# Social-Network Graphs

# Last Time …

- Social Network Graphs
- Betweenness
  - Girvan-Newman Algorithm
- Graph Laplacian
  - Spectral Bisection
  - $\lambda_2, w_2$
- Eigenvalue problems

# Projects

- Yelp data challenge
  - http://www.yelp.com/dataset_challenge
- Global Disease Monitoring and Forecasting with Wikipedia
  - Recent paper, PLoS Nov '14

# Direct discovery of communities

- Although partitioning the graph using betweenness is effective, it has some drawbacks
  - Not possible to place an individual in two different communities
  - Everyone is assigned a community
- Alternatively, discover communities by looking for subsets of the nodes that have a relatively large number of edges among them
  - Finding cliques → NP Complete
  - Easier to find complete bipartite subgraphs
  - Counting tringles

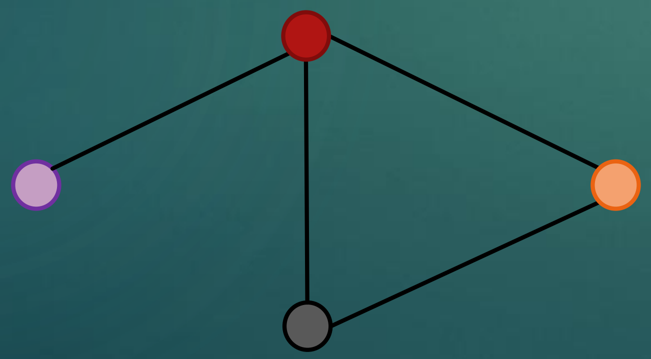
# Why count triangles

- Clustering coefficient:

  given an undirected graph $G = (V, E)$
  cc(v) = fraction of v's neighbors who are neighbors themselves

$$= \frac{|\{(u, w) \in E \mid u \in N(v) \ \land \ w \in N(v)\}|}{\binom{d_v}{2}}$$

number of $\Delta$s incident on $v$

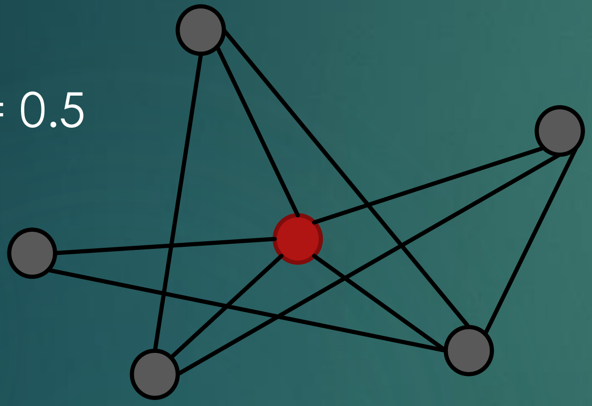


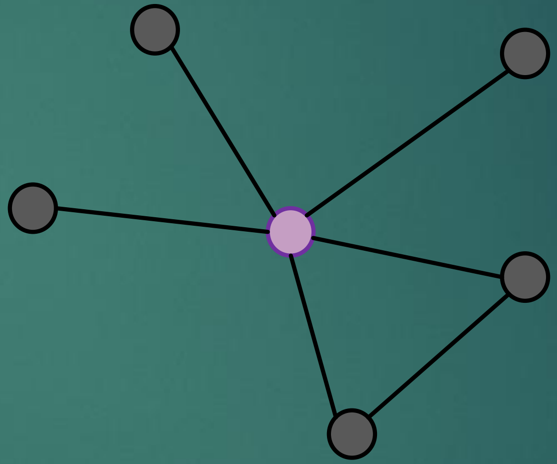
cc ( ) = N/A
cc ( ) = 1/3
cc ( ) = 1
cc ( ) = 1

# Why clustering coefficients?

Captures how tight-knit the network is around a node

cc ( ) = 0.5





cc ( ) = 0.1

Network Cohesion

- Tightly knit communities foster more trust, social norms

# How to count triangles



Sequential

foreach $v \in V$

  foreach $u, w \in N(v)$

    if $(u, w) \in E$

      triangles[v]++

$$\sum_{v \in V} d_v^2$$

Even for sparse graphs can be quadratic if one vertex has high degree

# Parallel Version



Parallelize the edge checking phase

- Map 1: foreach $v$ generate $(v, N(v))$
- Reduce 1: Input $(v, N(v))$

  Output: all 2 paths $((v_1, v_2), u)$ where $v_1, v_2 \in N(u)$

- Map 2: generate $((v_1, v_2), u)$ and $((v_1, v_2), \phi)$ for $(v_1, v_2) \in E$
- Reduce 2: input $((v_1, v_2), u_1, u_2, \ldots, u_k, \phi)$

  If $\phi$ is part of the input, then increment triangle count by 1/3

# Data skew

- How much parallelism can we achieve?
  - Generate all paths to check in parallel
  - The runtime becomes $\max_{v \in V} d_v^2$
- Naïve parallelization does not help with data skew
  - Some nodes will have very high degree
  - Remember power-log distribution
  - Most reducers will be done quickly
  - A few will take forever
    - Curse of the last reducer

# Adapting the algorithm

- Dealing with skew directly
  - Currently each triangle is counted 3 times
  - Running time is quadratic in the degree of the vertex
  - Idea: count each triangle once, by the lowest degree vertex

# How to count Δs better

Sequential version [Shank '07]

foreach $v \in V$

    foreach $u, w \in N(v)$

        if $d(u) > d(v)$ & $d(w) > d(v)$

            if $(u, w) \in E$

                triangles[v]++

# Dealing with skew



Why does it help?

- Partition nodes into two groups:
  - Low: $\mathcal{L} = \{v : d_v \leq \sqrt{m}\}$
  - High: $\mathcal{H} = \{v : d_v > \sqrt{m}\}$
- There are at most $n$ low nodes; each produces at most $m$ paths
- There are at most $2\sqrt{m}$ high nodes

- These two are identical
- no mapper can produce substantially more work than others
- Total work is $\mathcal{O}(m^{3/2})$ , which is optimal

# Triangles with all $\mathcal{H}$ nodes

- There are only $\mathcal{O}(\sqrt{m})$ $\mathcal{H}$ nodes
- Therefore, there are at most $\mathcal{O}\left(m^{3/2}\right)$ triangles with all $\mathcal{H}$ nodes
- Using $E$, check if these triangles exist in $\mathcal{O}(1)$ time

- Total time – $\mathcal{O}\left(m^{3/2}\right)$

# Triangles with all at least 1 $\mathcal{L}$ node

- Consider all $m$ edges - $\mathcal{O}(m)$
  - Given an edge $(v_1, v_2)$
    - Ignore if $v_1, v_2 \in \mathcal{H}$
    - Consider the smaller node, say $v_1 \prec v_2$
      - This node has $k$ nodes in its adjacency list, with $k < \sqrt{m}$ - $\mathcal{O}(\sqrt{m})$
      - Count triangle with node $u_i$ iff edge $(v_2, u_i)$ exists and $v_1 \prec u_i$
- $\mathcal{O}\left(m^{3/2}\right)$