$\lambda = \frac{Q_{net}}{L}$, $\sigma = \frac{Q_{net}}{A}$, $\rho = \frac{Q_{net}}{Volume}$

Exam ~~1~~ Equations!

Force:

$F = \frac{k q_0 Q}{r^2}$, $F = q_0 E$

Electric Field:

$E = \frac{kQ}{r^2}\hat{r}$

$E = \frac{F}{q_0}$

$E_{dipole} = \frac{2kp}{y^3}$ ← qd; y ← distance from point to charge

$E_{inf\ plane} = \frac{\sigma}{2\varepsilon_0}$

$E_{line} = 2k\frac{\lambda}{r}\hat{r} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$

$E_x = -\frac{\partial V}{\partial x}$

Flux:

area of gauss surface

$\Phi = \oint \vec{E}\cdot d\vec{A} = \oint \vec{E}\cdot\hat{n}\, dA$

$\Phi = \vec{E}\cdot d\vec{A} = |E||A|\cos\phi$ ← angle between $\hat{n}$ + $\vec{E}$

uniform "E"

$\Phi_{net} = \frac{Q_{encl}}{\varepsilon_0}$

Work:

remember loss in potential for stuff

$W_{a\to b} = \int_{r_a}^{r_b} \vec{F}\,d\vec{r} = \frac{kqq_0}{r_a} - \frac{kqq_0}{r_b} = U_a - U_b = W_{net}$

$W_{a\to b} = Fd = q_0 E d$

Pot Energy to a q_0 = $U = q_0 k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots\right)$

Pot Energy of a system: $U = \frac{kqq}{r}$ add potential energy for every unique radius connecting

$\Delta U = V q_r$, $\Delta U = \Delta V q_0$

Voltage:

*how much charge wants to move and { if I move a charge this distance, this is how much work will be done

$V = k\frac{q}{r}$ ← distance from charge to where potential is being measured

$V_a - V_b = \frac{W_{net}}{q_0} = \int_a^b \vec{E}\cdot d\vec{l}$

$V = k\int \frac{1}{r}\,dq$

$\Delta V_{line} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_a}{r_b}\right)$ ← for line

$V_a - V_b$ between spheres $= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$

$\cdot\ C = \frac{Q}{V} \rightarrow C_{plate} = \frac{Q}{V} = \frac{\varepsilon_0 E A}{E d}$ $\leftarrow$ $\int E \cdot dA = \frac{Q_{encl}}{\varepsilon_0}$

$\leftarrow V = \int E \cdot dl$

$\cdot\ U_{pot} = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \rightarrow u = \frac{1}{2}\varepsilon_0 E^2$

series $\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

parallel $C_{tot} = C_1 + C_2$

series same charge

parallel same voltage

dielectrics: $K = \frac{E_0}{E_d} = \frac{C_d}{C_0} = \frac{Q_d}{Q_0}$ *Voltage constant before and after dielectric

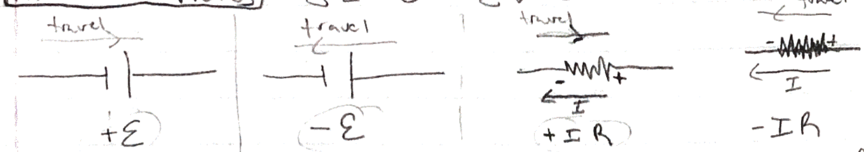
$I = \frac{dQ}{dt} = n q v_d A \rightarrow J = n q v_d$

$\rho = \frac{|E|}{|J|}$ $\rho(T) = \rho_0[1 + \alpha(T - T_0)]$

$R(T) = R_0[1 + \alpha(T - T_0)]$

$R = \frac{L}{A}\rho$ $V = IR$

$\boxed{\text{Kirchoff Rules}}$ $\Sigma I = 0$ $\Sigma V = 0$



$V_{ab} = \varepsilon - IR$

* pick a battery to "win" current

$\boxed{\text{RC circuits}}$ charging: $q = C\varepsilon(1 - e^{-t/RC}) = Q_0(1 - e^{-t/RC})$; $i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC} = I_0 e^{-t/RC}$

discharging: $q = Q_0 e^{-t/RC}$ $[I_0 = -\frac{Q_0}{RC}]$ $i = -\frac{Q_0}{RC} e^{-t/RC}$

$P = \varepsilon i = i^2 R$ $P_{bat} = P_{diss} + P_{capacitor}$

I, V, Q NOT CONSTANT

MAGNETS MAGNETS MAGNETS MAGNETS MAGNETS MAGNETS MAGNETS MAGNETS

$F = q\vec{v} \times \vec{B}$ and $F = I\vec{l} \times \vec{B}$, ϕ = angle $\vec{B}$ & $\vec{l}$

$R = \frac{mv}{qB}$ (circular stuff)

$qE = qvB \rightarrow v = \frac{E}{B}$ (Velocity selector)

$B = \frac{\mu_0}{4\pi}\left(\frac{q\vec{v} \times \hat{r}}{r^2}\right)$

$\Phi_B = \int \vec{B} \cdot d\vec{A} = 0$ if closed surface $= \mu_0 I_{encl}$

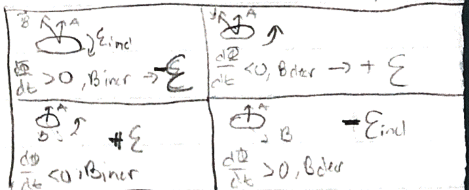
Torque $= I\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$ ϕ = angle $\vec{n}$ & $\vec{B}$

$\tau_{rotwire} = IBA\sin\phi$

$U_{pot} = -\vec{\mu} \times \vec{B}$ ϕ = angle $\vec{\mu}$ and $\vec{B}$

<u>Ampere's</u> <u>LAW</u>: $\oint \vec{B} \cdot d\vec{l}$ $B_{wire} = \frac{\mu_0 I}{2\pi r}$ $B_{sole} = \mu_0 n I$, $n = \frac{N}{L}$ $B_{circl} = B_x = N\frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$

<u>Faraday's</u> <u>Law</u>: $\varepsilon_{ind} = -\overset{N_{(coil)}}{} \frac{d\Phi_B}{dt} \Rightarrow \int \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$



Lenz Law: dir of ε_{ind} is to oppose cause of effect.

Slide wire: $\varepsilon_{ind} = BLV$

$\varepsilon_{ind} = vBL$

$\varepsilon_{ind} = \oint_{loop} (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$\varepsilon_{sole} = \mu_0 n A \frac{dI}{dt}$

$\boxed{\text{Induction}}$

toroidal spiral $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \rightarrow 2\pi r B = \mu_0 N i \Rightarrow B = \frac{\mu_0 N i}{2\pi r} \rightarrow \Phi = BA = \frac{\mu_0 N i A}{2\pi r}$

$L = \frac{N\Phi}{i} \rightarrow L = \frac{\mu_0 N^2 A}{2\pi r}$, $A = \pi r^2$

$\varepsilon = -L\frac{di}{dt}$

$L = \frac{N\Phi}{i}$

$U_{pot} = \frac{1}{2}Li^2$, $u = \frac{B^2}{2\mu_0}$

RL circuit: $i = \frac{\varepsilon}{R}(1 - e^{(-R/L)t})$ $\frac{di}{dt} = \frac{\varepsilon}{L}e^{(-R/L)t}$ $\leftarrow$ *battery connected | disconnected $i = I_0 e^{(-R/L)t}$

LRC circuit:

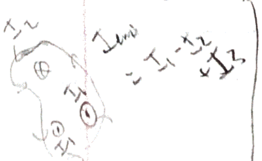
$q = A e^{(-R/2L)t} \cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right)$

ω

$T = \frac{1}{f}$

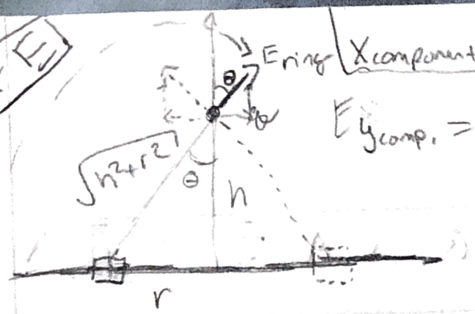
$\omega = 2\pi f$

ε_{ind} of rot wire rect $= BAN\omega\sin(\omega t)$



Proofs

Constant E



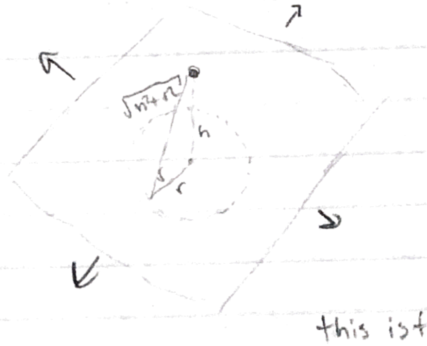
X components cancel!

$\cos\theta = \frac{h}{\sqrt{h^2+r^2}}$

$$E_{y\,comp.} = |E_1|\cos\theta = |E_1|\frac{h}{\sqrt{h^2+r^2}}$$

infinite plate (side view)

* charge density $\sigma = \frac{Q_{net}}{Area}$ $\quad Q_{net} = Area\,\sigma$



Charge of the ring

$$\boxed{Q_{ring} = 2\pi r\,dr\,(\sigma)}$$

$$F_{ring} = \frac{k\,Q_r\,q_{test}}{(\sqrt{h^2+r^2})^2} = \frac{k\,Q_r\,q_{test}}{h^2+r^2}$$

this is the magnitude of this vector $\quad |\vec{E}_{ring}| = \frac{F_{ring}}{q_{test}} = \frac{k\,Q_r}{h^2+r^2}$

But we want ycomp which is $|\vec{E}_{ring}|\cdot\cos\theta$

$$E_{y\,comp.} = |\vec{E}_{ring}|\cos\theta = \left[\frac{k\,Q_r}{h^2+r^2}\right]\cdot\left[\frac{h}{\sqrt{h^2+r^2}}\right] = \frac{k\,h\,\overbrace{(2\pi r\,\sigma)}^{Q_r}}{(h^2+r^2)^{3/2}}\,dr$$

* now we integrate r from 0 to ∞ for infinite plane *

$$dE_{ring(y)} = \frac{(k)(h)(2\pi r)(\sigma)}{(h^2+r^2)^{3/2}}\,dr \qquad u = h^2+r^2,\quad du = 2r\,dr \qquad u^{-3/2}$$

$$E_{ring(y)} = k\,h\,\pi\,\sigma\int_0^\infty \frac{2r}{(h^2+r^2)^{3/2}}\,dr = k\,h\,\pi\,\sigma\int_0^\infty \frac{1}{u^{3/2}}\,du$$

$$k\,h\,\pi\,\sigma\left[-2\,u^{-1/2}\right]_0^\infty \rightarrow k\,h\,\pi\,\sigma\left[-2\,\frac{1}{\sqrt{h^2+r^2}}\right]_0^\infty = k\,h\,\pi\,\sigma\left[0+\frac{2}{\sqrt{h^2}}\right]$$

$$\boxed{E_{inf\,plane} = 2k\pi\sigma = \boxed{\frac{\sigma}{2\varepsilon_0}}}$$

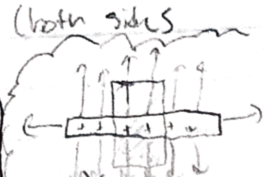
* strength of E not dependent on h! just σ

Proof Gauss law net flux

$Q_{net} = \sigma A$

① $\Phi = \oint \vec{E}\cdot d\vec{A}$ $\quad$ if it is uniform Electric Field $\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$

for a infinite sheet (uniform charge) → flux going through it (both sides



② $E = \frac{\sigma}{2\varepsilon_0}$ $\quad$ ③ since there is 2 sides ⟶ $\Phi = 2EA$

and $\sigma = \frac{Q_{net}}{A}$

so $E = \frac{Q_{net}}{2\varepsilon_0 A}$

④ $\Phi = \frac{2\,Q_{net}\,A}{2\,\varepsilon_0\,A} = \boxed{\Phi_{tot} = \frac{Q_{net}}{\varepsilon_0}}$

* TRUE ~~for any closed surface~~ *

$$C = \frac{Q}{V} \leftarrow \text{ that } V \text{ is } V_a - V_b \text{ (potential difference}$$

Capacitance

(from Gauss), from $V_a - V_b = \int_0^d E \, dl$

$$C_{plate} = \frac{Q}{V} = \frac{\varepsilon_0 E A}{E d}$$

for spheres and cylinders do same prescription.

Circuits

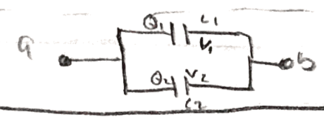
parallel = $\boxed{\text{Same voltage}}$ $\Rightarrow V_{net} = V_1 = V_2 \rightarrow \frac{Q_1}{V_1} + \frac{Q_2}{V_2} = \frac{Q_{net}}{V_{net}}$ so $Q_1 + Q_2 = Q_{net}$

same

if $Q_1 + Q_2 = Q_{net}$ then $C_1 V + C_2 V = C_3 V \rightarrow \underline{C_1 + C_2 + \dots = C_{net}}$

on same line so V's add

series: $\boxed{\text{Same charge}}$ $\Rightarrow V_1 + V_2 = V_{net} \rightarrow \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{net}}$ so $\frac{1}{C_1} + \frac{1}{C_2} + \dots = \frac{1}{C_{net}}$



parallel

Work to charge a plate $= \int dW \rightarrow W_{net} = Vq \Rightarrow \int V dq \rightarrow \int_0^Q \frac{q}{C} dq$

thus: $\frac{1}{2}\frac{Q^2}{C}$ (plugging in $C = \frac{Q}{V}$) $\rightarrow \boxed{\frac{1}{2} QV = \frac{1}{2} CV^2 = U_{pot}}$

Volume between

u = energy density $= \frac{U_{pot}}{Vol} \rightarrow U_{pot} = \frac{1}{2} QV = \frac{1}{2}\varepsilon_0 EA\, Ed = \frac{1}{2}\varepsilon_0 E^2 (Ad)$

so $u = \frac{U_{pot}}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

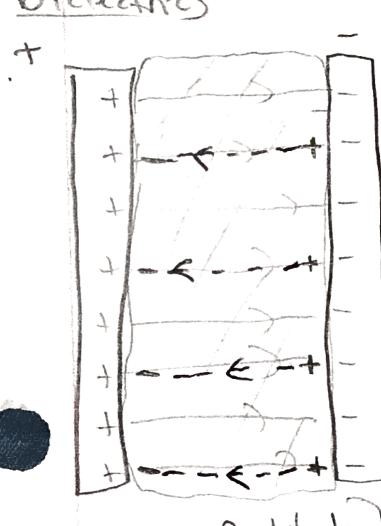
Concept:

[capacitors provide a quick surge of energy, faster than a batter
* see "How capacitors work" by National Maglab on Youtube.]

to charge a plate you need to apply a potential across it causing one side to build charge. (needs to have build up of charge

Dielectrics

the amount any dielectric affects C is K



$\boxed{K \geq 1}$ $K = \frac{E_0}{E_d}$ $C_d = \frac{Q}{V} = \frac{Q}{E_d d} = \frac{Q}{\frac{E_0 d}{K}} = K\frac{Q}{V_0} = K C_0$ so $\boxed{C_d = C_0 \cdot K}$

also using $C = \frac{Q}{V}$ in $C_d = C_0 \cdot K \rightarrow \boxed{K = \frac{Q_d}{Q_0}}$

is V constant before and after dielectric?

if $V_0 = V_d \rightarrow V_d = \frac{Q_d}{C_d} = \frac{Q_d}{KC_0}$ then $V_d = V_0$ $\frac{Q_d}{KC_0} = \frac{Q_0}{C_0} \rightarrow K = \frac{Q_d}{Q_0}$ ✓

since V is constant then

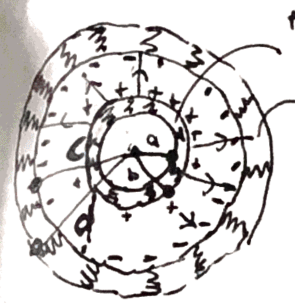
$V_0 = V_d$ and $E_0 \cancel{d} = E_d \cancel{d} \rightarrow E_0 = E_d$

[* weakens field by creating induced field] → if V is constant then it causes more charge to build on the capacitor so C goes up

Gaussian surfaces and charge within insulators and conductors

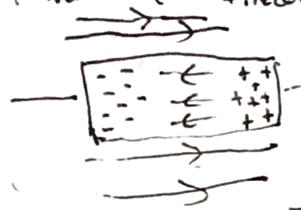
examples ~ to build intuition hopefully

(zigzag) means conductor



[A] $E_{r<a} = 0$ * the electric field around is 0 because the field lines cancel
↳ a charge wouldnt move...

(B) $E_{a<r<b} = 0$ * inside a field the surface for example
• so within the field the charges balance



[C] $E_{b<r<c} = ?$

using shell theorem

$E = k\frac{+2q}{r^2}$

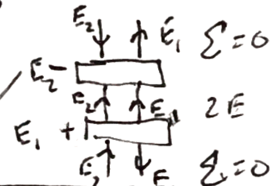
or

$EA = \frac{Q_{net}}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}\right)$ → so we can think of the Electric field as if you condensed all of the charge down

$4\pi r^2$

^ in the formula its the distance between 2 points? well idk just not from center.

[D] $E_{r>d} = 0$ bec/



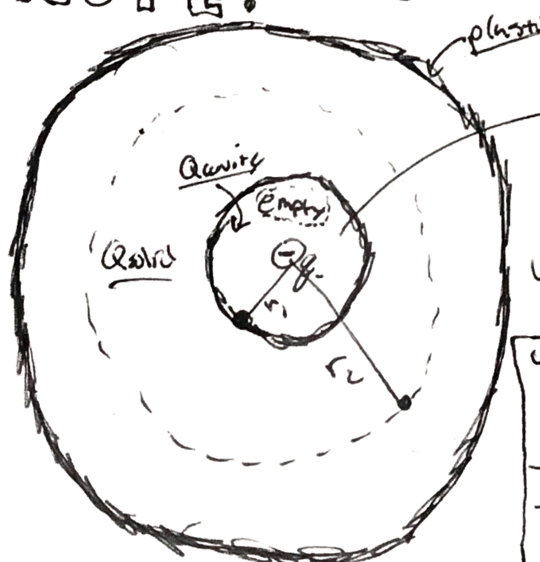
in $\rho = \frac{Q_{net}}{Volume} = \frac{3\, Q_{net}}{4\pi r^3}$ ← • if its outside insolator its the radius of the insolator
• if its insize the insolator its the radius of the gaussian surface
* this is just because if you are inside it doesnt matter about insolater (NO EFFECT)

in $EA = \frac{Q_{net}}{\varepsilon_0}$

A = surface area of gaussian surface

→ if your within insolator you are at a shell within ρ so you have use the r to gauss

NOTE: if you have



scooped out!

* $E = \frac{Q_{net}}{4\pi r_2^2 \varepsilon_0} \rightarrow Q_{net} = Q_{cavity} + Q_{solid}$
(Q_{cavity} ↑ given, Q_{solid} ↑ find)

* $\rho_{solid} = \frac{Q_{solid}}{V_{solid}} \Rightarrow Q_{solid} = \rho V_{solid} = \rho \frac{4}{3}\pi(r_2^3 - r_1^3)$

now you know Q_{net} cuz Q_{cavity} is given

use $E = \frac{Q_{solid} + Q_{cavity}}{4\pi r_2^2 \varepsilon_0}$

what if gaussian was inside the scooped out part

$E = k\frac{q}{r_2^2}$? | for not scooped: $\frac{Q_{net}}{\varepsilon_0} = EA$ and $\rho = \frac{3\, Q_{net}}{4\pi r_2^3}$ solve for E
($A = 4\pi r_1^2$)

think more about what charges want to do and what fields they would produce (ie. what would happen

* Voltage is pretty much saying if you put some q_0 in a field it will have U potential energy.

Voltage notes

$$\frac{W_{a\to b}}{q_0} = \frac{-\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a)$$

$$V = \frac{U}{q_0} \text{ or } U = q_0 V \quad , \quad V \frac{J}{C}$$

$V_a = \frac{U_a}{q_0}$ is the potential energy per unit of charge at point a.

$V_{ab} = V_a - V_b$ (voltage), V_{ab}, the potential in V of a with respect to b, equals the work done by the electric force, when a UNIT (1C) of charge moves from a to b.

Point a

point b

the voltage of this battery = the difference in potential from its positive end to negative end

sidenote * $\frac{J}{C}$, 100C or 1000C etc... more charge you add the more work done

$$V = k\frac{q}{r} \xrightarrow{\text{from}} U = \frac{k q q_0}{r} \rightarrow \frac{U}{q_0} = k\frac{q}{r} = V$$

value of point charge

distance from point charge to where potential is being measured

$$V = k\int \frac{1}{r}\,dq$$

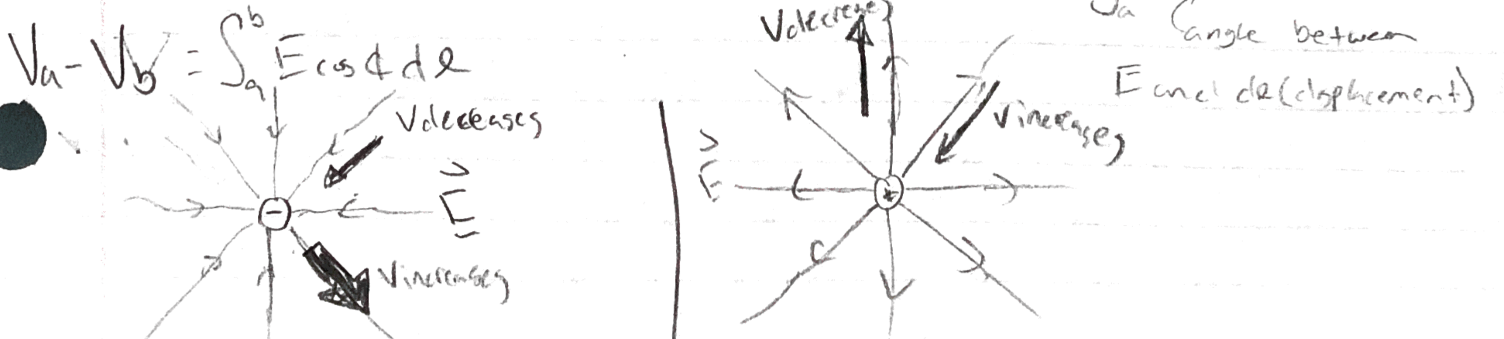
electric potential due to a collection of point charges, scalar sum of the potentials due to each charge

integral over charge distribution

Recall! $W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0 \vec{E}\cdot d\vec{l}$ | $\frac{W_{a\to b}}{q_0} = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl$

angle between E and dl (displacement)

$$V_a - V_b = \int_a^b E\cos\phi\, dl$$



How to actually make sense theorem:

$\vec{E}$ field comes in
→ mess with electrons,
→ the electrons move to
→ balance out creating their own $\vec{E}$ field, opposite & it cancels

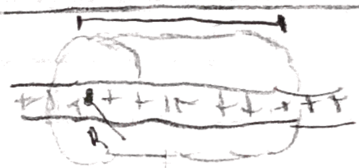
* $E\left(\frac{N}{C}\right) = E\left(\frac{Volt}{m}\right)$
* $1eV = 1.602 \cdot 10^{-19} J$

* When all charges are at rest, the entire solid volume of a conductor is at the same potentiall.

Line of charge derivation

+ + + + + + + + + + + +

r_b • B
r_a • A
$\Delta V_{A \to B} = ?$



$\lambda = \frac{Q_{enc}}{L}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

$E \, 2\pi R \cancel{L} = \frac{\lambda \cancel{L}}{\varepsilon_0}$

$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$

$E = k\frac{Q}{r^2} \rightarrow Er^2 = kQ$ $V\cancel{r} = Er^{\cancel{2}}$
$V = k\frac{Q}{r} \rightarrow Vr = kQ$ $V = Er$

$V = ER$, $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$

$dV = E\,dr$ * have to use $d\cancel{V}$ + dr because the force isnt const.

$dV = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}\,dR$

negative sign because we are pushing the charge up so we are losing energy

$V = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{R}\,dR$

$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\ln R\Big|_{r_a}^{r_b} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_a}{r_b}\right)}$

- When using gauss E = constant because you are going around the same distance, so the force stays the same

- When doing inf line stuff the force changes as you go to it, the E is changing

End of chap 28 and start of chap 29

Amperes law.

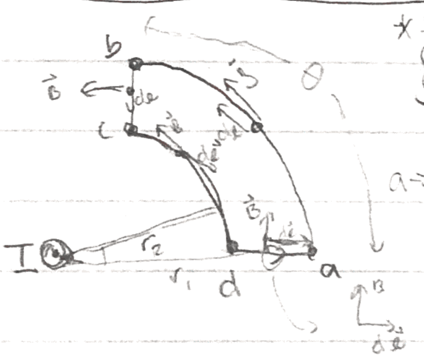
for highly symetric current distrobutions use Amperes law like I did with gauss law for electric Field. Or you can add up $d\vec{B}$'s to a current element and summing them up.

But Amperes law isnt about flux more about a line interal along a closed path.

$\oint \vec{B} \cdot d\vec{\ell}$ is for a long straight conductor $B = \frac{\mu_0 I}{2\pi r}$

sign depends on direction current relative to direction of integration

curl fingers in direction you walk, thumb is in direction of positive current direc



* for an integration path that doesnt enclose the conductor $\oint \vec{B} \cdot d\vec{\ell} = 0$

$a \to b$ $B_{\parallel} = B_1 = \frac{\mu_0 I}{2\pi r_1}$ from $c \to d$ $B_{\parallel} = -B_2 = \frac{-\mu_0 I}{2\pi r_2}$

$b \to c$ and $d \to a$ have $d\ell \perp B$ so $= 0$

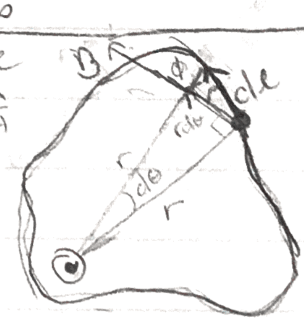
$$= \frac{\mu_0 I}{2\pi r_1}(r_1 \theta) + 0 - \frac{\mu_0 I}{2\pi r_2}(r_2\theta) + 0 = 0$$

with $\int_a^b d\ell$ and $\int_c^d d\ell$

$\vec{B}$ is greater on cd but the arc is less so cd = ab

more generally $\vec{B} \cdot d\vec{\ell} = B\,d\ell\cos\phi$ — angle between B and $d\ell$

from figure $d\ell\cos\phi = r\,d\theta$ ← distance of $d\ell$ from conductor



$\cos\phi = \frac{r\,d\phi}{d\ell}$ $\frac{A}{H}$

$d\ell\cos\phi = r\,d\phi$

just arc distance formula $s = R\theta$

$$\oint \vec{B} \cdot d\vec{\ell} = \oint \frac{\mu_0 I}{2\pi r} d\ell\cos\phi = \oint \frac{\mu_0 I}{2\pi r} r\,d\theta$$

$$\frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\theta = \mu_0 I = \mu_0 I_{encl}$$

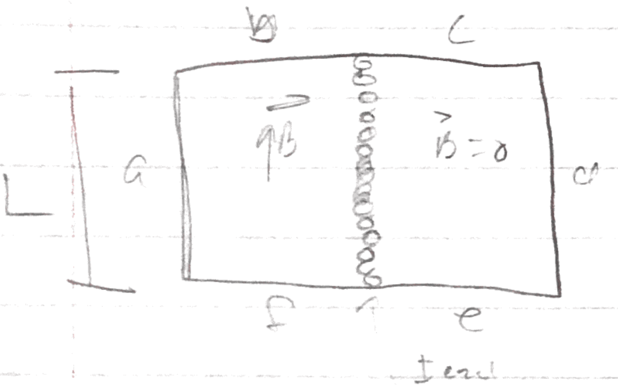
if I is outside loop then net change in θ is 0

* curl fingers along path → thumb points +I

$I_{encl} = I_1 - I_2 + I_3$

$\vec{B}$

solenoid



c, d, e in $B = 0$ so toss these

b, f $\perp$ $\cos(90) = 0$

* just care about A *

$$B\int_0^L dl = BL = \mu_0 I_{encl}$$

- if L is whole length
- N is total turns

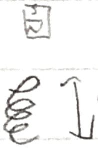
$I_{encl} = I \cdot N$

$\rightarrow$ $B = \mu_0 I \cdot \frac{N}{L}$ (one I)

or $\frac{N}{L} = \frac{\text{turns}}{\text{meter}} = n$ so $B = \mu_0 n I$

Chap 29

up and down = current in coil



up and down = current this means theres an emf

You get current either because B changes with time or B isnt uniform. And this changes the flux through the area

Faradays Law

(angle between B + A)

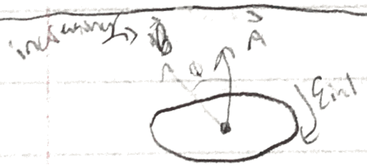
$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\, dA \cos\phi$

and Faradays Law states:

$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}$$

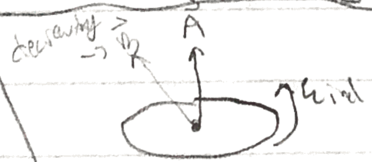
How to figure out direction of $\mathcal{E}_{ind}$ or current.

1. define + dir for $\vec{A}$
2. from dir $\vec{A}$ and $\vec{B}$ determine the sign of the magnetic flux and its rate of change $\frac{d\Phi_B}{dt}$. $\quad -\mathcal{E} = \frac{d\Phi_B}{dt}$
3. if flux is increasing, $+\frac{d\Phi_B}{dt}$, then $\mathcal{E}_{ind}$ or current is negative
   if flux is decreasing, $-\frac{d\Phi_B}{dt}$, the $\mathcal{E}_{ind}$ or current is positive $\quad \mathcal{E}_{ind} = \frac{d\Phi_B}{dt}$
4. Now use RHR, thumb w/ $\vec{A}$ $\begin{cases} \text{if } +\mathcal{E}_{ind} \text{ then same dir as fingers} \\ \text{if } -\mathcal{E}_{ind} \text{ then opposite dir as fingers} \end{cases}$

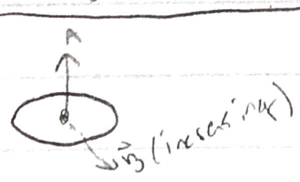


(aligned) with $\vec{B}$

- $\Phi_B > 0$ • $\mathcal{E}_{ind} = -$
- $\frac{\Phi_B}{dt} > 0$



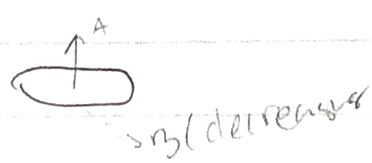
- $\Phi_B > 0$ • $\mathcal{E}_{ind} > 0$
- $\frac{\Phi_B}{dt} < 0$



$\Phi_B < 0$

$\frac{\Phi_B}{dt} < 0$ (aligned)

$\mathcal{E}_{ind} > 0$



$\Phi_B < 0$ $\quad \mathcal{E}_{ind} < 0$

$\frac{d\Phi_B}{dt} > 0$

Or Φ through 1

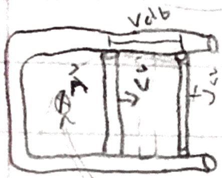
number of turns

Flux through a coil is $N \cdot \int \vec{B} \cdot d\vec{A} = \Phi_{net} \rightarrow N \cdot \Phi_{1\,turn} = \Phi_{net}$

$\mathcal{E}_{ind} = -N \frac{d\Phi}{dt}_{1\,turn}$

X B field

Slide Wire Generator; * creates emf bcz the bar moves changing the Area



$\mathcal{E} = -\frac{d\Phi}{dt} = -B\frac{dA}{dt} = \frac{-B(LVdt)}{dt} = -BLV$ ← says emf is CCW

Area $= l \cdot w = L \cdot l = L \cdot V \cdot dt$

→ chose Area to line up with $\vec{B}$, so put thumb with Area and fingers curl in emf dir., if you get a minus sign its opposite of fingers.

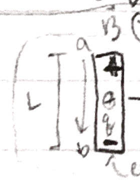
Lenz Law * only gives direction

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

Using Lenz law on slide wire direction. USE RHR and pick a dir of current (pointer finger), Then (middle finger for B) then F B your thumb, (pick which makes sense) *think energy*

if your thumb is with $\vec{v}$ it creates infinite energy, so current is other way.

Moving rod;



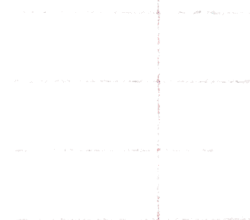
← we move it with constant v

$F = q\vec{v} \times \vec{B}$

- Charges experience $F_B = qvB$ this is upward
- excess separation of charge → creates E → $F_E = qE$ this builds till counters F_B exactly.

$qE = qvB \Rightarrow E = vB$ and $V_{ab} = E \cdot L = vBL$

General Form;

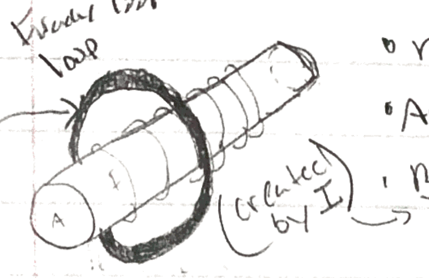


$d\mathcal{E}_{ind} = (\vec{V} \times \vec{B}) \cdot d\vec{l} \rightarrow \mathcal{E}_{ind} = \oint (\vec{V} \times \vec{B}) \cdot d\vec{l}$

or $\vec{F} = q\vec{v} \times \vec{B} \rightarrow \underbrace{\vec{F} \cdot d\vec{l}}_{work} = q(\vec{v} \times \vec{B}) \cdot d\vec{l}$ divide by q $\frac{Work}{q} = \mathcal{E}$

$$\mathcal{E}_{ind} = \oint_{loop} (\vec{V} \times \vec{B}) \cdot d\vec{l}$$

29 cont...



• n turns per unit length.

• Area • $\boxed{I = \text{current in solenoid}}$ $\boxed{\text{INCREASING OVERTIME}}$

• $B = n \cdot (\mu_0 I)$ so flux = $\Phi = B \cdot A = nA(\mu_0 I)$

↑ turns ↑ amperes law

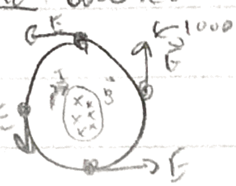
if I is changing w/ time then theres an induced emf.

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -\mu_0 n A \frac{dI}{dt}$$ (induced emf in loop)

creates a current $I = \frac{\mathcal{E}}{R}$, what causes the charges to move?, Not magnetic bc/ loop isn't even in magnetic field

So theres an induced electric field caused by changing magnetic flux

$$\oint \vec{E} \cdot d\vec{l} = \mathcal{E} = \frac{-d\Phi_B}{dt}$$



$$E \cdot 2\pi r = \left|\frac{d\Phi_B}{dt}\right|$$

$$E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$

Maxwells Eq

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = \frac{-d\Phi_B}{dt}$$ changing magnetic flux ← need to study derivation w/ tutor

$$\oint \vec{B} \cdot d\vec{l} = I_{encl}\mu_0$$ or $= \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt}\right)_{enc}$

↑ conducting current through path

Chapter 30 Inductance

imagine 2 coils

This produces $\vec{B}$, and if i is changing it creates a change in Φ_B causing $\mathcal{E}_{ind}$ in coil 2

N = turns

i_1 and coil 2 has Φ_{B2}

$$\mathcal{E}_{2(ind)} = -N_2 \frac{d\Phi_{B2}}{dt}$$

could write as $\Phi_{B2} = (\text{constant})i$ — more convient like this

since i causes a flux number of turns matters,

$$N_2\Phi_{B2} = M_{21} i_1 \quad \rightarrow M = \text{mutual inductance}$$

Φ_{B2} — flux through a single turn,

time derivative

$$N_2 \frac{d\Phi_{B2}}{dt} = M_{21}\frac{di_1}{dt} \longrightarrow \mathcal{E}_2 = -M_{21}\frac{di_1}{dt}$$

* Change in current i_1 from coil 1 induces an emf in coil 2 that is directly proportional to rate of change of i_1 *

also $M_{21} = \frac{N\Phi_{B2}}{i_1}$, M_{21} depends only on size, shape, # turns, orientation, distance

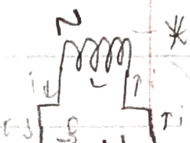
writing for both coils:

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt}, \quad M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}$$

Mutual Inductance is from 2 circuits, Lets consider a single isolated circuit

- as the current changes through this circuit, it changes flux $\xrightarrow{\text{creates}}$ emf.

*By Lenz's law it opposes the current so it makes it more difficult for variations in current



we define self inductance as: $L = \frac{N\Phi_B}{i}$

- if i changes so does flux so: $\underbrace{N\frac{d\Phi_B}{dt}}_{\mathcal{E}} = L\frac{di}{dt}$

so → $\boxed{\mathcal{E} = -L\frac{di}{dt}}$

their purpose is to oppose any variation in current in the circuit.

Next →

Chapter 30 continued

Inductors

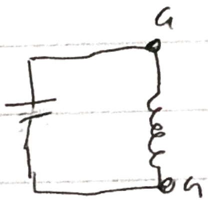
Using Kirchoff rules:

- Kirchoff rule stated that the algebraic sum of voltages in a loop is zero bec/ the electric field produced by charges distributed around the circuit is conservative. Called (E_c) w/ inductor its different.
- The magnetically induced electric field within the coils of the inductor NOT conservative (E_N)

$E_c + E_N = 0$, there has to be accumulations of charge on the terminals of the inductor to produce this field.

$\mathcal{E}_{emf} = \int_a^b \vec{E}_N \cdot d\vec{l} = -L\frac{di}{dt}$ and $\vec{E}_N = -\vec{E}_c \rightarrow \int_a^b -\vec{E}_c \cdot d\vec{l} = -L\frac{di}{dt}$

* neg cancel



this integral is just V_{ab} of a point a with respect to b $\rightarrow$

$V_{ab} = V_a - V_b = L\frac{di}{dt}$ thus we are allowed to use Kirchoff rule.

ex $L = ?$ Toroidal solenoid

$L = \frac{N\Phi_B}{i}$

$\boxed{L = \frac{\mu_0 N^2 A}{2\pi r}}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \rightarrow$ $\boxed{B 2\pi r = \mu_0 N i \quad B = \frac{\mu_0 N i}{2\pi r}}$

$\Phi = BA = \frac{\mu_0 N i A}{2\pi r}$

$A = \pi r^2$

Voltage betw terminals of inductor is $\boxed{L\frac{di}{dt}}$

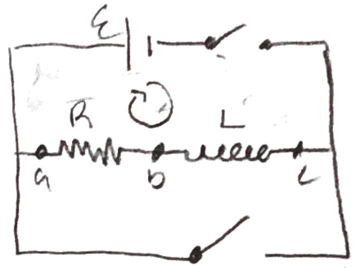
$P = V \cdot i = Li\frac{di}{dt}$ and $P = \frac{dU}{dt}$ so $dU = Li\,di \rightarrow U = L\int_0^{I_{final}} i\,di$ I_{final}

$\boxed{U = \frac{1}{2}LI^2}$ Using L from here $\boxed{U = \frac{1}{2}\frac{\mu_0 N^2 A}{2\pi r} I^2}$, lets look a $u = \frac{U}{\text{Volume}}$

$u = \frac{U}{\underset{\text{length wire}}{2\pi r} A} = \frac{1}{2}\mu_0 \frac{N^2 I^2}{(2\pi r)^2}\frac{\cancel{A}}{\cancel{A}}$ so $\boxed{u = \frac{1}{2}\mu_0 \frac{(NI)^2}{(2\pi r)^2}}$ recall which can be rewritten

as $\frac{B^2}{\mu_0^2} = \frac{(NI)^2}{(2\pi r)^2}$ so $\boxed{u = \frac{B^2}{2\mu_0}}$

RL circuits



Current begins to grow at a rate that depends on value of L

$V_{ab} = iR$ $\quad V_{bc} = L\frac{di}{dt}$

$\mathcal{E} - iR - L\frac{di}{dt} = 0$ solve for $\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{\mathcal{E}}{L} - \frac{R}{L}i$

- at $t=0$ $V_{ab}=0$ so $\frac{di}{dt} = \frac{\mathcal{E}}{L}$ @ $t=0$
- eventually it reaches a steady state value I where $\frac{di}{dt} = 0$

$0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \rightarrow I = \frac{\mathcal{E}}{R}$

Solution $\frac{di}{dt} = -\frac{R}{L}\left(i - \frac{\mathcal{E}}{R}\right) \rightarrow \frac{1}{\left(i - \frac{\mathcal{E}}{R}\right)} di = -\frac{R}{L} dt$

$\int_0^i \frac{1}{i - \frac{\mathcal{E}}{R}} = -\int_0^t \frac{R}{L} dt' \rightarrow \ln\left(\frac{i - \mathcal{E}/R}{-\mathcal{E}/R}\right) = -\frac{R}{L}t$

$i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$ and $\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t}$

Current Decay: now disconnect from battery. without $\mathcal{E}$

$i = I_0 e^{-(R/L)t}$

per 100u L-C circuit:

On Other Notes