Intro to MPI

Last Time …

Intro to Parallel Algorithms

- Parallel Search
- Parallel Sorting
  - Merge sort
  - Sample sort

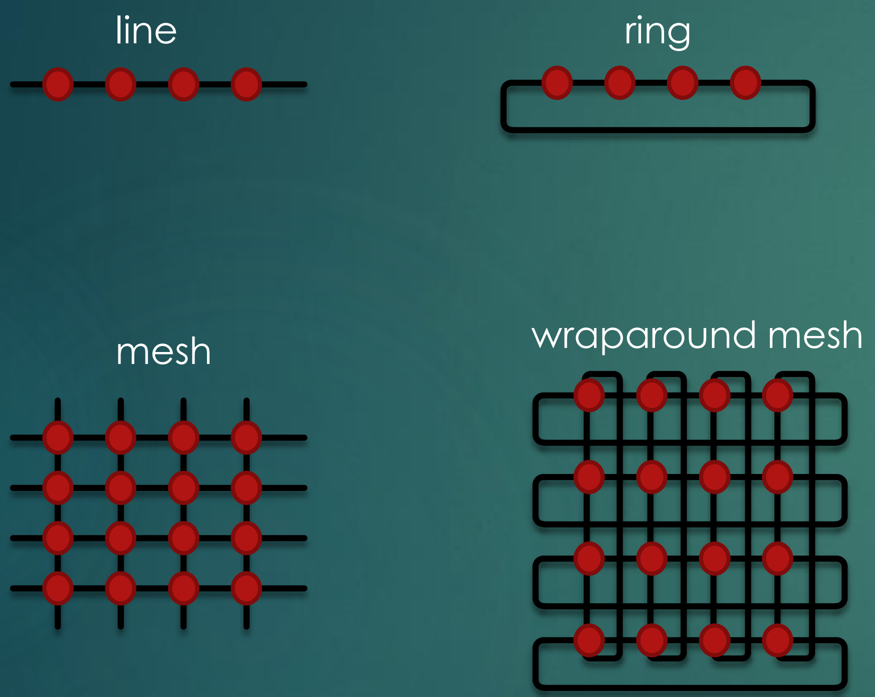
Today

- Network Topology
- Communication Primitives
  - Message Passing Interface (MPI)
- Randomized Algorithms
  - Graph Coloring

Network Model

- Message passing model
- Distributed memory nodes
- Graph $\mathcal{G} = (N, E)$
  - nodes → processors
  - edges → two-way communication link
- No shared RAM
- Asynchronous
- Two basic communication constructs
  - `send (data, to_proc_i)`
  - `recv (data, from_proc_j)`

Network Topologies



Link: one task pair, bidirectional

Send-Recv time:
→ latency + message/bw

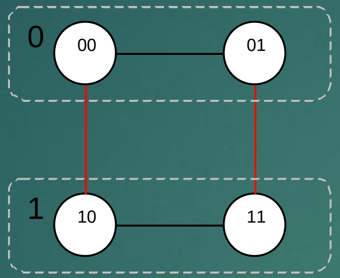
Node: one send/recv

Hypercube?

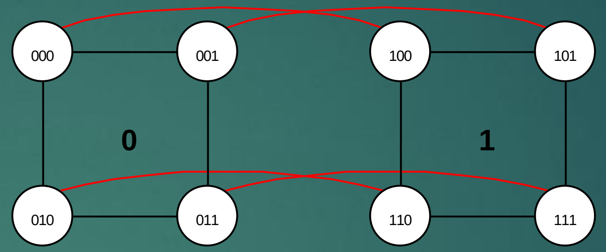
Hypercube

generic term 3-cube, 4-cube, . . . , q-cube

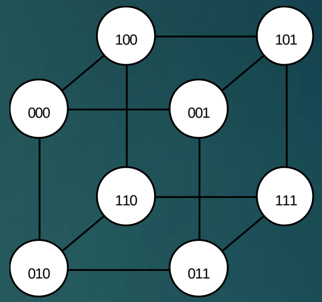
(a) Binary 1-cube, built of two binary 0-cubes, labeled 0 and 1

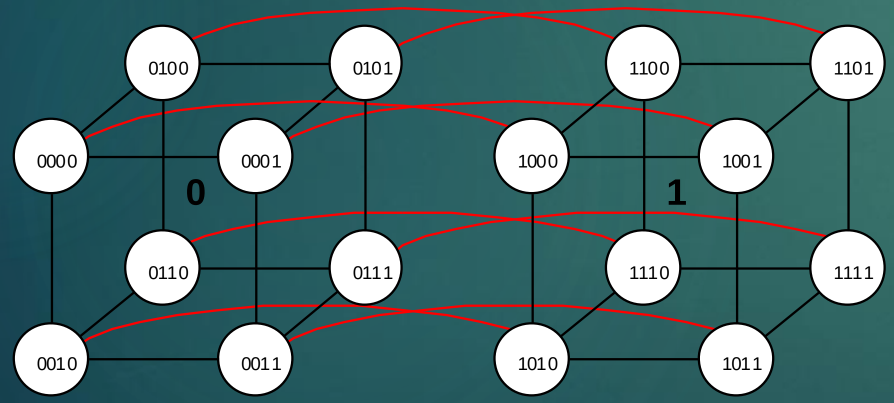


(b) Binary 2-cube, built of two binary 1-cubes, labeled 0 and 1

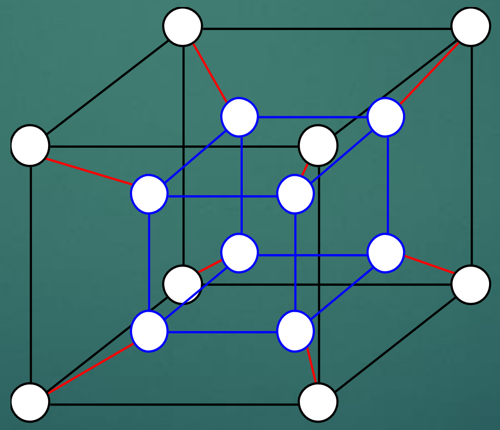


(c) Binary 3-cube, built of two binary 2-cubes, labeled 0 and 1





(d) Binary 4-cube, built of two binary 3-cubes, labeled 0 and 1

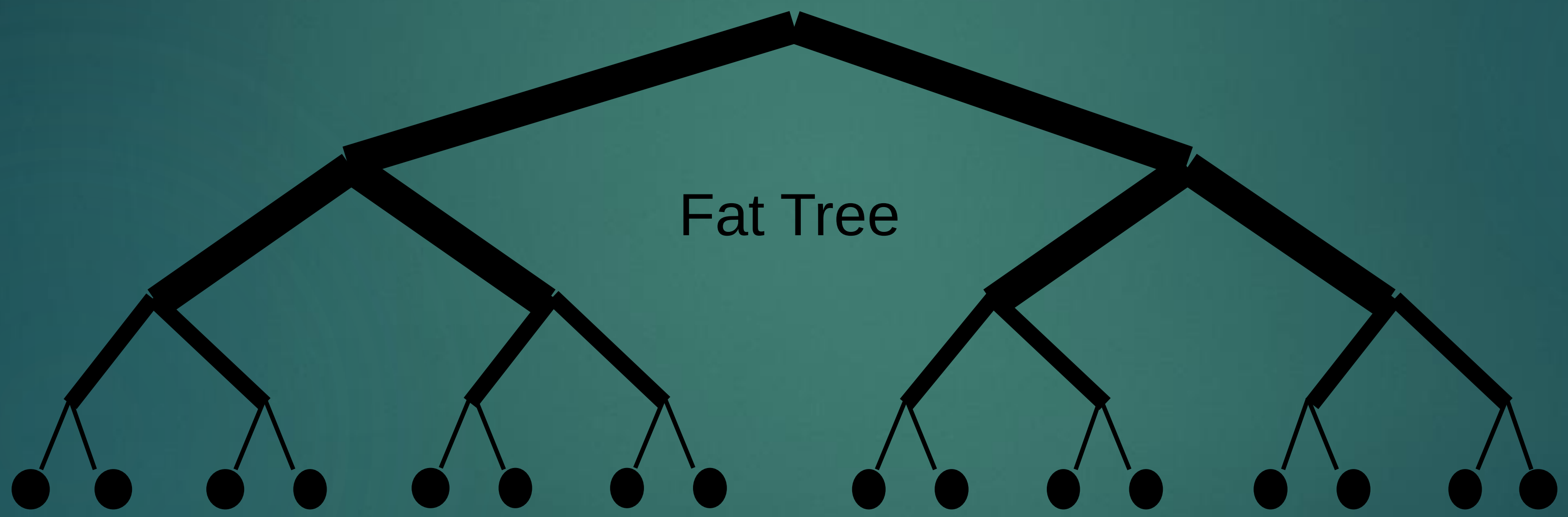


d dimensions, 2^d processes

two processes connected iff they differ in only one bit

distance = $\log p$

Network topologies



Basic concepts in networks

- Routing (delivering messages)
- Diameter
  - maximum distance between nodes
- Communication costs
  - Latency: time to initiate communication
  - Bandwidth: channel capacity (data/time)
    - Number and size of messages matters
- Point-to-point and collective communications
  - Synchronization, all-to-all messages

Network Models

Allow for thinking and measuring communication costs and non-locality

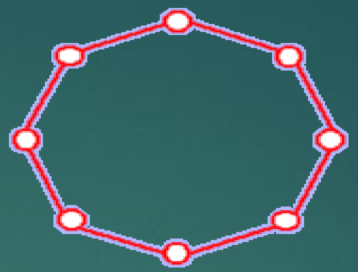
p2p cost in MPI

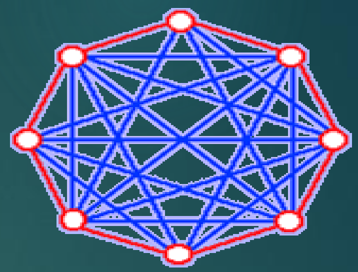
- start-up time: t_s
  - add header/trailer, error correction, execute the routing algorithm, establish the connection between source and destination
- per-hop time: t_h
  - Time to travel between two directly connected nodes
    - *node latency*
- per-word transfer time: t_w

We will assume that the cost of sending a message of size m is:

$$t_{comm} = \underbrace{t_s}_{\text{latency}} + \underbrace{t_w}_{\frac{1}{\text{bandwidth}}} m$$

Network Metrics





- Diameter
  - Max distance between any two nodes
- Connectivity
  - Number of links needed to remove a node
- Bisection width
  - Number of links to break network into equal halves
- Cost
  - Total number of links

- Ring, Fully connected ring
  - Diameter

    $\frac{p}{2}$ $\qquad$ 1
  - Connectivity

    2 $\qquad$ $p-1$
  - Bisection Width

    2 $\qquad$ $p^2/4$
  - Cost

    $p-1$ $\qquad$ $\frac{p(p-1)}{2}$

Network Metrics

- Diameter
  - Max distance between any two nodes
- Connectivity
  - Number of links needed to remove a node
- Bisection width
  - Number of links to break network into equal halves
- Cost
  - Total number of links

- Hypercube
  - Diameter

    $\log p$
  - Connectivity

    $\log p \; (= d)$
  - Bisection Width

    $p/2$
  - Cost

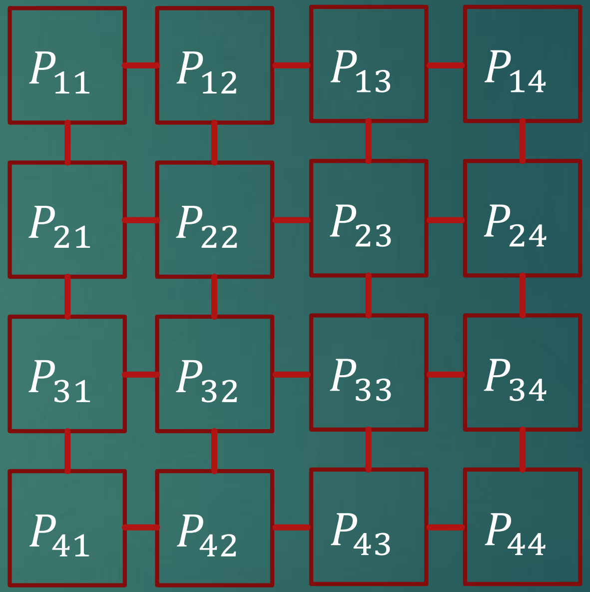
    $\frac{p \log p}{2}$

Example problem

- how would you perform a fully distributed matrix multiplication (`dgemm`)
  - Assume a 2D grid topology
  - Need to compute $C = A * B$

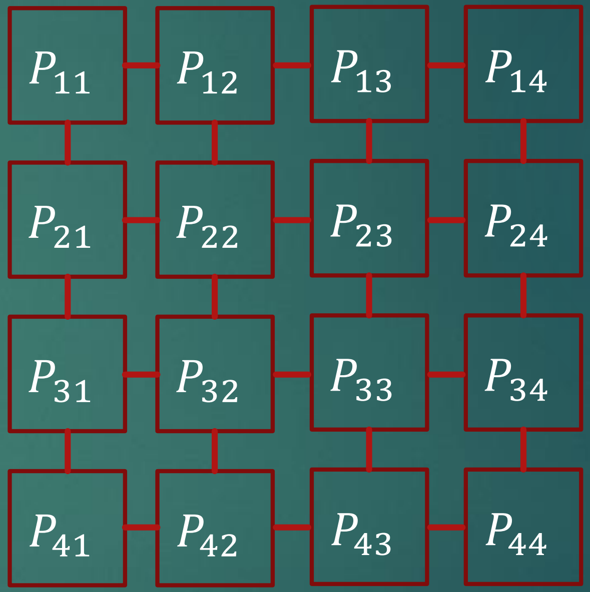
$$C_{ij} = \sum_{k} A_{ik} B_{kj}$$

  - How to divide data across processors?
  - What is the local work?
  - Communication? Costs?



Matrix-Matrix Multiplication on 2D Mesh

- Compute $C = AB$, $p = n \times n$
- Rows of A fed from left
- Columns of B fed from right
- Operation is synchronous
- ij indexing of processor ids
- Each ij processor computes C_{ij}



Matrix-Matrix Multiplication

```
% i,j process id
% p=n, 1 element per process
for k=1:n
    recv ( A(i, k), (i-1, j  ) )
    recv ( B(k, j), (i  , j-1) )

    C(i,j) += A(i,k)*B(k,j);

    send ( A(i,k), (i+1, j  ) )
    send ( B(k,j), (i  , j+1) )
```

$B(4,4)$

$B(4,3)\ B(3,4)$

$B(4,2)\ B(3,3)\ B(2,4)$

$B(4,1)\ B(3,2)\ B(2,3)\ B(1,4)$

$B(3,1)\ B(2,2)\ B(1,3)$

$B(2,1)\ B(1,2)$

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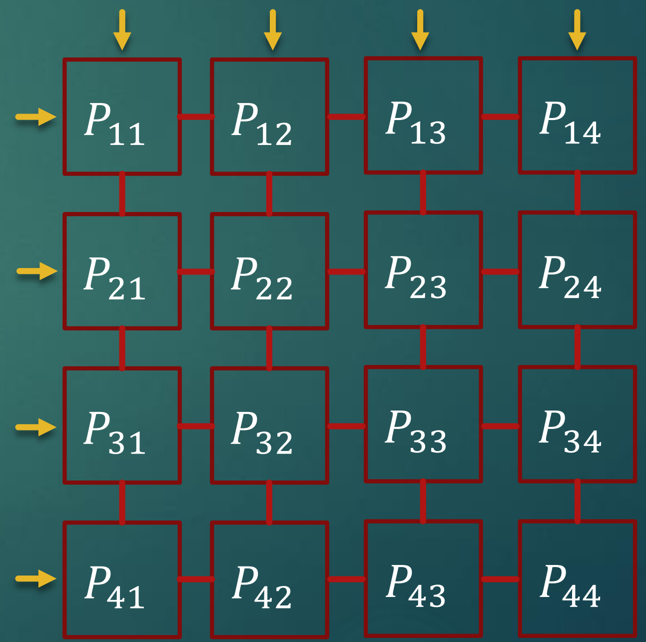


Collective Communications

MPI Collectives

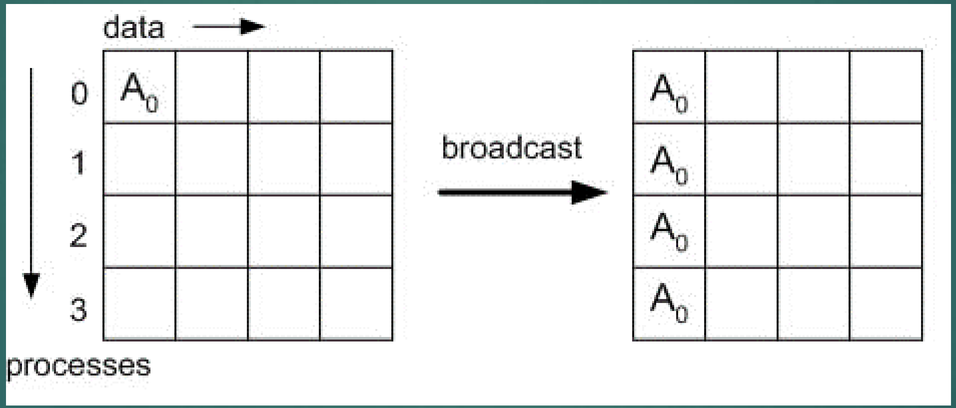
- Communicator (`MPI_Comm`)
  - determines the scope within which a point-to-point or collective operation is to operate
  - Communicators are dynamic
    - they can be created and destroyed during program execution.
    - `MPI_COMM_WORLD`
- `int MPI_Barrier(MPI_Comm comm);` **avoid**
  - Synchronize all processes within a communicator

Data Movement

- broadcast
- gather(v)
- scatter(v)
- allgather(v)
- alltoall(v)

Broadcast

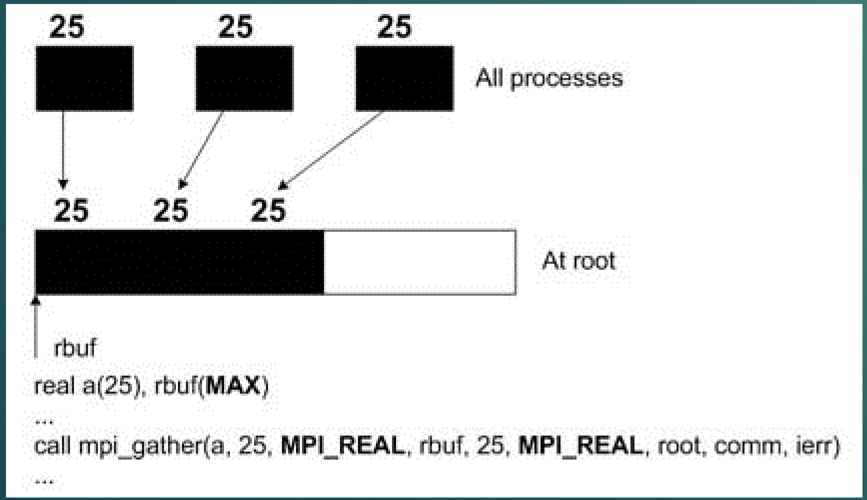
```
int MPI_Bcast(void* buffer, int count, MPI_Datatype datatype, int root, MPI_Comm comm)
```

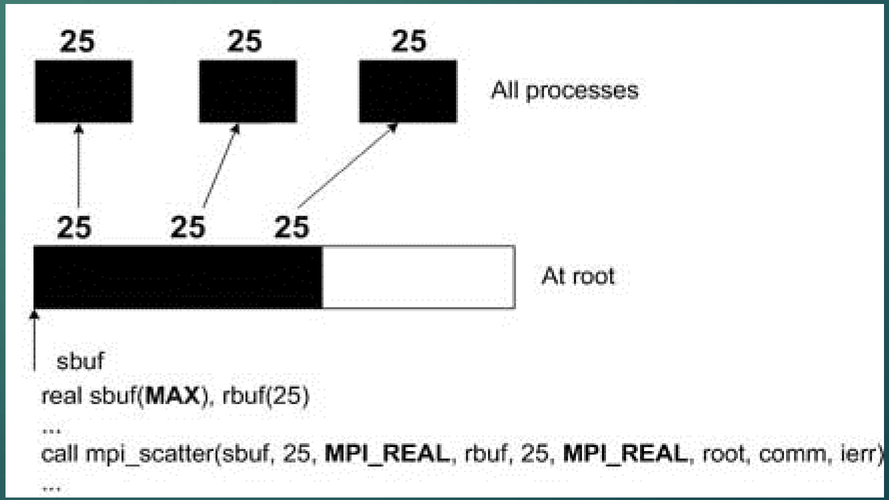


Gather & Scatter

```
int MPI_Gather(void* sbuf, int scount, MPI_Datatype stype,
               void* rbuf, int rcount, MPI_Datatype rtype,
               int root, MPI_Comm comm )

int MPI_Scatter(void* sbuf, int scount, MPI_Datatype stype,
                void* rbuf, int rcount, MPI_Datatype rtype,
                int root, MPI_Comm comm)
```





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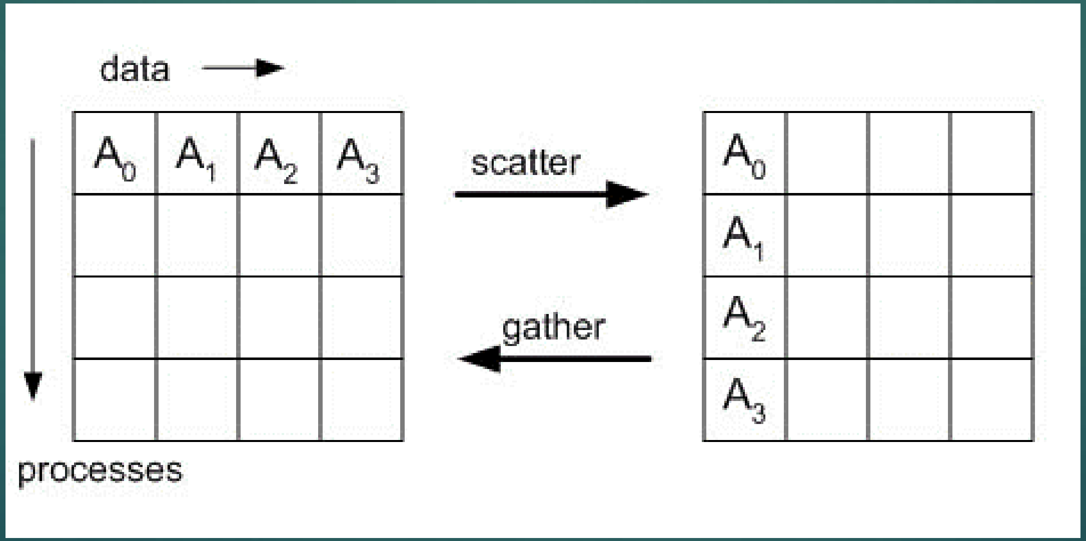
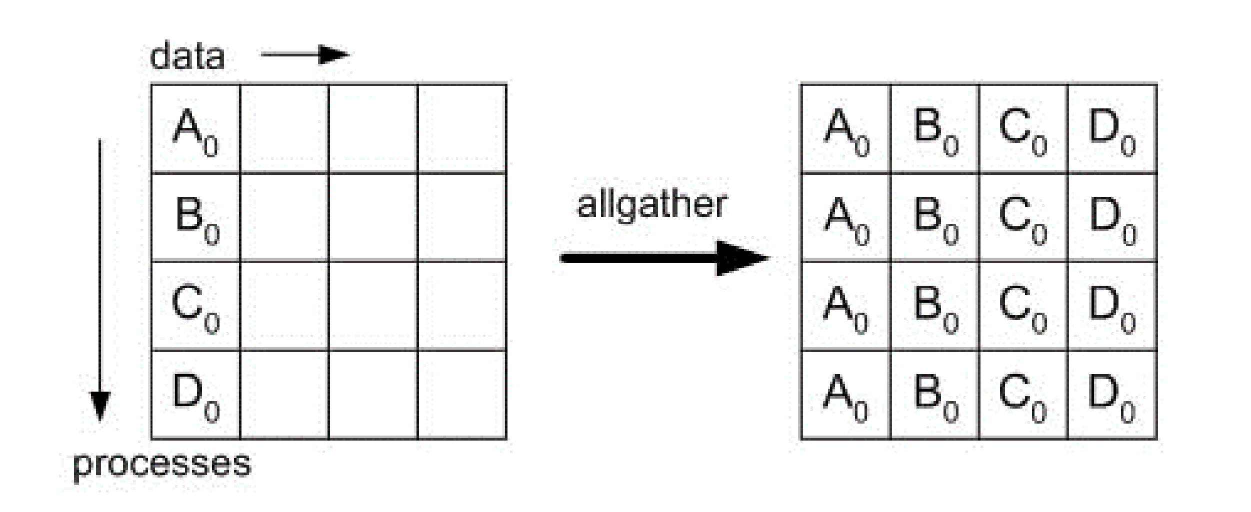


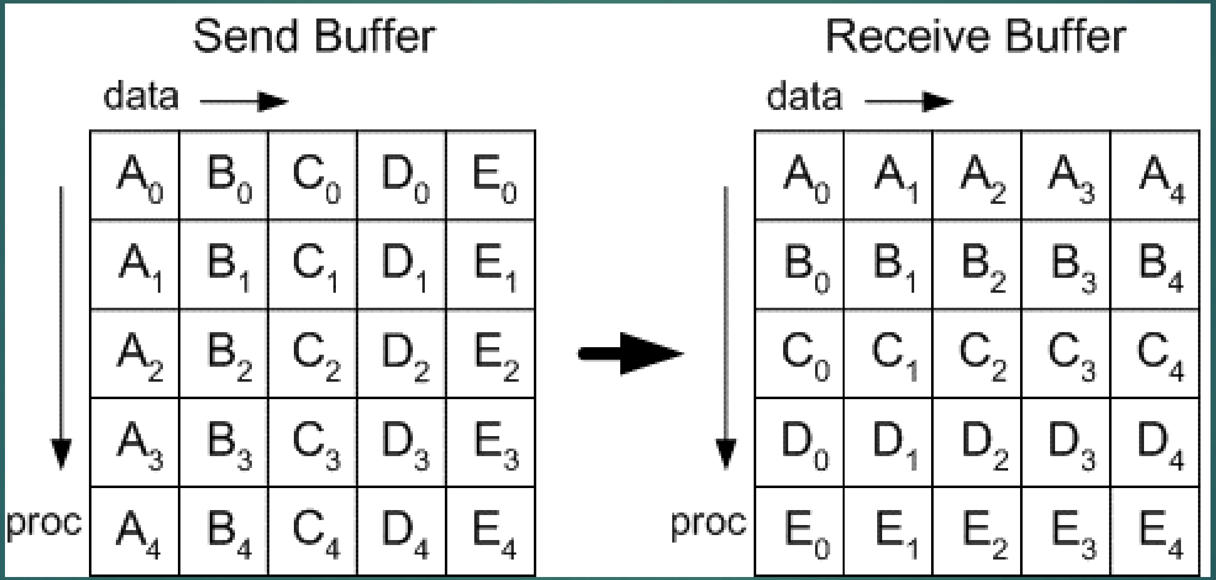
Allgather

```
int MPI_Allgather(void* sbuf, int scount, MPI_Datatype stype,
                  void* rbuf, int rcount, MPI_Datatype rtype, MPI_Comm comm)
```



All to All

```
int MPI_Alltoall(void* sbuf, int scount, MPI_Datatype stype,
                 void* rbuf, int rcount, MPI_Datatype rtype, MPI_Comm comm)
```



Global Computation

- Reduce
- Scan

Reduce

```
int MPI_Reduce(void* sbuf, void* rbuf, int count,
               MPI_Datatype stype, MPI_Op op, int root, MPI_Comm comm)

int MPI_Allreduce(void* sbuf, void* rbuf, int count
                  MPI_Datatype stype, MPI_Op op, MPI_Comm comm)

int MPI_Reduce_scatter(void* sbuf, void* rbuf, int* rcounts,
                       MPI_Datatype stype, MPI_Op op, MPI_Comm comm)
```

Scan (prefix-op)

```
int MPI_Scan(void* sbuf, void* rbuf, int count,
             MPI_Datatype datatype, MPI_Op op, MPI_Comm comm)
```

Assignment 1 – Problem 1

Implement samplesort in parallel

- Samplesort

1. Local sort, pick samples
2. Gather samples at root
3. Sort samples, pick splitters
4. Broadcast splitters
5. Bin data into buckets
6. AlltoAllv data
7. Local sort