Quantum Prep and Motivation

(complex numbers and Probability Density Functions)

Motivation

photo electric effect + particle nature of light → In the photo electric effect when light comes in and releases electrons, classically if you increase the intensity the electrons will "pop off harder", in reality just more electrons come off, so thinking about light as a particle fits better

↳ the strongest motivation however is the ultra-violet catastrophe → when looking at the energy density for black body
↳ classically w/ no probability assumptions it was wrong

↳ classically → discrete, no probabilistic nature

↳ quantum → particles have a boltzmann distribution, $\sum P_n = 1$ (n = energy levels) e you'll find particles in all energy levels
↳ then find average energy level to get energy density

- De broglie → said that particles have an intrinsic wave nature, has been shown in diffraction experiments

$\lambda = \frac{h}{p}$ (for particles)

↳ small momentum = $\underbrace{\text{large wavelength}}_{\text{uncertain location}}$, big p = $\underbrace{\text{small } \lambda}_{\text{know its location better}}$

there is this trade off between momentum and position uncertainties.
↳ no new thing for waves, but these are particles behaving this way!

Heisenberg Uncertainty Principle

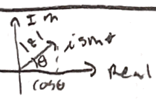
$$p = \frac{h}{\lambda} \rightarrow dp = h\frac{d\lambda}{\lambda^2} \rightarrow \Delta p = \frac{h\Delta\lambda}{\lambda^2}$$

now use $\Delta x \Delta \lambda = \varepsilon \lambda^2$ ← derivation in 213 notes: comes from wave not having a defined position

↳ $\Delta x \Delta p \frac{\cancel{\lambda^2}}{h} \sim \varepsilon \cancel{\lambda^2}$

$$\boxed{\Delta x \Delta p = \varepsilon h}$$

Complex numbers



$$z = a + ib = |z|[\cos\theta + i\sin\theta] = |z|e^{i\theta}$$

$$z^* = a - ib = |z|[\cos\theta - i\sin\theta]$$

$$= |z|[\cos(-\theta) + i\sin(-\theta)] = |z|e^{-i\theta}$$

, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$,

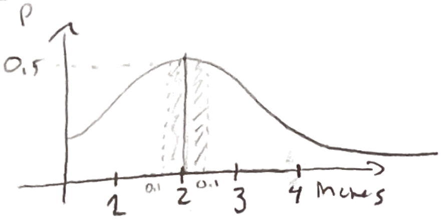
$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$|z_1 z_2| = |z_1||z_2|$$

$zz^* = a^2 + b^2$: magnitude

Probability Density Function

ex Y = exact amount of rain tommorrow



What is the probability that there is 2 in of rain? , $P(Y=2)=0.5$

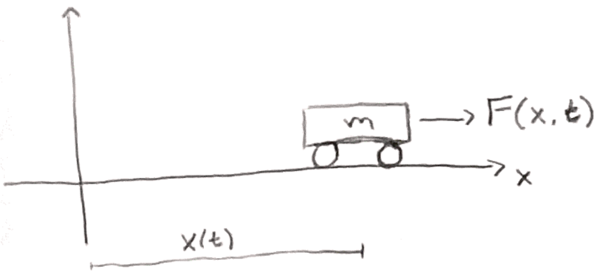
But this continuous & probability of getting an age at exactly 2 in is 0.

$P(1.9 < Y < 2.1)$

$= \int_{1.9}^{2.1} f(x)\,dx$

w discrete you do $\frac{N(j)}{N} = P$, so you'd look to the height of P

Schrödinger Equation

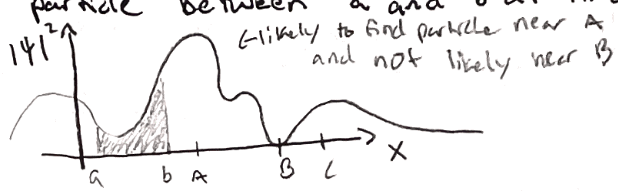


* classically we want to find the EOM to find $x(t)$.
  - ↳ we normally use $F = ma$
  - ↳ $F = -\frac{\partial V}{\partial x}$ on microscopic levels
  - ↳ $m\ddot{x} = -\frac{\partial V}{\partial x}$, this w/ initial cond determines x

* In quantum mechanics we look for a particles <u>wave function</u> instead of position
  - ↳ we use the Schrödinger eq to do so → $\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi}$, $\hbar = \frac{h}{2\pi} = 1.05 \cdot 10^{-34}\ \mathrm{J \cdot s}$
  - ↳ initial cond → $\psi(x,0)$ ↗ determines $\psi(x,t)$

[You can perform a mathematical operation on ψ to make it useful] → $\boxed{\int_a^b |\psi(x,t)|^2 dx}$

* gives you the probability of finding a particle between a and b at time t



Probability

Ex) People in a room

$N(j)$ = # number of people at age j

1 person = 14 yrs	→ $N(14) = 1$
1 person = 15 yrs	→ $N(15) = 1$
3 person = 16	→ $N(16) = 3$
2 person = 22	→ $N(22) = 2$
2 person = 24	→ $N(24) = 2$
5 person = 25	→ $N(25) = 5$

$N_{tot} = \sum_{j=0}^{\infty} N(j) = 14$ people

Q: Probability of picking someone who is 15? $P(15) = \frac{1}{14}$, $P(16) = \frac{3}{14}$

In general: $P(j) = \frac{N(j)}{N}$ and $\sum_{j=0}^{\infty} P(j) = 1$

Average age

$$\frac{1(14)+1(15)+3(16)+2(22)+2(24)+5(25)}{14} = \frac{294}{14} = 21$$

in general $\boxed{\langle j \rangle = \frac{\sum j N(j)}{N} = \sum j P(j)}$

$\langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$, in general $\langle f(j) \rangle = \sum_{j=0}^{\infty} f(j) P(j)$

Deviation from the mean

$\Delta j = j - \langle j \rangle \rightarrow \langle \Delta j \rangle = \sum (j - \langle j \rangle) P(j) = \underbrace{\sum j P(j)}_{\langle j \rangle} - \langle j \rangle \underbrace{\sum P(j)}_{1} = 0$

[• got 0, gross, bec the dist will always be even about the mean, so lets make it an absolute value, those are hard to deal w/ so lets do the square]

plugging in and simplifying

$\boxed{\sigma^2 = \langle (\Delta j)^2 \rangle = \langle j^2 \rangle - \langle j \rangle}$, $\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle}$

Continuous Probability

$$P_{ab} = \int_a^b \rho(x)\,dx$$ ← continuous version of the discrete example, instead of doing just the height / # of people we find the area

$$\langle x \rangle = \int_{-\infty}^{\infty} x\rho(x)\,dx$$

Ex: Drop a rock off a cliff, what is the average of all the distances? (or time average of the distance traveled)

$X(t) = \frac{1}{2}gt^2$, $\frac{dx}{dt} = gt$ and the time it takes to hit ground $T = \sqrt{\frac{2h}{g}}$

[Probability that the camera flashes in interval dt] → $\frac{dt}{T} = \frac{dx}{gt}\sqrt{\frac{g}{2h}} = \underbrace{\frac{1}{2\sqrt{hx}}}_{\rho(x)}dx$, $\rho(x) = \frac{1}{2\sqrt{hx}}$ $(0 \le x \le h)$

analogous to $\frac{N(j)}{N}$ but inf small interval, not discrete, so integrate to get full

$\int_0^h \frac{1}{2\sqrt{hx}}dx = 1$ checks out and $\langle x \rangle = \int_0^h \frac{x}{2\sqrt{hx}} = \frac{1}{2\sqrt{h}}\left(\frac{2}{3}x^{3/2}\right)\Big|_0^h = \frac{h}{3}$

$\rho = |\Psi(x,t)|^2$ for quantum $\quad i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$

$\boxed{\text{Normalizing } \Psi}$, Ψ = a solution and $A\Psi$ = solution also so we can always normalize or make it so $\int|\Psi|^2 = 1$ is true

What if you normalize at $t=0$ will it stay normalized?

$$\frac{d}{dt}\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = \int_{-\infty}^{\infty}\frac{\partial}{\partial t}|\Psi(x,t)|^2dx \rightarrow \frac{\partial}{\partial t}|\Psi|^2 = \frac{\partial}{\partial t}(\Psi^*\Psi) = \Psi^*\frac{\partial\Psi}{\partial t} + \frac{\partial\Psi^*}{\partial t}\Psi$$

Schrodinger eq:

$$\frac{\partial\Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2} - \frac{i}{\hbar}V\Psi \quad \text{and} \quad \frac{\partial\Psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2\Psi^*}{\partial x^2} + \frac{i}{\hbar}V\Psi^*$$

Now plug in!

← if you expand out by doing $\partial/\partial x$ two x terms will cancel

$$\frac{\partial|\Psi|^2}{\partial t} = \frac{i\hbar}{2m}\left(\Psi^*\frac{\partial^2\Psi}{\partial x^2} - \frac{\partial^2\Psi^*}{\partial x^2}\Psi\right) = \frac{\partial}{\partial x}\left[\frac{i\hbar}{2m}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \frac{\partial\Psi^*}{\partial x}\Psi\right)\right]$$

using F.T.C.

$$\frac{d}{dt}\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = \frac{i\hbar}{2m}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \frac{\partial\Psi^*}{\partial x}\Psi\right)\Big|_{-\infty}^{\infty}$$

The Ψ must go to zero as $x \to \pm\infty$, it has to exist somewhere,

$$\frac{d}{dt}\int_{-\infty}^{\infty}|\Psi|^2dx = 0$$, integral is a constant, independent of time

.omentum

or a particle in state ψ, $\langle x \rangle = \int_{-\infty}^{\infty} x |\psi(x,t)|^2 dx$

Remember taking a measurement collapses the wave function to a spike
↳ so you need a bunch of particles at state ψ or be able to return the particle to ψ

ψ is time dependent so over time $\langle x \rangle$ will change.

← two terms cancel when you expand

$$\frac{d\langle x \rangle}{dt} = \int x \frac{\partial}{\partial t}|\psi|^2 dx = \frac{i\hbar}{2m}\int x \frac{\partial}{\partial x}\left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x}\psi\right)dx$$

Now use integration by parts: $\int_a^b f \frac{dg}{dx} dx = -\int_a^b \frac{df}{dx} g \, dx + fg\Big|_a^b$

$$\frac{d\langle x \rangle}{dt} = -\frac{i\hbar}{2m}\int \underset{\frac{df}{dx}}{1} \cdot \left(\psi^* \frac{\partial \psi}{\partial x} - \underbrace{\frac{\partial \psi^*}{\partial x}\psi}_{\text{integrate this by parts}}\right)dx + x(\psi \text{ junk} \ldots)\Big|_{-\infty}^{\infty}$$

ψ is 0 at infinity

$$\frac{d\langle x \rangle}{dt} = -\frac{i\hbar}{2m}\left[\int \psi^* \frac{\partial \psi}{\partial x} dx + \int \psi^* \frac{\partial \psi}{\partial x} dx + 0\right] = \boxed{-\frac{i\hbar}{m}\int \psi^* \frac{\partial \psi}{\partial x} dx}$$

↳ $\langle p \rangle = m\frac{d\langle x \rangle}{dt} = -i\hbar \int \psi^* \frac{\partial \psi}{\partial x} dx$

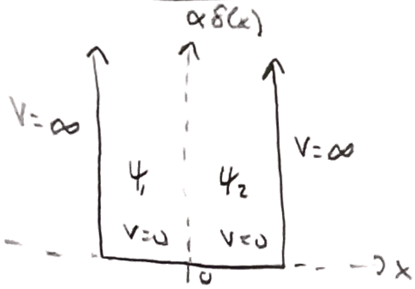
$$\boxed{\begin{aligned} \langle x \rangle &= \int \psi^* x \psi \, dx \\ \langle p \rangle &= \int \psi^* \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi \, dx \end{aligned}}$$

operates on ψ, to calculate $T = \frac{p^2}{2m}$ or $L = \vec{r} \times \vec{p}$
we replace every p by $\frac{\hbar}{i}\frac{\partial}{\partial x}$ + put between ψ^* and ψ

$$\langle T \rangle = -\frac{\hbar^2}{2m}\int \psi^* \frac{\partial^2}{\partial x^2}\psi \, dx$$

$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$

Or just do it generally uhg



$\psi_1(x) = A\sin(kx) + B\cos(kx) \qquad -a < x < 0$

$\psi_2(x) = C\sin(kx) + D\cos(kx) \qquad 0 < x < a$

Boundary conditions

(1) $\psi_1(-a) = 0 \rightarrow -A\sin(ka) + B\cos(ka) = 0 \xrightarrow{\text{Using } (3)} -A\sin(ka) + B\cos(ka) = 0$

(2) $\psi_2(a) = 0 \rightarrow C\sin(ka) + D\cos(ka) = 0 \xrightarrow{\text{using } (3)} C\sin(ka) + B\cos(ka) = 0$

(3) $\psi_1(0) = \psi_2(0) \rightarrow B = D$

(4) $-\frac{\hbar^2}{2m}\int_{-\varepsilon}^{+\varepsilon} \frac{d^2\psi}{dx^2}dx + \int_{-\varepsilon}^{+\varepsilon} \alpha\delta(x)\psi dx = 0 \rightarrow \left.\frac{d\psi}{dx}\right|_{-\varepsilon}^{+\varepsilon} = \frac{2m}{\hbar^2}\alpha\psi_1(0)$

$\hookrightarrow \left.\frac{d\psi_2}{dx}\right|_0 - \left.\frac{d\psi_1}{dx}\right|_0 = Ck - Ak = \frac{2m}{\hbar^2}\alpha B \rightarrow C - A = \frac{2m\alpha}{k\hbar^2}B$

Plug (1) into (2) $\rightarrow C\sin(ka) + A\sin(ka) = 0 \rightarrow (C+A)\sin(ka) = 0$

Either $C = -A$ or $\sin(ka) = 0$;

$\hookrightarrow \boxed{ka = n\pi}$

$B = 0$

$\hookrightarrow A = C$

$-A\sin(ka) + B\cos(ka) = 0$
$-A\sin(ka) + B\cos(ka) = 0$ redundant

$\hookrightarrow$ just use this one $\rightarrow B = A\tan(ka)$

(4) $-2A = \frac{2m\alpha}{k\hbar^2} A\tan(ka)$

$$\boxed{\tan(ka) = -\frac{k\hbar^2}{m\alpha}}$$

can also solve (1) for $B = A\tan(ka)$

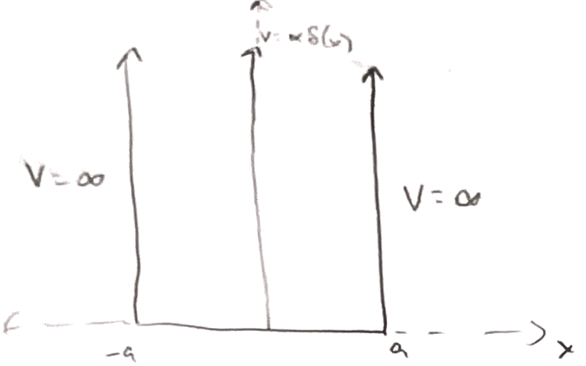
and plug into (4)

do same for (2) solve for $\tan(ka)$, plug into (4)

get ratio C/A and A/C

$C^2 = A^2$

$C = \pm A$



Inside the well you will have a solution of the form $\psi(x) = \gamma\cos(kx) + \beta\sin(kx)$ but we know there will be even and odd solutions so lets use that info directly.

$$\psi_{even}(x) = \begin{cases} A\sin(kx) + B\cos(kx) & \text{for } x<0 \\ -A\sin(kx) + B\cos(kx) & \text{for } x>0 \end{cases}$$

← general wave function

← plug $x<0$ explicitly bec $\psi(x) = \psi(-x)$

<u>Check:</u> (needs to be true for all x)

$\psi = \psi(-x)$

[If you swap these then you have a change in the sign of A]

$\psi = A\sin(kx) + B\cos(kx) \rightarrow \psi(-x) = -A\sin(kx) + B\cos(kx)$

→ $x>0$: $\psi = -A\sin(kx) + B\cos(kx)$

→ $x<0$: $\psi = A\sin(kx) + B\cos(kx)$

[Now you can take derivatives of $+\varepsilon, -\varepsilon$ and $\psi(0)=\psi(0)$, $\psi(a)=0$, same as $\psi(-a)=0$] → with little to no shenanigans

Lets do ψ_{odd}!

$\psi(x) = -\psi(-x)$

?

$\psi = A\sin(kx) + B\cos(kx) \rightarrow$ apply $-\psi(-x) \rightarrow \psi = A\sin(kx) - B\cos(kx)$

→ $x>0$: $\psi = A\sin(kx) - B\cos(kx)$

→ $x<0$: $\psi = -A\sin(kx) - B\cos(kx)$

Chapter 2

Schrodinger equation

getting $\Psi(x,y)$: $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$

$\Psi(x,t) = \psi(x)\phi(t)$ (seperation of variables) * pg 25 for why seperation of variables

plug in seperation of variables → $\frac{\partial \Psi}{\partial t} = \psi \frac{d\phi}{dt}$, $\frac{\partial^2 \Psi}{\partial x^2} = \frac{d^2\psi}{dx^2}\phi$

$i\hbar \frac{d\phi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\phi + V\psi\phi$

↳ divide by $\psi\phi$ → $\underbrace{i\hbar \frac{1}{\phi}\frac{d\phi}{dt}}_{\text{just } t} = \underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V}_{\text{just } x}$

[Each can be equal to their own constant] $E = \frac{i\hbar}{\phi}\frac{d\phi}{dt}$, $\frac{d\phi}{dt} = -\frac{iE}{\hbar}\phi$

↳ plug back into Schrodinger eq → $E = V - \frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}$ → $\boxed{E\psi = V\psi - \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}}$

Time dependent

$\frac{d\phi}{dt} = \frac{\phi E}{i\hbar}$ → $\frac{1}{\phi}d\phi = \frac{-iE}{\hbar}dt$ → $\phi = e^{-iEt/\hbar} = \cos(\frac{Et}{\hbar}) + i\sin(\frac{Et}{\hbar})$

• seperation is good because for probability density function, time dependence falls out

$|\Psi(x,t)|^2 = \Psi(x,t)^*\Psi(x,t) = |\psi(x)|^2$ cuz $\phi = e^{-iEt/\hbar}$

Hamiltonian

$H(x,p) = \frac{p^2}{2m} + V(x)$ → $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ → $\hat{H}\psi(x) = E\psi(x)$

$\langle H \rangle = \int \psi(x)^*\hat{H}\psi(x)\,dx = \int \psi^* E\psi\,dx = E\int|\psi(x,t)|^2 dx = E$ (normalize it)

→ $\hat{H}^2\psi(x) = \hat{H}(\hat{H}\psi) = E^2$

→ $\langle H^2 \rangle = \int \psi^*\hat{H}^2\psi\,dx = E^2 \overbrace{\int|\psi|^2 dx}^{1} = E^2$ → $\sigma_H^2 = E^2 - E^2 = 0$

Linear combination

• time independent leads to inf soln, $\psi_1, \psi_2, \psi_3, \dots$ w/ $E_1, E_2, E_3, \dots$

time dependent has property that any linear combination is a solution, Ψ.

↳ general solution $\Psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)e^{-iE_nt/\hbar}$ pg 29 for more info

$|c_n|^2$ is the probability that a measurement of the energy would return E_n , $\sum_{n=1}^{\infty}|c_n|^2 = 1$

$\langle H \rangle = \sum_{n=1}^{\infty}|c_n|^2 E_n$

Ex particle is a linear combo of two states
stationary state, just depends on x
$\Psi(x,0) = c_1\psi_1(x) + c_2\psi_2(x)$ → find for all times t

↳ $\Psi(x,t) = c_1\psi_1(x)e^{-iE_1t/\hbar} + c_2\psi_2 e^{-iE_2t/\hbar}$ → you can see how the prob density fluctuates

$|\Psi(x,t)|^2 = (c_1\psi e^{iE_1t/\hbar} + c_2\psi_2 e^{iE_2t/\hbar})(c_1\psi_1 e^{-iE_1t/\hbar} + c_2\psi_2 e^{-iE_2t/\hbar})$

$= c_1^2\psi_1^2 + c_2^2\psi_2^2 + 2c_1c_2\psi_1\psi_2\cos[(E_2-E_1)t/\hbar]$ ← not a stationary state

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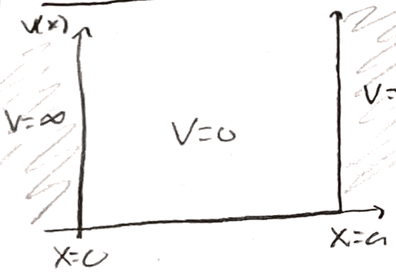
Infinite square well

$V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$, outside the well $\psi(x)=0$



inside

$k^2 = \frac{\sqrt{2mE}}{\hbar}$

$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$ → $\frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi$ → $\frac{d^2\psi}{dx^2} = -k^2\psi$

[assume a solution of] → $\psi(x) = A\sin(kx) + B\cos(kx)$, and $\psi(0) = \psi(a) = 0$

↳ $\psi(0) = A\sin(0) + B\cos(0) = 0$ → $B = 0$ and $\psi(a) = A\sin(ka) = 0$ so $ka = n\pi$

↳ $\psi(x) = A\sin\left(\frac{n\pi}{a}x\right)$ we also know $k = \frac{\sqrt{2mE}}{\hbar}$ (from our diff eq we assumed from)

↳ $E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$] integer values of E, not continuous

Normalization

$\int_0^a |A|^2\sin^2(kx)\,dx = |A|^2\frac{a}{2} = 1$ → $|A|^2 = \frac{2}{a}$ → $\boxed{\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$

Can a particle be found in more than one state?

always integer so = 0   always integer so = 0

$\int \psi_m^*\psi_n(x)\,dx = \frac{2}{a}\int_0^a \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}x\right)dx = \frac{1}{\pi}\left[\frac{\sin((m-n)\pi)}{m-n} - \frac{\sin((m+n)\pi)}{m+n}\right] = 0$

Only works if $m \ne n$, → $\int \psi_m^*\psi_n\,dx = \delta_{mn}$, we say ψ are orthonormal

...finite well

We found:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right), \quad E_n = \frac{(n\pi\hbar)^2}{2ma^2} \quad \text{and} \quad \int \psi_m^* \psi_n \, dx = \delta_{mn}$$

Completeness - any function can be expressed as a linear combination of:

$$f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x) = \sqrt{\frac{2}{a}}\sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{a}x\right) \quad \text{(fourier series)}$$

a trick

when $n=m$
$\delta = 1$

$$\int \psi_m^*(x) f(x)\,dx = \sum_{n=1}^{\infty} c_n \int \psi_m^* \psi_n \, dx = \sum c_n \delta_{mn} = c_m$$

[then the n^{th} coeff] $\rightarrow$ $c_n = \int \psi_n^* f(x)\,dx$

Four properties

1. even and odd, ψ_1 is even, ψ_2 is odd and so on (true if V is symetric)
2. as you go up in energy each state has one more node ψ_1 has none, ψ_2 has one (universal)
3. Orthonormal because $\int \psi_m^* \psi_n \, dx = \delta_{mn}$ (proof of general in chapter 3)
4. Completness, any function can be represented as a linear combo ~~of~~ $c_n\psi_n$

$$\Psi(x,t) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right) e^{-i(n^2\pi^2\hbar/2ma^2)t} \quad (\psi(x)\phi(t))$$

$\hookrightarrow$ [most general solution] $\rightarrow$ $$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right) e^{-i(n^2\pi^2\hbar/2ma^2)t}$$

Initial cond $\rightarrow$ $\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi(x)$] same form as $f(x) = \sum_{n=1}^{\infty} c_n\psi_n$ which is complete

use $\rightarrow$ $c_n = \int \psi_n^* f(x) = \sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\Psi(x,0)\,dx$

c_n is the coefficient on our linear combination which is always a solution $\rightarrow$ fundamental principle of QM.

$\hookrightarrow$ using other mathematical tools we have found ways to calculate c_n

ex $\Psi(x,0) = Ax(a-x)$, $0 \le x \le a$)

4

normalize

$$1 = \int_0^a |\Psi(x,0)|^2 = A^2 \int_0^a x^2(a-x)^2 dx \rightarrow A = \sqrt{\frac{30}{a^5}}$$

Now use: $$c_n = \int \psi_n^* f(x) = \int_0^a \underbrace{\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)}_{\text{general solution for an infinite well}} \overbrace{\sqrt{\frac{30}{a^5}}\, x(a-x)}^{\text{functional form of } \Psi(x,0)} dx$$

functional form of $\Psi(x,0)$, in which can be by a superposition of solutions inf well

$$= \frac{4\sqrt{15}}{(n\pi)^3}\left[\overset{1}{\cos(0)} - \overset{\pm 1}{\cos(n\pi)}\right] = \begin{cases} 0 & n \text{ even} \\ \frac{8\sqrt{15}}{(n\pi)^3} & n \text{ odd} \end{cases}$$

(general solution) $\Psi(x,t) = \sqrt{\frac{30}{a}}\left(\frac{2}{\pi}\right)^3 \sum_{n=1,3,5} \frac{1}{n^3} \sin\left(\frac{n\pi}{a}x\right) e^{-in^2\pi^2\hbar t/2ma^2}$

Proof of normalization of $|c_n|^2$:

$$1 = \int |\Psi(x,0)|^2 dx = \int \left(\sum_{m=1}^{\infty} c_m \psi_m(x)\right)^* \left(\sum_{n=1}^{\infty} c_n \psi_n(x)\right) dx$$ pull sums & constants out

$$= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_m^* c_n \int \psi_m^* \psi_n dx = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_m^* c_n \delta_{mn} = \sum_{n=1}^{\infty} |c_n|^2$$

Connection w/ Total energy:

$\hat{H}\psi_n = E_n \psi_n$

uses than $\Psi = c_n \psi_n$ of stationary states

$$\langle H \rangle = \int \Psi^* \hat{H} \Psi dx = \int \left(\sum c_m \psi_m\right)^* \hat{H}\left(\sum c_n \psi_n\right) dx = \sum\sum c_m^* c_n E_n \int \psi_m^* \psi_n dx$$

$$\langle H \rangle = \sum |c_n|^2 E_n$$

Probability of probing the ground state: $\Psi(x, t=0) = A(\psi_1 + 2\psi_2)$

$\sum |c_n|^2 = 1$ $\quad \hookrightarrow A^2 + 4A^2 = 1 \rightarrow A = \frac{1}{\sqrt{5}}$

$c_1 = \frac{1}{\sqrt{5}} \rightarrow c_1^2 = \frac{1}{5}$

probability

4, Linear combo? Tieing it all together hopefully

(5)

time independent schrodinger eq → inf amount of solutions, $\psi_1(x), \psi_2(x), \psi_3(x), \cdots \psi_n(x)$

$$\Psi_n(x,t) = \psi_n \underbrace{e^{-iE_nt/\hbar}}_{\phi_n(t)}$$

After given a $V(x)$ and $\Psi(x,0)$ we want to find $\Psi(x,t)$
↳ first get $\psi_n(x)$ from TISE
↳ has its own associated energies

[Any linear combination of itself is a solution] → $\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_nt/\hbar}$

↑ with an associated constant

$$\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

$\sum_{n=1}^{\infty} |c_n|^2 = 1$, $|c_n|^2 =$ probability that a measurement of the energy would return the value E_n

probability of get a particular value of E_n

↓

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \overbrace{\psi_n(x) e^{-iE_nt/\hbar}}^{\Psi_n(x,t)}$$

Infinite Well

$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$, orthogonality → $\int \psi_m^* \psi_n \, dx = \delta_{mn}$

Completeness

$f(x) = \sum c_n \psi_n(x) = \Psi(x,0)$; any other function $f(x)$ can be written as a linear combination of ψ_n w/ c_n [proof is it expanded $f(x)$ as an inf sum of sine functions]

↳ [multiply both sides by ψ_m^* and integrate] $\boxed{\int \psi_m^* f(x)\,dx} = \sum_{n=1}^{\infty} c_n \int \psi_m^* \psi_n \, dx = \sum_{n=1}^{\infty} c_n \delta_{mn} = \boxed{c_m}$

↳ $\boxed{c_n = \int \psi_n^* f(x)\,dx}$ $\xrightarrow[f(x) = \Psi(x,0)]{\text{recall}}$ $c_n = \sqrt{\frac{2}{a}} \int_0^a \sin\left(\frac{n\pi}{a}x\right) \Psi(x,0)\,dx$

Full solution?

$$\Psi_n(x,t) = \psi_n(x)\,\phi(t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) e^{\frac{-iE_nt}{\hbar}}$$

↳ $$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) e^{-iE_nt/\hbar}$$

Harmonic Oscillator

Classically: $F = -kx = m\frac{d^2x}{dt^2}$, $x(t) = A\sin(\omega t) + B\cos(\omega t)$, $\omega \equiv \sqrt{\frac{k}{m}}$

↳ more importantly the potential → $V(x) = \frac{1}{2}kx^2$ → $V(x) = \frac{1}{2}m\omega^2x^2$

Quantum Problem

↳ solve SE for $V(x) = \frac{1}{2}m\omega^2x^2$

$\hat{p} \equiv -i\hbar\frac{d}{dx}$

↳ $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$ → rewrite: $\frac{1}{2m}\left[\hat{p}^2 + (m\omega x)^2\right]\psi = E\psi$

like if you did $u^2+v^2 = (iu+v)(-iu+v)$

[Going to define an operator that will show its use] → $\hat{a}_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x)$

makes final result nicer

$\hat{a}_-\hat{a}_+ = \frac{1}{2\hbar m\omega}(i\hat{p} + m\omega x)(-i\hat{p} + m\omega x)$

$= \frac{1}{2\hbar m\omega}\left[\hat{p}^2 + (m\omega x)^2 - im\omega(x\hat{p} - \hat{p}x)\right]$

commuter of x, p

$[\hat{x},\hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} \neq 0$ cuz $\hat{x}\hat{p} \neq \hat{p}\hat{x}$

$\hat{a}_-\hat{a}_+ = \frac{1}{2\hbar m\omega}\left[\hat{p}^2 + (m\omega x)^2\right] - \frac{i}{2\hbar}[\hat{x},\hat{p}]$

* have a test function, operators are slippery *

$[x,\hat{p}]f(x) = \left[x(-i\hbar)\frac{d}{dx}f - (-i\hbar)\frac{d}{dx}(xf)\right] = -i\hbar\left(x\frac{df}{dx} - x\frac{df}{dx} - f\right) = i\hbar f(x)$

↳ $\boxed{[\hat{x},\hat{p}] = i\hbar}$

$\hat{a}_-\hat{a}_+ = \frac{1}{\hbar\omega}\hat{H} + \frac{1}{2}$ or $\hat{H} = \hbar\omega(\hat{a}_-\hat{a}_+ - \frac{1}{2})$

↳ (if we redid what we just did) → for $\hat{a}_+\hat{a}_- = \frac{1}{\hbar\omega}\hat{H} - \frac{1}{2}$ $\xrightarrow{\text{then}}$ $[\hat{a}_-,\hat{a}_+] = 1$

$\hat{H} = \hbar\omega(\hat{a}_+\hat{a}_- + \frac{1}{2})$ → in $\hat{H}\psi = E\psi$ → $\hbar\omega(\hat{a}_\pm\hat{a}_\mp \pm \frac{1}{2})\psi = E\psi$

$\hat{H} = \hbar\omega(\hat{a}_-\hat{a}_+ - \frac{1}{2})$

↳ claim: if ψ satisfies schrodinger equation w/ energy E, $\hat{H}\psi = E\psi$

→ $\hat{a}_+\psi$ satisfies schrodingers w/ $(E+\hbar\omega)$ → $\hat{H}(\hat{a}_+\psi) = (E+\hbar\omega)(\hat{a}_+\psi)$

→ $\hat{a}_-\psi$ satisfies w/ $(E-\hbar\omega)$

↳ Proof

Proof from $\hat{H}\psi = E\psi \rightarrow \hat{H}(\hat{a}_+\psi) = (E+\hbar\omega)(\hat{a}_+\psi)$ $\{ \hat{a}_+(\hat{a}_-\hat{a}_+) = (\hat{a}_+\hat{a}_-)\hat{a}_+$

$\hat{H}(\hat{a}_+\psi) = \hbar\omega(\hat{a}_+\hat{a}_- + \frac{1}{2})(\hat{a}_+\psi) = \hbar\omega(\hat{a}_+\hat{a}_-\hat{a}_+ + \frac{1}{2}\hat{a}_+)\psi = \hbar\omega\hat{a}_+(\hat{a}_-\hat{a}_+ + \frac{1}{2})\psi$

* use $\hat{a}_-\hat{a}_+ = \hat{a}_+\hat{a}_- + 1$ (commutator relation) *

$\hookrightarrow \hat{H}(a_+\psi) = \hat{a}_+[\hbar\omega(\hat{a}_+\hat{a}_- + 1 + \frac{1}{2})\psi] = \hat{a}_+(\hat{H} + \hbar\omega)\psi = \hat{a}_+(E+\hbar\omega)\psi$ $\quad \hat{H}\psi = E\psi$

$\hookrightarrow \boxed{\hat{H}(a_+\psi) = (E+\hbar\omega)(\hat{a}_+\psi)}$ → by same logic $\boxed{\hat{H}(\hat{a}_-\psi) = (E-\hbar\omega)(\hat{a}_-\psi)}$

$\hat{a}_\pm$ = ladder operators, allows us to climb up and down in energy

→ you cant lower forever, $\hat{a}_-\psi_0 = 0$ (lowest rung)

→ use this to determine $\psi_0(x)$

$\hookrightarrow \frac{1}{\sqrt{2\hbar m\omega}}(\hbar\frac{d}{dx} + m\omega x)\psi_0 = 0 \rightarrow \frac{d\psi_0}{dx} = -\frac{m\omega}{\hbar}x\psi_0$ (separate)

$\hookrightarrow \boxed{\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}}$ → (normalize this bitch) $1 = |A|^2\int_{-\infty}^{\infty} e^{-\frac{m\omega x^2}{\hbar}} = |A|^2\sqrt{\frac{\pi\hbar}{m\omega}}$

$\hookrightarrow \boxed{\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}}$ determine → E → $\hbar\omega(\hat{a}_+\hat{a}_- + \frac{1}{2})\psi_0 = E_0\psi_0$ $\quad$ * $\hat{a}_-\psi_0 = 0$ *

$\frac{\hbar\omega}{2}\psi_0 = E_0\psi_0$

$\boxed{E_0 = \frac{1}{2}\hbar\omega}$

$\psi_n(x) = A_n(\hat{a}_+)^n\psi_0(x)$ w/ $E_n = (n+\frac{1}{2})\hbar\omega = E + E_0$ $\quad$ $n\hbar\omega$, keep adding $\hbar\omega$ as you go up

<u>Finding An</u>

$\left.\begin{aligned}\hat{a}_+\psi_n &= c_n\psi_{n+1}\\ \hat{a}_-\psi_n &= d_n\psi_{n-1}\end{aligned}\right]$ $\quad \int_{-\infty}^{\infty} f^*(\hat{a}_\pm g)dx = \int_{-\infty}^{\infty}(\hat{a}_\mp f)^* g\, dx$ $\quad$ | $\hat{a}_\mp$ = hermitian conjugate of $a_\pm$

<u>Proof</u>:

$\int_{-\infty}^{\infty} f^*(\hat{a}_\pm g)dx = \frac{1}{\sqrt{2\hbar m\omega}}\int_{-\infty}^{\infty} f^*(\mp\hbar\frac{d}{dx} + m\omega x)g\, dx$, integrate by parts $\int f^*\frac{dg}{dx}dx = -\int(\frac{df}{dx})^* g\, dx$ to

$= \frac{1}{\sqrt{2\hbar m\omega}}\int_{-\infty}^{\infty}\left[(\pm\hbar\frac{d}{dx} + m\omega x)f\right]^* g\, dx$ $\quad$ $\{\int_{-\infty}^{\infty} f^*(\mp\hbar\frac{dg}{dx}) + m\omega x f^* g\, dx$ integrate this by parts & pull out $f^* g$

$= \int_{-\infty}^{\infty}(\hat{a}_\mp f)^* g\, dx$

$\int_{-\infty}^{\infty} f^*\hat{Q}g\, dx = \int(\hat{a}^\dagger f)^* g$

$\left(\frac{d}{dx}\right)^\dagger = -\frac{d}{dx}$

pg 4(?)

have: $\hat{a}_+\psi$

e have:

$\hat{a}_+ \psi_n = c_n \psi_{n+1}$

$\hat{a}_- \psi_n = d_n \psi_{n-1}$ and $\int_{-\infty}^{\infty} f^*(\hat{a}_\pm g)\,dx = \int_{-\infty}^{\infty} (\hat{a}_\mp f)^* g\,dx$

normalization

$$\int_{-\infty}^{+\infty} \underbrace{(\hat{a}_\pm \psi_n)^*}_{f} \underbrace{(\hat{a}_\pm \psi_n)}_{g}\,dx = \int_{-\infty}^{\infty} (\hat{a}_\mp \underbrace{\hat{a}_\pm \psi_n}_{f})^* \psi_n\,dx$$

look at

$\hbar\omega\left(\hat{a}_\pm \hat{a}_\mp \pm \frac{1}{2}\right)\psi = E\psi$ and $\psi_n(x) = A_n (\hat{a}_+)^n \psi_0(x)$

Use $E_n = (n+\frac{1}{2})\hbar\omega$ and $a_- a_+ = \frac{\hat{H}}{\hbar\omega} + \frac{1}{2}$

$\hookrightarrow a_- a_+ \psi_n = \frac{1}{\hbar\omega} \underbrace{\hat{H}\psi_n}_{E_n} + \frac{1}{2}\psi_n$

$= (n+\frac{1}{2})\psi_n + \frac{1}{2}\psi_n \rightarrow a_- a_+ \psi_n = (n+1)\psi_n$

Use $a_+ a_- = \frac{\hat{H}}{\hbar\omega} - \frac{1}{2}$ for other version.

$$\boxed{\hat{a}_+ \hat{a}_- = n\psi_n \quad \text{and} \quad \hat{a}_- \hat{a}_+ \psi_n = (n+1)\psi_n}$$

So

$$\int_{-\infty}^{\infty} (\hat{a}_+ \psi_n)^* (\hat{a}_+ \psi_n)\,dx = |c_n|^2 \overbrace{\int |\psi_{n+1}|^2 dx}^{1}$$

and $= \int_{-\infty}^{\infty} (a_- a_+ \psi_n)^* \psi_n = \int_{-\infty}^{\infty} (n+1) \underbrace{|\psi_n|^2 dx}_{1}$ $\Big\rangle$ $|c_n|^2 = n+1$

alternatively

$$\int_{-\infty}^{\infty} (\hat{a}_- \psi_n)^* (\hat{a}_- \psi_n)\,dx = |d_n|^2 \int |\psi_{n-1}|^2 dx$$

$= \int_{-\infty}^{\infty} (a_+ a_- \psi_n)^* \psi_n = n\int |\psi_n|^2 dx$ $\Big\rangle$ $|d_n|^2 = n$

From $\rightarrow \int_{-\infty}^{\infty} (\hat{a}_+ \psi_n)^* (\hat{a}_+ \psi_n)\,dx = (n+1)\int_{-\infty}^{\infty} \psi_{n+1}^* \psi_{n+1}\,dx \rightarrow$

From $\rightarrow \int_{-\infty}^{\infty} (\hat{a}_- \psi_n)^* (\hat{a}_- \psi_n)\,dx = n\int_{-\infty}^{\infty} \psi_{n-1}^* \psi_{n-1}\,dx \rightarrow$

plug it in it works dammit!

$$\hat{a}_+ \psi_n = \sqrt{n+1}\,\psi_{n+1}$$

$$\hat{a}_- \psi_n = \sqrt{n}\,\psi_{n-1}$$

Now we can finally [illegible]

$$\psi_n = A_n (\hat{a}_+)^n \psi_0(x)$$

same pattern

$\hat{a}_+ \psi_n = \sqrt{n+1}\, \psi_{n+1}$, $\hat{a}_- \psi_n = \sqrt{n}\, \psi_{n-1}$

$\hat{a}_+ \psi_0 = \sqrt{1}\, \psi_{1+0}$, $\hat{a}_+ \psi_1 = \sqrt{2}\, \psi_2 \rightarrow \psi_2 = \frac{1}{\sqrt{2}} \hat{a}_+ \psi_1 = \frac{1}{\sqrt{2}} \hat{a}_+^2 \psi_0$

$\psi_3 = \frac{1}{\sqrt{3}} \hat{a}_+ \psi_2 = \frac{1}{\sqrt{3 \cdot 2}} (\hat{a}_+)^3 \psi_0$

thus $\boxed{\psi_n = \frac{1}{\sqrt{n!}} (\hat{a}_+)^n \psi_0}$

* read & understand bottom of pg 46?

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operator Q and its Hermitian conjugate $\hat{Q}^\dagger$

$$\int_{-\infty}^{\infty} f^* \hat{Q} g \, dx = \int_{-\infty}^{\infty} (\hat{Q}^\dagger f)^* g \, dx \rightarrow a_+^\dagger = a_-$$

$A^2 + 4A^2 = 1$

$5A^2 = 1$

$A = \frac{1}{\sqrt{5}}$

$|c_1|^2 = \frac{1}{5}$

$|c_2|^2 = \frac{4}{5}$

$E_n = (n + \frac{1}{2})\hbar\omega$

$E_0 = \frac{\hbar\omega}{2}$

$E_1 = \frac{3}{2}\hbar\omega$

$\frac{1}{10} + \frac{12}{10}$

?monic Oscillator Analytically

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}m\omega^2x^2\psi=E\psi$$ → define $\mu \equiv \sqrt{\frac{m\omega}{\hbar}}\,x$ and $K \equiv \frac{2E}{\hbar\omega}$

$$\hookrightarrow \frac{d^2\psi}{d\mu^2}=(\mu^2-K)\psi$$

Start that for large μ, μ^2 is the dominant term over K (constant)

$$\hookrightarrow \frac{d^2\psi}{d\mu^2}\approx\mu^2\psi \rightarrow \psi(\mu)\approx Ae^{-\mu^2/2}+\cancel{Be^{\mu^2/2}}$$ → blows up as $\mu\to\infty$

$\hookrightarrow \psi(\mu)\to(\text{something})\,e^{-\mu^2/2}$ at large μ, so $\psi(\mu)$ decays exponentially

$\psi(\mu)=h(\mu)e^{-\mu^2/2}$ (this is exact, first part motivate the $e^{-\mu^2/2}$ out of our solution, isolating $h(\mu)$)

↳ differentiate twice: $\frac{d^2\psi}{d\mu^2}=\left(\frac{d^2h}{d\mu^2}-2\mu\frac{dh}{d\mu}+(\mu^2-1)h\right)e^{-\mu^2/2}$ (plug into SE)

drops out

$$\hookrightarrow \boxed{\frac{d^2h}{d\mu^2}-2\mu\frac{dh}{d\mu}+(K-1)h=0}$$ (now we look for general solutions)

$h(\mu)=a_0+a_1\mu+a_2\mu^2+\dots=\sum_{j=0}^{\infty}a_j\mu^j$ (power series, any function can be represented as a power series like Taylor expansion)

$$\frac{dh}{d\mu}=\sum_{j=0}^{\infty}ja_j\mu^{j-1}=a_1+2a_2\mu+3a_3\mu^2+\dots$$

$$\frac{d^2h}{d\mu^2}=2a_2+2\cdot3a_3\mu+3\cdot4a_4\mu^2+\dots=\sum_{j=0}^{\infty}(j+1)(j+2)a_{j+2}\mu^j$$

(Putting these sums into boxed eq)

$$(j+1)(j+2)a_{j+2}-2ja_j+(K-1)a_j=0 \rightarrow a_{j+2}=\frac{(2j+1-K)}{(j+1)(j+2)}a_j$$

Start w/ a_0: even coeff $a_2=\frac{1-K}{2}a_0$, $a_4=\frac{(5-K)}{12}a_2=\frac{(5-K)(1-K)}{24}a_0$, ...

Start w/ a_1: odd coeff $a_3=\frac{(3-K)}{6}a_1$, $a_5=\frac{(7-K)}{20}a_3=\frac{(7-K)(3-K)}{120}a_1$, ...

$h(\mu)=h_{even}(\mu)+h_{odd}(\mu)$ where $h_{even}\equiv a_0+a_2\mu^2+a_4\mu^4+\dots$

$h_{odd}\equiv a_1\mu+a_3\mu^3+a_5\mu^5+\dots$

(These determine $h(\mu)$ in terms of two arbitrary functions a_0 and a_1, as expected for a 2nd order ODE)

We get

Recall for large μ:

$$\psi(\mu) \approx A e^{-\mu^2/2} + \underbrace{B e^{\mu^2/2}}_{\text{not normalizable solution}}$$

$$a_{j+2} = \frac{(2j+1-k)}{(j+1)(j+2)} a_j$$

We were able to write the full $\psi(\mu)$ in terms of $e^{-\mu^2/2}$

$\hookrightarrow \psi(\mu) = h(\mu) e^{-\mu^2/2}$ but the $h(\mu)$ since this form is exact contains the bad part

$\hookrightarrow$ this form is completely equivalent to the SE, it came from the diff eq of the SE and we plugged in power series to reduce it to this form (solving ODE).

[Lets look at all solutions, specifically ones w/ large j]

$a_{j+2} \approx \frac{2}{j} a_j$

$a_j \approx \frac{C}{(j/2)!}$] assume its right

$$h(\mu) \approx C \sum \frac{1}{(j/2)!} \mu^j \approx C \sum \underbrace{\frac{1}{j!} \mu^{2j}}_{\text{Power series of } e^{\mu^2}} \approx C e^{\mu^2}$$

Series must truncate, meaning for some j (call it n) $a_{j+2} = 0$

$\therefore 0 = (2n+1-k) \rightarrow K = 2n+1 \rightarrow E_n = (n+\frac{1}{2})$ for $n = 0, 1, 2, \ldots$

For allowed values of K:

$$a_{j+2} = \frac{-2(n-j)}{(j+1)(j+2)} a_j$$

[Recall if we start w/ a_1 we get odd solution] and [if we start w/ a_0 we get even soln]

$\underline{n=0}$ (we get even a's, so $a_1 = 0$ so $h(\mu)$ odd isnt infinite)

$a_{j+2} = \frac{+2j}{(j+1)(j+2)} a_j$ and $h_{even}(\mu) = a_0 + a_2\mu^2 + a_4\mu^4 + \ldots$

if $j=0$ then $a_2 = 0 \rightarrow h(\mu) = h_{even} + \cancel{h_{odd}}^{0} = a_0$ bec all other terms are then 0.

$\psi_0 = a_0 e^{-\mu^2/2}$

if $\underline{n=1}$: $a_0 = 0$ so all untouched h_{even}s are 0 (not inf series) or $\underline{\text{just } h_{odd} \text{ is valid here}}$

for $j=1$: $h_1(\mu) = a_1\mu \Rightarrow \psi_1(\mu) = a_1 \mu e^{-\mu^2/2}$

$\hookrightarrow a_3 = 0$

$\psi_2(\mu) = a_0(1-2\mu^2)e^{-\mu^2/2}$

$a_1 = 0$ so odds are dead

for $n=2$, $j=0$ gives $a_2 = -2a_0$, $j=2$ gives $a_4 = 0$ $\overset{n-j}{(2-2)} \rightarrow h_2(\mu) = a_0 \underbrace{(1-2\mu^2)}_{\text{hermite poly if you keep going}}$

$$\boxed{\text{normalized} \quad \psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\mu) e^{-\mu^2/2}}$$

H_n's on table 2.1