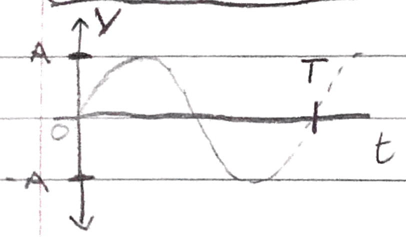
AC, capacitance review

Circuits notes



sinusoidal signal

height / time "t" — phase — radians / sec

(1) $V = A\sin(2\pi f\, t + \phi) = A\sin(\omega t + \phi)$

↑ how sinusoidal, $T = \frac{1}{f}$

Ways to specify amplitude → A, peak to peak → $A_{pp} = 2A$, $A_{rms} = \frac{A}{\sqrt{2}}$

(ex) power dissipated in a resistor given the sinusoidal varying voltage,

↳ can't use $P = IV = \frac{V^2}{R} = I^2R$ since voltage is changing constantly.

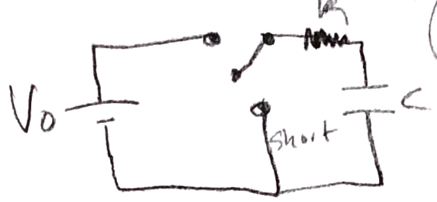
↳ $\boxed{\text{instead calculate the Average power}}$ $P = \frac{1}{T}\int_0^T \frac{V^2}{R}\,dt =$ plug in (1) and compute $= \frac{A^2}{2R} = \frac{A_{rms}^2}{R}$

*remember A corresponds to peak voltage so we can use $P = \frac{V^2}{R}$ If we only use A_{rms}.

$\boxed{\text{Capacitor Review}}$ *a capacitor is 2 conducting plates $\left(\frac{\text{coulombs}}{\text{volt}}\right)$

- for parallel plates $C = \varepsilon\frac{A}{d}$, A is area, d is distance, ε is dielectric constant
- a capacitor is a charge storage device
- when a voltage V is applied, a charge Q is stored
  ↳ $\boxed{Q = CV}$
- In electronics we are usually concerned w/ currents → $I = \frac{dQ}{dt}$, $\frac{dQ}{dt} = C\frac{dV}{dt} = I$

Series $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$ | Parallel $C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$



(Switched RC circuit)

→ can be switched on or connected to a short

→ if connected to short $V=0$.

RC circuits Review

KVL $V = IR + \frac{Q}{C}$, $V = V_0$ or 0

(take time derivative) $0 = R\frac{dI}{dt} + \frac{I}{C}$ ← same for either case ⇒ $\boxed{\frac{dI}{I} = -\frac{dt}{RC}}$

integration: $\ln(I) = -\frac{t}{RC} + K$ ← constant of integration

$I = e^{\left(-\frac{t}{RC}+K\right)} = e^{K} e^{-t/RC} = \boxed{I_0 e^{-t/RC} = I}$ gonna call it this for now

[Voltage across resistor is just IR] → [Voltage across capacitor?] → $V_c = ?$

(either V_0 or 0)

KVL: $V_c = V - IR$

plugging in

eq 1 → $\boxed{V_c = V - I_0 R e^{-t/RC}}$

[or general form when $V_1 = -I_0 R$ and $V = V_2$ are constant] $V_c = V_1 e^{-t/RC} + V_2$

Charging → V_c is 0 at $t=0$ and $V=V_0$	discharging → now connect short at $t=0$, $V_c = V_0$ and $V=0$
from eq 1: $0 = V_0 - I_0 R$ so $I_0 = \frac{V_0}{R}$	$V_0 = 0 - I_0 R$ → $I_0 = -\frac{V_0}{R}$ so
then $I = \frac{V_0}{R} e^{-t/RC}$ and $V_c = V_0\left[1 - e^{-t/RC}\right]$	$I = -\frac{V_0}{R} e^{-t/RC}$ and $V_c = V_0 e^{-t/RC}$
	current is backwards; voltage is decreasing

[ALTERNATING Current]

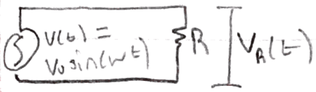
whether or not use imaginary or trig get same answer but look different

($V_{in} = V_p \sin(\omega t)$) ($V_{in} = V_p \cos(\omega t)$)

→ You can have a sin drive or a cos drive, both give you different final solutions. They will have the same magnitude though. $V(t) = \sin(\omega t)$ for all

$I_0 \sin(\omega t + \phi)$

Pure Resistive load



KVL!

$V(t) - V_R(t) = 0$, $V(t) = I_R(t) R = V_R(t)$ $V_0 = V_{R0}$ too

also: $I(t) = \frac{V_R(t)}{R} = \frac{V_{R0}}{R} \sin(\omega t) = I_{R0} \sin(\omega t)$

I, V are in phase → $\phi = 0$

$I_{R0} = \frac{V_{R0}}{R}$, doesn't depend on ω → $X_R = 0$ not sure

Pure Capacitive load

KVL: $V(t) - V_C(t) = 0$ → $V(t) = \frac{Q(t)}{C}$ ($V(t) = V_C(t)$)

$Q(t) = CV(t) = CV_C(t) = CV_{C0} \sin(\omega t)$

$\frac{dQ(t)}{dt} = I_C(t) = \omega C V_{C0} \cos(\omega t)$ {convert to sin bec V is sin}

$I_C(t) = \omega C V_{C0} \sin(\omega t + \frac{\pi}{2})$ $\phi = -\frac{\pi}{2}$

↳ $I_{C0} = \omega C V_0$ → $X_C = \frac{1}{\omega C}$

$V_{C0} = I_{C0} X_C$

Pure Inductive load

KVL: $V(t) - V_L(t) = 0$, $V(t) - L\frac{dI}{dt} = 0$

$I_L(t) = \int \frac{V_0}{L} \sin(\omega t)\, dt = I_L(t) = \frac{-V_0}{\omega L} \cos(\omega t)$

using $-\cos(\theta) = \sin(\theta - \frac{\pi}{2})$ like before

$I_L(t) = \frac{V_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$, $\phi = +\frac{\pi}{2}$

↳ $I_{L0} = \frac{V_0}{\omega L}$ → $X_L = \omega L$

X_X = **Reactance** its the real value and its as if the C, L, R was a resistor by Ohms Law
↳ imaginary part of impedance, for AC current only, (how much device fights AC) (Resistor doesn't)

Impedance

$V(t) = V_p \cos(\omega t)$ → $\hat{V}(t) = V_p e^{j\omega t}$

$$\boxed{\hat{Z} = \frac{\hat{V}(t)}{\hat{i}(t)} = \frac{V_p}{I_p} e^{-j\theta_i}}$$

this waiting to be taken $Re()$. $Re(e^{i\theta}) = \cos\theta + 0$ proj onto Real axis

$Im(\hat{Z}) = X_{reactance}$

* can take Re() before or after derivative

$\frac{d}{dt} Re(e^{i\theta}) = Re(i e^{i\theta})$ prove by subbing, $e^{i\theta} = \cos\theta + i\sin\theta$

$\cos\theta = -\sin\theta$

$Z_R = R$ (no Img part)

Capacitor Impedance

$q(t) = CV(t)$ → $\frac{dq}{dt} = C\frac{dV(t)}{dt}$

$\hat{i}(t) = C\frac{d\hat{V}(t)}{dt}$ ← $\hat{V}(t) = V_p e^{j\omega t}$

$\hat{i}(t) = CV_p j\omega e^{j\omega t} = Cj\omega \hat{V}(t)$

$$\hat{Z}_C = \frac{\hat{V}(t)}{\hat{i}(t)} = \frac{1}{jC\omega}$$

Inductor Impedance

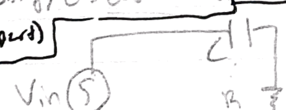
$V(t) = L\frac{di(t)}{dt}$, $\hat{i}(t) = I_p e^{j(\omega t + \phi)}$

$\frac{d\hat{i}(t)}{dt} = I_p j\omega e^{j(\omega t + \phi)} = j\omega \hat{i}(t)$

$\hat{V}(t) = Lj\omega \hat{i}(t)$

$$\hat{Z}_L = \frac{\hat{V}(t)}{\hat{i}(t)} = \hat{j}L\omega$$

RC Circuit w/ trig



$V_{in} = V_p \sin(\omega t)$
$I = I_p \sin(\omega t + \phi)$

(abhorrent dont do this way)

$\sin(\omega t + \phi) = \sin(\omega t)\cos\ldots$
$\cos(\omega t + \phi) = \cos(\omega t)\cos(\phi) \ldots$

KVL! $V_{in} = \frac{Q}{C} + IR$ → $\frac{dV_{in}}{dt} = R\frac{dI}{dt} + \frac{I}{C}$ plug eq's in above:

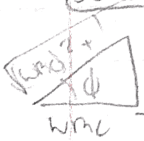
$\omega V_p \cos(\omega t) = R\omega I_p \cos(\omega t + \phi) + \frac{I_p}{C}\sin(\omega t + \phi)$ → Isolate I_p, ϕ using

$$\left(\cos\phi + \frac{1}{\omega RC}\sin\phi - \frac{V_p}{I_p R}\right)\cos(\omega t) + \left[-\sin\phi + \frac{1}{\omega RC}\cos\phi\right]\sin(\omega t) = 0$$

valid for all t's

$\boxed{\omega t = \frac{\pi}{2}}$ $-\sin\phi + \frac{1}{\omega RC}\cos\phi = 0$

$\tan\phi = \frac{1}{\omega RC}$



$\boxed{t = 0}$ $\cos\phi + \frac{1}{\omega RC}\sin\phi - \frac{V_p}{I_p R} = 0$ use right triangle

$\frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} + \frac{1}{\omega RC} \cdot \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{V_p}{I_p R}$ finally some ALG EBRA ...

$$I_p = \frac{\omega C V_p}{\sqrt{1 + (\omega RC)^2}}, \quad I = \frac{\omega C V_p}{\sqrt{1 + (\omega RC)^2}} \sin(\omega t + \phi), \quad \phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

→ could use $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ but that would be harder than trig

→ but $Re(e^{i\theta}) = \cos\theta$ is MUCH EASIER, on back.

$e^{j\theta} = \cos\theta + j\sin\theta$, $\hat{z} = |z| e^{j\theta}$

maybe not ⊥ Idk

RC w/ complex

KVL: $\boxed{R\frac{dI}{dt} + \frac{I}{C} = \frac{dV_{in}}{dt}}$

this step is semi illegal, but works out rigorously you would hold onto Re and pull constants

$V_{in} = V_p\cos(\omega t) = \mathrm{Re}(V_p e^{j\omega t})$

$I = I_p\cos(\omega t + \phi) = \mathrm{Re}(I_p e^{j(\omega t + \phi)}) = \mathrm{Re}(\hat{I}_p e^{j\omega t})$, a nice substitution of $\hat{I}_p = I_p e^{j\phi}$

plug V_{in}, I into KVL $\rightarrow R\hat{I}_p j\omega \cancel{e^{j\omega t}} + \frac{1}{C}\hat{I}_p \cancel{e^{j\omega t}} = j\omega V_p \cancel{e^{j\omega t}} \Rightarrow R\hat{I}_p j\omega + \frac{1}{C}\hat{I}_p = j\omega V_p$ (solve for $\hat{I}_p$)

$\hat{I}_p = \frac{\omega C V_p}{\omega RC - j}$; $\hat{z} = a + bj$; $a = \omega RC$, $b = -1$ $\rightarrow |z| = \sqrt{(\omega RC)^2 + 1} \rightarrow |z|e^{j\theta} = \hat{z} \rightarrow \hat{z} = \sqrt{(\omega RC)^2+1}\, e^{j\theta}$; $\tan\theta = -1/\omega RC$

then $\hat{I}_p = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}} e^{-j\theta} \rightarrow I = \mathrm{Re}(\hat{I}_p e^{j\omega t}) = \mathrm{Re}\left[\frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}} e^{-j\theta} e^{j\omega t}\right] = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}} \mathrm{Re}(e^{j(\omega t - \theta)})$

$I = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}\cos(\omega t - \theta)$, $\tan\theta = \frac{-1}{\omega RC}$ * different from trig bec cosine drive

GENERALIZATIONS

Real part of complex impedance → resistance

Complex part of complex impedance → reactance (X)

$\hat{Z}_{series} = \sum_i \hat{Z}_i$, $\frac{1}{\hat{Z}_{parallel}} = \sum_i \frac{1}{\hat{Z}_i}$

Generalized complex Ohms law: $\hat{V} = \hat{I}\hat{Z}$ bec of how we define $\hat{Z}$.

$\hat{Z} = R + jX = |\hat{Z}| e^{j\theta}$, $\theta = \tan^{-1}\left(\frac{X}{R}\right)$, $R = |z|\cos(\theta)$, $X = |z|\sin\theta$

Steps for any circuit:

1. $\hat{V} = \hat{I}\hat{Z}$ (all complex)
2. find $\hat{Z}_{tot}$
3. calculate $\hat{I} = \frac{\hat{V}}{\hat{Z}_{tot}}$
4. get in form $a + jb = \hat{z}$
5. convert to $|\hat{z}|e^{j\theta} = \hat{z}$
6. plug in $\hat{V}$
7. take real part of $\hat{I}$

RC again

② $Z_{tot} = R + \frac{1}{j\omega C}$

③ $\hat{I} = \hat{V}/(R + \frac{1}{j\omega C})$

④ $\hat{I} = \frac{\omega C \hat{V}}{\sqrt{(\omega RC)^2+1}} e^{-j\theta}$ using z components $\theta = \tan^{-1}\left(\frac{-1}{\omega RC}\right)$

⑤ $\hat{V}$ can be $\mathrm{Re}(V_p e^{j\omega t})$, $V_p\cos(\omega t)$, or $\mathrm{Im}(V_p e^{j\omega t})$, $\sin(\omega t)$

let's do $\cos\theta$ $\hat{I} = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}} e^{j(\omega t - \theta)}$

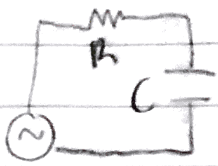
⑦ $I = \mathrm{Re}(\hat{I}) = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}\cos(\omega t - \theta)$, $\tan\theta = \frac{-1}{\omega RC}$

different than trig bec this is cos drive not sine, if you had sine:
$\sin(\omega t - \theta)$ same $\sin(\omega t + \phi)$
$\theta = \tan^{-1}(-1/\omega RC) \Leftrightarrow \phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$

Bonus side notes

- to get V_C, use $\frac{dQ}{dt} = I$ and integrate I then divide by C, or shortcut $V_C = \frac{I(t)}{\omega C}$
- fancy thing in RC circuit $\hat{V}_R = \frac{Z_R}{Z_R + Z_C}\hat{V}(t)$, voltage divider!

RC circuit as differentiator and Integrator



(works for all)

$\hat{v}(t) = V_p e^{j\omega t}$

$\hat{V}_C(t) = \hat{V}(t)\dfrac{\hat{Z}_C}{\hat{Z}_R + \hat{Z}_C}$ = (start w/ I: $\frac{dq}{dt}$ and integrate and divide by C for V_C)

$\hat{V}_R(t) = \hat{V}(t)\dfrac{Z_R}{Z_R + Z_C} = \hat{I}(t)R$

$\hat{Z}_R = R$, $\hat{Z}_C = 1/j\omega C$

differentiator

→ if $\boxed{|\hat{Z}_R| << |\hat{Z}_C|}$ for $\hat{V}_R(t) \approx \hat{V}(t)\dfrac{\hat{Z}_R}{\hat{Z}_C} = \hat{V}(t)\, j\omega RC$

→ if you look at $\dfrac{d\hat{v}(t)}{dt} = j\omega \hat{v}(t)$, so if Z_R is very small you can use RC factor to differentiate.

$$\boxed{\hat{V}_R(t) = \frac{d\hat{V}(t)}{dt}RC} = \hat{v}(t)\, j\omega RC \rightarrow \frac{d\hat{V}(t)}{dt} = \frac{1}{RC}\hat{V}_R(t)$$

Integrator

→ if $\boxed{|\hat{Z}_R| >> |\hat{Z}_C|}$ $\hat{V}_C(t) \approx \dfrac{\hat{Z}_C}{\hat{Z}_R}\hat{v}(t) = \hat{V}(t)\dfrac{1}{j\omega RC}$

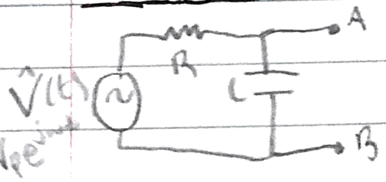
→ if you look at $\int \hat{v}(t)\,dt = \frac{1}{j\omega}\hat{v}(t)$ which is very similar to

$$\boxed{\hat{V}_C(t) = \frac{1}{RC}\int \hat{v}(t)\,dt = \frac{1}{j\omega RC}\hat{V}(t)} \qquad \int \hat{v}(t)\,dt = RC\,\hat{V}_C(t)$$

High Pass/Low Pass filters

LOW PASS:

$|V_{AB}| = \left|\dfrac{Z_C}{Z_R + Z_C}\right| |\hat{V}(t)| = \dfrac{V_p}{\sqrt{(\omega RC)^2 + 1}}$ (no cos part) (just amplitude part) ↳ $A\cos(\cancel{\omega t} - \theta)$



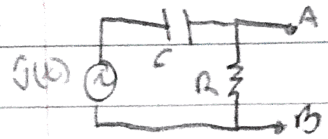
at $\omega \to 0$, $\lvert V_{AB}\rvert = \lvert\hat{V}\rvert$
at $\omega \to \infty$ $\lvert V_{AB}\rvert \to 0$

So at low frequencies voltage is passed but at high it blocked

(same)

HIGH PASS

$|V_{AB}| = \left|\dfrac{Z_R}{Z_C + Z_R}\right| |\hat{V}(t)| = V_p \dfrac{R\omega C}{\sqrt{(R\omega C)^2 + 1}}$ (no cos part just V_p)

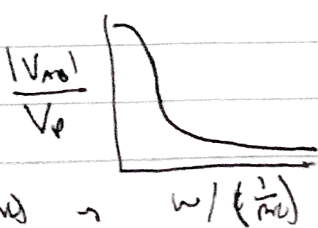


$\omega \to 0$, $\lvert V_{AB}\rvert = 0$
$\omega \to \infty$ $\lvert V_{AB}\rvert \to \lvert\hat{V}\rvert$

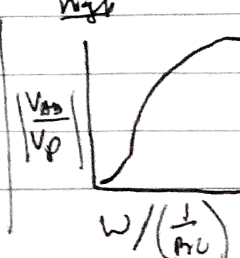
High frequency pass filter

$T(\omega_c) = \frac{1}{\sqrt{2}}$ (when you plug $\omega_c = \frac{1}{RC}$)

low



high



(ωRC) →

Transmission

(low): $T(\omega) = \dfrac{V_{out}}{V_{in}} = \dfrac{1}{\sqrt{(\omega RC)^2 + 1}} = \dfrac{V_{AB}}{V_p}$

*just output voltage V_{AB} w/ V_p (Amplitude)

(high): $T(\omega) = \dfrac{V_{out}}{V_{in}} = \dfrac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} = \dfrac{V_{AB}}{V_p}$

*output voltage is voltage divider (what's transmitted)

Decibels compares amplitudes:

for $P_{avg} = A^2/2R = \frac{A_{rms}}{R}$

$$dB = 20\log_{10}\left(\frac{A_2}{A_1}\right) = 10\log_{10}\left(\left(\frac{A_2}{A_1}\right)^2\right) = 10\log_{10}\left(\frac{P_2}{P_1}\right)$$ ← other constants cancel

ex) if $A_2 = 2A_1$ then → $20\log(2) \approx 6$ so A_2 is 6 decibels higher

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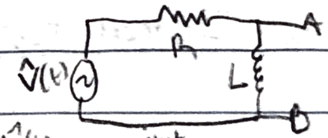
What about Inductors



$\hat V(t) = V_p e^{j\omega t}$

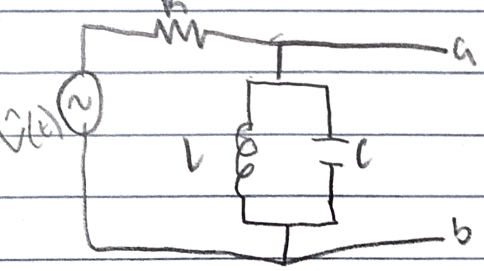
$$\hat V_{AB} = \hat V(t)\frac{\hat Z_L}{\hat Z_L + \hat Z_R} = \hat V\frac{j\omega L}{R + j\omega L}$$

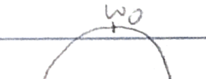
full detail: $\hat V_{AB} = \hat V(t)\frac{-\omega L}{Rj - \omega L}$, $a = -\omega L$, $b = R$ → $\hat V_{AB} = \frac{-\omega L}{\sqrt{R^2+(\omega L)^2}} e^{-j\theta} V_p e^{j\omega t} = \frac{-V_p\omega L}{\sqrt{R^2+(\omega L)^2}} e^{j(\omega t - \theta)}$

$Re(\hat V_{AB}) = \boxed{\frac{-V_p\omega L}{\sqrt{R^2+(\omega L)^2}}}\cos(\omega t - \theta)$, lets look at amplitude: → V_{pAB}

$$T = \frac{V_{pAB}}{V_p} = \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$$ $\boxed{\omega \to 0, T = 0, \quad \omega \to \infty, T \to 1}$ (High Pass)

Band Pass Filters





$$Z_{LC} = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{j\omega L}{\omega^2 LC - 1}$$

$$\hat V_{AB} = \hat V\frac{Z_{LC}}{Z_{LC} + Z_R} = \hat V\frac{-j\omega L}{R(\omega^2 LC - 1) - j\omega L}$$

Both 0!

Convert to Real look at ratio of A

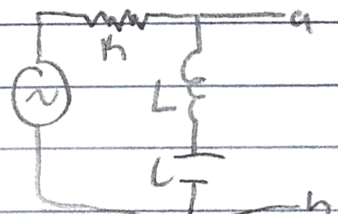
$$T = \frac{V_{pAB}}{V_p} = \frac{\omega L}{\sqrt{R^2(\omega^2 LC - 1)^2 + (\omega L)^2}}$$

$\omega = 0$	$T = 0$
$\omega \to \infty$	$T \to 0$

→ if you do $\frac{dT}{d\omega} = 0$ → $\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{R}{L}\cdot\frac{1}{RC}}$ and $\frac{R}{L}$ = ω for inductor (High pass), $\frac{1}{RC}$ = ω for capacitor (low pass)

Band width $= \Delta\omega = \omega_C - \omega_L$

Notch Filter



$$\hat Z_{LC} = j\left(\omega L - \frac{1}{\omega C}\right)$$

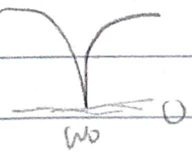
$$\hat V_{ab} = \hat V\frac{\hat Z_{LC}}{\hat Z_R + \hat Z_{LC}} = \hat V\frac{j(\omega^2 LC - 1)}{R\omega C + j(\omega^2 LC - 1)}$$

take real part & look at amplitude ratio

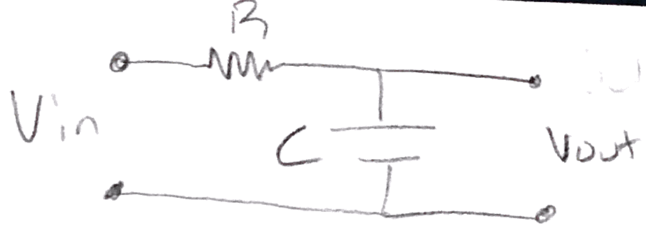
really T is $\left|\frac{V_{pab}}{V_p}\right|$

$$T = \left|\frac{V_{pab}}{V_p}\right| = \left|\frac{\omega^2 LC - 1}{\sqrt{(R\omega C)^2 + (\omega^2 LC - 1)^2}}\right|$$

$\omega = 0$	$T = 1$
$\omega = \infty$	$T = 1$



0 happens at $\omega = \frac{1}{\sqrt{LC}} = \omega_0 = \sqrt{\omega_C \cdot \omega_L}$ again



RC circuits Part 2

Capacitor charging and discharging unified:

KVL: $V_{in} - \frac{q}{C} - \frac{dq}{dt}R = 0 \rightarrow V_{in} = \frac{q}{C} + \frac{dq}{dt}R$

general sol'n: $q = Ae^{-t/RC} + B$ and $V_c = \frac{q}{C}$

C is constant so $\frac{A}{C} = a$

$V_c = ae^{-t/RC} + b$ $V(t=0) = V_{initial}$ and $V(t=\infty) = V_{final}$

$a + b = V_{initial}$ $b = V_{final}$

$a = V_{initial} - V_{final} = -\Delta V$

$V_c = V_{final} - \Delta V e^{-t/RC}$

(V_{final} is source voltage after switching
↳ meaning the capacitor's final voltage is the source)

RC circuits as integrator

$V_{in} - V_R - V_c = 0$ and $V_R = IR \rightarrow I = \frac{V_R}{R} = \frac{V_{in} - V_c}{R}$, $V_c = \frac{q}{C} = \frac{1}{C}\int I\,dt$

$V_c = \frac{1}{C}\int I\,dt = \frac{1}{C}\int \frac{V_{in} - V_c}{R}\,dt$ → and if there is any initial voltage $V_c = V_{initial} + \frac{1}{C}\int I\,dt$

ALTERNATING Current

For any circuit:

$\hat{V} = \hat{I}\hat{Z}$ (complex ohm's law)

- find $\hat{Z}_{tot}$
- calculate $\hat{I} = \frac{\hat{V}}{Z_{tot}}$
- get in form $a + \hat{j}b$
- convert complex to real w/ $|\hat{z}|e^{j\theta}$
- take real part

ex RC circuit:

② $\hat{Z}_{tot} = R + \frac{1}{j\omega C}$

③ $\hat{I} = \hat{V}/(R + \frac{1}{j\omega C})$

④ $\hat{I} = \frac{\omega C\hat{V}}{\sqrt{(\omega RC)^2+1}}e^{-j\theta}$, using $|z| \rightarrow$ the legs of triangle $\tan\theta = \left(\frac{-1}{\omega RC}\right)$

⑤ for $\hat{V}$ can be either $V_p\cos(\omega t)$ or $V_p\sin(\omega t)$ in either case we can use $V_p e^{j\omega t}$ with the proviso that at the end we will take real part for cosine drive or Imaginary part for sin drive

$\hat{I} = \frac{\omega CV_p}{\sqrt{(\omega RC)^2+1}}e^{j(\omega t - \theta)}$

⑥ $I = Re(\hat{I}) = \frac{\omega CV_p}{\sqrt{(\omega RC)^2+1}}\cos(\omega t - \theta)$

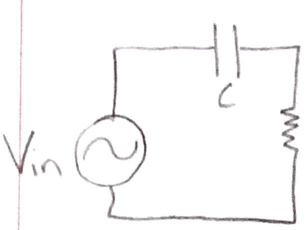
Big side note: when taking Im part your answer will look a lot different from the trig way or back. ex

$\sin(\omega t - \theta)$ vs $\sin(\omega t + \theta)$

$\theta = \tan^{-1}\left(\frac{-1}{\omega RC}\right)$ $\theta = \tan^{-1}\left(1/\omega RC\right)$

Bode notes

RC circuit Response to a sine wave



KVL: $V_{in} = \frac{Q}{C} + IR$

[take time derivative] $\frac{dV_{in}}{dt} = R\frac{dI}{dt} + \frac{I}{C}$ ①

$\frac{dV_{in}}{dt}$ not $\neq 0$ bec/ Voltage isnt constant

essentially solving Diff eq by Unit step and guessing right answer...

- $V_{in} = V_p \sin(\omega t)$, V_p is amplitude sin is changing that w/ time,
- assume current follows same sinusoidal pattern

$I = I_p \sin(\omega t + \phi)$, I_p, ϕ are constants to be determined

plugging into ① taked $\frac{d}{dt}$ where necess $\rightarrow \boxed{\omega V_p \cos(\omega t) = R\omega I_p \cos(\omega t + \phi) + \frac{I_p}{C}\sin(\omega t + \phi)}$

[isolating I_p, ϕ trig identities:] $\sin(\omega t + \phi) = \sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)$; $\cos(\omega t + \phi) = \cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)$ {now divide by $R\omega I_p$ and apply the identities}

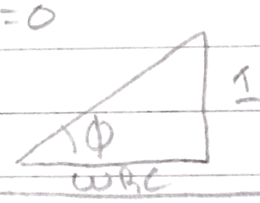
$\hookrightarrow (\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)) + \frac{1}{\omega RC}(\sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)) = \frac{V_p}{I_p R}\cos(\omega t)$

$\hookrightarrow \left[\cos\phi + \frac{1}{\omega RC}\sin\phi - \frac{V_p}{I_p R}\right]\cos(\omega t) + \left[-\sin\phi + \frac{1}{\omega RC}\cos\phi\right]\sin(\omega t) = 0$ {this is valid for all times so lets choose some that are convinient}

$t=0$: $\cos\phi + \frac{1}{\omega RC}\sin\phi - \frac{V_p}{I_p R} = 0$

$\boxed{\omega t = \frac{\pi}{2}}$: $-\sin\phi + \frac{1}{\omega RC}\cos\phi = 0$

$\hookrightarrow \boxed{\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)}$



Using right triangle:

$\sin\phi = \frac{1}{\sqrt{1+(\omega RC)^2}}$, $\cos\phi = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$

Using these yields $\frac{\omega RC}{\sqrt{1+(\omega RC)^2}} + \frac{1}{\omega RC}\cdot\frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{V_p}{I_p R}$ ALGEBRA $\rightarrow \boxed{I_p = \frac{\omega C}{\sqrt{1+(\omega RC)^2}} V_p}$

Thus the final sol'n $\rightarrow I = \frac{\omega C V_p}{\sqrt{1+(\omega RC)^2}}\sin(\omega t + \phi)$, $\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$

Same thing w/ complex #'s

$R\frac{dI}{dt} + \frac{I}{C} = \frac{dV_{in}}{dt}$, $\begin{cases} I = I_p\cos(\omega t + \phi) = Re(I_p e^{j(\omega t + \phi)}) = Re(\hat{I}_p e^{j\omega t}) \\ V_{in} = V_p\cos(\omega t + \phi) = Re[(V_p)e^{j\omega t}] \end{cases}$

$\hat{I}$ = complex current amplitude? $\boxed{I_p e^{j\phi} = \hat{I}_p}$ a nice substitution

plug in: $R\hat{I}_p j\omega \cancel{e^{j\omega t}} + \frac{1}{C}\hat{I}_p \cancel{e^{j\omega t}} = j\omega V_p \cancel{e^{j\omega t}} \Rightarrow R\hat{I}_p j\omega + \frac{1}{C}\hat{I}_p = j\omega V_p$ [solve for $\hat{I}_p$]

$\hat{I}_p = \frac{\omega C V_p}{\omega RC - j}$ $\hat{z} = \boxed{a+bj}$ $a = \omega RC$, $b = -1$ | convert to $|z|e^{j\theta} = \hat{z}$ | $|\hat{z}| = \sqrt{(\omega RC)^2+1}$, $\hat{z} = \sqrt{(\omega RC)^2+1}\,e^{j\theta}$ $\rightarrow \hat{I}_p = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}e^{-j\theta}$

Now take "Real part" $\rightarrow I = Re\left[\hat{I}_p e^{j\omega t}\right] = Re\left[\frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}e^{-j\theta}e^{j\omega t}\right] = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}\left(e^{j(\omega t - \theta)}\right)$ take the Re of this

original: $I = I_p\cos(\omega t + \phi)$

answer: $I = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}Re\left(e^{j(\omega t - \theta)}\right)$

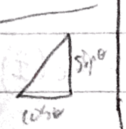
bec/ $I_p Re\left(e^{j(\omega t - \theta)}\right) = I_p\cos(\omega t - \theta)$

$\tan\theta = \frac{-1}{\omega RC}$

so $\boxed{I = \frac{\omega C V_p}{\sqrt{(\omega RC)^2+1}}\cos(\omega t - \theta)}$

$Re(e^{i\theta}) = \frac{e^{i\theta} + e^{-i\theta}}{2}$

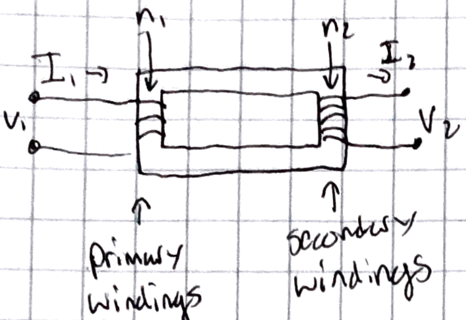
$e^{i\theta} = \cos\theta - j\sin\theta$



take real part proj on the axis

$Re(e^{i\theta}) = \cos\theta + 0$

Transformers



→ Faradays law states that a time varying magnetic field inside of a coil of water induces a voltage this voltage is $\boxed{V_2}$

equation is → $V_2 = \left(\frac{n_2}{n_1}\right) V_1$

{if a resistor or another load is attached} (to the secondary coil a current I_2 can flow)

flipped so if ↑V then I↓

energy has to be cons.

$I_2 = \left(\frac{n_1}{n_2}\right) I_1$

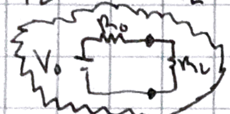
"turns ratio"

Impedence matching

{Voltage source = V_0, internal Resistence = R_0} connect to load resistor = R_L → [how should R_0 and R_L be related if we want max power to the load resistor]

ohms law w/ total R

$$P_L = I^2 R_L = \left(\frac{V_0}{R_0 + R_L}\right)^2 R_L$$



{pretty much trying to find right balance of I and V to max power, balance happens when $R_L = R_0$}

to find extrema: $$\frac{dP_L}{dR_L} = V_0^2\left[\frac{1}{(R_0+R_L)^2} - \frac{2R_L}{(R_0+R_L)^3}\right] = 0$$

→ [you get $R_L = R_0$ for *Max* power transfer]

[source resistance must match load]!

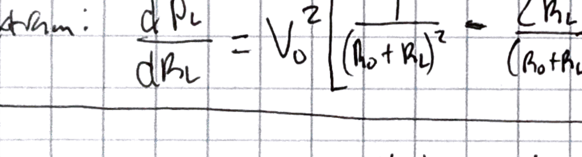
$\boxed{\text{BUT}}$ not everything will have that desired Resistence so to match impedence we use transformers.

$$V_2 = I_2 R_L \rightarrow V_1\left(\frac{n_2}{n_1}\right) = I_1\left(\frac{n_1}{n_2}\right) R_L \rightarrow V_1 = I_1\left[\left(\frac{n_1}{n_2}\right)^2 R_L\right]$$

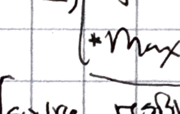
which also shows $\frac{V_1}{I_1} = R_{eff} = \left(\frac{n_1}{n_2}\right)^2 R_L$

so what was R_L before is now R_{eff}

the point is the transformer has changed the resistace the source "sees" from R_L to R_{eff}. By choosing a transformer w/ correct n_1/n_2 you can match R_{eff} to whatever source resistence you have, insuring max power.



so

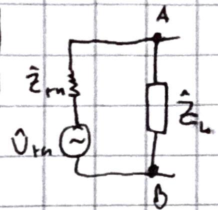


this is ideal case but $R_L \neq R_0$ always equal R_0, so we use transformers.

Source produces current + has an R_{eff} resistance, source causes current + voltage by Faradays law. We cant chang # windings to match to R_L

Example

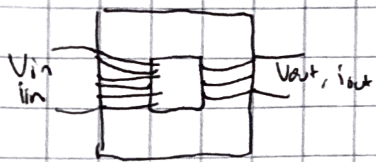
[Before we were power impedance matching but there's voltage impedance matching (give same answer)]



$\hat{V}_{out} = \hat{V}_{oc}\left(\frac{Z_L}{Z_L + Z_{th}}\right)$

[in order for V_{oc} to be delivered to Z_L, $Z_L >> Z_{th}$, if Z_L is too small the source gets too much voltage, so we use transformers]

$Z_{in} = \frac{V_{in}}{i_{in}}$, $Z_{out} = \frac{V_{out}}{i_{out}}$ $\quad \frac{Z_{out}}{Z_{in}} = \frac{V_{out}\, i_{in}}{V_{in}\, i_{out}} = \left(\frac{N_{out}}{N_{in}}\right)^2$



$\frac{V_{out}}{V_{in}} = \frac{N_{out}}{N_{in}}$, energy has to be conserved $\} V_{in} i_{in} = V_{out} i_{out}$

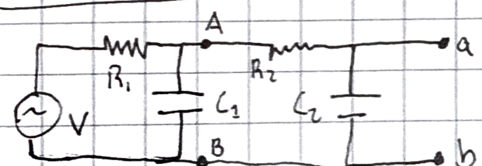
[an audio source has an output impedance of 200Ω, Design a filter that Transmits more than 90% of frequencies above 15kHz but less than 10% of frequencies below .2kHz.]

→ R should be $10 \cdot R_{th}$ and $T > 90\%$ at 15kHz, $T < 10\%$ at .2kHz

→ T for a high pass filter

$T = \frac{R\omega C}{\sqrt{(R\omega C)^2 + 1}}$ → plug in $\left.\begin{matrix} R = 2k\Omega \\ \end{matrix}\right\} \begin{matrix} T = .9 \text{ @} 15kHz \rightarrow \\ T = .1 \text{ @} .2kHz \rightarrow \end{matrix}$ $\boxed{\begin{matrix} C > 1.1 \cdot 10^{-8} = 11nF \\ C < 4.0 \cdot 10^{-8} = 40nF \end{matrix}}$

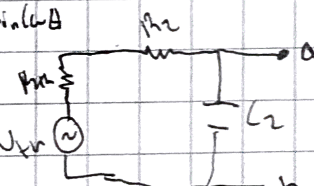
(Thevenin AC)



$V = V_p \sin(\omega t)$

$V_{AB} = ?$ ($V_{AB} = V_{oc}$ in this case)

$|V_{AB}| = \left|\frac{Z_{C_1}}{Z_R + Z_{C_1}}\right| |V| = \frac{V_p}{\sqrt{(R_1\omega C_1)^2+1}}$, $|Z_{th}| = \left|\frac{R_1\left(\frac{-j}{\omega C_1}\right)}{R_1 - \frac{j}{\omega C_1}}\right| = \frac{R_1}{\sqrt{(R_1\omega C_1)^2+1}}$

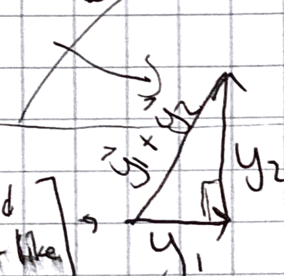


$V_{AB} =$

$|V_{ab}| = |V_{AB}| \left|\frac{R_{C_2}}{R_{C_2} + (R_{th} + R_2)}\right|$

$|Z_{th2}| = \frac{(R_2 + R_{th}) Z_{C_2}}{Z_{C_2} + Z_{th} + Z_{R_2}}$

If this varies in the it can make shapes!



Lissajous patterns

[for our voltage signals you can look at interesting patterns]

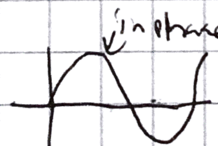
-take our 'cos' signal

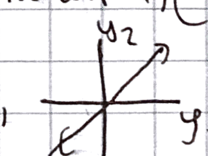
→ $y_1 = \cos(\omega t)$ ← this could be like voltage signal

$y_2 = \cos(\omega t + \phi)$ ← this could be like current, [you could look at it like]

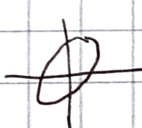
$y_1 + y_2 = \cos\left(\frac{2\pi t}{T}\right) + \cos\left(\frac{2\pi t}{T} + \phi\right)$
$= 2\cos\left(\frac{\pi t}{T} + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)$

ex $\phi = 0$
$y_1 = \cos(\omega t)$
$y_2 = \cos(\omega t)$

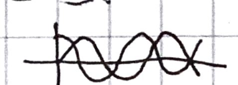


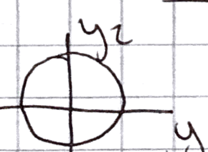


$\phi = \frac{\pi}{4}$

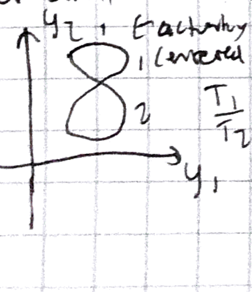


$\phi = 90°$
$y_1 = \cos(\omega t)$
$y_2 = \cos(\omega t + \frac{\pi}{2}) = \sin(\omega t)$

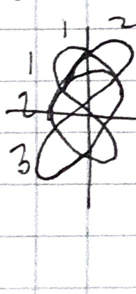




for different frequencies



$\frac{T_1}{T_2} = \frac{1}{2}$



$\frac{T_1}{T_2} = \frac{2}{3}$

$\frac{T_1}{T_2} = \frac{N_1}{N_2}$ ← number of x-axis lobes ← number of vertical y-axis lobes