VP6 Damped Oscillator and Coupled Oscillator [empty class, list representation]

I. List Representation

1. range(). Notice the number in range should be integer (int)

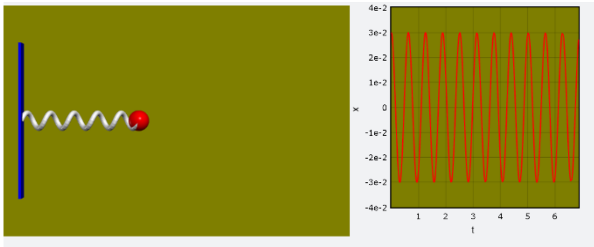
```
L = range(5)                    # list(L) = [0, 1, 2, 3, 4].
L = range(4, 9)                 # list(L) = [4, 5, 6, 7, 8]
L = range(1, 6, 2)              # list(L) = [1, 3, 5]    1 to 6 every other 2 numbers
```

2. list representation. Sometimes we want to generate a list with some conditions, e.g.

```
L = [i**2  for  i  in  range(5)]              # =  [0, 1, 4, 9, 16]
L = [0.1*i*pi   for  i   in   range(-3, 3)]   # =  [-0.3*pi, -0.2*pi, -0.1*pi, 0, 0.1*pi, 0.2*pi]
L = [i**2  for  i  in  range (5)  if  i != 3] # =  [0, 1, 4, 16]
```

3. List representation can be used in a nested structure, or for dictionary or tuple. e.g.

```
L = [i*10 + j for i in range(3) for j in  range(5) ]   # =  [0, 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24]
D = {i:i**2  for i  in [0, 1, 2]}                      # = {0:0,   1:1, 2:4}
```

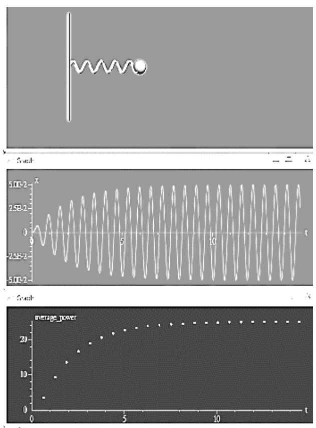


II. Practice (SHM)

```
from vpython import *
size, m = 0.02, 0.2   # ball size =  0.02 m, ball mass = 0.2kg
L, k = 0.2, 20           # spring original length = 0.2m, force constant = 20 N/m
amplitude = 0.03
b = 0.05 * m * sqrt(k/m)

scene = canvas(width=600, height=400, fov = 0.03, align = 'left', center=vec(0.3, 0, 0), background=vec(0.5,0.5,0))
wall_left = box(length=0.005, height=0.3, width=0.3, color=color.blue)          # left wall
ball = sphere(radius = size,  color=color.red)                                   # ball
spring = helix(radius=0.015, thickness =0.01)
oscillation = graph(width = 400, align = 'left', xtitle='t',ytitle='x',background=vec(0.5,0.5,0))
x=gcurve(color=color.red,graph = oscillation)

ball.pos = vector(L+amplitude, 0 , 0)                                            # ball initial position
ball.v = vector(0, 0, 0)                              # ball initial velocity
ball.m = m
spring.pos = vector(0, 0, 0)
t, dt = 0, 0.001
while True:
    rate(1000)
    spring.axis = ball.pos - spring.pos       # spring extended from spring endpoint A to ball
    spring_force = - k * (mag(spring.axis) - L) * norm(spring.axis)         # spring force vector
    ball.a = spring_force / ball.m                # ball acceleration = spring force /m - damping
    ball.v +=   ball.a*dt
    ball.pos += ball.v*dt
    t += dt
    x.plot(pos=(t,ball.pos.x - L))
```

Modified from hw3, the above code simulates the horizontal oscillation of a given amplitude.

1. Now, in addition to the restoring force from the spring, add the air resistance force $\vec{f} = -b\vec{v}$ to the ball with **damping factor** $b = 0.05m\sqrt{k/m}$.



2. Follow Practice 1, but instead of letting the ball to oscillate from the initial amplitude, i.e. ball.pos = vector(L+amplitude, 0, 0), now let the ball to be initially at rest at $x = L$, i.e. ball.pos = vector(L, 0, 0), and allow a sinusoidal force $\vec{F} = f_a \sin(\omega_d t)\,\hat{x}$ ($\hat{x}$ is the unit vector in x-axis) applied on the ball, with f_a = 0.1 and ω_d = $\sqrt{k/m}$. We know when a force $\vec{F}$ exerts on an object of velocity $\vec{v}$, the power on the object by the force is $P = \vec{F} \cdot \vec{v}$. Find by your simulation P_T, the power averaged over a period $T = 2\pi / \omega_d$ at the end of each period. In additional to the curve graph for ball's position versus time, plot a dot graph for P_T versus time. Observe the results with different settings, such as ω_d =0.8 $\sqrt{k/m}$, 0.9 $\sqrt{k/m}$, 1.1 $\sqrt{k/m}$, or 1.2 $\sqrt{k/m}$ and/or with different *b* values. Think about the

results. Before proceeding to Practice 3, change ω_d and *b* back to the original values.

3. Often, we do not want an animated simulation, which is slow due to the animation, but only the calculation results. We can modify the above computer codes easily for such purpose. We can just delete the code that creates the canvas, the plot, and the graphs (i.e., those bold blackened codes), delete rate(1000), and replace codes that generate visual objects by the following codes that generate objects from an empty class.

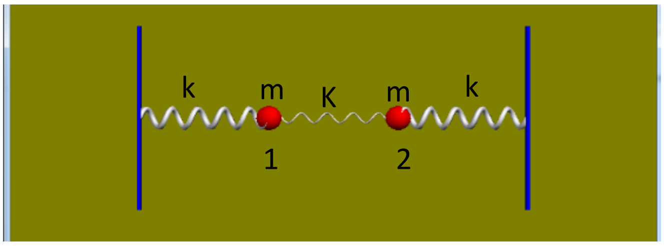
```
class obj:   pass

wall_left, ball, spring = obj(), obj(), obj()
```

With these, we still do the same simulation but do not animate them. This will speed up the simulation. Instead of plotting P_T, now only print P_T every period. You can see after certain number of periods, the system reaches a **steady state** and the P_T is almost a constant. Also notice that how much faster the simulation can run without the animation. (NOTICE: for this technique to work, you need to have all the proper parameters set for every object after they have been created by the empty class obj.)

III. Homework

(must) Let **omega = [0.1*i + 0.7*sqrt(k/m) for i in range(1, int(0.5*sqrt(k/m)/0.1))]** and by using "**for omega_d in omega:**", perform the calculation for steady-state P_T as practice 3 for different ω_d. Do not print the result. Instead, for each ω_d, when the system reaches steady state, add the latest result of the steady-state P_T to the plot of the "steady-state P_T versus omega_d" and then calculate for the steady-state P_T the next ω_d. You will get something similar to the figure shown here, which shows clearly the system's response to different driving frequency ω_d. In additional to plotting steady-state P_T versus ω_d, also print the optimal ω_d such that steady-state P_T has the highest value.

(optional) Continue from your complete code for Practice 2, add an addition ball, two more springs, and a right wall.



All the parameters are the same as in the original program except for the following: the middle spring is of constant K = 5.0. The damping factor for ball 1 (the left one) is $b_1 = 0.05m\sqrt{k/m}$ and for ball 2 (the right one) is $b_2 = 0.0025m\sqrt{k/m}$. The external force $\vec{F} = f_a \sin(\omega_d t)\,\hat{x}$ is exerted on ball 1 with f_a = 0.1 and ω_d = $\sqrt{(k+K)/m}$. This is similar to Practice 2 except that now the driving frequency is corresponding to the resonance frequency of the combining effect due to the left and the middle springs. As in Practice 2, simulate the system and calculate by your simulation P_T the power averaged over a period $T = 2\pi/\omega_d$ at the end of each period. In this optional homework, you need to show the simulation animation, the curve graph for ball's position versus time, and a dot graph for period-averaged power P_T versus time. In the simulation, you will see a very interesting phenomena, that the periodic force $\vec{F} = f_a \sin(\omega_d t)\,\hat{x}$ is exerted on ball 1, but in the end, ball 1 barely oscillates, think about what is the physics behind this.

Working out the optional part, we will observe the principle behind "EIT" (Electromagnetically Induced Transparency), a very interesting and advanced research topic in optics that won many awards. You may read https://en.wikipedia.org/wiki/Electromagnetically_induced_transparency for the mechanism of EIT and see the similarity between the EIT and our simulation here.