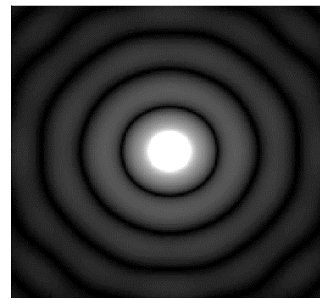
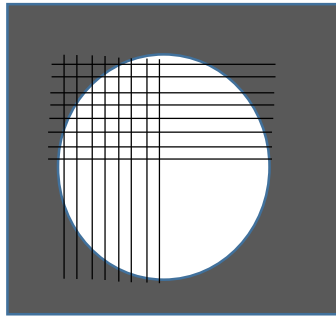


## Computer Work 14 Circular Aperture Diffraction



This homework is to find the diffraction pattern of a circular aperture far from the screen. Assume there is a circular aperture of diameter  $d=100\text{ }\mu\text{m}$  and there is a spherical screen at  $R=1\text{ m}$  away. The light source is of wavelength  $\lambda = 500\text{ nm}$ . To obtain the diffraction pattern, you can assume there are many point light sources at the lattice points, sitting at the cross points of the  $N=100$  vertical lines and  $N=100$  horizontal lines, each separated by  $s=d/N=1\text{ }\mu\text{m}$  apart. If the lattice point's position is within the circular aperture, then it is allowed to radiate light. Then on the screen you add all the electric field from the light sources of the grid points and you will be able to obtain the diffraction intensity pattern by squaring the electric field. (The detailed derivation is listed in the next page.) In addition to generating the intensity plot of the diffraction pattern, also find and print the radius of the first dark ring and check whether the Rayleigh criterion  $\theta = 1.22\lambda / d$  is satisfied. Also notice that, to calculate intensity is to do the square of the amplitude. However, the 'real' diffraction pattern is really faint to be observed on the computer screen. Therefore, we also do a 'false' intensity image of the diffraction pattern by taking the absolute value of the amplitude as the intensity. Of course, when you want to calculate the radius of the first dark ring, you should use the 'real' intensity image.

```
from vpython import *
from numpy import *
```

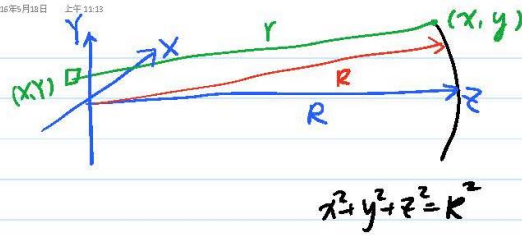
```
N = 100
R, lamda = 1.0, 500E-9
d = 100E-6
```

```
dx, dy = d/N, d/N
scene1 = canvas(aligned = 'left', height=600, width=600, center = vector(N*dx/2, N*dy/2, 0))
scene2 = canvas(aligned = 'right', x=600, height=600, width=600, center = vector(N*dx/2, N*dy/2, 0))
scene1.lights, scene2.lights = [], []
scene1.ambient, scene2.ambient = color.gray(0.99), color.gray(0.99)
side = linspace(-0.01*pi, 0.01*pi, N)
x,y = meshgrid(side,side)
```

```
E_field = cos(10000*((x-0.005)**2 + (y-0.002)**2)) # change this to calculate the electric field of diffraction of the aperture
```

```
Inte = abs(E_field) ** 2
maxl = amax(Inte)
for i in range(N):
    for j in range(N):
        box(canvas = scene1, pos=vector(i*dx, j*dy, 0), length = dx, height= dy, width = dx,
            color=vector(Inte[i,j]/maxl,Inte[i,j]/maxl,Inte[i,j]/maxl))
```

```
Inte = abs(E_field)
maxl = amax(Inte)
for i in range(N):
    for j in range(N):
        box(canvas = scene2, pos=vector(i*dx, j*dy, 0), length = dx, height= dy, width = dx,
            color=vector(Inte[i,j]/maxl,Inte[i,j]/maxl,Inte[i,j]/maxl))
```



$$\begin{aligned}
 r &= [(x-X)^2 + (y-Y)^2 + z^2]^{\frac{1}{2}} \\
 &= [x^2 + y^2 + z^2 - 2xX - 2yY + x^2 + y^2]^{\frac{1}{2}} \\
 &= R \left[ 1 - \frac{2xX + 2yY}{R^2} + \frac{x^2 + y^2}{R^2} \right]^{\frac{1}{2}} \\
 &\because \text{small aperture } \therefore x^2 + y^2 \ll R^2 \\
 &\approx R \left[ 1 - \frac{2xX + 2yY}{R^2} \right]^{\frac{1}{2}} \\
 &= R \left[ 1 - \frac{2xX + 2yY}{2R^2} \right]
 \end{aligned}$$

The amplitude of the electric field at position  $(x, y)$  on the screen gets its contribution from all point source sitting inside the aperture:

$$E(x, y) = \iint_{\text{Aperture}} \frac{1}{r} \sin(\omega t - kr) dXdY \approx \iint \frac{1}{R(1 - \frac{xX + yY}{R^2})} \sin\left(\omega t - kR\left(1 - \frac{xX + yY}{R^2}\right)\right) dXdY$$

$$\text{Let } \theta_x = x/R, \theta_y = y/R, k_x = k\theta_x, k_y = k\theta_y$$

$$E(\theta_x, \theta_y) \approx \iint \frac{1}{R} \sin(\omega t - kR + k_x X + k_y Y) dXdY$$

$$E(\theta_x, \theta_y) \approx \iint \frac{1}{R} \sin(\omega t - kR) \cos(k_x X + k_y Y) dXdY + \iint \frac{1}{R} \cos(\omega t - kR) \sin(k_x X + k_y Y) dXdY$$

$$E(\theta_x, \theta_y) \approx \sin(\omega t - kR) \iint \frac{1}{R} \cos(k_x X + k_y Y) dXdY + \cos(\omega t - kR) \iint \frac{1}{R} \sin(k_x X + k_y Y) dXdY$$

In this problem,  $\iint \frac{1}{R} \sin(k_x X + k_y Y) dXdY = 0$  due to the symmetry of the integration on the 'circular' aperture. Therefore,

$$E(\theta_x, \theta_y) \approx \sin(\omega t - kR) \iint \frac{1}{R} \cos(k_x X + k_y Y) dXdY = A \cdot \sin(\omega t - kR)$$

$$A = \iint_{\text{Aperture}} \frac{1}{R} \cos(k_x X + k_y Y) dXdY$$

is the amplitude of the electric field on the spherical screen, which is the one you should calculate by summation over all the grid point on the aperture.