SYNTHESIZING GROUND MOTION USING BAYESIAN OPTIMIZATION IN VARIATIONAL AUTO-ENCODER

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In earthquake engineering, accurately predicting the response of a structure to ground motion is indispensable for the design of earthquake-resistant structures. The primary objective of this research is to develop an advanced method for synthesizing and optimizing ground motions using Bayesian optimization. By integrating Bayesian Optimization, the research seeks to enhance structural responses' predictive accuracy and reliability under extreme seismic events. The study is organized into three case studies, each of which focuses on different features or conditions for ground motion synthesis. All cases employ a dataset processed by the short-time Fourier transform with the same Variational Auto-encoder, intending to optimize specific features while ensuring that the response spectrum closely aligns with the design spectrum. The first case examines the optimization of peak ground acceleration through the incremental adjustment of parameters within the latent space in a uniform direction. The second case study extends the optimization approach to optimize the peak ground acceleration within a 95% confidence level. The third case study is concerned with the optimization of the maximum displacement of a Single-Degree-of-Freedom system within a 95% confidence interval. The integration of machine learning and Bayesian optimization aims to enhance the exploration of the latent space of the ground motion dataset trained by the Variational Auto-encoder, thereby reducing the computational costs. The results show that the synthesized seismic motions not only realistically reflect actual ground motion but are also optimized for the specific features we targeted. This study additionally focuses on ways to save computational resources and improve methods of ground motion synthesis in terms of reliability and accuracy in earthquake engineering.
keyword: *Ground Motion, Bayesian Optimization, Variational Auto-encoder*

1. INTRODUCTION

Disaster resilience is crucial in today's rapidly changing world. The requirement for resilience will be more pressing in earthquake-prone areas since severe consequences can lead to huge casualties and property damage. In Taiwan, the 1999 Chi-Chi earthquake revealed deficiencies in building design and necessitated a reassessment of the seismic code[1)]. In response, the government enacted a more comprehensive seismic design code that requires using materials and construction techniques to address the seismic activity patterns characteristic of Taiwan. Following the Tohoku earthquake, a thorough reevaluation of Japanese society was conducted. Because of the magnitude of the damage this earthquake inflicted, Japan has encouraged the use of innovative technology[2)]. Nevertheless, there is considerable scope for further improvement. In the coming decades, seismic design will continue to be a significant concern in the field of civil engineering.

Seismic design is based on the structural analysis of the seismic performance of a structure and design improvement. There are many other types of analysis methods currently available, such as linear dynamic and static analysis, and nonlinear dynamic and static analysis. Among these techniques, nonlinear dynamics analysis is the most accurate method to evaluate the performance of a building in an earthquake. Here is a threshold that although nonlinear dynamics analysis is accurate, the analysis is costly and requires a lot of computational resources to evaluate complex structures. All other techniques try to simplify the analysis at the expense of accuracy. With the increasing focus on performance-based design, these methods allow the evaluation of the strength, deformability, and damping capacity of structures[3)]. The other optimization processes such as Incremental Dynamic Analysis (IDA), engineers can predict the threshold of structural failure by progressively increasing the seismic intensity. Each approach offers specific advantages, making dynamic analysis an essential aspect of creating resilient and efficient seismic designs.

The primary objective of this research is to develop an advanced method for selecting and optimizing ground motions using Bayesian optimization. By integrating Bayesian Optimization, the research seeks to enhance structural responses' predictive accuracy and reliability under extreme seismic events. The study is organized into three case studies, each of which focuses on different features or conditions for ground motion synthesis.

2. SHORT-TIME FOURIER TRANSFORM DATASET AND VARIATIONAL AUTO-ENCODER

(1) Short-Time Fourier Transform dataset

The original dataset used in this study is 20,000 ground motion records collected from K-NET. has already processed spectrum fitting on this dataset. For each ground motion record, a 60-second segment from the acceleration time history that encompasses the major component of the wave is selected. Because the seismograph's sampling rate is 100 Hz, a single ground motion record is 60 seconds. It is necessary to preprocess the dataset before its utilization to train a Variational Auto-encoder model.

The short-time Fourier transform (STFT) is a commonly used approach in signal processing, especially in ground motion analysis. This will result in the modification of the time-domain signal into the frequency domain, allowing for the viewing of frequencies at different points in time by applying STFT to acceleration data. This approach employs a fixed-size window that slides across the full signal in STFT, specifying the signal components at each time point. On each segment of data within the window, a Fast Fourier Transform (FFT) is executed to transform the acceleration data from the time domain into the frequency domain for the specified duration. The results of the FFT process, which are complex values combined by multiple frequency components, are converted into their amplitude-phase form.

The amplitude component may be utilized for the generation of a spectrogram, which is a three-dimensional representation of time, frequency, and amplitude. The amplitude spectrum represents the amount of energy present in each time window and at each frequency point. In earthquake engineering, the amplitude spectrum indicates which frequencies were active during an earthquake. Spectrograms are typically color-coded to indicate different amplitudes. Higher amplitudes are represented by brighter colors, while time and frequency correspond to the horizontal and vertical axes of the plot, respectively.

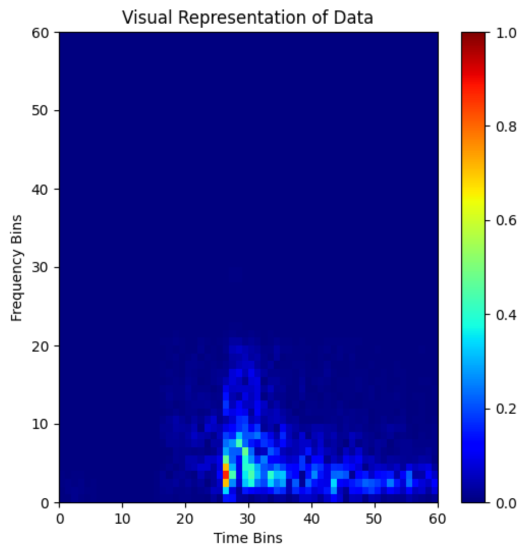

Figure 1. The STFT spectral of GM record No.323.

(2) Variational Auto-encoder

The following variable autoencoder (VAE) model consists of two main parts, encoder and decoder. Using ground motion data STFT (Short Time Fourier Transform) spectral forms to train the model. The following are the details of the VAE model:

The input data is the spectral format of the ground motion STFT dataset. This dataset is input to the encoder through an input layer of shape 60x60x1. This data passes through several convolutional layers and MaxPooling2D layer in the Encoder and fits with ReLU activation functions to extract features. As the network gets deeper, the convolution layers become more complex. At each step, more complex feature selection can be performed on the input ground motion STFT data.

The dense layers in the encoder, within the 26-dimension latent space, have as main components `z_mean` and `z_log_var`. The former estimates the latent distribution mean, while the latter computes the logarithmic variance, both of which are fundamental to the latent dimensionality of the data. It is completed with a sampling layer that uses the reparameterization strategy to sample the parameter of the latent space using the values of `z_mean` and `z_log_var`, which will generate new synthetic ground motion data from the learned distributions.

The decoder architecture commences with an input layer shaped as the sampled latent vector from the latent space. It applies convolutional and UpSampling2D layers to rebuild the original input shape, essentially functioning as the mirror opposite of the encoder. This structure allows for the reconstruction of ground motion data.

3. METHODOLOGY

(1) Bayesian optimization

Bayesian optimization (BO) is an efficient global optimization algorithm that is widely used in machine learning, engineering design, automatic parameter adjustment, and other fields. The core idea is the use of the Gaussian Process (GP) as a surrogate model to approximate the unknown target function and the use of the acquisition function to intelligently select the next evaluation point, to find the global optimal value of the target function in as few trials as possible. It consists mainly of selecting the initial data points, constructing the Gaussian process model, updating the post-test distribution, estimating the sampled functions, and then selecting new points to estimate.

Gaussian process is usually denoted as:

$$f(x) \sim \mathrm{GP}(m(x), k(x, x')) \tag{1}$$

where $m(x)$ is a mean function describing the expected output in and $k(x, x')$ is a covariance function describing the correlation of the output between any two points x and x'.

(2) Tree-structured Parzen Estimation

Tree-structured Parzen Estimation (TPE) is a type of Bayesian Optimization (BO) approach and a sampler in the Optuna framework. Like Bayesian Optimization, TPE focuses on hyperparameter optimization by developing a probabilistic model that maps the relationship between hyperparameters and the target function.

The core of the TPE algorithm involves two distinct probabilistic models[4)]:

- High-performance model $l(x)$, estimated as:

$$l(x) = p(x \mid y > y^*)$$

- Low-performance model $g(x)$, estimated as:

$$g(x) = p(x \mid y \leq y^*)$$

where y^* is the cutoff point, typically the best-observed value of the objective function so far.

The TPE algorithm infers new hyperparameters to maximize the Expected Improvement (EI), calculated as (Watanabe, 2023):

$$EI(x) = \int_{-\infty}^{y^*} (y^* - y) p(y \mid x) \, dy \tag{2}$$

This integral is performed over all values of y that are less than the current optimal value y^*, capturing the region of improvement relative to y^*. The product $(y^* - y)$ indicates the degree of improvement, while $p(y \mid x)$, the conditional probability distribution of the target function y given hyperparameter x, adjusts the degree of improvement based on the likelihood that x will achieve y.

In our research, we choose the Tree-structured Parzen Estimator (TPE) as a sampler in Bayesian optimization. TPE is especially useful for multi-parameter optimization problems, especially when it is necessary to quickly converge to the optimal solution from a large number of possible solutions. This is particularly important for the expensive calculation problem, which typically involves a large number of input parameters and complex model structures.

4. RESULTS

(1) Case 1: Peak Ground Acceleration

Groups of data were selected from both the top and bottom 10% of Peak Ground Acceleration (PGA) values. These data sets were processed through the encoder of a Variational Auto-encoder (VAE), resulting in the calculation of the mean latent space parameters for these groups. By gradually adjusting the upper bound of searching space for upward adjustment from the mean of the bottom 10% of groups by adding a fraction, the difference between the means of the top and bottom 10% of groups. In each group, the searching space of the lower bound remains fixed while the upper bound changes gradually.

Figure 2 shows all the iterations for this case, plotting the distribution of t and the value of peak ground acceleration. The distribution of Peak Ground Acceleration across the vector can be visualized in this diagram. In this study, the results from the first five groups demonstrated high consistency, with all groups showing a maximum value of 8.80, all derived from an optimization t around 0.0564. However, the results of the sixth group were not the same although expected to be similar to those of the first five groups. Similarly, groups seven to ten also showed consistency, with each group achieving a peak value of 10.47 from a sampling point of 0.666. Figure 3 and Figure 4 show the synthesized ground motion and comparison of the optimized GM response spectrum and design spectrum.

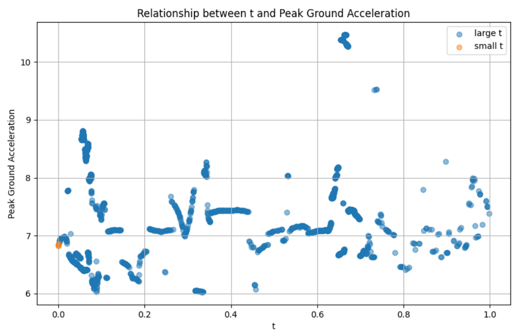

Figure 2. Relationship between t and Peak Ground Acceleration

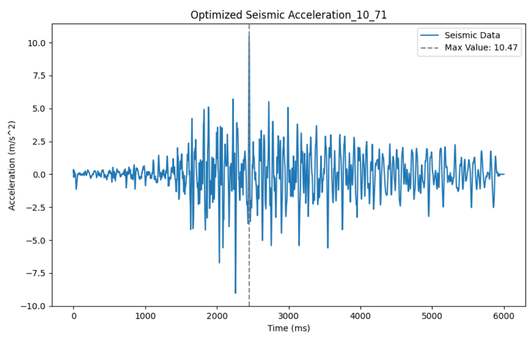

Figure 3. The Optimized Ground Motion of Case 1

These inconsistencies may be due to inherent weaknesses in the TPE algorithm. Although TPE can handle complex parameter spaces and multi-parameter optimization with relative efficiency, its performance may be limited when considering single-parameter optimization. The algorithm did not explore the parameter space enough or reach optimal convergence, which explains why some results fall short of the expected optimization efficiency. These results suggest that the proper choice of the optimization strategy and the configuration of its parameters

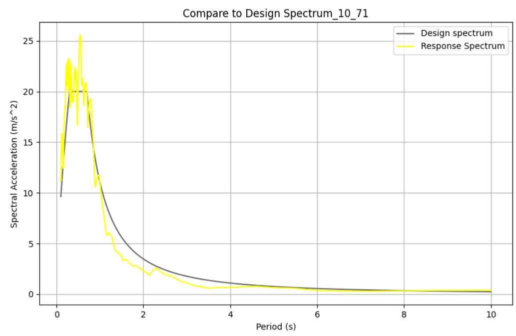

Figure 4. Comparison of Optimized GM Response Spectrum and Design Spectrum

are important to achieve the desired optimization results in any sampler. Therefore, these results will be important in guiding the choice and application of optimization algorithms.

(2) Case 2: Optimize Peak Ground Acceleration in the 95% confidence interval

Given a set of STFT data that is processed through an encoder to predict the latent space values, we calculate the mean and variation of all the data to establish the 95% confidence interval. In this case, the search space was defined as the 95% confidence interval, encompassing 26 dimensions. Using the formula:

$$\text{CI} = \mu \pm z \cdot \frac{\sigma}{\sqrt{n}} \tag{3}$$

where:

- μ is the mean of the latent space values.
- σ is the standard deviation of the latent space values.
- n is the number of STFT dataset.
- z is the z-score corresponding to the 95% confidence level, approximately 1.96 for a two-sided interval.

As a result, TPE is an optimization method that quickly indicates the optimal value. It produced a sample value of 0.971 at the 28th iteration and increased it further to 0.977 at the 297th iteration, showing that TPE would explore the hyperparameter optimization space efficiently. Despite this, there are no more increased values after the iterations. This means that once a point is reached that is close to the optimal region, further improvement becomes gradual and difficult. The results also suggest that the peak ground acceleration values of ground motion in the optimization process increase progressively with time along the slow path. The optimization process is also subtle, as initial results can be exciting, while further improvements require more careful tuning. Some optimization algorithms very quickly reach their limits of improvement, reflecting what often happens in complex optimization scenarios: the phenomenon of diminishing returns. Figure 5 and Figure 6 show the synthesized ground motion and comparison of the optimized GM response spectrum and design spectrum.

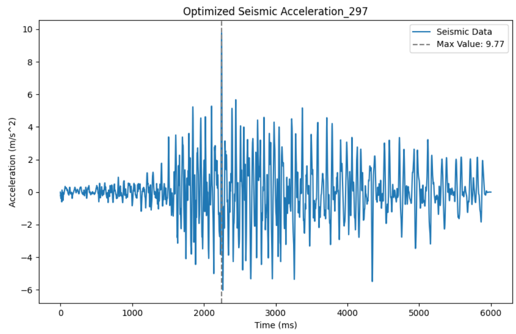

Figure 5. The Optimized Ground Motion of Case 2

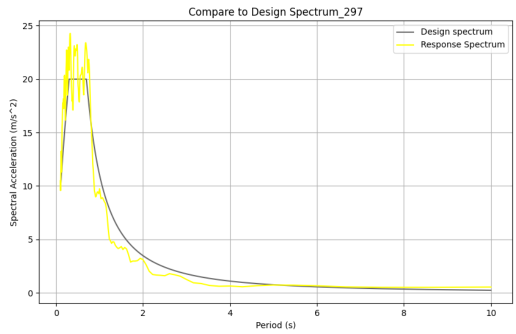

Figure 6. Comparison of Optimized GM Response Spectrum and Design Spectrum

(3) Case 3: Optimize the maximum displacement of SDOF analysis in the 95% confidence interval

Given a set of STFT data that is processed through an encoder to predict the latent space values, we calculate the mean and variation of all the data to establish the 95% confidence interval. In this case, the search space was defined as the 95% confidence interval, encompassing 26 dimensions. Same as in case 2.

In this case, the displacement response of a Single-Degree-of-Freedom (SDOF) system is set as an objective function. In principle, an SDOF system is always considered a simplified model of more complex Multiple-Degrees-of-Freedom systems. By studying the displacement response of the SDOF system, the basic behavior of complex structures can be understood and predicted. The displacement is an index used for the evaluation or assessment of any structure in terms of performance and safety. The displacements of the SDOF system could ensure good performance during earthquakes and meet the expected safety standards of the structure.

When configuring a Single Degree of Freedom (SDOF) model for analyzing high-rise steel or concrete structures, it's crucial to set the natural frequencies between 0.5 Hz and 1.5 Hz. This range is sufficient to portray the actual dynamic behaviors of high-rise steel or concrete buildings, as these structures typically possess natural frequencies within this spectrum. Another fundamental issue in earthquake engineering is resonance, which occurs when the natural frequency of a structure aligns with the dominant frequency of seismic waves. In this case, we set an SDOF model to evaluate possible resonance scenarios and search for the maximum displacement in the latent space.

The SDOF model is configured as follows:

- Stiffness, $k = 2.5$ N/m
- Damping coefficient, $c = 0.1592$ kg/s
- Mass, $m = 0.2533$ kg
- Time step, $\Delta t = 0.01$ sec

Calculation of the natural frequency of the SDOF system:

$$\omega_N = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5}{0.2533}} \approx 3.142\,\text{rad/s} \approx 0.500\,\text{Hz}$$

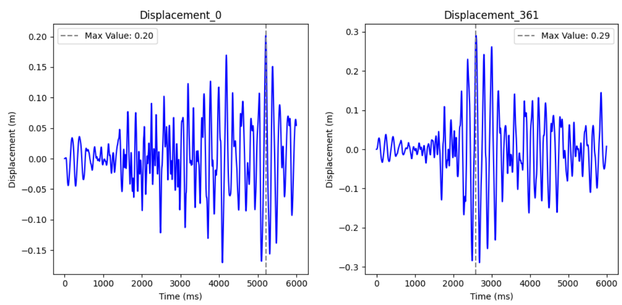

Figure 7. Comparison of Initial and Optimized Maximum Displacement in SDOF System

In the diagram 7 shows the result that the displacement of the SDOF system exhibited minimal variation from its value of 0.20 in the initial iteration to its optimized value of 0.29 in the 361st iteration. Although the maximum displacement of the SDOF system appears almost the same in some groups, the synthesized ground motions are very different.

Figure 8 and Figure 9 show the synthesized ground motion and comparison of the optimized GM response spectrum and design spectrum.

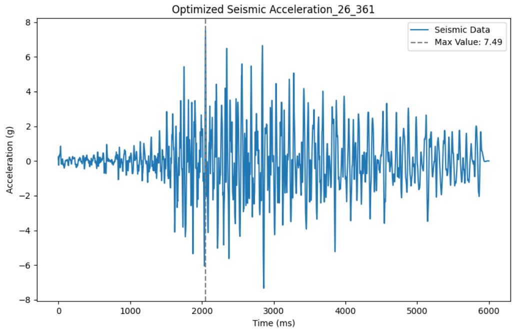

Figure 8. The Optimized Ground Motion of Case 3

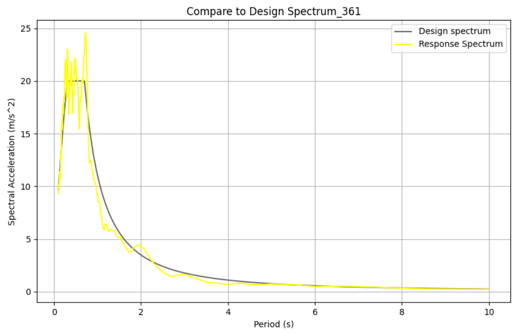

Figure 9. Comparison of Optimized GM Response Spectrum and Design Spectrum

5. CONCLUSIONS

To achieve the objective, the selection and optimization of appropriate ground motions for seismic analysis were explained using Bayesian Optimization with the Tree-structured Parzen Estimator (Optuna). The majority of this research has been devoted to the development and implementation of a methodology that employs machine-learning techniques to enhance the reliability of predictions regarding structural responses by synthesizing ground motion for dynamic analysis.

Case 1 study consisted of peak ground acceleration optimization. In this case study, we used Variational Auto-encoder and Bayesian Optimization to study the optimization of ground motions to maximize the Peak Ground Acceleration while fitting the target response spectrum as closely as possible. By using the STFT dataset and exploring latent space, it is possible to search the ground motion with the constraint of Design Spectrum. Case 2 extends the optimization process to the peak ground acceleration at the 95% confidence level. The inherent variability and therefore uncertainty in the seismic data could be accounted for in the synthesis of the ground motions. In Case 3, the maximum possible displacement of an SDOF system was optimized and kept within the 95% confidence interval. The optimized process and how the maximum displacement alternative could be observed.

This study shows the efficiency of search processes in the modeling of seismic ground motions by integrating the Variational Auto-encoder with Bayesian Optimization. This study synthesizes ground motions that are highly representative of potential seismic events, thereby ensuring that structures designed to withstand real earthquakes have optimized safety margins. The application of Bayesian Optimization, more specifically TPE, is an effective method for rapidly exploring complex parameter space in latent space, while simultaneously achieving significant computational savings.

REFERENCES:

1) Yu-Chi Sung, Kuo-Chun Chang, Dyi-Wei Chang, Hsiao-Hui Hung, and Kuang-Yen Liu. Damage investigation and seismic retrofit of bridges in Taiwan after 921 Chi-Chi earthquake. In *Proceedings of the International Symposium on Engineering Lessons Learned from the 2011 Great East Japan Earthquake*, pages 1–4. Tokyo, Japan, 2012.
2) Yutaka Nakamura and Keiichi Okada. Review on seismic isolation and response control methods of buildings in Japan. *Geoenvironmental Disasters*, 6, 06 2019.
3) Satyabrata Choudhury. Performance-based seismic design of structures. *CRC Press*, May 2024, New York.
4) Shuhei Watanabe. Tree-structured Parzen estimator: Understanding its algorithm components and their roles for better empirical performance, 2023.