
- Background survey released today. Complete by Friday (part of class participation grade)

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- **Word error rate (WER) computation**
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Evaluation

- How to evaluate the word string output by a speech recognizer?

Word Error Rate

- Word Error Rate =

$$\frac{100\ (\text{Insertions}+\text{Substitutions} + \text{Deletions})}{\text{Total Word in Correct Transcript}}$$

- Aligment example:

```
REF:  portable ****  PHONE UPSTAIRS last night so
HYP:  portable FORM  OF    STORES   last night so
Eval           I     S     S
```

- WER = 100 (1+2+0)/6 = 50%

NIST sctk scoring software: Computing WER with sclite

- http://www.nist.gov/speech/tools/
- Sclite aligns a hypothesized text (HYP) (from the recognizer) with a correct or reference text (REF) (human transcribed)

```
id: (2347-b-013)
Scores: (#C #S #D #I) 9 3 1 2
REF:  was an engineer SO I   i was always with **** **** MEN UM and they
HYP:  was an engineer ** AND i was always with THEM THEY ALL THAT and they
Eval:                 D  S                     I    I    S   S
```

Sclite output for error analysis

```
CONFUSION PAIRS                  Total                 (972)
                                 With >=  1 occurrances (972)
   1:     6  ->  (%hesitation) ==> on
   2:     6  ->  the ==> that
   3:     5  ->  but ==> that
   4:     4  ->  a ==> the
   5:     4  ->  four ==> for
   6:     4  ->  in ==> and
   7:     4  ->  there ==> that
   8:     3  ->  (%hesitation) ==> and
   9:     3  ->  (%hesitation) ==> the
  10:     3  ->  (a-) ==> i
  11:     3  ->  and ==> i
  12:     3  ->  and ==> in
  13:     3  ->  are ==> there
  14:     3  ->  as ==> is
  15:     3  ->  have ==> that
  16:     3  ->  is ==> this
```

Better metrics than WER?

- WER has been useful
- But should we be more concerned with meaning ("semantic error rate")?
  - Good idea, but hard to agree on
  - Has been applied in dialogue systems, where desired semantic output is more clear

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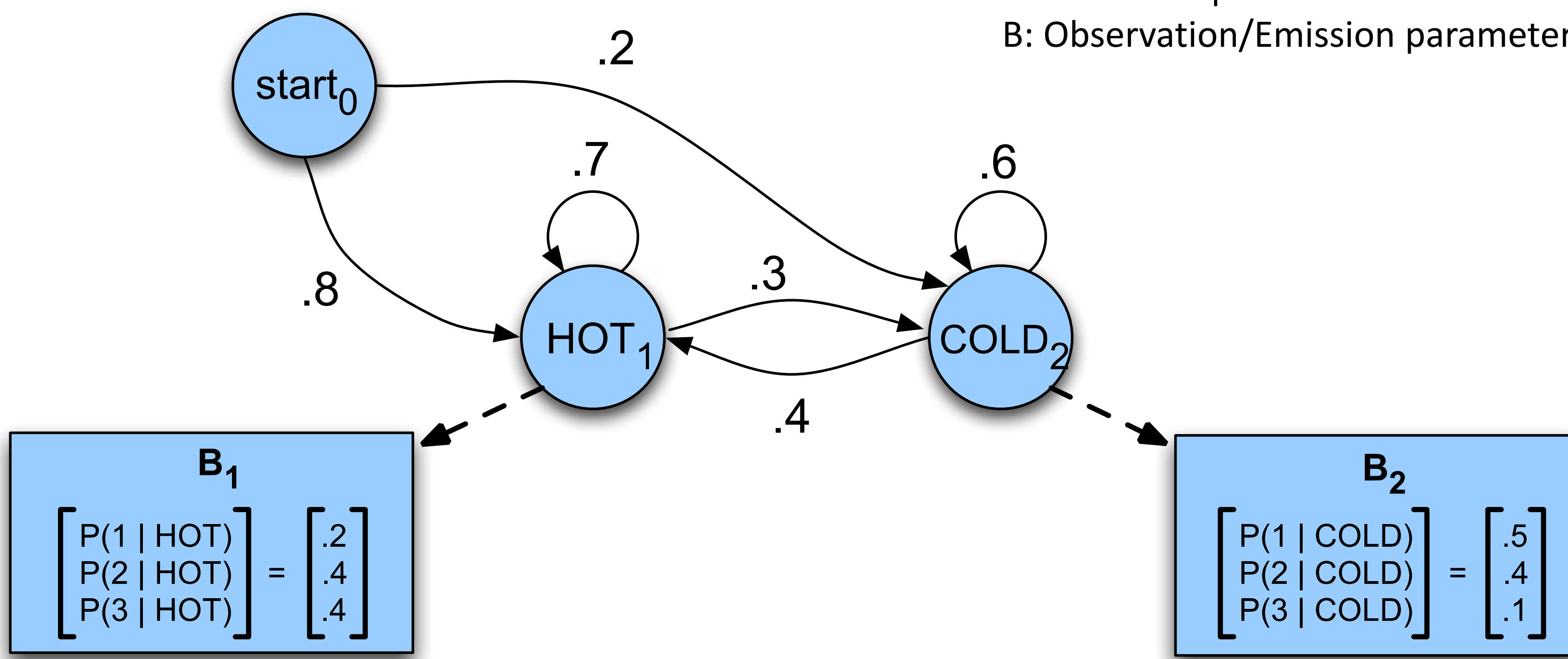
HMM for ice cream

A: Transition parameters
B: Observation/Emission parameters



The Learning Problem

Learning: Given an observation sequence O and the set of possible states in the HMM, learn the HMM parameters A and B.

- Baum-Welch = Forward-Backward Algorithm (Baum 1972)
- Is a special case of the EM or Expectation-Maximization algorithm (Dempster, Laird, Rubin)
- The algorithm will let us train the transition probabilities $A = \{a_{ij}\}$ and the emission probabilities $B = \{b_i(o_t)\}$ of the HMM

The Learning Problem

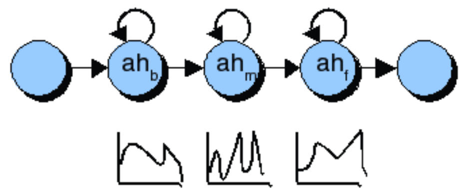
- Baum-Welch / EM enables maximum likelihood training of (A,B)
- In practice we do not train A
  - Why?

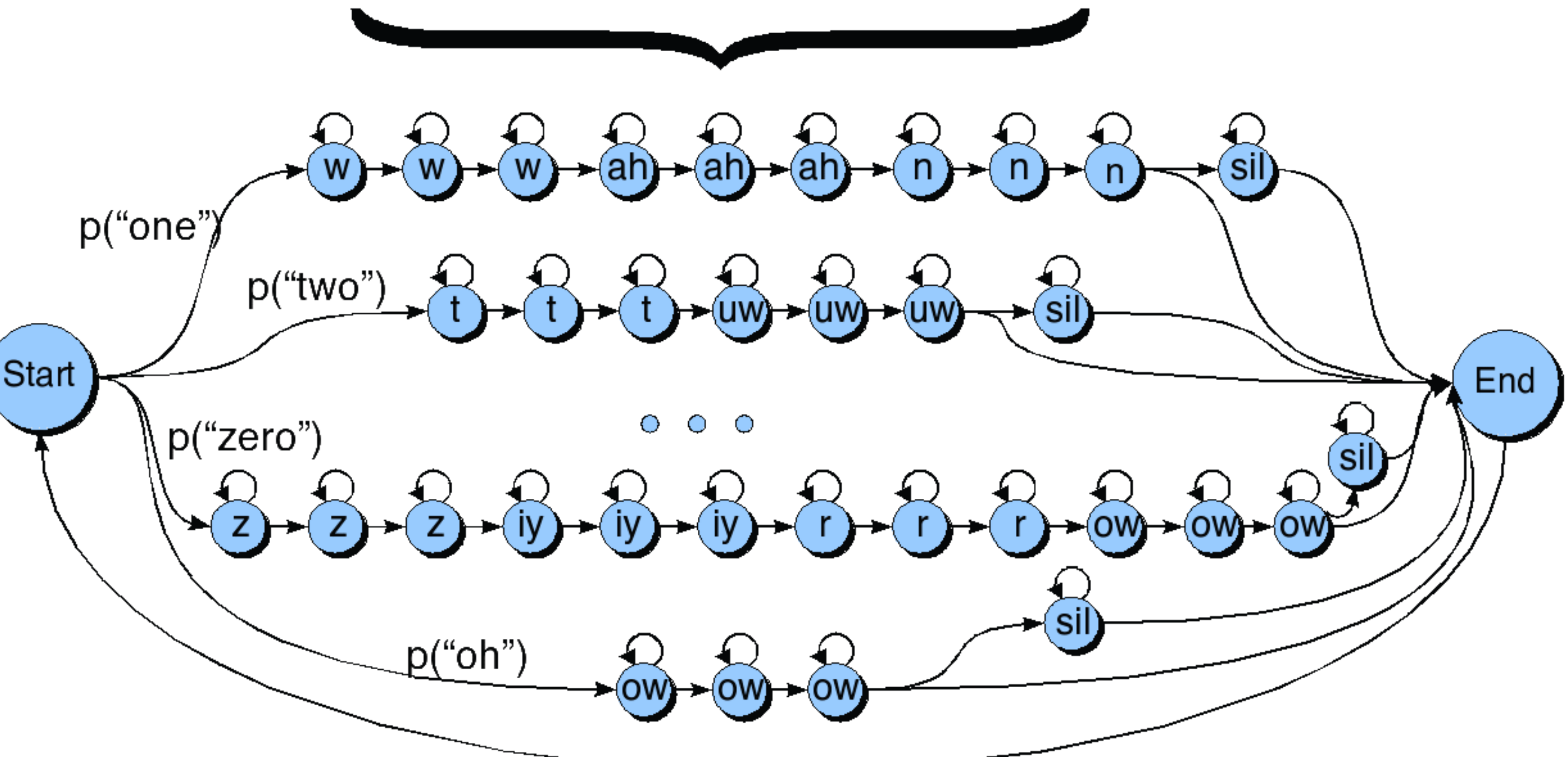
HMM for the digit recognition task

Lexicon

one	w ah n
two	t uw
three	th r iy
four	f ao r
five	f ay v
six	s ih k s
seven	s eh v ax n
eight	ey t
nine	n ay n
zero	z iy r ow
oh	ow

Phone HMM

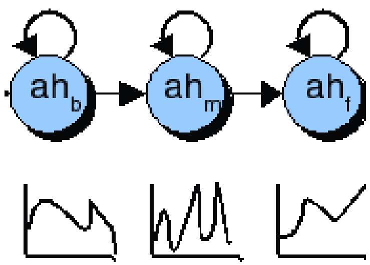




The Learning Problem

- Baum-Welch / EM implicitly does "soft assignment" of hidden states when updating observation/emission model parameters
- In practice we use "hard assignment" / Viterbi training

Estimating hidden states in training



ah_f	0	0.1	0.1	0.95
ah_m	0	0.15	0.5	0.05
ah_b	1.0	0.8	0.4	0
	o_1	o_2	o_3	o_4

- Updating parameters in EM/Soft Assignment
  - $B_{ahm} \sim 0{*}o_1 + 0.15{*}o_2 + 0.5{*}o_3 + 0.05{*}o_4$
- Updating parameters with Viterbi/Hard Assignment
  - $B_{ahm} \sim o_3$

Typical training procedure in LVCSR

- Generate a forced alignment with existing model
  - Viterbi decoding with a very constrained prior (the transcript)
  - Assigns observations to HMM states
- Create new observation models from update alignments
- Iteratively repeat the above steps, occasionally introducing a more complex observation model or adding more difficult training examples

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  - How we train LVCSR systems in practice
- **Advanced decoding**

Advanced Search (= Decoding)

- How to weight the AM and LM
- Speeding things up: Viterbi beam decoding
- Multipass decoding
  - N-best lists
  - Lattices
  - Word graphs
  - Meshes/confusion networks
- Finite State Methods

What we are searching for

- Given Acoustic Model (AM) and Language Model (LM):

AM (likelihood) LM (prior)

$$(1) \quad \hat{W} = \underset{W \in L}{\arg\max}\, P(O \mid W)\, P(W)$$

Combining Acoustic and Language Models

- We don't actually use equation (1)

$$(1)\quad \hat{W} = \underset{W \in L}{\arg\max}\, P(O|W)P(W)$$

  - AM underestimates acoustic probability
    - Why? Bad independence assumptions
    - Intuition: we compute (independent) AM probability estimates; but if we could look at context, we would assign a much higher probability. So we are underestimating
    - We do this every 10 ms, but LM only every word.
    - Besides: AM isn't a true probability
  - AM and LM have vastly different dynamic ranges

Language Model Scaling Factor

- Solution: add a language model weight (also called language weight LW or language model scaling factor LMSF

$$(2)\quad \hat{W} = \underset{W \in L}{\arg\max}\, P(O|W)P(W)^{LMSF}$$

$$= \text{argmax} \log P(O|W) + LMSF * \log P(W)$$

- Value determined empirically, is positive (why?)
- Often in the range 10 +/- 5.
- Kaldi uses an acoustic model scaling factor instead, but it achieves the same efffect

Language Model Scaling Factor

- As LMSF is increased:
  - More deletion errors (since increase penalty for transitioning between words)
  - Fewer insertion errors
  - Need wider search beam (since path scores larger)
  - Less influence of acoustic model observation probabilities

Slide from Bryan Pellom

Word Insertion Penalty

- But LM prob P(W) also functions as penalty for inserting words
  - Intuition: when a uniform language model (every word has an equal probability) is used, LM prob is a 1/V penalty multiplier taken for each word
  - Each sentence of N words has penalty N/V
  - If penalty is large (smaller LM prob), decoder will prefer fewer longer words
  - If penalty is small (larger LM prob), decoder will prefer more shorter words
- When tuning LM for balancing AM, side effect of modifying penalty
- So we add a separate word insertion penalty to offset

$$(3) \quad \hat{W} = \operatorname*{argmax}_{W \in L} P(O|W)P(W)^{LMSF}\, WIP^{N(W)}$$

Word Insertion Penalty

- Controls trade-off between insertion and deletion errors
  - As penalty becomes larger (more negative)
  - More deletion errors
  - Fewer insertion errors
- Acts as a model of effect of length on probability
  - But probably not a good model (geometric assumption probably bad for short sentences)

Log domain

- We do everything in log domain
- So final equation:

$$(4)\quad \hat{W} = \underset{W \in L}{\operatorname{argmax}} \log P(O \mid W) + LMSF \log P(W) + N \log WIP$$

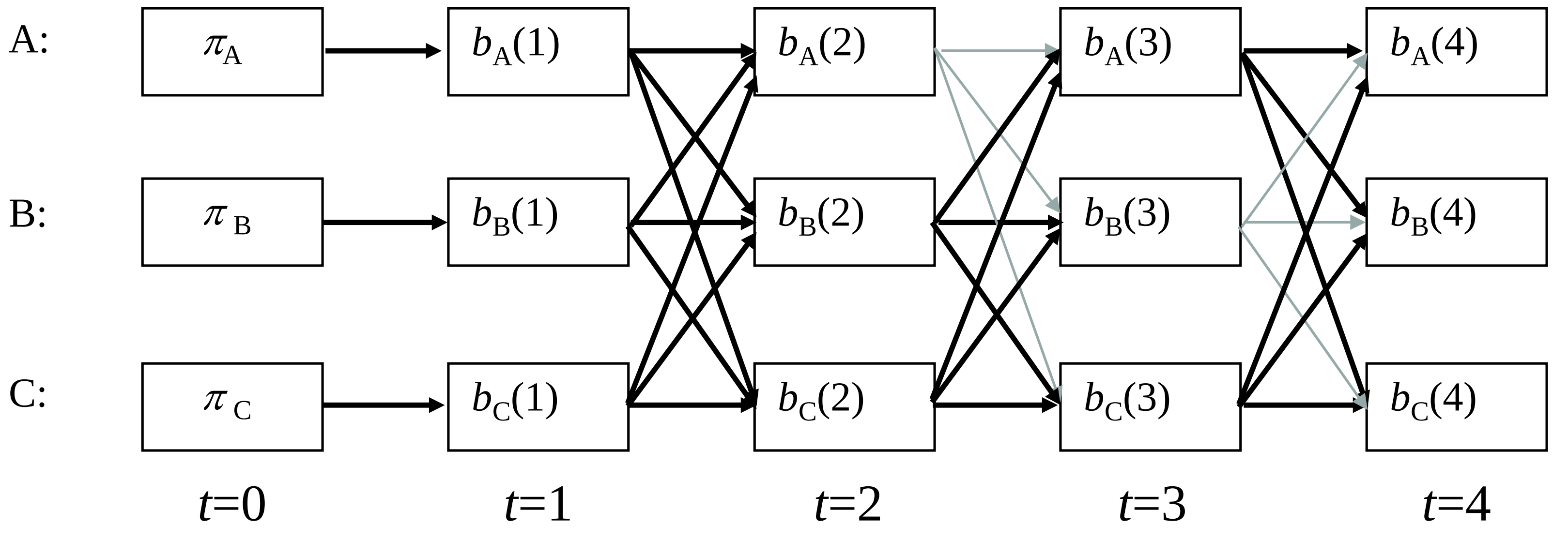
Speeding things up

- Viterbi is $O(N^2T)$, where N is total number of HMM states, and T is length
- This is too large for real-time search
- A ton of work in ASR search is just to make search faster:
  - Beam search (pruning)
  - Fast match
  - Tree-based lexicons

Beam search

- Instead of retaining all candidates (cells) at every time frame
- Use a threshold T to keep subset:
  - At each time t
  - Identify state with lowest cost Dmin
  - Each state with cost > Dmin+ T is discarded ("pruned") before moving on to time t+1
  - Unpruned states are called the active states

Viterbi Beam Search



Slide from John-Paul Hosom

Viterbi Beam search

- Most common search algorithm for LVCSR
  - Time-synchronous
    - Comparing paths of equal length
  - Two different word sequences W1 and W2:
    - We are comparing $P(W1|O_{0t})$ and $P(W2|O_{0t})$
    - Based on same partial observation sequence O_{0t}
    - So denominator is same, can be ignored
  - Time-asynchronous search (A*) is harder

Viterbi Beam Search

- Empirically, beam size of 5-10% of search space
- Thus 90-95% of HMM states don't have to be considered at each time t
- Vast savings in time.

On-line processing

- Problem with Viterbi search
  - Doesn’t return best sequence til final frame

- This delay is unreasonable for many applications.

- On-line processing
  - usually smaller delay in determining answer
  - at cost of always increased processing time.

On-line processing

- At every time interval I (e.g. 1000 msec or 100 frames):
  - At current time *tcurr*, for each active state *qtcurr*, find best path P(*qtcurr*) that goes from from *t0* to *tcurr* (using backtrace (ψ))
  - Compare set of best paths P and find last time *tmatch* at which all paths P have the same state value at that time
  - If tmatch exists {

    Output result from t0 to tmatch

    Reset/Remove ψ values until tmatch

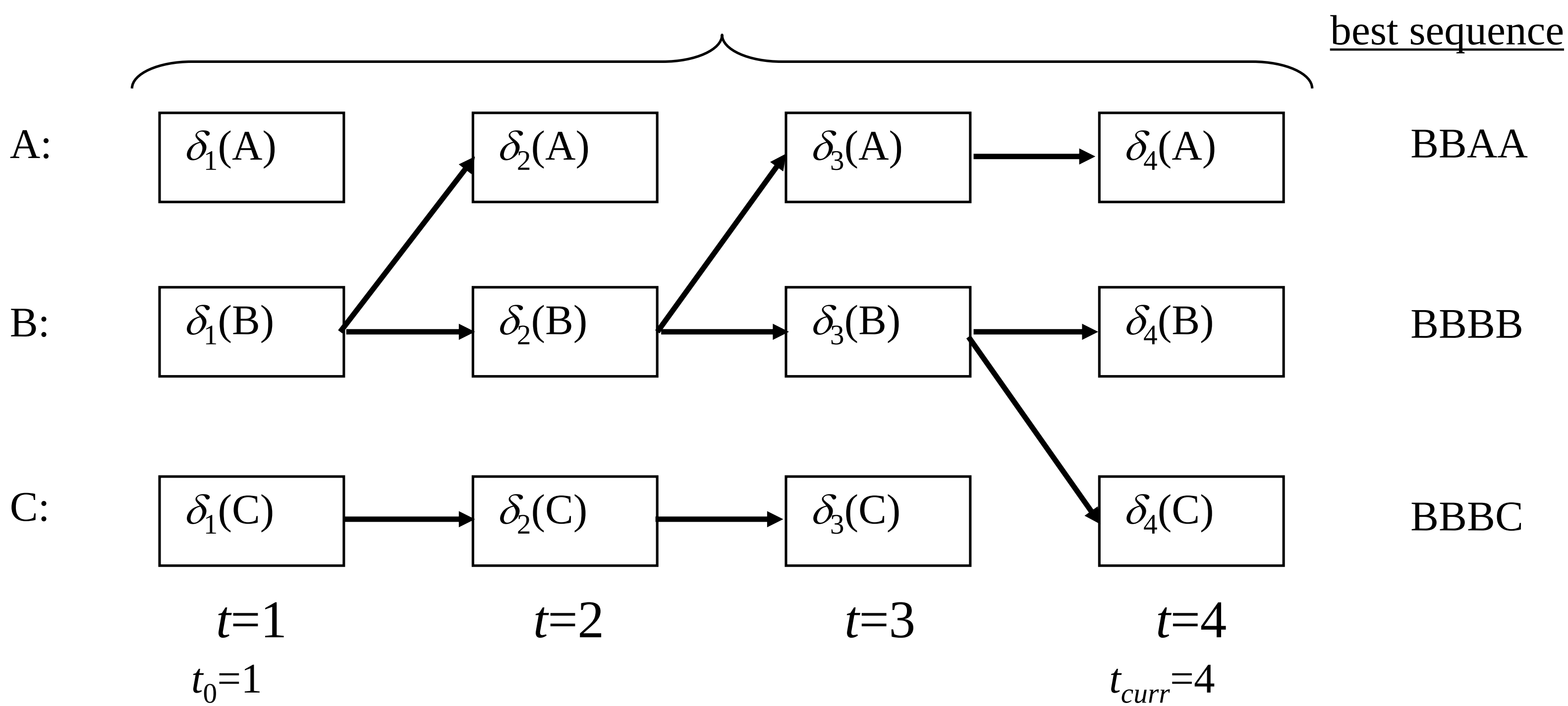
    Set t0 to tmatch+1

    }
- Efficiency depends on interval I, beam threshold, and how well the observations match the HMM.

Slide from John-Paul Hosom

On-line processing

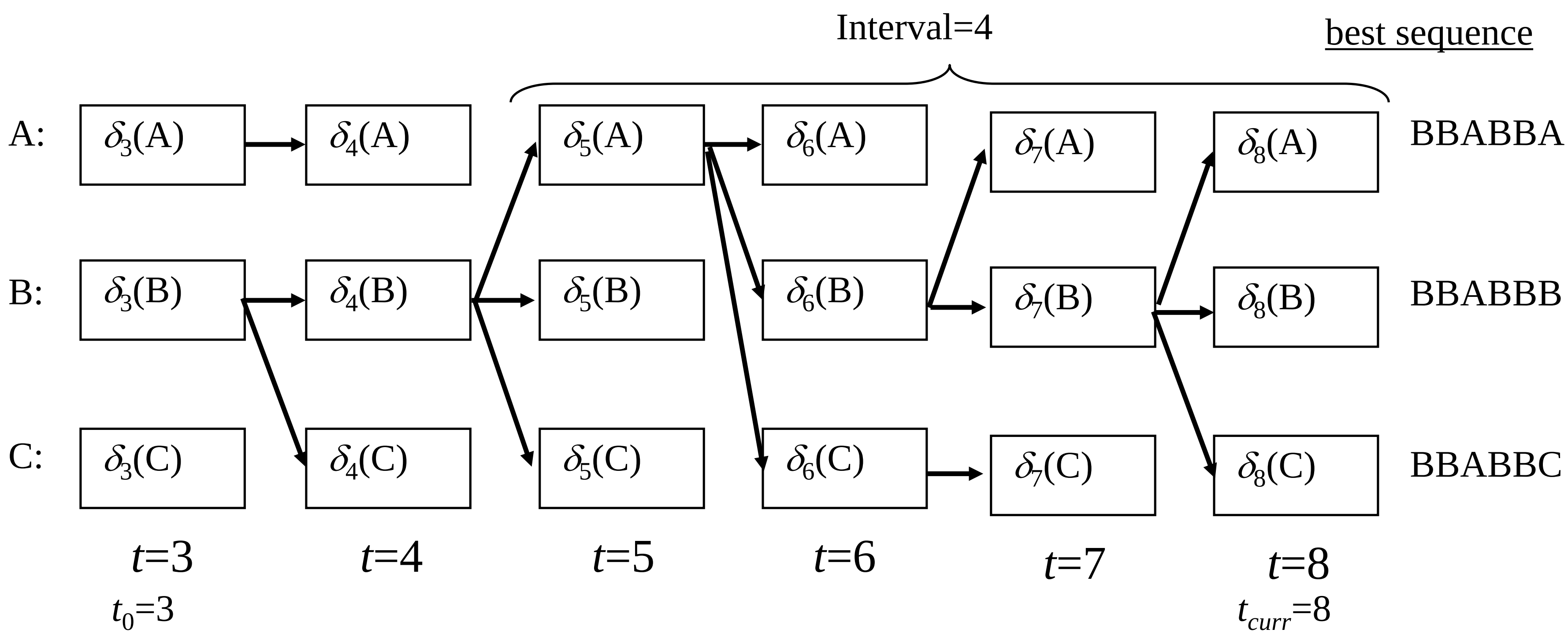
- Example (Interval = 4 frames):



- At time 4, all best paths for all states A, B, and C have state B in common at time 2. So, tmatch = 2.
- Now output states BB for times 1 and 2, because no matter what happens in the future, this will not change. Set t0 to 3

Slide from John-Paul Hosom

On-line processing



- Now t_{match} = 7, so output from t=3 to t=7: BBABB, then set t_0 to 8.

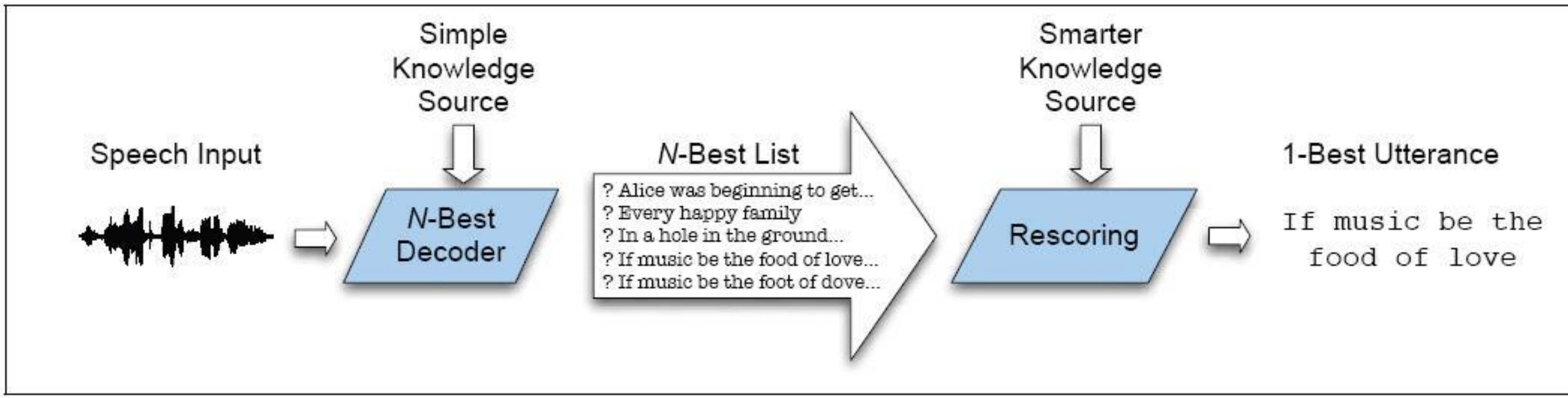
- If T=8, then output state with best δ_8, for example C. Final result (obtained piece-by-piece) is then BBBBABBC

Slide from John-Paul Hosom

Problems with Viterbi

- It’s hard to integrate sophisticated knowledge sources
  - Trigram grammars
  - Parser-based or Neural Network LM
    - long-distance dependencies that violate dynamic programming assumptions
  - Knowledge that isn’t left-to-right
    - Following words can help predict preceding words
- Solutions
  - Return multiple hypotheses and use smart knowledge to rescore them
  - Use a different search algorithm, A* Decoding (=Stack decoding)

Multipass Search



Ways to represent multiple hypotheses

- N-best list
  - Instead of single best sentence (word string), return ordered list of N sentence hypotheses
- Word lattice
  - Compact representation of word hypotheses and their times and scores
- Word graph
  - FSA representation of lattice in which times are represented by topology

Another Problem with Viterbi

- The forward probability of observation given word string

$$P(O|W) = \sum_{S \in S_1^T} P(O,S|W)$$

- The Viterbi algorithm makes the "Viterbi Approximation"

$$P(O|W) \approx \max_{S \in S_1^T} P(O,S|W)$$

- Approximates P(O|W)
  - with P(O|best state sequence)

Solving the best-path-not-best-words problem

- Viterbi returns best path (state sequence) not best word sequence
  - Best path can be very different than best word string if words have many possible pronunciations
- Two solutions
  - Modify Viterbi to sum over different paths that share the same word string.
    - Do this as part of N-best computation
      - Compute N-best word strings, not N-best phone paths
  - Use a different decoding algorithm (A*) that computes true Forward probability.

Sample N-best list

Rank	Path	AM logprob	LM logprob
1.	it's an area that's naturally sort of mysterious	-7193.53	-20.25
2.	that's an area that's naturally sort of mysterious	-7192.28	-21.11
3.	it's an area that's not really sort of mysterious	-7221.68	-18.91
4.	that scenario that's naturally sort of mysterious	-7189.19	-22.08
5.	there's an area that's naturally sort of mysterious	-7198.35	-21.34
6.	that's an area that's not really sort of mysterious	-7220.44	-19.77
7.	the scenario that's naturally sort of mysterious	-7205.42	-21.50
8.	so it's an area that's naturally sort of mysterious	-7195.92	-21.71
9.	that scenario that's not really sort of mysterious	-7217.34	-20.70
10.	there's an area that's not really sort of mysterious	-7226.51	-20.01

N-best lists

- Again, we don't want the N-best paths
- That would be trivial
  - Store N values in each state cell in Viterbi trellis instead of 1 value
- But:
  - Most of the N-best paths will have the same word string
    - Useless!!!
  - It turns out that a factor of N is too much to pay

Computing N-best lists

- In the worst case, an admissible algorithm for finding the N most likely hypotheses is exponential in the length of the utterance.
  - S. Young. 1984. "Generating Multiple Solutions from Connected Word DP Recognition Algorithms". Proc. of the Institute of Acoustics, 6:4, 351-354.
- For example, if AM and LM score were nearly identical for all word sequences, we must consider all permutations of word sequences for whole sentence (all with the same scores).
- But of course if this is true, can't do ASR at all!

Computing N-best lists

- Instead, various non-admissible algorithms:
  - (Viterbi) Exact N-best
  - (Viterbi) Word Dependent N-best
- And one admissible
  - A* N-best

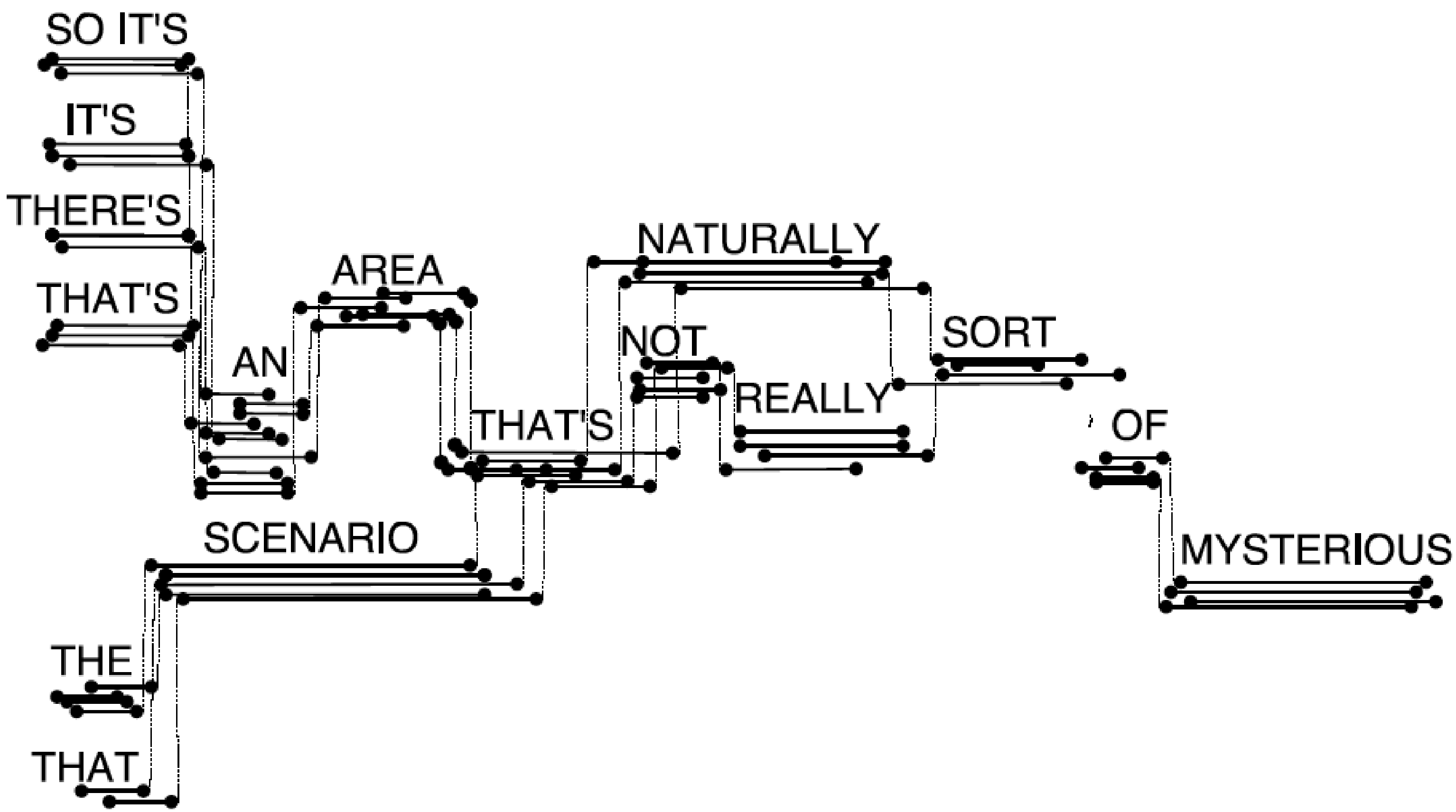
Word-dependent ('bigram') N-best

- Intuition:
  - Instead of each state merging all paths from start of sentence
  - We merge all paths that share the same previous word
- Details:
  - This will require us to do a more complex traceback at the end of sentence to generate the N-best list

Word-dependent (‘bigram’) N-best

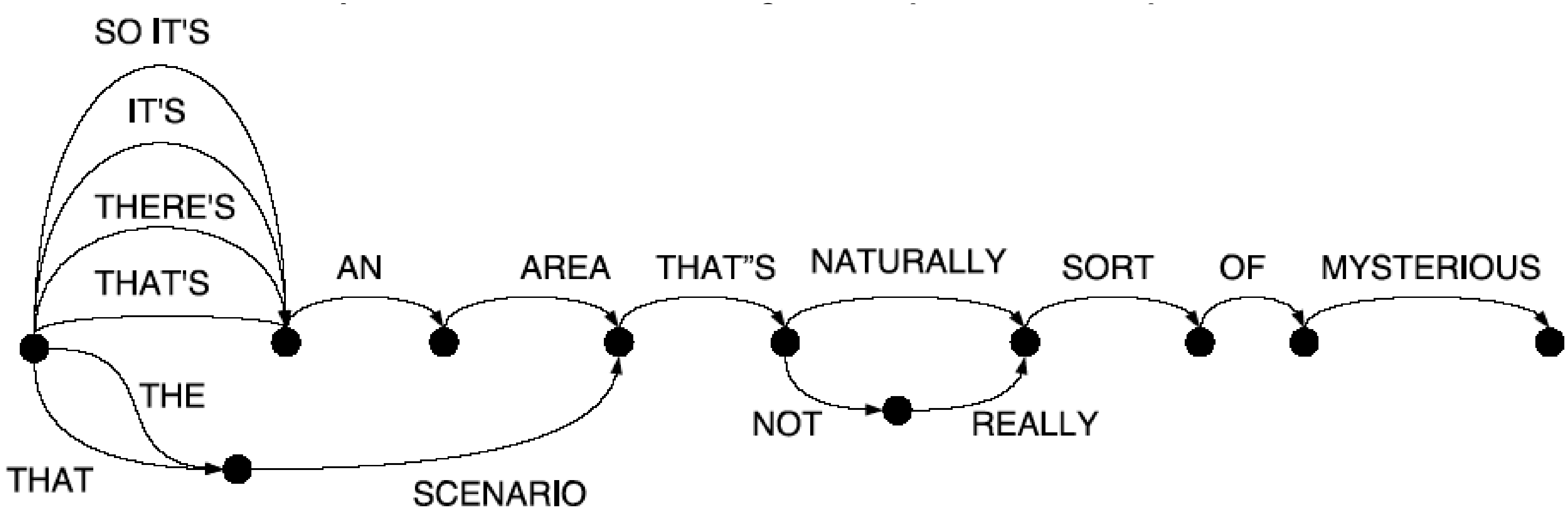
- At each state preserve total probability for each of k << N previous words
  - K is 3 to 6; N is 100 to 1000
- At end of each word, record score for each previous word hypothesis and name of previous word
  - So each word ending we store “alternatives”
- But, like normal Viterbi, pass on just the best hypothesis
- At end of sentence, do a traceback
  - Follow backpointers to get 1-best
  - But as we follow pointers, put on a queue the alternate words ending at same point
  - On next iteration, pop next best

Word Lattice



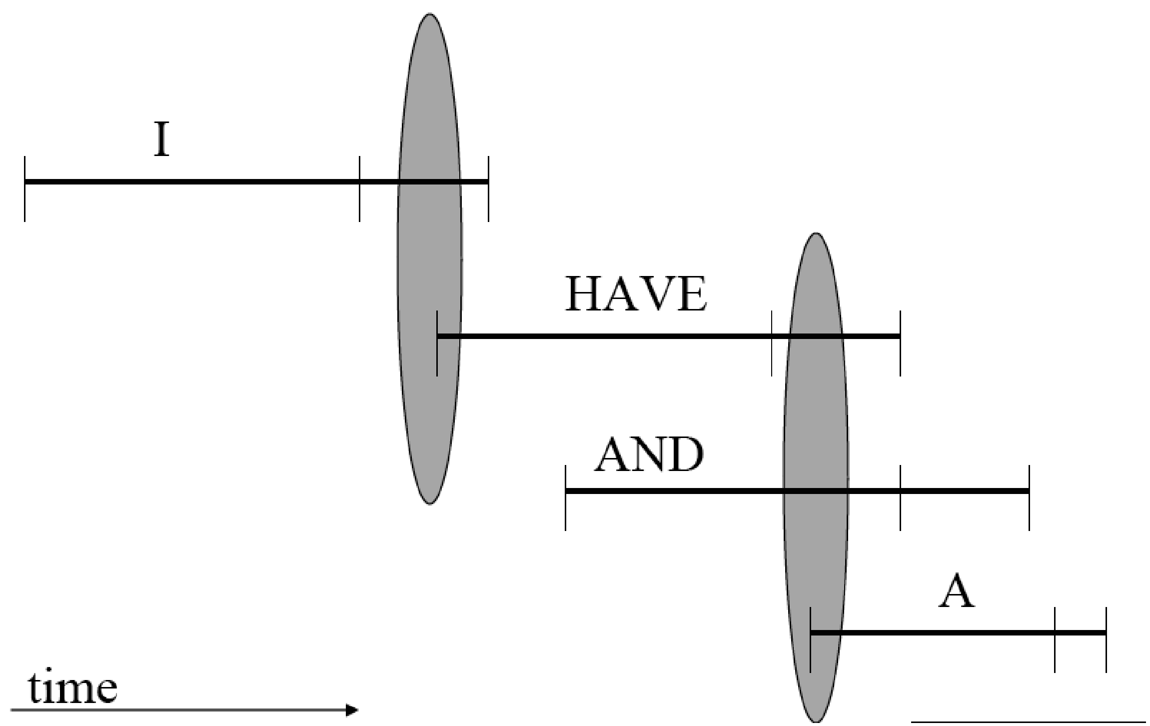
Word Graph

- Timing information removed



Converting word lattice to word graph

- Word lattice can have range of possible end frames for word
- Create an edge from (w_i,t_i) to (w_j,t_j) if t_{j-1} is one of the end-times of w_i



Slide from Bryan Pellom

Lattices

- Some researchers are careful to distinguish between word graphs and word lattices
- But we'll follow convention in using "lattice" to mean both word graphs and word lattices.
- Two facts about lattices:
  - Density: the number of word hypotheses or word arcs per uttered word
  - Lattice error rate (also called "lower bound error rate"): the lowest word error rate for any word sequence in lattice
    - Lattice error rate is the "oracle" error rate, the best possible error rate you could get from rescoring the lattice.
    - We can use this as an upper bound

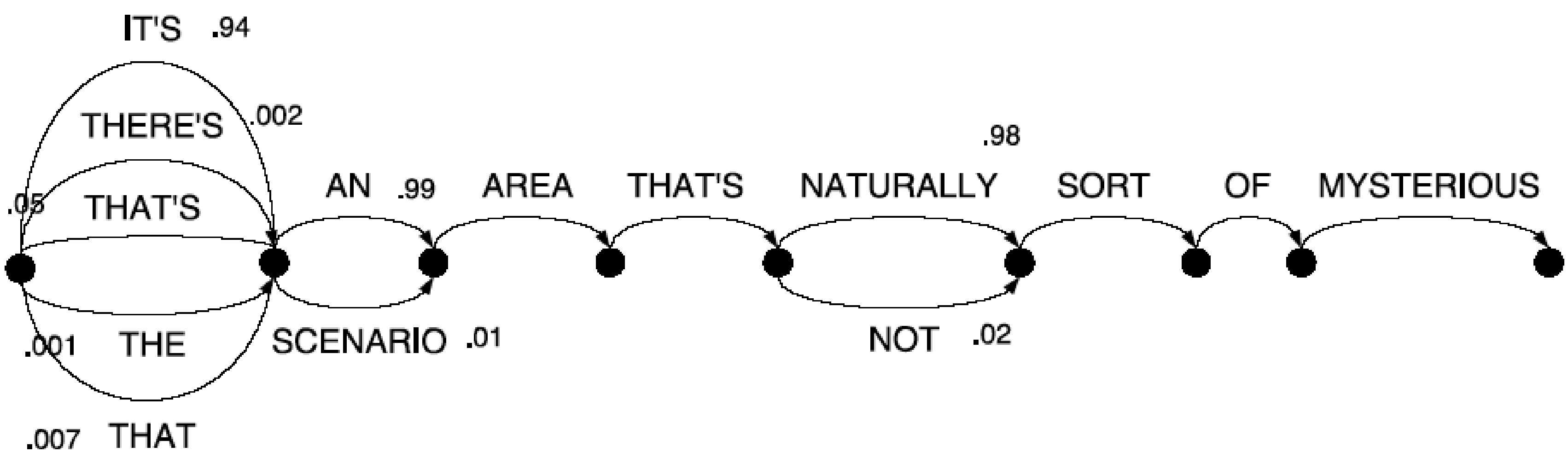
Posterior lattices

- We don’t actually compute posteriors:

$$\hat{W} = \underset{W \in \mathcal{L}}{\operatorname{argmax}} \frac{P(O|W)P(W)}{P(O)} = \underset{W \in \mathcal{L}}{\operatorname{argmax}} P(O|W)P(W)$$

- Why do we want posteriors?
  - Without a posterior, we can choose best hypothesis, but we can’t know how good it is!
  - In order to compute posterior, need to
    - Normalize over all different word hypothesis at a time
  - Align all the hypotheses, sum over all paths passing through word

Mesh = Sausage = pinched lattice



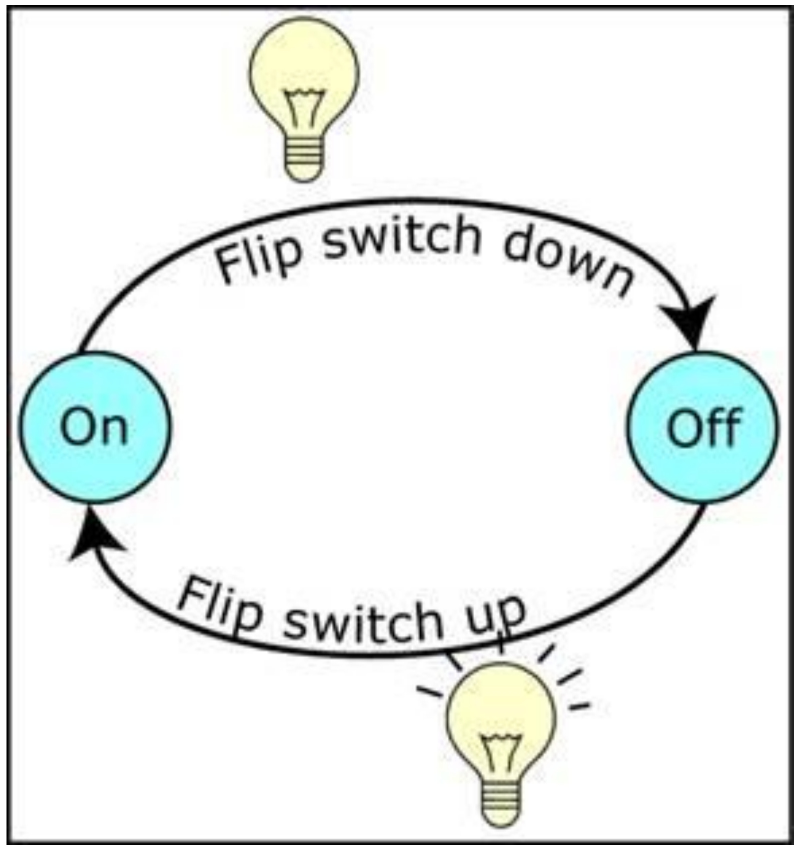
Summary: one-pass vs. multipass

- Potential problems with multipass
  - Can't use for real-time (need end of sentence)
    - (But can keep successive passes really fast)
  - Each pass can introduce inadmissible pruning
    - (But one-pass does the same w/beam pruning and fastmatch)
- Why multipass
  - Very expensive KSs. (NL parsing, higher-order n-gram, etc.)
  - Spoken language understanding: N-best perfect interface
  - Research: N-best list very powerful offline tools for algorithm development
  - N-best lists needed for discriminant training (MMIE, MCE) to get rival hypotheses

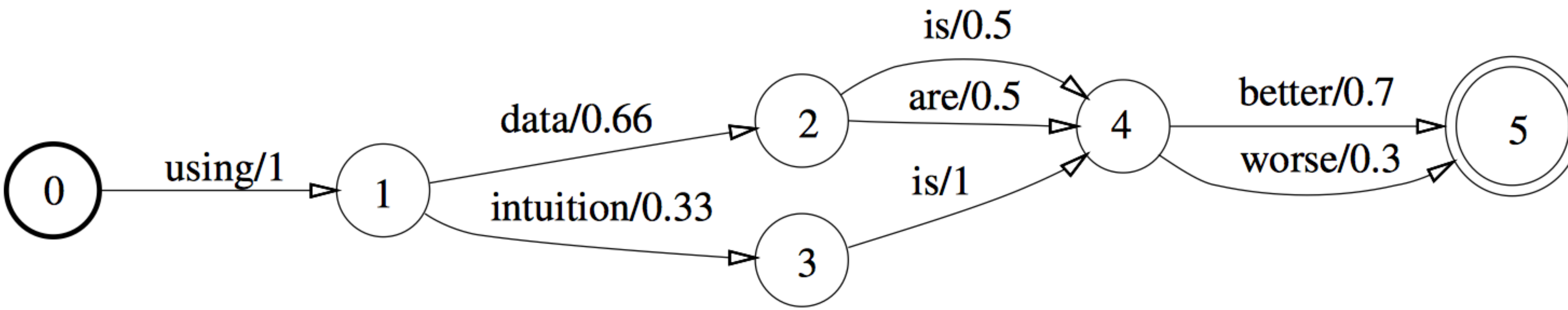
Weighted Finite State Transducers for ASR

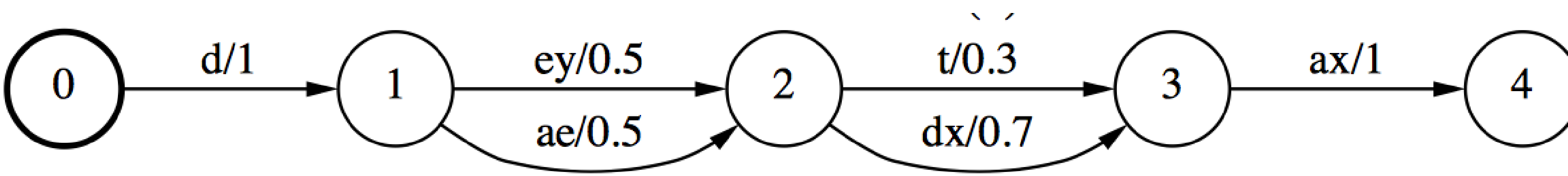
- The modern paradigm for ASR decoding
- Used by Kaldi
- Weighted finite state automaton that transduces an input sequence to an output sequence
- Mohri, Mehryar, Fernando Pereira, and Michael Riley. "Speech recognition with weighted finite-state transducers." In *Springer Handbook of Speech Processing*, pp. 559-584. Springer Berlin Heidelberg, 2008.
- http://www.cs.nyu.edu/~mohri/pub/hbka.pdf

Simple State Machine

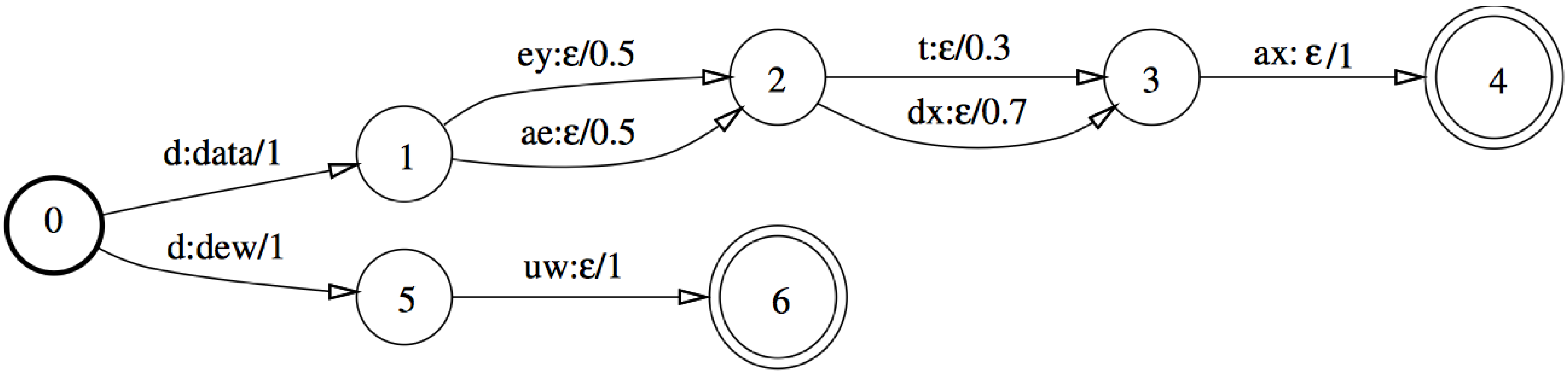


Weighted Finite State Acceptors





Weighted Finite State Transducers



WFST Algorithms

Composition: combine transducers at different levels.

If G is a finite state grammar and P is a pronunciation dictionary, $P \circ G$ transduces a phone string to word strings allowed by the grammar

Determinization: Ensures each state has no more than one output transition for a given input label

Minimization: transforms a transducer to an equivalent transducer with the fewest possible states and transitions

slide from Steve Renals

WFST-based decoding in Kaldi: HCLG

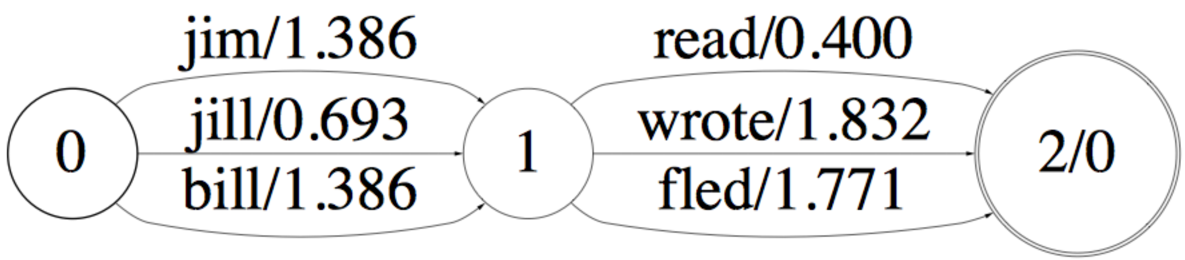
- Represent the following components as WFSTs:
  - H: HMM structure
  - C: Phonetic context dependency
  - L: Lexicon (Pronunciation dictionary)
  - G: Grammar (Language model)
  - The decoding network is defined by their composition:

$H \circ C \circ L \circ G$

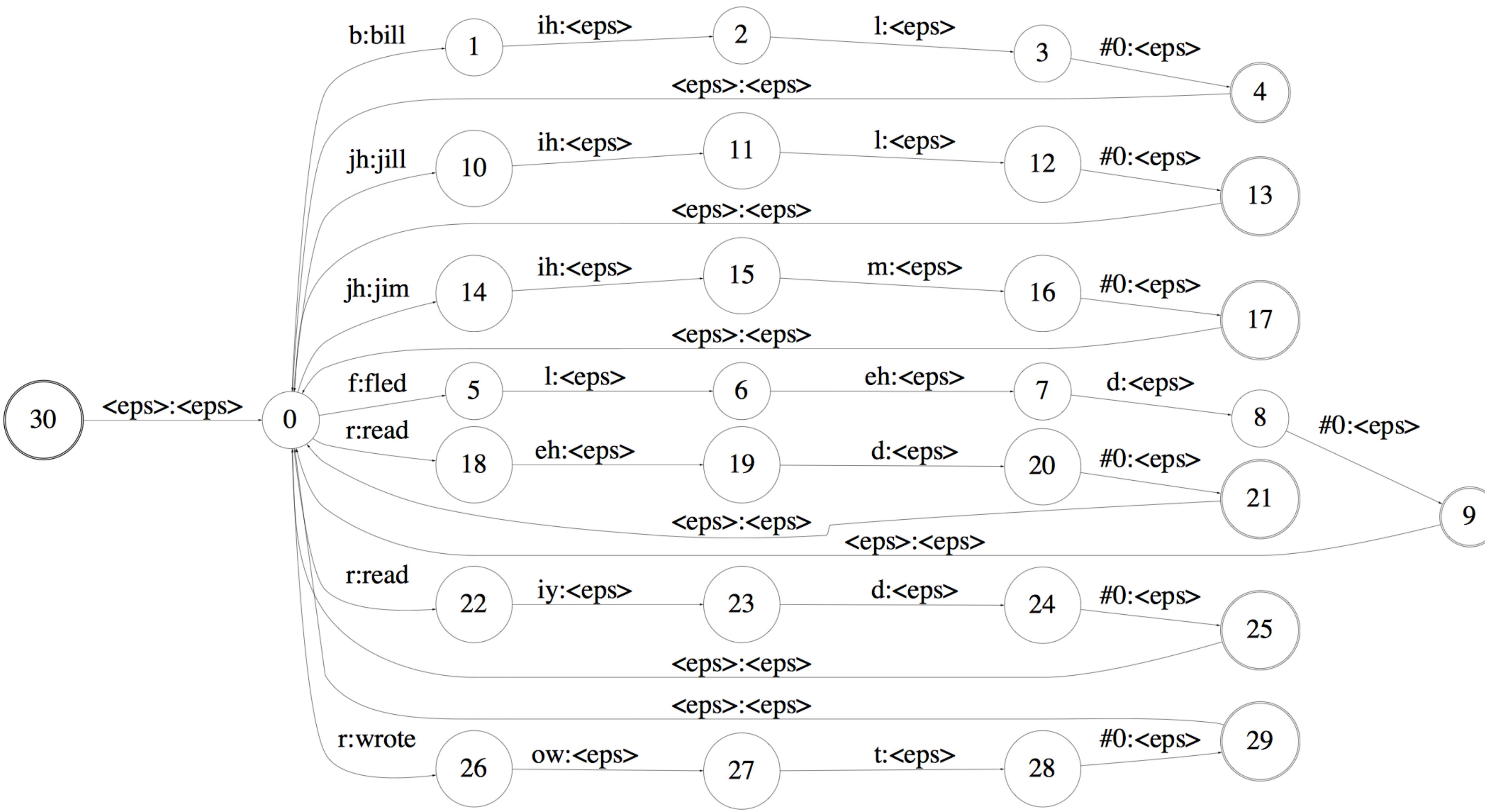
- Successively determinize and combine the component transducers, then minimize the final network

slide from Steve Renals

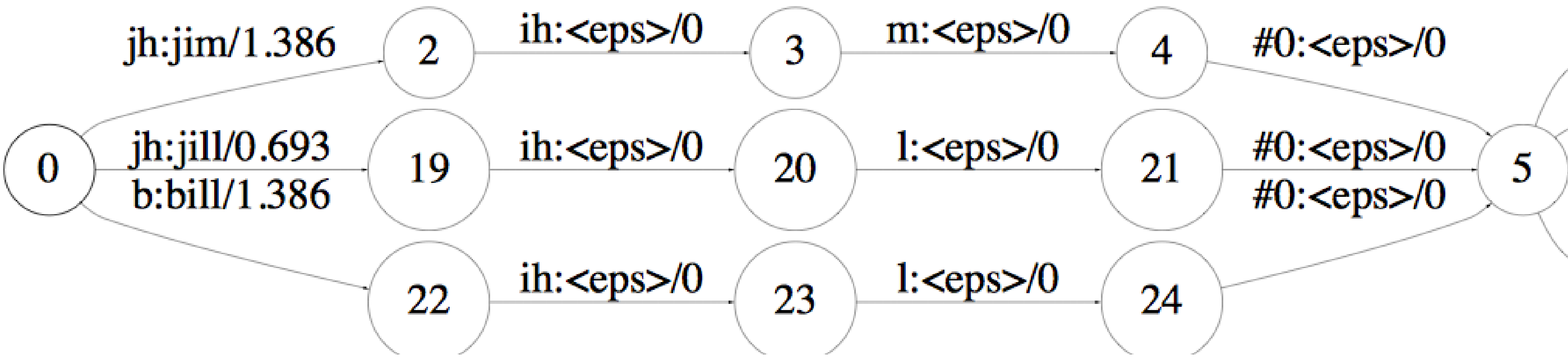
G (Language model)



L (Pronunciation dictionary)



G o L



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m:<eps>/0
#0:<eps>/0
jh:jill/0.693
b:bill/1.386
ih:<eps>/0
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#0:<eps>/0
#0:<eps>/0
ih:<eps>/0
l:<eps>/0
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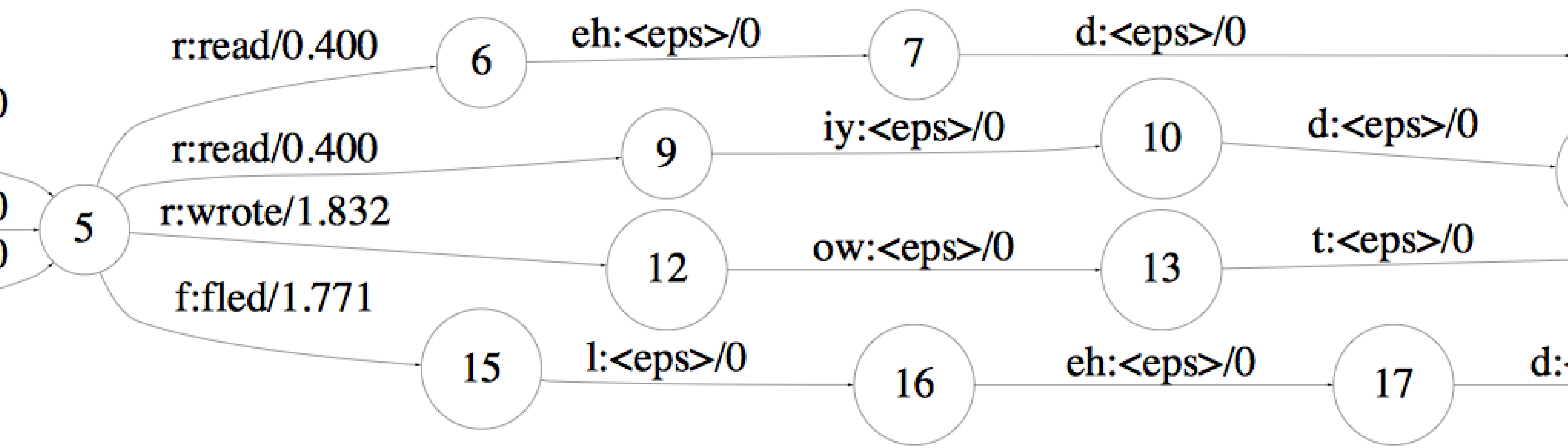

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eh:<eps>/0
d:<eps>/0
r:read/0.400
iy:<eps>/0
d:<eps>/0
r:wrote/1.832
ow:<eps>/0
t:<eps>/0
f:fled/1.771
l:<eps>/0
eh:<eps>/0
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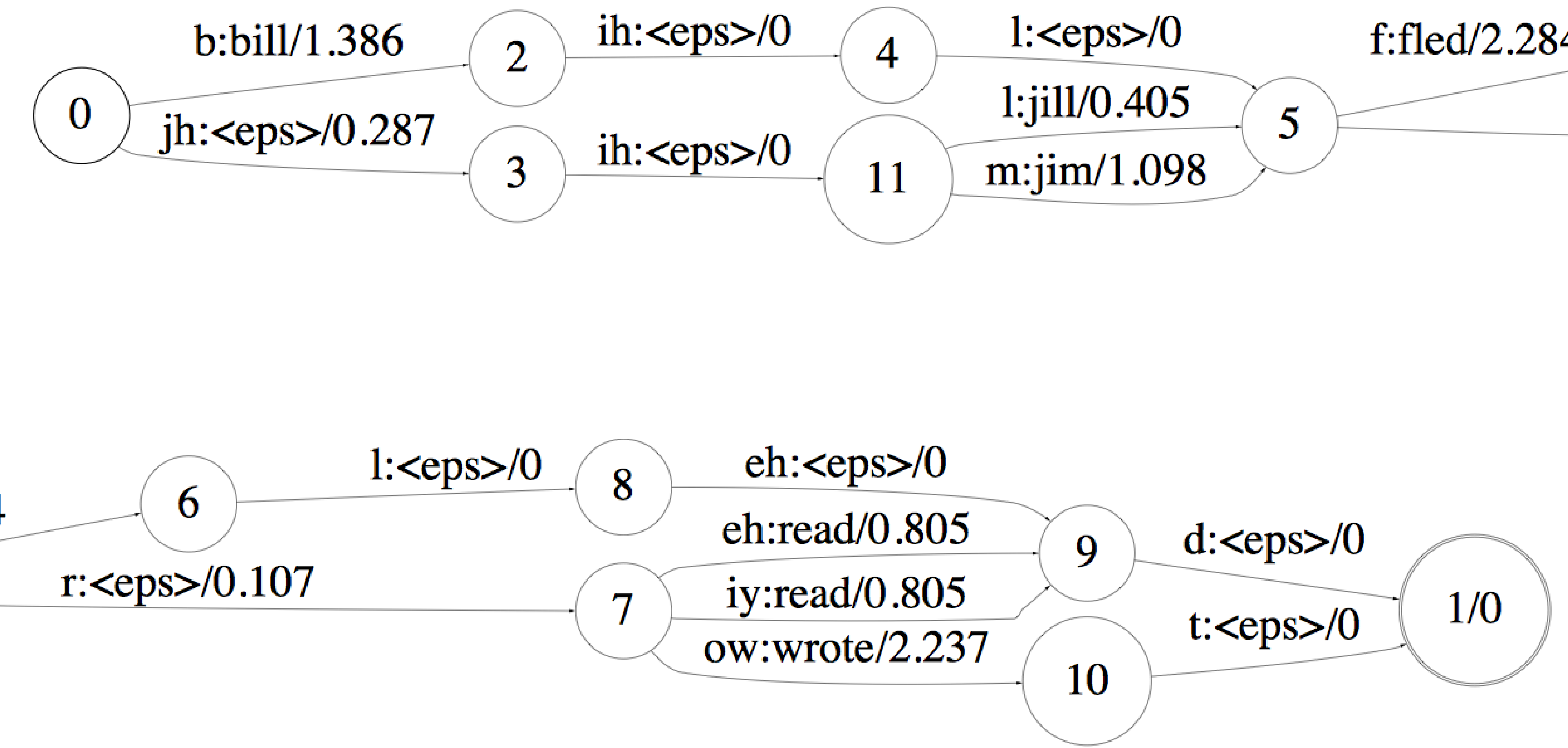
12
13
15
16
17

min(det(L o G))


0
b:bill/1.386
2
ih:<eps>/0
4
l:<eps>/0
f:fled/2.28
jh:<eps>/0.287
3
ih:<eps>/0
11
l:jill/0.405
m:jim/1.098
5
6
l:<eps>/0
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9
d:<eps>/0
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7
iy:read/0.805
ow:wrote/2.237
10
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Advanced Search (= Decoding)

- How to weight the AM and LM
- Speeding things up: Viterbi beam decoding
- Multipass decoding
  - N-best lists
  - Lattices
  - Word graphs
  - Meshes/confusion networks
- Finite State Methods
  - For a more thorough introduction to WFST decoding in Kaldi:

    http://danielpovey.com/files/Lecture4.pdf

Appendix: Baum-Welch Training

Input to Baum-Welch

- O unlabeled sequence of observations
- Q vocabulary of hidden states

- For ice-cream task
  - O = {1,3,2,...,}
  - Q = {H,C}

Starting out with Observable Markov Models

- How to train?
- Run the model on observation sequence O.
- Since it's not hidden, we know which states we went through, hence which transitions and observations were used.
- Given that information, training:
  - $B = \{b_k(o_t)\}$: Since every state can only generate one observation symbol, observation likelihoods B are all 1.0
  - $A = \{a_{ij}\}$:

$$a_{ij} = \frac{C(i \rightarrow j)}{\sum_{q \in Q} C(i \rightarrow q)}$$

Extending Intuition to HMMs

- For HMM, cannot compute these counts directly from observed sequences
- Baum-Welch intuitions:
  - Iteratively estimate the counts.
    - Start with an estimate for a_{ij} and b_k, iteratively improve the estimates
  - Get estimated probabilities by:
    - computing the forward probability for an observation
    - dividing that probability mass among all the different paths that contributed to this forward probability

The Backward algorithm

- We define the backward probability as follows:

$$b_t(i) = P(o_{t+1}, o_{t+2}, \ldots o_T, | q_t = i, \Phi)$$

- This is the probability of generating partial observations O_{t+1T} from time *t+1* to the end, given that the HMM is in state *i* at time *t* and of course given Φ.

The Backward algorithm

1. **Initialization:**

$$\beta_T(i) \;=\; a_{i,F}\,, \quad 1 \le i \le N$$

2. **Recursion** (again since states 0 and q_F are non-emitting):

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij}\, b_j(o_{t+1})\, \beta_{t+1}(j), \quad 1 \le i \le N, 1 \le t < T$$

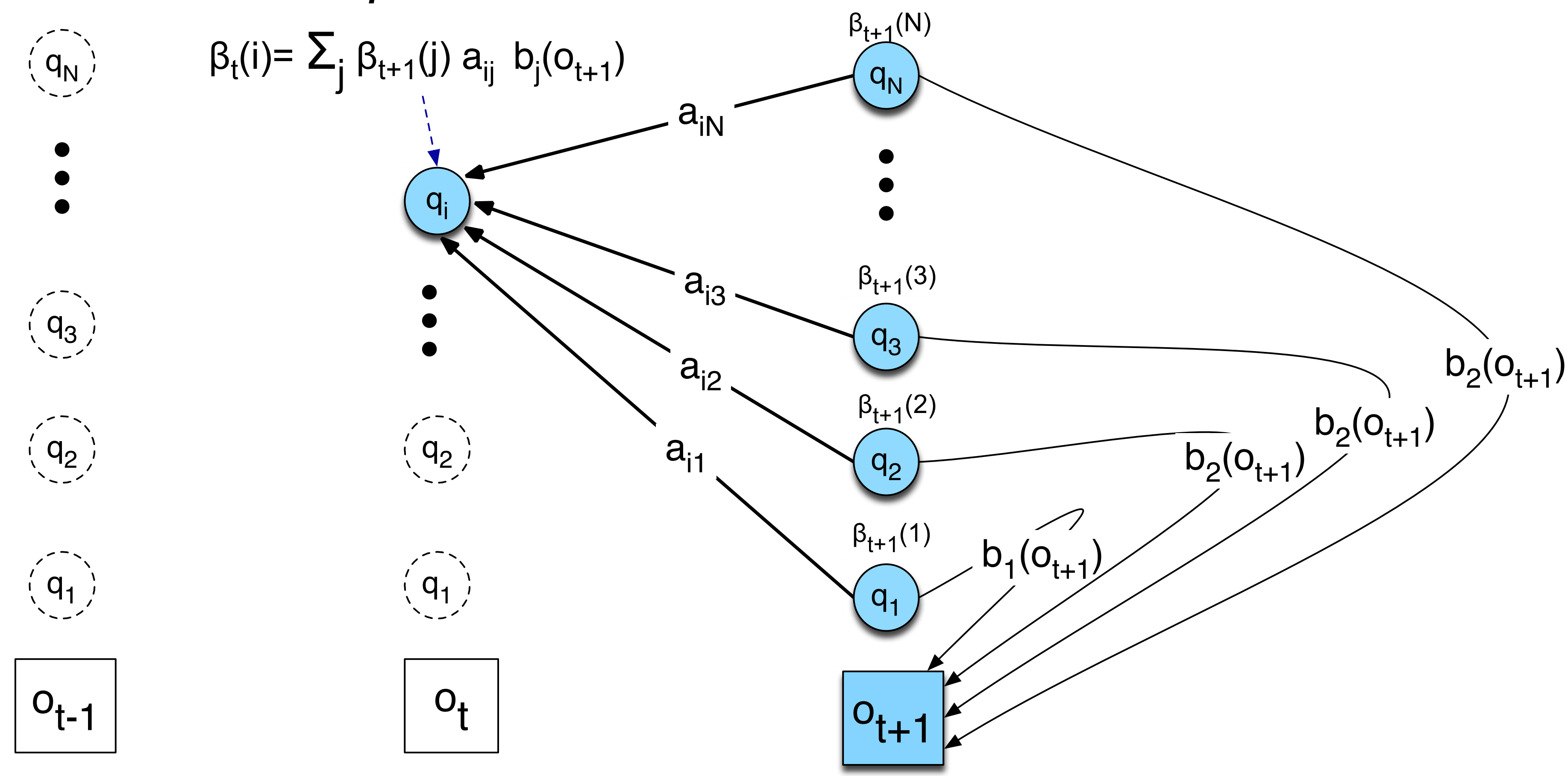
3. **Termination:**

$$P(O|\lambda) = \alpha_T(q_F) = \beta_1(\overset{q_0}{\cancel{0}}) = \sum_{j=1}^{N} a_{0j}\, b_j(o_1)\, \beta_1(j)$$

Inductive step of the backward algorithm

- Computation of $\beta_t(i)$ by weighted sum of all successive values $\beta t+1$

$$\beta_t(i)= \Sigma_j\ \beta_{t+1}(j)\ a_{ij}\ \ b_j(o_{t+1})$$



Intuition for re-estimation of aij

- We will estimate $\hat{a}_{ij}$ via this intuition:

$$\hat{a}_{ij} = \frac{\text{expected number of transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

- Numerator intuition:
  - Assume we had some estimate of probability that a given transition i→j was taken at time *t* in observation sequence.
  - If we knew this probability for each time t, we could sum over all *t* to get expected value (count) for i→j.

Re-estimation of aij

- Let ξ_t be the probability of being in state *i* at time *t* and state *j* at time *t+1*, given $O_{1..T}$ and model Φ:

$$x_t(i,j) = P(q_t = i, q_{t+1} = j \mid O, /)$$

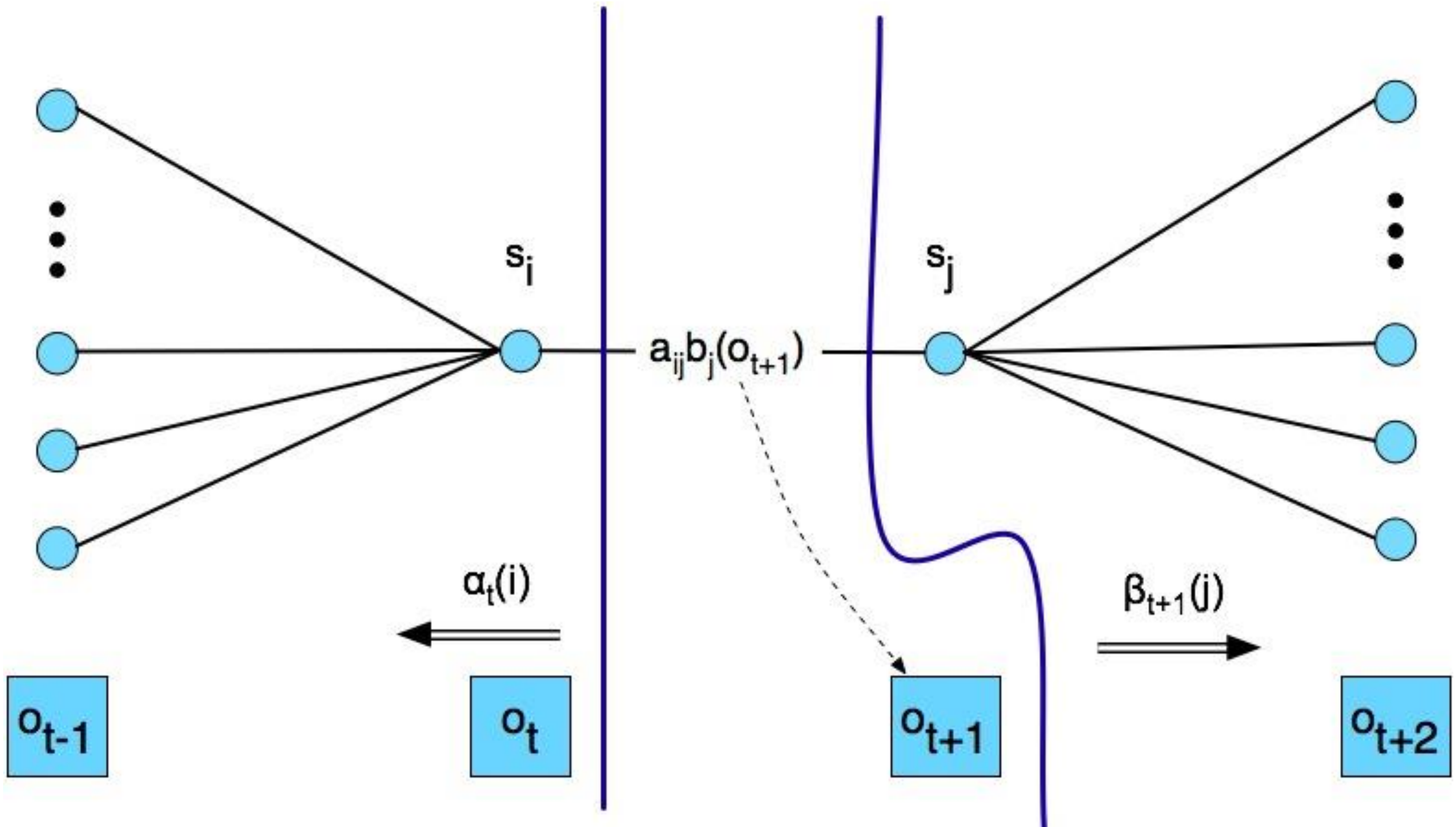
- We can compute ξ from not-quite-ξ, which is:

$$not\_quite\_x_t(i,j) = P(q_t = i, q_{t+1} = j, O \mid /)$$

Computing not-quite-ξ

The four components of $P(q_t = i, q_{t+1} = j, O \mid /)$: a, b, a_{ij} and $b_j(o_t)$

$$\text{not-quite-}\xi_t(i,j) = \alpha_t(i)\, a_{ij}\, b_j(o_{t+1})\, \beta_{t+1}(j)$$



From not-quite-ξ to ξ

- We want:

$$x_t(i,j) = P(q_t = i, q_{t+1} = j \mid O, /)$$

- We've got:

$$not\_quite\_x_t(i,j) = P(q_t = i, q_{t+1} = j, O \mid /)$$

- Which we compute as follows:

$$\text{not-quite-}\xi_t(i,j) = \alpha_t(i)\, a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$

From not-quite-ξ to ξ

- We want:

$$x_t(i,j) = P(q_t = i, q_{t+1} = j \mid O, /)$$

- We've got:

$$not\_quite\_x_t(i,j) = P(q_t = i, q_{t+1} = j, O \mid /)$$

- Since:

$$P(X \mid Y, Z) = \frac{P(X, Y \mid Z)}{P(Y \mid Z)}$$

- We need:

$$x_t(i,j) = \frac{not\_quite\_x_t(i,j)}{P(O \mid /)}$$

From not-quite-ξ to ξ

$$x_t(i,j) = \frac{not\_quite\_x_t(i,j)}{P(O \mid l)}$$

$$\text{not-quite-}\xi_t(i,j) = \alpha_t(i)\, a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$

$$P(O|\lambda) = \alpha_T(q_F) = \beta_T(q_0) = \sum_{j=1}^{N} \alpha_t(j)\beta_t(j)$$

$$\xi_t(i,j) = \frac{\alpha_t(i)\, a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)}{\alpha_T(q_F)}$$

From ξ to a_{ij}

$$\hat{a}_{ij} = \frac{\text{expected number of transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

- The expected number of transitions from state *i* to state *j* is the sum over all *t* of ξ
- The total expected number of transitions out of state *i* is the sum over all transitions out of state *i*
- Final formula for reestimated a_{ij}:

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{k=1}^{N} \xi_t(i,k)}$$

Re-estimating the observation likelihood b

- This is the probability of a given symbol v_k from the observation vocabulary V, given a state j: $\hat{b}_j(v_k)$.

$$\hat{b}_j(v_k) = \frac{\text{expected number of times in state j and observing symbol } v_k}{\text{expected number of times in state } j}$$

We'll need to know $\gamma_t(j)$: the probability of being in state *j* at time *t:*

$$\gamma_t(j) = P(q_t = j | O, \lambda)$$

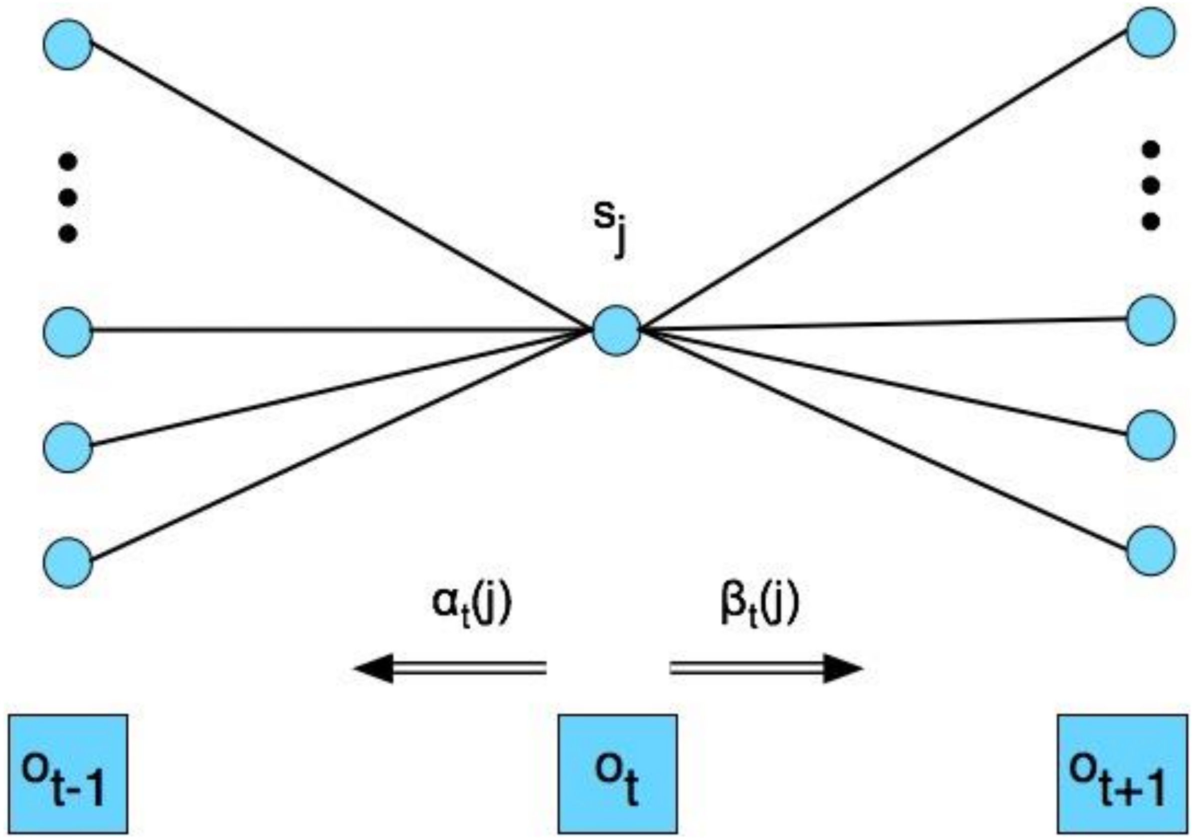
$$\gamma_t(j) = \frac{P(q_t = j, O | \lambda)}{P(O|\lambda)}$$

$$\gamma_t(j) = \frac{\alpha_t(j)\beta_t(j)}{P(O|\lambda)}$$

Computing γ

$$\gamma_t(j) = \frac{P(q_t = j, O|\lambda)}{P(O|\lambda)}$$

$$\gamma_t(j) = \frac{\alpha_t(j)\beta_t(j)}{P(O|\lambda)}$$



$$\hat{b}_j(v_k) = \frac{\sum_{t=1 s.t. O_t=v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}$$

Summary

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{k=1}^{N} \xi_t(i,k)}$$

The ratio between the expected number of transitions from state i to j and the expected number of all transitions from state i

$$\hat{b}_j(v_k) = \frac{\sum_{t=1 s.t. O_t=v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}$$

The ratio between the expected number of times the observation data emitted from state j is v_k, and the expected number of times any observation is emitted from state j

The Forward-Backward Algorithm

function FORWARD-BACKWARD(*observations* of len T, *output vocabulary* V, *hidden state set* Q) **returns** *HMM=(A,B)*

initialize A and B
iterate until convergence

E-step

$$\gamma_t(j) = \frac{\alpha_t(j)\beta_t(j)}{\alpha_T(q_F)} \ \forall\, t \text{ and } j$$

$$\xi_t(i,j) = \frac{\alpha_t(i)\,a_{ij}b_j(o_{t+1})\beta_{t+1}(j)}{\alpha_T(q_F)} \ \forall\, t,\ i, \text{ and } j$$

M-step

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1}\xi_t(i,j)}{\sum_{t=1}^{T-1}\sum_{k=1}^{N}\xi_t(i,k)} \qquad \hat{b}_j(v_k) = \frac{\sum_{t=1 s.t.\ O_t=v_k}^{T}\gamma_t(j)}{\sum_{t=1}^{T}\gamma_t(j)}$$

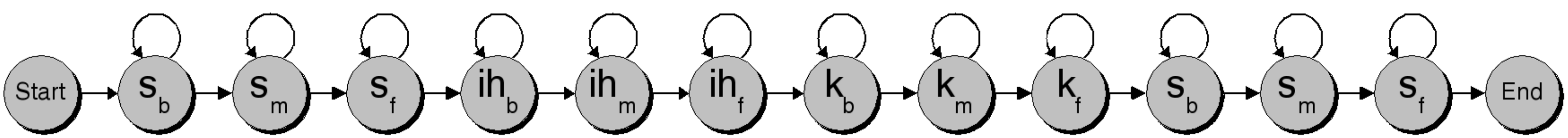
return A, B

Summary: Forward-Backward Algorithm

- Intialize Φ=(A,B)
- Compute α, β, ξ
- Estimate new Φ' =(A,B)
- Replace Φ with Φ'
- If not converged go to 2

Applying FB to speech: Caveats

- Network structure of HMM is always created by hand
  - no algorithm for double-induction of optimal structure and probabilities has been able to beat simple hand-built structures.
  - Always Bakis network = links go forward in time
  - Subcase of Bakis net: beads-on-string net:



- Baum-Welch only guaranteed to return local max, rather than global optimum
- At the end, we through away A and only keep B