2D Compressible MHD Code for Tearing Mode

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Abstract

To get more experience on coding fluid codes, I try to write this 2D compressible MHD code for tearing mode (magnetic reconnection).

In fact, I wrote the first version of this code in June 2010. Unfortunately, it doesn't work well. The framework of this code is from [Fu1995], section 7.4 and Appendix 2.

1 2D Compressible MHD Equations

All variables are in (x, z) plane, $\partial/\partial y=0$.

1.1 Original equations

Original equations (no Hall term, η, γ, υ constants):

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} = -\mathbf{u}\cdot\nabla\rho - \rho\nabla\cdot\mathbf{u},\\
&\frac{\partial p}{\partial t} = -\mathbf{u}\cdot\nabla p - \gamma p\nabla\cdot\mathbf{u},\\
&\rho\frac{\partial \mathbf{u}}{\partial t} = -\rho\mathbf{u}\cdot\nabla\mathbf{u} - \nabla p - \frac{1}{\mu_0}\nabla\frac{B^2}{2} + \frac{1}{\mu_0}\mathbf{B}\cdot\nabla\mathbf{B} + \upsilon\nabla^2\mathbf{u},\\
&\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla\times(\mathbf{u}\times\mathbf{B})}_{-\mathbf{u}\cdot\nabla\mathbf{B}-\mathbf{B}\nabla\cdot\mathbf{u}+\mathbf{B}\cdot\nabla\mathbf{u}} - \underbrace{\nabla\times[\eta(\nabla\times\mathbf{B})]}_{\eta\nabla^2\mathbf{B}}.
\end{aligned}
\tag{0.1}
$$

Where, we have used,

$$
\begin{aligned}
&\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\\
&\mathbf{J} = \nabla\times\mathbf{B},\\
&\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J}.
\end{aligned}
\tag{0.2}
$$

Here, considering that $\nabla\cdot\mathbf{B}=0$, we use A_y instead of $\mathbf{B}$, i.e.,

$$
\mathbf{B} = \nabla\times\mathbf{A} = \nabla\times(0, A_y, 0) = (-\partial A_y/\partial z,\ 0,\ \partial A_y/\partial x).
\tag{0.3}
$$

And the last equation in (0.1) changes to,

$$\nabla\times\left(\frac{\partial\mathbf{A}}{\partial t}\right)=\nabla\times\{\mathbf{u}\times(\nabla\times\mathbf{A})-\eta[\nabla\times(\nabla\times\mathbf{A})]\},\tag{0.4}$$

Or,

$$\frac{\partial\mathbf{A}}{\partial t}=\mathbf{u}\times(\nabla\times\mathbf{A})-\eta[\nabla\times(\nabla\times\mathbf{A})].\tag{0.5}$$

Rewrite (0.1) and (0.5) to explicit form in 2D (with normalization),

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}=-u_x\frac{\partial\rho}{\partial x}-u_z\frac{\partial\rho}{\partial z}-\rho\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right),\\
&\frac{\partial p}{\partial t}=-u_x\frac{\partial p}{\partial x}-u_z\frac{\partial p}{\partial z}-\gamma p\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right),\\
&\frac{\partial u_x}{\partial t}=-u_x\frac{\partial u_x}{\partial x}-u_z\frac{\partial u_x}{\partial z}-\frac{1}{\rho}\frac{\partial}{\partial x}\left(p+\frac{B^2}{2}\right)+\frac{1}{\rho}\left(B_x\frac{\partial B_x}{\partial x}+B_z\frac{\partial B_x}{\partial z}\right)+\frac{1}{\rho}\upsilon_m\left(\frac{\partial^2u_x}{\partial x^2}+\frac{\partial^2u_x}{\partial z^2}\right),\\
&\frac{\partial u_z}{\partial t}=-u_x\frac{\partial u_z}{\partial x}-u_z\frac{\partial u_z}{\partial z}-\frac{1}{\rho}\frac{\partial}{\partial z}\left(p+\frac{B^2}{2}\right)+\frac{1}{\rho}\left(B_x\frac{\partial B_z}{\partial x}+B_z\frac{\partial B_z}{\partial z}\right)+\frac{1}{\rho}\upsilon_m\left(\frac{\partial^2u_z}{\partial x^2}+\frac{\partial^2u_z}{\partial z^2}\right),\\
&\frac{\partial A_y}{\partial t}=-u_x\frac{\partial A_y}{\partial x}-u_z\frac{\partial A_y}{\partial z}+\eta_m\left[\frac{\partial^2A_y}{\partial x^2}+\frac{\partial^2A_y}{\partial z^2}\right].
\end{aligned}\tag{0.6}$$

1.2 Normalization

In (0.6), the normalization for density, pressure, length, magnetic field, velocity and time are $\rho_0, p_0, L_0, B_0, u_A=B_0/\sqrt{\mu_0\rho_0}, \tau_A=L_0/u_A$, respectively.

Parameters are

$$\upsilon_m\to\frac{\upsilon}{u_AL_0\rho_0},\ \eta_m\to\frac{\eta}{u_AL_0},\tag{0.7}$$

Lundquist number $S=1/\eta_m$.

1.3 Discrete

In the right hand side of (0.6), for spatial derivative, we use

$$\begin{aligned}
&\frac{\partial f}{\partial x}\to\frac{f_{i+1}-f_{i-1}}{2\Delta x},\\
&\frac{\partial f}{\partial z}\to\frac{f_{j+1}-f_{j-1}}{2\Delta z},\\
&\frac{\partial^2 f}{\partial x^2}\to\frac{f_{i+1}+f_{i-1}-2f_i}{\Delta x^2},\\
&\frac{\partial^2 f}{\partial z^2}\to\frac{f_{j+1}+f_{j-1}-2f_j}{\Delta z^2}.
\end{aligned}\tag{0.8}$$

In the left hand side, for time derivative, we use 4-th order Runge-Kutta scheme.

2 Code Structure

Variables array x(ni,nj,5), for ρ, p, u_x, u_z, A_y, respectively.

3 Simulation Results

4 References

[Fu1995] 傅竹风 & 胡友秋, 空间等离子体数值模拟, 安徽科学技术出版社, 1995.

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