托卡马克边界微观湍流的数值模拟

Numerical Simulations of Micro-turbulence in Tokamak Edge

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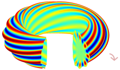
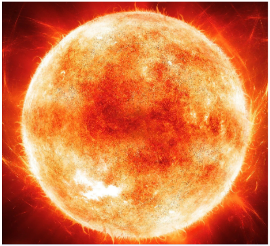


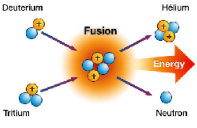
Content

I. Motivations & Background

Fusion on the Sun ... and the Earth...

- Controlled fusion $\rightarrow$ to solve the energy problem
- High temperature ($>$ 1keV) $\rightarrow$ plasma (charged particles + e.m. field)
- Equations: kinetic (Vlasov/Boltzmann) + Maxwell (e.m.) / Possion (e.s.)





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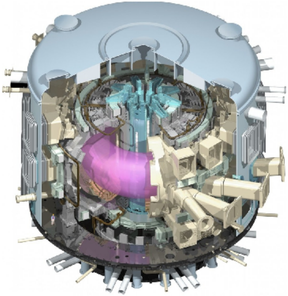
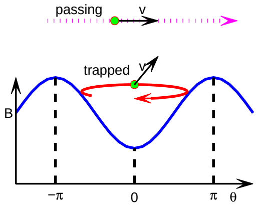
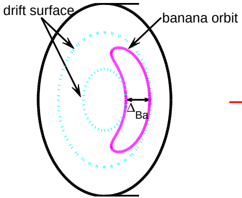


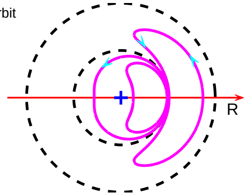
Here, we study **tokamak** (magnetic confinement) plasma.

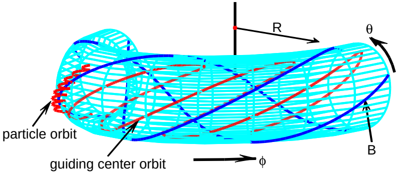
Magnetic confinement

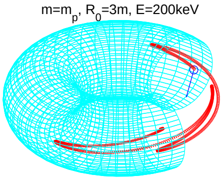
Charged particles **well confinement** in tokamak equilibrium magnetic field





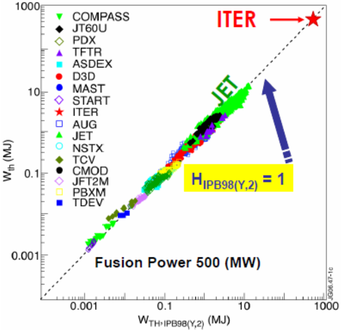




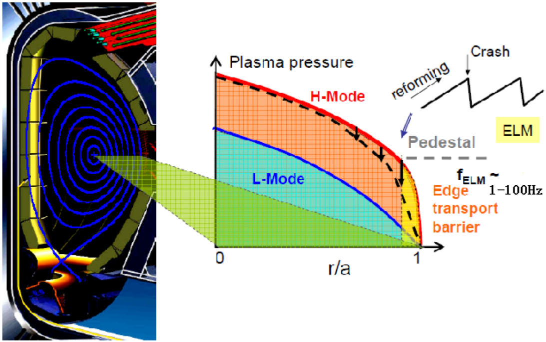


Collision / perturbation (turbulence) lead transport

H-mode operation of tokamak



$$\tau_{E,th}^{ELMy} = 0.0562 I^{0.93} B^{0.15} P^{-0.69} n^{0.41} \times M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}$$

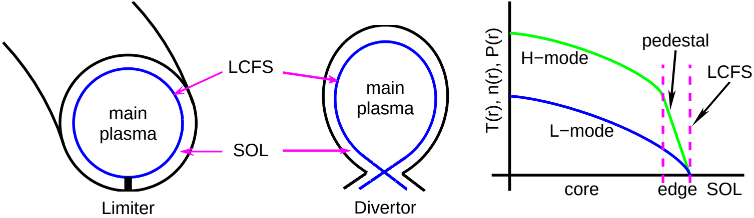


Y. F. Liang, 4TH ITER International Summer School, Austin, Texas USA, 2010

- H-mode (Wagner *et al.*, 1982) is ITER baseline scenario
- Energy stored in H-mode is **twice** or more than L-mode
- Two **'phases'**: L-mode - weak gradient; H-mode - strong gradient

First principle studies of the edge physics still lacking

Physics: a. core - comprehensively studied; **b. edge - current frontier;** c. SOL - more complicated (atom/molecule process)

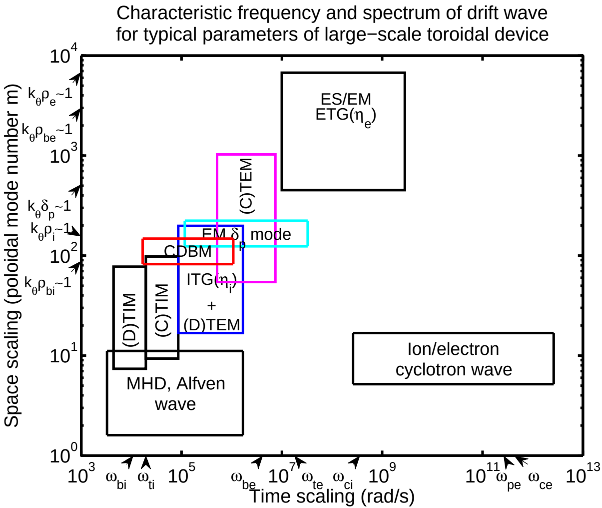


Existed studies of edge H-mode and ELM

- **Kinetic: beginning stage (GYRO, GTC, GEM, $\cdots$), challenged**
- Fluid models: BOUT++, JOREK, $\cdots \rightarrow$ no kinetic physics
- Simplified models (e.g., ODEs): bifurcation (Itoh-Itoh), prey-predator (Diamond) $\rightarrow$ qualitative at most

$\Rightarrow$ **We focus on edge kinetic (first principle) physics**

Edge mirco-instabilities / turbulence



Edge: **ITG, TEM, KBM,** $\cdots$ (temperature/pressure gradient, trapped particles)

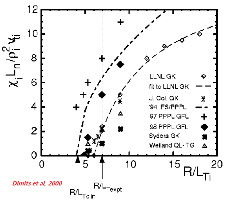
II. Edge Electrostatic Physics (Main)

Here and below, simulations are mainly based on GTC code.
Linear theories are mainly based on eigen solutions.

1. Background and initial setups

Define heat flux $q_j = \int dv^3(\frac{1}{2}m_j v^2 - \frac{3}{2}T_j)\delta v_r \delta f_j \equiv n_j \chi_j \nabla T_j$, $j = i, e$. H-mode unknown physics issues:

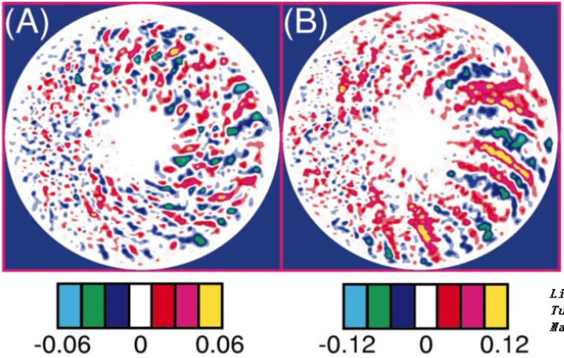
- L-H transition is still not fully understood.
- **How will transport coefficient χ_j changes with ∇T_j increasing?** Does mixing length ($D \sim l_c^2/\tau_c$) estimation really valid? Or, how to estimate l_c and τ_c? A simplest one $D \sim (\gamma_k/k_\perp^2) \propto \gamma_k$. Taroni-Bohm (Horton2012 book) gives $\chi_e \propto \nabla T_e$.

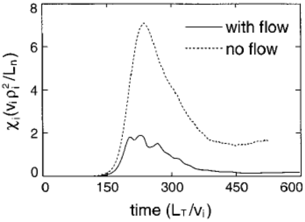


- Is **zonal flow** still important?
- How important the **mode coupling** can be in the nonlinear evolutions?

Next, we focuses on **edge electrostatic** physics.

Physics understandings in L-mode (weak gradient) still hold in H-mode strong gradient stage?





Lin, Z. et al., Science, 1998, 281, 1835
Turbulent Transport Reduction by Zonal Flows: Massively Parallel Simulations

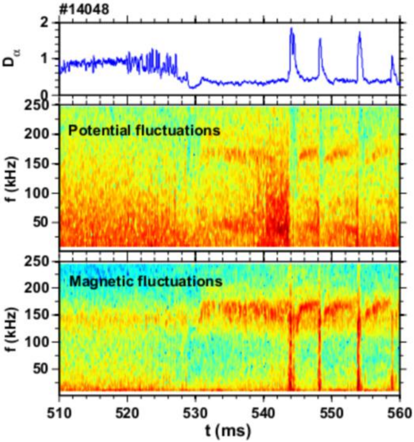
Also for H-mode?

It is believed that (at least in **L-mode** stage or **core** plasmas):

- **Zonal flow** important to reduce transport (eg., Chen01, Waltz08)
- **Mode coupling** important for nonlinear cascading (eg., Lin05, Chen05)
- Larger gradient → **larger** transport coefficients

HL-2A H-mode experiments

Typical HL-2A H-mode exp. signal (#14048, from D. F. Kong)



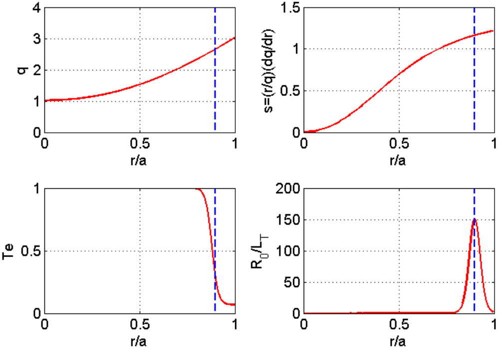
- ES: low frequency → **this work**
- EM: high frequency → (my) simulations not succeed yet, $f^{GTC} \gg f^{exp.}$

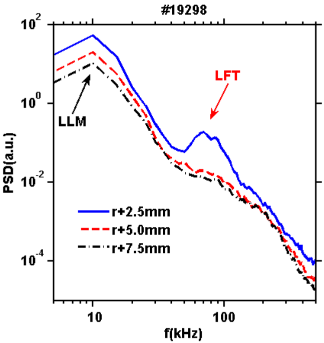
GTC edge simulation parameters

GTC edge simulation parameters are taken from recent H-mode exp. of HL-2A (#19298, from D. F. Kong)

- $f \sim 80$kHz, $m \sim 10$-33

HL-2A **typical L-mode** $R_0 L_T^{-1} < 40$**,** **typical H-mode** $R_0 L_T^{-1} > 80$

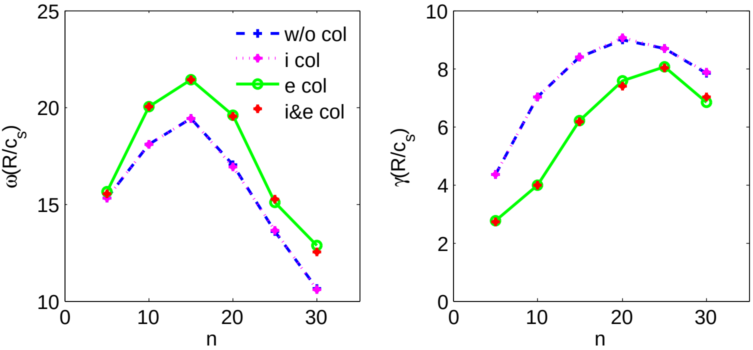




$B_0 = 1.35T$, $a = 40cm$, $R_0 = 165cm$, $q = 2.5 - 3.0$, $s = 0.3 - 1.0$, $R_0/L_T = 80 - 160$, $T_e(r) = T_i(r)$, $n_e(r) = n_i(r)$, $\eta = L_n/L_T \simeq 1.0$.

2. Linear results and eigen theory

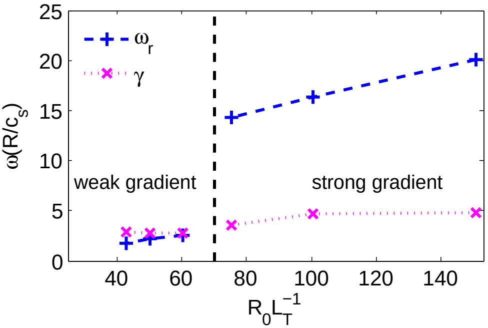
Scanning of n, γ^{max}: $n \simeq 25 \rightarrow m \simeq nq \simeq 60 \gg m^{exp.} \simeq 10-33 \Rightarrow$ **nonlinear required** (later)

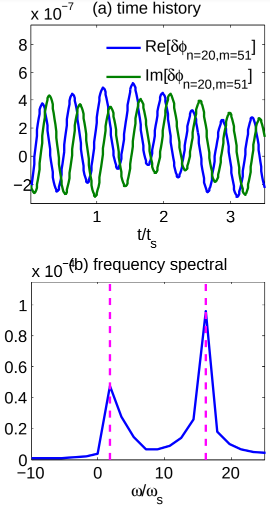


- Adiabatic electron: no instability → not ITG
- Kinetic electron: unstable, electron diamagnetic direction → **TEM**
- Collision: e col. small damping → CTEM not DTEM

Two Trapped Electron Mode (TEM) branches

$n = 20$, $T_e = 200eV$ (Right figure: $R/L_T = 75$)





Most unstable micro-instabilities under weak and strong gradients are in different branches: (H) $\omega_r > 10\omega_s$, $\omega_r \gg \gamma$; (L) $\omega_r < 3\omega_s$, $\omega_r < \gamma$.

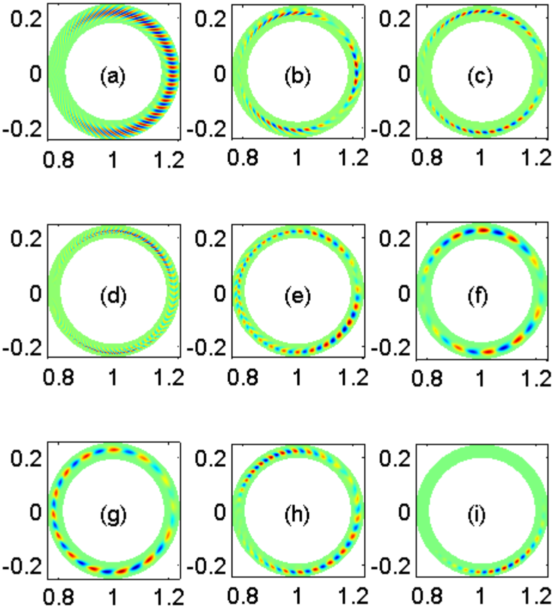
Various mode structures

Single-n ($n = 5 - 30$)

- **(a) weak gradient** L-mode parameter gives **conventional ballooning structures** appendix of TEM in GTC simulation
- **(b)-(i) strong gradient** H-mode parameters give **unconventional structures** of TEM.

Mostly unexpected:

- a. **anti-ballooning**, $|\theta_p| > \pi/2$
- b. **multi-peak**

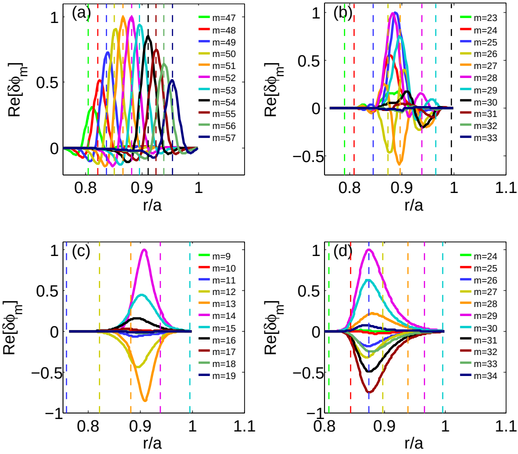


Fourier components $\delta\phi_m(r)$ of TEM

$$\delta\phi(r,\theta,\zeta) = e^{in\zeta}\sum_m \delta\phi_m(r)e^{-im\theta}$$

Corresponding polodial cross section mode structures of (a)-(d) are taken from previous (a), (b), (g) and (i), respectively.

- Unconventional mode structures (especially anti-ballooning, $u_m \simeq -u_{m+1}$, i.e., a 180° phase shift for neighboring Fourier) can **reduce the effective correlation length**. We can expect that **H-mode can have better confinement**.



Strong gradient $k_\parallel \propto |nq - m| > 1 \neq 0$

Model linear theory

- **Model** eigenmode equation for unconventional structure of drift wave

$$\Big[\rho_i^2\frac{\partial^2}{\partial x^2}-\frac{\sigma^2}{\omega^2}\Big(\frac{\partial}{\partial\theta}+ik_\theta sx\Big)^2-\frac{2\epsilon_n}{\omega}\Big(\cos\theta+\frac{i\sin\theta}{k_\theta}\frac{\partial}{\partial x}\Big)-\frac{\omega-1}{\omega+\eta_s}-k_\theta^2\rho_i^2\Big]\delta\phi(x,\theta)=0, \quad (1)$$

$\sigma=\epsilon_n/(qk_\theta\rho_i)$, $\eta_s=1+\eta_i$, $x=r-r_s$, poloidal wave number $k_\theta=nq/r$

- **1D**: Corresponding 1D equation in ballooning space (normalization: ω_{*e})

$$\Big\{\frac{\sigma^2}{\omega^2}\frac{d^2}{d\vartheta^2}+k_\theta^2\rho_i^2[1+s^2(\vartheta-\vartheta_k)^2]+\frac{2\epsilon_n}{\omega}[\cos\vartheta+s(\vartheta-\vartheta_k)\sin\vartheta]+\frac{\omega-1}{\omega+\eta_s}\Big\}\delta\hat{\phi}(\vartheta,\vartheta_k)=0, \quad (2)$$

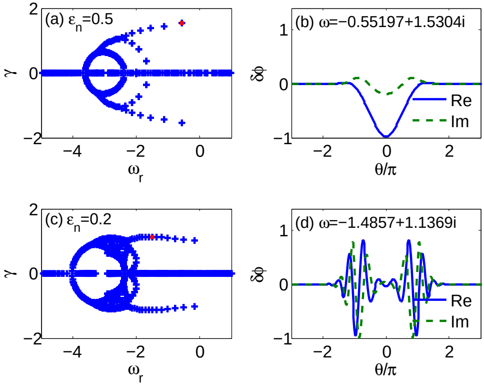
ϑ_k ballooning-angle parameter.

l: 'quanta' number

- Approximate to Weber equation $u''+(bx^2+a)u=0$, eigenvalues $a(\omega)=i(2l+1)\sqrt{b(\omega)}$, eigenfunctions $u(x)=H_l(i\sqrt{b}x)e^{-ibx^2/2}$, H_l is l-th Hermite polynomial ($l=0,1,2,...$), **series eigenstates**.

1D eigen solutions to drift instability

- **Weak gradient ($\epsilon_n \equiv L_n/R = 0.5$), most unstable solution ground state (a&b)**, conventional structure.
- **Strong gradient ($\epsilon_n = 0.2$), most unstable solution not ground state (c&d)**, unconventional.
- **Condition $\epsilon_n < \epsilon_c$**, critical gradient parameter ϵ_c depends on other parameters.



Xie&Xiao, Phys. Plasmas, 22, 090703 (2015).

Eq.(2), series solutions exist. ($s = 0.8$, $k_\theta \rho_i = 0.4$, $q = 1.0$, $\eta_s = 3.0$ and $\vartheta_k = 0$)

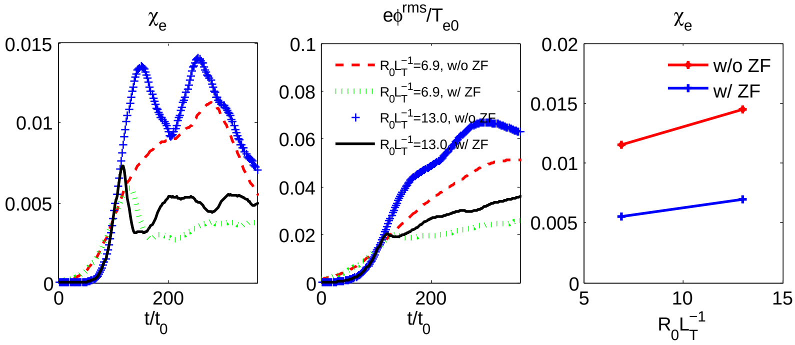
Solved by companion matrix method

appendix $\Rightarrow$ **All solutions.**

Linear: Eigenstates jump!!!

3. Nonlinear results: normal turbulent transport understandings in L-mode/weak gradient

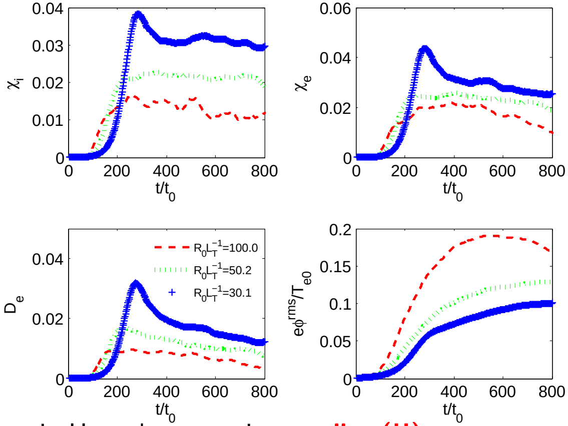
Agree with usual understandings / theoretical models slide 11



- Stronger gradients in L-mode stage give larger transport coefficients
- Zonal flow can reduce the transport coefficients significantly

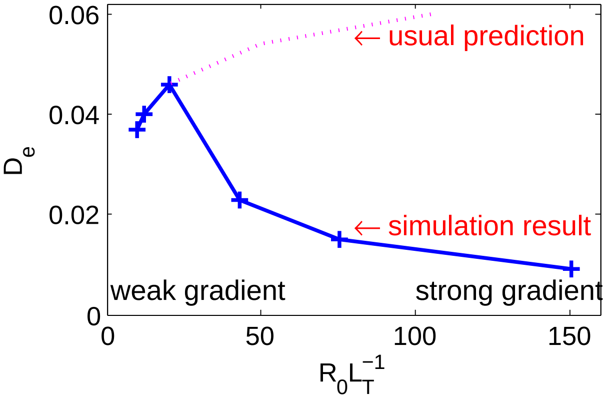
Reverse trend of turbulent transport: H-mode/strong gradient

heat conductivity χ_j, particle diffusivity D_j



Stronger gradients in H-mode stage give **smaller (!!)** transport coefficients of particles and energy, though the root mean square of e.s. potential still higher.

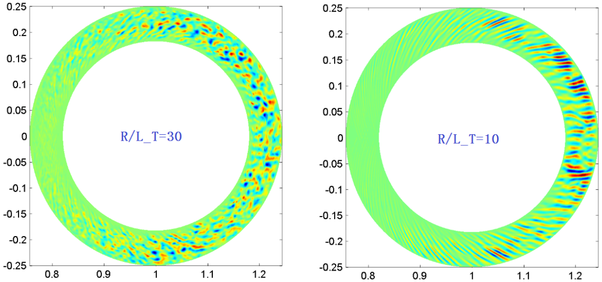
Nonlinear critical gradient exist



A turning point (**critical gradient**) exists for the reverse trend of the transport coefficients.

Eddy sizes - correlation length

Estimate the radial correlation length l_c from the eddy size.



Strong gradient ($R_0 L_T^{-1}$=30) small eddy size. Weak gradient ($R_0 L_T^{-1}$=10) large eddy size. Assume correlation time τ_c not change too much $\rightarrow$ $D \sim l_c^2/\tau_c \propto l_c^2 \rightarrow D \downarrow$.

Why stronger gradient has a small eddy size? The **unconventional mode structures**!!

Discussions

- Strong gradient (H-mode) eigen state $l \neq 0$ v.s. weak gradient (L-mode) $l = 0$, indicate different transport behaviors between H-mode and L-mode.
- Unconventional mode structures can reduce the effective correlation length. We can expect that **H-mode can have better confinement**.
- Nonlinear simulations confirm that the **transport coefficients decrease** with gradient increasing.

Thus ... see diagram

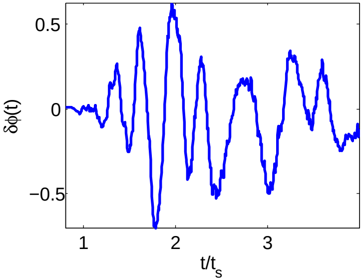
Provides some hints to L-H transition and H-mode transport mechanism by first-principle gyrokinetic simulations.

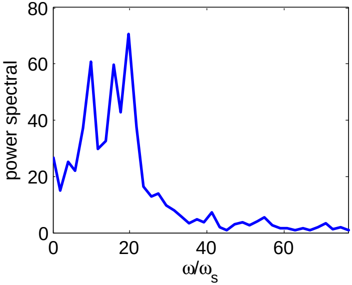
$$\mathrm{L} \rightleftharpoons \mathrm{H}$$

Eigenstates jump ! vs. **'phase' transition ?**

Nonlinear frequency

Diagnose at a fixed point $(r = r_c, \theta = \pi/2, \zeta = 0)$, $\omega \simeq 16\omega_s$



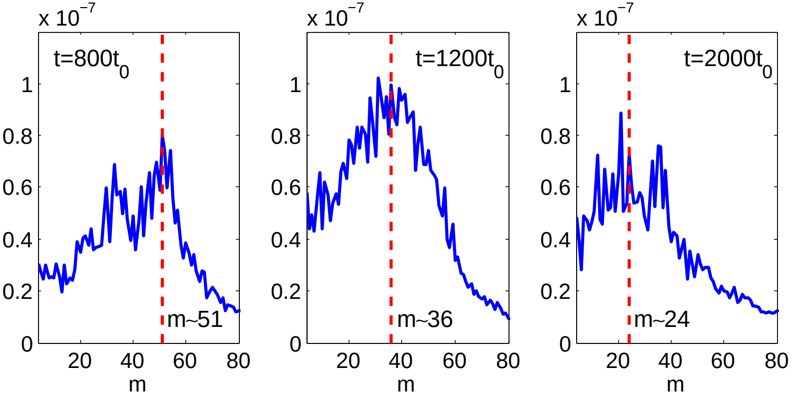


if $T_e \simeq 50eV \Rightarrow f^{sim.} \simeq 78kHz + f^{doppler}$, if $|f^{doppler}| < 10kHz$
$\Rightarrow f^{sim.} \simeq f^{exp.} \simeq 80kHz$.

Nonlinear **frequency agrees exp. !!**

Nonlinear evolutions of the poloidal spectral

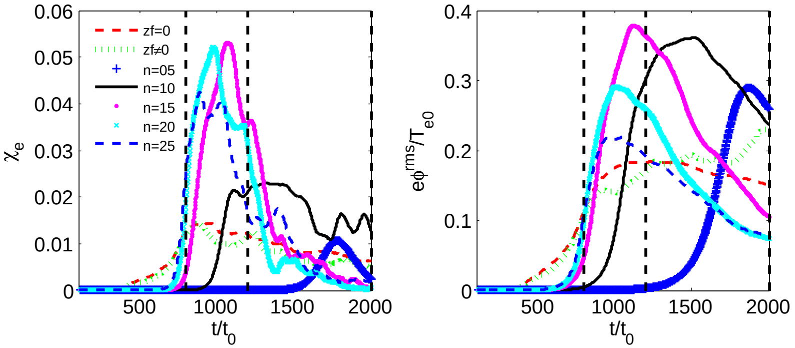
$m^{sim.} \simeq 10 - 40$ **vs.** $m^{exp.} \simeq 10 - 33$, nonlinear poloidal **spectral agrees exp. !!**



Reverse cascading from high to low m mode number.

Mode coupling and zonal flow are less important in strong gradient

multi-n (w/ & w/o zonal flow) vs. single-n



$t = 800t_0$, dominate is $n \simeq 20 - 25$ gives $m \simeq nq \simeq 57$; $t = 1200t_0$, $n \simeq 15$ gives $m \simeq nq \simeq 40$; $t = 2000t_0$, $n \simeq 10$ gives $m \simeq nq \simeq 26$.

Close to multi-n (previous slide) results, **reveal multi-mode-coupling not important for m downshift as in L-mode** [e.g., Wang07, Lang08].

III. Electromagnetic Physics (Preliminary)

1. Global ideal ballooning mode

AMC[1] code solves the linear ideal MHD vorticity equation (shear Alfvén law)

$$\underbrace{\nabla\cdot\left(\frac{\omega^2}{v_A^2}\nabla_\perp\delta\phi\right)}_{\text{inertial}}+\underbrace{\mathbf{B}\cdot\nabla\left(\frac{1}{B^2}\nabla\cdot B^2\nabla_\perp Q\right)}_{\text{field line bending}}-\underbrace{\nabla\left(\frac{J_\|}{B}\right)\cdot(\nabla Q\times\mathbf{B})}_{\text{kink / parallel equilibrium current}}+\underbrace{2\frac{\kappa\cdot(\mathbf{B}\times\nabla\delta P)}{B^2}}_{\text{ballooning}}=0, \quad (3)$$

$\kappa=\mathbf{b}\cdot\nabla\mathbf{b}$, $Q=(\mathbf{b}\cdot\nabla\delta\phi)/B$, $\delta P=(\mathbf{b}\times\nabla\delta\phi\cdot\nabla P)/B$, $J_\|=\mathbf{b}\cdot\nabla\times\mathbf{B}$.

Shifted circular geometry. Second order for $\epsilon=r/R\ll 1$, $\beta\sim O(\epsilon^2)$.

Terms separated well, good for theoretical study.

[1] H. S. Xie & Y. Xiao, Phys. Plasmas, 22, 022518 (2015).

We solve below coupled equation

$\delta\phi = \sum \delta\phi_m(r)\exp(in\zeta - im\theta)$, expanding Eq.(3) to $O(\epsilon^2)$, to a coupled equation

$$\mathbf{L_{m,m-1}}\delta\phi_{\mathbf{m-1}} + \mathbf{L_{m,m}}\delta\phi_{\mathbf{m}} + \mathbf{L_{m,m+1}}\delta\phi_{\mathbf{m+1}} = \mathbf{0}, \tag{4}$$

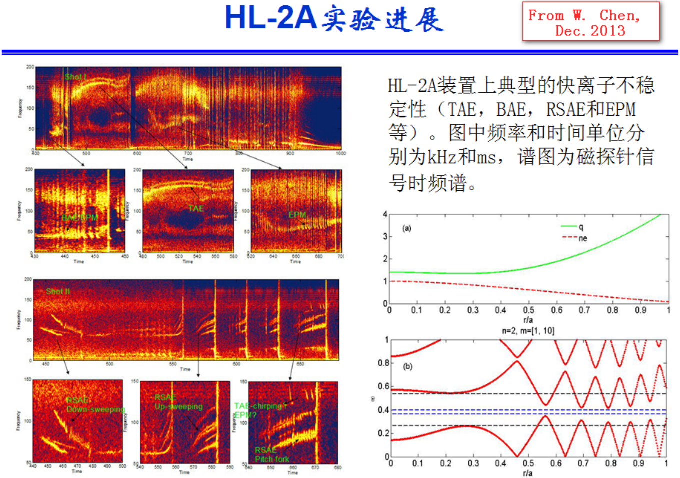
$$L_{m,m} = \frac{\partial}{\partial r}\Big[\frac{(1+4\epsilon\Delta')}{v_A^2}\bar{\omega}^2 - k_m^2 - c_s^2\Big] r\frac{\partial}{\partial r} + (k_m^2)' - \frac{m^2}{r}\Big\{\frac{[1-4\epsilon(\epsilon+\Delta')]}{v_A^2}\bar{\omega}^2 - k_m^2 - c_s^2 - \bar{\kappa}_r\alpha/q^2\Big\}, \tag{5}$$

$$L_{m,m\pm1} = \bar{\omega}^2\Big\{\frac{\partial}{\partial r}\frac{(2\epsilon+\Delta')}{v_A^2}r\frac{\partial}{\partial r} - \frac{(\epsilon-\Delta')}{v_A^2}\frac{m(m\pm1)}{r} \mp \frac{[\epsilon+(r\Delta')']}{v_A^2}m\frac{\partial}{\partial r}\Big\} - \Big\{\frac{\partial}{\partial r}r\Delta' k_m k_{m\pm1}\frac{\partial}{\partial r} - \frac{m^2}{r}(\epsilon+\Delta')k_m k_{m\pm1} \mp m[\epsilon+(r\Delta')']k_m k_{m\pm1}\frac{\partial}{\partial r}\Big\} - \frac{m\alpha}{2q^2}\Big(\frac{m}{r} \mp \frac{\partial}{\partial r}\Big). \tag{6}$$

$\bar{\omega} = \omega/(V_A/R_0)$, $V_A = \langle v_A(r,\theta)\rangle$, $k_m = (n - m/q)$.
$\mathbf{AX} = \lambda\mathbf{BX}$, $\omega^2 = \lambda$, $\mathbf{X} = [\cdots, \delta\phi_{m-1}, \delta\phi_m, \delta\phi_{m+1}, \cdots]^T$. $\Rightarrow$ **All solutions.**

Successfully applied to experiments for AEs

AMC frequencies and RSAE sweeping agree with HL-2A experiment.



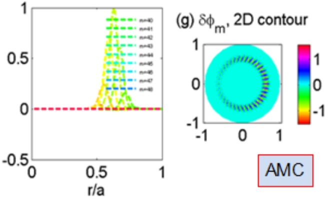
Others: [1] EPL, 107, 25001 (2014); [2] Chen et al, 2015 (submitted); [3] Also J-TEXT (HUST) and SUNIST (Tsinghua).

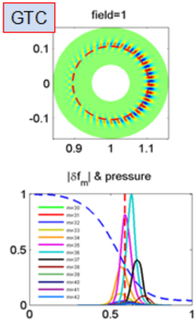
IBM: first step to KBM

Ideal Ballooning

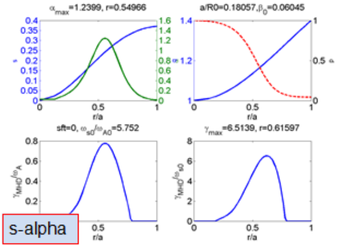
	gamma	r/a (position)
s-alpha, n-> infty	6.51	0.62
gtc, n=30	6.7	0.60
amc-reduce, n=30	5.75	0.63

w/o shift





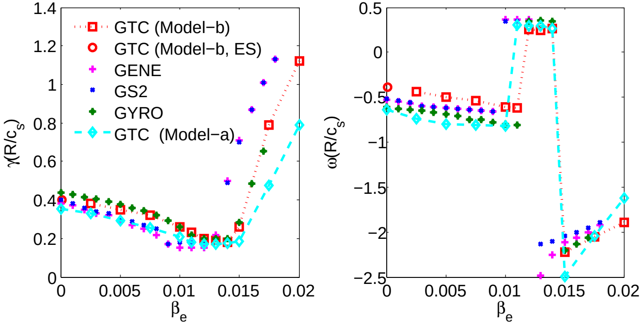
well agree



IBM benchmark: AMC, local s-α, GTC. Well agree $\Rightarrow$ **linear IBM has been well understood**.

2. Global kinetic ballooning mode

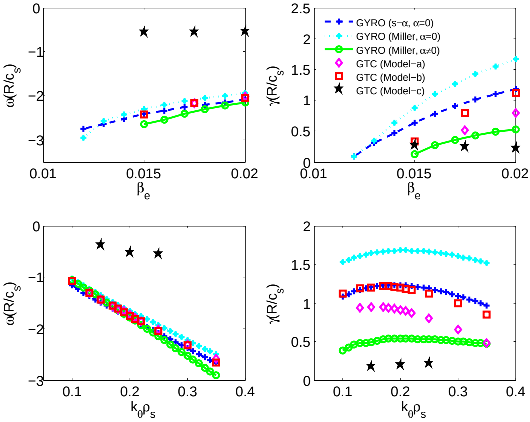
β increasing (Cyclone parameters): ITG $\rightarrow$ TEM $\rightarrow$ KBM



Equilibrium implementations do not affect ITG and TEM significantly, but affect the KBM branch significantly.

Different equilibrium implementations

Model-a: $\theta=\theta_s$. Model-b: $\Delta=0$, $\theta=\theta_s-\epsilon\sin\theta_s$. Model-c: $\Delta\neq0$, $\theta=\theta_s-(\epsilon+\Delta')\sin\theta_s$. (GYRO data from E. A. Belli)



⇒ **Accurate global equilibrium model would be crucial** (especially, global Shafranov shift included) to validate experiments with the gyrokinetic simulation.

IV. Summary & Future works

Summary: Turbulent transport in L vs. H

Different transport behavior between H-mode and L-mode

	new understandings (strong gradient)	conventional understandings (weak gradient)
linear	series eigenstates non-ground state	one eigenstate ground state
zonal flow	less important	very important
mode-coupling	less important	important
$\nabla T \uparrow$	χ_j or $D_j \downarrow$	χ_j or $D_j \uparrow$

Novel linear and nonlinear physics in H-mode strong gradient, **limits the scope of the conventional understandings** of the turbulent transport in L-mode.

Compare: experiment, theory and simulation

"Extraordinary claims require extraordinary evidence." – Carl Sagan

How much confidence?

1. Exp. and simulation under weak and strong gradient are both low and high frequency (i.e., **eigenstates jump**), respectively. Mode numbers and frequencys agree **quantitatively**. Eigen theory agree **qualitatively**. Global quantitatively eigen theory is very complicated and is under development.

2. The **transport turning point** under strong gradient simulations of course exist in experiment which leads H-mode. The quantitatively comparing is much difficult due to that many other factors should be involved.

3. The less importance of **zonal flow** has also been found in some recent experiments (Kobayashi2013PRL), which can reasonably be understood from unconventional mode structures.

4. The less importance of **mode coupling** requires further evidences to be confirmed.

Summary: EM

1. A new fast and easy used global eigen code **AMC** is developed to study AEs and IBM, which has also been succeed used for domestic experiments.

2. Showing that the **equilibrium implementation** is crucial to validate experiments with the gyrokinetic KBM simulation.

3. Some yet unsolved linear & nl issues for KBM simulations are examined and listed. (appendix)

Future works

1. Developing the **quantitatively eigen theory**. Giving more comprehensive **physical pictures**.

2. Further checking the **nonlinear results**.

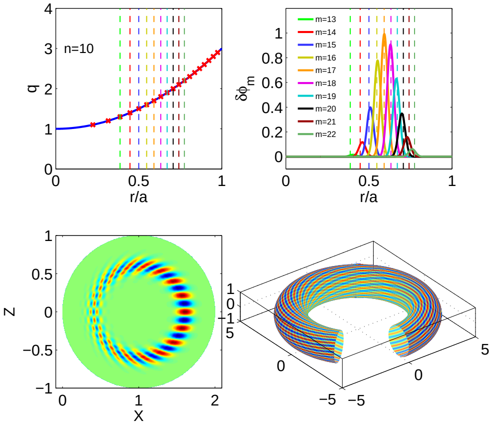
3. **Electromagnetic** studies are still challenged. Solutions are required.

Thank you!

Conventional ballooning structure

$$\delta\phi(r,\theta,\zeta,t) = e^{-i(n\zeta-\omega t)} \sum_m \delta\hat{\phi}_m(r) e^{im\theta}$$

back to GTC TEM



$nq(r_m) = m$, $k_\parallel = (nq-m)/R_0 \simeq 0$, $\delta\hat{\phi}_m(r-r_m) \simeq \delta\hat{\phi}_{m+1}(r-r_{m+1})$,
$\delta\hat{\phi}_m(r) = A(r)\exp[-(r-r_m)^2/\Delta r_c^2]$, $A(r) = \exp[-(r-r_c)^2/\Delta A^2]$

Companion matrix method to solve eigenvalue problem

Discrete forms of Eqs.(1) and (2) back can be written as

$$(\mathbf{M}_0+\omega\mathbf{M}_1+\omega^2\mathbf{M}_2+\omega^3\mathbf{M}_3)\cdot\mathbf{X}=0. \tag{7}$$

Using the transformation

$$\mathbf{X}=\mathbf{X}_1\Rightarrow\begin{cases}\omega\mathbf{X}_1=\mathbf{X}_2\\ \omega\mathbf{X}_2=\mathbf{X}_3\\ \omega\mathbf{M}_3\mathbf{X}_3=-\mathbf{M}_2\mathbf{X}_3-\mathbf{M}_1\mathbf{X}_2-\mathbf{M}_0\mathbf{X}_1,\end{cases} \tag{8}$$

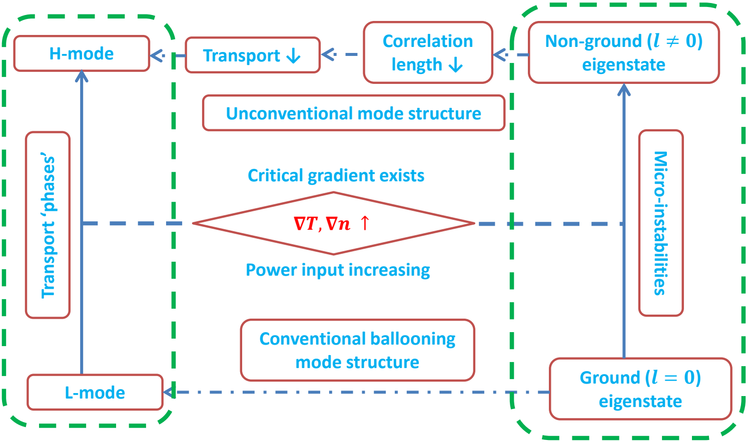
yields an equivalent standard matrix eigenvalue problem

$$\omega\begin{bmatrix}\mathbf{I}&\mathbf{O}&\mathbf{O}\\ \mathbf{O}&\mathbf{I}&\mathbf{O}\\ \mathbf{O}&\mathbf{O}&\mathbf{M}_3\end{bmatrix}\cdot\begin{pmatrix}\mathbf{X}_1\\ \mathbf{X}_2\\ \mathbf{X}_3\end{pmatrix}=\begin{bmatrix}\mathbf{O}&\mathbf{I}&\mathbf{O}\\ \mathbf{O}&\mathbf{O}&\mathbf{I}\\ -\mathbf{M}_0&-\mathbf{M}_1&-\mathbf{M}_2\end{bmatrix}\cdot\begin{pmatrix}\mathbf{X}_1\\ \mathbf{X}_2\\ \mathbf{X}_3\end{pmatrix}. \tag{9}$$

which can gives all solutions of the discrete system.

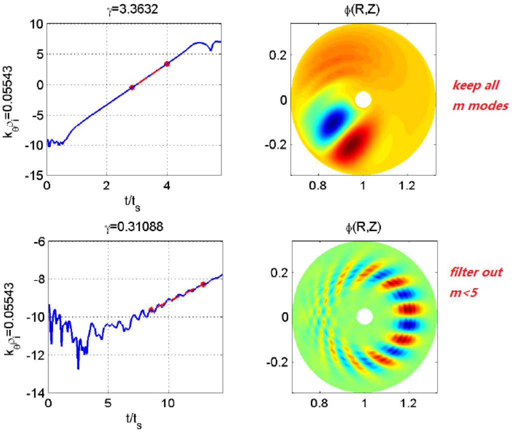
Diagram for new picture of L-H transition

back



Numerical challenges for both linear and nonlinear

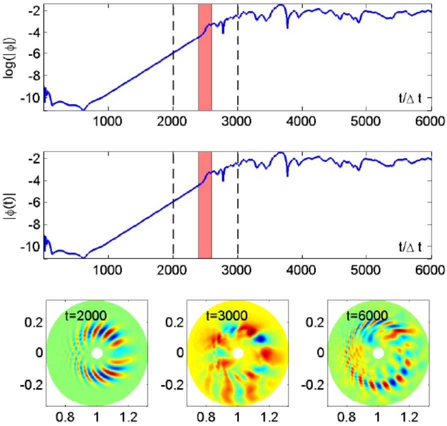
- Linear: growth rate (also frequency) jump ($\gamma = 0.31 \to 3.4$, similar, Wan12) at $k_\theta \rho_i < 0.1$ ($n = 10$, $q = 1.4$), **reason(s) unknown**.



Gathering existing challenges should be very important for next step.

A rare succeed nonlinear example

- Nonlinear: crash for most cases.



Although some successes have been obtained in this decade, the nonlinear (and even linear) gyrokinetic simulations of KBM are still challenged. Physics? Models? Numerical? $\Rightarrow$ **In this work, we stop the E.M. studies here.**