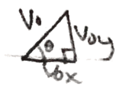
# Kinematics

center of mass = $\frac{\Sigma_i m r}{\Sigma_i m}$

$a = \frac{dv}{dt}$

$v = \frac{dx}{dt}$ $\quad V_f = V_0 + at \quad V_f^2 = V_0^2 + 2a\Delta y \quad V = \frac{d}{t}$

$X = V_{ox} t \quad y_f - y_0 = \frac{1}{2}at^2 + V_{oy} t$

- same place same time = $X_{obj\,1}(T_{hit}) = X_{obj\,2}(T_{hit})$
- if shot at angle  can be put into components



## Circular Kinematics

$S_{arc\ length} = R\theta \quad V_{tan} = R\omega \quad a_{tan} = \alpha R \quad f = \frac{cycles}{sec},\ T = \frac{sec}{cycle}$

$a_{rad} = \frac{v^2}{r} = \omega^2 R$ (by plugging in $V_{tan}$)

$\omega_f = \omega_0 + \alpha t \quad \left(V = \frac{2\pi r}{T} = \frac{\text{circumference}}{\text{time to go around circ}}\right)$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ if there is no $\alpha$ then $\Delta\theta = \omega t$

$\omega = 2\pi f \quad T_{period} = \frac{1}{f} = \frac{2\pi}{\omega} \quad t = \frac{\omega}{\alpha}$

$\omega = \frac{2\pi}{T}$

$I_p = I_{cm} + M_{tot} d^2$ — distance from cm

$I = \int r^2 dm \ [I_{ring} = mr^2,\ I_{disc} = \frac{1}{2}mr^2,\ I_{rod} = \frac{1}{12}mL^2,\ I_{sphere} = \frac{2}{5}mr^2,\ I_{ringshell} = \frac{2}{3}mR^2$

## Periodic Motion

phase angle → what point in the cycle the motion was at t=0

$X = A\cos(\omega t + \phi) \quad \phi = \tan^{-1}\left(-\frac{V_{ox}}{\omega X_0}\right)$

$a = -\omega^2 x \quad V_x = -A\omega\sin(\omega t + \phi) \quad \omega^2 X_0^2 + V_{ox}^2 = A^2\omega^2$

$V_x = \omega\sqrt{A^2 - x} \quad \ddot{X} + \omega^2 x = 0$

$\omega_{spring} = \sqrt{\frac{k}{m}} \quad \omega_{pend} = \sqrt{\frac{g}{L}} \quad \omega_{torsion} = \sqrt{\frac{K_{tor}}{I}}$ ← torque for torsion = $-K\theta_0$

## Torque

$\Sigma \tau_{torque} = I\alpha \quad \tau = r \times F \rightarrow \tau = rF\sin\theta$

axis of rotation to where Fapp is

F ← only $F_\perp$ matters

$V = \frac{d}{t}$

## Turn stuff into stuff equations

- $S_{arc} = R\theta$, $V_{tan} = R\omega$, $a_{tan} = \alpha R$
- $a_{rad} = \frac{V^2}{R} = \omega^2 R$

  $t = \frac{\omega}{\alpha}$

- $2\pi f = \omega$ $\quad T = \frac{1}{f} = \frac{2\pi}{\omega}$

  $\omega = \frac{2\pi}{T}$

- $V = \frac{2\pi r}{T}$

for springs $a = -\omega^2 x$ the x function

$\omega_{spring} = \sqrt{\frac{k}{m}}$

$\omega_{pend} = \sqrt{\frac{g}{L}}$ $\quad \omega_{phys} = \sqrt{\frac{mgd}{I}}$ distance from cm

$I_{ring} = mr^2$ $\quad I_{rod} = \frac{1}{12} m L^2$

$I_{disc} = \frac{1}{2} m r^2$ $\quad I_{sphere} = \frac{2}{5} m R^2$

$I_{thin sphere} = \frac{2}{3} m R^2$

$I_p = I_{cm} + M_{tot} d^2$ distance from "cm"

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**Kinematics:** $a = \frac{dv}{dt}$ $\quad V = \frac{dx}{dt}$ | $V_f^2 = V_0^2 + 2a\Delta y$

$X = V_{0x} t$ $\quad y_f = y_0 - \frac{1}{2} a t^2 + V_{0y} t$

**Circular:** $\omega_f = \omega_0 + \alpha t$ $\quad \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \to$ no $\alpha = \omega t$

**Forces:** $\Sigma F = ma$ $\quad F = -G \frac{Mm}{r^2}$ $\quad F = \frac{dp}{dt}$

**Energy** scalar $\quad \boxed{\text{Gravity}}$

$\Delta E = W_{non cons}$ $\quad \Delta E_k = W_{net}$ $\quad U = -\frac{GMm}{r}$ $\quad$ Energy lost $= \frac{E_0 - E_f}{E_0}$

$E_k = \frac{1}{2} m V^2$ $\quad E_g = mgh$ $\quad E_{roll} = \frac{1}{2} I \omega^2$

**Torque:** rotation of axis of rotation to point of application of $\vec{F}$

$\Sigma \tau = I\alpha$ $\quad \tau = \vec{r} \times F$ $\quad \tau = m(\vec{r} \times \vec{a})$

$\tau = Fr\sin\theta$

**Momentum** vector

$p = mv$ $\quad \Delta p = p_f - p_i$ $\quad \Delta p = F\Delta t$

angular Momentum: $L = I\omega$ $\quad \vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = mvr$ (for circular orbit)

$L_f - L_i = 0$

**Periodic Motion**

$x = A\cos(\omega t + \phi)$ $\quad \phi = \tan^{-1}\left(-\frac{V_{0x}}{\omega x_0}\right)$ $\quad \omega^2 x_0^2 + V_{0x}^2 = A^2\omega^2$

$V_x = -\omega A \sin(\omega t + \phi)$ $\quad \ddot{x} + \omega^2 x = 0$

$V_x = \omega\sqrt{A^2 - x^2}$

## Fluids

$P = \frac{F}{A}$ $\quad P = P_0 + \rho g h$ $\quad$ area velocity $A_{in} V_{in} = A_{out} V_{out}$

$P_{in} + \frac{1}{2}\rho V_{in}^2 + \rho g h = P_2 + \frac{1}{2}\rho V_{out}^2 + \rho g h_2$

Bouyancy force $= \rho_{fluid}\, g\, V_{dis}$

Submerged $F_g - F_{bou} = m_{obj}\, a_{obj}$ $\quad m_{obj} \to \rho_{obj} V_{obj}$

intensity $= \frac{\text{Power}}{\text{Area}}$

intensity $= \frac{\text{Power}}{4\pi R^2}$

## Sound and Waves

$\lambda = \frac{4L}{n}$ for one end closed

left to right wave speed

$V = f\lambda$ $\quad f_n = \frac{V}{\lambda_n}$ $\quad \lambda_n = \frac{2L}{n}$

$V = \sqrt{\frac{T}{\mu}}$ tension; $\mu$ for string $\frac{m}{L}$

$K = \frac{2\pi}{\lambda}$

$P_{Avg} = \frac{FA^2 k\omega}{2T}$

fixed: $y_R = -A\cos(Kx + \omega t)$

free $y_R = A\cos(Kx + \omega t)$

node: $x = \frac{n}{k}\pi = n\frac{\lambda}{2}$ | When $\sin(k) = \pm 1$ $\quad Kx = (n + \frac{1}{2})\pi$

antinode $x = (n + \frac{1}{2})\frac{\lambda}{2}$

$V = \sqrt{\frac{B}{\rho}}$ $\quad B = \gamma P$ $\quad \gamma: \left(\frac{\text{specific energy at constant pressure}}{\text{specific heat energy}}\right) = \sqrt{\frac{\gamma R T}{M}}$

doppler: $f_{obs} = \frac{(V_{sound} + V_{obs})}{(V_{sound} - V_{source})} f_{actual}$