Chapter 7
Using Indicator and Interaction Variables

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Applied Regression Analysis:
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7.1 Using and Interpreting Indicator Variables

- Suppose some observations have a particular characteristic or attribute, while others do not.
- We can include this information in the regression model by using dummy or indicator variables.

Add the info thru a coding scheme

Use a binary (dummy) variable to "indicate" when the characteristic is present

$D_i = 1$ if observation i has the attribute

$D_i = 0$ if observation i does not have it

An Example

$D_i = 1$ if individual i is employed

$D_i = 0$ if individual i is not employed

We could do it the other way and use the "1" to indicate an unemployed individual.

Multiple Categories

- For multiple categories, use multiple indicators.
- For example, to indicate where a firm's stock is listed, we could define 3 indicator variables; one each for the NYSE, AMEX and NASDAQ.
- For computational reasons, we would include only two of these in the regression.

Example 7.1 Employment Discrimination

If two groups have apparently different salary structures, you first need to account for differences in education, training and experience before any claim of discrimination can be made.

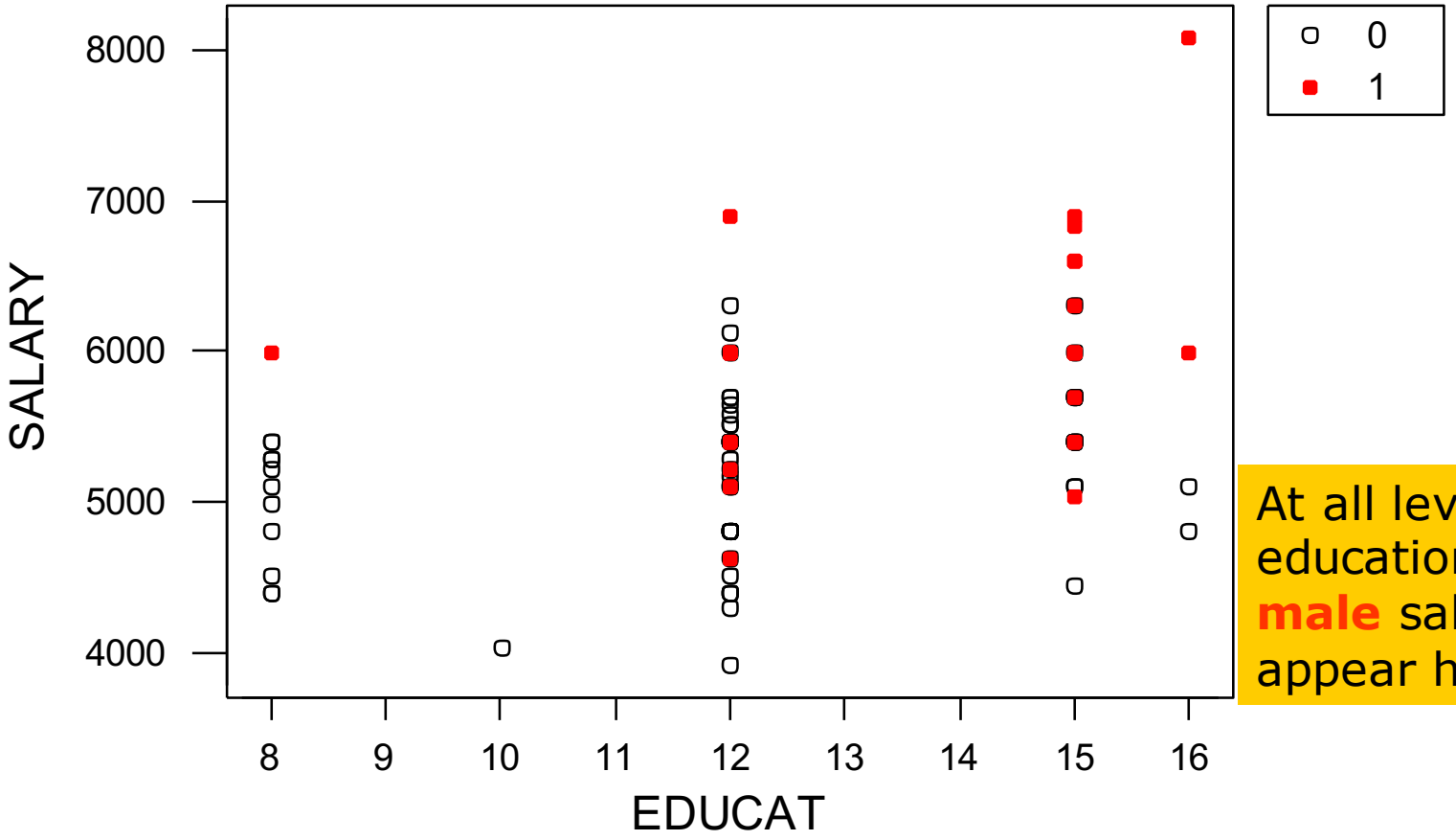
Regression analysis with an indicator variable for the group is a way to investigate this.

Treasury Versus Harris

The data set HARRIS7 contains information on the salaries of 93 employees of the Harris Trust and Savings Bank. They were sued by the US Department of Treasury in 1981.

Here we examine how salary depends on education, also accounting for gender.

Salary Versus Years of Education



At all levels of education, the **male** salaries appear higher.

Regression Analysis

The regression equation is

SALARY = 4173 + 80.7 EDUCAT + 692 MALES

Predictor	Coef	SE Coef	T	P
Constant	4173.1	339.2	12.30	0.000
EDUCAT	80.70	27.67	2.92	0.004
MALES	691.8	132.2	5.23	0.000

S = 572.4 R-Sq = 36.3% R-Sq(adj) = 34.9%

How do we interpret this equation?

An Intercept Adjuster

For an indicator variable, the b_j is not really a slope. To see this, evaluate the equation for the two groups.

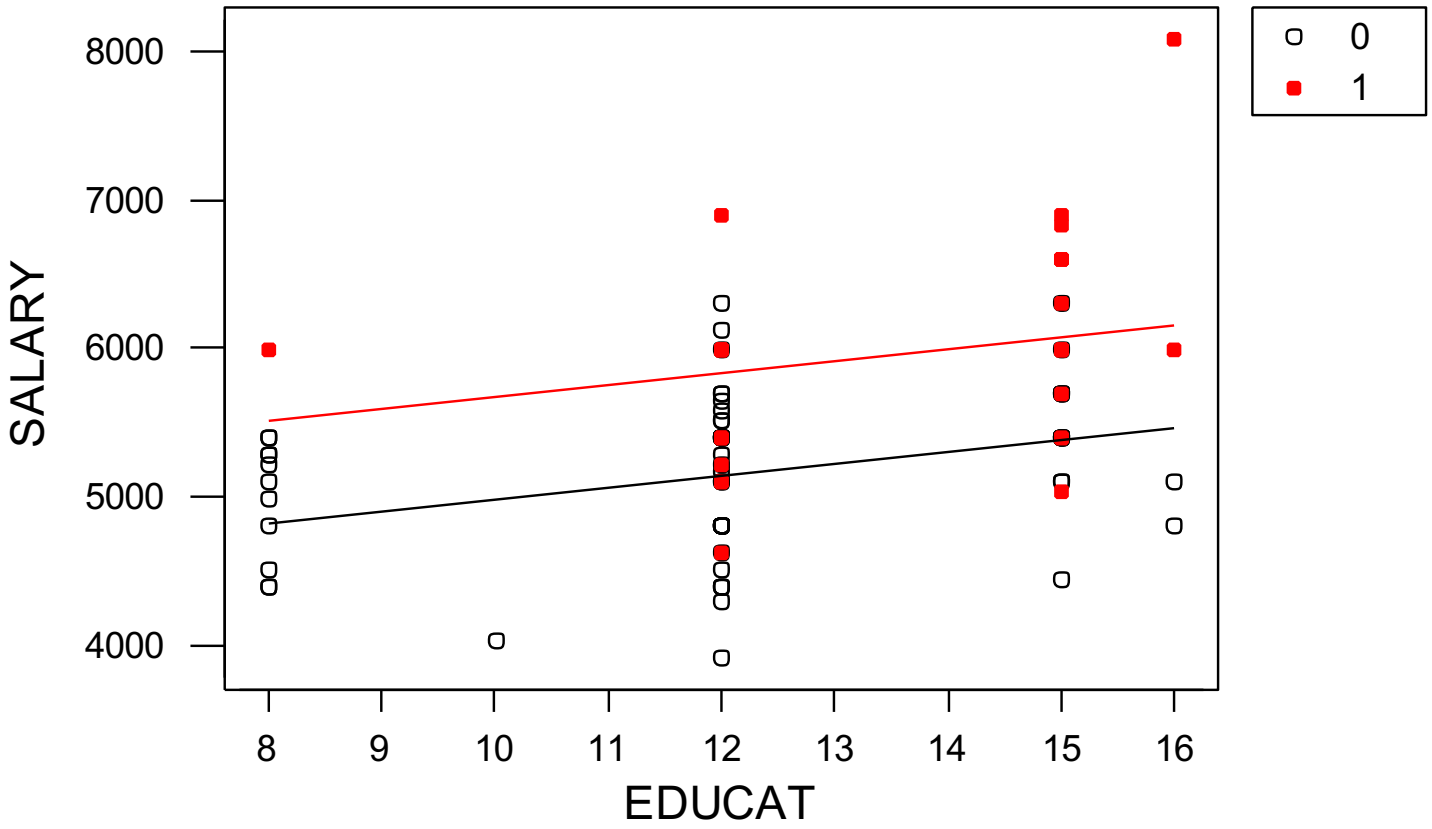
FEMALES (MALES = 0)

SALARY = 4173 + 80.7 EDUCAT + 692 MALES
= 4173 + 80.7 EDUCAT + 692 (0)
= 4173 + 80.7 EDUCAT

MALES (MALES = 1)

SALARY = 4173 + 80.7 EDUCAT + 692 MALES
= 4173 + 80.7 EDUCAT + 692 (1)
= 4173 + 80.7 EDUCAT + 692
= 4865 + 80.7 EDUCAT

Parallel Salary Equations



Is The Difference Significant?

H_0: $\beta_{MALES} = 0$ (After accounting for years of education, there is no salary difference)

H_a: $\beta_{MALES} \neq 0$ (After accounting for education, there IS a salary difference)

Use $t = b/SE_b$ as usual

$t = 5.23$ is significant

What if the Coding Was Different?

- If we had an indicator for females and used it, the equation would be:

SALARY = 4865 + 80.7 EDUCAT - 692 FEMALES

- The difference between the groups is the same. For females, the intercept in the equation is 4865 – 692 = 4173

Multiple Categories

- Pick one category as the "base category".
- Create one indicator variable for each other category.
- In general, if there are *m* categories, use *m – 1* indicator variables.

Example 7.3 Meddicorp Sales

Y = Sales in one of 25 territories

X_1 = advertising in territory

X_2 = bonuses paid in territory

Also Region: 1 = South

2 = West

3 = Midwest

How do you use region?

What happens if you just put it in the model?

Sales = -84 + 1.55 ADV + 1.11 BONUS + 119 Region

R^2 = 92.0% and S_e = 68.89

SE(Region) = 28.69 so t_{stat} = 4.14 is significant

Region as an X

This implies the difference between Region 3 (MW) and Region 2 (W) = b_3 = 119

And the difference between Region 2 (W) and Region 1 (S) is also 119

The sales differences may not be equal but this **forces** them to be estimated that way

A more flexible approach

- Use two indicator variables to tell the three regions apart
- Can use any one of the three as the "base" category.
- Here is what it looks like if Midwest is selected as the base.

Coding scheme

Region	D_1 South	D_2 West
SOUTH	1	0
WEST	0	1
MIDWEST	0	0

Results

SALES = 435 + 1.37ADV + .975 BONUS
- 258 South – 210 West

R^2 = 94.7 and S_e = 57.63

Both indicators are significant

This Defines Three Equations

SALES = 435 + 1.37ADV + .975 BONUS
- 258 South – 210 West

S: SALES = 177 + 1.37ADV + .975 BONUS

W: SALES = 225 + 1.37ADV + .975 BONUS

MW: SALES = 435 + 1.37ADV + .975 BONUS

Is Location Significant?

- Because location is measured by two variables in a group, we need to do a partial F test.
- The full Model has ADV, BONUS, SOUTH and WEST and has $R^2 = 94.7$
- The reduced model has only ADV and BONUS, with $R^2 = 85.5$

Output For F-Test

FULL MODEL

S = 57.63 R-Sq = 94.7% R-Sq(adj) = 93.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	4	1182560	295640	89.03	0.000
Residual Error	20	66414	3321		
Total	24	1248974			

REDUCED MODEL

S = 90.75 R-Sq = 85.5% R-Sq(adj) = 84.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	1067797	533899	64.83	0.000
Residual Error	22	181176	8235		
Total	24	1248974			

Partial F Computations

$$F = \frac{(SSE_R - SSE_F) / (K - L)}{MSE_F}$$

$$= \frac{(181176 - 66414)/ (4-2)}{3321} = 17.3$$

7.2 Interaction Variables

- Another type of variable used in regression models is an interaction variable.
- This is usually formulated as the product of two variables; for example, $x_3 = x_1x_2$
- With this variable in the model, it means the level of x_2 changes how x_1 affects Y

Interaction Model

With two x variables the model is:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + e$$

If we factor out x_1 we get:

$$y = \beta_0 + (\beta_1 + \beta_3 x_2) x_1 + \beta_2 x_2 + e$$

so each value of x_2 yields a different slope in the relationship between y and x_1

Sales on ADV & BONUS

$$SALES = \beta_0 + \beta_1(ADV) + \beta_2(BONUS) + \beta_3(ADV * BONUS) + \epsilon$$

$$SALES = \beta_0 + (\beta_1 + \beta_3 * BONUS)(ADV) + \beta_2(BONUS) + \epsilon$$

(note) ADV*BONUS may be not significant

Interaction Involving an Indicator

If one of the two variables is binary, the interaction produces a model with two different slopes.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + e$$

When $x_2 = 0$

$$y = \beta_0 + \beta_1 x_1 + e$$

When $x_2 = 1$

$$y = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) x_1 + e$$

Example 7.4 Discrimination (again)

- In the Harris Bank case, suppose we suspected that the salary difference by gender changed with different levels of education.
- To investigate this, we created a new variable MSLOPE = EDUCAT*MALES and added it to the model.

Regression Output

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The regression equation is
SALARY = 4395 + 62.1 EDUCAT - 275 MALES + 73.6 MSLOPE

Predictor         Coef     SE Coef           T         P
Constant        4395.3       389.2       11.29     0.000
EDUCAT           62.13       31.94        1.95     0.055
MALES           -274.9       845.7       -0.32     0.746
MSLOPE           73.59       63.59        1.16     0.250

S = 571.4         R-Sq = 37.3%     R-Sq(adj) = 35.2%
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How do we interpret the equation this time?

$$SALARY = \beta_0 + \beta_1(EDUCAT) + \beta_2(MALES) + \beta_3(EDUCAT * MALES) + e$$

$$SALARY = \beta_0 + \beta_1(EDUCAT) + (\beta_2 + \beta_3 * EDUCAT)(MALES) + e$$

$$\hat{\beta}_2 = -274.9 \qquad \hat{\beta}_3 = 73.59 \qquad EDUCAT = 8 \sim 16$$

$$\hat{\beta}_2 + \hat{\beta}_3 * EDUCAT > 0$$

A Slope Adjuster

To see the interaction effect, once again evaluate the equation for the two groups.

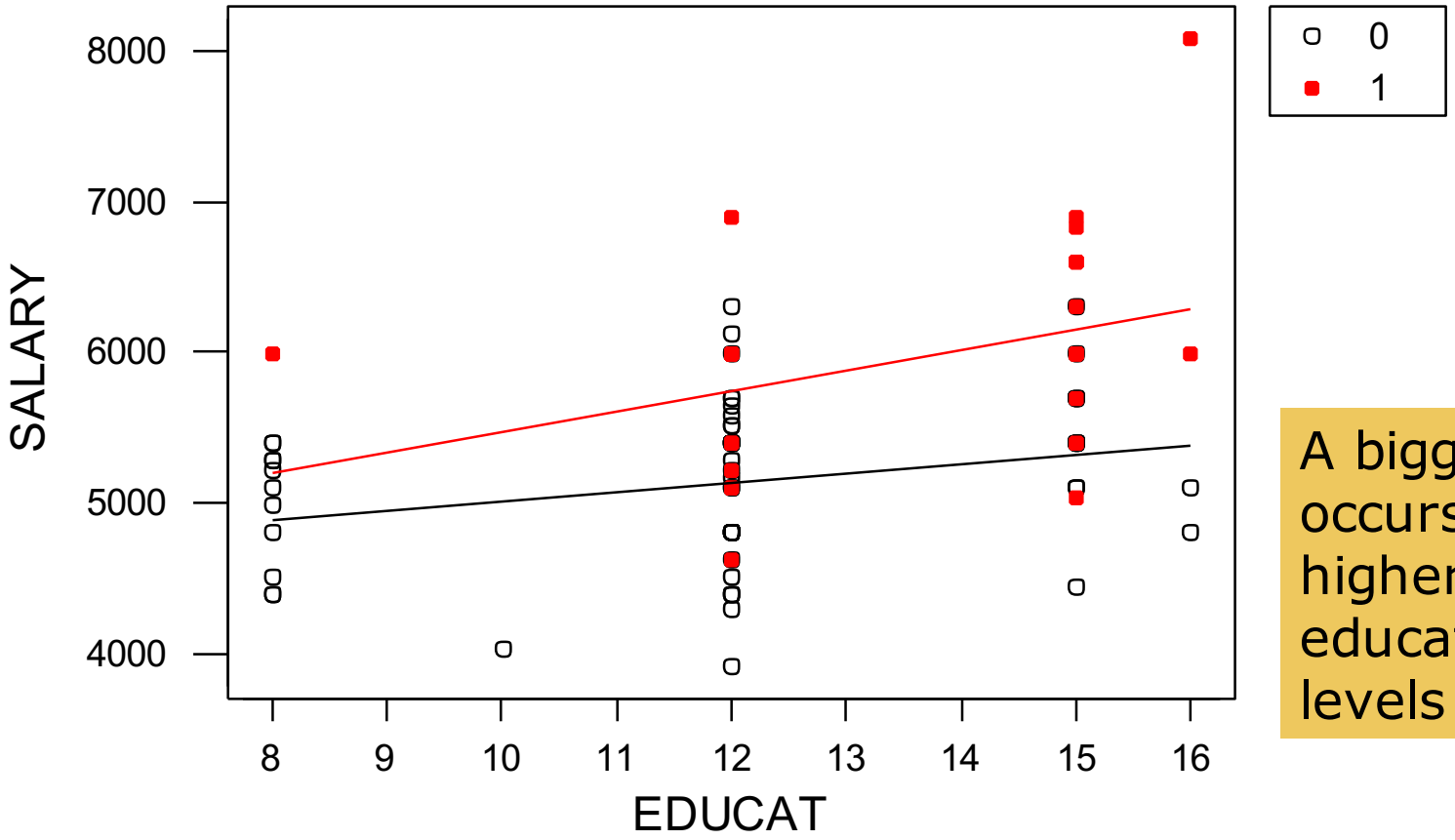
FEMALES (MALES = 0)

SALARY = 4395 + 62.1 EDUCAT - 275 MALES + 73.6 MSLOPE
= 4395 + 62.1 EDUCAT - 275 (0) + 73.6 (EDUCAT*0)
= 4395 + 62.1 EDUCAT

MALES (MALES = 1)

SALARY = 4395 + 62.1 EDUCAT - 275 MALES + 73.6 MSLOPE
= 4395 + 62.1 EDUCAT - 275 (1) + 73.6 (EDUCAT*1)
= 4395 + 62.1 EDUCAT – 275 + 73.6 EDUCAT
= 4120 + 135.7 EDUCAT

Lines With Two Different Slopes



A bigger gap occurs at higher education levels

Tests in This Model

$$SALARY = \beta_0 + \beta_1(EDUCAT) + \beta_2(MALES) + \beta_3(EDUCAT * MALES) + e$$

- Although the slope adjuster implies the salary gap increases with education, this effect is not really significant ($t_{MSLOPE} = 1.16$).
- The overall affect of gender is now contained in two variables, so a partial F test would be needed to test for differences between male and female salaries.

$$H_0 : \beta_2 = \beta_3 = 0$$

7.3 Seasonal Effects in Time Series Regression

- Data collected over time (say quarterly)
- If we think the Y variable depends on the calendar can do a kind of "seasonal adjustment" by adding quarter dummies
- Q1 = 1 if this was first quarter, Q2 = 1 if a second quarter, Q3 = 1 if third
- Don't use Q4 since that is the "base"

Seasonal Adjustment

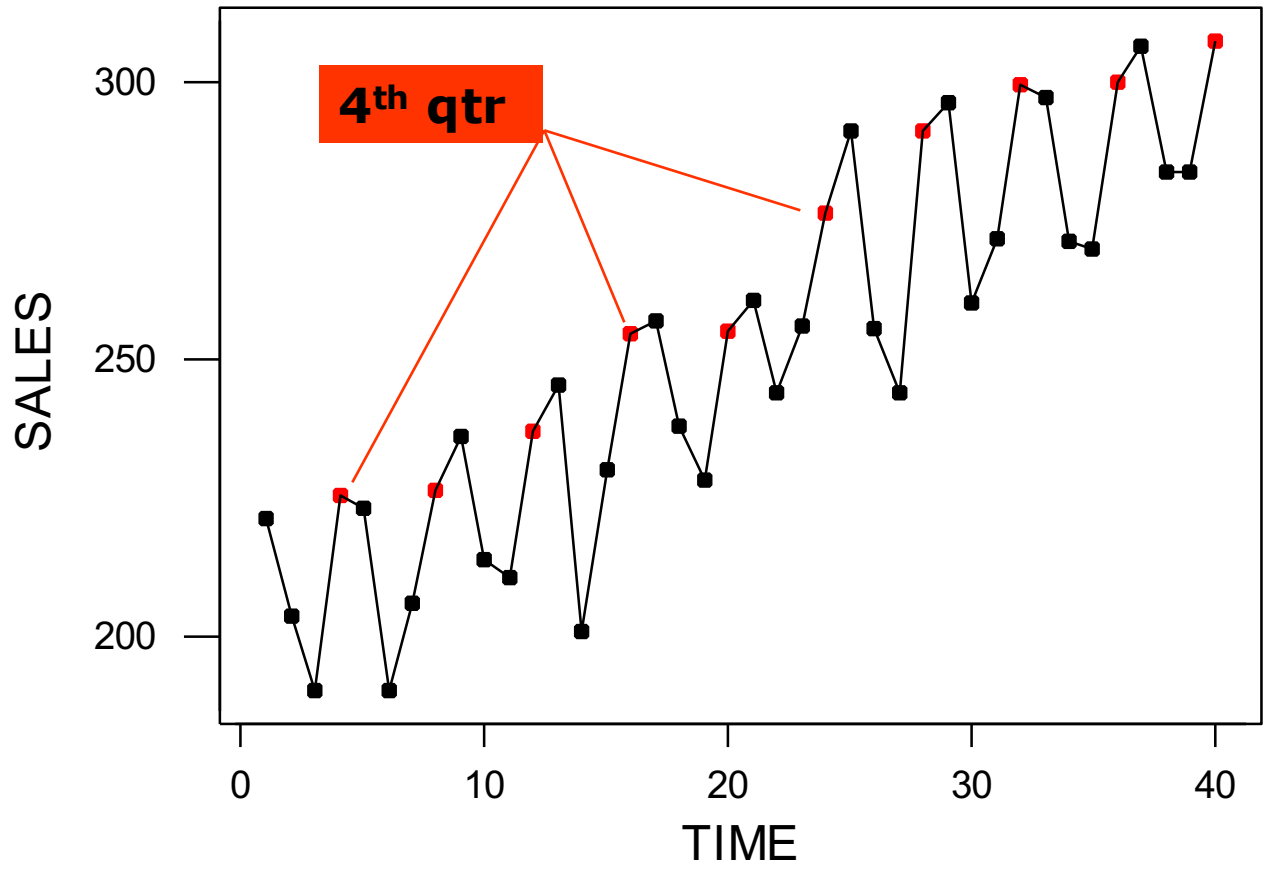
	Q1	Q2	Q3
1st Quarter	1	0	0
2nd Quarter	0	1	0
3rd Quarter	0	0	1
4th Quarter (BASE)	0	0	0

$$SALES = \beta_0 + \beta_1(TIME) + \beta_2(Q1) + \beta_3(Q2) + \beta_4(Q3) + e$$

Example 7.5 ABX Company Sales

- We fit a trend to these sales in Example 3.11 by regressing sales on a time index variable.
- Because this company sells winter sports merchandise, including seasonal effects should markedly improve the fit.

ABX Company Sales



$$SALES = \beta_0 + \beta_1(TIME) + \beta_2(Q1) + \beta_3(Q2) + \beta_4(Q3) + e$$

Two Regressions

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The regression equation is
SALES = 199 + 2.56 TIME

Predictor        Coef     SE Coef          T        P
Constant      199.017       5.128      38.81    0.000
TIME           2.5559      0.2180      11.73    0.000

S = 15.91       R-Sq = 78.3%     R-Sq(adj) = 77.8%


The regression equation is
SALES = 211 + 2.57 TIME + 3.75 Q1 - 26.1 Q2 - 25.8 Q3

Predictor        Coef     SE Coef          T        P
Constant      210.846       3.148      66.98    0.000
TIME          2.56610     0.09895      25.93    0.000
Q1              3.748       3.229       1.16    0.254
Q2            -26.118       3.222      -8.11    0.000
Q3            -25.784       3.217      -8.01    0.000

S = 7.190       R-Sq = 95.9%     R-Sq(adj) = 95.5%
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Are the Seasonal Effects Significant?

- The strong t-ratios for Q2 and Q3 say "yes" and the model R^2 increased by 17.6% when we added the seasonal indicators.
- With evidence this strong we probably don't need to test further.
- In general, however, we would need another partial F test to see if the overall seasonal effect is significant.

$$H_0 : \beta_2 = \beta_3 = \beta_4 = 0$$