New Paradigm for Turbulent Transport Across a Steep Gradient in Toroidal Plasmas

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Max Planck Institut für Plasmaphysik, Garching, Germany, Jun. 22, 2017

This work is mainly done at IFTS-ZJU. My current address: FSC-PKU. The presentation largely bases on: [1] Xie, Xiao & Lin, Phys. Rev. Lett., 118, 095001, 2017; [2] Xie & Xiao, Phys. Plasmas, 22, 090703, 2015; [3] H. S. Xie, PhD thesis, Zhejiang University, 2015.

Ackn.: D. F. Kong, L. Chen, GTC team, G. Y. Fu, P. H. Diamond, X. Q. Xu, G. S. Xu, H. Q. Wang, J. Cheng, Z. C. Feng, H. T. Chen, T. Xie, Z. X. Lu, D. Dickinson, F. Zonca, B. Li, Z. M. Sheng, · · ·

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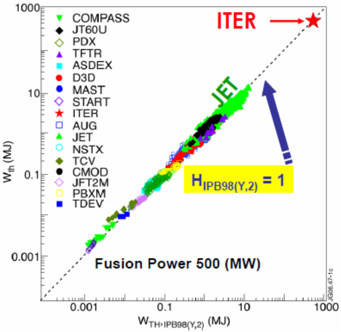
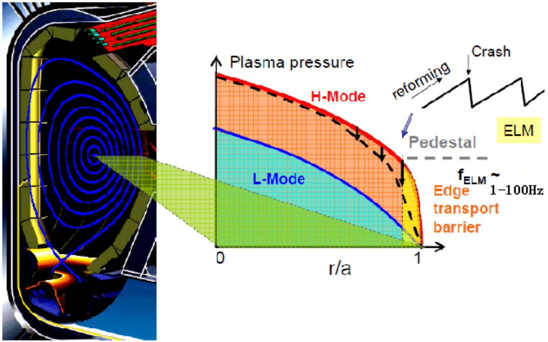
Introduction

1. Background: H-mode operation of tokamak



$$\tau_{E,th}^{ELMy} = 0.0562 I^{0.93} B^{0.15} P^{-0.69} n^{0.41} \times M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}$$

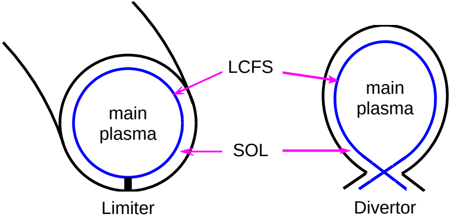


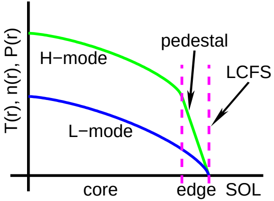
Y. F. Liang, 4TH ITER International Summer School, Austin, Texas USA, 2010

- H-mode (Wagner *et al.*, 1982) is ITER baseline scenario
- Energy stored in H-mode is **twice** or more than L-mode
- Two **'phases'**: L-mode - weak gradient; H-mode - strong gradient

First principle studies of the edge physics still lacking

Physics: a. core - comprehensively studied; **b. edge - current frontier;** c. SOL - more complicated (atom/molecule process)





Existed studies of edge H-mode and ELM

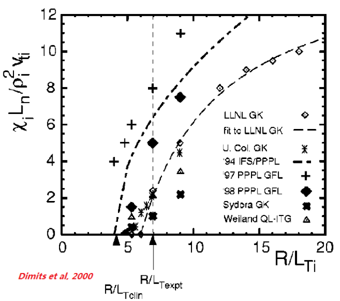
- **Kinetic: beginning stage (GYRO, GTC, GEM, $\cdots$), challenged**
- Fluid models: BOUT++, JOREK, $\cdots \rightarrow$ limited kinetic physics
- Simplified models (e.g., ODEs): bifurcation (Itoh-Itoh), prey-predator (Diamond) $\rightarrow$ qualitative at most

$\Rightarrow$ **We focus on edge kinetic (first principle) physics**

H-mode unknown physics issues

Define heat flux $q_j = \int dv^3(\frac{1}{2}m_j v^2 - \frac{3}{2}T_j)\delta v_r \delta f_j \equiv n_j \chi_j \nabla T_j$, $j = i, e$.

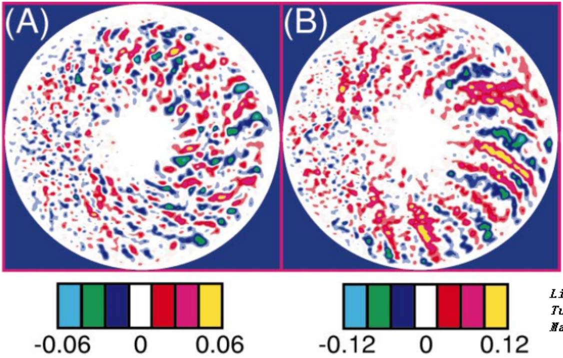
- L-H transition is still not fully understood.
- **How will transport coefficient χ_j changes with ∇T_j increasing?** Does mixing length ($D \sim l_c^2/\tau_c$) estimation really valid? Or, how to estimate l_c and τ_c? A simplest one $D \sim (\gamma_k/k_\perp^2) \propto \gamma_k$. Taroni-Bohm (Horton2012 book) gives $\chi_e \propto \nabla T_e$.

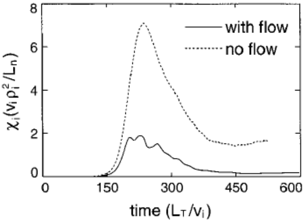


- Is **zonal flow** still important?
  - How important the **mode coupling** can be in the nonlinear evolutions?

Next, we focuses on **edge electrostatic** physics.

Physics understandings in L-mode (weak gradient) still hold in H-mode strong gradient stage?





Lin, Z. et al., Science, 1998, 281, 1835
Turbulent Transport Reduction by Zonal Flows: Massively Parallel Simulations

Also for H-mode?

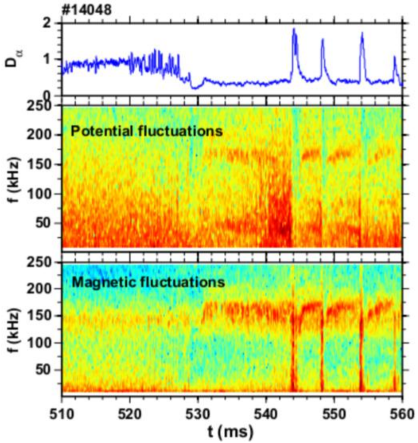
It is believed that (at least in **L-mode** stage or **core** plasmas):

- **Zonal flow** important to reduce transport (eg., Chen01, Waltz08)
- **Mode coupling** important for nonlinear cascading (eg., Lin05, Chen05)
- Larger gradient $\rightarrow$ **larger** transport coefficients

Gyrokinetic simulations

HL-2A H-mode experiments

Typical HL-2A H-mode exp. signal (#14048, from D. F. Kong)



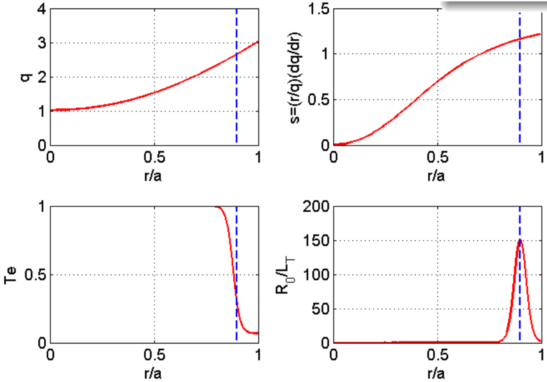
- ES: low frequency → **this work**

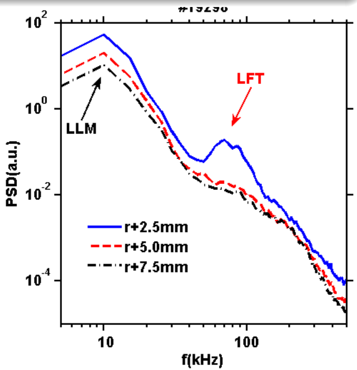
2. Nonlinear transport: GTC edge simulation parameters

GTC edge simulation parameters are taken from recent H-mode exp. of HL-2A (#19298, from D. F. Kong)

- $f \sim 80$kHz, $m \sim 10$-33

HL-2A **typical L-mode** $R_0 L_T^{-1} < 40$, **typical H-mode** $R_0 L_T^{-1} > 80$

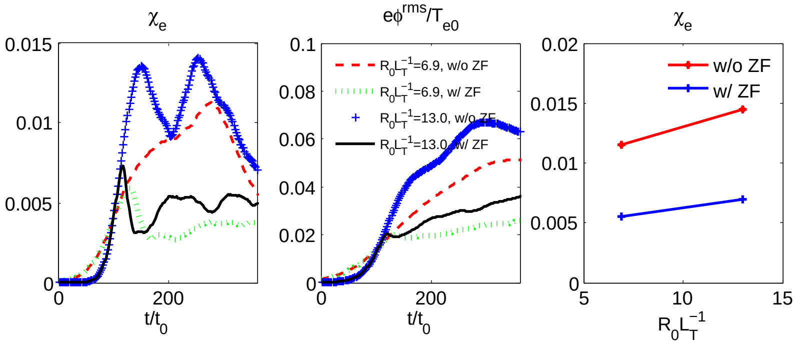




$B_0 = 1.35T$, $a = 40cm$, $R_0 = 165cm$, $q = 2.5 - 3.0$, $s = 0.3 - 1.0$, $R_0/L_T = 80 - 160$, $T_e(r) = T_i(r)$, $n_e(r) = n_i(r)$, $\eta = L_n/L_T \simeq 1.0$.

Normal turbulent transport understandings in L-mode/weak gradient

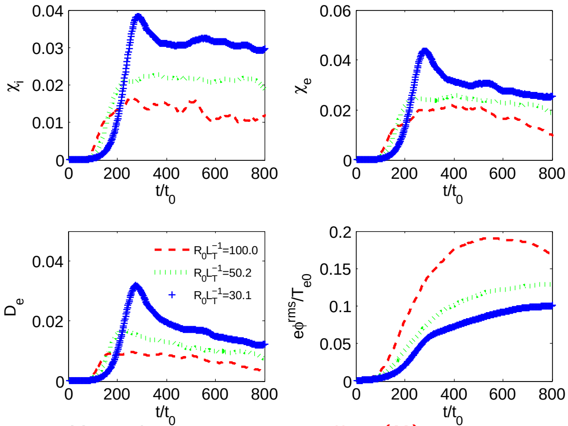
Agree with usual understandings / theoretical models



- Stronger gradients in L-mode stage give larger transport coefficients
- Zonal flow can reduce the transport coefficients significantly

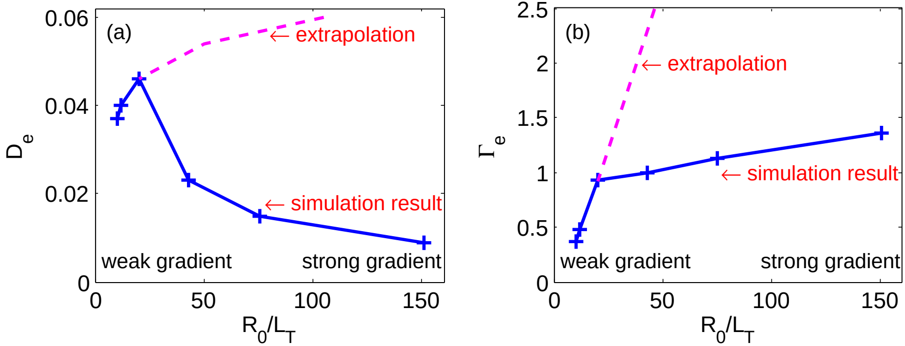
Reverse trend of turbulent transport: H-mode/strong gradient

heat conductivity χ_j, particle diffusivity D_j



Stronger gradients in H-mode stage give **smaller (!!)** transport coefficients of particles and energy, though the root mean square of e.s. potential still higher.

Nonlinear critical gradient exist

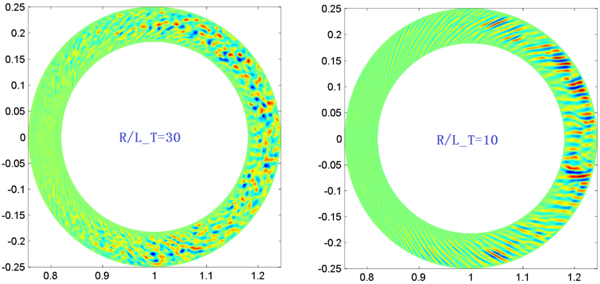


A turning point (**critical gradient**) exists for the reverse trend of the transport coefficients. [Similar to second order phase transition (suggested to add by one of the PRL referee) of Landau1937.]

Xie, Xiao & Lin, Phys. Rev. Lett., 118, 095001 (2017).

Eddy sizes - correlation length

Estimate the radial correlation length l_c from the eddy size.



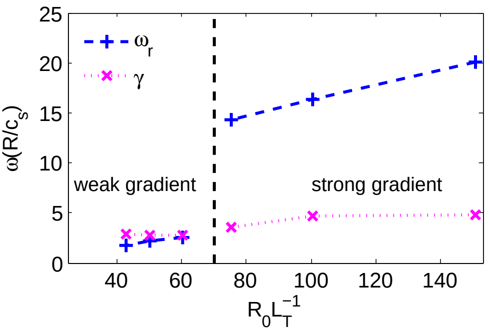
Strong gradient ($R_0 L_T^{-1}$=30) small eddy size. Weak gradient ($R_0 L_T^{-1}$=10) large eddy size. Assume correlation time τ_c not change too much $\rightarrow$ $D \sim l_c^2/\tau_c \propto l_c^2 \rightarrow D \downarrow$.

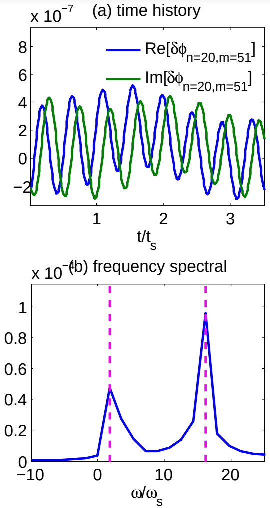
Next: **Why** stronger gradient has a small eddy size? The formation of the **mode structures** should be examined carefully.

Linear results and theory

3. Linear: two Trapped Electron Mode (TEM) branches

$n = 20$, $T_e = 200eV$ (Right figure: $R/L_T = 75$)





Most unstable micro-instabilities under weak and strong gradients are in different branches: (H) $\omega_r > 10\omega_s$, $\omega_r \gg \gamma$; (L) $\omega_r < 3\omega_s$, $\omega_r < \gamma$.

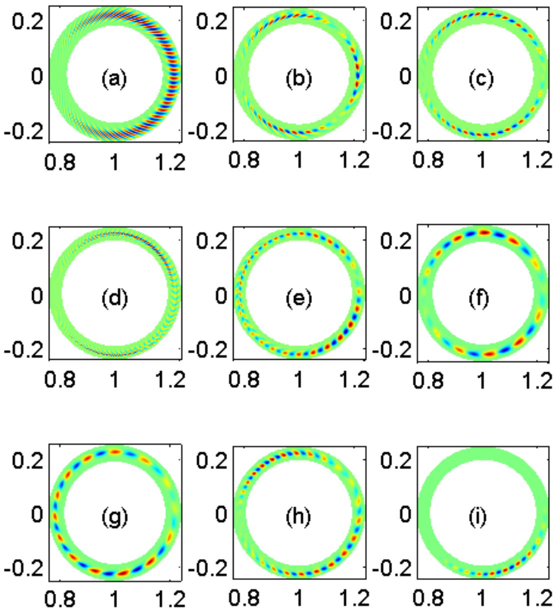
Various mode structures

Single-n ($n = 5 - 30$)

- **(a) weak gradient** L-mode parameter gives **conventional ballooning structures** of TEM in GTC simulation
- **(b)-(i) strong gradient** H-mode parameters give **unconventional structures** of TEM.

Mostly unexpected:

- a. **anti-ballooning**, $|\theta_p| > \pi/2$
- b. **multi-peak**

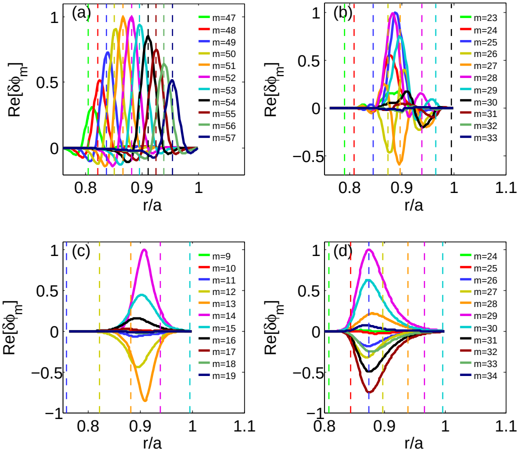


Fourier components $\delta\phi_m(r)$ of TEM

$\delta\phi(r,\theta,\zeta) = e^{in\zeta}\sum_m \delta\phi_m(r)e^{-im\theta}$

Corresponding polodial cross section mode structures of (a)-(d) are taken from previous (a), (b), (g) and (i), respectively.

- Unconventional mode structures (especially anti-ballooning, $u_m \simeq -u_{m+1}$, i.e., a 180° phase shift for neighboring Fourier) can **reduce the effective correlation length**. We can expect that **H-mode can have better confinement**.



Strong gradient $k_{\|} \propto |nq - m| > 1 \neq 0$

Model linear theory

- **Model** eigenmode equation for unconventional structure of drift wave

$$\left[\rho_i^2\frac{\partial^2}{\partial x^2}-\frac{\sigma^2}{\omega^2}\left(\frac{\partial}{\partial\theta}+ik_\theta sx\right)^2-\frac{2\epsilon_n}{\omega}\left(\cos\theta+\frac{i\sin\theta}{k_\theta}\frac{\partial}{\partial x}\right)\right.$$
$$\left.-\frac{\omega-1}{\omega+\eta_s}-k_\theta^2\rho_i^2\right]\delta\phi(x,\theta)=0, \qquad (1)$$

$\sigma=\epsilon_n/(qk_\theta\rho_i)$, $\eta_s=1+\eta_i$, $x=r-r_s$, poloidal wave number $k_\theta=nq/r$

- **1D**: Corresponding 1D equation in ballooning space (normalization: ω_{*e})

$$\left\{\frac{\sigma^2}{\omega^2}\frac{d^2}{d\vartheta^2}+k_\theta^2\rho_i^2[1+s^2(\vartheta-\vartheta_k)^2]+\frac{2\epsilon_n}{\omega}[\cos\vartheta\right.$$
$$\left.+s(\vartheta-\vartheta_k)\sin\vartheta]+\frac{\omega-1}{\omega+\eta_s}\right\}\delta\hat{\phi}(\vartheta,\vartheta_k)=0, \qquad (2)$$

ϑ_k ballooning-angle parameter.

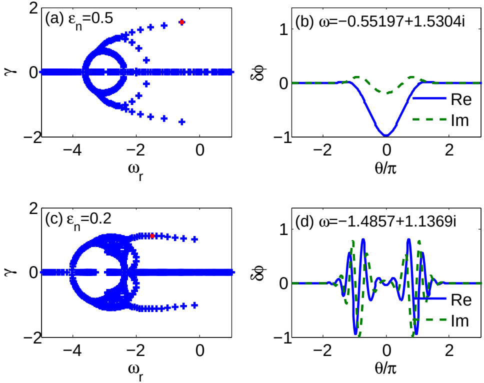
l: 'quanta' number

- Approximate to Weber equation $u''+(bx^2+a)u=0$, eigenvalues $a(\omega)=i(2l+1)\sqrt{b(\omega)}$, eigenfunctions $u(x)=H_l(i\sqrt{b}x)e^{-ibx^2/2}$, H_l is l-th Hermite polynomial ($l=0,1,2,...$), **series eigenstates**.

1D eigen solutions to drift instability

- **Weak gradient ($\epsilon_n \equiv L_n/R = 0.5$), most unstable solution ground state (a&b)**, conventional structure.
- **Strong gradient ($\epsilon_n = 0.2$), most unstable solution not ground state (c&d)**, unconventional.
- **Condition $\epsilon_n < \epsilon_c$**, critical gradient parameter ϵ_c depends on other parameters.

Xie&Xiao, Phys. Plasmas, 22, 090703 (2015).



Eq.(2), series solutions exist. ($s = 0.8$, $k_\theta \rho_i = 0.4$, $q = 1.0$, $\eta_s = 3.0$ and $\vartheta_k = 0$)

Linear: Eigenstates jump!!!

Discussions

- Strong gradient (H-mode) eigen state $l \neq 0$ v.s. weak gradient (L-mode) $l = 0$, indicate different transport behaviors between H-mode and L-mode.
- Unconventional mode structures can reduce the effective correlation length. We can expect that **H-mode can have better confinement**.
- Nonlinear simulations confirm that the **transport coefficients decrease** with gradient increasing.

Thus ... see diagram

Provides some hints to L-H transition and H-mode transport mechanism by first-principle gyrokinetic simulations.

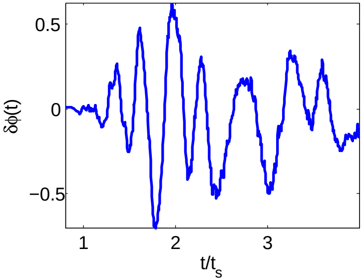
$$L \rightleftharpoons H$$

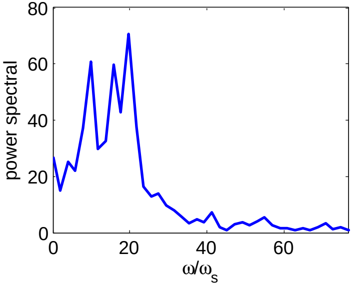
Eigenstates jump ! vs. **'phase' transition ?**

More nonlinear results

4. Compare with experiment: nonlinear frequency

Diagnose at a fixed point $(r = r_c, \theta = \pi/2, \zeta = 0)$, $\omega \simeq 16\omega_s$



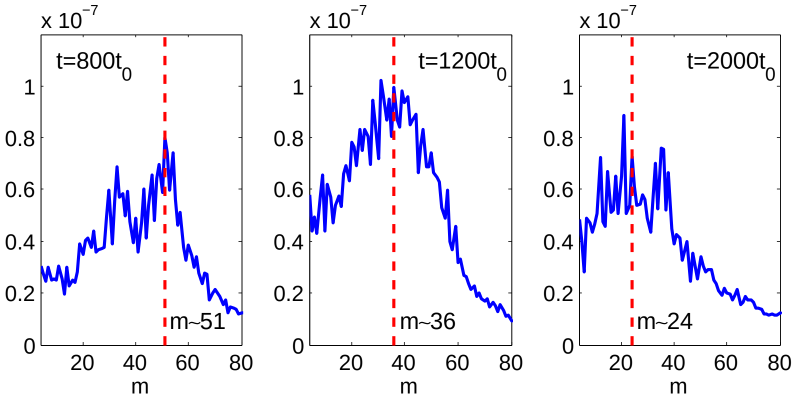


if $T_e \simeq 50eV \Rightarrow f^{sim.} \simeq 78kHz + f^{doppler}$, if $|f^{doppler}| < 10kHz$
$\Rightarrow f^{sim.} \simeq f^{exp.} \simeq 80kHz$.

Nonlinear **frequency agrees exp. !!**

Nonlinear evolutions of the poloidal spectral

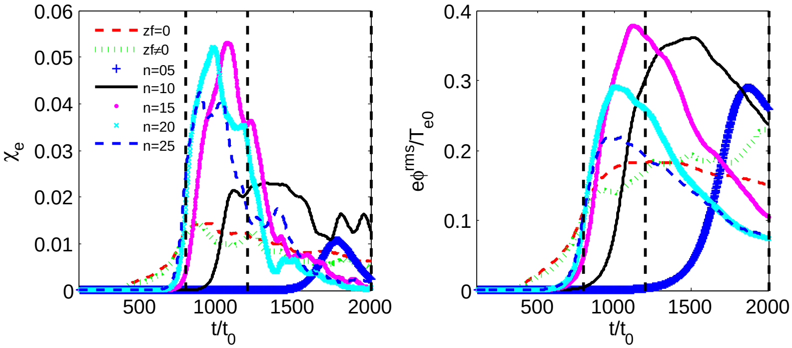
$m^{sim.} \simeq 10 - 40$ **vs.** $m^{exp.} \simeq 10 - 33$, nonlinear poloidal **spectral agrees exp. !!**



Reverse cascading from high to low m mode number.

Mode coupling and zonal flow are less important in strong gradient

multi-*n* (w/ & w/o zonal flow) vs. single-*n*



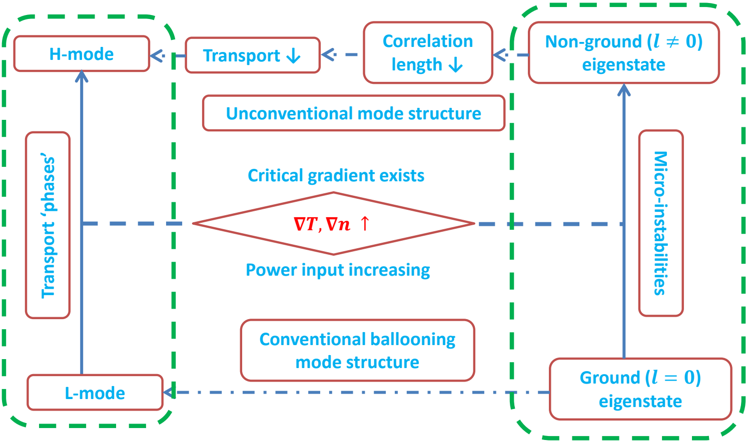
$t = 800t_0$, dominate is $n \simeq 20 - 25$ gives $m \simeq nq \simeq 57$; $t = 1200t_0$, $n \simeq 15$ gives $m \simeq nq \simeq 40$; $t = 2000t_0$, $n \simeq 10$ gives $m \simeq nq \simeq 26$.

Close to multi-n (previous slide) results, **reveal multi-mode-coupling not important for *m* downshift as in L-mode** [e.g., Wang07, Lang08].

Summary

5. Summary: diagram for new picture of L-H transition

back



without invoking shear flow or zonal flow

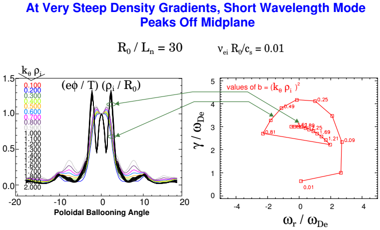
Related works / Backup

6. Related works

- Gyrokinetic (mainly fixed profiles):
  [1] H. S. Xie & Y. Xiao, *Unconventional ballooning structures for toroidal drift waves*, Physics of Plasmas, 22, 090703 (2015).
  [2] H. S. Xie, "*Numerical Simulations of Micro-turbulence in Tokamak Edge*", PhD thesis, Zhejiang University, 2015. http://hsxie.me/files/thesis
  [3] H. S. Xie, Y. Xiao & Z. Lin, Phys. Rev. Lett., 118, 095001 (2017).
  [4] C. S. Chang et al, XGC results, PRL, 118, 175001 (2017).
- Fluid (profile evolution):
  Rogers et al., Phys. Rev. Lett., 81, 4396 (1998).
  X. Q. Xu et al. Phys. Plasmas 7, 1951 (2000).
  Park et al. Phys. Plasmas 22, 032505 (2015).
  L. Chone et al, Nuclear Fusion, 55, 073010 (2015).
  B. Li et al. Phys. Plasmas 24, 055905 (2017).· · ·
- Models:
  Bifurcation: Itoh & Itoh, 1990s
  Prey-predator: Diamond, 1990s-2010s

Evidences/facts gathering

- Unconventional structures: GEM, GYRO (WangE2012, local), GTC (Fulton2014, global)
- Multi-eigenstates: local GS2 (**Ernst2005, APS, below figure**), theoretically know at 1960s (Pearlstein&Berk1969, Chen1980, Horton1981)
- 2D eigen in model (fluid) equation[1]: Dickinson2014, XieT2012, McDevitt2015APS (haven't shown that they are most unstable).
- Local is not conclusive ($\theta_k \neq \theta_p$) and previous works have not told what they are, why and how important of them.



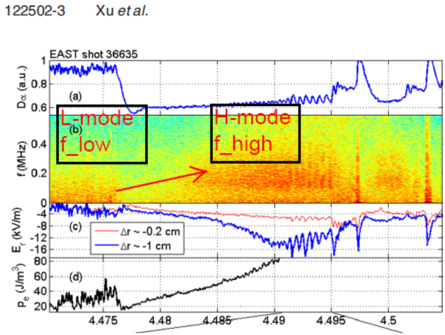
A complete picture should include: global, critical gradient, unconventional mode structures, eigenstates jump, consequences & **physical understanding**

More evidences are gathering, more understandings are required. What about EM (e.g., KBM)?

[1] Preliminary global theory: Xie&Li, PoP, 23, 082513 (2016).

Experimental frequency jump before and after L-H transition

- **HL-2A** (LiuF2010PoP L-mode, Xie2015 H-mode)
- Experimental frequencies (usually TEMs) jumps from low to high before and after L-H transition have also been reported in **EAST** (Xu2012PoP, Wang2012NF)



More quantitatively and qualitatively experimental evidences are required to **support or exclude** the new kinetic eigenstates jump picture to L-H transition.

More

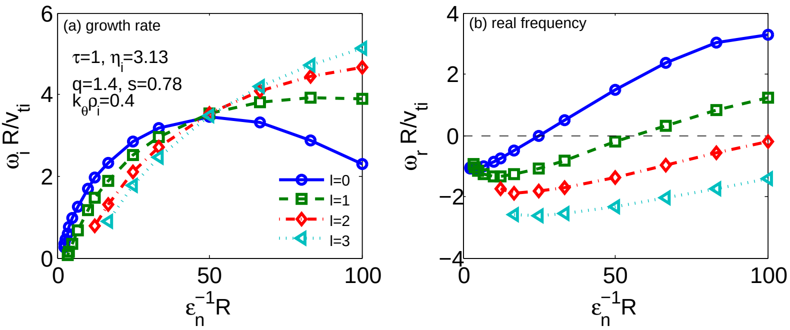
Abstract: First principle gyro-kinetic study of the edge turbulent transport suggests a completed new possible mechanism, without invoking shear flow or zonal flow, for the the low (L) to high (H) confinement modes transition. At H-mode strong gradient the most unstable micro-instabilities are non-ground eigenstates with unconventional mode structures which significantly reduce the effective correlation length and thus reverse the transport trend. Both linear and nonlinear critical gradients exist, which lead **discontinuous jump** as required to explain the L-H transition.

- The **relation** of this **kinetic picture** to traditional **fluid picture and model theory picture**, where flow shear is usually very important, are **not clear** yet.
- Our studies are based on first-principle model **without artificial parameters** and thus can provide **quantitative outputs** to compare with experiments. How important this new mechanism can be in the past and future **experiments can be checked directly**.
- **Flow**, **electromagnetic effects** and **self-consistent evolutions of the profiles** can be considered for the next step to give more quantitative outputs for comparing with experiments.

Gyrokientic eigen solutions

Gyrokinetic Eigen solutions

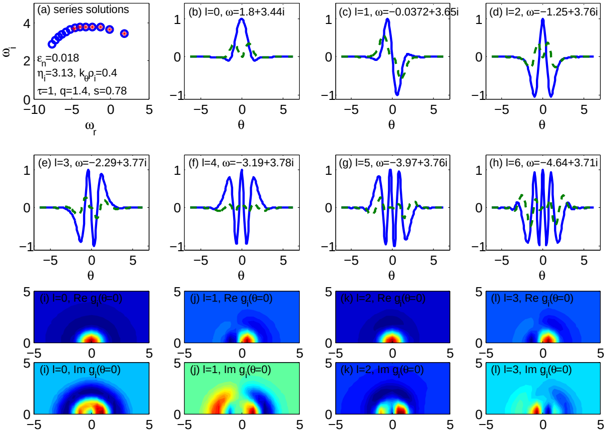
Gyrokinetic-Poisson equation (s-α model)[2]



At strong gradient, the most unstable ITG mode transit from $l = 0$ ground state even mode to $l \geq 1$ high order ITG modes at $\epsilon_n^{-1} R \simeq 50$. The real frequency can transit to electron direction!! $\rightarrow$ the propagation direction is not a decisive criteria for the experimental diagnosis of turbulent mode at the edge plasmas.

[2] Xie et al, PoP, 2017, arXiv 1706.03914.

Series higher order ITGs[3]

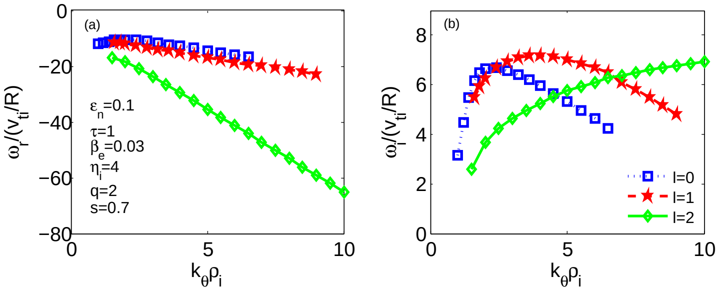


For cyclone parameters, $k_\perp \rho_i = 0.4$ and $\epsilon_n = 0.018$. Multi-eigenmodes are shown, where the most unstable modes are around quantum number $l \simeq 2 - 5$.

[3] See also: M. K. Han, Z. X. Wang, J. Q. Dong and H. R. Du, NF, 2017, 57, 046019.

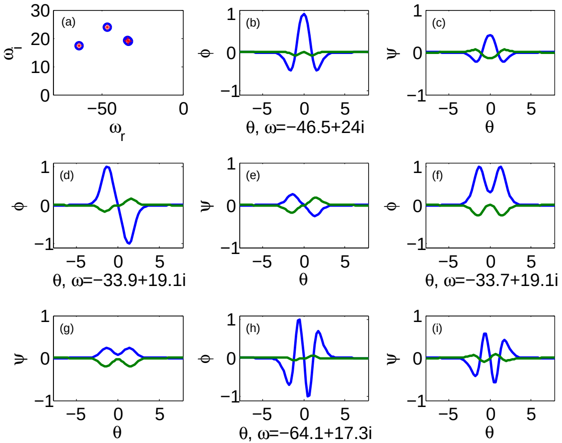
Gyrokinetic Electromagnetic model

Gyrokinetic Electromagnetic model (with $\delta\phi$, $\delta A_{\parallel}$, remove $\delta B_{\parallel}$, adiabatic electron, $\alpha = 0$), eigen solution

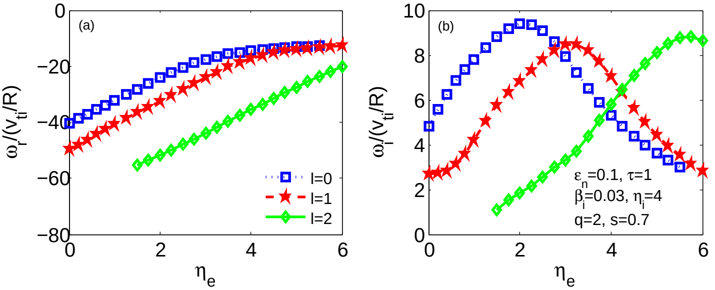


Multi KBMs co-exist, for $l = 0, 1, 2$. The $l = 1$ KBM (i.e., MTM) dominates at $3 \lesssim k_\theta \rho_i \lesssim 7$, and the $l = 2$ KBM dominates at $k_\theta \rho_i \gtrsim 7$.

Microtearing mode (MTM) can merely be $l = 1$ KBM!



With increasing gradient, i.e., $\epsilon_n = 0.05$, $\eta_i = 8.5$, $l = 3$ (h&i) KBM can also be found, and the most unstable one is $l = 2$ (b&c) under these parameters ($k_\theta \rho_i = 6.0$, $\beta_i = 0.05$, $s = 0.78$, $q = 1.4$, $\tau = 1.0$).

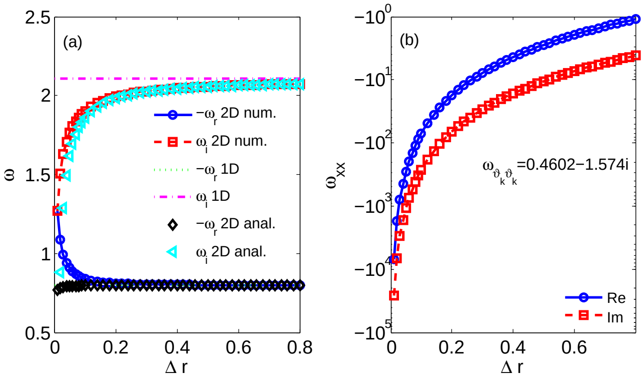


Scanning of η_e for $l = 0, 1, 2$ KBMs with $k_\theta \rho_i = 5.0$. For $\eta_e = 0$, the $l = 0, 1$ KBMs are still unstable, which means that electron temperature gradient is not a must for $l = 1$ KBM (i.e., MTM).

Also slab tearing parity AITG: Z. Gao, J. Q. Dong, G. J. Liu and C. T. Ying, Phys. Plasmas, **9**, 1692 (2002).

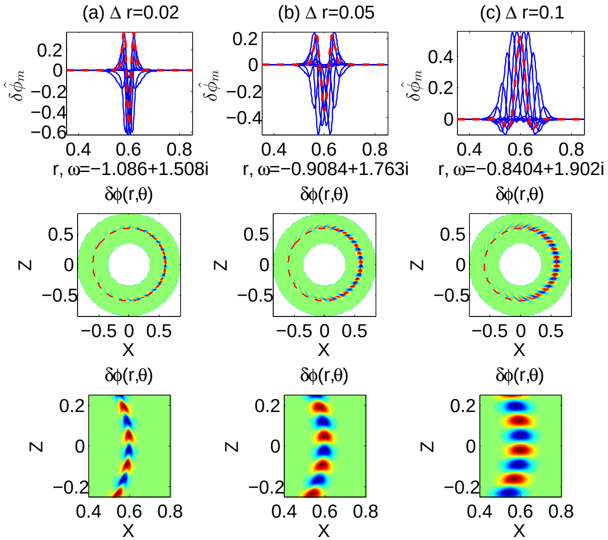
Global effect

Fluid electrostatic model[4]



Steep gradient leads growth rate reduction. $\epsilon_n^{-1} = \epsilon_{n0}^{-1} e^{-(r-r_s)^2/\Delta r^2}$

[4]Xie&Li, PoP, 23, 082513 (2016).

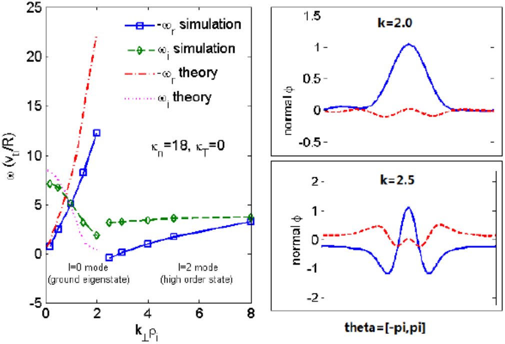


And also twisting (triangle-like) mode structure, due to imag part of $b = k_\theta^2 s^2 \omega_{xx}/\omega_{\vartheta_k \vartheta_k}$. Fast particle is not a must!

Other configurations

Strong gradient high order mode also in other configurations, e.g., dipole

Scan $k_\perp \rho_i$ (gkd1d code, f90 + mpi)



Against to previous result of $k_\| \sim 0$ mode dominate, a high order $k_\| \neq 0$ mode is most unstable at larger $k_\perp \rho_i$ for strong gradient $\kappa_n = 18$. New!!