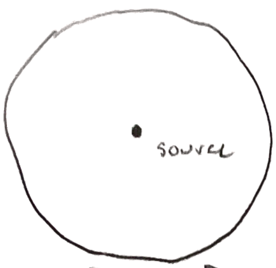
Radiation

- when charges accelerate their fields can transport energy
- assume the source is localized
- Find the energy being radiated

charges by the EM force is equal to the decrease in energy remaining in the fields, less energy flowed through the surface

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)d\tau - \frac{1}{\mu_0}\oint_S \vec{E}\times\vec{B}\cdot d\vec{a}$$

Imagine a gigantic sphere: $\vec{S} \equiv \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{\text{energy}}{\text{time}\cdot\text{area}}$ transported by the fields



[Power Passing through surface] $\rightarrow P(r,t) = \oint \vec{S}\cdot d\vec{a} = \frac{1}{\mu_0}\oint \vec{E}\times\vec{B}\cdot d\vec{a}$

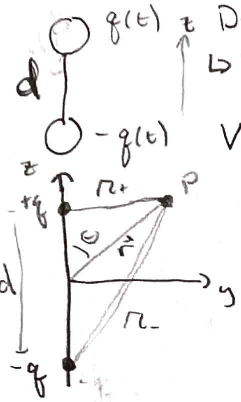
- energy that reaches the surface came from the source at $t_0 = t - \frac{r}{c}$

$P_{radiation}(t_0) = \lim_{r\to\infty} P(r, t_0 + \frac{r}{c})$, $E \propto \frac{1}{r^2}$, $B \propto \frac{1}{r^2}$, $S \propto \frac{1}{r^4}$

so for $\underbrace{r\to\infty}$, static charges don't radiate | Jefimenko's eq have $\dot{\rho}, \dot{J}$ terms $\propto \frac{1}{r}$

bec fields go to ∞

Electric Dipole Radiation



Drive charge back and forth: $q(t) = q_0\cos(\omega t)$

↳ leads to an oscillating electric dipole: $\vec{p}(t) = p_0\cos(\omega t)\hat{z}$ $\quad p_0 = q_0 d$

$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}', t_r)}{\mathscr{r}}d\tau' = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i(\vec{r}', t_r)}{\mathscr{r}_i} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_0\cos(\omega(t - \frac{\mathscr{r}_+}{c}))}{\mathscr{r}_+} - \frac{q_0\cos(\omega(t-\frac{\mathscr{r}_-}{c}))}{\mathscr{r}_-}\right]$$

[law of cosines] $\rightarrow \mathscr{r}_\pm = \sqrt{r^2 \mp rd\cos\theta + (\frac{d}{2})^2}$, $d \ll r$, $\mathscr{r}_\pm \approx r\left(1 \mp \frac{d}{2r}\cos(\theta)\right)$

taylor expansion x2, $\sqrt{\sim}$, $\frac{1}{x}$

$$\boxed{\frac{1}{\mathscr{r}_\pm} \approx \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)}$$

$$\cos\left(\omega\left(t - \frac{\mathscr{r}_\pm}{c}\right)\right) \approx \cos\left(\omega(t - r/c) \pm \frac{\omega d}{2c}\cos\theta\right) = \cos\left(\omega(t-\tfrac{r}{c})\right)\overbrace{\cos\left(\tfrac{\omega d}{2c}\cos\theta\right)}^{\approx 1} \mp \sin\left(\omega(t-\tfrac{r}{c})\right)\sin\left(\tfrac{\omega d}{2c}\cos\theta\right)$$

$\approx \frac{\omega d}{2c}\cos\theta$

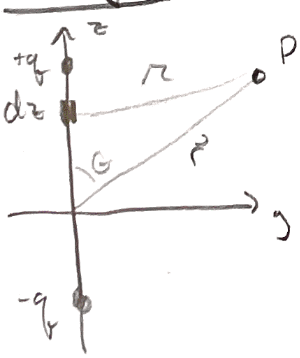
$\boxed{d \ll \frac{c}{\omega}} \rightarrow \lambda = \frac{2\pi c}{\omega} \rightarrow d \ll \lambda \rightarrow \boxed{\cos\left(\omega\left(t - \frac{\mathscr{r}_\pm}{c}\right)\right) \approx \cos\left(\omega(t - \tfrac{r}{c})\right) \mp \frac{\omega d}{2c}\cos\theta\sin\left(\omega(t-\tfrac{r}{c})\right)}$

$$\boxed{V(r,\theta,t) = \frac{p_0\cos\theta}{4\pi\varepsilon_0}\left[-\frac{\omega}{c}\sin\left(\omega(t-\tfrac{r}{c})\right) + \frac{1}{r}\cos\left(\omega(t-\tfrac{r}{c})\right)\right]}$$

for $\omega \to 0$ $V = \frac{p_0\cos\theta}{4\pi\varepsilon_0 r^2}$, stationary dipole

[We want the fields at large distances from the source], $r \gg \frac{c}{\omega}$, $r \gg \lambda$, $\boxed{V(r,\theta,t) = \frac{-p_0\omega}{4\pi\varepsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left(\omega(t-\tfrac{r}{c})\right)}$

Getting $\vec{A}$



$\vec{I}(t) = \frac{dq}{dt}\hat{z} = -q_0\omega\sin(\omega t)\hat{z}$ $\quad$ first order appr. let $\mathscr{r} = r$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{-d/2}^{d/2}\frac{-q_0\omega\sin(\omega(t-\frac{\mathscr{r}}{c}))\hat{z}}{\mathscr{r}}dz,$$

$$\boxed{\vec{A}(r,\theta,t) = -\frac{\mu_0 p_0\omega}{4\pi r}\sin\left(\omega(t-\tfrac{r}{c})\right)\hat{z}}$$

Now compute the fields from the Potentials

$$\vec\nabla V = \frac{\partial V}{\partial r}\hat r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat\theta = \frac{-P_0\omega}{4\pi\varepsilon_0 c}\left[\cos\theta\left(\frac{-1}{r^2}\sin(\omega(t-\tfrac{r}{c})) - \frac{\omega}{rc}\cos(\omega(t-\tfrac{r}{c}))\right)\hat r - \frac{\sin\theta}{r^2}\sin(\omega(t-\tfrac{r}{c}))\hat\theta\right]$$

$$\approx \frac{P_0\omega^2}{4\pi\varepsilon_0 c^2}\left(\frac{\cos\theta}{r}\right)\cos\left(\omega(t-\tfrac{r}{c})\right)\hat r$$

r very big compared to sin

$$\frac{\partial \vec A}{\partial t} = -\frac{\mu_0 P_0\omega^2}{4\pi r}\cos\left(\omega(t-\tfrac{r}{c})\right)(\cos\theta\,\hat r - \sin\theta\,\hat\theta)$$

$$\boxed{\vec E = -\vec\nabla V - \frac{\partial \vec A}{\partial t} = \frac{-\mu_0 P_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left(\omega(t-\tfrac{r}{c})\right)\hat\theta}$$

$$\vec\nabla\times\vec A = \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat\phi = \frac{-\mu_0 P_0\omega}{4\pi r}\left[\frac{\omega}{c}\sin\theta\cos\left(\omega(t-\tfrac{r}{c})\right) + \frac{\sin\theta}{r}\sin\left(\omega(t-\tfrac{r}{c})\right)\right]\hat\phi$$

$1/r^2$ so

$$\boxed{\vec B = \vec\nabla\times\vec A = \frac{-\mu_0 P_0\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left(\omega(t-\tfrac{r}{c})\right)\hat\phi}$$

- these represent monocromatic waves of frequency ω traveling radially, $\frac{E_0}{B_0} = c$
- spherical waves, amplitude decreases like $1/r$

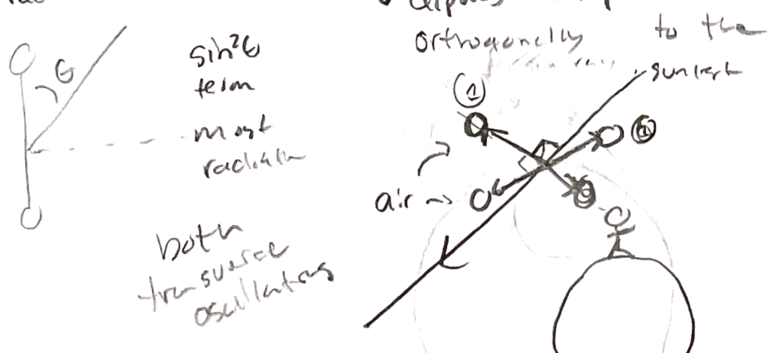
$$\vec S(\vec r, t) = \frac{1}{\mu_0}(\vec E\times\vec B) = \frac{\mu_0}{c}\left(\frac{P_0\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left(\omega(t-\tfrac{r}{c})\right)\right)^2\hat r, \quad \frac{1}{T}\int_0^T \vec S\, dt$$

$$\boxed{\langle S\rangle = \left(\frac{\mu_0 P_0^2\omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat r}$$

$$\langle P\rangle = \int\langle\vec S\rangle\cdot d\vec a = \frac{\mu_0 P_0^2\omega^4}{32\pi^2 c}\int\frac{\sin^2\theta}{r^2}r^2\sin\theta\, d\theta\, d\phi = \boxed{\frac{\mu_0 P_0^2\omega^4}{12\pi c}}$$

Ex

- Strong dependence of ω in the $\langle P\rangle = \frac{\mu_0 P_0^2}{12\pi c}\omega^4$ is what accounts for blue sky
- sunlight passing through atmosphere stimulates atoms to oscilate like tiny dipoles
- Sunlight → white light → broad range of frequencies
- Energy radiated by dipoles is stronger because of ω^4 term, blue, purple have higher ω than red.



- dipoles being stimulated by the sunlight oscillate orthogonally to the rays.

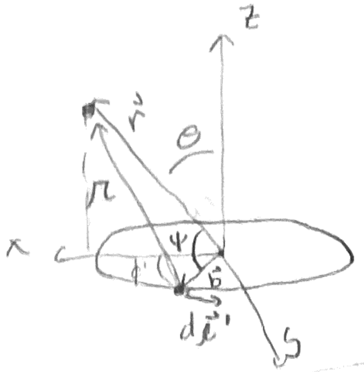
① oscillates straight on, $\theta = 0$ so no light seen

② oscillates ⊥ $\theta = \pi/2$ so see it
* radiates big" and stronger, bec of ω^4 term, big ω → small λ
* more power radiated by big ω, small λ

* If the light is being reradiated by all the light in the sky, if it goes through a lot of air, then most the blue light is scattered away!!

Magnetic Dipole Radiation

$I(t) = I_0 \cos(\omega t)$, oscillating magnetic dipole

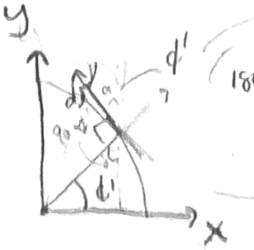


$\hookrightarrow \vec{m} = \pi b^2 I(t) = m_0 \cos(\omega t)\,\hat{z}$, $m_0 = \pi b^2 I_0$

(Not charged) $\rightarrow \nabla V = 0$

[Just need to calculate $\vec{A}$] $\rightarrow \vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{I_0\cos(\omega(t-\frac{\mathscr{R}}{c}))}{\mathscr{R}}\,dl'$

$d\vec{l}' = b\cos\phi'\,d\phi'\,\hat{y}$ (for a point directly above the x-axis, has to point in y dir (direction of current))



→ so indeed $b\cos\phi' = d\vec{l}\cdot\hat{y}$

$180 = 90 - \phi' + 90 + \alpha$

$\phi' = \alpha$

$\hookrightarrow \vec{A}(\vec{r},t) = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\frac{\cos(\omega(t-\frac{\mathscr{R}}{c}))}{\mathscr{R}}\cos\phi'\,d\phi'$

"get all contributions along x-axis!"

$\mathscr{R} = \sqrt{r^2 + b^2 - 2rb\cos\psi}$ and $\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z}$

$\vec{b} = b\cos\phi'\,\hat{x} + b\sin\phi'\,\hat{y}$

only x-comp!

$\vec{r}\cdot\vec{b} = rb\sin\theta\cos\phi'$

$= rb\cos\psi$

$\rightarrow \boxed{\mathscr{R} = \sqrt{r^2 + b^2 - 2rb\sin\theta\cos\phi'}}$

Approximation 1: $b \ll r$, want the loop very small so its like a dipole!

$\mathscr{R} \approx r\left(1 - \frac{b}{r}\sin\theta\cos\phi'\right) \rightarrow \frac{1}{\mathscr{R}} \approx \frac{1}{r}\left(1 + \frac{b}{r}\sin\theta\cos\phi'\right)$ (taylor expansions)

$\cos(\omega(t - \frac{\mathscr{R}}{c})) = \cos(\omega(t - \frac{r}{c}) + \frac{\omega b}{c}\sin\theta\cos\phi') = \cos(\omega(t-\frac{r}{c}))\cos(\frac{\omega b}{c}\sin\theta\cos\phi') - \sin(\omega(t-\frac{r}{c}))\sin(\frac{\omega b}{c}\sin\theta\cos\phi')$

$b \ll \frac{c}{\omega}$, wavelength long compared to radius of dipole:

$\cos(\omega(t - \frac{\mathscr{R}}{c})) \approx \cos(\omega(t - \frac{r}{c})) - \frac{\omega b}{c}\sin\theta\cos\phi'\sin(\omega(t-\frac{r}{c}))$

(Now insert into integral) $\rightarrow \vec{A}(\vec{r},t) \approx \frac{\mu_0 I_0 b}{4\pi r}\hat{y}\int_0^{2\pi}\left(\cos(\omega(t-\frac{r}{c})) + b\sin\theta\cos\phi'\,\frac{1}{r}\left(\cos(\omega(t-\frac{r}{c})) - \frac{\omega}{c}\sin(\omega(t-\frac{r}{c}))\right)\right)\cos\phi'\,d\phi'$

$\int_0^{2\pi}\cos\phi'\,d\phi' = 0$ $\quad \int_0^{2\pi}\cos^2\phi'\,d\phi' = \pi$

$\vec{A}(r,\theta,t) = \frac{\mu_0 m_0}{4\pi}\frac{\sin\theta}{r}\left(\frac{1}{r}\cos(\omega(t-\frac{r}{c})) - \frac{\omega}{c}\sin(\omega(t-\frac{r}{c}))\right)\hat{\phi}$

"throw out $1/r$"

For $r \gg \frac{c}{\omega}$: $\boxed{\vec{A}(r,\theta,t) = -\frac{\mu_0 m_0 \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin(\omega(t-\frac{r}{c}))\,\hat{\phi}}$

in general $\vec{A}$ is in $\hat{\phi}$

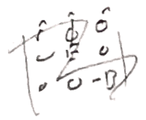
Now get the fields:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left(\omega\left(t - \tfrac{r}{c}\right)\right)\hat{\phi}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos\left(\omega\left(t - \tfrac{r}{c}\right)\right)\hat{\theta}$$ (using $r \gg \frac{c}{\omega}$

[And mutually perpendicular!] $\rightarrow$ $\frac{E_0}{B_0} = c$, in phase $\vec{E}\times\vec{B} = \hat{r}$

Same as electric dipole radiation but $\vec{B}$ is $\hat{\theta}$ and E is $\hat{\phi}$ instead of other way around



$$\vec{S}(\vec{r},t) = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{\mu_0}{c}\left(\frac{m_0\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left(\omega\left(t-\tfrac{r}{c}\right)\right)\right)^2\hat{r}$$

$$\langle\vec{S}\rangle = \left(\frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3}\right)\frac{\sin^2\theta}{r^2}\hat{r}$$

$$\langle P\rangle = \int\langle\vec{S}\rangle\cdot d\vec{a} = \frac{\mu_0 m_0^2\omega^4}{32\pi^2 c^3}\int_{\text{sphere}}\frac{\sin^2\theta}{r^2}r^2\sin\theta\, d\theta\, d\phi = \boxed{\frac{\mu_0 m_0^2\omega^4}{12\pi c^3}}$$

[Lets look at ratios of power] $\rightarrow$ $\frac{P_{magnetic}}{P_{electric}} = \left(\frac{m_0}{p_0 c}\right)^2$ $p_0 = qd \rightarrow I_0 = q_0\omega$; $m_0 = \pi b^2 I_0$ let $d = \pi b$ for comparison

$$\frac{P_{mag}}{P_{elec}} = \left(\frac{I_0\pi b^2}{q\pi b c}\right)^2 = \left(\frac{I_0\pi b^2}{\pi b}\frac{\omega}{I_0 c}\right)^2 = \left(\frac{\omega b}{c}\right)^2$$ $\leftarrow$ we said this was small in approx.

Radiation Reaction

- accelerating charge radiates
  - ↳ that energy comes at the expense of kinetic energy

→ Under the influence of a force a charged particle accelerates less than a neutral one of the same mass → The radiation exerts a force $\vec{F}_{rad}$ back on the charge, recoil force

Deriving the Radiation Reaction

[Total Power Radiated by an acc particle] → $P = \frac{\mu_0 q^2 a^2}{6\pi c}$ → Conservation of energy suggests this is also the rate the particle loses energy → $\vec{F}_{rad} \cdot \vec{v} = -P = -\frac{\mu_0 q^2 a^2}{6\pi c}$

↳ This is actually wrong because when calculating P we were looking at the energy radiated away, But there's still energy in the velocity fields that we discarded. (velocity fields just dont transport it to ∞)

* as a particle accelerates + decelerates energy is exchanged through the particles kinetic Energy + the velocity fields + radiated away by acc fields

↳ $\vec{F}_{rad} \cdot \vec{v} = -P_{rad}$ accounts for the energy radiated away

[Energy lost by the particle must equal the energy carried away by radiation plus extra energy in the velocity fields] → $\vec{E} = \vec{E}_{vel} + \vec{E}_a$ $\quad U \propto \vec{E}\cdot\vec{E}$ so

$$E^2 = \underbrace{E_v^2 + 2\vec{E}_v \cdot \vec{E}_a}_{\text{these terms go like } 1/r^{3+}} + E_a^2$$

Is ~~this saying~~ that accelerative charge or moving charge, puts ~~energy into its~~

yes!

(or circular motion (spring

Consideration of periodic motion

- consider a time interval where a system returns to its original state at both ends
  - ↳ energy in velocity fields is the same at both ends
  - ↳ the only energy that get away is the acceleration fields

$$\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \vec{v}\, dt = \frac{-\mu_0 q^2}{6\pi c}\int_{t_1}^{t_2} a^2\, dt \;,$$

[state of the sys is the same at t_1 + t_2] → $\int_{t_1}^{t_2} a^2\, dt = \int_{t_1}^{t_2} \frac{d\vec{v}}{dt}\cdot\frac{d\vec{v}}{dt}\, dt = \cancel{\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\right)\Big|_{t_1}^{t_2}}^{\,0} - \int_{t_1}^{t_2} \frac{d^2\vec{v}}{dt^2}\cdot\vec{v}\, dt$

Boundary term drops bc, $\vec{v}$ and $\vec{a}$ are the same at t_1 + t_2

$$\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \vec{v}\, dt = \frac{+\mu_0 q^2}{6\pi c}\int_{t_1}^{t_2} \dot{\vec{a}} \cdot \vec{v}\, dt \rightarrow \int_{t_1}^{t_2}\left(\vec{F}_{rad} - \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}\right)\cdot\vec{v}\, dt = 0$$

↳ $\boxed{\vec{F}_{rad} = \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}}$

Radiation Damping of a charged particle on a spring

$m\ddot{x} = F_{spring} + F_{rad} + F_{driving} = -m\omega_0^2 x + m\tau\dddot{x} + F_{driving}$, $\tau = \frac{\mu_0 q^2}{6\pi c}$

Particle oscillates at $x(t) = x_0\cos(\omega t + \phi)$

↳ $\dddot{x} = -\omega^2\dot{x}$, let $\gamma = \omega^2\tau$

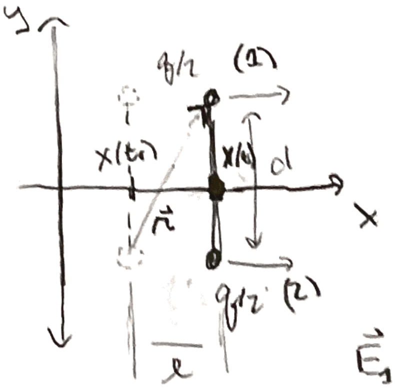
↳ $m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = F_{driving}$

Mechanism Responsible for Radiation Reaction

- point charge fields → ∞ at center so consider distribution of charge + take the limit
- in general EM force on one part (A) on another (B) is not equal + opposite to force of B on A

↳ If distribution is divided up into infinitesimal pairs the result is a net force of the charge on itself. (self-force)

$\boxed{\text{Consider a dumbell of charge}}$



$\vec{E}_{\text{at 1 due to 2}} = \vec{E}_1 = \frac{(q/2)}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left[(c^2 + \vec{\mathscr{r}}\cdot\vec{a})\vec{u} - (\vec{\mathscr{r}}\cdot\vec{u})\vec{a}\right]$, remember this tells you the field at a point due to signal emitted at the retarded time

$\vec{u} = c\hat{\mathscr{r}}$, $\vec{\mathscr{r}} = \ell\hat{x} + d\hat{y}$, [V=0 at rest at the retarded time]

↳ $\vec{\mathscr{r}}\cdot\vec{u} = c\mathscr{r}$ and $\vec{\mathscr{r}}\cdot\vec{a} = \ell a$ and $\mathscr{r} = \sqrt{\ell^2+d^2}$

↳ Only care about x-comp, y-components will cancel

↳ Same reason for magnetic forces

$\vec{E}_1 = \frac{q/2}{4\pi\epsilon_0}\frac{\mathscr{r}}{(c\mathscr{r})^3}\left((c^2+\ell a)c\hat{\mathscr{r}} - (c\mathscr{r})\vec{a}\right)$, $\hat{\mathscr{r}}\cdot\hat{x} = \frac{\vec{\mathscr{r}}}{\sqrt{\ell^2+d^2}}\cdot\hat{x} = \frac{\ell}{\mathscr{r}}$

↳ $E_{1x} = \frac{q}{8\pi\epsilon_0}\frac{\mathscr{r}}{(c\mathscr{r})^3}\left((c^2+\ell a)\frac{c\ell}{\mathscr{r}} - c\mathscr{r}a\right) = \frac{q}{8\pi\epsilon_0}\frac{1}{c^3\mathscr{r}^2}\left(\frac{c^3\ell}{\mathscr{r}} + \frac{ca\ell^2}{\mathscr{r}} - c\mathscr{r}a\right)$

$= \frac{q}{8\pi\epsilon_0}\frac{1}{c^2\mathscr{r}^2}\left(\frac{c^2\ell}{\mathscr{r}} + \frac{a\ell^2}{\mathscr{r}} - \frac{\mathscr{r}^2 a}{\mathscr{r}}\right)$, $c^2\ell - a\ell^2 - (\ell^2+d^2)a$

$c^2\ell - ad^2$

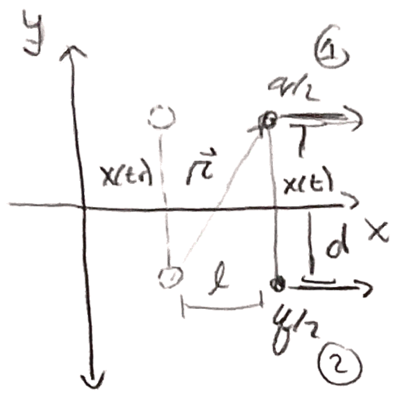
$$\boxed{E_{1x} = \frac{q}{8\pi\epsilon_0 c^2}\frac{(\ell c^2 - ad^2)}{(\ell^2+d^2)^{3/2}}}$$

Same E field in x-dir for $\vec{E}_{\text{at 2 due to 1}} \Rightarrow E_{2x}$

The total force on itself is $\frac{q}{2}E_1 + \frac{q}{2}E_2$

$\vec{F}_{self} = \frac{q}{2}(\vec{E}_1 + \vec{E}_2)$ → $\vec{F}_{self} = \frac{q^2}{8\pi\epsilon_0 c^2}\frac{(\ell c^2 - ad^2)}{(\ell^2+d^2)^{3/2}}\hat{x}$

Mechanism of Radiation Reaction



$$\boxed{\vec{F}_{self} = \frac{q^2}{8\pi\epsilon_0 c^2}\frac{(lc^2 - ad^2)}{(l^2+d^2)^{3/2}}\hat{x}}$$

• we want to let $d \to 0$ so taylor expand

$r^2 = l^2 + d^2 \to d = \sqrt{r^2 - l^2}$, [notice the following trick if we expand $x(t)$ about t_r]

→ $v = 0$ at t_r

$$x(t) = x(t_r) + \dot{x}(t_r)(t-t_r) + \frac{1}{2}\ddot{x}(t_r)(t-t_r)^2 + \frac{1}{3!}\dddot{x}(t_r)(t-t_r)^3 + \dots$$

↳ $x(t) - x(t_r) = l = \frac{1}{2}a(t-t_r)^2 + \frac{1}{6}\dot{a}(t-t_r)^3 + \dots$

Plug into d and pull out $c(t-t_r)$! w/ $r^2 = c^2(t-t_r)^2$

$$d = c(t-t_r)\sqrt{1 - \left(\frac{a(t-t_r)}{2c} + \frac{\dot{a}(t-t_r)^2}{6c} + \dots\right.} = c(t-t_r) - \frac{a^2}{8c}(t-t_r)^3 + (\;)(t-t_r)^4 + \dots$$ writes like

↳ tells us d in terms of T but we want it the other way
we expand it terms of d

$d \approx c(t-t_r) \to (t-t_r) \approx \frac{d}{c}$ → plug into $\frac{a^2(t-t_r)^3}{8c}$ term → $(t-t_r) \approx \frac{d}{c} + \frac{a^2 d^3}{8c^5}$

↳ $(t-t_r) = \frac{d}{c} + \frac{a^2}{8c^5}d^3 + (\;)d^4 + \dots$ [Plug this into $\vec{F}_{self}$]

$$l = \frac{1}{2}a(t-t_r)^2 + \frac{1}{6}\dot{a}(t-t_r)^3 + \dots = \frac{1}{2}\frac{a}{c^2}d^2 + \frac{\dot{a}}{6c^3}d^3 + (\;)d^4 + \dots$$

$c^2(t-t_r)^2 = d^2 + l^2$ → do expan of $\frac{1}{x^3}$, $x = \sqrt{d^2+l^2} = c(t-t_r)$

↳ $\vec{F}_{self} = \frac{q^2}{4\pi\epsilon_0}\left[-\frac{a}{4c^2 d} + \frac{\dot{a}}{12c^3} + (\;)d + \dots\right]\hat{x}$, that is the acceleration at retarded time

$a(t_r) = a(t) + \dot{a}(t)(t_r - t) + \dots = a(t) - \dot{a}(t)\frac{d}{c} + \dots$ (taylor expand about t)

$$\vec{F}_{self} = \frac{q^2}{4\pi\epsilon_0}\left[-\frac{a(t)}{4c^2 d} + \underbrace{\frac{\dot{a}(t)}{3c^3}} + (\;)d + \dots\right]\hat{x}$$

$F^{int}_{rad} = \frac{\mu_0 q^2 \dot{a}}{12\pi c}$, survives $d \to 0$ b/c no d dependence

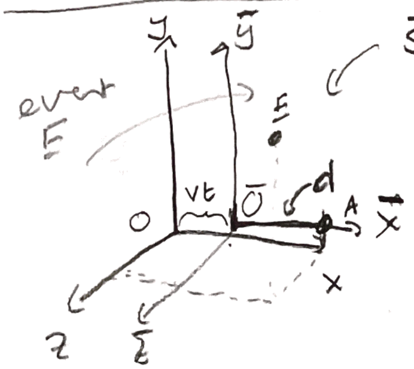
force between 1 and 2 still need force between each and itself

↳ $F_{rad} = \frac{\mu_0 q^2 \dot{a}}{6\pi c}$

The Lorentz Transformations

event: takes place at x, y, z, t

Galilean Transformation



$\bar{S}$ slides along x axis at speed v

*start the clock when origins coincide *

$$x = \underbrace{d}_{\bar{x}} + vt$$

$\bar{x}$ ← distance from origin for $\bar{O}$

* but not in relativity *

assumed lengths are same for all observers

so $\bar{x} = x - vt$

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = t$ ← assumed time is same for all observers

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformations

* instead substitute $d = \frac{1}{\gamma}\bar{x}$ (length contraction formula)

↳ $x = \frac{1}{\gamma}\bar{x} + vt$ → $\boxed{\bar{x} = \gamma(x - vt)}$

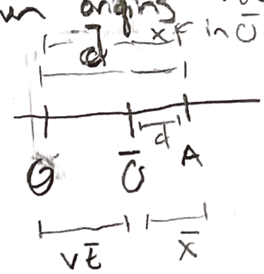
if $\bar{S}$ starts $\bar{t}$ when origins are lined up, O will be a distance $v\bar{t}$ from $\bar{O}$

$\bar{x} = \bar{d} - v\bar{t}$

↓

$\bar{d} = \frac{1}{\gamma}x$

$\bar{O}$ sees x contracted



$\bar{d}$ is length $\bar{O}$ sees O is from A.

$x \neq \bar{d}$ see $\bar{O}$ is looking at a moving frame,

$x = \gamma(\bar{x} + v\bar{t})$

↑

we know this

so

$$\boxed{\begin{aligned} \bar{x} &= \gamma(x - vt) \\ \bar{y} &= y \\ \bar{z} &= z \\ \bar{t} &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}}$$

$\boxed{\text{ex}}$

event A	event B	
$x_A = 0$	$x_B = b$	simultaneous in S
$t_A = 0$	$t_B = 0$	

event A

$\bar{x}_A = \gamma(0 - 0) = 0$

$\bar{t}_A = \gamma(0 - \frac{v}{c^2} 0) = 0$

event B

$\bar{x}_B = \gamma(b + 0) = \gamma b$

$\bar{t}_B = \gamma(0 - \frac{v}{c^2} b) = -\frac{\gamma v}{c^2} b$

$\hookrightarrow \bar{t} = -\gamma \frac{v}{c^2} x \leftarrow$ generally

$x < 0$ $\quad \bar{t}$ is ahead

$x > 0$ $\quad \bar{t}$ is behind

Suppose S watches a single clock in $\bar{S}$ frame at some constant $\bar{x}$ distance

$$t_i = \gamma(\bar{t}_i + \frac{v}{c^2} \underbrace{\bar{x}}_{\text{constant}})$$

$$t_f = \gamma(\bar{t}_f + \frac{v}{c^2} \bar{x})$$

$$t_f - t_i = \Delta t = \gamma \Delta \bar{t}$$

Proper Time and Proper Velocity

$v = \frac{d}{t}$

$d\tau = \sqrt{1 - u^2/c^2}\, dt$, $d\tau$ = is the proper time or the time associated w the moving object

u = velocity of me

dt = what someone watching you say

↳ [τ time my watch registers
t = time a person watching sees my watch go] it's going more slowly

* On a flight to LA and pilot announces planes velocity is $\frac{4}{5}c$

↳ [Pilot means displacement / time] → $\vec{u} = \frac{d\vec{l}}{dt}$, [velocity relative to the ground $d\vec{l}$, dt are measured by ground observer]

* good to know for being on time to an appointment

* But If I want to know if I will be hungry I would want to know my distance covered per unit proper time.

$d\vec{\bar{l}} = \gamma\, d\vec{l}$

length I see, $d\vec{l}$ ground distance, $\frac{4}{5}c$ is ground velocity

Proper Velocity: $\vec{\eta} \equiv \frac{d\vec{l}}{d\tau}$, $\vec{\eta} = \gamma \vec{u}$

could do l/η to see how long trip is

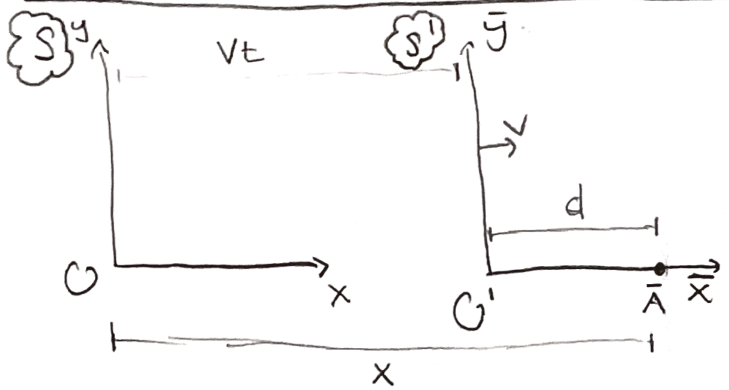
velocity of airplane relative to ground person

$\eta^\mu \equiv \frac{dx^\mu}{d\tau}$, $\eta^0 = \frac{dx^0}{d\tau} = c\frac{dt}{d\tau} = \frac{c}{\sqrt{1-u^2/c^2}}$

Lorentz Transformation

definition of an event: (x, y, z, t) → just now worry about (x, t)
↳ happens at a specific place at a specific time

Modification of the Galilean Transformation



$x = d + vt$ $\xrightarrow{\text{galilean assertion}}$ $d = \bar{x}$

*really O would see d contracted while O' would just see it as $\bar{x}$

↳ $\bar{x} = x - vt$

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = t$ ← clocks start at similar times

Applying the postulates of Special Relativity:

speed of light is the same for all inertial observers regardless of relative motion

↳ O' would measure c and O too

↳ [Galilean transformation needs a modification] → 1) $\bar{x}' = \gamma(x - vt)$ 2) $x = \gamma(\bar{x}' + v\bar{t})$ — lengths need to change by a factor γ to keep c = same

apparently $t \neq \bar{t}$, $x \neq \bar{x}$

[Light pulse in both reference frames] → O says light took t seconds to travel x meters: $x = ct$; O' says light took $\bar{t}$ seconds to travel $\bar{x}'$ meters: $\bar{x} = c\bar{t}$ $\Big\}\ \frac{x}{t} = \frac{\bar{x}}{\bar{t}}$

[multiply 1) and 2) together] → $x\bar{x} = \gamma^2(x\bar{x} + xv\bar{t} - \bar{x}vt - v^2t\bar{t})$ now plug in $x = ct$, $\bar{x} = c\bar{t}$

↳ $c^2 t\bar{t} = \gamma^2(c^2 t\bar{t} + vct\bar{t} - vc\bar{t}t - v^2t\bar{t})$ → $c^2 = \gamma^2(c^2 + vc - vc - v^2)$

↳ $\gamma^2 = \frac{c^2}{c^2 - v^2}$ → $\boxed{\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}}$

$x = \gamma(\bar{x} + v\bar{t})$ → $x - \gamma\bar{x} = \gamma v\bar{t}$ $\bar{x} = \gamma(x - v$

↳ $\bar{t} = \frac{x}{\gamma v} - \frac{\bar{x}}{v}$ → $\bar{t} = \gamma\left(\frac{x}{\gamma^2 v} - \frac{\bar{x}}{v\gamma}\right)$,

↳ $\bar{t} = \gamma\left(\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right) = \gamma\left(\frac{x}{v}\underbrace{\left(\frac{1}{\gamma^2} - 1\right)}_{-v^2/c^2} + t\right)$

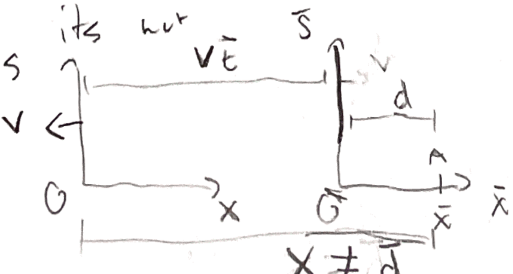
$\bar{t} = \gamma(t - \frac{v}{c^2}x)$

Now $\boxed{\bar{x} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(x - vt)}$ and

and $\boxed{\bar{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}x\right)}$

To understand: $x = \gamma(\bar{x} + v\bar{t})$

its not



$d = \bar{O}$ to $\bar{A}$ measured in S

$\bar{x} = \bar{O}$ to $\bar{A}$ measured in $\bar{S}$ ← (measured) at rest in $\bar{S}$ but moving in S

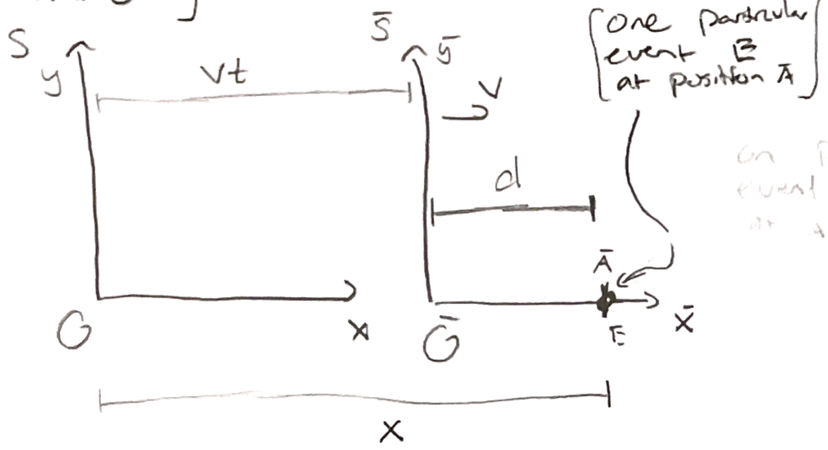
↳ calls for length contraction! $d = \frac{1}{\gamma}\bar{x}$ → $\bar{x} = \gamma(x - vt)$

POV of $\bar{S}$ at $\bar{t}$:

all on line

Continued

[O is rest Frame]



d is the distance O to A measured in S

$\bar{x}$ is the distance $\bar{O}$ to $\bar{A}$ measured in $\bar{S}$

*measured in 2 different frames

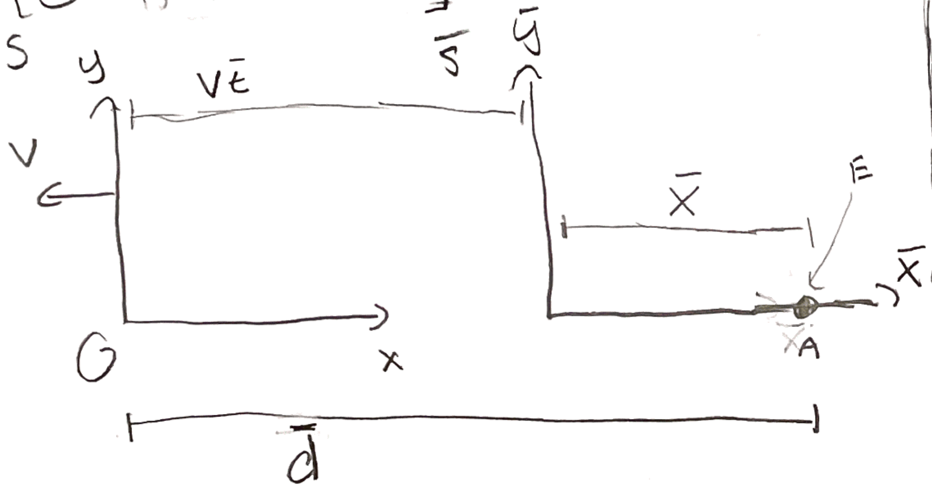
[$\bar{x}$ is at rest in $\bar{S}$, obviously but not to S, its moving and thus must be length contract

$\boxed{d = \frac{1}{\gamma}\bar{x}}$ → plug in to galilean for $x = d + vt$ 1) in my derivation

↳ $\boxed{\bar{x} = \gamma(x - vt)}$

*depicted at time $\bar{t}$ *

[O' is rest Frame]



[t and $\bar{t}$ represent the same physical instant at E but not elsewhere bec of simultaneity]

*assume $\bar{S}$ starts clock when origins coincide then at $\bar{t}$ O will be $v\bar{t}$ from $\bar{O}$

↳ $\bar{x} = \bar{d} - v\bar{t}$ (more intuitively $\bar{d} - \bar{x} = v\bar{t}$)

But whats $\bar{d}$

↳ $\bar{d}$ is the distance from O to A in $\bar{S}$ at time $\bar{t}$ ↙ A is when E occurs

↳ classical says $x = \bar{d}$ but not,

↳ x is distance from O to A in S ← x is at rest in S, so its moving for $\bar{S}$ then length contraction again

↳ $\bar{d}$ is O to A in $\bar{S}$

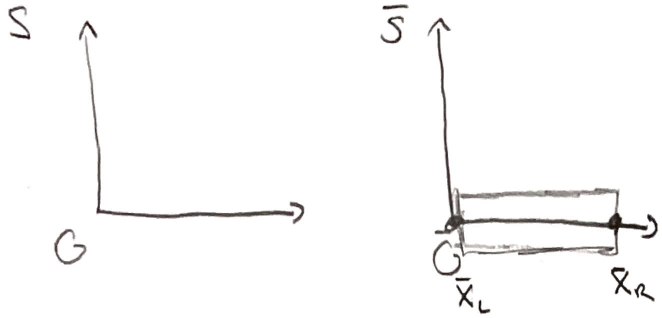
$\boxed{\bar{d} = \frac{1}{\gamma}x}$

Thus $\bar{x} = \frac{1}{\gamma}x - v\bar{t}$ → $\boxed{x = \gamma(\bar{x} + v\bar{t})}$

The rest follows from my proof, finding γ and transformation

Using Lorentz Transformation | Time dilation, length contraction, vel addition

Length Contraction



G is making a measurement of a stick that's at rest in $\bar{G}$

↳ $\bar{G}$ measures $\bar{X}_R - \bar{X}_L = \Delta\bar{X}$

[But S measures the X_L and X_R too, how does it change?]

Well

$$\bar{X}_R = \gamma(X_R - Vt_R)$$
$$\bar{X}_L = \gamma(X_L - Vt_L)$$

So if I know what X_R & t_R are when measured, in S, we can transform to $\bar{S}$

↳ S would make the length measurement at the same time

↳ $\bar{X}_R - \bar{X}_L = \gamma(X_R - X_L) - \gamma V\cancel{(t_R - t_L)}$ same time in S

↳ $\Delta\bar{X} = \gamma\Delta X \rightarrow \Delta x = \frac{1}{\gamma}\Delta\bar{x}$ ← rest length

length measured by S

Time Dilation and Simultaneity

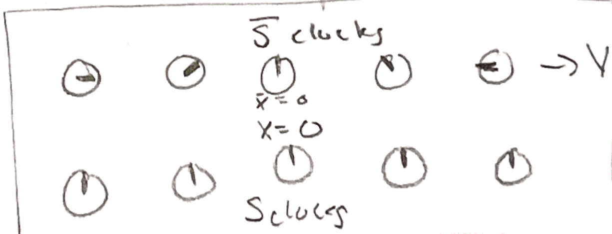
Event A	Event B
$X_A = 0,\ t_A = 0$	$X_B = b,\ t_B = 0$] simultaneous in S
$\bar{X}_A = \gamma(0 - V\cdot 0) = 0$	$\bar{X}_B = \gamma b - 0 = \gamma b$
$\bar{t}_A = \gamma(0 - \frac{V}{c^2}0) = 0$	$\bar{t}_B = \gamma(0 - \frac{Vb}{c^2}) = -\gamma\frac{V}{c^2}b$
	* NOT simultaneous in $\bar{S}$

transform frame S to $\bar{S}$

Lorentz transformation

$$\bar{X} = \gamma(X - Vt)$$
$$\bar{t} = \gamma(t - \frac{V}{c^2}X)$$

Event A, Event B: to S they explode at same time $t_A = t_B = 0$ but to $\bar{S}$ they don't



S [now examining all clocks in $\bar{S}$] → $\bar{t}_B = -\gamma\frac{V}{c^2}X$ [looking at how time changes w/ different X]

further away more/less its behind/ahead

→ for negative X, clock is ahead

→ for positive X, clock is behind

note) * for $\bar{S}$ its S clocks that are out of sync

[Suppose we look at one clock in $\bar{S}$, at one point in space] → [What time does S see on the clock in $\bar{S}$] → use $t = \gamma(\bar{t} + \frac{V}{c^2}\bar{x})$

↳ elapsed time

$$\Delta t = \gamma\Delta\bar{t} + \frac{\gamma V}{c^2}\Delta\bar{x}$$

$\Delta\bar{x} = 0$ so

→ $\Delta t = \gamma\Delta\bar{t}$

(Δt ↑ what S sees; $\Delta\bar{t}$ ↑ rest time)

such $\bar{x}_2 - \bar{x}_1 = 0$

$\Delta\bar{x} = 0$

↳ $\boxed{\Delta\bar{t} = \frac{1}{\gamma}\Delta t}$

Velocity Addition + example

• Suppose a particle w/ speed u is moving in S (u is measured by S)

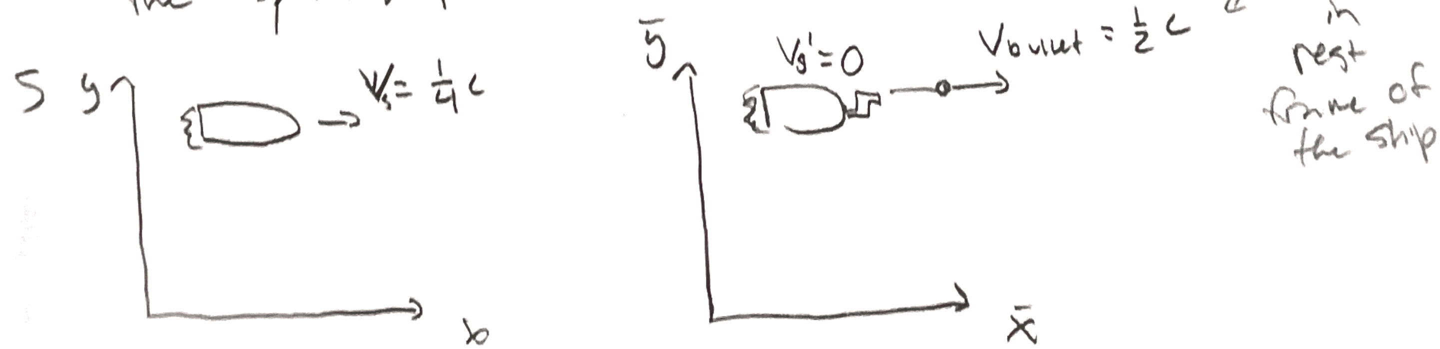
↳ $u = \frac{dx}{dt}$ → [transform to $\bar{S}$] → $d\bar{x} = \gamma(dx - v\,dt)$, $d\bar{t} = \gamma(dt - \frac{v}{c^2}dx)$ → $\frac{d\bar{x}}{d\bar{t}} = \bar{u}$

$$\boxed{\bar{u} = \frac{u - v}{1 - uv/c^2}}$$

u = speed of particle in S, $\bar{u}$ = speed of particle in $\bar{S}$

v = speed of relative frames

ex) Say bullet is fired from a spaceship at $\frac{1}{2}c$ at the spaceship moving at $\frac{1}{4}c$ what is the bullets velocity relative to the planet that measures the spaceships velocity



We want u, the speed of the particle in S

↳ reasons $u = \frac{\bar{u} + v}{1 + \frac{\bar{u}v}{c^2}} = \frac{\frac{1}{2}c + \frac{1}{4}c}{1 + \frac{1}{c^2}\frac{1}{8}c^2} = \frac{3/4}{9/8}c = \frac{3}{4}\cdot\frac{8}{9}c = \frac{2}{3}c$