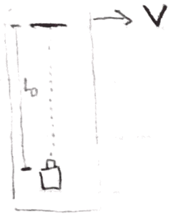
SPECIAL THEORY OF RELATIVITY

- physics are the same in all inertial RF (reference frame)
- nothing can go faster than light, c is the same in all RFs

Time Dilation

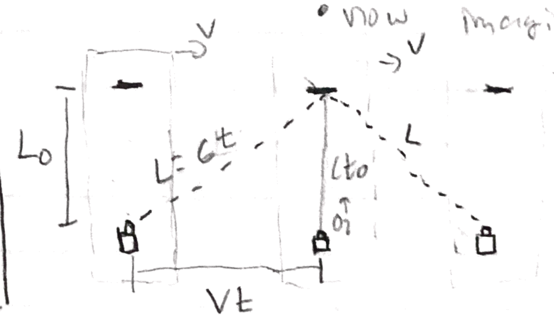


- imagine this light clock is on a rocket

$\Delta t_0 = \frac{2L_0}{c}$ ← proper time

- the person on the rocket measures this time
- now imagine you are watching this rocket move

Lets take a look at that time



if $ct = L$
$\Delta t = \frac{L}{c}$
(the longer distance)

ct = distance light travels in t sec
vt = distance rocket travels in t sec
ct_0 = distance light travels relative to the person on the rocket

* the reason you can say $L_0 = ct_0$ is because $c = \frac{L_0}{t_0}$ no matter what with t_0 being relative to rocket man

- so if c cant change and L_0 cant change t_0 has to change
- but both observers can agree on L_0

$$(ct)^2 = (vt)^2 + (ct_0)^2 \rightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Δt_0 ← time person on rocket measures

Δt ↑ time observer measures

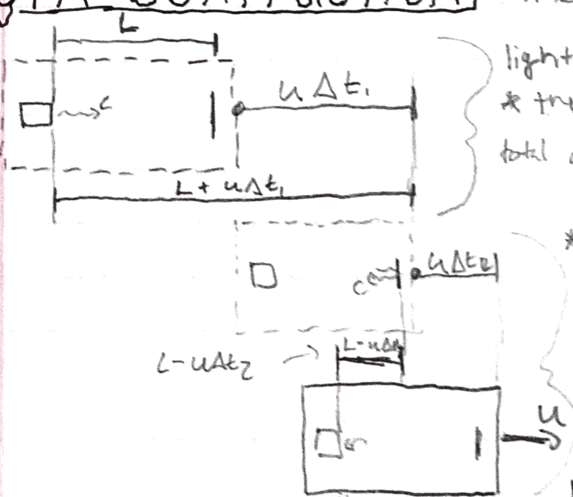
$(ct)^2$, $(vt)^2$: me watching I can get these two

$(ct_0)^2$: you get same L_0 cant change and $L_0 = ct_0$ (c cant change also)

c moves less distance in inertial RF compared to in motion so only thing that can change is time

Length Contraction

* Δt_1 is time to hit mirror Δt_2 is time to reflect & return



light had to travel $L + u\Delta t_1$ ← where this is how far the rocket traveled in Δt_1

* the ship is running away from it

total distance light traveled = $c\Delta t_1 = L + u\Delta t_1$ [A] → $\boxed{\Delta t_1 = \frac{L}{c+u}}$

* the ship is running into the light, so in the time Δt_2 for it to return to the emitter, the ship travels $u\Delta t_2$. So the total distance light traveled is $L - u\Delta t_2$

$c\Delta t_2 = L - u\Delta t_2$ [B] → $\boxed{\Delta t_2 = \frac{L}{c-u}}$

the total return time $= \Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-u} + \frac{L}{c+u}$

- now use Δt from time dilation because
and $\Delta t_0 = \frac{2L_0}{c}$ set them equal.
solve for L,

length person not moving → $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

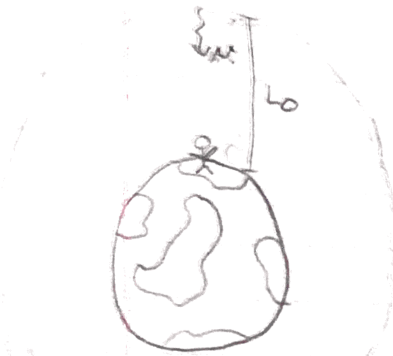
↑ length person who moving

Time dilation and length contraction.
length contraction in one RF is a time dilation in another

$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$, $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

REMEMBER c has to be same in all RF so:
RF 1 $c = \frac{L_0}{\Delta t}$ and RF 2 $c = \frac{L}{\Delta t_0}$ | $c = \frac{L_0}{\Delta t_0}$ and $c = \frac{L}{\Delta t}$ don't make sense

moon hitting the earth



Time dilation: person is at rest watching the moon
→ the muon measures its lifetime (proper life) to be $(2.2\cdot10^{-6}s)$
→ the person measures the height of earth (proper length) to be (100km)

Find the minimum speed μ has to travel to reach Earth.
person is stationary ref frame

$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ ← $(2.2\cdot10^{-6}s)$ solving for this → $333\cdot10^{-6}s = \frac{2.2\cdot10^{-6}s}{\sqrt{1-\frac{v^2}{c^2}}}$ ⇒ $\boxed{V = .999978c}$

recall! $\Delta t = \frac{L}{c}$ ← long length or length person sees μ go, in this case $L = L_0$
$\Delta t = \frac{L_0}{c} = \frac{100km}{3\cdot10^8} = 333\cdot10^{-6}s$ (use 'c' cause it must live for atleast this amount of Δt)

Length contraction! → person moving with muon (or Earth flying toward us)
⇒ muon ~~stationary~~ stationary ref frame
- Earth speed is $V = .999978c$
- for observer on ~~Earth~~ we travel 100km (L_0)

height of atmosphere from our pov

$L = L_0\sqrt{1-\frac{v^2}{c^2}}$ = .66 km or 660m

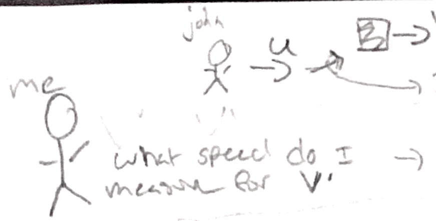
*moving means moving relative to stationary observer

time dilation: figure out what the moving thing's Δt_0 is (aka what moving thing's time measure)

time that stationary observer sees → $\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ ← time moving thing measure; velocity of moving thing

length contraction: L_0 = length a person watching at rest measures.

length moving thing sees → $L = L_0\sqrt{1-\frac{v^2}{c^2}}$ velocity of moving thing



$\xrightarrow{u}$ $\boxed{B} \to v'$

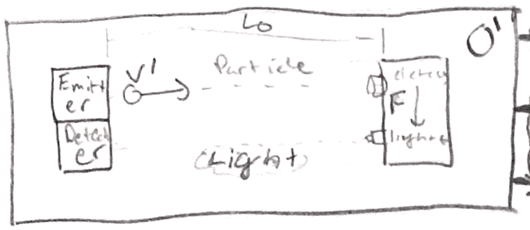
→ John measures box to be v'

$$V = \frac{v'+u}{1+\frac{v'u}{c^2}}$$

what speed do I measure for v' → this speed is V

Consequences of Special Relativity

VELOCITY ADDITION



O'
- emitter releases particle w/ v' in O'
- one cycle is 1 tick $= \Delta t_0$ in O'

$\to u$

thus: $\Delta t_0 = \Delta t_{emit} + \Delta t_{light} = \frac{L_0}{v'} + \frac{L_0}{c}$

O

- for O (and O' is moving at u) it sees at speed V (not v')

now just for particle to reach F in Δt_1

→ (1) $V\Delta t_1 = L + u\Delta t_1$ → $D_{tot} = D_{contracted} + D_u$

[contracted length bc/ we see it moving] → (2) now for light side get time in Δt_2 seconds

$c\Delta t_2 = L - u\Delta t_2$ → $D_{tot} = D_{length} - D_u$ [doesn't have to go the inch the shuttle moved since emission]

$$\Delta t_1 = \frac{L}{V-u}, \quad \Delta t_2 = \frac{L}{c+u} \to \Delta t = \frac{L}{V-u} + \frac{L}{c+u} \text{ (for O)}$$

1. Use time dilation formula to plug Δt and use $\Delta t_0 = \frac{L_0}{v'} + \frac{L_0}{c}$
2. then use Length Contraction formula to relate L & L_0

↓you↓ get ↓this↓

$$\boxed{V = \frac{v'+u}{1+\frac{v'u}{c^2}}}$$

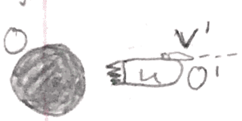
V = speed O sees of particle
v' = speed O' sees of particle
u = speed of O' in O RF

notice if $\frac{v'u}{c^2}$ is small you get normal velocity addition

(only for velocity components in direction of u)

* if O' measures the particle $v' = c$ then both observers measure c, required by Einstein

(Example)



→ O measures rocket moving away at .8c
→ u fires missile measures its speed .6c relative to it
→ what speed does O measure v'?

$$V = \frac{v'+u}{1+\frac{v'u}{c^2}} = .95c$$, O only measures .95c and not 1.4c!! yay :)

The Relativistic Doppler Effect

(heard by O) (freq emitted by source) (speed of waves in medium) (speed of observer)

Classical Doppler Effect: $f' = f \dfrac{(V \pm V_o)}{V \pm V_s}$ ← (velocity of source)

* relative to medium

Since velocities dont add like this and light doesnt need a medium

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→ consider a source of waves at rest in O. (same velocity)

(more like the waves are O RF but this makes it easy to see)

→ O' moves at u relative to source

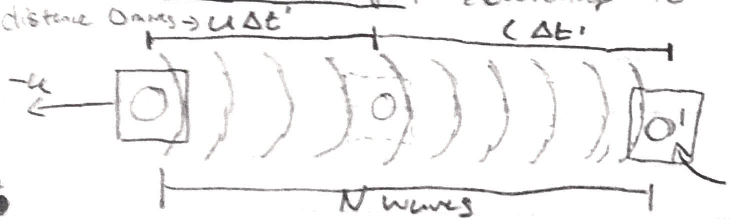
→ In this situation were in RF of O' (more explanation below)

→ O observes the source to emit N waves at freq f. According to O it takes time interval $\boxed{\Delta t_0 = \frac{N}{f}}$, this is proper time in RF O

→ in same time interval for O' O moves $u\Delta t'$ (assumes even distribution)

→ the total wavelength according to $\boxed{O' \text{ is}}$ $\boxed{\frac{\text{total length interval}}{\text{number waves}}}$

Since u = speed of O then O' sees waves go away at speed u



FOR O'!

[1] $\lambda' = \dfrac{c\Delta t' + u\Delta t'}{N}$ since O sees $f = \dfrac{N}{\Delta t_0}$: $\dfrac{\text{total waves}}{\text{total length of time}}$ → $\lambda' = \dfrac{c\Delta t' + u\Delta t'}{Nf}$ ← (O's freq)

[2] according to O' $f' = \dfrac{c}{\lambda'}$ $\xrightarrow{\text{algebra}}$ $f' = f \dfrac{\Delta t_0}{\Delta t'} \left(\dfrac{1}{1 + \frac{u}{c}} \right)$

(best way for me to think abt it is $c = \lambda \cdot f$ cause how often they hit times dist between is speed)

(time O measures is proper cause its the time measured by the moving object cause O' is stationary in this RF)

(O' is stationary in this RF so the time it measures the time dilate?)

→ Using time dilation eq for $\dfrac{\Delta t_0}{\Delta t'}$, and algebra

$$\boxed{f' = f\sqrt{\dfrac{1 - \frac{u}{c}}{1 + u/c}}}$$ ← $\boxed{\text{NOTE}}$!!!!! $\boxed{\text{assumes source and observer are seperating; if approaching then } f' = f\sqrt{\dfrac{1 + \frac{u}{c}}{1 - \frac{u}{c}}}}$

Example

A galaxy is moving away from the Earth such that the blue Hydrogen line of 434nm on Earth is recorded at 600nm What is speed of the galaxy?

Knowing $f = \frac{c}{\lambda}$ and $f' = \frac{c}{\lambda'}$ then

$\dfrac{c}{\lambda'} = \dfrac{c}{\lambda}\sqrt{\dfrac{1 - u/c}{1 + u/c}}$ ⇒ plugging in = $u = .31c$

(length when source is moving) (length when source isnt moving)

← similar to length contraction

Lorentz transformation derivation

- an event is something that happens at a definite time and place.
  - → lets assign event with coordinates (x, t) (a fire cracker going off)
  - → you moving to the right at speed u give it (x', t')

(assumed $x' = 0$ + $x = 0$ are at $t = t' = 0$)

Lets look at Galilean:

sychronized → $t = t' = 0$ → thus $t = t'$

you say the event is at $\boxed{x' = x - ut}$

* because you've been moving to right at u the event will be less by ut

[Since velocity of light is said to be the same for each obsv we have to modify.]

Thus if I predict you will say event is at $x' = x - ut$ you will say lengths need to be chan by γ

Getting γ

$$x' = \gamma(x - ut) \quad [1] \qquad \text{like wise} \qquad x = \gamma(x' + ut') \quad [2]$$

this leaves open times being different

Suppose when our origins coincided we sent a light pulse to set off fire cracker. The light took t seconds to travel x meters according to me. According to you it took t', x' to go but we both agree on c.

thus: $x = ct \qquad x' = ct'$

now → multiply (1) and (2) then use these cause we are transforming between two events.

$$xx' = \gamma^2(xx' + xut' - x'ut - u^2tt')$$

$$c^2tt' = \gamma^2(c^2tt' + uctt' - uct't - u^2tt')$$ take out γ cancel tt'

$$\gamma^2 = \frac{1}{1 - \frac{u^2}{c^2}} \rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$ now plug in → $x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}$

if you isolate t'

$$t' = \frac{t - \frac{ux}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

← remember $1 - \frac{1}{\gamma^2} = \frac{u^2}{c^2}$

Using these equations to get Length contraction & time dilation

$$x' = \frac{x - ut}{\sqrt{1-\frac{u^2}{c^2}}} \quad y' = y \quad z' = z \quad , \quad t' = \frac{t - \left(\frac{u}{c^2}\right)x}{\sqrt{1-\frac{u^2}{c^2}}}$$

Length contraction

O' [rod in O'] $\rightarrow u$

$O \rightarrow$ measures

O' measures $L_0 = x'_2 - x'_1$

O measures $L = x_2 - x_1$

definition of L_0

$\boxed{\text{For } O \text{ to make the measurement } O \text{ must measure at same time.}}$

→ lets say O measures a flash by O' at one end then the other

first flash: O' measures x'_1, t'_1 and O measures x_1, t_1

second flash: O' measures x'_2, t'_2 and O measures x_2, t_2

$$x'_1 = \frac{x_1 - ut_1}{\sqrt{1-\frac{u^2}{c^2}}} \qquad x'_2 = \frac{x_2 - ut_2}{\sqrt{1-\frac{u^2}{c^2}}}$$

same thats how you chose time measure it

O' must arrange flashes so they appear simultaneous

subtracting ...

$$x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1-\frac{u^2}{c^2}}} - \cancel{\frac{u(t_2 - t_1)}{\sqrt{1-\frac{u^2}{c^2}}}}$$

$$L = L_0\sqrt{1-\frac{u^2}{c^2}}$$

→ The length that the observer who doesn't need to measure at the same time measures

Time dilation

$\beta = \sqrt{1-\frac{u^2}{c^2}}$

$O' \rightarrow$ → imagine O' has a clock ticking

$O \rightarrow$ → each tick occurs at same place

$x'_2 - x'_1 = 0 \in$ definition of Δt_0!

$$t' = \frac{t - \frac{ux}{c^2}}{\beta}$$

$$x' = \frac{x - ut}{\beta} \rightarrow x = x'\beta + ut$$

thus $\Delta x = \cancel{\Delta x'\beta}^{\,0} + \Delta t\, u$

$$\Delta t_0 = \Delta t' = \frac{\Delta t - \frac{u\Delta x}{c^2}}{\beta}$$

plugging in

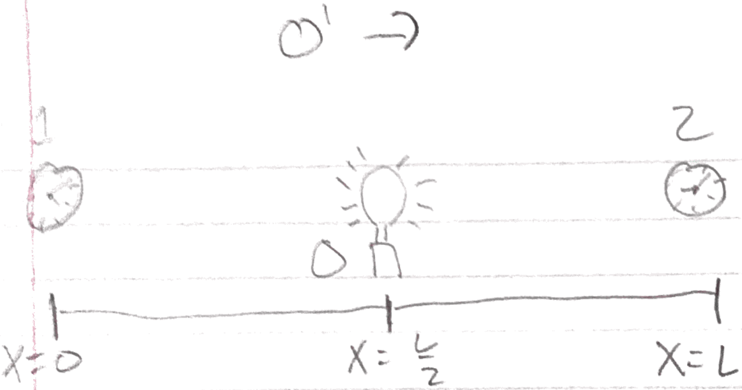
$$\beta\Delta t_0 = \Delta t - \frac{u\Delta x}{c^2} \rightarrow \beta\Delta t_0 = \Delta t\left(1 - \frac{u^2}{c^2}\right)$$

thus $$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}}}$$

<u>proper time!</u> is the time for observer, with speed u relative to another observer measures, and Δx between ticks is zero

<u>proper length!</u> is the length the observer at rest relative to an observer with speed u measures, and Δt between measurements is zero

Simultaneity & clock synchronization



important notes!

- location of O' doesn't matter but velocity does
- dir of velocity is import if he moves other way the order is reversed

→ flash of light starts two clocks simultaneously in O
→ O' measures 2 ahead of 1
heres why

light takes $\frac{L}{2c}$ to reach the clock

→ examining from moving observer O'

in RF O it measures:

clock [1] at $x_1 = 0$, $t_1 = \frac{L}{2c}$
clock [2] at $x_2 = L$, $t_2 = \frac{L}{2c}$

using lorentz transformation, we find O' observes clock 1 to receive

$$t_1' = \frac{t_1 - \left(\frac{u}{c^2}\right)x_1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{L/2c}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$t_2' = \frac{t_2 - \left(\frac{u}{c^2}\right)x_2}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{L/2c - (u/c^2)L}{\sqrt{1-\frac{u^2}{c^2}}}$$

thus t_2' is smaller than t_1' so clock 2 appears to receive signal earlier

$$\Delta t' = t_1' - t_2' = \frac{uL/c^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

→ O' observes both clocks to run slow due to time dilation
→ O' also observes clock 2 ahead of clock 1

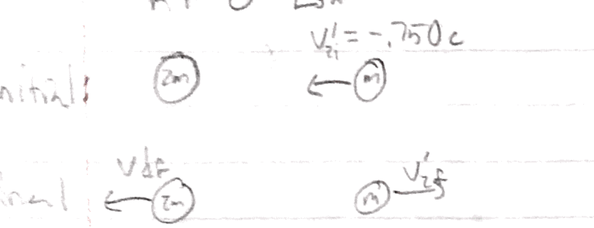
so 2 events simultaneous in one RF isn't in another

NO DERIVATIONS

Lime

Relativity with momentum and energy

RF O'

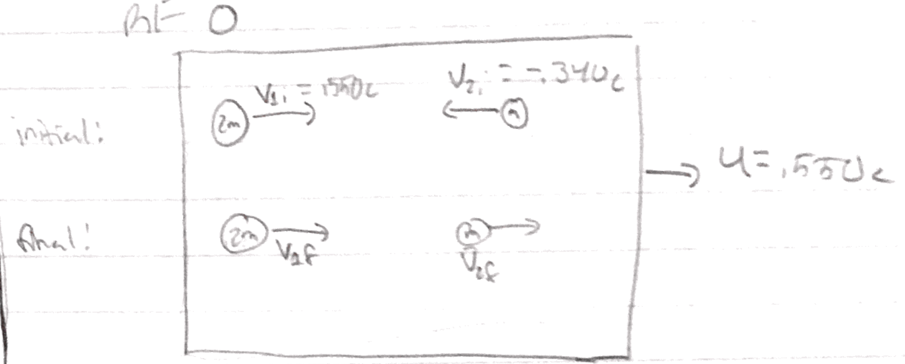


$p'_i = m_1 V'_{1i} + m_2 V'_{2i} = -.750mc$

$p'_f = m_1 V'_{1f} + m_2 V'_{2f} = -.750mc$

conserved

RF O



using velocity transformations to get right V's

$p_i = m_1 V_{1i} + m_2 V_{2i} = .760mc$

$p_f = m_1 V_{1f} + m_2 V_{2f} = .641mc$

not conserved

Have to find a new formula to satisfy Einstein.

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow p_x = \frac{mv_x}{\sqrt{1-\frac{v^2}{c^2}}} \quad p_y = \frac{mv_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

V is velocity of the particle as measured in inertial RF, its not u

O'

$$p'_i = \frac{m_1 V'_{1i}}{\sqrt{1-\frac{V'^2_{1i}}{c^2}}} + \frac{m_2 V'_{2i}}{\sqrt{1-\frac{V'^2_{2i}}{c^2}}} = \frac{2m(0)}{\sqrt{1-0}} + \frac{m(-.750c)}{\sqrt{1-(.750)^2}} = -1.134mc$$

conserved

$$p'_f = \frac{m_1 V'_{1f}}{\sqrt{1-V'^2_{1f}/c^2}} + \ldots = -1.134mc$$

Same for momentum in O

Energy → If you blindly apply old formulas to this example, it won't work

$$K = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \qquad K \to \infty \quad v \to c$$

or

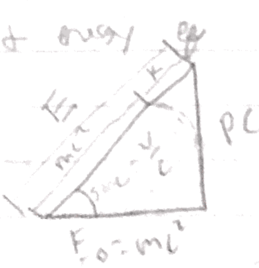
$$\boxed{K = E - E_0 \quad \text{and} \quad E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and} \quad E_0 = mc^2}$$

relativistic total energy

total energy measured in RF in which particle is at rest

manipulating momentum + energy

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$



$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$, $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$, $K = E - mc^2$ mass $= \frac{MeV}{c^2}$ $E = MeV$

$\rightarrow E^2 = (pc)^2 + (mc^2)^2$, $m_{invariant}^2 = \left(\frac{E}{c^2}\right)^2 - \left(\frac{p}{c^2}\right)$ $p = \frac{MeV}{c}$

very useful bec. velocities (are constant) but p & E are! $\rightarrow$

$E_{tot}^2 = m_{inv}^2 c^4 + p_{tot}^2 c^2$

$\boxed{\text{Relativistic energy and momentum}}$ → examples, examples.

Momentum of a proton w/ $v_p = .86c$ → $p = \frac{m_p v_p}{\sqrt{1-\frac{v_p^2}{c^2}}}$ → $pc = \frac{m_p c^2}{\sqrt{1-\frac{v_p^2}{c^2}}} \cdot \frac{v}{c}$ ← makes calc easy

$E_{0p^+} = m_p c^2 = 938\ MeV$

$pc = 1580\ MeV$ → $p = 1580\ \frac{MeV}{c}$ ← c's cancel out!

Relativistic Energy → $E_{tot} = \sqrt{(pc)^2 + (mc^2)^2} = \sqrt{(1580)^2 + (938)^2}$

$K = E_{tot} - E_{0p^+} = 899\ MeV$

$I_{sun} = 1.4 \cdot 10^3 \frac{W}{m^2}$ at earth

• at what rate the mass of the sun is decreasing

imagine the sphere with "r" so the total sun output is $(4\pi r^2)(I_{sun})$
$= 4 \cdot 10^{26}\ J/s$

sun loses energy by a loss in rest energy,

$\frac{\Delta E_0}{c^2} = \Delta m = 4.4 \cdot 10^9\ kg/s$ ← 4 billion kg every second.

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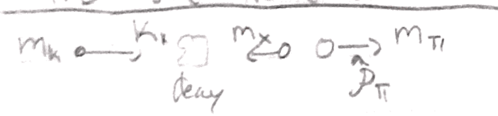
conservation laws in decays and collisions

→ a loss in kinetic energy is accompanied by a gain in rest energy

→ but the total relativistic energy (kinetic + rest energy) is constant. ($E = K + E_0$)

Ex 1



$K_K = 77\ MeV$ $m_\pi = 139.6\ \frac{MeV}{c^2}$

$m_K = 497.7\ \frac{MeV}{c^2}$ $p_\pi = 381.6\ MeV/c$

a) find momentum and total relativistic energy of unknown particle (m_x)

total energy → $E_K = K_K + m_K c^2 = 574.7\ MeV$ | (for π meson) | $E_\pi = \sqrt{(cp_\pi)^2 + (m_\pi c^2)^2} = 406.3\ MeV$

Kmeson and p_K $p_K c = \sqrt{E_K^2 - m_K c^2} = 287.4\ MeV/c$ now use momentum to solve problem

(initial) (final)

$p_K = p_\pi + p_x \Rightarrow p_x = p_K - p_\pi \rightarrow p_x = -94.2\ MeV/c$

$E_{initial} = E_{final} \rightarrow E_K = E_\pi + E_x \rightarrow E_x = E_K - E_\pi = 574.7 - 406.3$

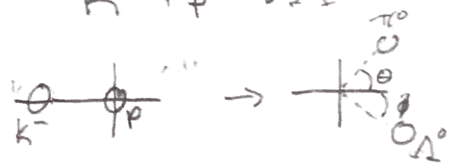
$E_x = 168.4\ MeV$

you can find mass by using $m_x c^2 = \sqrt{E_x^2 - (cp_x)^2} = 139.6\ \frac{MeV}{c^2}$

m_x = pi meson

$K^- + p \to \Lambda^0 + \pi^0$ $\quad m_p = 938.3\ \frac{MeV}{c^2}$ $\quad m_K = 938.3\ \frac{MeV}{c^2}$

$m_\Lambda = 1115.7\ \frac{MeV}{c^2}$ $\quad m_\pi = 135.0\ \frac{MeV}{c^2}$



ex2

$K_K = 152.4$ MeV $\quad K_\Lambda = ?$

$K_\pi = 254.8$ MeV $\quad \theta = ?$

$\phi = ?$

$E_{initial} = E_K + E_p = (K_K + m_K c^2) + m_p c^2$ ← proton at rest

$E_{final} = E_\Lambda + E_\pi = (K_\Lambda + m_\Lambda c^2) + (K_\pi + m_\pi c^2)$

$E_{initial} = E_{final}$ everything known except K_Λ

$K_\Lambda = 78.9$ MeV

to get direction information use conservation of mom

$p_{initial}$ = just m_K $\Rightarrow cp_K = \sqrt{E_K^2 - (m_K c^2)^2}$ and $E_K = K_K + m_K c^2$

↑ known ↑ known

$p_{initial} = p_K = 416.8\ \frac{MeV}{c}$

$cp_\Lambda = \sqrt{E_\Lambda^2 - (m_\Lambda c^2)^2} = 426.9$ and $cp_\pi = \sqrt{E_\pi^2 - (m_\pi c^2)^2}$

↑ $E_\Lambda = K_\Lambda + m_\Lambda c^2$ $\qquad$ ↑ $E_\pi = K_\pi + m_\pi c^2$

momentum of two particles is...

$p_{xf} = p_\Lambda \cos\theta + p_\pi \cos\phi$ $\qquad p_{yf} = p_\Lambda \sin\theta - p_\pi \sin\phi$

$p_{xi} = p_K$ $\qquad p_{yi} = 0$

set $p_{xf} = p_{xi}$

set $p_{yi} = p_{yf}$

$\phi = \cos^{-1}\left(\frac{p_K^2 + p_\pi^2 - p_\Lambda^2}{2 p_\pi p_K}\right) = 65.7°$ $\qquad \theta = \sin^{-1}\left(\frac{p_\pi \sin\phi}{p_\Lambda}\right) = 51.3°$

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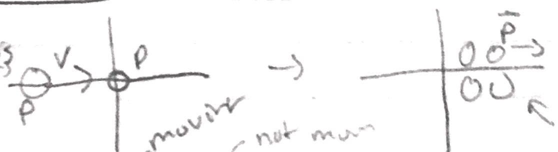
2 $\quad p + p \to p + p + p + \bar{p}$

find threshhold kinetic energy for this



each particle has same p + E_p'

← move as unit

total initial energy of 2 p is $E_p + m_p c^2$

has rest energy but unknown in momentum

total final energy of 4 part. $E_p' + E_p' + E_p' + E_p' = 4E_p'$

so: $E_p + m_p c^2 = 4E_p'$

conservation of momentum ⟶ $p_p + 0 = 4p_p' \to \sqrt{E_p^2 - (m_p c^2)^2} = 4\sqrt{E_p'^2 - (m_p c^2)^2}$

do algebra...

$E_p = 7 m_p c^2$

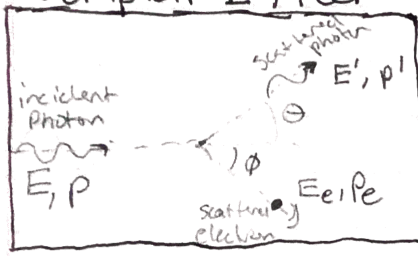
↓

$K_p = E_p - m_p c^2 = 6 m_p c^2$

$= 5.626$ GeV

Interaction of Photons with electrons, and atoms, and other combinations!

Compton Effect



photon: ← einstein's theory of quantized light

$E = hf = \frac{hc}{\lambda}$ and $p = \frac{E}{c}$ ← from $E^2 = (pc)^2 + (mc^2)^2$ photon   initial

$E' = hc/\lambda'$ $p' = E'/c$ and moves with angle θ final

electron:

total Energy $= E_e$, and tot momentum p_e with angle ϕ

rest energy $\rightarrow m_e c^2$ ✱ 3 eq & 4 unknowns

$E_{initial} = E_{final} \rightarrow E + m_e c^2 = E' + E_e$

$P_{x\,initial} = P_{x\,final} \rightarrow p = p_e \cos\phi + p'\cos\theta \rightarrow p_e\cos\phi = p - \cos\theta$

$P_{y\,initial} = P_{y\,final} \rightarrow 0 = p_e \sin\phi - p'\sin\theta \rightarrow p_e\sin\phi = p'\sin\theta$

...ugh ...n be ...l speed ...ok if it matters or why they did that

→ square both momentum eq →
$$\left[\begin{aligned} p_e^2\cos\phi &= p + (p')^2\cos^2\theta - 2pp'\cos\theta \\ p_e^2\sin\phi &= p_2^2\sin\theta \end{aligned}\right. \quad > \sin^2\phi + \cos^2\phi = 1$$

→ combine: $p_e^2 = p^2 + (p')^2 - 2pp'\cos\theta$

$\boxed{\text{now use Energy}}$ $E + m_e c^2 = E' + E_e \rightarrow E + m_e c^2 - E' = E_e = \sqrt{(p_e c)^2 + (m_e c^2)^2}$

Plug in and ALGEBRA!

then: $(E + m_e c^2 - E')^2 = (p_e c)^2 + (m_e c^2)^2$

$\rightarrow \frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$ or → $\boxed{\underset{\text{scattered}}{\lambda'} - \underset{\text{incident}}{\lambda} = \frac{h}{m_e c}(1 - \cos\theta)}$

$\boxed{\text{Also}}$

K_e of electron is whatever is left over between both photons

$K_e = E - E'$

recall x: $p = p_e\cos\phi + p'\cos\theta$
y: $p_e\sin\phi = p_2\sin\theta$

$\tan\phi = \frac{\sin\phi}{\cos\theta} \overset{\text{(algebra)}}{=} \frac{p'\sin\theta}{p - p'\cos\theta} \underset{E = pc}{\overset{\text{using}}{=}} \frac{E_2\sin\theta}{E_1 - E_2\cos\theta}$

ex. Xray w/ $\lambda = .2400\,nm$ are Compton scattered at $\theta = 60°$

Find: (a) (λ') scatter wavelength
b) energy of scatter X-ray photons
c) K_e & d) direction of electrons

(a) $\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta)$

b) $E' = \frac{hc}{\lambda'}$ -

c) $K_e = E - E'$

d) $\phi = \tan^{-1}\left(\frac{E'\sin\theta}{E - E'\cos\theta}\right)$

Photon + atom

atom release photon

$E_i = E_f + K + E$

initial energy of atom

final leftover energy of atom

recoil energy of atom

very small

photon energy emitted

has to have recoil energy because of conservation of momentum.

atom absorbs proton w/ Energy E.

$E_i + E = E_f + K$

Bremsstrahlung and X-ray Production

→ when an electron is accelerated it radiates electromagnetic Energy. Or emits photons

- beam of electrons accelerated by a ΔV, thus experience loss in pot. eng. $-e\Delta V$, gain $K = e\Delta V$
- when they hit a target they make many collisions with the atoms
- momentum transfered to atom, electron slow, Photon emitted

(recoil energy of atom so small can be neglected)

→ K_e of electron before collision, K' of electron after.

→ photon energy $E = hf = \frac{hc}{\lambda}$

$K_e - K' = \text{leftover energy} = \frac{hc}{\lambda}$

electron will make many collisions emitting λ from target λ (small energy) top to $hf_{max} = K$

meaning electron loses all its energy K in a single encounter.

→ the smallest wavelength is determined by max Energy loss

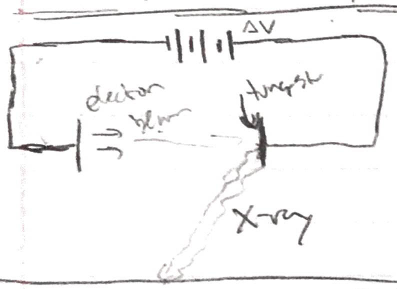
$K_e - K' = \frac{hc}{\lambda}$ → $K_e = \frac{hc}{\lambda_{min}}$

no energy after it loses all its K_e

$\lambda_{min} = \frac{hc}{K_e}$

$K_e = e\Delta V$

$\boxed{\lambda_{min} = \frac{hc}{e\Delta V}}$



"inverse photoelectric effect"

Pair Production and Annihilation. Photon → electron + positron or electron + positron → photon

$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_{ph_1} + E_{ph_2}$

for positron + electron at rest → $E_{ph_1} = E_{ph_2} = m_e c^2$

why 2 photons? momentum conservation requires at least two photons

Questions

- why are photons emitted
- can you create a machine that "cancels out" light waves
- why don't light all around mess with each other