MLE, LRT, Hotelling's T^2 & Testing Hypothesis

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HUFS

순서

- 확률함수와 우도함수
- 우도함수와 최우추정량(MLE)
- 우도함수와 우도비(LRT) 검정통계량
- 다변량정규분포의 우도함수
- 다변량 정규분포의 우도비(LRT) 검정통계량
- 다변량 정규분포와 Hotelling's T^2 검정통계량
- 일반선형가정과 Hotelling's T^2 검정통계량

확률함수 $P(x|\theta)$ vs. 우도함수

Probability ft.(확률함수): $P(x|\theta)$

(e.g) 동전 2개를 던져서 앞면이 x 개 나올 확률

$$P\,(x\,|\,\theta) =$$

$$\xrightarrow{\theta = \frac{1}{2}}$$

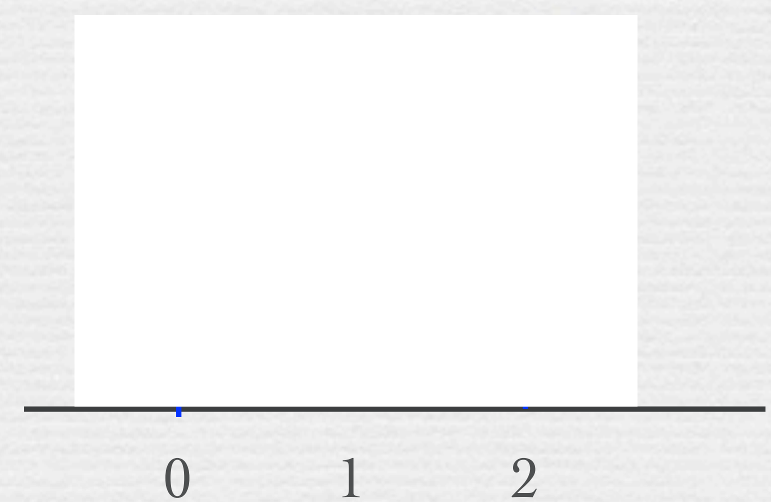
$$P(x|\theta = 1/2) =$$

$$x = 2 \qquad P(x = 2|1/2) =$$

$$x = 1 \qquad P(x = 1|1/2) =$$

$$x = 0 \qquad P(x = 0|1/2) =$$

$$\longrightarrow$$



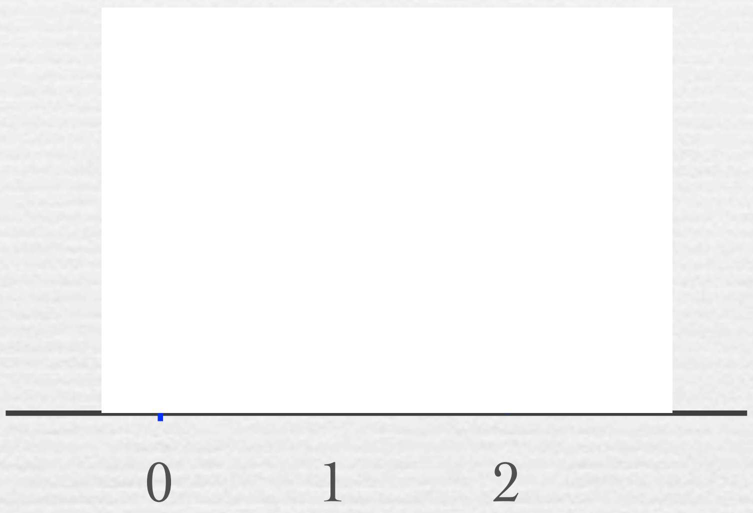
$$P\,(x \mid \theta) = \binom{2}{x} \theta^x (1-\theta)^{2-x} \xrightarrow{\theta = \frac{1}{4}} P(x|\theta = 1/2) =$$

$$x = 2 \qquad P(x = 2|1/4) =$$

$$x = 1 \qquad P(x = 1|1/4) =$$

$$x = 0 \qquad P(x = 0|1/4) =$$

$\longrightarrow$

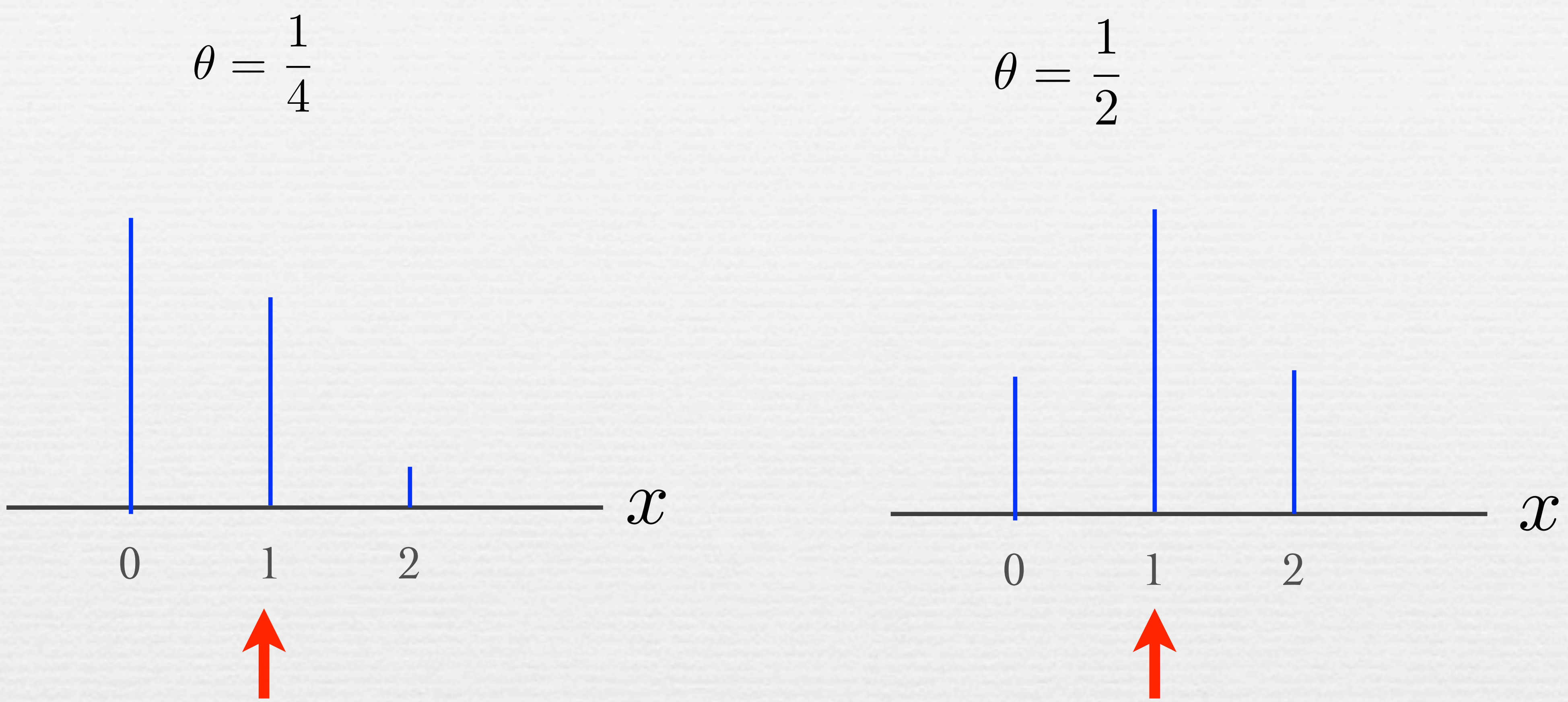


$$\boxed{P(x|\theta) =}$$

$\longrightarrow$

$\longrightarrow$

But, we don't know θ ...



If $x = 1$,

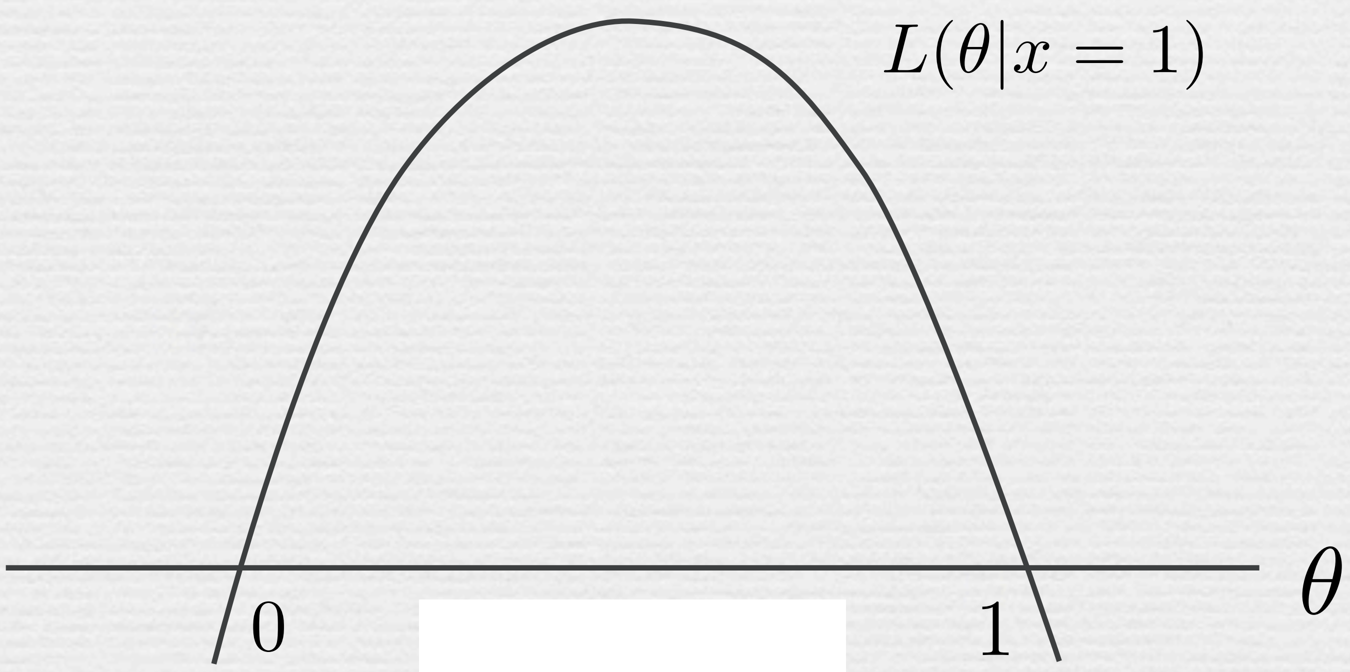
Given x,

need to say

우도함수(likelihood ft.)와 최우추정량 (MLE)

(e.g) 동전 2개를 던져서 앞면이 x=1 개 나왔을 때의 우도 : $L(\theta|x)$

$$L(\theta \mid x) = \binom{2}{x}\theta^{x}(1-\theta)^{2-x} \xrightarrow{x=1} L(\theta \mid x=1) =$$



$$\boxed{L(\theta|x) = \text{a function of } \theta}$$

우도비검정(Likelihood Ratio Test: LRT)

$$H_0 : \theta \in \omega \quad \text{vs.} \quad H_1 : \theta \in \Omega - \omega$$

(e.g.) $H_0 : \mu = 10 \quad \text{vs.} \quad H_1 : \mu \neq 10$

$\Omega = \text{all real values of } \mu$

$\sup_{\theta \in \omega} L(\theta) \rightarrow$

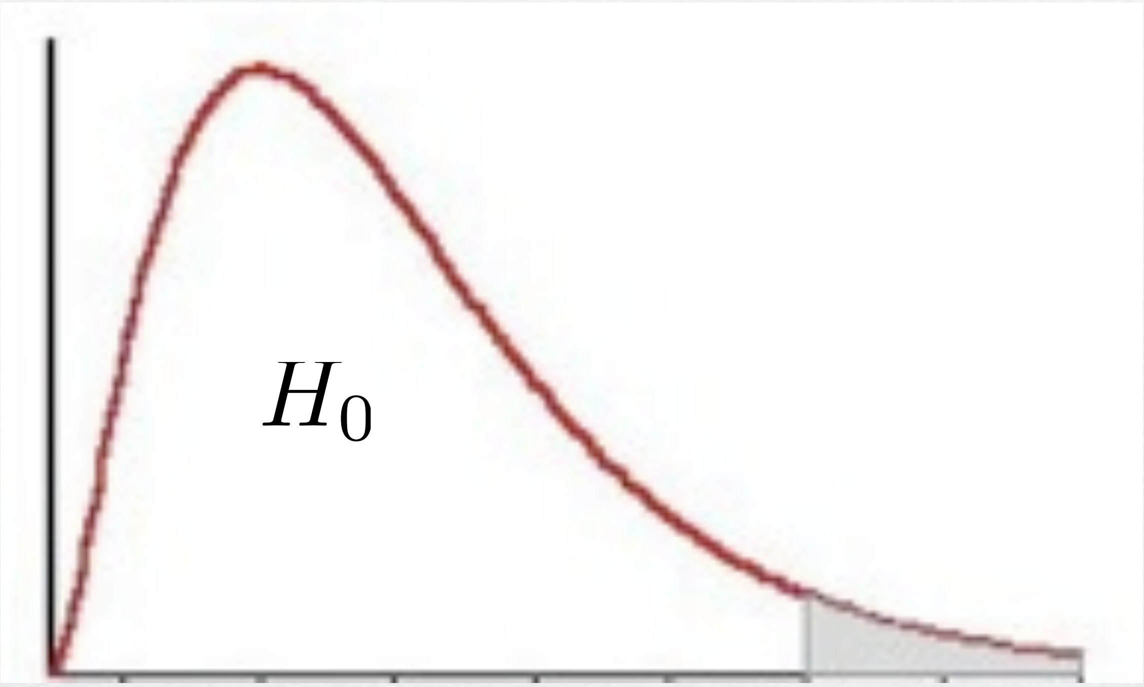
$\sup_{\theta \in \Omega} L(\theta) \rightarrow$

LRT 통계량

If H_0 is true $\rightarrow$

$\nu = \dim(\Omega) - \dim(\omega)$

우도비검정(Likelihood Ratio Test: LRT)



$-2\log\lambda$

If $\; -2\log\lambda$

다변량 정규분포의 우도함수

$$\boldsymbol{x}_i \sim N_p(\boldsymbol{\mu}, \Sigma)$$

우도함수(likelihood function)

$$L(\boldsymbol{\mu}, \Sigma) =$$

로그우도함수(log-likelihood function)

$$l(\boldsymbol{\mu}, \Sigma) = \log L(\boldsymbol{\mu}, \Sigma) = -\frac{n}{2}\log|2\pi\Sigma| - \frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{x}_i - \boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{x}_i - \boldsymbol{\mu})$$

$$\frac{\partial l(\boldsymbol{\mu}, \Sigma)}{\partial \boldsymbol{\mu}} = \mathbf{0}$$

$$\frac{\partial l(\boldsymbol{\mu}, \Sigma)}{\partial \Sigma} = \mathbf{0}$$

$\longrightarrow$

(S is a sample covariance matrix)

다변량 정규분포와 우도비 검정통계량 LRT

$$\boxed{H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0}$$

$$\text{(e.g.)} \quad H_0 : \boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} = \boldsymbol{\mu}_0$$

$$\sup_{\mu \in \Omega} L(\mu) = L(\boldsymbol{\mu} = \bar{\boldsymbol{x}}, \quad \Sigma = S = \qquad)$$

$$\sup_{\mu \in \omega} L(\mu) = L(\boldsymbol{\mu} = \boldsymbol{\mu}_0, \quad \Sigma = \qquad)$$

$$\lambda = \qquad T_0^2 = n(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)^T S^{-1} (\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)$$

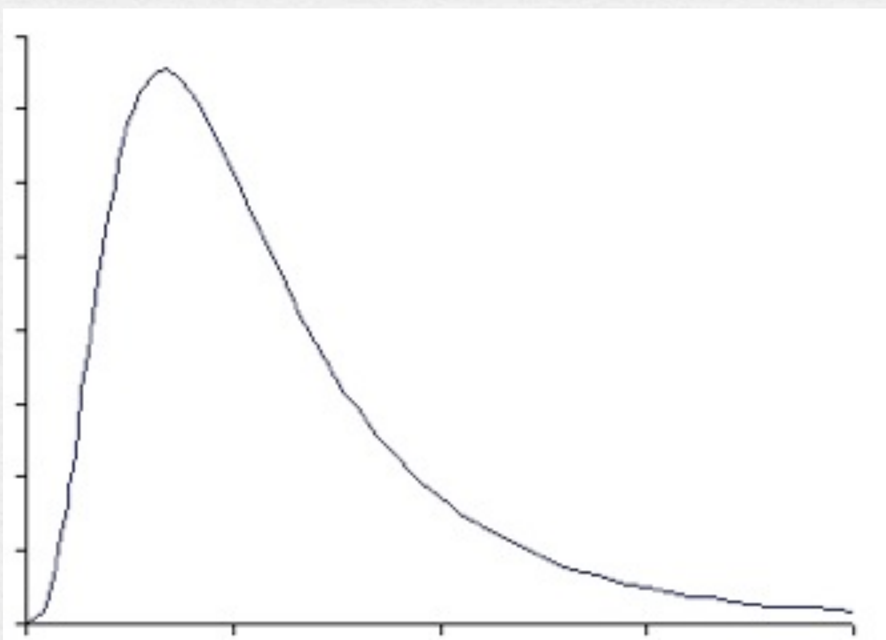
$\rightarrow$ If $-2 \log \lambda$

다변량 정규분포와 Hotelling's T^2 통계량

$$H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0$$

$$T_0^2 = n(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)^T S^{-1} (\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)$$

호텔링의 T^2 통계량



$$\frac{n-p}{p(n-1)} T_0^2$$

If $\frac{n-p}{p(n-1)} T_0^2 >$

example (sweat data) p.69

$$H_0 : \boldsymbol{\mu} = \left(\quad \right)$$

- Y1=sweat rate = [(pre-run weight)-(post-run weight)]*16 (oz).. after 1 hr running
- Y2=Na
- Y3=Ka

```
data b; set a;

v1=y1-4; v2=y2-50; v3=y3-10;

proc glm;

model v1 v2 v3 = /nouni;

manova h=INTERCEPT ; run;
```

Manova Test Criteria and Exact F Statistics for
the Hypothesis of no Overall INTERCEPT Effect
H = Type III SS&CP Matrix for INTERCEPT   E = Error SS&CP Matrix

S=1   M=0.5   N=7.5

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.66112774	2.9045	3	17	0.0649
Pillai's Trace	0.33887226	2.9045	3	17	0.0649
Hotelling-Lawley Trace	0.51256699	2.9045	3	17	0.0649
Roy's Greatest Root	0.51256699	2.9045	3	17	0.0649

일반선형가정 and Hotelling's T^2

일반선형가정 $H_0 : A\boldsymbol{\mu} = \boldsymbol{b}$ $\longleftrightarrow$

$\boldsymbol{x}_i \sim N_p(\boldsymbol{\mu}, \Sigma) \longrightarrow A\boldsymbol{x}_i - \boldsymbol{b} \sim$

If H_0 is true,

$$T_0^2 = n(A\bar{\boldsymbol{x}} - \boldsymbol{b})^T (ASA^T)^{-1} (A\bar{\boldsymbol{x}} - \boldsymbol{b})$$

(e.g.) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \sim N_3\left(\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{pmatrix}\right)$

$H_0 : \begin{pmatrix} \mu_2 = 12\mu_1 + 2 \\ \mu_2 = 5\mu_3 \end{pmatrix} \longleftrightarrow$

example (sweat data) p.69

```
data b; set a;

v1=12*y1-y2+2;

v2=y2-5*y3;

proc glm;

model v1 v2 = /nouni;

manova h=INTERCEPT ;  run;
```

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of No Overall Intercept Effect
H = Type III SSCP Matrix for Intercept
E = Error SSCP Matrix

S=1 M=0 N=8

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.67374574	4.36	2	18	0.0286
Pillai's Trace	0.32625426	4.36	2	18	0.0286
Hotelling-Lawley Trace	0.48423944	4.36	2	18	0.0286
Roy's Greatest Root	0.48423944	4.36	2	18	0.0286