

Statistical Inference - course project part 1

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Overview

In this project, i will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$.

The followings are the instruction for this projecct:

1. `lambda = 0.2` for all of the simulations.
2. Investigate the distribution of averages of 40 exponentials
3. Need to do a thousand simulations.

Simulations

Set the conditions

```
lambda <- 0.2  
n <- 40  
sims <- 1000
```

Set seed for the reproducibility, and simulate exponential distribution

```
set.seed(1)  
sim_exp <- replicate(sims, rexp(n, lambda))  
dim(sim_exp)  
## [1] 40 1000
```

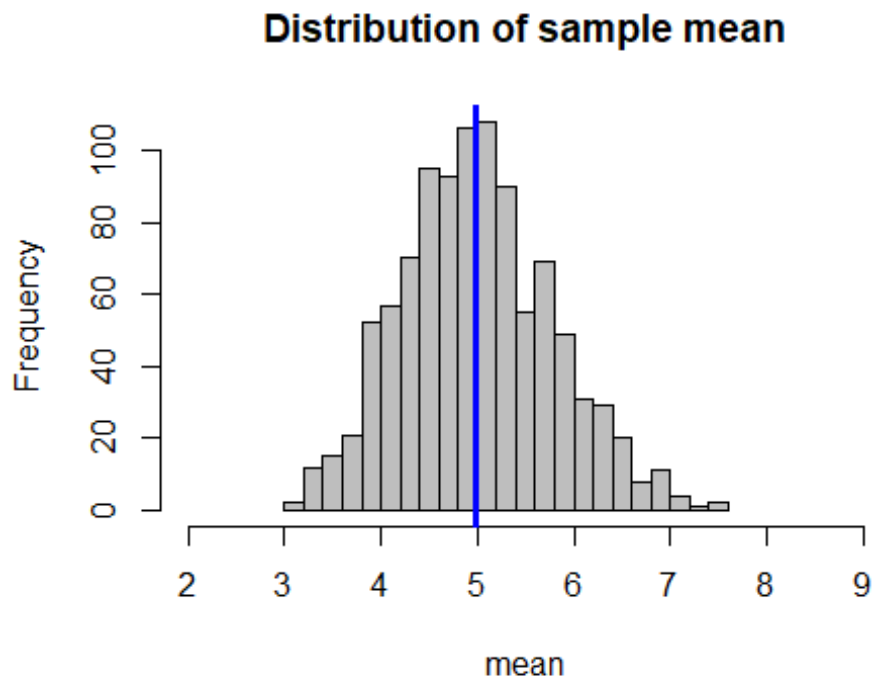
Calculate the mean

```
mean_exp <- apply(sim_exp, 2, mean)
```

1. Sample Mean versus Theoretical Mean

Plot histogram of the means of the exponential distribution.

```
hist(mean_exp, main = "Distribution of sample mean", xlab = "mean",
     xlim = c(2, 9), breaks = 30, col = "grey")
abline(v = mean(mean_exp), lwd = "3", col = "blue")
```



The sample mean is as follow:

```
mean(mean_exp)
## [1] 4.990025
```

The theoretical mean of exponential distribution is $1/\lambda$.

```
theo_mean <- 1/lambda
theo_mean
## [1] 5
```

2. Sample Variance versus Theoretical Variance

The sample variance is as follow:

```
var(mean_exp)
## [1] 0.6111165
```

The theoretical variance of exponential distribution is $((1/\lambda)/\sqrt{n})^2$.

```
theo_var <- ((1/lambda)/sqrt(n))^2
theo_var

## [1] 0.625
```

Distribution

The exponential distribution of 1000 simulations is approximately normal. Due to the Central Limit Theorem, the means of the sample simulations should follow a normal distribution.

```
hist(mean_exp, main = "Distribution of sample mean", xlab = "mean",
     xlim = c(2, 9), breaks = 30, col = "grey", probability = TRUE)

# add density plot
lines(density(mean_exp), lwd = 3, col = "red")

# add normal distribution line
x <- seq(min(mean_exp), max(mean_exp), length = 2 * n)
y <- dnorm(x, mean = 1/lambda, sd = sqrt(((1/lambda)/sqrt(n))^2))
lines(x, y, col = "black", lwd = 3, lty = 2)
```

