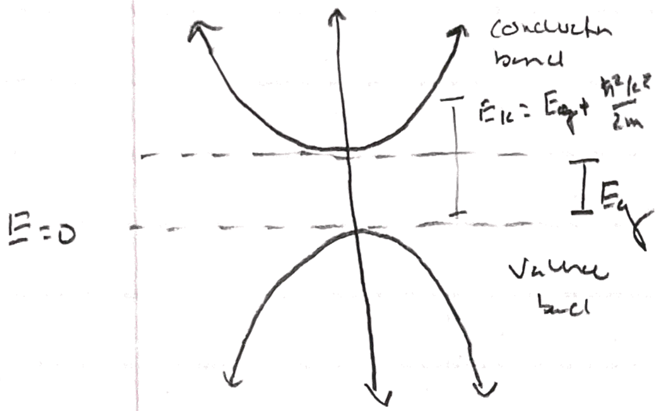
Semiconductors



*simplified view of the band edge *
*structure of a direct gap semiconductor *
↳ direct gap semiconductor w/ band edges at the center of the brillouin zone

$$E_{conduction} = E_g + \frac{\hbar^2 k^2}{2m_e^*}$$

$$E_{valence} = \frac{-\hbar^2 k^2}{2m_h^*}$$

[Probability distribution of thermally excited charge carriers are] → $f(E) = \dfrac{1}{e^{(E-E_F)/k_BT} + 1}$, $f(E_F) = 1/2$

*for semiconductors E_F lies within the gap → $\underbrace{(E-E_F)}_{\approx 2\text{-}3\,eV} \gg \underbrace{k_BT}_{\approx 0.025\,eV}$

↳ make approximation $f(E) = e^{E_F/k_BT} e^{-E/k_BT}$ $\qquad \mu \approx E_F$

◦ intrinsic semiconductor → charge neutral condition → $\bar{e}$ concentration = hole$^+$ concentration

Density of States: $g_e(E) = \dfrac{1}{2\pi^2}\left(\dfrac{2m^*}{\hbar^2}\right)^{3/2}\left(\overset{\hbar^2k^2/2m}{E_c - E_g}\right)^{1/2}$ ← as it should be by def!

Thermally excited $\acute{e}$ density: $n = \int_{E_{c1}}^{E_{c2}} g_e(E) f(E)\, dE$ $\qquad E_c = E_g + \overbrace{\frac{\hbar^2k^2}{2m_e^*}}^{E}$

for entire concentration

↳ $n_e = \int_{E_{c1}}^{E_{c2}} \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E-E_g)^{1/2} e^{E_F/k_BT} e^{-E/k_BT}\, dE$

$$n_e = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} e^{E_F/k_BT}\int_{E_g}^{\infty}(E-E_g)^{1/2} e^{-E/k_BT}\, dE$$

let $x = \frac{E-E_g}{k_BT}$, ↳ $\int_0^\infty x^{1/2} e^{-x}\, dx = \frac{\sqrt{\pi}}{2}$

$$\boxed{n_e = 2\left(\frac{m_e k_BT}{2\pi\hbar^2}\right)^{3/2} e^{\frac{E_F}{k_BT}} e^{-E_g/k_BT}}$$

What about holes?

$$f_h = 1 - f(E) = 1 - \frac{1}{e^{(E-E_F)/k_BT}+1} = \frac{1}{e^{(E_F-E)/k_BT}+1} \approx e^{-E_F/k_BT}\, e^{E/k_BT}$$

$$E_{\text{valence}} = \frac{-\hbar^2 k^2}{2m_h^*}$$

$$n_e = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(E_F - E_g)/k_BT}$$

hole density

$$g_h = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(-E)^{1/2} \rightarrow p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_BT}$$

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-E_g/k_BT}$$ ← independent of fermi level!

$\left[\begin{array}{l}\text{For intrinsic conductor} \\ n = p\end{array}\right]$ → solve for $E_F = \frac{1}{2}E_g + \frac{3}{4}k_BT \ln\left(\frac{m_h}{m_e}\right)$

plug into above

$$n = p = \sqrt{np} = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4} e^{-E_g/2k_BT}$$

More doping

Intrinsic semiconductor: $n = p = n_i = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_e m_h)^{3/4} e^{-E_g/2k_BT}$

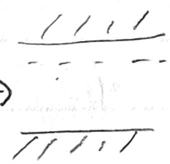
Extrinsic semiconductor: contribution of impurities exceeds those supplied by interband transitions

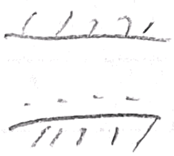
$n = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(E_F - E_g)/k_BT} = n_i e^{(E_F - E_i)/k_BT}$, $N_{conduction} = 2\left(\frac{m_e k_BT}{2\pi\hbar^2}\right)^{3/2}$

$p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_BT} = n_i e^{(E_i - E_F)/k_BT}$, $N_{valence} = 2\left(\frac{m_h k_BT}{2\pi\hbar^2}\right)^{3/2}$

for intrinsic semiconductor $E_i = E_F$

↳ $n_i = N_c e^{(E_i - E_g)/k_BT}$, $p_i = N_v e^{-E_i/k_BT}$

[Primary donor type] →  E_d $n_i \ll N_d$ electron concentration dominates $n \sim N_d$

(Acceptor type) →  E_g $n_i \ll N_a$ hole concentration dominates $p \sim N_c$

At low temp, $E_d > k_BT$ thermal energy is insufficient to raise e^- to donor or acceptor levels

$n \approx \sqrt{N_c N_d}\, e^{-E_d/2k_BT}$ ← like a semiconductor between donor level + conduction band!

↳ $p_i = n_i = \sqrt{N_c N_v}\, e^{-E_g/2k_BT}$ ← no doping comparison

$N_{conduction} = N_c$, $N_{donors} = N_d$

$N_{valence} = V_v$, $N_{acceptors} = N_a$

Doping

↳ potential is like that of electron screened in dielectric

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\varepsilon_r r}$$ silicon $\varepsilon_r = 11.7$ germanium $\varepsilon_r = 16.0$

Binding energy (E_d) of Hydrogen $= 13.6\,eV = \underbrace{\frac{e^4 m}{2(4\pi\varepsilon_0\hbar)^2}}$

if $\varepsilon_r = 10$, $\frac{m_e}{m_0} = 0.2$

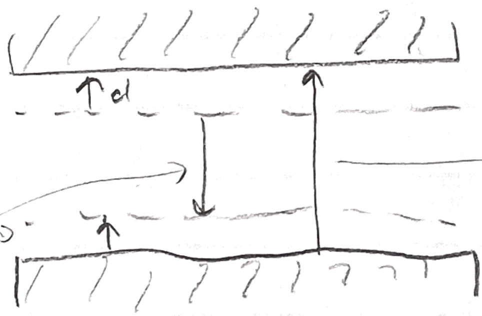
Binding Energy of doped $e^- = \frac{1}{\varepsilon_r^2}\left(\frac{m_e}{m_H}\right) E_{d,H} \approx 0.01\,eV$

Bohr radius: $r_d = \varepsilon_r \left(\frac{m}{m_e}\right) a_0 \approx 30\,Å$ ($a_0 = 0.53\,Å$) much bigger than interatomic spacing

↳ just below, like easily thermally excited

$n_m = \frac{1}{\frac{4}{3}\pi r_d^3}$ → concentration at which impurity band forms & permits conductivity by hopping between them

Electronic Processes



$$n_i = n = p = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4} e^{-E_g/2k_BT}$$

↑ intrinsic concentration

$n_i \gg (N_d - N_a)$

↑ intrinsic carrier concentration MUCH smaller than donor/acceptor electron/hole concentration

$N_d \gg N_a \rightarrow$ n-~~type~~ $\in$ negative charge
$N_a \gg N_d \rightarrow$ p-type $\in$ hole charge

$J = \sigma E$, $J = -nev$

Electrical Conductivity

$$\sigma = \frac{ne^2 \tau_e}{m_e} \quad , \quad \vec{V}_{drift,e} = -\frac{e\tau_e}{m_e}\vec{E}$$

Define mobility: $\mu_e = \frac{e\tau_e}{m_e}$, $\sigma = ne\mu$

$$\mu_{hole} = \frac{e\tau_h}{m_h}$$

Total conductivity: $\sigma = ne\mu_e + pe\mu_h$

HW # 11

Problem 1

$$\rho_i = \begin{pmatrix} \rho_i & -R_i B \\ R_i B & \rho_i \end{pmatrix}, \quad \sigma_i = \frac{1}{\rho_i^2 + (R_i B)^2} \begin{pmatrix} \rho_i & R_i B \\ -R_i B & \rho_i \end{pmatrix}$$

$$\sigma_{tot} = \frac{1}{\rho_{tot}} = \frac{1}{\rho_e} + \frac{1}{\rho_h} \qquad \frac{1}{\rho_{tot}} = \text{inverse of matrix}$$

$$= \frac{1}{\rho_e^2 + (R_e B)^2} \begin{pmatrix} \rho_e & R_e B \\ -R_e B & \rho_e \end{pmatrix} + \frac{1}{\rho_h^2 + (R_h B)^2} \begin{pmatrix} \rho_h & R_h B \\ -R_h B & \rho_h \end{pmatrix}$$

$$\sigma_{tot} = \begin{pmatrix} \underbrace{\frac{\rho_e}{\rho_e^2 + (R_e B)^2} + \frac{\rho_h}{\rho_h^2 + (R_h B)^2}}_{a} & \underbrace{\frac{R_e B}{\rho_e^2 + (R_e B)^2} + \frac{R_h B}{\rho_h^2 + (R_h B)^2}}_{b} \\ \underbrace{\frac{-R_e B}{\rho_e^2 + (R_e B)^2} + \frac{-R_h B}{\rho_h^2 + (R_h B)^2}}_{-b} & \underbrace{\frac{\rho_e}{\rho_e^2 + (R_e B)^2} + \frac{\rho_h}{\rho_h^2 + (R_h B)^2}}_{a} \end{pmatrix}$$

$$\rho_{tot} = \begin{pmatrix} \rho & R_H B \\ -R_H B & \rho \end{pmatrix} \leftarrow \text{need to get to } \rho_{tot} = \sigma_{tot}^{-1}$$

$$\rho_{tot} = \frac{1}{a^2 + b^2} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} \rho & -R_H B \\ R_H B & \rho \end{pmatrix}$$

$$\boxed{R_H B = \frac{b}{a^2 + b^2}}$$

b) $R_e = \frac{-1}{ne}$, $R_h = \frac{1}{pe}$,

$\sigma_h = pe\mu_h$, $\sigma_e = ne\mu_e$, $\rho = \frac{1}{\sigma}$

Problem 3 20/20

- intrinsic Si: doped w/ acceptors $N_a = N_0 e^{-ax}$

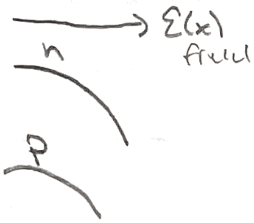
a) find expression for build in E-field, $N_a >> n_i$

Below are notes for problem

$$J_e = \overbrace{e\mu_e n}^{\sigma} \mathcal{E} + eD_e \frac{\partial n}{\partial x} = \text{field current} + \text{diffusion current}$$

$$J_h = e\mu_p p \mathcal{E} - eD_h \frac{\partial p}{\partial x} = \longrightarrow$$

(How diffusion & Drift currents work)



Drift current always in direction of Electric field

$\rightarrow J_{drift} = e\mu_e n \mathcal{E} + e\mu_p p \mathcal{E} = +\hat{x}$

diffusion current

$J_{e,diff} = eD_e \frac{\partial n}{\partial x}$

$J_{h,diff} = -eD_h \frac{\partial p}{\partial x}$

depends on slope of density

$\frac{\partial n}{\partial x} < 0, \quad \frac{\partial p}{\partial x} < 0$

$J_{e,diff} = -\hat{x} \quad J_{h,diff} = +\hat{x}$

$\mathcal{E}(x)$ = electric field $= -dV/dx = -\frac{d}{dx}\left(\frac{E_i}{-e}\right) = \frac{1}{e}\frac{dE_i}{dx}$, E_i = energy

at equillibrium:

$$J_h = 0 = e\mu_p p \mathcal{E} - eD_h \frac{\partial p}{\partial x} = 0 \rightarrow \mathcal{E} = \frac{D_h}{\mu_p}\frac{1}{p}\frac{dp}{dx}$$

field causing a balance between drift current & diffusion current

(Same for J_e) $\rightarrow \mathcal{E} = -\frac{D_e}{\mu_e}\frac{1}{n}\frac{dn}{dx}$

For n-type, low hole (p) concentrat

$$p = \frac{n_i^2}{N_d} = n_i e^{(E_i - E_f)/k_BT}$$

concentration of donor from n-dope

$$\hookrightarrow \mathcal{E} = \frac{D_h}{\mu_p}\frac{1}{k_BT}\left(\frac{dE_i}{dx} - \cancel{\frac{dE_F}{dx}}^{\,0}\right) \rightarrow \frac{D_h}{\mu_p} = \frac{k_BT}{e} \text{ (Einstein Relation)}$$

$\frac{1}{e}\frac{dE_i}{dx} = \mathcal{E}$

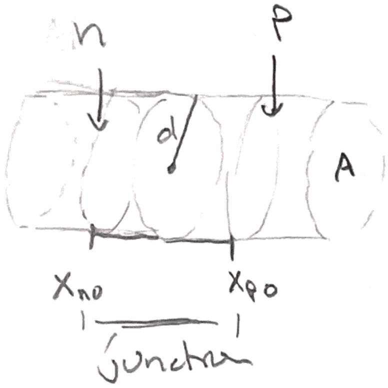
Problem 5

18/20

$N_d = 10^{16}\ cm^{-3}$

$N_A = 4 \cdot 10^{18}\ cm^{-3}$

$d = 0.02 in$



Calculate: V_0, x_{n0}, x_{p0}, Q_+, $\mathcal{E}_0$, sketch $\mathcal{E}(x)$

Below is just notes + equations to solve problem

$Q_+ = |Q_-|$

* junction must have an equal # carriers on either side *

$$e(\underbrace{A x_{po}}_{\text{volume of depletion}})\underbrace{N_a}_{\text{charge per vol}} = e(A X_{no})N_d$$

Poisson's Equation: $\nabla^2 V = \vec{\nabla} \cdot \vec{\nabla} V = \vec{\nabla} \cdot \vec{\mathcal{E}} = \frac{\rho}{\varepsilon} = \frac{e}{\varepsilon}(p - n + N_d - N_a)$

* N_d & N_a have flipped sign because we are in the depletion region, the carrier filled a hole or hole got filled in doped region leaving the other charge due to the doping atom (extra or missing proton) (neglect p, n] most $\mathcal{E}$ from N_d, N_a

$$\hookrightarrow \frac{d\mathcal{E}}{dx} = \begin{cases} \frac{e}{\varepsilon} N_d & 0 \le x < x_{no} \\ -\frac{e}{\varepsilon} N_a & -x_{po} < x \le 0 \end{cases}$$

integrate w/ B.C. $\mathcal{E}(x_{no}) = 0$, $\mathcal{E}(-x_{po}) = 0$

$$\rightarrow \boxed{\mathcal{E}(x) = \begin{cases} \frac{e}{\varepsilon_0} N_d(x - x_{no}) & 0 < x < x_{no} \\ -\frac{e}{\varepsilon_0} N_a(x + x_{po}) & -x_{po} < x < 0 \end{cases}}$$

also don't forget! * $\cancel{e}\, x_{po} N_a = \cancel{e}\, x_{no} N_d$ *

$$V_0 = -\int \vec{\mathcal{E}} \cdot d\vec{\ell} = -\int_{-x_{po}}^{x_{no}} \mathcal{E}(x)\,dx \overset{\text{graphically or}}{=} -\left[\int_0^{x_{no}} \frac{e}{\varepsilon_0} N_d(x - x_{no})dx + \int_{-x_{po}}^{0} -\frac{e}{\varepsilon_0} N_a(x + x_{po})dx\right]$$

$$= -\left[\frac{eN_d}{\varepsilon_0}\left(\frac{1}{2}x_{no}^2 - x_{no}^2\right) + \frac{e}{\varepsilon_0}N_a\left(-\frac{x_{po}^2}{2} + \frac{x_{po}^2}{2}\right)\right] = \frac{eN_d}{\varepsilon_0}\frac{x_{no}^2}{2} + \frac{eN_a}{\varepsilon_0}\frac{x_{po}^2}{2} = \frac{e}{2\varepsilon_0}(N_d x_{no}^2 + N_a x_{po}^2)$$

$$= \frac{e}{2\varepsilon_0}(N_d x_{no}^2 + N_d x_{no} x_{po}) = \frac{e}{2\varepsilon_0} N_d x_{no}(x_{no} + x_{po}) = \boxed{\frac{1}{2}\mathcal{E}_{max} \underset{\uparrow}{W}}$$

width of junction

[Dielectric Equations]

$E_d = \frac{1}{\varepsilon_r^2}\left(\frac{m_e}{m}\right) E_{d,H}$ ← 13.6 eV ,

$r_d = \varepsilon_r \left(\frac{m}{m_e}\right) a_0$ ← 0.53 Å ,

Overlap cond.: $n = \frac{1}{\frac{4}{3}\pi (r_d)^3}$

Master Equation Sheet

$$n = \int_{E_g}^{\infty} g_e f \, dE = 2\left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{3/2} e^{(E_F - E_g)/k_B T}$$

$$p = \int_{-\infty}^{0} g_h f_h \, dE = 2\left(\frac{m_h k_B T}{2\pi \hbar^2}\right)^{3/2} e^{-E_F/k_B T}$$

$$g_e = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E - E_g)^{1/2}$$

$$g_h = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(-E)^{1/2}$$

$$f = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

Intrinsic: $n_i = p_i = \sqrt{np} = 2\left(\frac{k_B T}{2\pi \hbar^2}\right)^{3/2}(m_e m_h)^{3/4} e^{-E_g/2k_B T}$

$E_F = \frac{1}{2}E_g + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$ ← get by $p = n$ & solve

Doping: * use n, p from original

↳ [Typically for problems its small] →

$n \approx N_d = n_i e^{(E_F - E_{i,F})/k_B T}$ → $E_F = E_{i,F} + k_B T \ln\left(\frac{N_d}{n_i}\right)$

$p = N_A = n_i e^{(E_{i,F} - E_F)/k_B T}$ → $E_F = E_{i,F} - k_B T \ln\left(\frac{N_A}{n_i}\right)$

minority charge in n-type: $p = n_i^2/N_d$ minority charge in p-type: $n = n_i^2/N_A$

Drode Theory Equations

$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} + \vec{F}$ $\mu_e = \frac{\tau e}{m}$ $j = -env$ $\sigma = \frac{ne^2\tau}{m} = n\mu_e e$

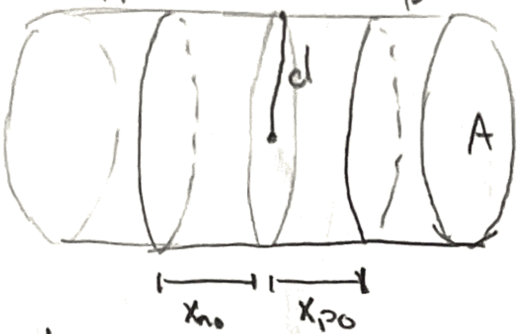
$\sigma = 1/\rho$ $\sigma_{tot} = \sigma_e + \sigma_h = n\mu_e e + p\mu_h e$

↳ $(\vec{F} = -e\vec{E})$ → $v = \frac{\tau e}{m}E$, $j = \frac{ne^2\tau}{m}E$

↳ $\left(\vec{F} = -e\left(\vec{E} + \frac{\vec{p}}{m}\times\vec{B}\right)\right)$ → $R_H = \frac{E_y}{j_x B}$, $R_H = -\frac{1}{ne}$, $R_H = \frac{\rho_e R_h + \rho_h^2 R_e + R_e R_h (R_e + R_h) B^2}{(\rho_e + \rho_h)^2 + (R_e + R_h)^2 B^2}$

↳ to get last eq → know $\rho_i = \begin{pmatrix} \rho_i & -R_i B \\ R_i B & \rho_i \end{pmatrix}$ and $\vec{\sigma}_{tot} = \vec{\sigma}_e + \vec{\sigma}_h$, remember inverses $\sigma_e = \frac{1}{|\rho_e|}\begin{bmatrix} \rho_i & R_e \\ -R_e B & \rho_i \end{bmatrix}$

$R_H = -\frac{1}{(n_e + n_h)e}$



Charge Equilibrium (continuity)

$$Q_+ = |Q_-|$$

$$e(A\,x_{po})N_a = e(A\,x_{no})N_d$$

PN Junction

In equilibrium $j_n = 0$

[Has drift + Diff curr] $\rightarrow$

$$j_e = (e\mu_e n)\mathcal{E} + eD_e\frac{\partial n}{\partial x}$$

$$j_n = (e\mu_n p)\mathcal{E} - eD_h\frac{\partial p}{\partial x}$$

$\hookrightarrow \mathcal{E} = \frac{D_h}{\mu_p}\frac{1}{p}\frac{\partial p}{\partial x}$, $\frac{D_h}{\mu_p} = \frac{k_B T}{e}$

equilibrium electric field

thanks to Einstein Relation

from $\rightarrow \vec{\mathcal{E}} = \vec{\nabla} V \rightarrow \mathcal{E} = dV/dx$

[Potential Drop across PN-Junction]

$$\Delta V = -\frac{k_B T}{e}\int_{p=N_a}^{p=n_i^2/N_d}\frac{1}{p}\,dp \rightarrow \Delta V = \frac{k_B T}{e}\ln\left(\frac{N_a}{n_i^2/N_d}\right) = \frac{E_{F,p} - E_{F,n}}{e}$$

[Poisson Eq]

$$\nabla^2 V = \vec{\nabla}\cdot\vec{\mathcal{E}} = \frac{\rho}{\varepsilon_0} = \frac{e}{\varepsilon_0}(p - n + N_d - N_a) \rightarrow \frac{d\mathcal{E}}{dx} = \begin{cases} \frac{eN_d}{\varepsilon_0} & 0 \le x \le x_{no} \\ -\frac{eN_a}{\varepsilon_0} & -x_{po} < x < 0 \end{cases}$$

Integrate

$$\rightarrow \mathcal{E} = \begin{cases} \frac{e}{\varepsilon_0}N_d(x - x_{no}) \\ -\frac{e}{\varepsilon_0}N_a(x + x_{po}) \end{cases}$$

$$\hookrightarrow V_0 = \int_{-x_{no}}^{x_{po}} \mathcal{E}(x)\,dx = \frac{1}{2}\mathcal{E}_{max}W, \quad W = (x_{no} + x_{po}), \quad \mathcal{E}_{max} = \frac{eN_d}{\varepsilon_0}x_{no} = \frac{eN_a}{\varepsilon_0}x_{po}$$

use continuity

$$\hookrightarrow N_d(x_{po} + x_{no}) = WN_d \rightarrow x_{po} = \frac{N_d}{N_a + N_d}W, \quad x_{no} = \frac{N_a}{N_a + N_d}W$$

$\hookrightarrow W =$ equate ΔV and V_0 $= \sqrt{\frac{2\varepsilon_0 V_0}{e}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$ $\leftarrow$ from $V_0 = \frac{1}{2}\frac{e}{\varepsilon_0}N_d x_{no}^2$ + solve

Capacitance: $C = \frac{dQ}{dV}$, $Q = eA\,x_{no}N_d$, $x_{no} = \frac{N_a}{N_a + N_d}W$, W depends on V

Work function: $dJ_x = -eV_x\,dN$, $(dk)^3 = \frac{dN}{2}\left(\frac{2\pi}{L}\right)^3$,

$\hookrightarrow dN = \left(\frac{L}{2\pi}\right)^3 2f(E)\,d^3k$, integrate

$\hookrightarrow$ [Condition to leave metal] $\rightarrow \frac{\hbar^2 k_{x,0}^2}{2m} \ge E_F + e\underbrace{\Phi}_{\text{work fun}}$

$p = \hbar k$, $p = mv$

$\hbar = \frac{h}{2\pi}$

$p = \hbar k$

$p = \frac{h}{\lambda}$

$\frac{h}{\lambda} = \hbar k$

$\frac{2\pi}{\lambda} = k$

De Haas-van Alphen Effect

$\vec{p} = \hbar\vec{k} + q\vec{A}$

Bohr-Sommerfeld relation: $\oint \vec{p}\cdot d\vec{r} = (n+\frac{1}{2})2\pi\hbar$

↳ [comes from Bohr's theory] → $\int p\,dx = \int \hbar k\,dx = 2\pi\hbar n$ ← condition for constructive interference of wave

↳ $\oint \vec{p}\cdot d\vec{r} = \underbrace{\oint \hbar\vec{k}\cdot d\vec{r}} + q\oint \vec{A}\cdot d\vec{r}$

EOM of particle in B-field: $\hbar\frac{d\vec{k}}{dt} = q\frac{d\vec{r}}{dt}\times\vec{B}$ ← if you write out every comp. w/ $\vec{B}$ = constant

↳ integrate w.r.t time: $\hbar\vec{k} = q\int_0^t \frac{d\vec{r}}{dt}\times\vec{B}\,dt = q\vec{r}\times\vec{B}$

↳ $\oint \hbar\vec{k}\cdot d\vec{r} = q\oint \overset{A}{\vec{r}}\times\overset{B}{\vec{B}}\cdot\overset{C}{d\vec{r}} = q\overset{B}{\vec{B}}\cdot\oint d\vec{r}\times\vec{r} = -q\vec{B}\cdot\oint\vec{r}\times d\vec{r}$

$\vec{B}\oint\vec{r}\times d\vec{r}$ for circular loop: $\vec{r} = R\hat{r}$, $d\vec{r} = dr\,\hat{\phi} = R\,d\phi\,\hat{\phi}$, say $\vec{B} = B\hat{z}$

↳ $-qB\hat{z}\cdot\oint R\hat{r}\times(R\,d\phi\,\hat{\phi})$, $\hat{z}\cdot(\hat{r}\times\hat{\phi}) = \sin\theta$

↳ $-qB\int_0^{2\pi} R^2\sin\theta\,d\phi = -qB\,2\pi R^2\sin\theta$

Φ = Flux = $\vec{B}\cdot\vec{A} = BA\,\hat{z}\cdot\hat{n} = BA\cos\alpha = BA\sin\theta$, $A = \pi R^2$

So! $\oint \hbar\vec{k}\cdot d\vec{r} = -2q\Phi$

$q\oint\vec{A}\cdot d\vec{r} \overset{\text{Stokes}}{=} q\int(\vec{\nabla}\times\vec{A})\cdot d\vec{a} = q\int\vec{B}\cdot d\vec{a} = q\Phi$

↳ Together: $\oint\vec{p}\cdot d\vec{r} = -q\Phi = (n+\frac{1}{2})2\pi\hbar$

$$\Phi_n = \left(n+\frac{1}{2}\right)\frac{2\pi\hbar}{e}$$

$\left[\begin{matrix}\text{Area in realspace} \\ \text{to the area in} \\ \text{reciprocal space}\end{matrix}\right] \rightarrow \frac{\hbar}{q}\vec{k} = \vec{r}\times\vec{B} \rightarrow \underset{\perp \text{ plane}}{\frac{\hbar}{q} k = r B}$

$\hookrightarrow r = \frac{\hbar}{eB} k$ $\rightarrow$ Area in k-space S_n

real space $\downarrow$ $\pi k^2 = S_n$

$$A_n = \pi r^2 \rightarrow A_n = \left(\frac{\hbar}{eB}\right)^2 S_n$$

$$\Phi = A_n B = \left(\frac{\hbar}{eB}\right)^2 S_n B = \underbrace{\left(\frac{\hbar}{e}\right)^2 \frac{1}{B} S_n = \left(n+\frac{1}{2}\right)\frac{2\pi\hbar}{e}}$$

$$S_n = \left(n+\frac{1}{2}\right)\frac{2\pi e}{\hbar} B$$

find ΔB such that area is same

$$S\left(\frac{1}{B_{n+1}} - \frac{1}{B_n}\right) = \frac{2\pi e}{\hbar}\left(n+\frac{3}{2}-n-\frac{1}{2}\right) = \frac{2\pi e}{\hbar}$$

* increments of $1/B$ reproduce similar orbits

$\pi k^2 = \frac{N}{2}\left(\frac{2\pi}{L}\right)^2$, $k^2 = \frac{2mE}{\hbar^2}$ $\rightarrow$ $\frac{dN}{dE}$

$$\cancel{\pi}\left(\frac{2mE}{\hbar^2}\right) = N\frac{2\pi^{\cancel{2}}}{L^2} \rightarrow N = \frac{L^2}{2\pi}\frac{2mE}{\hbar^2} = \frac{L^2}{\pi}\frac{mE}{\hbar^2}$$

$$g(E) = \frac{L^2}{\pi}\frac{m}{\hbar^2}$$

$$g(E)\,\hbar\frac{eB}{m} = \frac{L^2}{\pi}\frac{eB}{\hbar}$$

De Haas-van Alphen Effect

* oscillation of the magnetic moment of a metal as a function of static B-field intensity

$$\Delta S = S_n - S_{n-1} = \frac{2\pi e}{\hbar} B$$

Area in k-space $= (2\pi/L)^2$ (Landau Level)

(# of free electrons that coalese in a single Magnetic level) $= D = \left(\frac{2\pi e}{\hbar} B\right) \frac{1}{(L/2\pi)^2} = \frac{eL^2}{2\pi\hbar} B$