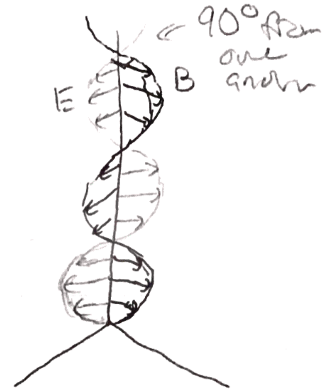
Electromagnetic Waves



(if charges are accelerated or current varrys over time) → produces EM wave

$\vec{E} = E_0 \sin(kz - \omega t)$

$\vec{B} = B_0 \sin(kz - \omega t)$

$k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$, $c = \lambda f$, $\frac{\omega}{k} = c$! $\quad B_0 = \frac{E_0}{c}$

flux: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$, for a plane wave: $\vec{S} = \frac{1}{\mu_0} E_0 B_0 \sin^2(kz - \omega t)\, \hat{k} \left(\frac{W^2}{m^2}\right)$ ← Power/Area

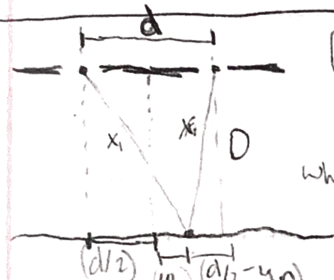
Power: $P = S \cdot A \rightarrow P = \frac{1}{\mu_0 c} E_0^2 A \sin^2(kz - \omega t)$ ~ using $B_0 = \frac{E_0}{c}$ ~

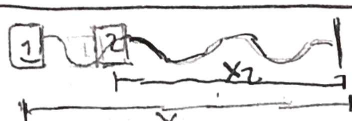
the intensity ($S = \frac{P}{A}$) fluctuates very quickly (vis light 10^{15} osc/s) we observe avg cycles

T is observation time: $P_{av} = \frac{1}{T}\int_0^T P\,dt \rightarrow I = \frac{P_{av}}{A} = \frac{1}{2\mu_0} \frac{E_0^2}{c}$

$\boxed{\text{increase in intensity means increase in Electric field}}$

WRONG Old Classical MODEL





← where is constructive interference?

→ $x_1 - x_2 = 0, \lambda, 2\lambda, 3\lambda$ etc

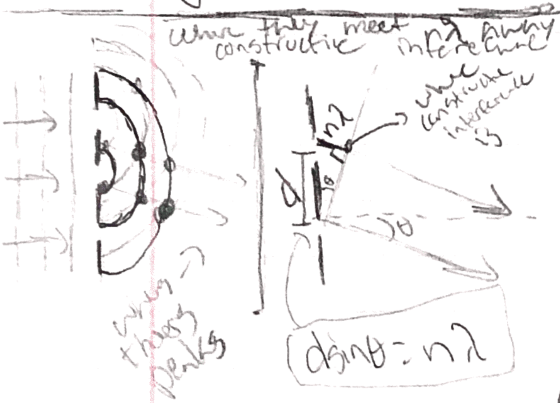
when you push 2 so it lines up $(x_1 - x_2 = n\lambda)$

y_n is distance from center to a maximum

$x_1^2 = D^2 + \left(\frac{d}{2} + y_n\right)^2$, $x_2^2 = D^2 + \left(\frac{d}{2} + y_n\right)^2 \xrightarrow{\text{algebra}} y_n = \frac{x_1^2 - x_2^2}{2d} = \frac{(x_1 + x_2)(x_1 - x_2)}{2d}$

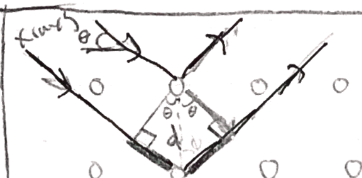
$x_1 \approx D$, $x_2 \approx D$ if usually D is 1 m and d, $y_n \approx$ 1mm

$y_n = (x_1 - x_2)\frac{D}{d} \rightarrow$ constructive interference $\rightarrow y_n = n\lambda \frac{D}{d}$



$(d\sin\theta = n\lambda)$

those lines can be thought of where peaks are, think of those as your x_1 and x_2 in 1 2 from above



R_2 travels $2d\sin\theta$ more

$R_1 + 2d\sin\theta = R_2$

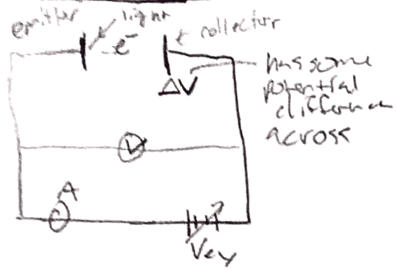
[rays represent where all constructive interference happens] → lets say $R_1 = 0\lambda$, $R_2 = \lambda$

$2d\sin\theta = \lambda$

if it was one more d down (another layer of atoms) that length would be $2(2d\sin\theta)$ but it'd be the second wavelength so it'd cancel

* idk why general form is $2d\sin\theta = n\lambda$ tho?

Photo electric effect



electrons going from emitter to collector

*electrons with $K_e < \Delta u$ get pushed back to emitter

* the voltage point that repels the most energetic electrons is stopping potential

$\Delta u = -e\Delta V$ (pos quantity)

lose that amount of K_e

$$\boxed{K_{max} = eV_s}$$

→ an increase in intensity, is in increase in electric field, thus increase in Force

→ $F = qE$, increase in force increases K_{max}. $\frac{1}{2\mu_0 c} E_0^2$

ex: laser w/ $I = 120\ W/m^2$ shot at Na, takes 2.3eV of energy to release e^-. assume e^- confined to a radius of .10nm

how long will take to release an electron

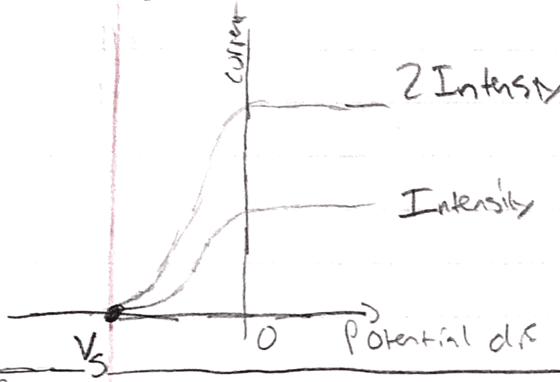
$P_{av} = IA$ $A = \pi r^2$

$P_{av} = \frac{\Delta \text{Energy}}{\Delta t}$

work function, how much it takes to pop an electron off

$\Delta t = \frac{\Delta \text{Energy}}{P_{av}} = \frac{\phi}{IA} = .10$ secs

WHERE THIS CLASSICAL THEORY FAILS



from $K_{max} = eV_s$, doubling intensity increases K_{max}*, but if you increase K_{max}, V_s should change. But it doesnt.

classical says a brighter light (more intense) will knock the e^- off harder generating more of a current, meaning you'll need higher V_s

if you think about it as particle, brighter means more photons so only more electrons will come off not greater Kenergy.

Speed does affect current but they will never reach the plate b/c they are being reflected by a V_s

Quantum Theory of Photoelectric effect.

em wave not continously distributed but instead in small little bundles (quanta)

→ Energy of a photon associated with some λ or f → $E = hf$ or $E = \frac{hc}{\lambda}$

$h = 6.57 \cdot 10^{-34}\ J \cdot s$

photons energy from relativity is $E = pc$ (rest mass = 0) thus $p = \frac{h}{\lambda}$

super simple, a single photon delivered to electron, if photon energy > ϕ then electrons produced

$K_{max} = hf - \phi$ ← so left overs of the two is how much energy electron has

↑ total energy ↑ energy need to release

doubling intensity means twice as many photons strike, thus twice as many electrons. But they will all have the same energy.

if a photon supplies exactly ϕ, the corresponding f (cut off f_c), there is no excess K_{max} so

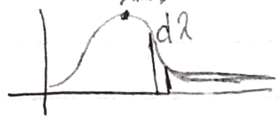
$0 = hf_c - \phi \rightarrow hf_c = \phi \rightarrow \lambda_c = \frac{hc}{\phi}$

cut off λ_c represents the largest wavelength for which the photoelectric effect can be observed for with a work function ϕ.

Thermal Radiation

- blackbody absorbs all radiation incident on it, reflects none $\frac{c}{4}u(\lambda)$

$I(\lambda)$ = intensity as a function of λ

$I = \int_0^\infty I(\lambda)\, d\lambda \rightarrow$ 

stephan boltzmann
stephan constant

$$\left[\begin{array}{l} \text{Stephans law} \quad I(\overset{\text{temperature}}{T}) = \int_0^\infty I(\lambda)\, d\lambda = \sigma T^4 \leftarrow \underline{\text{for blackbody}} \\ \frac{dI(\lambda)}{d\lambda} = 0 \rightarrow \text{Wiens Displacement law: } \lambda_{max} \text{ at which } I(\lambda) \text{ releases } \lambda_{max} \\ \lambda_{max} T = 2{,}898 \cdot 10^{-3}\ \text{m} \cdot \text{K} \end{array}\right]$$

if not a perfect black body it behaves as a proportion of a black body

$I = \varepsilon \sigma T^4$, $\varepsilon = 1$ blackbody, $\varepsilon < 1$ non ideal blackbody

blackbody (the hole), walls are in thermal eq

$u(\lambda)$ = energy density $\rightarrow I(\lambda) \propto c\, u(\lambda)$

$I(\lambda)$ = radiation intensity

*if only 1/4 EM radiation exits blackbody the $\rightarrow \boxed{I(\lambda) = \frac{1}{4} c\, u(\lambda)}$ on average

$n = 4$ (goes $1 \to \infty$) if there 4 peaks over distant L

$\lambda = \frac{2L}{n}$

for a small interval of $d\lambda$, n can be treated as cont. func.

$N(\lambda)\, d\lambda = 2\left|\frac{dn}{d\lambda}\right| d\lambda = 4L \frac{d\lambda}{\lambda^2}$] for standing wave

due to both oscillations of light (polarization)

number of standing waves between $\lambda + d\lambda$

3D: $n = \frac{4\pi L^3}{3}\left(\frac{1}{\lambda}\right)^3$, $N(\lambda)\, d\lambda = 2\left|\frac{dn}{d\lambda}\right| = \frac{8\pi V}{\lambda^4}\, d\lambda$

small lambda van

$u = \frac{(\text{number of modes})(E_{av})}{\text{volume}}$ $\quad E_{av} = \frac{1}{N}\int_0^\infty E\, e^{-E/kT}\, dE = kT$

for each degree of freedom its $\frac{1}{2}kT$ (energy per mode: kT 2 degree of freedom per mode)

$u(\lambda) = \frac{8\pi kT}{\lambda^4}$

$\frac{du}{d\lambda}$?

Better derivation of $u(\lambda)$

mode → when you have all the components of the wave be sinusoidal at a fixed f

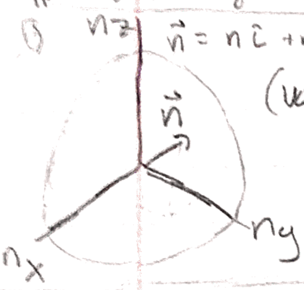
now we need to get # modes that can meet this condition, so you need to count all possible combos of $\vec{n}$. approx is treating it like its the volume of a sphere in (n space)

① from the wave eqn → $\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \frac{\partial^2 E}{c^2 \partial t^2}$ and knowing this eqn must give amplitude $=0$ on walls, to form standing wave

The boundary cond. are $E = E_0 \sin\left(\frac{n_1 \pi x}{L}\right)\sin\left(\frac{n_2 \pi y}{L}\right)\sin\left(\frac{n_3 \pi z}{L}\right)$

Subst'uting this into wave eqn $\sin\left(\frac{2\pi c t}{\lambda}\right)$

$$\boxed{n_1^2 + n_2^2 + n_3^2 = \frac{4L^2}{\lambda^2}} \leftarrow \text{sphere (black body)}$$



$\vec{n} = n\hat{i} + n\hat{j} + n\hat{k}$ $\qquad \vec{n} = \sqrt{n_1^2 + n_2^2 + n_3^2}$

$(\text{volume of } n\text{'s}) = \frac{4\pi}{3}(n_1^2 + n_2^2 + n_3^2)^{3/2}$

PROBLEM PROBLEM PROBLEM

- # modes is only positive but we used negative parts of sphere.
- 8 sections of sphere in 3D grid, we only want (+) so mult by $1/8$

+ 2 polirizations

for each polirization, mult by 2,

$$\text{volume} = \left(\frac{2}{8}\right)\left(\frac{4\pi}{3}\right)\left(\frac{4L^2}{\lambda^2}\right)^{3/2} = \boxed{\frac{8\pi L^3}{3\lambda^3}}$$

this # of modes (N)

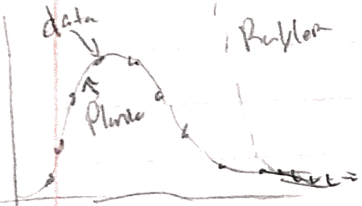
how does the # Modes change per λ? → $\left|\frac{dN}{d\lambda}\right| = \left|\frac{d}{d\lambda}\left[\frac{8\pi L^3}{3\lambda^3}\right]\right| = \frac{8\pi L^3}{\lambda^4}$

$$\frac{du}{d\lambda} = \frac{1}{\text{volume}}\left(\frac{dE}{d\lambda}\right) = \frac{1}{(\text{volume})}\frac{kT \cdot dN}{d\lambda}$$

$$= \frac{8\pi kT}{\lambda^4}$$

E_{av}, each Number of modes

works only in high f, ultraviolet catastophe



Planck guessed $u(\lambda) = \frac{8\pi}{\lambda^4}\frac{hc/\lambda}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$

$E = hf$ → from Einstein

but Energy at a level is. $E_m = m\,hf$

hf I $\quad m = (0, 1, 2 \ldots)$ $\quad$ ex $m = 2$ $E_2 = 2hf$

What is the chance of findint a particle here?

→ these particles have a distribution given by Boltzmann Prob distr. (P_m)

1) $P_m = A e^{-\frac{E_m}{kT}} = A e^{-\frac{hfm}{kT}}$ .. P_m is a probability → $\sum_{m=0}^{\infty} P_m = 1$ ← you'll find 100% the particles in all energy levels

2) plugging in: $\sum_{m=0}^{\infty} A e^{-\frac{hfm}{kT}} = 1$ → $A = \frac{1}{\sum e^{-\frac{hfm}{kT}}}$

from the definition of mean

1) $m_{avg} = \sum_{m=0} m P_m = \frac{\sum m e^{-\frac{hfm}{kT}}}{\sum e^{-\frac{hfm}{kT}}}$

2) using $\sum_{m=0} x^m = \frac{1}{1-x}$, $\sum_{m=0} m x^m = \frac{x}{(1-x)^2}$, $x = e^{-\frac{hf}{kT}}$ ← (NO m)

3) $m_{avg} = (1-x)\frac{x}{(1-x)^2}$ $\qquad f = c/\lambda$

4) $m_{avg} = \frac{1}{e^{hc/\lambda kT} - 1}$

5) $\boxed{E_{avg} = m_{avg}\,hf \rightarrow \frac{hc/\lambda}{e^{\frac{hc}{\lambda kT}} - 1}}$

* $\frac{dN}{d\lambda}$ vs $N(\lambda)$ not sure the error but.....

recall: $u(\lambda) = \frac{(\text{number of modes or oscillators})\,E_{avg}}{\text{Volume}} = \frac{N(\lambda)\,E_{av}}{V} = \frac{\left(\frac{8\pi V}{\lambda^4}\right)E_{av}}{V}$

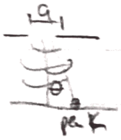
$$\boxed{u(\lambda) = \frac{8\pi}{\lambda^5}\,\frac{hc}{e^{\frac{hc}{\lambda kT}} - 1}}$$

Stephans Law: $I = \int_0^\infty I(\lambda)\,d\lambda = \int_0^\infty \frac{c}{4}u(\lambda)\,d\lambda$ $\rightarrow \frac{2}{15}\frac{\pi^5}{c^2}\frac{k^4}{h^3}T^4$

Weins Law: $I(\lambda) = \frac{c}{4}u(\lambda)$, λ_{max} at $\frac{dI(\lambda)}{d\lambda} = 0$

Davidson & Germer, experiments looking to confirm idea through diffraction,

De Broglie's Hypothesis



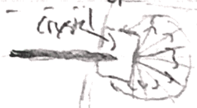
applying $E=hf$, einsteins idea for energy of light in packets, to particles.

de Broglie said ehh lets do it $p=\frac{h}{\lambda}$ or $\lambda=\frac{h}{p}$ for a particle

↳ this wave nature shows itself in diffraction experiments,

$a\sin\theta=n\lambda$ for peaks at certain angles * most intense in middle where each add up on the sides its only itself.

Doing this on an electron



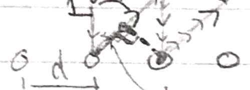
← shows wave nature

instead of through a slit, its on a crystal like

non relativistic → a beam of electrons is accelerated from rest through a ΔV, acquiring a $K=e\Delta V$ and $p=\sqrt{2mK}$

$\frac{U}{q}=\Delta V$, $\Delta U=-\Delta K$ (maybe a minus idk)

$\boxed{\text{for Davidson \& Germer}}$;



so if its a wave you'll get the peaks happening at that "beam", and thus will be a maximum.

$n\lambda$ comes from the phase difference d.f. to eq to θ, const. interf. at 2λ for example.

$d\sin\phi$ is how much further electron beam 1 goes

$(d\sin\theta=n\lambda)$ → they used nickel w/ $d=.215\text{nm}$, peak at $50°$, ($n=1$ no other peaks)

→ so $\underline{\lambda=d\sin\phi=.165\text{nm}}$, they accelerate $\bar{e}$ through a ΔV of $54V$ so $K=e\Delta V=54eV$

and a $p=\sqrt{2mk}=\frac{1}{c}\sqrt{2mc^2k}=\frac{1}{c}\sqrt{2(511000eV)(54eV)}=\frac{1}{c}7430eV$ (Now using energy to get momentum to see de broglie and how it compares to diff.)

↑ rest mass of e^-

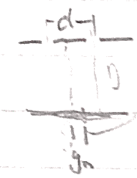
$\boxed{\text{for de Broglie}}$ → $\lambda=\frac{hc}{pc}=\frac{1240eV\cdot nm}{7430eV}=.167nm$

notice how close $.165nm$ & $.167nm$ are, shows wave nature of particles. (if $\sin\phi$ goes over $90°$ you got max n, or the incident angle)

Double slits w/ particles

from double slit eq $y_n=n\lambda\frac{D}{d}$ ← distance from max & center, we will see how neutrons do,

$\Delta y=y_{n+1}-y_n=\lambda\frac{D}{d}$ → $\lambda=\frac{\Delta y d}{D}$ ← distance between slit, ← distance from screen



if you plug

diffraction $=1.89\text{nm}$ for neutron and de broglie $=1.85\text{nm}$

$\boxed{ex}$ through what ΔV do you have to acc electrons to see a 12nm virus, the diameter is our de Broglie λ

$p=\frac{h}{\lambda}$; $E_{tot}=\sqrt{(pc)^2+(mc^2)^2}$ → $\sqrt{E_{tot}^2-(mc^2)^2}=pc$, $E_{tot}=K+mc^2$

$pc=\sqrt{(K+mc^2)^2-(mc^2)^2}$ if you mult out you get $pc=\sqrt{2Kmc^2+K^2}$, and $pc=\frac{hc}{\lambda}$

$\sqrt{K^2+2Kmc^2}=\frac{hc}{\lambda}$, solve for K & algebra by quadratic formula you get

only plus root makes sense

$K=-mc^2\pm\sqrt{(mc^2)^2+(\frac{hc}{\lambda})^2}$ → $K=-mc^2+\sqrt{(mc^2)^2+(\frac{hc}{\lambda})^2}$ recall $\Delta U=-\Delta K$ (error)

$\frac{\Delta U}{q}=\Delta V$ so $\Delta V=\frac{-\Delta K}{+e^-}=\frac{1}{e}\left(-mc^2+\sqrt{(mc^2)^2+(\frac{hc}{\lambda})^2}\right)$

$10m \pm 1m$ $\Delta\lambda$

7.5 .2
.0256

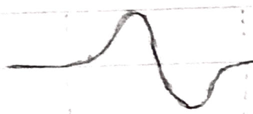
$T = \frac{\lambda}{2}$
$\frac{1}{f} = \frac{\lambda}{2}$

Heisenberg's uncertainty Principle.

idea: $\lambda = \frac{h}{p}$ and it works for particles. If you have a small momentum the more its spread out and the less you know its location. If you have big p the less spread out and the more you know its location.

Classical uncertainty, wave

Δ = size, Δ = uncertainty, Δ can be uncertainty?



$\leftarrow \Delta x \approx \lambda \rightarrow$

① since our λ is so short, all uncertainty $\Delta\lambda$ — same order magnitude

this our uncertainty $\boxed{\Delta\lambda \sim \varepsilon\lambda}$ meaning our uncertainty is roughly as a fraction • actual

ex: $10 \pm 1m$, $\lambda = 10m$, $\Delta\lambda = \pm 1m$ so $\Delta\lambda \sim \varepsilon\lambda$, and $\varepsilon = .1$ in this case.

② the size of our Δx in this case is: $\Delta x \approx \lambda$ and $\Delta\lambda = \varepsilon\lambda$ (doesn't always work out to be like λ

the product is $\Delta x \, \Delta\lambda \sim \varepsilon\lambda^2$ → $10m = 10m$ checks out (ex $\Delta\lambda = .2$, $\lambda = 7.5$ works out 1.5

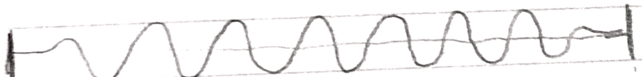
10 1 .1 · 100

$\downarrow \Delta x$ means $\Delta\lambda\uparrow$ or our uncertainty when we try to confine the packet.

Classical wave uncertainty

For a larger wave packet.

$\Delta\lambda \sim \frac{\varepsilon\lambda}{N}$ ← just divides by the number of times we count our wavelengths which is the same as the Δ of our count our uncertainty



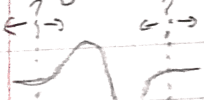
$\Delta x \approx N\lambda$

$\Delta x \cdot \Delta \sim (N\lambda)\left(\frac{\varepsilon\lambda}{N}\right) = \varepsilon\lambda^2$

Same as our smaller case! ($T = \frac{\lambda}{v}$ could plug in and it'd work)

Frequency time Uncertainty

Classical uncertainty wave uncertainty



$\leftarrow \Delta t = T \rightarrow$

→ time to complete one cycle.

Trying to measure Period, difficulty knowing where to stop and start cycle.

$\Delta T \sim \varepsilon T$ same logic as above

Δt is like our Δx which is like → close to the actual size or period / plugging in you get same formula!

$\Delta t = T$, $\Delta t \cdot \Delta T \sim \varepsilon T^2$ (or $\Delta x \cdot V_{wave} = \Delta t$, $\Delta\lambda \cdot V_{wave} = \Delta T$ and $\lambda^2 = V_{wave}^2 \cdot T^2$

lets get it in f, $f = \frac{1}{T}$ Δf related to ΔT? not $\Delta f \neq \frac{1}{\Delta T}$, should be directly rel

to get relationship $df = -\frac{dT}{T^2}$, converting to finite intervals, $\Delta f = \frac{\Delta T}{T^2}$ (interval is like +)

plugging into $\Delta t \, \Delta T \sim \varepsilon T^2$ → $\boxed{\Delta f \, \Delta t \sim \varepsilon}$

→ says longer duration of wave packet more precisely we can measure frequency.

HEISENBERG UNCERTAINTY RELATIONSHIPS

These relationships apply to all waves and thus de Broglie waves.

using $p = \frac{h}{\lambda}$ to find uncertainty do $dp = h\frac{d\lambda}{\lambda^2}$ → $\Delta p = h\frac{\Delta\lambda}{\lambda^2}$

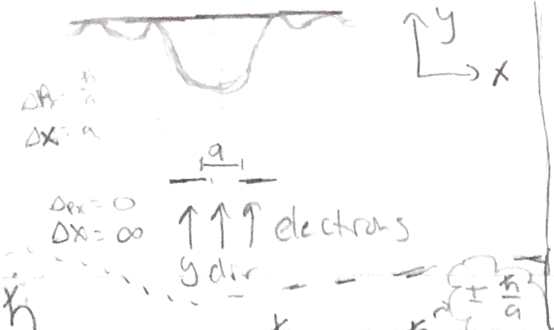
using $\Delta x \, \Delta\lambda \sim \varepsilon\lambda^2$ → $\Delta x \, \Delta p \frac{\lambda^2}{h} \sim \varepsilon\lambda^2$ → $\boxed{\Delta x \, \Delta p = \varepsilon h}$

the smallest possible value of $\Delta x \Delta p$ is $= \frac{h}{4\pi} = \frac{1}{2}\hbar$

so $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ or $\Delta y \Delta p_y \geq \frac{\hbar}{2}$ or $\Delta z \Delta p_z \geq \frac{\hbar}{2}$

115

Some examples



electrons before slit

we know $\Delta P_x = 0$ so

$\Delta x = \infty$

electrons after slit

$\Delta x = a$ ← our uncertainty is confined to the slit width

→ by $\Delta p \Delta x \sim \hbar$

ΔP_x will then become $\frac{\hbar}{\Delta x} = \frac{\hbar}{a}$, so now theres uncertainty in momentum, no longer know if its going straight. So it spreads ← uncertainty = magnitude ↳ more below

→ an angle that specifies where a particle with $p_x = \frac{\hbar}{a}$ lands

$\theta = \sin\theta \approx \tan\theta = \frac{p_x}{p_y} = \frac{\hbar/a}{p_y}$, using de Broglie $\lambda = h/p_y$, $\sin\theta = \frac{\lambda}{2\pi a}$

energy and time uncertainty energy and time unce

from $E = hf \rightarrow \Delta E = h\Delta f \rightarrow \Delta E \Delta t \sim \varepsilon h \rightarrow \Delta E \Delta t \geq \frac{\hbar}{2}$

idea: particle w/ short lifetime $\Delta t \to 0$, measurement of its rest energy will be imprecise $\Delta E \to \infty$

a particle thats stable $\Delta t = \infty$, Energy can be measured w/ ∞ precision, $\Delta E = 0$.

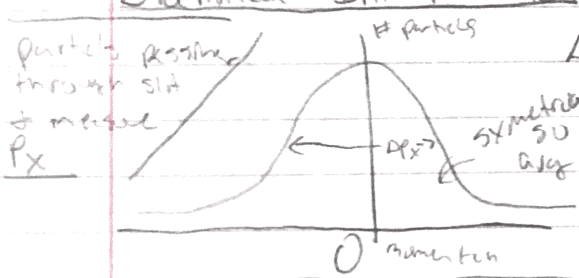
ex an electron moves in x dir with $V = 3.6 \cdot 10^6$ m/s can measure its speed with 1% precision.

$p_x = mV_x = 3.3 \cdot 10^{-24}$ m/s·kg

whats Δx ?, $* \Delta p_x = .01 \cdot 3.3 \cdot 10^{-24}$ thus

$\Delta x \sim \frac{\hbar}{\Delta p_x} = \frac{1.05 \cdot 10^{-34}\ \text{J.s}}{3.3 \cdot 10^{-26}} = 3.2\ \text{nm}$

Statistical Interpretation



Δp_x is similar to standard deviation

↳ $\sigma = \sqrt{(A^2)_{avg} - (A_{avg})^2}$

symmetrical so avg is 0, mean is 0, $P_{x,avg} = 0$

By analogy: $\Delta P_x = \sqrt{(P_x)^2_{avg} - 0} \rightarrow \Delta P_x = \sqrt{(P_x)^2_{avg}}$

rough overall magnitude of p_x

Δp_x = magnitude

ex. Particle of mass m is confined to a 1D box. What's minimum KE of this particle?

$\Delta x = L \rightarrow \Delta x \Delta p \geq \hbar/2 \rightarrow \{\Delta p \geq \frac{\hbar}{2L}\}$

$(\Delta p)^2 = (p^2)_{avg} - (p_{avg})^2 \approx 0$

average x, y components is 0, if squared first, dont cancel

$E_{k\,avg} = \frac{(p_x^2)_{avg}}{2m} = \frac{(\Delta p)^2}{2m} \geq \frac{\hbar^2}{8mL^2}$

ex nuclear beta decay, electrons are ejected from the nucleus. Occasionally one escapes. Range of ke should electron have?

$\Delta p_x \sim \hbar / \Delta x = 19.7\ \text{MeV}/c$

↳ diameter of nucleus

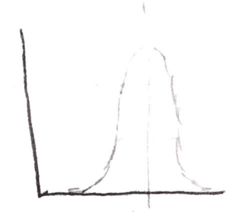
$K = \sqrt{(pc)^2 + E_{rest}^2} - E_{rest} = 19\ \text{MeV}$

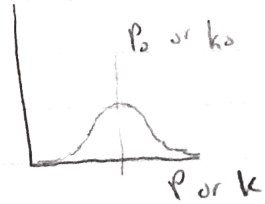
electrons emitted from nucleus have $K_e \sim 1$ MeV much small than the typical spread in energy required by uncertainty principle for electrons confined to nucleus. Suggests electrons cant be in nucleus.

in detail missed → Week 4. V2 pg 25, estimate Δx, square it,
↳ pg 35

Wave Packets & Quantum

- particle is a wave packet bc/ it has finite spatial extension
- cant be just a simple plane wave with definite P and E must be described as a wave packet of a distribution of momenta.
- wave packet is the sum of ∞ # of plane waves each weighted by $A(p)$





$$\Psi(x) = \int_{-\infty}^{\infty} A(k)\cos(kx)\,dk$$

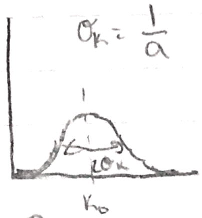
A(p) is distribution function of p also correlated to the amplitude when constructing packets

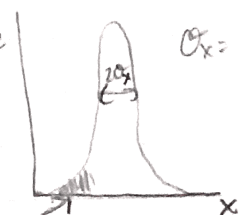
ex of Gauss dist

using $y(x) = \int_{-\infty}^{\infty} A(k)\cos kx\, dk$ w/ $A(k) = C e^{-\frac{a^2}{2}(k-k_0)^2}$ → A(k)

$$y(x) = \frac{2C\sqrt{\pi}}{a} e^{\frac{-2x^2}{a^2}} \cos(k_0 x)$$

$$|y(x)|^2 = \frac{4C^2\pi}{a^2} e^{\frac{-4x^2}{a^2}} \cos^2(k_0 x)$$





* $\cos^2(k_0x)$ hut (not?) comes due to fast oscillator

probability density $= p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(x-\text{mean})}{2\sigma^2}\right)}$ * to find probability $\int_{\text{low end}}^{\text{high end}} p(x)\,dx$

k_0 the center!

you look find by inspection again from $A(k)^2$, $\sigma_x = \frac{1}{2}a$

↳ by inspection $a = \frac{1}{\sigma_k}$ → $\sigma_k = \frac{1}{a}$ if you square $y(x)$ which squares $A(k)$ ·

→ probability density too, more later w/ Schrodinger. Just Notice it has same shape as $A(k)$ making it a probability density

definition of Uncertainty:

σ = uncertainty so → [comes from real space, $y(x)^2$] $\Delta x = \sigma_x$, [amplitude $A(k)$, associated w/ momentum] $\Delta k = \sigma_k$

↳ lets right $p = \frac{h}{\lambda}$ in terms of k, $k = \frac{2\pi}{\lambda}$ so $p = \frac{h}{2\pi}k = \boxed{\hbar k = p}$

$$\Delta x \Delta p = \sigma_x \hbar \sigma_k$$

$$\Delta x \Delta p = \left(\frac{a}{2}\right)\hbar\left(\frac{1}{a}\right) \rightarrow \Delta x \Delta p = \frac{\hbar}{2}$$

← for gaussian distribution its $= \frac{\hbar}{2}$

for any other distribution $> \frac{\hbar}{2}$

Group Velocity & Phase velocity, with Quantum

$$V_p = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \frac{2\pi\, E/h}{2\pi p/h} = \frac{E}{p}$$

$$V_G = \left(\frac{d\omega}{dk}\right)_{k=k_0} \rightarrow \left(\frac{dE}{dp}\right)_{p=p_0}$$

V_p depends on p, $E(p)$ or $E(k)$ is called the dispersion relation

V_G is often evaluated at the center of momentum of $A(p)$

ex for a low speed particle w/ V_0, whats phase & group velocity

Group

$$E_k = \frac{1}{2} m V_0^2 = \frac{p_0^2}{2m} \rightarrow V_G = \left.\frac{dE}{dp}\right|_{p=p_0} = \left.\frac{p}{m}\right|_{p=p_0} = V_0$$

wow, the particle moves w/ wave packet

* the classical velocity corresponds to the group velocity of the packet !!!! !

Phase

$$V_p = \frac{E}{p} = \frac{p}{2m}$$ → depends on p, no classical counterpart makes sense.

Consider a wave packet that represents a confined particle at $t=0$

initial cond: Δx_0, Δp_{x0} at time t,

$X = V_x t \qquad V_x = V_{x_0} \pm \Delta V_{x_0} \rightarrow \Delta x = \sqrt{(\Delta x_0)^2 + (\Delta V_{x_0} t)^2}$

$\Delta V_{x_0} = \Delta p_{x_0}/m$

and $\Delta p_{x0} = \frac{\hbar}{\Delta x_0}$ → $\Delta x = \sqrt{(\Delta x_0)^2 + \left(\frac{\hbar t}{m \Delta x_0}\right)^2}$

the narrower we can spacially confine the packet the quicker it spreads

---

Construction of Wave Packets

Wave Packets

$-\infty$ to ∞, particle
→ could be found anywhere

- building wave packets, pure sinusoidal waves not useful.
→ add waves of different λ together

A = amplitude k = wave number $= 2\pi/\lambda$
λ = wavelength

ex add together 2 waves

wave function?

$$y(x) = A_1\cos(k_1 x) + A_2\cos(k_2 x) = A_1\cos\left(\frac{2\pi}{\lambda_1}x\right) + A_2\cos\left(\frac{2\pi x}{\lambda_2}\right)$$

still $-\infty$ to ∞ → gets you wave that has sorta localized spots but not completely, if you add more it gets better.

→ the regions still repeat even if you add a lot so you need to find a acceptable area.

→ if $A_1 = A_2 = A$ then $y(x)$ can be rewritten like this (lots of trig)

$$y(x) = 2A\cos\left(\frac{\pi x}{\lambda_1} - \frac{\pi x}{\lambda_2}\right)\cos\left(\frac{\pi x}{\lambda_1} + \frac{\pi x}{\lambda_2}\right)$$

→ if λ_1 and λ_2 are close together, that is $\Delta\lambda = \lambda_2 - \lambda_1$ is much smaller than λ_1 or λ_2

$$y(x) = 2A\cos\left(\frac{\Delta\lambda\pi x}{\lambda_{avg}^2}\right)\cos\left(\frac{2\pi x}{\lambda_{avg}}\right)$$ where $\lambda_{avg} = \frac{(\lambda_1 + \lambda_2)}{2} \approx \lambda_1$ or λ_2

Any finite combo of waves with discrete λ will have $-\infty$ to ∞, need to add a part that is large where we want to confine our particle, but goes to 0 as $x \to \pm\infty$

→ $1/x$ would work → $y(x) = \frac{2A}{x}\sin\left(\frac{\Delta\lambda\pi x}{\lambda_0^2}\right)\cos\left(\frac{2\pi x}{\lambda_0}\right)$, λ_0 is central wavelength
↳ otherwise cos blows up at, $x=0$

another type is Gaussian form ← makes wave, while e^x part pulls wave and makes it drop off
↳ keeps it localized

$$y(x) = Ae^{-2(\Delta\lambda\pi x/\lambda_0^2)^2}\cdot\cos\left(\frac{2\pi x}{\lambda_0}\right)$$

$$\boxed{k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f}$$

Generally $\quad y(x) = \sum A_i \cos(k_i x)$

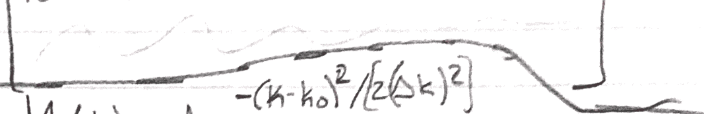
if we have a cont set of wave numbers this sum becomes an integral

(1) $y(x) = \int A(k)\cos(kx)\,dk$ ← integral is carried out over whatever wave numbers is permitted.

[ex if the range is] → $k_0 - \Delta k/2$ to $k_0 + \Delta k/2$, cont distrib cent at k_0, if all have same amplitude A_0

$$y(x) = \frac{2A_0}{x}\sin\left(\frac{\Delta k}{2}x\right)\cos(k_0 x) \leftarrow$$

[identical to $k_0 = 2\pi/\lambda_0 + \Delta k = 2\pi \Delta\lambda/\lambda_0^2$]



a better apprx. can be made by letting $A(k) = A_0 e^{-(k-k_0)^2/[2(\Delta k)^2]}$

↳ gives a range of wave numbers with largest contribution at center, k_0, and falls off to zero with characteristic width of Δk.

Using (1) to this case with k ranging from $-\infty$ to ∞ gives

$$y(x) = A_0 \Delta k \sqrt{2\pi}\, e^{-(\Delta k x)^2/2} \cos(k_0 x)$$

Motion of wave Packet.

→ * for added $V_{phase} = \frac{\omega_{avg}}{k_{avg}}$

$y(x,t) = \cos(kx - \omega t)$ lets follow a peak, when $kx - \omega t = 0, 2\pi, 4\pi$

↳ the speed is taking time derivative $\frac{d}{dt}(kx - \omega t) = \frac{d}{dt}(\text{constant})^{\to 0} \to \frac{dx}{dt} = \frac{\omega}{k} = V_{phase}$

When you add waves w/ different λ or k's you get

$y(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 - \omega_2 t)$

$= \underbrace{2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)}_{\text{tells you about envelope}} \underbrace{\cos(k_{ave}x - \omega_{ave}t)}_{\text{tells you about phase velocity}}$

$\Delta k = k_2 - k$

$k_{ave} = \frac{k_1 + k_2}{2}$

$\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$ tells you how its rolling but how fast?,

$\frac{d}{dt}\left(\frac{\Delta k x}{2} - \frac{\Delta\omega}{2}t\right) = \frac{d}{dt}(\text{const})$ following peaks $0, 2\pi, 4\pi$

↳ $\frac{\Delta\omega}{\Delta k} = \frac{dx}{dt} = V_{group}$ use infinitesimals, $\frac{d\omega}{dk}$.