# MapReduce & PageRank

# Last Time …

- Map Reduce
  - Overview
  - Matrix multiplication
  - Complexity theory

# Today …

- Assignment 1 – deadline extended – due Oct 6
- Complexity theory for MapoReduce
- Page Rank

# Complexity Theory for mapreduce

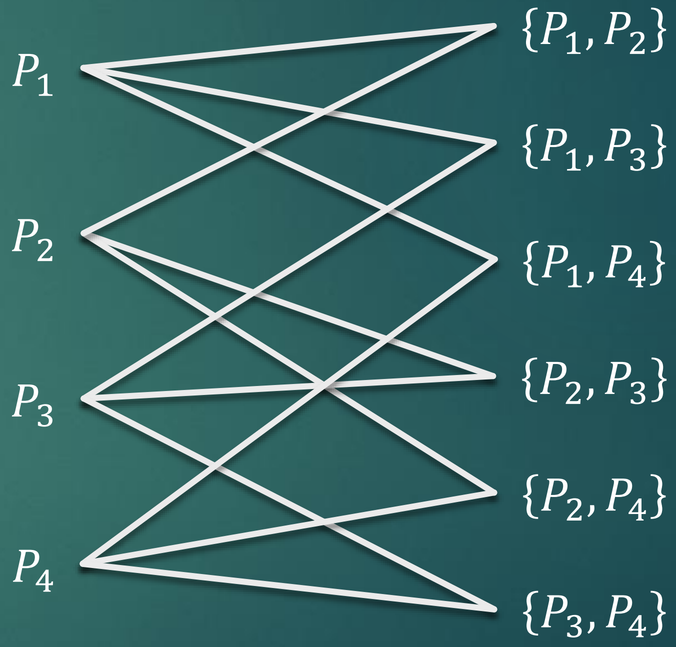
# Reducer size & Replication rate

- Reducer size $(q)$
  - Upper bound on the number of values that are allowed to appear in the list associated with a single key
    - By making the reducer size small, we can force there to be many reducers
      - High parallelism → low wall-clock time
    - By choosing a small $q$ we can perform the computation associated with a single reducer entirely in the main memory of the compute node
      - Low synchronization (Comm/IO) → low wall clock time
- Replication rate $(r)$
  - number of $(k, v)$ pairs produced by all the Map tasks on all the inputs, divided by the number of inputs
  - $r$ is the average communication from Map tasks to Reduce tasks

# Graph model for mapreduce problems

- Set of inputs
- Set of outputs
- many-many relationship between the inputs and outputs, which describes which inputs are necessary to produce which outputs.

- Mapping schema
  - Given a reducer size $q$
  - No reducer is assigned more than $q$ inputs
  - For every output, there is at least one reducer that is assigned all input related to that output



# Grouping for Similarity Joins

- Generalize the problem to $p$ images
- $g$ equal sized groups of $\frac{p}{g}$ images
- Number of outputs is $\binom{p}{2} \approx \frac{p^2}{2}$
- Each reducer receives $\frac{2p}{g}$ inputs $(q)$
- Replication rate $r = g - 1$

- $r = \frac{2p}{q}$
- The smaller the reducer size, the larger the replication rate, and therefore higher the communication
  - communication $\leftrightarrow$ reducer size
  - communication $\leftrightarrow$ parallelism

# Lower bounds on Replication rate

1. Prove an upper bound on how many outputs a reducer with $q$ inputs can cover. Call this bound $g(q)$
2. Determine the total number of outputs produced by the problem
3. Suppose that there are $k$ reducers, and the $i^{th}$ reducer has $q_i < q$ inputs. Observe that $\sum_{i=1}^{k} g(q_i)$ must be no less than the number of outputs computed in step 2
4. Manipulate inequality in 3 to get a lower bound on $\sum_{i=1}^{k} q_i$
5. 4 is the total communication from Map tasks to reduce tasks. Divide by number of inputs to get the replication rate

$$r \geq \frac{p}{q}$$

$$\binom{q}{2} \approx \frac{q^2}{2}$$

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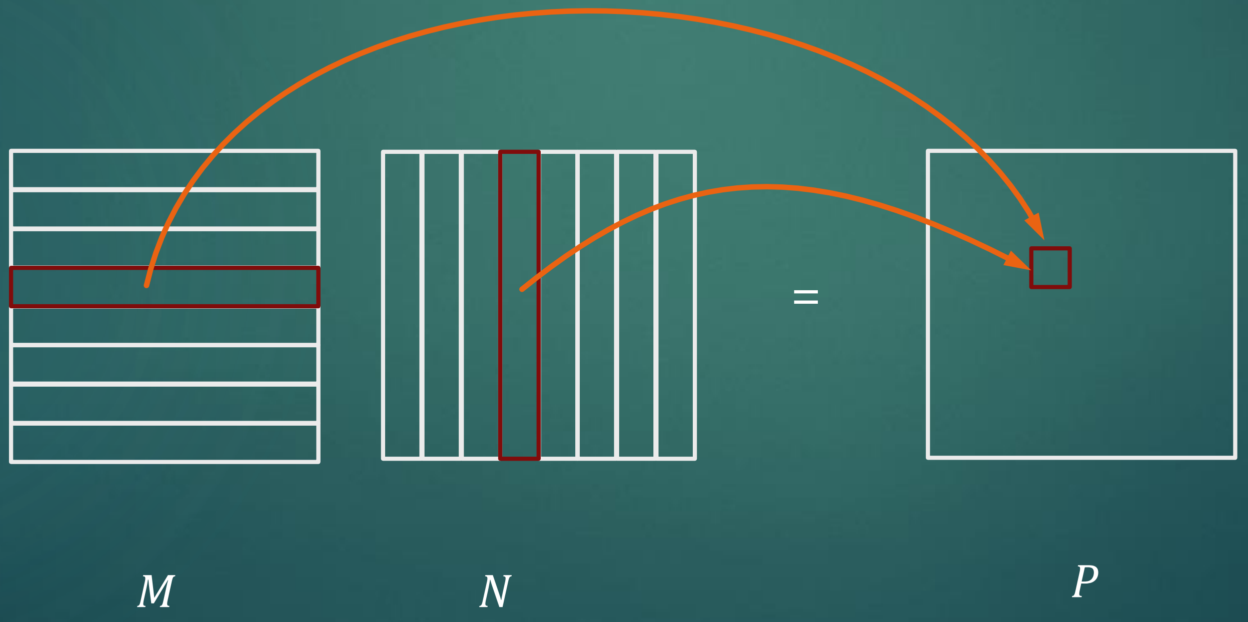
$$\sum_{i=1}^{k} \frac{q_i^2}{2} \geq \frac{p^2}{2}$$

$$q \sum_{i=1}^{k} q_i \geq p^2$$

$$\sum_{i=1}^{k} q_i \geq \frac{p^2}{q}$$

# Matrix Multiplication

- Consider the one-pass algorithm → extreme case
- Lets group rows/columns into bands → $g$ groups → $n/g$ columns/rows




$n \times n$

# Matrix Multiplication

- Map:
  - for each element of $M, N$ generate $g$ $(k, v)$ pairs
  - Key is group paired with all groups
  - Value is $(i, j, m_{ij})$ or $(i, j, n_{ij})$
- Reduce:
  - Reducer corresponds to key $(i, j)$
  - All the elements in the $i^{th}$ band of $M$ and $j^{th}$ band of $N$
  - Each reducer gets $n\left(\frac{n}{g}\right)$ elements from 2 matrices
  - $q = \frac{2n^2}{g}$ , $r = g$ → $r = \frac{2n^2}{q}$

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- Each reducer receives $k$ rows from $M$ and $N$ → $q = 2nk$ and produces $k^2$ outputs → $g(q) = \frac{q^2}{4n^2}$
- $n^2$
- $\sum_{i=1}^{k} \frac{q_i^2}{4n^2} \geq n^2$

$$\sum_{i=1}^{k} q_i^2 \geq 4n^4$$

- $\sum_{i=1}^{k} q_i \geq \frac{4n^4}{q}$

$$r = \frac{1}{2n^2} \sum_{i=1}^{k} q_i = \frac{2n^2}{q}$$

# Matrix Multiplication

LET US REVISIT THE TWO-PASS APPROACH

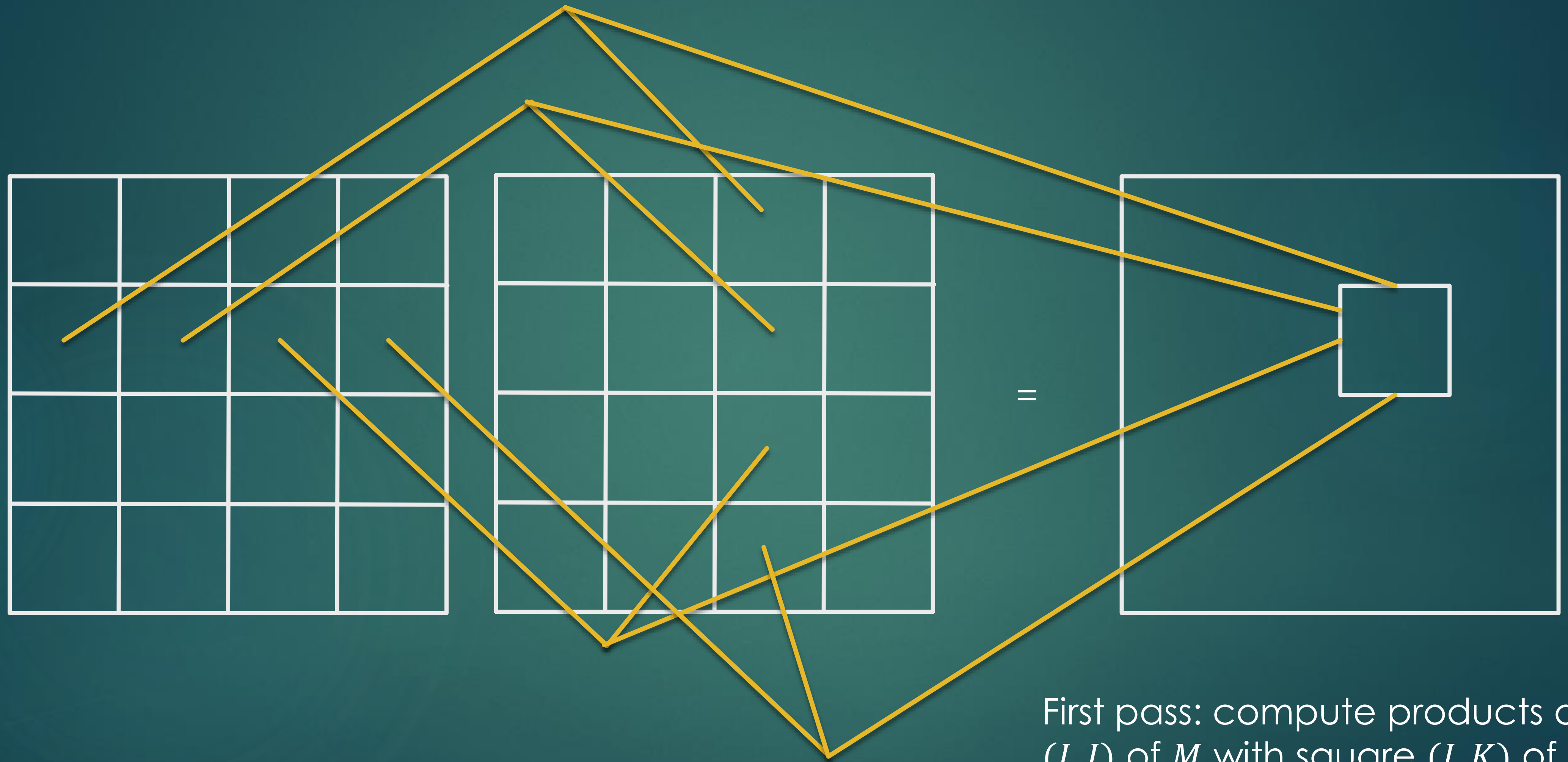
# Matrix-vector multiplication

- $n \times n$ matrix $M$ with entries $m_{ij}$
- Vector $\boldsymbol{v}$ of length $n$ with values $v_j$
- We wish to compute

$$x_i = \sum_{j=1}^{n} m_{ij} v_j$$

- If $\boldsymbol{v}$ can fit in memory
  - Map: generate $(i, m_{ij} v_j)$
  - Reduce: sum all values of $i$ to produce $(i, x_i)$
- If $\boldsymbol{v}$ is too large to fit in memory? Stripes? Blocks?
- What if we need to do this iteratively?

# Grouped two-pass approach




$g^2$ groups of $\frac{n^2}{g^2}$ elements each

First pass: compute products of square $(I, J)$ of $M$ with square $(J, K)$ of $N$

Second pass: $\forall I, K$ sum over all $J$

# Grouped two-pass approach

- Replication rate for map1 is $g$ → $2gn^2$ total communication
- Each reducer gets $\frac{2n^2}{g^2}$ → $q = \frac{2n^2}{g^2}$ → $g = n\sqrt{\frac{2}{q}}$
- Total communication → $2\frac{\sqrt{2}n^3}{\sqrt{q}}$
- Assume map2 runs on same nodes as reduce1
  → no communication
- Communication → $gn^2$ → $\frac{\sqrt{2}n^3}{\sqrt{q}}$
- Total communication → $3\frac{\sqrt{2}n^3}{\sqrt{q}}$

# Comparison

$$\frac{n^4}{q} < \frac{n^3}{\sqrt{q}}$$

$q < n^2$

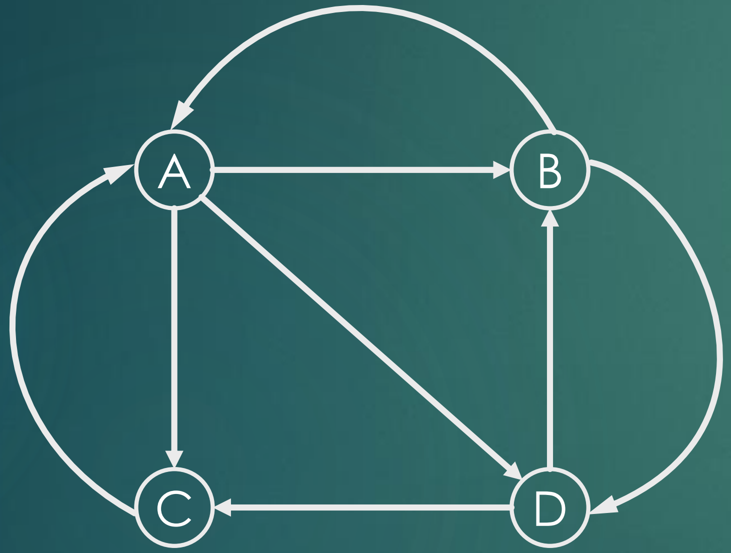
If $q$ is closer to the minimum of $2n$, two pass is better by a factor of $O(\sqrt{n})$

# Page Rank

# Webpage quality ranking

- Inverted web indexes help locate matching pages of search words
  - But there are too many matches and humans can’t read all
- Both relevance and quality are important in web search
- What is a high-quality web page?
- How to identify a high-quality web page?
  - Hard to spam
- Related to identifying high-quality scientific publications
  - But much bigger dataset

# Page Rank




Transition matrix

$$M = \begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

$\boldsymbol{v}$ → probability distribution for the location of a random surfer

$\boldsymbol{v} \leftarrow \left\{\frac{1}{n}\right\}^n$

Iterate on $\boldsymbol{v} \leftarrow M\boldsymbol{v}$

# Page Rank

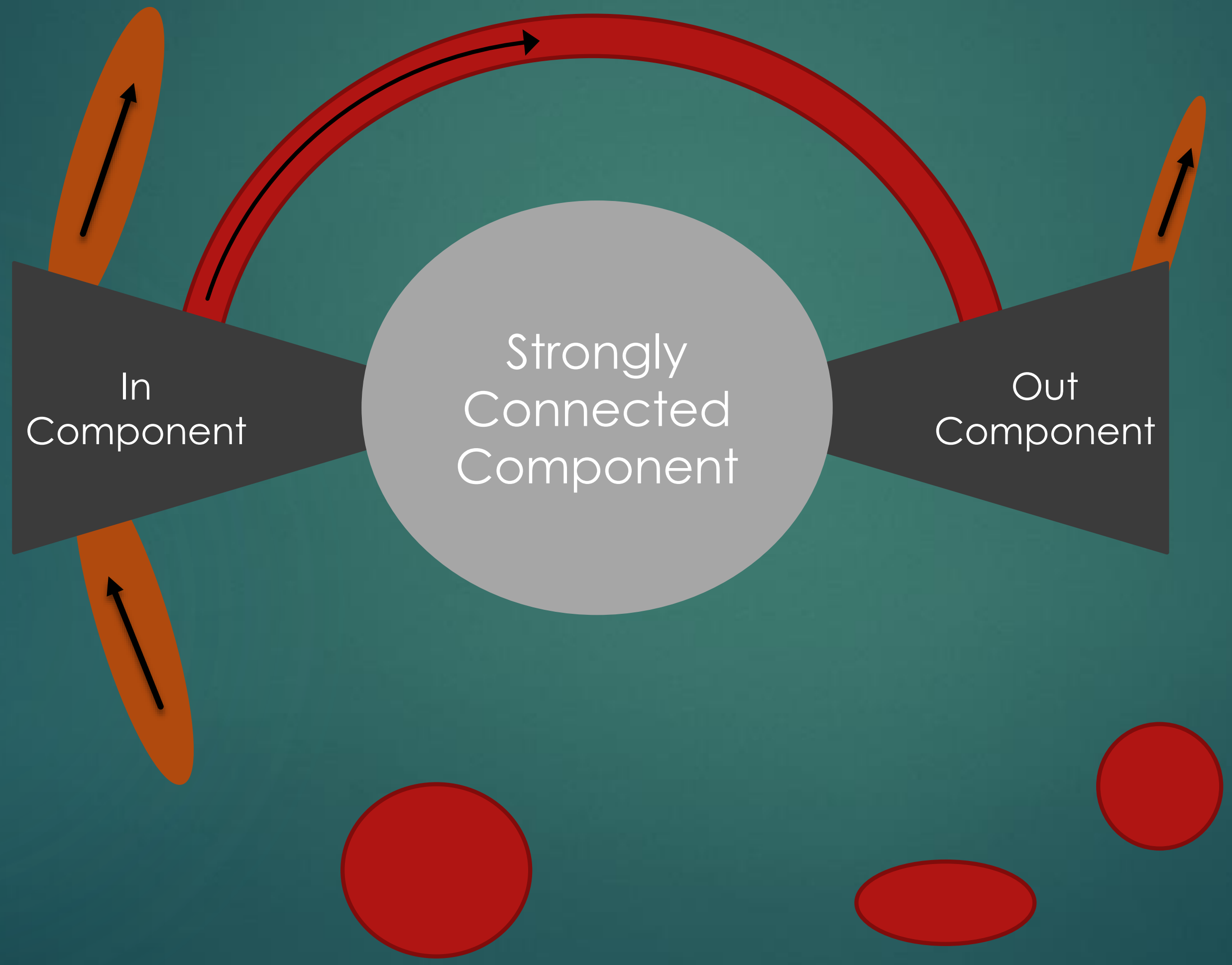
- Markov process
  - Limiting distribution
  - will converge if
    - Strongly connected
    - No dead ends
- Limiting $\boldsymbol{v}$ is an eigenvector of $M$
  - $\boldsymbol{v} = \lambda M \boldsymbol{v}$
  - $v$ is also the primary eigenvector
- Iterate a few times on $v \leftarrow Mv$ until $\|v_{i+1} - v_i\| < \epsilon$

# Solving Linear Systems

- $M\boldsymbol{x} = \boldsymbol{y} \;\; \Rightarrow \;\; \boldsymbol{x} = \boldsymbol{M}^{-1}\boldsymbol{y}$

- Gaussian Elimination → $\mathcal{O}(n^3)$
- Iterative approaches → $\mathcal{O}(kn^2)$
  - For sparse systems → $\mathcal{O}(kn)$
  - Use optimal solvers → $k$ independent of $n$

# Structure of the Web




**Dead Ends**

**Spider Traps**

# Dead Ends

- Remove dead ends from the graph
  - And incoming links
- Compute page-rank on strongly connected component
- Restore graph, retaining page ranks
- Use existing page ranks to compute ranks for dead-end nodes

# Spider traps & Taxation

- modify the calculation of PageRank by allowing each random surfer a small probability of teleporting to a random page

$$\boldsymbol{v}' = \beta M\boldsymbol{v} + \frac{(1-\beta)\boldsymbol{e}}{n}$$

- $\beta$ is a constant that represent the probability that the surfer follows a link on the page
- Approach will still be biased towards spider traps