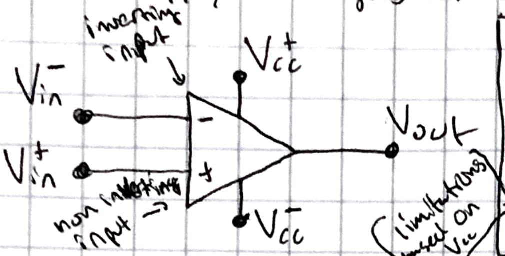
Operational Amplifiers

-black box (complicated)
-plus and minus dont specify polarity on V_{in} (V_{in}^-, V_{in}^+)

1

- Power supply voltages (V_{cc}^+ and V_{cc}^-)



$$V_{out} = G(V_{in}^+ - V_{in}^-)$$

$$V_{sat}^- \leq V_{out} \leq V_{sat}^+$$ → output voltage can only be within a range set by 2 saturation voltages

[you might think huge output voltages (G is like 10^6) but →] where $V_{sat}^+ \approx V_{cc}^+ - 1V$ (a little below pos power supply)
and $V_{sat}^- \approx V_{cc}^- + 1V$ (a little above the neg power supply)

* points of V_{in}^- and V_{in}^+ have very high impedance so very little current flows into these points
↳ output current is restricted | * takes a very small difference between V_{in}^- and V_{in}^+ to saturate

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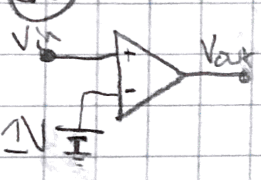
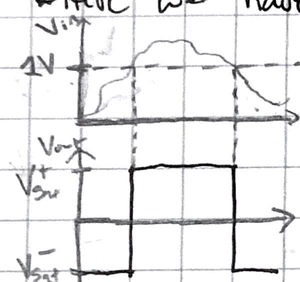
Use 1 Comparator → compares V_{in}^- and V_{in}^+ and gives pos or neg depending on which is larger

ex



non linear application

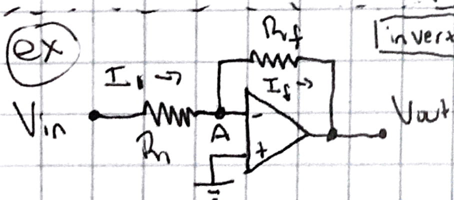


* Here we have a random signal V_{in} and 1V battery
* when $V_{in} < 1$ it hits V_{sat}^-, (large gain quickly brings it there)
when $V_{in} > 1$ it hits V_{sat}^+

[Here is what OpAmps are more generally used for] → Golden Rules
1. Output will make V_{in}^+ and V_{in}^- equal → $\frac{V_{out}}{G} \approx 0 = V_{in}^+ - V_{in}^-$
2. No current flows into the inputs

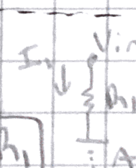
feedback circuits

ex [inverting amplifier]



($V_{in}^- = A$)

- $V_{in}^+ = 0$ so point A = 0 too
- since A is "ground" too $\boxed{V_{in} = I_1 R_1}$



KVL at point A: $V_A - I_f R_f - V_{out} = 0$ or (start at $V_{out} \to V_A$ = ground) $\boxed{V_{out} + I_f R_f = 0}$

Now use rule 2 that (opamp) draw little current so $I_1 = I_f$

then $$\boxed{V_{out} = -\frac{R_f}{R_1} V_{in}}$$ (- signal inverted)

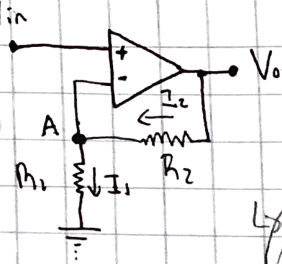
* input resistance is just the effective resistance between the input terminals.
↳ input terminals here are V_{in} and ground, since A is ground too $\boxed{\text{input resistance} = R_1}$

non inverting amplifier

using rules

2

ex



$V_{in} = V_A$ and $I_2 = I_1$

loop (− input to ground): $V_A = I_1 R_1$

loop Vout to ground: $V_{out} = I_2 R_2 + I_1 R_1$

↳ we know $V_A = V_{in}$ → $V_{out} = I_2 R_2 + V_{in}$ and $V_A = I_1 R_1 = I_2 R_1$

using $I_1 = I_2$ ↳

$$V_{out} = I_1 R_2 + V_{in} = V_{in}\frac{R_2}{R_1} + V_{in} = \boxed{V_{in}\left(1+\frac{R_2}{R_1}\right) = V_{out}}$$

idk why :
*input impedance $= \infty$

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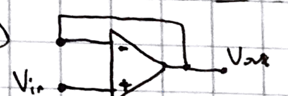
ex



[Buffer Circuit]

$V_{out} = V_{in}^-$ and by rules $V_{in}^+ = V_{in}^-$ so $\boxed{V_{out} = V_{in}}$

Used between a high impedance voltage source and a low impedance load to alleviate loading

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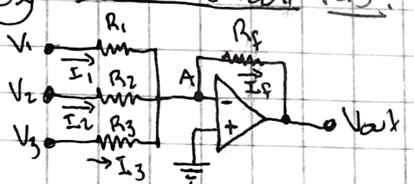
ex Adder Circuit rules: $V_A = 0$ and $I_f = I_1 + I_2 + I_3$



Ohms Law

$V_1 = I_1 R_1$
$V_2 = I_2 R_2$
$V_3 = I_3 R_3$

$$I_f = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

KVL $V_{out} \to (A=0) \to \boxed{V_{out} = -I_f R_f}$

$$\boxed{V_{out} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)}$$

→ we got a weighted sum based on resistors → if we wanted to add normally we make $R_1 = R_2 = R_3 = R_f$

(can easily extend to $\frac{R_f}{R_i}V_i$)

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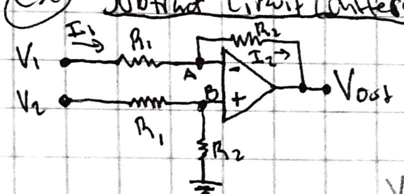
ex Subtract Circuit (differential) * $V_A = V_B$ and $I_1 = I_2$ *



① check bottom

$V_2 - IR_1 - IR_2 = 0$
$V_B = IR_2$

$$V_B = \left(\frac{R_2}{R_1+R_2}\right)V_2$$

② KVL: (goes to ground) V_B

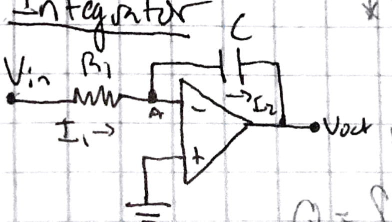
$V_1 - I_1 R_1 - V_A = 0 \to I_1 = \frac{V_1 - V_A}{R_1}$

$V_{out} + I_2 R_2 - V_A = 0$

[putting these 3 together]

$$V_{out} = \overbrace{V_A}^{V_B} - \overbrace{I_2}^{I_1} R_2 = \frac{R_2}{R_1+R_2}V_2 - \left(\frac{V_1}{R_1} - \frac{R_2}{R_1+R_2}\frac{V_2}{R_1}\right)R_2 = \boxed{\frac{R_2}{R_1}(V_2 - V_1) = V_{out}}$$

Integrator



* $V_A = 0$ $I_1 = I_2$ *

• $V_{in} = I_1 R_1$

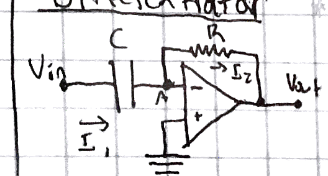
KVL $V_{out} - \frac{Q}{C} - V_A = 0$ ($V_A \to 0$)

$$Q = \int I_2 dt = \int I_1 dt$$

$$V_{out} = -\frac{Q}{C} = -\frac{1}{C}\int I_1 dt =$$

$$\boxed{V_{out} = -\frac{1}{RC}\int V_{in} dt}$$

Differentiator



* $V_A = 0$, $I_2 = I_1$ *

• $V_{in} - \frac{Q}{C} = 0$ and $V_{out} = -I_2 R$

• $\frac{dV_{in}}{dt} = \frac{I_1}{C}$

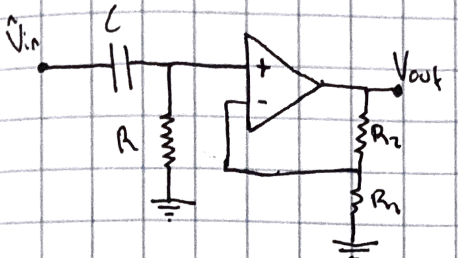
$$\boxed{V_{out} = -RC\frac{dV_{in}}{dt}}$$

(or integral of output same logic applies for integrator too, depends what u want)

Operational Amplifiers Part 2

High Pass

Golden rules: $V_+ = V_-$ and little current into the op-amp



KVL: ① $\hat{V}_{in} - \hat{i} Z_C - \hat{i} R = 0 \rightarrow V_{in} - \hat{i}(Z_C + R) = 0$

$\hat{V}_{in} - \hat{i} Z_C - V^+ = 0 \rightarrow \hat{i} = \dfrac{V_{in} - V^+}{Z_C}$

② $\hat{V}_{out} - \hat{i}_2 R_2 - \hat{i}_2 R_1 = 0 \rightarrow V_{out} = \hat{i}_2 (R_1 + R_2)$

$V^- - \hat{i}_2 R_1 = 0 \rightarrow \hat{i}_2 = \dfrac{V^-}{R_1}$

② $\hookrightarrow \hat{V}^- = \hat{V}_{out} \dfrac{R_1}{R_1 + R_2}$

① $\hookrightarrow \hat{V}_{in} = \dfrac{\hat{V}_{in} - V^+}{Z_C}(Z_C + R) \Rightarrow \hat{V}_{in} - \hat{V}_{in}\dfrac{Z_C}{Z_C + R} = V^+ \Rightarrow V^+ = \hat{V}_{in}\left(1 - \dfrac{Z_C}{Z_C + R}\right)$

$\hookrightarrow \hat{V}^+ = V_{in} \dfrac{R}{Z_C + R} = V_{in} \dfrac{R}{R - \frac{j}{\omega C}}$

simplifying

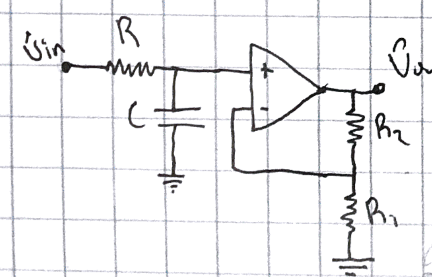
Combine ($V_+ = V_-$): $\hat{V}_{out} \dfrac{R_1}{R_1 + R_2} = V_{in} \dfrac{R}{R - \frac{j}{\omega C}} = V_{in} \dfrac{R\omega C(R\omega C + j)}{(R\omega C)^2 + 1}$

$\hookrightarrow \boxed{\hat{V}_{out} = \hat{V}_{in}\left(\dfrac{R_1 + R_2}{R_1}\right)\left(\dfrac{R\omega C^2 + jR\omega C}{(R\omega C)^2 + 1}\right)}$

when $\omega \to \infty$: $\hat{V}_{out} = \hat{V}_{in} \dfrac{R_1 + R_2}{R_1}$

Low Pass

Seeing how above its voltage divider!



- $\hat{V}^+ = \hat{V}_{in} \dfrac{Z_C}{Z_C + R}$ and $\hat{V}^- = \hat{V}_{out} \dfrac{R_1}{R_1 + R_2}$

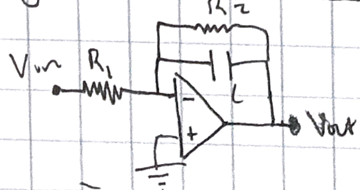
$\hookrightarrow \hat{V}^+ = \hat{V}_{in} \dfrac{-j}{-j + R\omega C} = \dfrac{\hat{V}_{in}}{R\omega C j + 1}$

Combine: $\boxed{\hat{V}_{out} = \hat{V}_{in}\left(\dfrac{R_1 + R_2}{R_1}\right)\left(\dfrac{1}{R\omega C j + 1}\right)}$

$\omega \to 0$

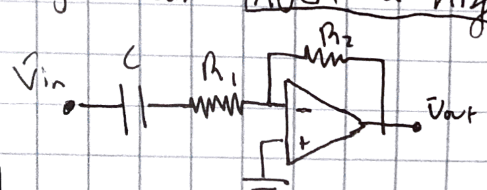
$\hat{V}_{out} = \hat{V}_{in} \dfrac{R_1 + R_2}{R_1}$

- you can invert a low pass



$\hat{V}_{out} = -\hat{V}_{in} \dfrac{R_2}{R_1} \dfrac{1}{R_2 \omega C j + 1}$

- you can invert a high pass

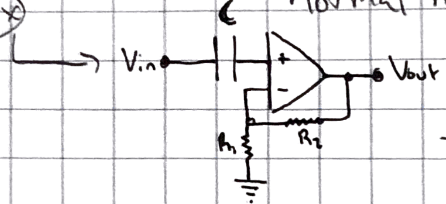


$\hat{V}_{out} = -\hat{V}_{in} \dfrac{R_2 \omega C}{R_1 \omega C - j}$

Practical considerations for the Operational Amplifier

- Actuall a small amount of current does flow into [op amp symbol]
  ↳ I_B^+ and I_B^- ($\sim 500\,nA$) ↔ but under certain conditions the can cause large effects ↔

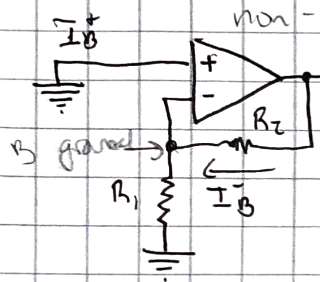
(ex) • normal non-inverting amplifier, but w/ a capacitor



- could be trying to filter out DC signal BUT
- → output will be saturated @ V_{sat}^+ and unresponsive to input
- → blocked dc current and OP won't work w/out it ????

What if we have:

non-inverting amplifier but the output is grounded



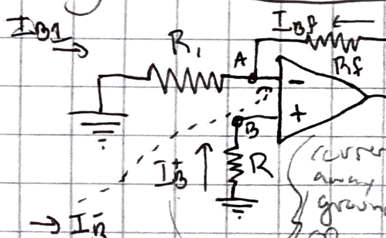
- could be testing the circuit
  ↳ if input = 0 we expect output = 0

Since point B is ground no current through R_1

$500\,nA \cdot 1\,M\Omega$ ← avoid large feedback resistors

$V_{out} = I_B^- R_2 = .5\,V$

You can minimize bias current (Buffer Circuit)



(current going away from ground bec op amp draws current?)

- grounded inputs since we are interested in bias effects

KVL $V_B^+ + I_B^+ R = 0 \rightarrow V_B^+ = -I_B^+ R$ (By rule 1 = A)

KCL $I_B^- = I_{Bf} + I_{B1}$

- $V_A + I_{B1} R_1 = 0 \rightarrow I_{B1} = -\frac{V_A}{R_1}$
- $V_{out} - I_{Bf} - V_A = 0 \quad I_{Bf} = \frac{V_{out} - V_A}{R_f}$

$I_B^- = -\frac{V_A}{R_1} + \frac{V_{out} - V_A}{R_f}$, $V_A = V_B^+$, $I_B^- = I_B^+ \frac{R}{R_1} + I_B^+ \frac{R}{R_f} + \frac{V_{out}}{R_f}$

↳ $V_{out} = I_B^- R_f - I_B^+ R \left(1 + \frac{R_f}{R_1}\right)$ and if you choose a resistor such that $R = \frac{R_1 R_f}{R_1 + R_f}$, $V_{out} = R_f (I_B^- - I_B^+)$ which is difference of current

[for real op amps max difference is specified as the input offset current] → $I_{os} = I_B^- - I_B^+$, {for 741 op amp $I_{os} < I_B^+ / 4$} some high quality op amps can do I_B^+ ≈ 0.05 nA

Slew Rate in the OP Amp

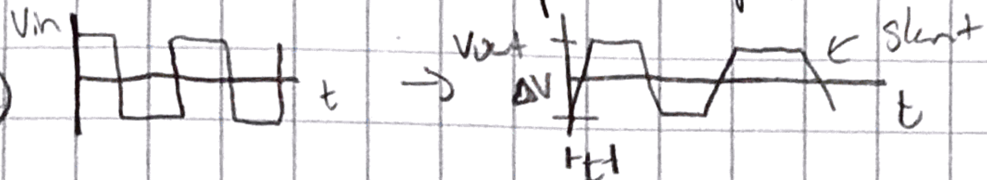
5

↑ slew is good

• suppose we suddenly change the input for our OP amp

<u>How fast will the output change?</u>

ex



[How long would it take the op-amp to switch from V_{sat}^- to V_{sat}^+]

↳ $V_{cc}^+, V_{cc}^- = \pm 15V$, $V_{sat}^{\pm} = \pm 14V$ → Slew rate = $\frac{\text{change in voltage}}{\text{change in time}} = \frac{dV}{dt}$

↑ rise time

↳ so the, $t = \frac{\Delta V}{\text{slew}} = \frac{28V}{.5 V/\mu s} = 56 \mu s$

ex imagine you have output $= A\sin(\omega t)$ → $\frac{dV}{dt} = \omega A\cos(\omega t)$

↳ to <u>avoid slew rate limiting</u> we want <u>max rate of change</u> < <u>slew rate</u>

you don't want the waveform changing faster than the slew rate

↳ $\omega A <$ slew → $f_{max} = \frac{\text{slew}}{2\pi A} \approx 8kHz$

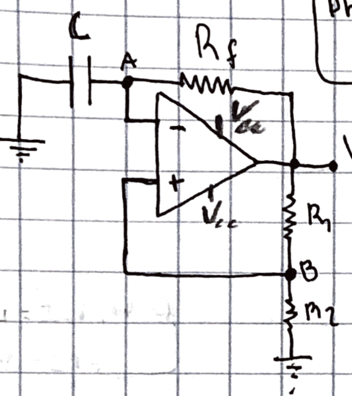
| above a certain f the output can't follow the input | "charge up the" |

↑

[decreasing amplitude makes it better] → when $A = V_{sat}$ is called "full Power Bandwidth"

Astable multivibrator

→ initially → $V_A = V_B = V_{out} = 0$ (not stable)



Phase 1

w/ some noise $V_B > V_A$: w/ $V_{out} = G(V_B - V_A)$

V_{out} • gets pushed to V_{sat}^+! so $V_{out} = V_{sat}^+$

↳ $V_B = V_{out}\frac{R_2}{R_1+R_2} = \boxed{V_{sat}^+ \frac{R_2}{R_1+R_2}}$ $\left(\begin{matrix} V_{out} = I(R_1+R_2) \\ V_B = IR_2 \end{matrix}\right)$

• this happens instantly....

• now the capacitor will start to charge! (slowly)

[↳ since $V_{out} > V_A$ and C is hooked up to ground a] current starts! → $V_A = \frac{Q}{C}$ (Voltage across capacitor by KVL)

[general equation for voltage on capacitor] → $V_A(t) = \underbrace{V_A(0)}_{\text{initial charge}} + \frac{Q(t)}{C} = V_A(0) + \frac{1}{C}\int I\,dt$

to find the current → KVL $V_{out} - IR_f - V_A = 0$ → $V_A(t) = V_A(0) + \frac{1}{C}\int \frac{V_{out} - V_A}{R_f}\,dt'$ ① charging

Phase 2

[now capacitor is charging by $V_A(t)$] when $V_A(t) \geq \left(V_B = V_{sat}^+ \frac{R_2}{R_1+R_2}\right)$ → w/ $V_{out} = G(V_B - V_A)$

$\boxed{\text{it goes to } V_{sat}^-}$!!

now $\boxed{V_B = V_{out}\frac{R_2}{R_1+R_2} = V_{sat}^- \frac{R_2}{R_1+R_2}}$ now the capacitor is discharging

!!! key (when it switches) * initial * !!!

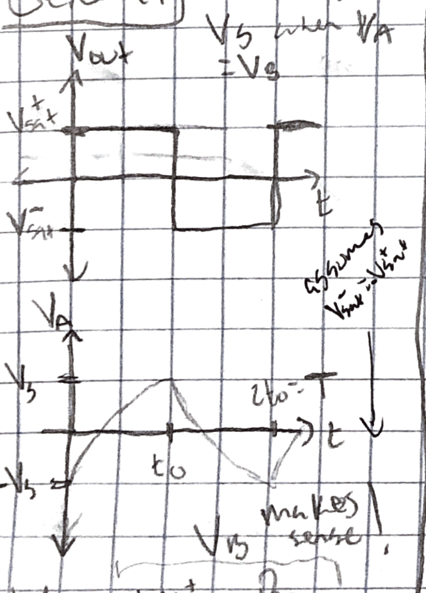
↳ $V_A(t) = (V_A = V_B) + \frac{1}{C}\int \frac{V_{out} - V_A}{R_f} = V_{sat}^+ \frac{R_2}{R_1+R_2} + \frac{1}{C}\int \frac{V_{sat}^- - V_A}{R_f}\,dt = V_A(t)$ ② discharging

repeat!

• it'll keep going when $V_A(t) \leq V_B = V_{sat}^- \frac{R_2}{R_1+R_2}$, $V_{out} = V_{sat}^+$, starts charging... $V_B = V_{sat}^+ \frac{R_2}{R_1+R_2}$

↳ $\boxed{V_A(t) = V_{sat}^- \frac{R_2}{R_1+R_2} + \frac{1}{C}\int \frac{V_{sat}^+ - V_A}{R_f}\,dt}$

General switches at V_S when $V_A = V_S$



V_S makes sense!

$V_S = \pm V_{sat}^{\pm} \frac{R_2}{R_1+R_2}$

[general eq for a capacitor discharging] $V_A = V_1 e^{-t/R_f C} + V_2$ and our limits

① $(t = 0 \to V_A = -V_S)$ and ② $(t \to \infty\ V_A \to V_{sat}^+)$ * never actually reached it switches before

② $V_2 = V_{sat}^+$ → and ① $-V_S = -V_{sat}^+ \frac{R_2}{R_1+R_2} = V_1 + (V_2 = V_{sat}^+)$

↳ $V_1 = -V_{sat}^+\left[\frac{R_2}{R_1+R_2} + 1\right]$ then plugging all in

$$\boxed{V_A = -V_{sat}^+\left[\frac{R_2}{R_1+R_2} + 1\right]e^{\frac{-t}{R_f C}} + V_{sat}^+}$$

the time t_0 when $V_A = \left(+V_S = V_{sat}^+ \frac{R_2}{R_1+R_2}\right)$

$$\boxed{t_0 = R_f C \ln\left(1 + \frac{2R_2}{R_1}\right)}$$

* no V_{sat} cancels out variations in power no effect!

Chap 7 2, 3, 5

[the other op amp is at $\frac{2}{3}V_{cc}$ so when it hits this V_{out} switches and discharges C]

[from general charging capacitor eq]

(2.) charges $V_{2,6} = \frac{-2V_{cc}}{3} e^{-t/(R_1+R_2)C} + V_{cc}$ (from $\frac{V_{cc}}{3}$ to $\frac{2}{3}V_{cc}$)

$\hookrightarrow \frac{2}{3} = \frac{-2}{3} e^{-t_0/\sim} + 1 \rightarrow \frac{1}{2} = e^{-t_0/\sim} \rightarrow \boxed{-(R_1+R_2)C\ln(\tfrac{1}{2}) = t_{on}}$

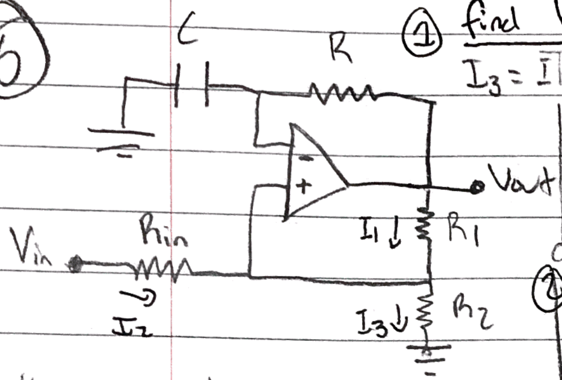
(3.) turn on for $4\text{hrs} = 14400\text{sec} = (R_1+R_2)C\ln(2)$

$R = R_1 = R_2 \rightarrow$ $10387.404 = RC$

$\boxed{C = 1\text{ Farad (can buy on amazon)} \quad R = 10.387\ k\Omega}$

$\boxed{V_{out} = G(V_+ - V_-)}$

(5)



(1) find V_+ | $V_{out} = I_1R_1 + (V_+ = I_3R_2)$ and $V_{in} - I_2R_{in} - V_+ = 0$

$I_3 = I_1 + I_2 \rightarrow \frac{V_+}{R_2} = I_1 + I_2 \rightarrow I_2 = \frac{V_+}{R_2} - I_1$

$\frac{V_{out} - V_+}{R_1} = I_1$ $\hookrightarrow V_{in} - \left(\frac{V_+}{R_2} - I_1\right)R_{in} - V_+ = 0$

and solve for V_+

(2) $\boxed{V_+ = \frac{R_2(R_1V_{in} - V_{out}R_{in})}{R_{in}R_1 + R_{in}R_2 + R_1R_2} = \frac{1}{5}(2V_{in} - V_{out})}$

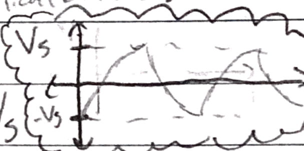
(capacitor gen eq)

(3) $V_- = V_1 e^{-t/RC} + V_2$

initial conditions: (won't actually reach V_{sat}^+) $t = \infty,\ V_- \rightarrow V_{sat}^+$

*not multiplication!!!

(4) $t = 0,\ V_- = V_+[V_{out} = V_{sat}^-] = -V_s$



(5) Right when charging starts $(-V_s)$ the capacitor just reached (V_{sat}^-)

$\hookrightarrow$ "done" happens when $V_+ < V_-$ but while its discharging it was $V_+ < V_- \rightarrow (V_{out} \rightarrow V_{sat}^-)$

t_0 [when it hits this it discharges]

$V_{out} = G(V_+ - V_-)$

$V_2 = V_{sat}^+$

(6) $V_+[V_{out} = V_{sat}^-] - V_{sat}^+ = V_1$

(7) $\boxed{V_- = (V_+[V_{out} = V_{sat}^-] - V_{sat}^+) e^{-t/RC} + V_{sat}^+}$

(8) (when does "t_0" happen?) (9) $V_- = V_+[V_{out} = V_{sat}^+] = +V_s$, [at its "done" point t_0, $V_+ < V_-$ (flips to V_{sat}^-)]

[But the capacitor just reached the point $(V_+[V_{sat}^+])$ when V_{out} flips to V_{sat}^+]

(10) $\hookrightarrow V_+[V_{out} = V_{sat}^+] = (V_+[V_{out} = V_{sat}^-] - V_{sat}^+) e^{-t_0/RC} + V_{sat}^+$

instant $\rightarrow V_{out} \rightarrow V_{sat}^+$

discharges

Capacitor reaches $V_+[V_{sat}^-]$

capacitor reaches $V_+[V_{sat}^+]$

charges $V_{out} \rightarrow V_{sat}^+$ instant

(11) algebra

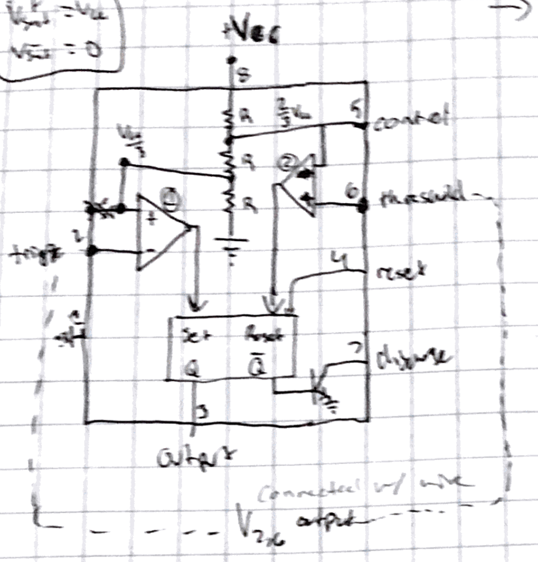
$$t_0 = RC\ln\left(\frac{\frac{R_2(R_1V_{in} - V_{sat}^-R_{in})}{R_{in}R_1 + R_{in}R_2 + R_1R_2} - V_{sat}^+}{\frac{R_2(R_1V_{in} - V_{sat}^+R_{in})}{R_{in}R_1 + R_{in}R_2 + R_1R_2} - V_{sat}^+}\right)$$

half period

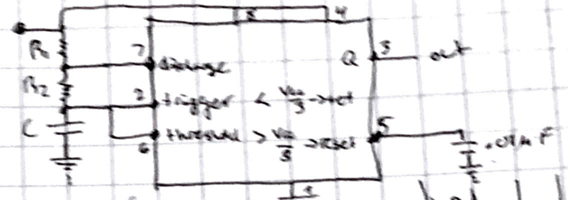
$T = 2t_0$

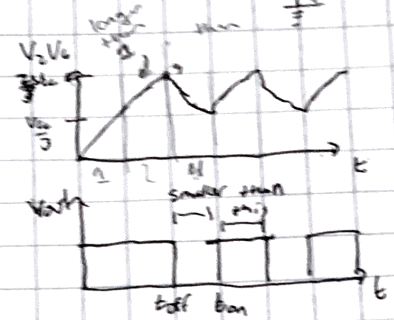
555 timer

$V_{sat}^{+} = V_{cc}$
$V_{sat}^{-} = 0$



Resistor & capacitor configuration to generate oscillator





Input			Q	$\bar{Q}$
set	Reset	State	digital output	
1	0	set	1	0
0	1	Reset	0	1
0	0	stay	previous Q and $\bar{Q}$	
1	1	not used	0	0

- $V_{out} = G(V_+ - V_-)$
- big figure is inside w/ Op amps,
- small figure is oscillator circuit

Op amp ①	Opamp ②
V_{out} → set	V_{out} → Reset
$V_+ \to \frac{V_{cc}}{3}$	$V_+ \to \frac{2}{3}V_{cc}$
$(V_2 = V_-)$ → trigger	$(V_6 = V_-)$ → threshold

as soon as one side is bigger than other it goes to ~~V_{sat}~~

$\boxed{1}$ Capasitor is uncharged. $0 < V_{2,6} < \frac{1}{3}V_{cc}$

Op amp ① $V_{out} = G(V_+ - V_-) \to V_- = 0$ and $V_+ = \frac{V_{cc}}{3} \to V_{out} = V_{sat}^{+}$

↳ Set = 1 [$V_2 = 0$ but the **high** V_{out} sets the flip flop]

Opamp ② $V_{out} = G(V_+ - V_-) \to V_+ = 0,\ V_- = \frac{2}{3}V_{cc} \to V_{out} = 0$

Reset = 0 [$V_6 = 0$ and V_{out} of 0 has no reset]

Starts to charge!

magnitude be V_{sat} must correspond to 0

the op amps voltage is the capacitor $V_{2,6}$ *

$\boxed{2}$ lets say Op amp ① voltage is pass $\frac{V_{cc}}{3}$ (but not $\frac{2}{3}V_{cc}$)

Set = 0 $V_- > V_+ \to V_{sat} = 0$

then op amp ② is still 0 $V_+ < V_- \to V_{sat} = 0$

Reset = 0

"Stay" keeps charging

$\boxed{3}$ Capacitor charges pass $\frac{2}{3}V_{cc}$! Op amp 1 still set = 0, $V_- > V_+ \to V_{sat} = 0$

Set = 0

[Capacitor now discharges through the transistor]

Op amp ② $V_+ > V_- \to V_{out} = V_{sat}$

Reset = 1

→ [discharges through stay phase until set turns to 1 again] or $V_{2,6} < \frac{V_{cc}}{3}$

- <u>charge</u> $V_{2,6} = -\frac{2V_{cc}}{3} e^{-t/(R_1+R_2)C} + V_{cc}$ (from $\frac{V_{cc}}{3}$ to $2\frac{V_{cc}}{3}$)
- <u>discharge</u> $V_{2,6} = \frac{2}{3}V_{cc}\, e^{-t/R_2 C}$ (from $\frac{2V_{cc}}{3}$ to $\frac{V_{cc}}{3}$)

$t_{on} = (R_1+R_2)C\ln(2)$ and $t_{off} = R_2 C \ln(2)$